

VSM COLLEGE OF ENGINEERING

RAMACHANDRA PURAM - 533 255



I B.TECH

PHYSICS

STUDY MATERIAL

(COMMON TO CSE, ECE & EEE)

	L	T	P	C
I Year - I Semester	4	0	0	3

APPLIED PHYSICS
 (CSE, ECE, EEE, IT, EIE, E.Com.E)

OBJECTIVES: Physics curriculum which is re-oriented to the needs of Circuital branches of graduate engineering courses offered by JNTUniv.Kkd. that serves as a transit to understand the branch specific advanced topics. The courses are designed to:

- Impart Knowledge of Physical Optics phenomena like Interference, Diffraction and Polarization involving required to design instruments with higher resolution.
- Teach Concepts of coherent sources, its realization and utility optical instrumentation.
- Study the concepts regarding the bulk response of materials to the EM fields and their analytically study in the back-drop of basic quantum mechanics.
- Understand the physics of Semiconductors and their working mechanism for their utility in sensors.

UNIT-I

INTERFERENCE: Principle of Superposition – Coherent Sources – Interference in thin films (reflection geometry) – Newton's rings – construction and basic principle of Interferometers.

UNIT-II

DIFFRACTION: Fraunhofer diffraction at single slit - Cases of double slit, N-slits & Circular Aperture (Qualitative treatment only)-Grating equation - Resolving power of a grating, Telescope and Microscopes.

UNIT-III

POLARIZATION: Types of Polarization – Methods of production - Nicol Prism -Quarter wave plate and Half Wave plate – Working principle of Polarimeter (Sacharimeter).

LASERS: Characteristics– Stimulated emission – Einstein's Transition Probabilities-Pumping schemes - Ruby laser – Helium Neon laser.

UNIT-IV

ELECTROMAGNETIC FIELDS: Scalar and Vector Fields – Electric Potential-Gradient, Divergence of fields – Gauss and Stokes theorems-Propagation of EM waves through dielectric medium.

UNIT-V

QUANTUM MECHANICS: Introduction - Matter waves – Schrödinger Time Independent and Time Dependent wave equations – Particle in a box.
FREE ELECTRON THEORY: Defects of Classical free electron theory –Quantum Free electron theory - concept of Fermi Energy.

UNIT-VI

BAND THEORY OF SOLIDS: Bloch's theorem (qualitative) – Kronig – Penney model – energy bands in crystalline solids – classification of crystalline solids– effective mass of electron & concept of hole.

SEMICONDUCTOR PHYSICS: Conduction – Density of carriers in Intrinsic and Extrinsic semiconductors – Drift & Diffusion – relevance of Einstein's equation- Hall effect in semiconductors

***Outcome:** Construction and working details of instruments, ie., Interferometer, Diffractometer and Polarimeter are learnt. Study EM-fields and semiconductors under the concepts of Quantum mechanics paves way for their optimal utility.*

Text Books:

1. A Text book of Engineering Physics – by Dr. M.N.Avadhanulu and Dr.P.G.Kshira sagar, S.Chand & Company Ltd., (2014)
2. 'Solid State Physics' by A.J.Dekker, Mc Millan Publishers (2011)
3. Engineering Physics by D.K.Bhattacharya and Poonam Tandon, Oxford press (2015)

Reference Books:

1. Applied Physics by P.K.Palanisamy, Scitech publications (2014)
2. Lasers and Non-Linear optics by B.B.Laud, New Age International Publishers (2008).
3. Engineering Physics by M. Arumugam, Anuradha Publication (2014)

The only Source of Knowledge & Experience

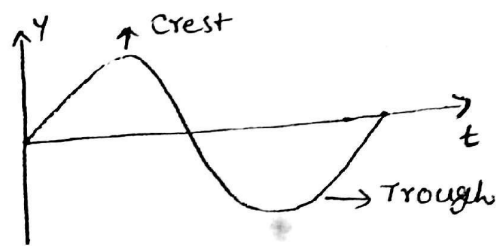
- Einstein

Interference

Definition Best Phenomena that provides direct evidence for the wave nature of light is interference. Observations in our day to day life are multiple colours on soap bubble as well as on thin layer of floating oil when viewed under Sun light.

wave Disturbance in the medium is known as wave. main characteristic of the wave is Energy can take transition from one end to another end

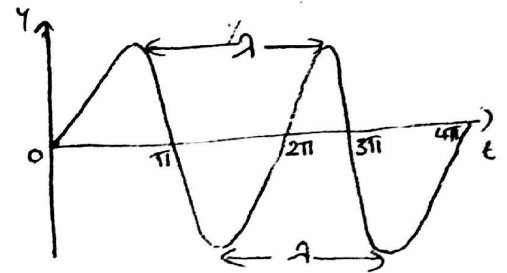
Crest:- The Section of the wave that rises above the undisturbed position is called the Crest.



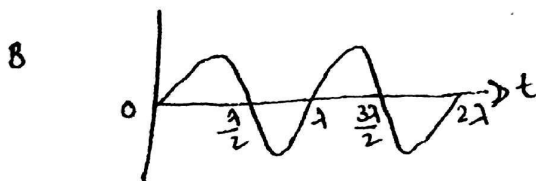
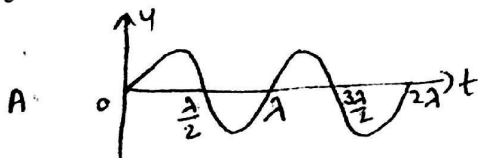
Trough:- The Section which lies below the undisturbed position.

Amplitude It is the maximum positive displacement from the undisturbed position of the medium to the top of the Crest.

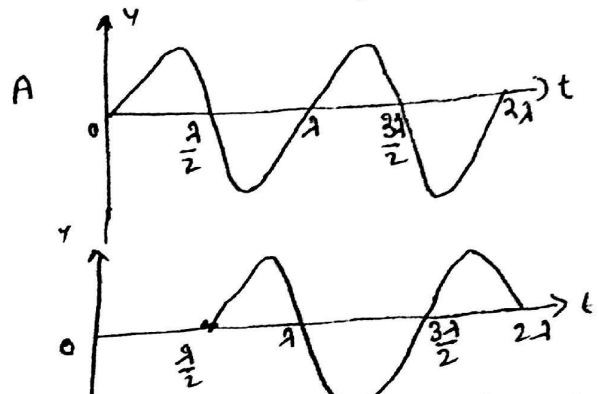
Wavelength Distance between two adjacent Crests or troughs are nothing but wavelength (λ).



Path difference:- The difference between the length of two paths.



A & B are start at same position '0'.
there is no path difference



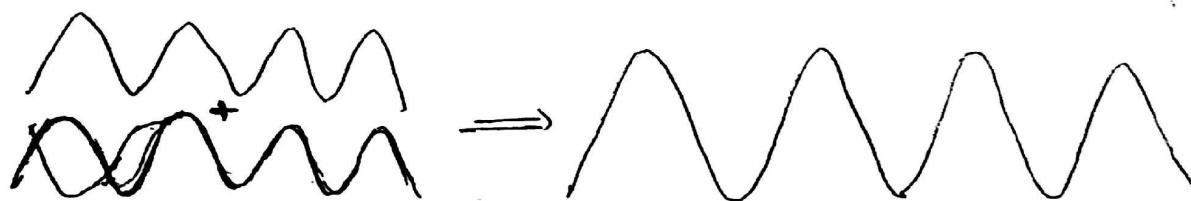
A starts from '0' & B starts from $\frac{\lambda}{2}$.
Path difference $\frac{\lambda}{2}$

When two or more waves travel simultaneously in a medium, the resultant displacement at any point is due to the algebraic sum of the displacements due to individual waves. This is principle of superposition.

Let us consider two waves travelling simultaneously in a medium. At any point let ' y_1 ' be the displacement due to one wave at any instant in the absence of the other and ' y_2 ' be the displacement of other wave at the same instant in the absence of the first wave.

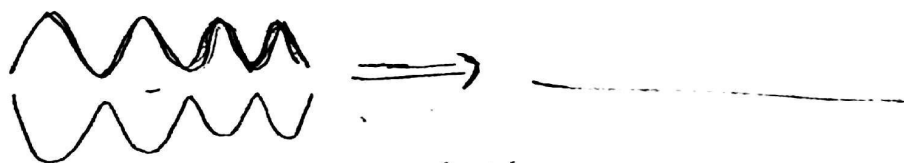
∴ The resultant displacement due to the presence of both waves is given by $y = y_1 \pm y_2$

+ve sign has to be taken when both the displacements y_1 & y_2 are in the same direction.



(a) waves in phase.

-ve sign has to be taken when they are in the opposite direction



(b) waves out of phase.

Interference:- modification of intensity due to the superposition of waves is called Interference of light.

Interference can be classified into two types

- (1) Constructive interference
- (2) Destructive interference.

Constructive interference:- If both the waves are in same phase (direction) the resultant displacement becomes maximum. i.e. one wave of the Crest coincide with the other wave of the Crest or one wave of the Trough coincide with the another wave of the trough

$$\therefore y = a + a = 2a$$

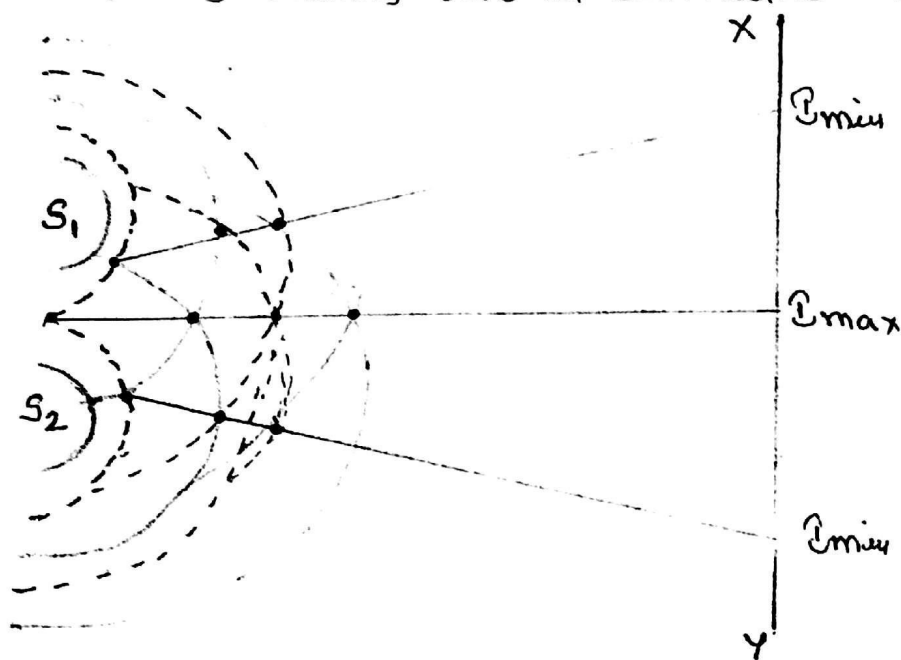
As the intensity is directly proportional to the Square of the amplitude $\therefore I = R^2 = 4a^2$

The intensity at these points is four times the intensity due to one wave. i.e. intensity becomes maximum in Constructive interference.

Destructive interference:- If both the waves are in out of phase the resultant displacement becomes zero. i.e. first wave of the Crest coincide with the second wave of Trough or first wave of trough coincide with the second wave of the Crest.

$$\therefore y = a - a = 0 \Rightarrow I = 0$$

i.e. intensity becomes zero in destructive interference.



Coherence:- Two waves are said to be coherent if their waves have

- (i) Same wavelength (λ)
- (ii) Same amplitude and
- (iii) Constant (or zero) phase difference

NOTE:- Two independent sources of light can never be coherent.

Conditions for interference.

- 1) The two interfering Sources, should be coherent i.e. the phase difference between them must remain constant with time.
- 2) The two waves should have same frequency.
- 3) If the interfering waves are polarized, they must be in the same state of polarization.
- 4) The separation between the light sources (d) should be as small as possible to get large fringe width.
- 5) The displacement (D) of the screen from the two sources should be quite large to obtain widely spaced fringes.
- 6) The amplitudes of the interfering waves should be equal.
- 7) The screen should be dark. If the screen is not dark, the minimum intensity will not appear to be zero, resulting a poor contrast between maxima and minima.

✓ Young's double slit experiment

Thomas Young demonstrated the concept of interference of light in 1802. Young in his experiment allowed sunlight to fall on a pinhole S. Spherical waves emerged out from S were made to fall on two pinholes A and B in an opaque screen. On the screen (xy) kept at certain distance from the opaque screen, he observed few coloured bright and dark bands. Later the white light was replaced by monochromatic source of light and pinholes were replaced by narrow slits.

A single vertical slit S was illuminated by light of wavelength λ from a monochromatic source such as Sodium Vapour lamp. A and B were two narrow slits accurately parallel to each other and parallel to S. The width of each slit was about 0.03 mm and they were about 0.3 mm apart. Since A and B

The distance between two consecutive fringes

$$x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

Condition for dark fringe:-

The point P will be dark if path difference is $(2n-1)\frac{\lambda}{2}$
 where $n = 1, 2, 3 \dots$ etc. or $\frac{x d}{D} = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow x_n = \frac{(2n-1)\lambda D}{2d}$$

The above equation gives the distances of the dark fringes from the Centre C. At distances $x_1, x_2, x_3 \dots$ from C, dark fringes corresponding to $n = 1, 2, 3 \dots$ etc are formed

$$\therefore \text{For } n=1 \quad x_1 = \frac{\lambda D}{2d}; \quad n=2 \quad x_2 = \frac{3\lambda D}{2d}; \quad n=3 \quad x_3 = \frac{5\lambda D}{2d}$$

$$\therefore x_n = (2n-1)\frac{\lambda D}{2d}$$

$\therefore$ The distance between two consecutive dark fringes

$$x_2 - x_1 = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

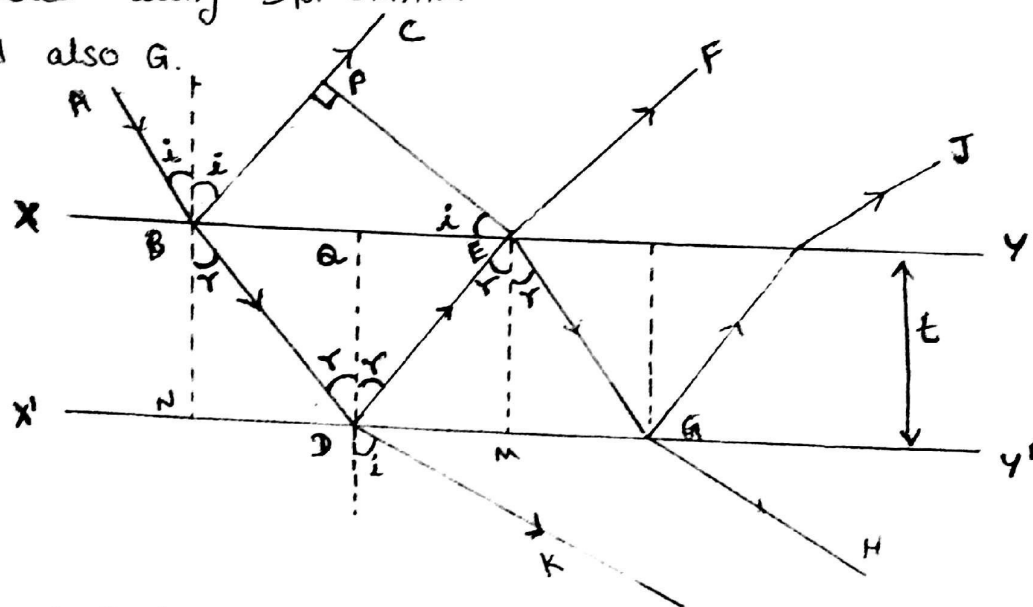
$\therefore$ The distance between two consecutive bright or dark fringes is called band width. From above equations bright and dark fringes have equal widths.

$\therefore$ The width of the interference fringe (β) is

- (i) directly proportional to the wavelength of light used (λ)
- (ii) directly proportional to the distance of the Screen from the two Sources (D)
- (iii) inversely proportional to the distance between the two coherent Sources (d)

Superposition of light reflected from the top and bottom surface of the film.

Let us consider a thin film of thickness t bound by two plane surfaces xy and $x'y'$ and let μ be the refractive index of material of the film. A ray of light AB incident on the surface xy at angle i is partly reflected along BC and partly refracted along BD . Let the angle of refraction be r . On the surface $x'y'$ the refracted ray is partly reflected along DE and partly refracted along DK . Similar reflection and refraction occur at E and also G .



Reflected System:- The rays BC and EF constitute reflected system. To find the path difference between these reflected rays EP is drawn perpendicular to BC .

$$\text{Path difference} = (BD + DE) \mu - BP \quad \text{--- (1)}$$

In $\triangle BDQ$

$$\cos r = \frac{BQ}{BD} = \frac{t}{BD}$$

$$\Rightarrow BD = \frac{t}{\cos r} = DE$$

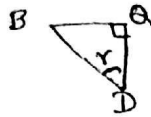
$$\therefore \text{Path difference} = \frac{2\mu t}{\cos r} - BP \quad \text{--- (1)}$$

In $\triangle BPE$

$$\sin i = \frac{BP}{BE} \Rightarrow BP = BE \sin i$$

$$= (BQ + QE) \sin i \quad \text{--- (2)}$$

In ΔBDQ



$$\tan r = \frac{BQ}{DQ} = \frac{BQ}{t}$$

$$\Rightarrow BQ = t \tan r = DQ$$

$$\therefore BP = 2t \tan r \sin i$$

$$= 2t \frac{\sin r}{\cos r} \mu \sin r \quad (\because \mu = \frac{\sin i}{\sin r})$$

$$\therefore BP = \frac{2\mu t \sin^2 r}{\cos r} \rightarrow (3)$$

(3)rd equation substitute in (1) equation then we can get

$$\text{Path difference} = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} = \frac{2\mu t (1 - \sin^2 r)}{\cos r}$$

$$\therefore \text{path difference} = 2\mu t \cos r \quad [1 - \sin^2 r = \cos^2 r]$$

Since the ray BC is reflected at the air-medium interface, it undergoes a phase change of π or path difference of $\frac{\lambda}{2}$.

$$\therefore \text{Path difference} = 2\mu t \cos r - \frac{\lambda}{2}$$

(i) Condition for bright band

The film will appear bright if the path difference is $n\lambda$

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda \Rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

where $n=0, 1, 2, 3, \dots$ etc

(ii) Condition for dark band

The film will appear dark if the path difference is $(2n+1)\frac{\lambda}{2}$.

$$\therefore 2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$(or) 2\mu t \cos r = (n+1)\lambda \quad \text{where } n=0, 1, 2, 3, \dots \text{ etc}$$

$$2\mu t \cos r = n\lambda \quad \text{where } n=1, 2, 3, \dots \text{ etc.}$$

Colours of thin films:-

When a thin film is exposed to a white light such as sun light, beautiful colours appear in the reflected light.

Ex: Soap bubble and thin oil layer.

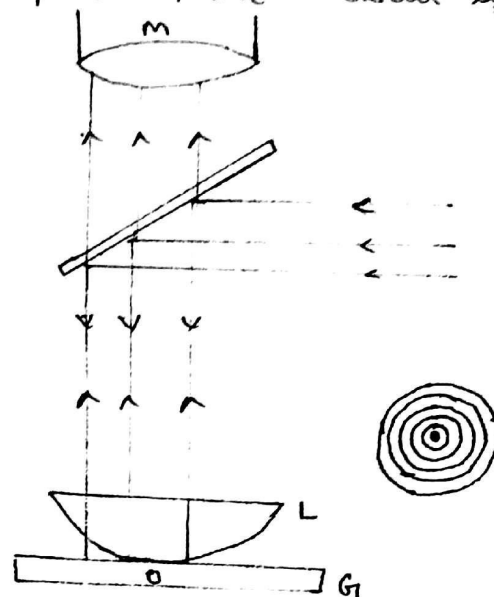
In the case of Soap bubble let us assume that thickness of the film t is a constant. Then in the formula $2\mu t \cos r$, μ and r are the variables since white light has varying λ value, μ also varies with λ . Due to curved nature of bubble angle of incidence varies for different points on the bubble and hence accordingly angle of refraction r varies. Hence in the formula $2\mu t \cos r = (2n+1)\frac{\lambda}{2}$, varying values of μ and r can satisfy the condition for constructive interference for a particular wavelength (λ) only. Accordingly that point will appear bright in that particular colour. In a similar way different points satisfy the conditions for constructive interference for different colours and hence appear multi coloured.

Let us now analyse the case of thin layer of oil film floating on water. In this case since the film is perfectly flat, when parallel rays such as sun light is incident, the angle of incidence i and hence angle of refraction r will remain constant, but for different λ values μ varies and also the thickness of the film t may not be constant throughout the film. Hence different points on the film satisfy the condition for constructive interference for different colours depending on the value of μ and t and hence appear multicoloured.

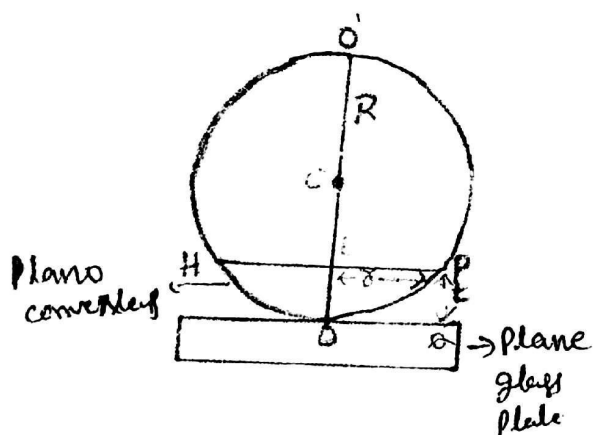
No dark band is seen because if a particular point satisfies condition for destructive interference for a particular wavelength, the same point may satisfy condition for constructive interference for different wavelength and hence throughout the film we see multi colours.

Newton's rings:- A plano convex lens (L) having long focal length f is placed with its convex surface on a plane glass plate (G). A gradually increasing thickness of air film will be formed between plane glass plate and convex surface of plano convex lens. The thickness of air film will be zero at the point of contact and symmetrically increases as we go radially from the point of contact.

A monochromatic light of wavelength ' λ ' is allowed to fall normally on the lens with the help of a glass plate ' G ' kept at 45° to the incident monochromatic beam. A part of the incident light rays are reflected up at the convex surface of the lens and the remaining light is transmitted through the air film. Again a part of this transmitted light is reflected at on the top surface of the glass plate. Both the reflected rays combine to produce an interference pattern in the form of alternate bright and dark concentric circular rings, known as Newton's rings, because Newton first demonstrated and showed these rings. The rings are circular because the air film has circular symmetry. These rings can be seen through the travelling microscope M .



Theory:-



Let R be the radius of the curvature of the lens L and let choose a point P at a distance r from O and let t be the thickness of air film at that point. Then the path difference between the light reflected at P and Q is $2t$. When the additional path difference

between the two reflected beams becomes $2t + \frac{\lambda}{2}$. When this path difference is $n\lambda$ constructive interference occurs.

The condition for bright rings is $2t + \frac{\lambda}{2} = n\lambda$

(or) $2t = (2n-1)\frac{\lambda}{2} \rightarrow (1)$ where $n=1, 2, 3, \dots$ etc.

Similarly for dark rings the condition is

$$2t + \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$\boxed{2t = n\lambda} \rightarrow (2)$$

Let us consider the curved surface of the lens as an arc of a circle whose centre is at C.

$$HE \times EP = OE \times EO'$$

$$\Rightarrow r^2 = t(2R - t)$$

$$= 2Rt - t^2$$

$$= 2Rt \text{ (approximately)}$$

$$\therefore \boxed{t = \frac{r^2}{2R}} \rightarrow (3)$$

Substituting equation (3) in the (1) equation for bright rings

$$\frac{2r^2}{2R} = (2n-1) \frac{\lambda}{2}$$

$$\Rightarrow r^2 = \frac{(2n-1)\lambda R}{2}$$

$$\text{or } \boxed{r_n = \sqrt{\frac{(2n-1)\lambda R}{2}}} \text{ where } n=1, 2, 3, \dots \text{ etc.} \rightarrow (4)$$

Substituting equation (3) in the (2) equation for dark rings

$$\frac{2r^2}{2R} = n\lambda \Rightarrow r^2 = nR\lambda \Rightarrow \boxed{r_n = \sqrt{nR\lambda}} \text{ where } n=1, 2, 3, \dots \text{ etc.} \rightarrow (5)$$

For $n=0$ the radius of the ring is zero which denotes the centre O. At the point of contact the thickness of air is zero, the ray reflected at the air-glass interface undergoes additional phase change π or path change of $\frac{\lambda}{2}$. Hence it appears dark.

From the equations (4) & (5) radius of rings are proportional to (i) $\sqrt{\lambda}$ and (ii) $\sqrt{R}$ and

$$\text{For the first dark ring } n=1 \quad r_1 = \sqrt{R\lambda}$$

$$\text{Second dark ring } n=2 \quad r_2 = \sqrt{2R\lambda}$$

Hence the spacing between the first and second dark rings

$$\sqrt{2R\lambda} \quad \sqrt{R\lambda} = (1.414 - 1) \sqrt{R\lambda}$$

$$\therefore r_2 - r_1 = 0.414 \sqrt{R\lambda}$$

For the third dark ring $n=3$ $r_3 = \sqrt{3R\lambda}$

Fourth dark ring $n=4$ $r_4 = \sqrt{4R\lambda}$

The spacing between third and fourth dark rings

$$r_4 - r_3 = \sqrt{4R\lambda} - \sqrt{3R\lambda} = (2 - 1.734) \sqrt{R\lambda}$$

$$\therefore r_4 - r_3 = 0.266 \sqrt{R\lambda}$$

we find that with increase of order, the rings get closer.

Determination of wavelength of Sodium light using Newton's rings:-

Let R be the radius of Curvature of the Curved Surface in contact with the glass plate and λ the wavelength of light used.

we know that the radius of n^{th} dark ring is given by

$$r_n = \sqrt{nR\lambda}$$

The diameter of the n^{th} dark ring is

$$D_n = 2r_n = 2\sqrt{nR\lambda} = \sqrt{4nR\lambda}$$

$$\text{Hence } D_n^2 = 4nR\lambda \rightarrow (1)$$

Similarly the diameter of the m^{th} dark ring is $D_m = \sqrt{4mR\lambda}$

$$D_m^2 = 4mR\lambda \rightarrow (2)$$

subtracting equation's 2 to (1)

$$D_m^2 - D_n^2 = 4mR\lambda - 4nR\lambda$$

$$\therefore \text{or } \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)}$$

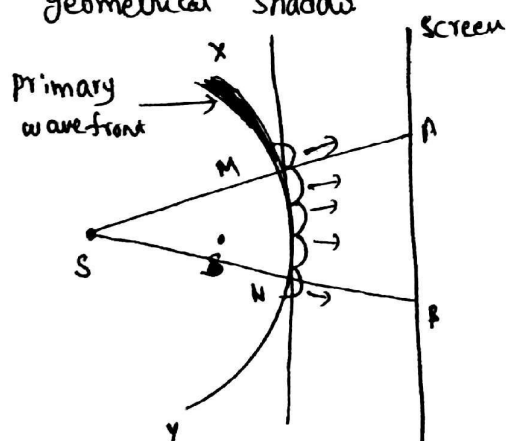
Hence by measuring the radius of rings of different orders and knowing the radius of Curvature of the lens R , we can determine the wavelength λ of the given monochromatic source of light.

Introduction:- The bending of light waves at an aperture or edge is known as diffraction.

When an object is placed in the path of light beam, the shadow of the object is formed, behind the object is due to diffraction.

Diffraction based on Huygen's wave theory:-

According to Huygen's wave theory every point on the primary wave front can act as second source of disturbance and secondary waves are generated from those points. The envelope of these secondary waves form secondary wavefront. Let primary wavefront XY from point source S of monochromatic light reach the aperture MN placed in its path. The light passing through the aperture has to cause brightness inside AB on the screen and regions beyond A and B have to be dark i.e., regions of geometrical shadow.



Every point on the wavefront reaching the aperture MN acts as secondary source of light and from those points light waves originate and thus secondary waves enter into the geometrical shadow region.

also. The waves from different parts of the primary wavefront travel different distances to reach any point on the geometrical shadow region and interfere to produce bright and dark fringes. Since the amplitude of secondary waves decrease while going away from A and B, the intensity gradually falls to zero.

Fresnel Diffraction

Fraunhofer Diffraction

1. Either a point source or an illuminated narrow slit is used
2. The wave front undergoing diffraction is either spherical or cylindrical.
3. The Source and the Screen are finite distances from the obstacle producing diffraction
4. No lens is used to focus the rays.

1. Extended source at infinite distance is used
2. The wave front undergoing diffraction is a plane wave front
3. The Source and Screen are at infinite distances from the obstacle producing diffraction.
4. Converging lens is used to focus parallel rays.

Difference between interference and diffraction :-

1. Superposition is due to two separate wave fronts originating from two coherent sources
2. The fringes normally have equal widths.
3. All the bright fringes have the same intensity
4. All the dark fringes have zero intensity

1. Superposition is due to secondary wavelets originating from different parts of the same wave front
2. The width between fringes is never equal
3. The intensity of bright fringes usually decrease with increase of order.
4. The intensity of dark fringes is not zero

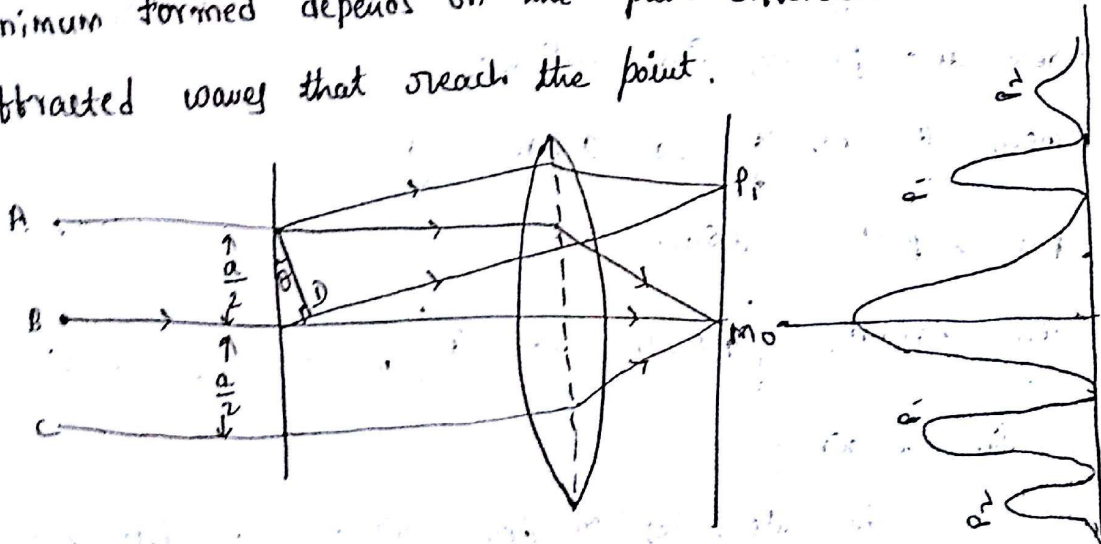
Fraunhofer Diffraction of Single Slit

Let us consider a narrow slit of width a .

A monochromatic light of wavelength λ is collimated by a lens. The collimated beam consists of plane wave fronts. When this beam is incident on the single slit, majority of the light beam passes through the slit without touching the slit edges, i.e., light transmits through the central portion of the slit. This undiffracted beam is converged at M_0 on the screen after passing through the lens. Maximum intensity is observed at M_0 , called the central maximum.

However, the light beam is diffracted at the two edges of the slit and produces a pattern of alternate maxima and minima with the central maxima at the centre of the pattern.

At a given point on the screen whether a maximum or minimum formed depends on the path difference between the diffracted waves that reach the point.



When slit is divided into two equal parts, $AB = a/2$; $BC = a/2$

AP_1 and BP_1 are two rays diffracted at an angle θ from the top of each half of the slit. $BP_1 > AP_1$

$\therefore$ Path difference $BP_1 - AP_1 = a \sin \theta$

(4) To get minimum intensity at P_1 , the path difference must be odd number multiple of $\lambda/2$.

From $\triangle ADB$ $\sin \theta = \frac{BD}{AB} = \frac{BD}{a/2}$

i.e. $BD = \frac{a}{2} \sin \theta$

but $BD = \lambda/2$ for minimum, then

$\frac{a}{2} \sin \theta = \lambda/2$ (or) $a \sin \theta = \lambda$

If the slit is divided into 4 equal parts, then

$\frac{a}{4} \sin \theta = \lambda/2$ (or) $a \sin \theta = 2\lambda$

for six equal parts $a \sin \theta = 3\lambda$

in general $a \sin \theta = n\lambda$ $n=1,2,3,\dots$ is condition for minima.

Suppose the slit is divided into 3 equal parts, such that the path difference between any two adjacent rays is $\lambda/2$. Then, two rays cancel each other and the 3rd reaches the screen and produces some intensity.

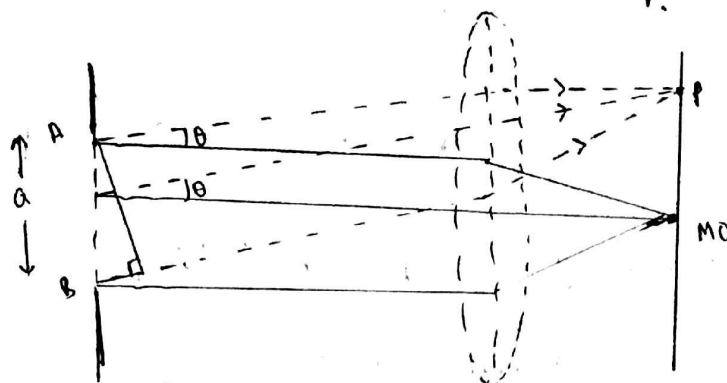
Then, $\frac{a}{3} \sin \theta = \lambda/2 \Rightarrow a \sin \theta = \frac{3\lambda}{2}$

For 5 parts $a \sin \theta = \frac{5\lambda}{2}$

In general, $a \sin \theta = (2n-1)\frac{\lambda}{2}$ $n=1,2,3,\dots$ is condition for maxima.

The intensities of these maxima are less than that of the central maximum decreases as we move away from the central maximum.

A monochromatic light beam of wavelength λ is incident on the single slit of width 'a'. The direct light is converged at P_0 on the screen by a lens. P_0 is the principle maximum. Light waves diffracted at an angle θ are focussed at P_1 .



BC is the path difference between the top ray and the bottom ray.

$$\text{From diagram } \sin\theta = \frac{BC}{AB}$$

$$\Rightarrow BC = AB \sin\theta = a \sin\theta$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \times a \sin\theta$$

Suppose the slit is divided into 'n' equal parts and amplitude of light in each part is A' . Then phase difference between two successive parts is $\frac{1}{n}$ of total phase difference.

$$\text{Say } \phi = \frac{1}{n} \left(\frac{2\pi}{\lambda} a \sin\theta \right)$$

Using the method of vector addition of amplitude discussed the resultant amplitude R is given by

$$R = \frac{A' \sin\left(\frac{n\phi}{2}\right)}{\sin\left(\frac{\phi}{2}\right)}$$

$$\Rightarrow R = \frac{A' \sin\left(\frac{n}{2} \cdot \frac{1}{n} \cdot \frac{2\pi}{\lambda} a \sin\theta\right)}{\sin\left(\frac{1}{n} \cdot \frac{2\pi}{\lambda} \cdot \frac{a \sin\theta}{2}\right)} = \frac{A' \sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a \sin\theta}{n\lambda}\right)}$$

then $R = \frac{A' \sin d}{\sin(\frac{d}{n})}$

As $\frac{d}{n}$ is very small $\sin(\frac{d}{n}) \approx \frac{d}{n}$

$\therefore R = A' \frac{\sin d}{\frac{d}{n}} = A' n \frac{\sin d}{d}$

$\therefore R = A \frac{\sin d}{d} \quad [A' n = A]$

$\therefore$ Intensity $I = R^2 = A^2 \left(\frac{\sin^2 d}{d^2} \right)$

condition for principal maxima:-

The expression for resultant amplitude R can be written in ascending powers of d as

$$R = \frac{A}{d} \left[d - \frac{d^3}{3!} + \frac{d^5}{5!} - \frac{d^7}{7!} + \dots \right]$$

$$\Rightarrow R = A \left[1 - \frac{d^2}{3!} + \frac{d^4}{5!} - \frac{d^6}{7!} + \dots \right]$$

If the negative terms vanish, the value R will be maximum

i.e. $d=0$

$$\therefore d = \frac{\pi a \sin \theta}{\lambda} = 0 \quad \text{or} \quad \sin \theta = 0 \quad \text{or} \quad \theta = 0$$

Now maximum value of R is A and intensity is proportional to A^2 . The condition $\theta=0$ means that this maximum is formed by those secondary wavelets which travel normally to the slit. The maximum is known as principal maximum.

minimum intensity positions:- the intensity will be minimum.

when $\sin d = 0$. The values of d which satisfy the equation are

$$d = \pm \pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots \text{etc} = \pm n\pi$$

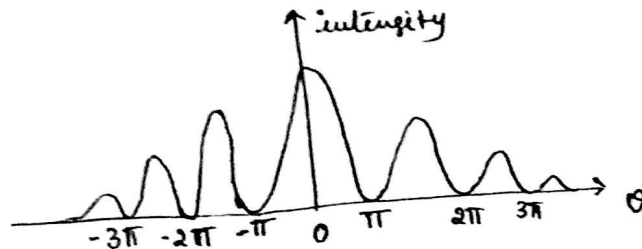
(7)

$$\text{or } \frac{\pi a \sin \theta}{\lambda} = \pm m\pi \quad \text{or} \quad \boxed{a \sin \theta = \pm m\lambda} \quad \text{--- (1)}$$

where $m = 1, 2, 3, \dots$ etc.

~~Intensity due to double slit~~

intensity distribution due to diffraction at a single slit



From equation (1) since θ is small $a \sin \theta = \pm \lambda$ ($m=1$)

$$\therefore \sin \theta = \theta = \frac{\lambda}{a} \rightarrow (2)$$

If x is the half width of central bright maxima and d is the distance between the slit and the screen then

$$\theta = \frac{x}{d} = \frac{\lambda}{a} \quad \text{or} \quad x = \frac{d\lambda}{a}$$

$\therefore$ width of the central maxima $2x = \frac{2d\lambda}{a}$

If the lens L_2 is very near the slit or the screen is far away from the lens L_2 , then

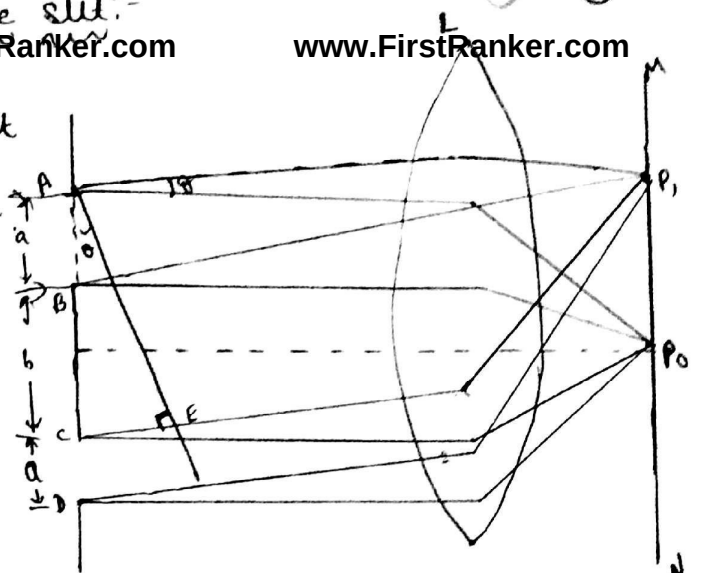
$$\sin \theta = \frac{x}{f} \quad [\because f \text{ is the focal length of lens } L_2]$$

$$\therefore \text{But from eqn (2) } \sin \theta = \frac{\lambda}{a}$$

$$\therefore \frac{x}{f} = \frac{\lambda}{a} \quad \text{or} \quad \boxed{a = \frac{f\lambda}{x}}$$

From above eqn with decrease of slit width, the fringe width increases, i.e. when slit becomes narrower, the fringe becomes wider.

Let S be a point source of monochromatic light. L_1 is collimating lens of focal length f at a distance f from the source so that the lens renders parallel rays.



Let AB and CD be two rectangular slits of width ' a ' and let ' b ' be the separation between the slits. Let a plane wavefront of wavelength λ fall normally on the slit. Let L be the collecting lens and MN be the screen.

When a plane wavefront falls normally on the slit, all the slits, all the secondary waves travelling in a direction parallel to OP_0 focus at P_0 . Hence P_0 corresponds to the position of the central bright maximum.

- (i) Interference due to diffracted secondary waves from corresponding points on the two slits.
- (ii) Diffraction due to individual slit

Missing order in the double slit diffraction pattern:

Certain directions the interference maxima may coincide with diffraction minima. Thus the combined effect of interference and diffraction result in missing of certain orders of interference maxima.

The Condition for interference maxima is given by

$$(a+b) \sin \theta = n \lambda \rightarrow (1)$$

The Condition for diffraction minima is given by

$$a \sin \theta = p \lambda \rightarrow (2)$$

where 'n' and 'p' are integers. If the values of 'a' and 'b' are such that both the conditions are satisfied simultaneously for the same value of θ , then in those directions certain interference maxima coincide with certain diffraction minima. Since this condition is satisfied for the same value of θ .

$$\therefore \sin \theta = \frac{n \lambda}{a+b} = \frac{p \lambda}{a} \Rightarrow \frac{n}{a+b} = \frac{p}{a}$$

Case i) Let $a=b$ Then $\frac{n}{2a} = \frac{p}{a}$ (or) $n=2p$

Since $p=1,2,3$ etc. Corresponding 'n' values are 2, 4, 6 etc. This means that in the diffraction pattern 2nd, 4th, 6th etc. orders of the interference maxima will be missing.

Case ii) Let $2a=b$ $\frac{n}{3a} = \frac{p}{a}$

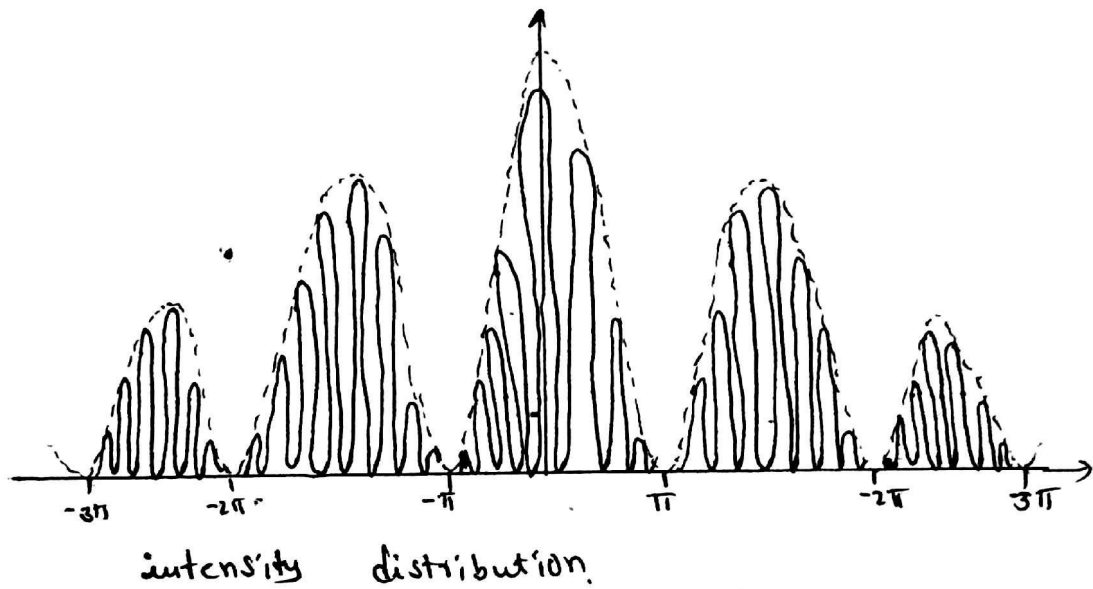
$$\Rightarrow n=3p$$

Since $p=1,2,3$ etc. Corresponding 'n' values are 3, 6, 9 etc. This means that in the diffraction pattern 3rd, 6th, 9th etc. orders of the interference maxima will be missing.

(iii) Let $a+b=a$ which means that $b=0$ Then

$$\frac{m}{a} = \frac{p}{a} \quad \text{or } m=p$$

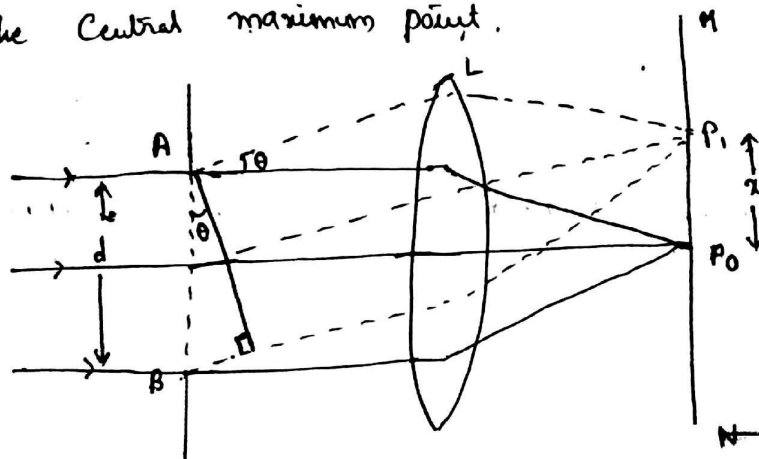
Since $p=1,2,3$ etc. Corresponding m values are also $1,2,3$ etc. This means that all the orders of interference maxima will be missing in the ~~diffraction~~ diffraction pattern. i.e., we will be observing only diffraction pattern corresponding to single slit of width $2a$.



The continuous curve represents ~~the~~ equally spaced interference maxima and minima while the dotted curve represents diffraction maxima and minima. The spacing of the interference maxima and minima depends on the values of ' a ' and ' b ' while the spacing between diffraction maxima depends on the value of ' a '.

① Fraunhofer Diffraction at a Circular Aperture:-

Let us consider a circular aperture of diameter d . A plane wave front falls normally on this aperture AB. Every point on the wave front in the plane of the circular aperture is a source of secondary wavelets. They spread out in all directions on passing through the aperture. Let the diffracted beam be focussed on the screen by a convex lens L. The waves travelling along the normal to the circular aperture are focussed at P_0 . Since all these waves travelled the same distance to reach P_0 , they all reinforce one another. Hence P_0 is the position of maximum intensity and this is the central maximum point.



Let us consider the waves travelling at angle θ with respect to the normal rays. They meet at the point P_1 on the screen. Let $P_0P_1 = x$. The path difference between the waves from A and B on reaching P_1 is given by AC.

$$\text{In } \triangle ABC \quad \sin \theta = \frac{BC}{AB} = \frac{BC}{d}$$

$$\Rightarrow BC = d \sin \theta$$

If this path difference is $\lambda, 2\lambda, 3\lambda \dots$ etc, then P_1 will be a

(12)

of maximum intensity i.e. $d \sin \theta_n = \frac{(2n+1)\lambda}{2}$ is the condition for constructive interference.

If P_1 is a point of minimum intensity, with P_0P_1 as radius and P_0 as centre if a circle is drawn, all points on the circle will have the same path difference and hence satisfy the condition for minimum intensity. Thus, the diffraction due to a circular aperture consists of a bright central disc surrounded by alternate dark and bright concentric rings. The bright central disc is called Airy's disc and the concentric rings are called Airy's rings.

If a collecting lens L of focal length f is used either very near to the slit or when the screen is at larger distance from the lens.

$$\sin \theta = \theta = \frac{x}{f}$$

But from the condition for first secondary minimum

$$\sin \theta = \theta = \frac{\lambda}{d}$$

$$\therefore \frac{x}{f} = \frac{\lambda}{d} \quad \text{or} \quad x = \frac{f\lambda}{d}$$

where x is the radius of the Airy's disc. Thus we find that with decrease of the diameter of the aperture, the radius of the Airy's disc increases.

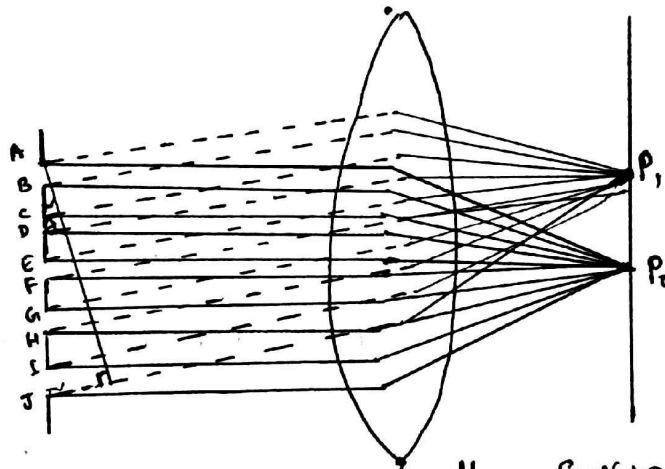
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Plane Diffraction Grating:- A diffraction grating is an optical device consists of large number of narrow, uniform spaced parallel slits and each slit is separated by an opaque region.

When a plane wavefront is incident normally on a grating surface, the incident light is transmitted through the slits and obstructed by opaque portions. Such type of grating is called as plane transmission diffraction grating.

The combined width of a ruling and a slit is called "grating element". Points on successive ~~width of a ruling~~ slits separated by a distance equal to the grating element are called "corresponding points".

THEORY:-



Let ABCDEFGHIJ represent the section of grating normal to the plane of the paper. AB, CD, EF... etc represent the slits of width 'a' each while BC, DE, FG... etc represent opaque rulings of width 'b' each. $a+b$ which is the combined width of a ruling and a slit is called grating element. It is also the distance between two successive slits. Any two points on successive slits separated by a distance $(a+b)$ are called corresponding points.

Let a plane wavefront be incident normally on the grating.

The points in the slits AB, CD, EF etc. act as secondary sources of light giving rise to secondary waves. These waves spread in all directions on the other side of the grating. The secondary waves in the same direction as that of the incident wave are focussed at P_0 on the screen. Since all these secondary waves have travelled equal distance to reach P_0 , they reinforce constructively and hence the point P_0 is the position of central bright maximum.

Now let us consider the secondary diffracted waves proceeding in a direction which makes an angle θ with respect to the normal to the grating. AMM₁ is drawn normal to the diffracted light. Let us consider the waves diffracted at the corresponding points A and C. The path difference the waves on reaching P_1 is CM. Since they travel equal path beyond AM.

$$\text{In } \triangle ACM \quad \sin \theta = \frac{CM}{AC}$$

$$\Rightarrow CM = (a+b) \sin \theta$$

The superposition of these waves at P_1 causes interference.

P_1 will be bright when $\boxed{(a+b) \sin \theta = n\lambda}$

where $n = 0, 1, 2, 3, \dots$ and angles $\theta_1, \theta_2, \theta_3$ etc. correspond to the directions of the principal maxima.

$$\therefore \sin \theta = \frac{n\lambda}{a+b} = nN\lambda \Rightarrow \boxed{\lambda = \frac{\sin \theta}{Nn}}$$

where $\frac{1}{(a+b)} = N$ is the number of grating elements or lines per unit width of the grating

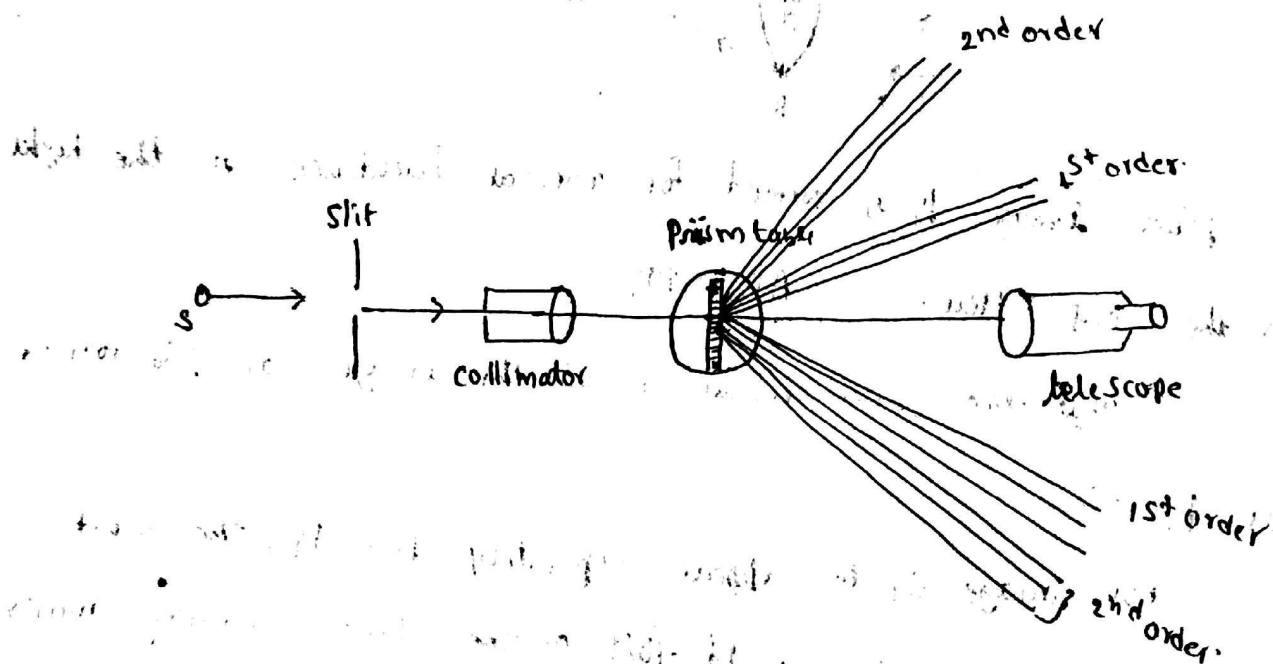
Maximum number of orders possible with a grating:-
 $n \leq \frac{1}{N\lambda}$

The maximum value that θ can have is 90° . Hence the maximum possible value of $\sin\theta$ is 1

$$: nN\lambda \leq 1 \quad \text{or} \quad n \leq \frac{1}{N\lambda}$$

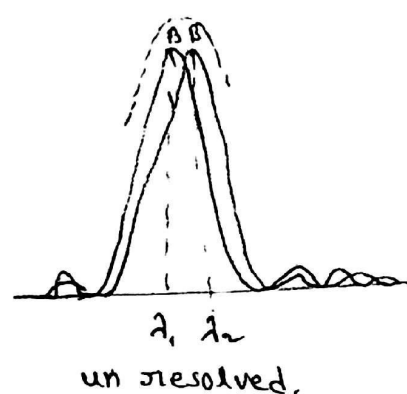
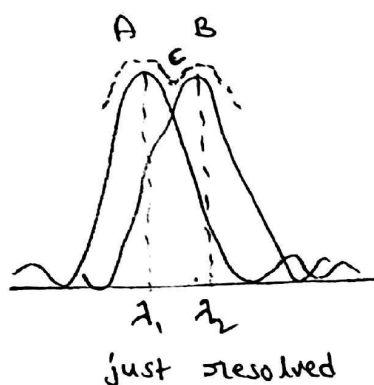
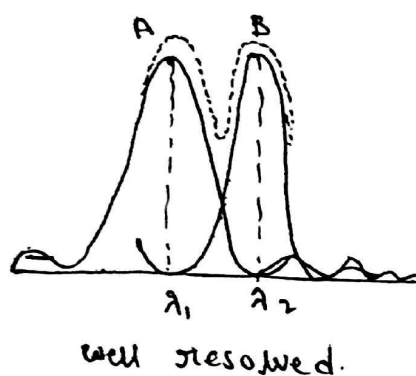
$$\text{or } \boxed{nN\lambda \leq 1}$$

Grating Spectrum:- If we replace monochromatic source of light with white ~~light~~ (mercury vapour) lamp which emits seven colours from violet to red. When the light from mercury vapour lamp falls normally on diffraction grating, diffraction spectra are produced by the grating and it consists of different orders of spectra on either of the central maximum. Each order of the spectrum consists of seven spectral lines.



Rayleigh's Criterion for Resolving power:-

According to him, two nearby images are said to be resolved if the position of the central maximum of one coincide with the first secondary minimum of the other and vice versa. A and B are the central maxima of the diffraction pattern of two spectral lines of wavelength λ_1 and λ_2 . The angle of diffraction of central maximum of the image B is greater than the angle of diffraction of first minimum of A. Hence both the optical line appear well resolved.



Case i):- A and B are the central maxima of the diffraction pattern of two spectral lines of wavelength λ_1 and λ_2 . The angle of diffraction of central maximum of the image B is greater than the angle of diffraction of minimum of A. Hence both the optical lines appear well resolved.

Case ii) The position of central maximum of wavelength λ_1 coincide with the position of the first minimum of wavelength λ_2 . Similarly the position of central maximum of wavelength λ_2 coincide with the position of the first minimum of wavelength λ_1 . In the resultant intensity curve, in the middle of the central maxima of A and B, a dip is seen at C. When we turn the spectrograph from A to B,

a noticeable decrease in intensity is observed at C. Thus the spectral

lines λ_1 and λ_2 can be distinguished from one another. According to Rayleigh, they are said to be just resolved.

Resolving power of Grating:-

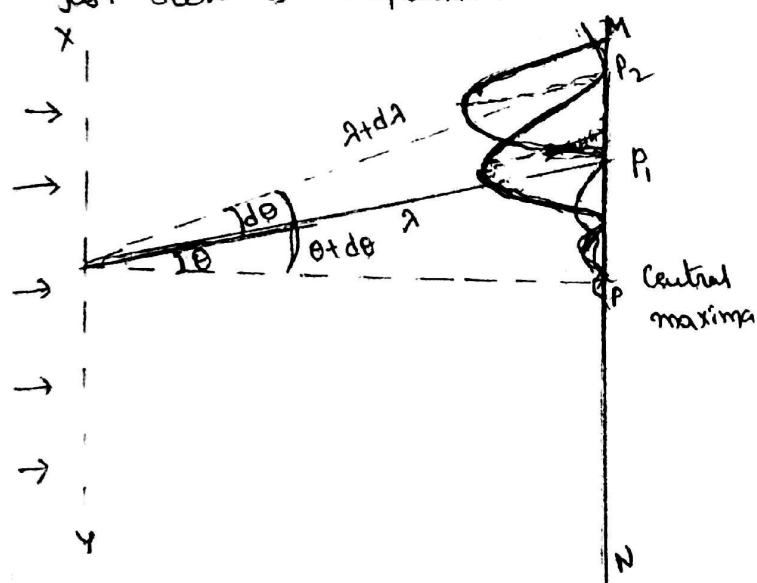
The method of seeing two close objects as separate using some optical instrument is called resolution. The capacity of the instrument to produce two separate images of very close objects is called resolving power.

"The resolving power of a grating is its ability to show two neighbouring lines in a spectrum as separate."

If we consider two very close spectral lines of wavelength λ and $\lambda + d\lambda$ its resolution is given by

$$\text{Spectral resolving power} = \frac{\lambda}{d\lambda}$$

"The resolving power of a diffraction grating may also be defined as the ratio of the wavelength of any spectral line to its difference of wavelengths between this line and a neighbouring line such that the two spectral lines can be just seen as separate."



XY is the grating surface. MN is the field of view of the telescope. Position P₁ corresponds to the n^{th} primary maximum of spectral line λ at angle of diffraction θ . Position P₂ corresponds to the n^{th} primary maximum of spectral line $(\lambda + d\lambda)$ at an angle of diffraction $\theta + d\theta$.

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The direction of n^{th} primary maximum of spectral line λ is given by $(e+d) \sin \theta = n\lambda \rightarrow (1)$

For N no. of slits $N(e+d) \sin \theta = Nn\lambda \rightarrow (2)$

The direction of $(n+1)^{\text{th}}$ primary maximum of spectral line $\lambda + d\lambda$, [angle $(\theta + d\theta)$]

$$N(e+d) \sin(\theta + d\theta) = Nn(\lambda + d\lambda) \rightarrow (3)$$

The direction of n^{th} minima in the direction of $\theta + d\theta$ having wavelength λ

$$N(e+d) \sin(\theta + d\theta) = (Nn+1)\lambda$$

$$N(e+d) \sin(\theta + d\theta) = Nn\lambda + \lambda \rightarrow (4)$$

from eqn's (3) & (4)

$$Nn\lambda + \lambda = Nn\lambda + Nnd\lambda$$

$$\therefore \lambda = Nnd\lambda$$

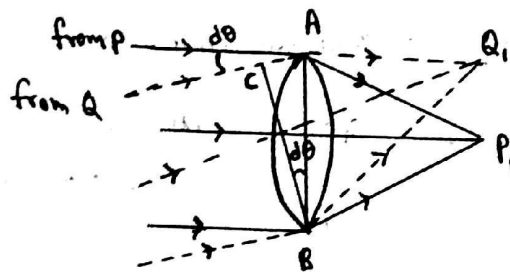
$$\therefore \boxed{\frac{\lambda}{d\lambda} = Nn}$$

From above equation resolving power is directly proportional to

- (1) Order of the spectrum
- (2) The total no. of lines per unit width of grating

The minimum distance by which two points on the object are separated from each other so that their images as produced by the telescope are just seen as separate is called the "limit of resolution" of telescope. The reciprocal of limit of resolution is known as the resolving power.

Let us consider a telescope having an objective of aperture $AB = d$. Let P & Q be two point objects lying on the object plane and having an angular separation θ . Parallel rays from these 2 point object P and Q give rise to two point-images P_1 and Q_1 lying in the focal plane of the objective of the telescope.



Point image P_1 is formed for normal incidence of the light on the lens. Hence $AP_1 = BP_1$.

Thus the first maximum of the image of P is formed at P_1 .

For image Q_1 to appear separately from P_1 , the first maximum of image Q_1 should fall on the first secondary minimum of the image P_1 . This condition is achieved if the path difference between the extreme rays from P and Q must be λ . To calculate this path difference BC is drawn normal to the rays coming from Q .

Now the path difference is AC . Let $AC = \lambda$.

Now let us divide the lens into two halves AO and OB .

If the path difference between the rays reaching Q , from A and B is λ , then the path difference between the rays reaching Q , from A and O , as well as from O and B will be $\frac{\lambda}{2}$. Hence the rays from all such corresponding points on the upper half of the lens AO and lower half of the lens OB interfere destructively forming first secondary minimum at Q .

From fig $AC = AB \cdot d\theta$ ($\because \angle CBA = d\theta$)

$$\text{or } d\theta = \frac{AC}{AB} = \frac{\lambda}{d}$$

where AB the aperture of the lens is equal to d .

$d\theta$ is called the limit of resolution of the telescope objective.

By more rigorous treatment Airy showed that λ should be replaced by 1.22λ .

$$\text{Thus } d\theta = \frac{1.22\lambda}{d}$$

$$\therefore \text{Resolving power} = \frac{1}{d\theta} = \frac{d}{1.22\lambda}$$

Resolving power of telescope depends upon

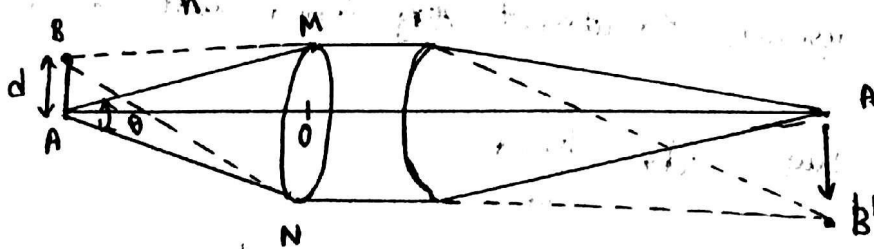
1. The diameter of the objective and

2. The wavelength of light used. Larger the diameter of the objective and smaller the wavelength of light used, higher the resolving power of the telescope.

Resolving power of a microscope:- the resolving power of a microscope is defined as the reciprocal of the linear distance between two microscopic objects that can be just resolved when seen through the microscope.

$$\therefore \text{Resolving power} = \frac{1}{d} = \frac{n \sin \theta}{\lambda}$$

where, d is the distance between two point objects, θ be semi-vertical angle of the cone formed by object at objective lens, λ is the wavelength of light, n the refractive index of the medium between the object and objective lens. To achieve high resolution, $n \sin \theta$ must be large which is called as numerical aperture of the optical system. The resolving power of a microscope increases with increase in the numerical aperture.



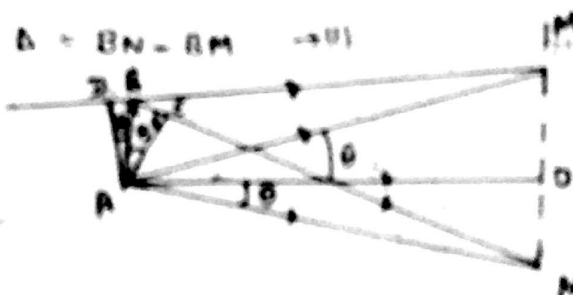
microscope is an optical instrument used to magnify the microscopic objects. MN is the aperture of the objective lens of the microscope. Let A and B be the two minute objects at a distance d . A' and B' be the corresponding Fraunhofer diffraction pattern of the two images of two objects. Assume that A' is the position of central maximum of A and B' is the position of

of the central maximum of B. According to Rayleigh's criterion, if the position of the central maximum of B coincides with the first minimum of the image A or vice versa, the two images are said to be just resolved.

The path difference Δ between the light rays resulting from the A' is given by

$$\Delta = (BN + NA') - (BM + MA')$$

but we have $NA' = MA'$



From fig AD is perpendicular to DM and AC is perpendicular to DN, then we have

$$BN - BM = (BC + CN) - (DM + DN) \rightarrow (1)$$

$$\text{But } CN = AN + AM + DM$$

$$\text{eqn (1) becomes } BN - BM = BC + DB \rightarrow (2)$$

From the $\triangle ACE$ and $\triangle ADB$

$$BC = AB \sin \theta = d \sin \theta \rightarrow (3)$$

and

$$DB = AE \sin \theta = d \sin \theta \rightarrow (4)$$

$$\text{From eqn (2) and (3) } \Delta = 2d \sin \theta \rightarrow (5)$$

If the eqn (5) is equal to 1.22λ , then A' is the first minimum of the image B', the two images are said to be just resolved.

$$2d \sin \theta = 1.22 \lambda$$

$$d = \frac{1.22 \lambda}{2 \sin \theta}$$

Here we have considered the object is self-luminous.

In the microscope, the objects are not self-luminous. According to Abbe, the least distance between two just resolvable object points A and B is given by:

$$d = \frac{\lambda}{2n \sin \theta} \rightarrow (5)$$

where λ is the wavelength of light, n is the refractive index of the medium between the object and the objective of the microscope.

$\therefore$ The reciprocal of equation (5) is

$$\frac{1}{d} = \frac{2n \sin \theta}{\lambda}$$

$\therefore$ In above equation $n \sin \theta$ is called the numerical aperture of the objective of the microscope. From above relation, it is clear that, to get the maximum resolution between two objects:

1. n , the refractive index must be large
2. The objective lens diameter should be large
3. Shorter wavelength of the light should be used.

they can be adjusted exactly perpendicular to each other. The mirror M_1 is mounted on a Carriage and can be moved exactly parallel to itself with a micrometer screw fitted with a graduated drum which can read a displacement $\sim 10^{-5}$ cm. The interference bands are observed with the help of telescope T.

working: Light from the Source S is made parallel with a collimating lens L and then it is made to fall on the glass plate G_1 . It is partly reflected at the back surface of G_1 along AC and partly transmitted along AB. The ray AC travels normally towards plane mirror M_1 and so it is reflected along the same path and comes out along AT. The refracted or transmitted light is received by the mirror M_2 normally, reflected along the same path and then moves along AT after reflection from the back surface of G_1 . So the two beams received along AT are produced from a single source by the division of amplitude and we may get interference patterns depending upon the distances travelled by them.

It is clear from diagram that a ray starting from Source S and suffering reflection at the mirror M_1 travels the glass plate G_1 twice whereas the ray reflected from mirror M_2 travels through the glass plate G_1 only once. To compensate it a second glass plate G_2 is introduced in the path AB and is made exactly parallel to G_1 .

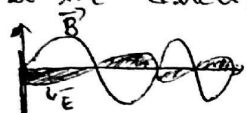
32. If the Resolving power of a grating in its first order is 1000, its resolving power in second order is **2000**
33. The concept of Resolving Power is invented by **Rayleigh**
34. Our eye can see two close objects as separated only if the angle subtended by them at the eye is greater than **5 minutes**
35. The method of seeing two close objects as separate using some optical instrument is called **Resolution**
35. Spectral resolving power is $\lambda/d\lambda$
36. The expression for resolving power of a grating is $\lambda/d\lambda = Nn$
37. For Telescope resolving power is $d/1.22 \lambda$
38. For microscope Resolving power is $2(NA) / \lambda$ ($NA = \text{Numerical Aperture}$)
39. Maximum number of orders possible with the grating is always an **Integer**
40. Polarization of light is evidence for **Transverse wave nature of light**
41. The phenomenon causing polarization of light is **Double refraction**
42. Polarization of light conclusively proves that **Light waves are transverse**
43. When the angle of incidence for O-ray on the Canada balsam is greater than the critical angle, then the phenomenon takes place is **Total internal reflection**
44. In elliptically polarized light **Amplitude of vibrations changes in magnitude and direction**
45. Nicol prism is based on the action of **Double refraction**
46. The phenomenon of double refraction is also called as **Birefringence**
47. Canada balsam acts as a rarer medium for O-Ray
48. Example for uniaxial crystal is **Topaz**
49. The line which is passing through the blunt corners of the crystal is called **Optic axis**
50. If an electron excites from lower state to higher state then the process is known as **absorption**
51. Which process does not require the inducement of energy **spontaneous emission**
52. Coherent light is emitted by **stimulated emission**
53. The life time of ground state is **infinite**
54. If N_1 & N_2 are populations of energy states E_1 & E_2 respectively in a system and $E_2 > E_1$ then condition for population inversion will be $N_1 < N_2$
55. Spontaneous emission is postulated by **Bohr**
56. Stimulated emission is postulated by **Einstein**
57. The population of the various energy levels of a system in thermal equilibrium is given by **Boltzmanns distribution law**
58. The lasing action is possible only if there is **population inversion**
59. Emission of photon when an electron jumps from higher energy state to lower energy state due to interaction with external energy is called **stimulated emission**
60. Among the following laser sources which source will give visible light radiation **He-Ne laser**
61. In He-Ne lasers, the ratio of He Ne mixture is **10:1**
62. Which of the following emits pulsed mode laser? **Ruby laser**
63. The doping percentage of chromium in Ruby is **0.05%**
64. In ruby laser the flash tube contains **Xenon gas**
65. In He-Ne laser, the emission takes place from which atoms? **Ne atoms**
66. The wavelength of radiation emitted by Ruby laser is **6943 Å**
67. The pumping process used in He-Ne gas laser is **Electric discharge**
68. The pumping process used in Ruby laser is
- Optical Pumping**
69. The colour of laser radiation that comes out of He-Ne laser is
- Red**
70. The light is guided through transparent glass fibers by
- total internal reflection**
71. Total internal reflection takes place only when the light travels from
- denser to rarer medium**

UNIT-3(A)

Polarization

Introduction:- Interference and diffraction are the phenomenon which confirmed the wave nature of light beyond any doubt but it could not establish whether light waves are longitudinal or transverse.

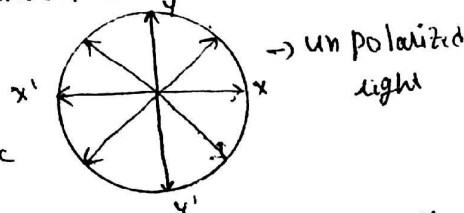
"The transverse nature of light has been established by polarization phenomenon". In transverse motion the particles of the medium execute periodic oscillations in a direction perpendicular to the direction of propagation of the wave.

 Light is nothing but an electromagnetic wave. The light emitting atoms are oscillating independently emitting individual wave trains. As a result the oscillations are at random. Hence in nature most light as it occurs is unpolarized.

"If the oscillations are confined to only one direction then it is called plane polarized light."

* In unpolarized light the electric field oscillates in all directions perpendicular to the direction of propagation of light.

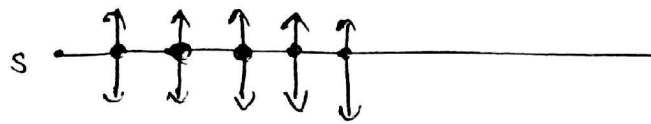
* If the oscillations of the electric field are confined to a only one direction, then it is called plane polarized light.



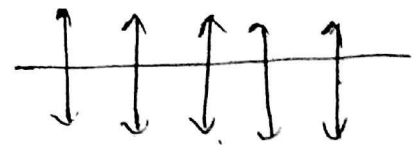
Representation of polarized and unpolarized light:-

Light waves are transverse and so have vibrations at right angles to the direction of propagation. Light emitting atoms are oscillating independently. The waves emitted have their own planes of vibration. Hence the waves vibrate in all possible

planes with equal probabilities. If the direction of propagation of light is considered to be consists of along the plane of paper then unpolarized light the vibrations are indicated by dots and arrows



un polarized light



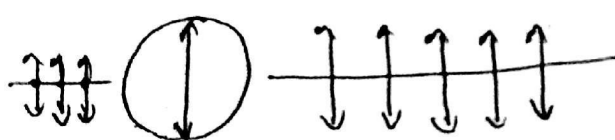
vertical vibrations



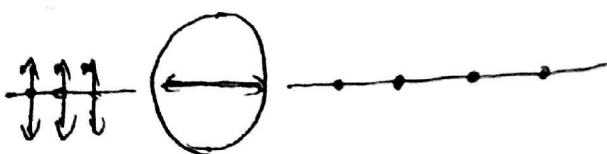
(Horizontal vibrations)

Types of polarized light:-

(1) plane polarized light:- If the vibrations are confined to a single plane then it is called plane polarized light (or) linearly polarized light.



(vertical vibrations)

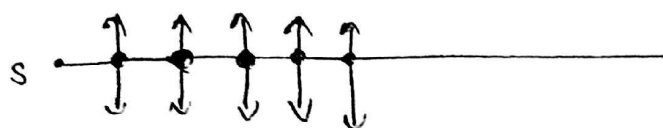


(Horizontal vibrations)

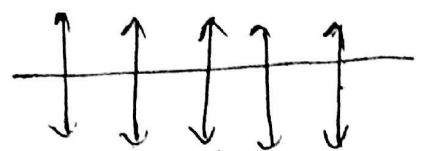
Note:- The vibrations along the paper are represented by arrows.
The vibrations along perpendicular to the plane of the paper are represented by dots.

Partially plane polarized light:- If the linearly polarized light contains small additional component of unpolarized light, it becomes partially plane polarized light. Then it is represented by either more

planes with equal probabilities. If the direction of propagation of light is considered to be consists of along the plane of paper then unpolarized light the vibrations are indicated by dots and arrows



un polarized light



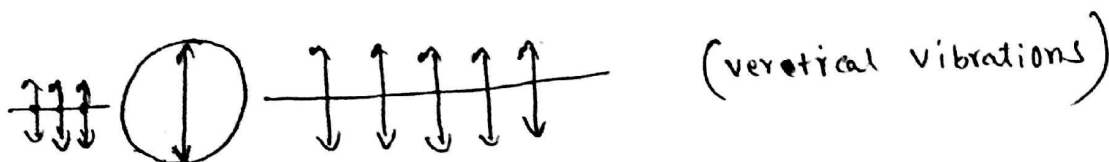
vertical vibrations



(Horizontal vibrations)

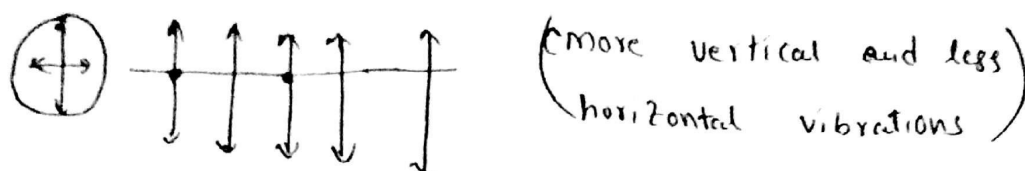
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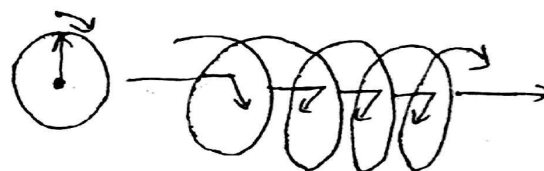
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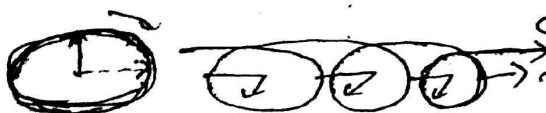
Circular polarized light:- In circular polarization, the electric vector of constant amplitude no longer oscillates but rotates while proceeding in the form of a helix. The projection of a wave on a plane intercepting the axis of propagation gives a circle with the amplitude of the vector remaining constant.

If the vector rotates in the clockwise direction with respect to the direction of propagation, it results in right circularly polarized light while the rotation in the anti-clockwise direction results in left circularly polarized light.



Elliptically polarized light:- In circular polarization, the electric vector of constant amplitude rotates while proceeding. If the amplitude of the electric vector is not a constant but varies periodically then it results in elliptically polarized light. For example,

If the electric vector has minimum amplitude while oscillating vertically and rotates while propagating to have maximum amplitude when oscillating horizontally.



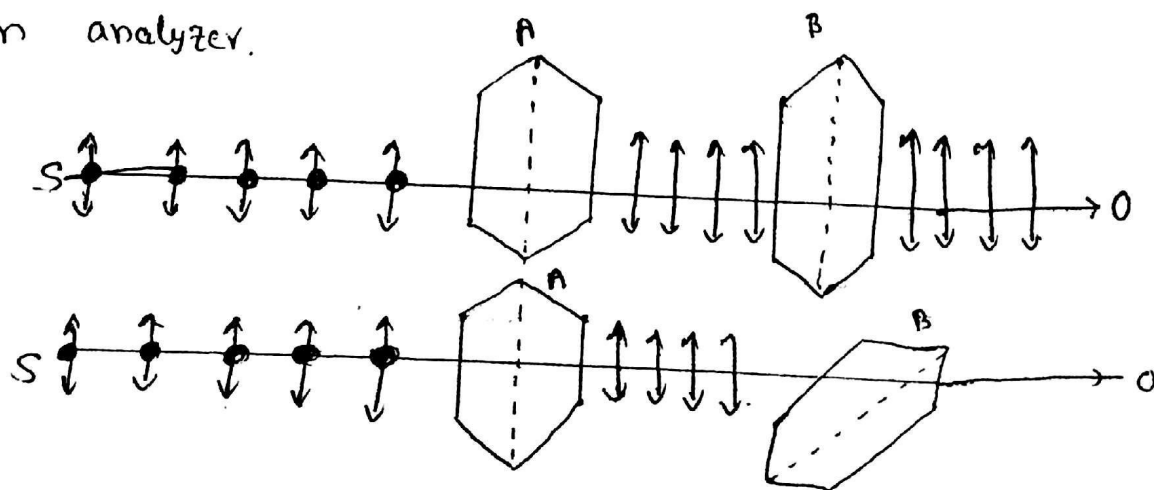
Production of polarized light:

Tourmaline Crystals are used for produce the plane polarized light. [These Crystals have the same behaviour with respect to light waves as Slits S_1 and S_2 have with respect to transverse waves in stretched string.]

Let us consider a tourmaline crystal A cut parallel to its crystallographic axis. Let unpolarized light from source S enter the crystal. The light emerging out of the crystal is plane polarized. When the crystal is rotated with emergent path of light as axis, the intensity of emergent beam does not vary. Now another crystal B is introduced. When the crystallographic axis of both A and B are parallel, light passes through B also without any fall of intensity.

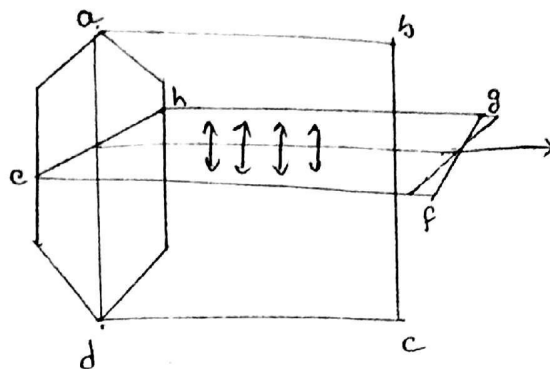
When crystal B is rotated, gradually intensity of light emerging out of B decreases and when the crystallographic axis of B and A are perpendicular, no light passes through B.

Hence crystal A is a polarizer and crystal B which is used to analyse whether the light emergent from A is polarized or not is an analyzer.



The property of plane polarized light differ with respect to two planes, one containing vibrations and another

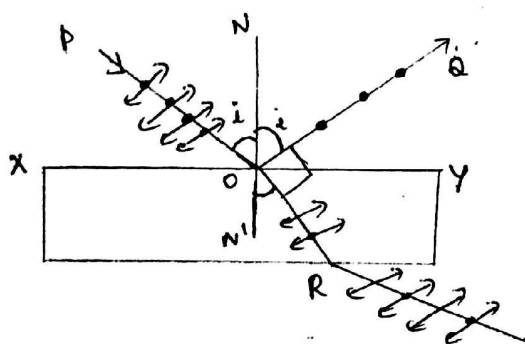
at right angles to it. The abcd contains oscillations of light and this plane is called plane of oscillations and the plane efgh perpendicular to the plane of oscillation is called plane of polarization.



Po

Polarization by reflection:-

In 1808, a French Scientist Malus found that when unpolarized light was reflected at the surface of some transparent medium such as glass, water etc. the reflected light was found to be partially plane polarized. The degree of polarization changed with the angle of incidence. For a particular angle of incidence the reflected light was found to be completely plane polarized.



The angle of incidence for which the reflected beam is completely plane polarized is known as polarizing angle (or) angle of polarization. This angle is also known as "Brewster's angle".

At this angle, the reflected and transmitted lights are at right angle to each other.

Brewster's law:- The refractive index of the material medium μ is equal to the tangent of the angle of polarization i_p .

$$\therefore \mu = \tan i_p$$

This is called Brewster's law. Applying this law to prove reflected and

refracted rays are at right angles.

Proof: XY represents the surface of transparent medium.
PO is the incident unpolarized light while OQ and OR are the reflected and refracted light rays.

$$\angle PON = i \text{ (angle of incidence)}$$

$$\angle N'OR = r \text{ (angle of refraction)}$$

From Snell's law $\mu = \frac{\sin i}{\sin r} \rightarrow (1)$

From Brewster's law $\mu = \tan i = \frac{\sin i}{\cos i} \rightarrow (2)$

from (1) & (2) we get

$$\frac{\sin i}{\sin r} = \frac{\sin i}{\cos i}$$

$$\cos i = \sin r \Rightarrow \sin(90-i) = \sin r$$

$$\therefore 90-i = r \Rightarrow \boxed{i+r=90} \rightarrow (3)$$

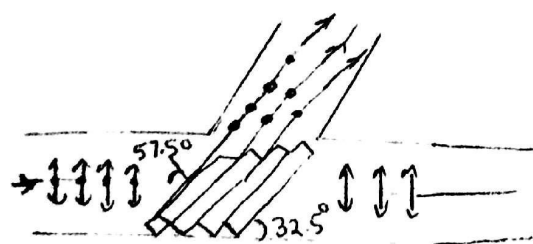
From Fig $\angle NOB + \angle QOR + \angle N'OR = 180^\circ$

$$i + \angle QOR + r = 180^\circ$$

$$\begin{aligned} \angle QOR &= 180 - (i+r) \\ &= 180 - 90 = 90^\circ \end{aligned}$$

$\therefore$ The reflected and refracted rays are at right angles.

Polarization by transmission:- When unpolarized light is incident at polarizing angle the reflected light is completely plane polarized and transmitted light is partially plane polarized. The

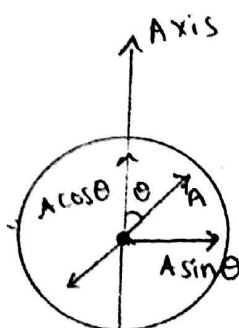


transmitted light contains a greater proportion of light vibrating parallel to the plane of incidence. If the process of reflection at polarized angle is repeated using number of

Plates all inclined at polarizing angle, finally the transmitted light becomes purely plane polarized. Such an arrangement is known as pile of plates.

The polarizing angle for glass is 57.5° . The pile of plates consists of number of glass plates fixed in a tube of suitable size inclined at angle of 32.5° to the axis of the tube so that the angle of incidence is 57.5° . A beam of monochromatic light entering the tube falls on the pile of plates at the polarizing angle. Each plate filters the vibrations at right angles to the plane of incidence by reflection. Hence after the beam has traversed about 15 plates, the transmitted light has vibrations only in the plane of incidence. Thus pile of plates acts as a polarizer.

Malus law:-



When unpolarized light passes through a polarizer, the transmitted light is plane polarized. When the polarized light is passed through an analyser, the intensity of transmitted light varies with the angle between the planes of polarizer and analyser.

Malus Stated that "The intensity of the polarized light transmitted through the analyser varies as the square of cosine of the angle between the plane of transmission of the analyser and the plane of polarizer".

Proof:- Polarized light can be resolved into two components

- (i) parallel to the plane of transmission of the analyser
- (ii) perpendicular to the plane of the analyser.

Let A be the amplitude of the incident plane polarized light with its plane of vibration at an angle θ with the axis of the analyser. The resolved component of amplitude A along optic axis is $A \cos \theta$. This component is transmitted through the analyser. The intensity of the transmitted light is given by

$$I = (A \cos \theta)^2 = A^2 \cos^2 \theta.$$

when $\theta = 0$ i.e. the plane of vibration is along the optic axis of the analyser.

$$I = A^2$$

Let this maximum value be $I_0 = A^2$

$$I = I_0 \cos^2 \theta.$$

When $\theta = 90^\circ$, i.e. the plane of vibration is normal to the optic axis of the analyser

$$I = I_0 \cos^2 \theta = 0.$$

The intensity of polarized light transmitted through the analyser varies as the square of $\cos \theta$, where θ is the angle between the plane of vibration of incident plane polarized light and optic axis of the transmitter.

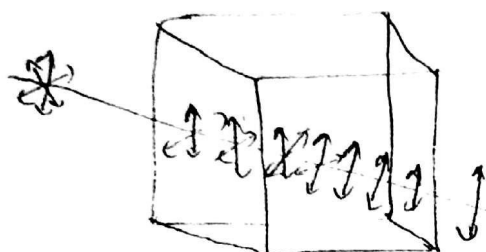
Polarization by scattering:-

When light beam passes through a medium consisting of very small particles, light is scattered. If the dimensions of the particles are smaller than wavelength of light, then the intensity of scattered light is found to be inversely proportional to the fourth power of wavelength. $I \propto \frac{1}{\lambda^4}$. If light is passed through such a medium and scattered light is analysed in a direction perpendicular to the direction of propagation of light, the scattered light

is found to be plane polarized.

Ex: blue colour of the sky

Polarization by Selective Absorption:-



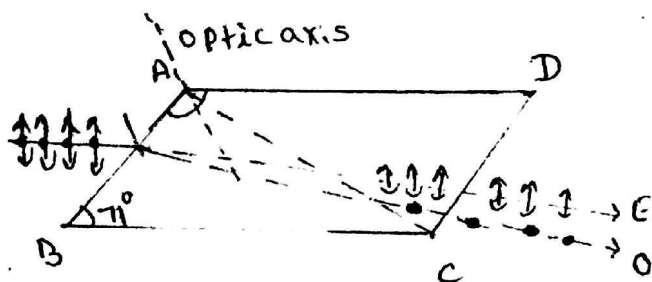
Light passing through the crystal is split into two components, the horizontal and vertical. The horizontal component is being completely

absorbed by the crystal and hence vertical component alone comes out. The light emerging from crystal, therefore is linearly polarized.

Double refraction: (birefringence):-

Unpolarized light has two components one vertical and another horizontal. When unpolarized light passes through certain anisotropic crystals like calcite or quartz, velocity of propagation of these two components vary. This means that the material exhibits two different refractive indices. Both the components have the same angle of incidence, they have different angle of refraction.

When unpolarized light passes through such crystals, we get two refracted beams and this phenomenon is called double refraction.



Calcite is a good example of a system of anisotropic crystals. It is rhombic in shape and the line AC joining the blunt corner A and C is called the crystal axis.

A line through 'A' which makes equal angles with the three edges gives the direction of "optic axis". Any line parallel to this line is also optic axis. Since optic axis is not a line but direction.

"When unpolarized light passes through the crystal, in a direction different from optic axis, it is split into ordinary ray (O-ray) and extra ordinary ray (E-ray). The

O-ray travels with the same velocity in all directions. The corresponding refractive index is called ordinary refractive index (μ_o). The extra ordinary ray travels with same velocity as that of ordinary ray along optic axis direction. In other directions, the velocity gradually changes and in a perpendicular direction to optic axis the change is maximum. The refractive index of extraordinary corresponding to this direction is called extra ordinary refractive index (μ_e).

Nicol's Prism - Nicol prism is one of the most important device used to produce plane polarized light. This was invented by William Nicol in 1828. Calcite crystal is modified such that it eliminates one of the two refracted rays by "total internal reflection".

A calcite crystal whose length is three times its breadth is taken. The two ends AB and CD of the crystal are cut, so that the angle ABC reduces from 71° to 68° . Then the crystal is cut into two halves along the plane A_1C_1 , which passes through the blunt corners and perpendicular to both the principal section and end faces. A_1C_1 makes an angle of 90° with C_1D and A_1B . The two cut faces are well polished and cemented together using

a thin layer of Canada balsam, a clear transparent material. It has refractive index 1.55 for $\lambda = 5893 \text{ \AA}$. When unpolarized Sodium light enters the Nicol prism, it splits into o-ray and e-ray. O-ray has the refractive index 1.6584 while the same for e-ray varies between 1.4864 to 1.6584. Inside the crystal when o-ray meets the thin layer of Canada balsam cement, it has to travel from denser medium to rarer medium.

Because of shaping of the crystal,

face, the o-ray is refracted more

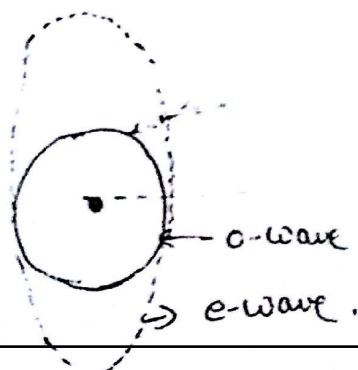
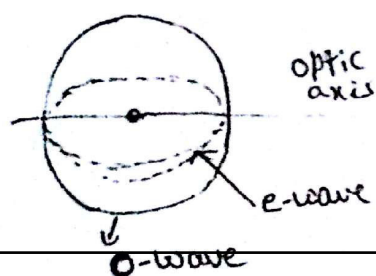
so that the angle of incidence at the Canada balsam interface is greater than the critical angle.

Hence it undergoes total internal reflection and leaves the crystal

through its side as shown. Hence e-ray alone emerges out of the other face of the prism.

Nicol prisms are good polarizers and analysers and can be used to produce and analyse plane polarized light.

wave plates:- Let us assume a point source inside the crystal at its centre to understanding of variation of velocity of e-ray with direction. The wave corresponding to o-ray travels with same velocity in all directions resulting in sphere. The



wave corresponding to e-ray travels with different velocities in different directions and the wave front advances

travel with same velocity and the difference is maximum along the direction perpendicular to optic axis.

* In some crystals velocity of e-ray is less than that of o-ray, the ellipsoidal wavefront of e-ray lies within the spherical wavefront of o-ray; the ellipsoidal wavefront of e-ray lies within the spherical wavefront o-ray. Such crystals are called positive uniaxial crystals.

Ex: Quartz Crystal

* In other crystals the ellipsoidal wavefront of e-ray lies away from the of o-ray. Such crystals are called negative uniaxial crystals.

Ex: Calcite.

Quarter and half plates:-

Let us consider

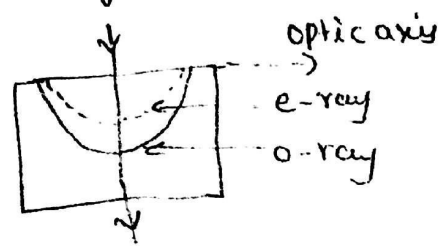
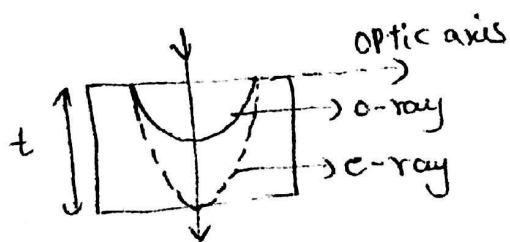
a double refracting crystal, in which its optic axis lies along its surface. Suppose an unpolarized light of wavelength λ incident normally on the crystal surface, the o-ray and e-ray travel with increasing path difference along their direction of propagation.

If t is thickness of the wave plate then

Optical path for o-ray = $\mu_o t$

Optical path for e-ray = $\mu_e t$

$\therefore$ Optical path difference $\Delta = (\mu_o - \mu_e)t$, where μ_o and μ_e are refractive indices of o-ray and e-ray respectively.



Hence optical path difference = $(\mu_o - \mu_e)t$

If the thickness of the plate is such that this path difference is $\frac{\lambda}{4}$, then the plate is called quarter wave plate

$$(\mu_o - \mu_e)t = \frac{\lambda}{4}$$

$$\therefore \boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}}$$

If the thickness of the plate is such that the path difference is $\frac{\lambda}{2}$, then the plate is called half wave plate.

$$(\mu_o - \mu_e)t = \frac{\lambda}{2}$$

$$\therefore \boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}}$$

Qunt

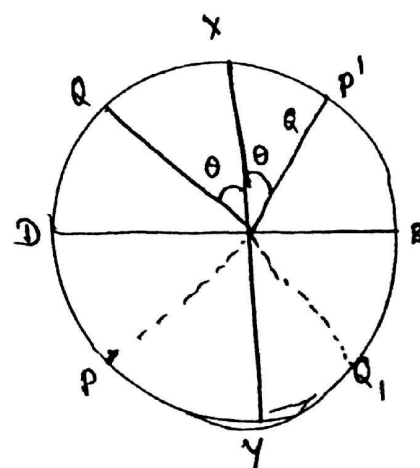
viewed through a telescope focused on the half-shade.

Action of Laurentz Half Shade plate:-

The Laurentz system consists of a half shade plate in two halves, one of the quartz cut parallel to its optic axis and the other, a matching plate of glass so chosen as to absorb and reflect the same amount light as the quartz plate.

The quartz plate is a half wave plate. It introduces a path difference of $\lambda/2$ between the ordinary and extraordinary rays in the transmission normal through it. Let the plane of vibration of the plane polarised light from the polariser P falls normally on half shade plate along CP. The light passing through the glass plate remains unaffected, while that falling on the quartz plate is broken up into two components e-ray (CX parallel to optic axis XY) and o-ray (perpendicular to the optic axis, that is along CB).

As in quartz o-ray travels faster. Hence on emergence, o-ray has vibrations along CD and e-ray has vibrations still along CX. Therefore, the emergent wave CQ is the resultant of vibrations along CD and CX. Hence Here $\angle P'CX = \angle QCX = \theta$. Thus



the angle b/w the vibration planes of light emerging from quartz, CQ and that of light emerging from glass, CP is 2θ

Thus there are two plane polarised lights one emerging from the glass with vibrations in the plane CP while other emerging from quartz with vibrations in the plane CQ. If the principal plane of the analyzing Nicol A is parallel to QCA', the light from the quartz plate will pass unobstructed. Thus, the quartz plate half will be brighter than the glass plate half. If the principle plane of the analyzing Nicol A is parallel to PCP', then the light from glass plate will pass unobstructed while the light from the quartz plate will be partially obstructed. Thus the right half will appear brighter as compared to the left half. But when the principle plane of the analyzing Nicol A is parallel to the optic axis XCY, the two halves appear equally illuminated.

Determination of the specific Rotation of Sugar Solution:-

To find the specific rotation of cane sugar or of an optically active substance, the glass tube T is first filled with clear water and the analyzer A is set in the position of equal brightness of the two halves of the field of view. The reading of the vernier is noted.

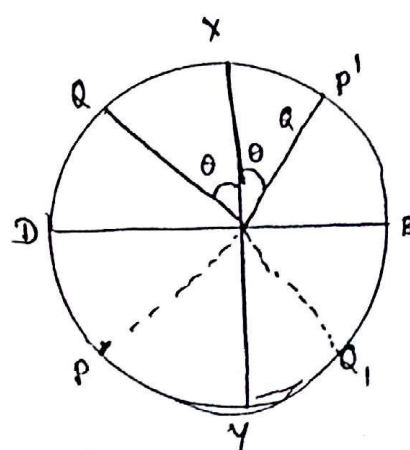
Now, the Sugar solutions of known concentration are filled in the tube and the tube is placed again in the same place. Due to optical activity of the solution, the vibrations on passing from the quartz half and the glass half is rotated. Therefore, on the introduction of the tube containing

viewed through a telescope T. The telescope is focussed on the half-shade.

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the angle b/w the vibration planes of light emerging from quartz, CQ and that of light emerging from glass, CP is 2θ

the Sugar solution, the field of view is not equally bright. The analyzer is rotated in the clockwise direction and is brought to a position so that the whole field of view again appears equally bright. The new position of the verniers on the circular scale is noted. The difference in the two readings of the analyzer gives the angle of rotation θ produced by the solution. In the actual experiment, the angles of rotation for the solutions various concentrations are measured. A

$\therefore$ The specific rotation of the cane sugar is determined by using the formula given.

$$S = \frac{\theta}{LC}$$

where L is the length of the tube in cm, C is the concentration of the solution in gm/cc and θ is the angle of rotation in degrees.

Biquartz polarimeter:-

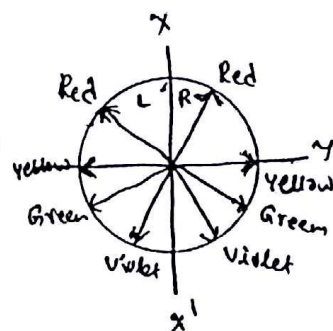
Biquartz polarimeter is an accurate instrument which is more sensitive than a half shade polarimeter used for finding the angle of rotation produced by an optically active substance. The experimental parts of biquartz polarimeter are the same as that of half shade polarimeter except that the half shade device is replaced by biquartz plate and monochromatic light is replaced by white light.

One piece of quartz produces right handed rotation and the other left handed.

when plane polarised white light is passed through each half of this plate, its different components will be rotated through different angle by each half but in the opposite directions. Each rotates the plane of polarisation of yellow light through 90° . Thus, the rotatory dispersion occurs in each plate. The left half plate rotates the plane of vibration in anti-clockwise direction and right half in the clockwise direction.

The red rays are rotated least, while the violet rays are rotated most. The intermediate rays are rotated through the angle lying between least of red and maximum of violet. For yellow the rotation is 90° .

If the principal plane of the analysing nicol be parallel to xx' , the yellow light will be completely quenched and the other colours will be present in the same direction proportion in each half. In this



position, the field of view appears grey (blue + red). This is called the tint of passage or transition tint. A slight rotation of the analysing nicol to one side from this position will change the colour of one half blue, while that of the other half to red. If the analysing Nicol is rotated to other side, the colours of the two halves are interchanged.

LASER

Light Amplification by Stimulated Emission of Radiation

Introduction:- The word "LASER" is an acronym for Light Amplification by Stimulated Emission of Radiation. In 1960 Maiman first achieved laser action at optical frequency in ruby. The power of the laser is several mw and size of the advanced laser system is one tenth of the diameter of a human hair.

Characteristics of the LASER:-

Directionality:- During the propagation of a laser its angular spreading will be less and occupies less area where it incidents. Hence it possesses high degree of directionality.

Ex:- Laser beam of 10cm diameter is operated on to the moon then it doesn't spread over not more than 5 km.

Monochromaticity:- The property of exhibiting a single wavelength by a light is called monochromaticity i.e., when it is sent through a prism then a single line will be appeared in the optical spectrum.

Brightness (Intensity):- The laser beam is highly bright as compared to the conventional light sources because more light energy is concentrated in a small region. Laser light is coherent, so at a time many photons are in phase and they superimpose to produce a wave of larger amplitude. The intensity is proportional to the square of amplitude. Hence, the intensity of the resultant laser beam is very high.

Coherence:- The property of existing either zero or constant phase angle difference between two or more wave is known as coherence.

Coherence is of two types.

Spatial coherence:- If a wave maintains a constant phase difference or in phase at two different points on the wave over a time t , then the wave is said to have spatial coherence.

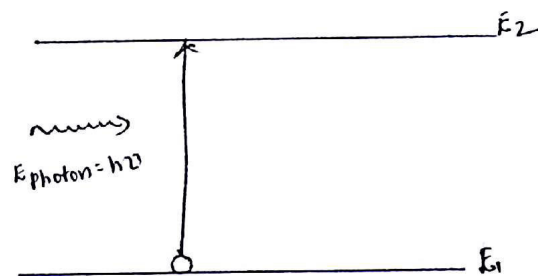
Temporal coherence:- If there exists either zero or constant phase angle difference between two light fields measured at two instants at the same point then wave is said to have temporal coherence.

Definitions:-

In lasers the interaction between matter and light is of three different types. They are absorption, spontaneous emission and stimulated emission. In these processes, two energy levels of atoms are involved. Let E_1 and E_2 be ground and excited states of an atom. Transition between these states involves absorption or emission of photon of energy $E_2 - E_1 = h\nu$, where 'h' is Planck's constant.

Absorption:-

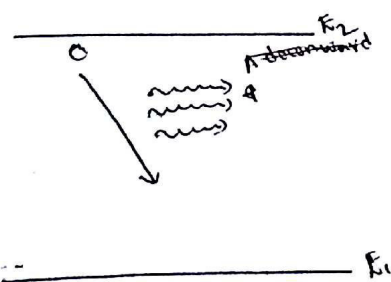
Let us consider a system in which two energy levels are present whose energies are E_1 and E_2 , where E_1 is ground state and E_2 is excited state.



Usually atoms are in the ground state as long as external forces are not applied. When a photon of energy $h\nu = E_2 - E_1$ is incident on the atom lying in ground state then it excites to higher state E_2 . This phenomenon is known as absorption.

Spontaneous Emission:-

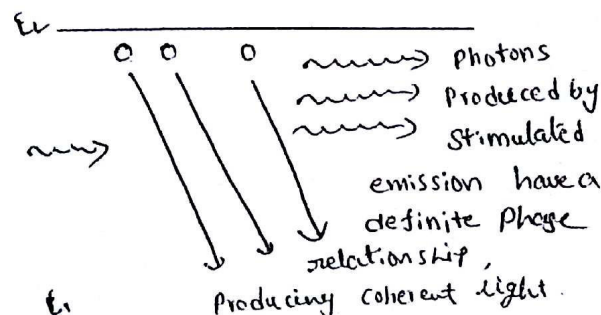
Let us assume that the atom is in the excited state E_2 . After the life time the atom decays to its ground state spontaneously emitting a photon of energy $h\nu = E_2 - E_1$. This phenomenon is known as spontaneous emission.



$$E_{\text{photon}} = h\nu = E_2 - E_1$$

Stimulated Emission:-

Let us assume that the atom is in the excited state E_2 , if a photon of energy $h\nu = E_2 - E_1$ is incident on it before the life time



it stimulates the atom from E_2 to E_1 , then a photon of energy $h\nu$ releases along with incident photon. These two photons will have same energy and phase. This phenomenon is known as stimulated emission. The emitted photons in this case have the single wavelength and they are in phase thus the photons are coherent.

Distinction between Spontaneous & Stimulated Emission :-

- Spontaneous Emission
1. Emission takes place without any stimulus energy
 2. Incoherent radiation
 3. Low intense and less directional
 4. polychromatic radiation
 5. This emission is postulated by Bohr.
 - 6) Ex: Light from Sodium or mercury Lamp

- Stimulated Emission
1. Emission takes place with the help of stimulus energy
 2. coherent radiation
 3. High intense and more directional
 4. monochromatic radiation
 5. This emission is postulated by Einstein
 - 6) Ex: Light from Ruby or He-Ne laser.

Life time:- The absorption of time spent by an atom in the excited state is known as lifetime of that energy state.

Ex. For Hydrogen atom life time is 10^8 sec.

Metastable State:- The excited state, which has long lifetime is known as metastable state. According to Heisenberg's uncertainty principle metastable state is an excited state of an atomic system whose energy level width is very small. So that the life time of the electrons is very large.

∴ Heisenberg's uncertainty principle

$$\Delta E \Delta t \geq \frac{h}{2\pi}$$

where ΔE is the width of the energy level and Δt is the

Life time of the electrons in that energy level.

Then $\Delta t \propto \frac{1}{\Delta E}$

Thus the life time of the electron in the metastable state will be very large due to its narrow size and hence it is easy to achieve population inversion at this level to start laser action.

Population:- The number of atoms per unit volume in an energy level is known as population of that energy level.

If N is no. of atoms per unit volume in an energy state E then expression for population (according to Boltzmann's distribution law)

$$N = N_0 \exp\left(\frac{-E}{K_B T}\right)$$

where N_0 is the population in the ground state,
 K_B is the Boltzmann's Constant and
 T = Temperature

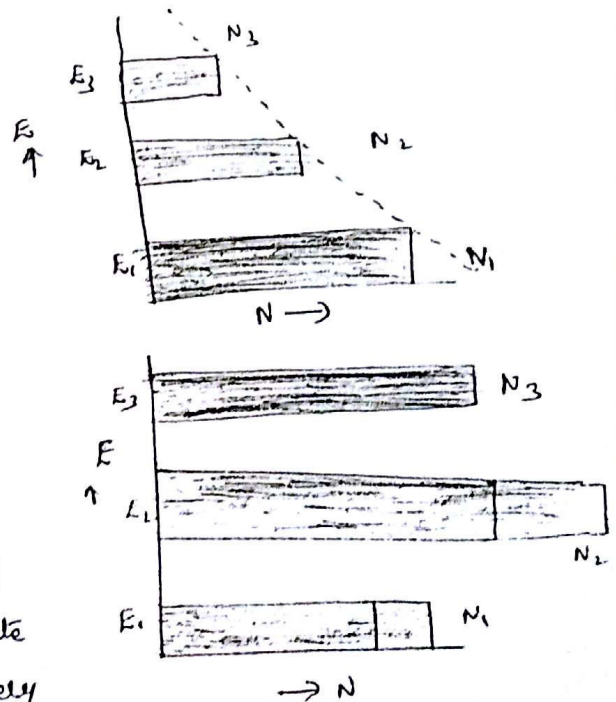
From the equation of population, it is maximum in the ground state and decreases exponentially as energy of energy level increases.

Population inversion:-

Usually in a system the number of atoms (N_1) present in the ground state (E_1) is larger than the number of atoms (N_2) present in the higher energy state. The process of making $N_2 > N_1$ is called population inversion.

Let E_1, E_2 and E_3 be ground state, metastable state and excited state of energies of the system respectively

Such that $E_1 < E_2 < E_3$. N_1, N_2, N_3 are the no. of populations of energy



levels E_1 , E_2 and E_3 respectively.

E_1 lifetime is unlimited and it is most stable state. E_3 lifetime is very less and it is the most unstable state. where E_2 has more life time.

When suitable form of energy is supplied to the system in a suitable way, then the atoms excite from ground state (E_1) to excited states (E_2 and E_3). Due to instability, excited atoms will come back to ground state after the lifetime of the respective energy states E_2 and E_3 . If this process is continued then atoms will excite continuously to E_2 & E_3 .

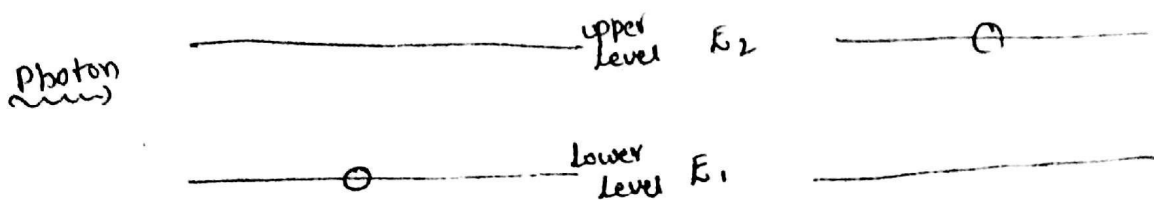
E_2 is the most unstable state. atoms will fall into E_1 immediately. At a stage the population in E_2 will become more than the population in ground state. This situation is called population inversion.

Pumping:- The process of ^{supplying} suitable form of energy to a system to achieve population inversion is called pumping.

Types (1) optical pumping (2) Electric discharge method
(3) Direct conversion (4) Chemical reactions.

Einstein's Coefficients:-

Stimulated Absorption:- In this a photon stimulates an electron to move from a lower energy state E_1 to higher state E_2 by means of absorption.



The no of stimulated upward transitions per unit volume per unit time per unit frequency is directly proportional to the incident energy density $u(\nu)$ and properties of energy level E_1 and E_2 .

$$P_{12} \propto u(\nu) \Rightarrow P_{12} = B_{12} u(\nu)$$

where B_{12} is proportionality constant, represents properties of energy levels known as Einstein's coefficient of absorption.

(i) Spontaneous emission:- The rate of probability to occur spontaneous emission process from energy level E_2 to energy level E_1 depends only on properties of energy levels E_1 & E_2 . This process is independent of energy density $U(\nu)$.

$$(P_{21})_{\text{spont}} = A_{21}$$

where A_{21} is proportional constant, represents properties of energy level known as Einstein's coefficient of spontaneous emission.

(ii) Stimulated Emission:-

The rate of probability to occur stimulated emission process from energy level E_2 to E_1 depends on properties of energy levels 1 and 2 as well as proportional to stimulated energy density $U(\nu)$ of frequency ν incident on the atom.

$$(P_{21})_{\text{stimulated}} \propto U(\nu)$$

$$\therefore (P_{21})_{\text{stim}} = B_{21} U(\nu)$$

where B_{21} is proportionality constant, represents properties of energy levels known as Einstein's coefficient of stimulated emission.

The total transition probability of atoms from energy level E_2 to energy level E_1 can be written as

$$P_{21} = (P_{21})_{\text{spontaneous}} + (P_{21})_{\text{stimulated}}$$

$$P_{21} = A_{21} + B_{21} U(\nu)$$

Relation Between Einstein's Coefficients:-

Let us consider N_1 and N_2 be populations in the energy levels E_1 and E_2 respectively in a system of atoms, which is at thermal equilibrium at a temperature T .

The number of atoms that take transitions per unit volume from energy level E_1 to energy level E_2 in unit time can be written as

$$N_1 P_{12} = N_1 B_{12} U(\nu) \rightarrow (1)$$

The number of atoms that take transitions per unit volume from energy level E_2 to Energy level E_1 in unit time can be written as

$$N_2 P_{21} = N_2 [A_{21} + B_{21} u(\nu)] \rightarrow (2)$$

At equilibrium, the number of transitions from energy level E_1 to Energy level E_2 will be equal to the number of transitions from Energy level E_2 to Energy level E_1 .

$$N_1 P_{12} = N_2 P_{21}$$

$$\Rightarrow N_1 B_{12} u(\nu) = N_2 [A_{21} + B_{21} u(\nu)] \quad [\text{from eqn's (1) \& (2)}]$$

$$\Rightarrow N_1 B_{12} u(\nu) - N_2 B_{21} u(\nu) = N_2 A_{21}$$

$$[N_1 B_{12} - N_2 B_{21}] u(\nu) = N_2 A_{21}$$

$$\Rightarrow u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{N_2 A_{21}}{N_2 B_{21} \left[\frac{N_1}{N_2} \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]}$$

$$\Rightarrow u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left[\frac{N_1}{N_2} \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \rightarrow (3)$$

According to Boltzmann's distribution law

$$N_1 = N_0 \exp\left(\frac{-E_1}{k_B T}\right) \quad N_2 = N_0 \exp\left(\frac{-E_2}{k_B T}\right)$$

$$\therefore \frac{N_1}{N_2} = \exp\left[\frac{-(E_1 - E_2)}{k_B T}\right] = \exp\left(\frac{+h\nu}{k_B T}\right) \quad [\because E_2 - E_1 = h\nu] \rightarrow (4)$$

Substituting equation (4) in equation (3) we get

$$u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left[\exp\left(\frac{h\nu}{k_B T}\right) \left(\frac{B_{12}}{B_{21}} \right) - 1 \right]} \rightarrow (5)$$

According to Planck's radiation law

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(\exp\left(\frac{h\nu}{k_B T}\right) - 1 \right)} \rightarrow (6)$$

Comparing equations (5) & (6)

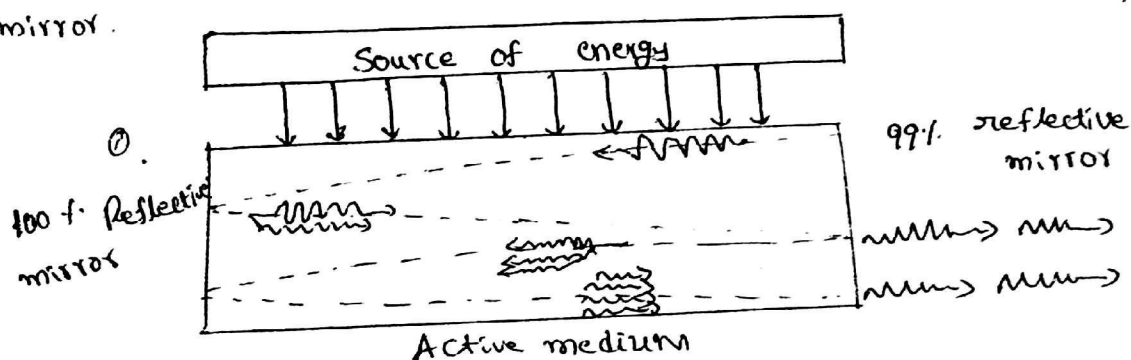
$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

$$\Rightarrow \frac{A_{21}}{B_{21}} \propto \nu^3 \quad \text{and} \quad B_{12} = B_{21}$$

The first relation shows that the ratio of Einstein's coefficients of A_{21} and B_{21} is proportional to cube of the frequency of incident photon. The second relation shows the rate of probability of induced emission and absorption are equal, when the system is in ~~an~~ equilibrium.

LASER SYSTEM:- It consists three parts.

- (i) Source of energy:- It supplies suitable form of energy to the active medium to achieve population inversion. i.e. it performs pumping process.
- (ii) Active medium:- It is a medium in which metastable state is present. In metastable state only the population inversion takes place. It can be a solid, liquid, gas or semiconductor diode junction.
- (iii) Optical Cavity:- It is an enclosure of active medium and essentially consists of two mirrors. One mirror is fully reflective and other one is partially transparent. Due to mirrors arrangement emitted laser takes back and forth reflections until it gains sufficient energy to come out. The output laser is emitted from partially transparent mirror.



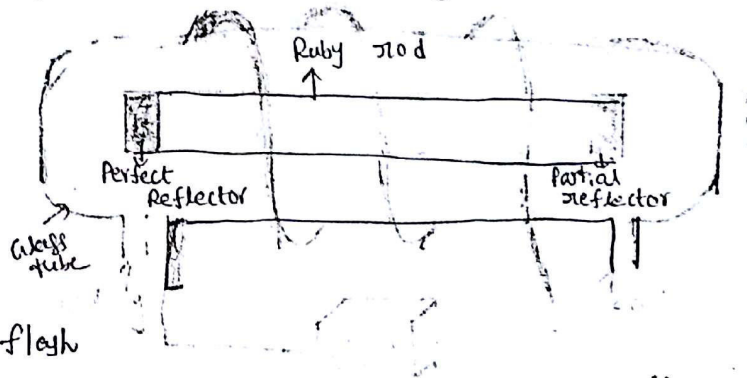
Ruby LASER:- Ruby laser is a solid state three level laser system demonstrated by Maiman in 1960. It produces pulsed laser which is useful for various industrial applications like surface hardening, hard facing, cladding of various industrial products. It is a high power laser, which has hundreds of mW. Each pulse will come out in duration of 10 nano seconds.

Source of energy: Xenon flash light

Active medium: Ruby Crystal rod

Optical Cavity: Arrangement of silver polished surface on either sides of the Ruby rod.

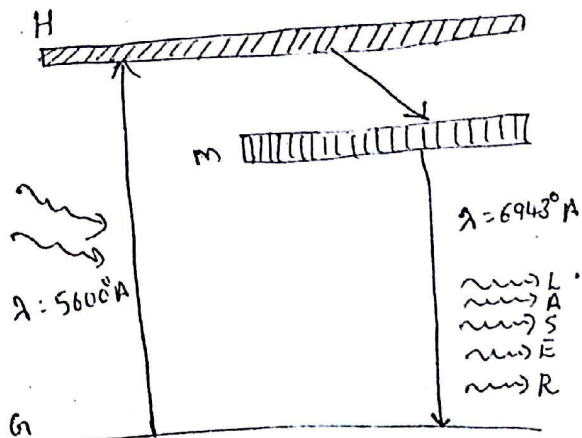
Construction:- Ruby laser is made up of cylindrical ruby crystal rod of composition Al_2O_3 and 0.05% of Cr_2O_3 , which length is few centimeters and diameter is 0.5 cm. It has partially silver polished surfaces on either sides through which laser is emitted. A Xenon flash tube is arranged around the ruby rod, which supplies green colour flash



light of wavelength $5600\text{\AA}$ to the active medium to achieve. Only a part of flashlight is used for the pumping the Cr^{+3} ions, while the rest heats up the apparatus. A cooling arrangement is provided to keep the experimental setup at normal temperature.

Working Principle:-

The Chromium atoms have three active energy levels, they are named ground state (G), meta-stable state (M) and Higher state (H). Due to the supply of Xenon flash light to the ruby rod, the Chromium atoms begin to excite from G state to excited states M & H.



Once Chromium atoms are excited to upper energy level H, they require two steps to return back to their ground state G. First step is from higher state H to metastable state M, which is shorter jump and energy emitted in this transition process to the crystal lattice as heat. This transition is called radiation less transition.

The Chromium atoms returned to M level can remain in this state for several milliseconds. The accumulation of excited atoms at M level increases the population at M level and then transition occurs from M to G level emitting out the photons randomly. Due to continuous working of flash lamp, the Chromium atoms are raised continuously to higher energy state and then to M level.

At a particular stage population of excited Chromium atoms will be reversed and more Chromium atoms at M than at G. At this position photons begin to interact with Chromium atoms at level M to a significant extent. This results in stimulated emission of other identical photons and a cascade begins.

The photons travelling parallel to the axes of the ruby rod are used for stimulation while the photons travelling in any direction other than this will pass out from the ruby rod. In the meantime, photons moving back and forth inside the ruby rod continue to build up until the intensity of radiation is greater enough to come out it bursts through the partial silver polished surface and it serves as output laser.

The output beam of wavelength is $6943 \text{ \AA}$ and frequency $7.32 \times 10^{14} \text{ Hz}$. This wavelength is in the red region of the visible spectrum.

Pumping Source In optical pumping, a light source such as a flash discharge tube is used. This method is adopted in solid state laser. In electric discharge method, the electric field causes ionization of the medium and raises it to the excited state. This technique is used in gas lasers. In semiconductor diode lasers, a direct conversion of electrical energy into light energy takes place.

ENGINEERING PHYSICS MODEL BITS

1. If a light ray travels a distance of s in a medium of refractive index μ , the equivalent distance in air medium is μs
2. The soap bubble appears multicoloured when exposed to **white light**
3. In order that a thin film of oil floating on the surface of water should show colours due to interference, the thickness of the oil film should be of the order **10000 Å**
4. Reflected light interference is **complimentary** to transmitted light interference
5. The path difference in case of a thin film of thickness t and refractive index μ when exposed to monochromatic light of wavelength λ is $2\mu t \cos r - (\lambda/2)$
6. The condition for constructive interference in case of a thin plane parallel film in reflected system is $2\mu t \cos r = (2n+1)\lambda/2$
7. The condition destructive interference in case of a thin plane parallel film in reflected system is $2\mu t \cos r = n\lambda$
8. When a thin film of oil or soap bubble is illuminated with white light, multiple colours appear. This is due to **interference**
9. If the film thickness is extremely small when compared to wavelength λ of light used to expose it, then the film appears **bright**
10. A parallel beam of light $\lambda = 5890 \text{ Å}$ is incident on a glass plate ($\mu = 1.5$) such that angle of refraction into plate is 60° . The smallest thickness of the plate which will make it appear dark by reflection would be **3927 Å**
11. In Newtons rings experiment the locus of points having constant thickness of air film forms **circle**
12. When the Newtons rings are viewed through a microscope, the central ring is seen dark because **phase difference of π due to phase change on reflection**
13. The air film in the Newtons rings apparatus is replaced by an oil film. The radius of the rings **decreases**
14. The convex lens in Newtons rings apparatus is replaced by an ordinary glass plate, then **shape the fringes is irregular**
15. In the Newtons rings apparatus the convex lens used should have **large** radius of curvature
16. If white light is used in the Newtons rings experiment, then the central spot will be
 - a) multi coloured
 - b) dark
17. If the glass plate of the Newtons rings apparatus is replaced by a plane mirror, then the fringes will **disappear**
18. In Newtons rings experiment the condition for bright fringes in case of reflected light is
 - a) $D_n \propto \sqrt{2n-1}$
19. In Newtons rings experiment the condition for dark fringes in case of reflected light is
 - d) $D_n \propto \sqrt{n}$
20. In Newtons rings experiment the diameter of 40th ring is 0.1 m with air film. When an oil film is formed the diameter of the same ring becomes 0.089 m. The refractive index of oil is **1.26**
21. The condition for maxima in the two slit diffraction is $(a+b) \sin \theta = n \lambda$
22. In a double slit diffraction, if the width of the slit is equal to the spacing between the slits, then **Even order interference maxima will be missing**
23. In a double slit diffraction, the fringe spacing on a screen 50cm away from the slits would be----if they are illuminated with blue light $\lambda = 480\text{nm}$, slit separation $b = 0.1\text{mm}$ and slit width $a = 0.020\text{mm}$ **2mm**
24. When white light is incident on a diffraction grating, the light diffracted more will be **Red**
25. Monochromatic light falling normally on a grating give rise to a diffracted second order beam at an angle of 30° . If the grating has 5000 lines/cm, the wavelength of light is
 - c) 500nm
26. Maximum number of orders possible with the grating is **Directly proportional to the grating element**
27. Points on a successive slits separated by a distance equal to the grating element are called **Corresponding points**
28. The combined width of a ruling and a slit is called **Grating element**
29. Number of orders visible would be ---- if the wavelength of light is 5000 Å and the number of lines per cm on the grating is 6655 Å
30. The condition for Diffraction minima is given by $a \sin \theta = p \lambda$
31. A grating is able to resolve two very close spectral lines of wavelength 5860 Å and 5896 Å in its first order

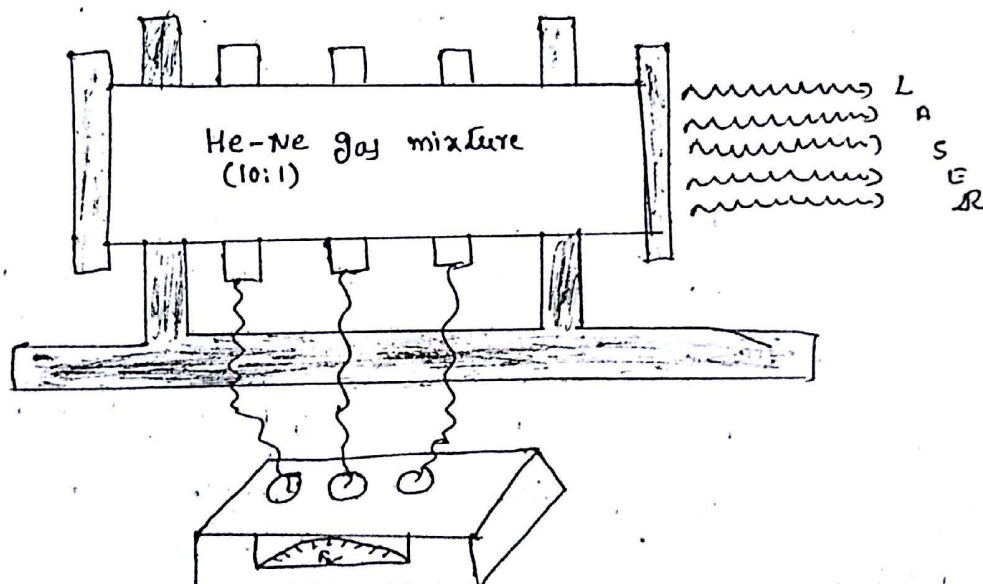
He-Ne LASER:- It is a gaseous laser system and is used to produce a continuous laser. This laser is highly directional, monochromatic, coherent and stable.

Source of energy: R.F oscillator

Active medium: He & Neon gas mixture

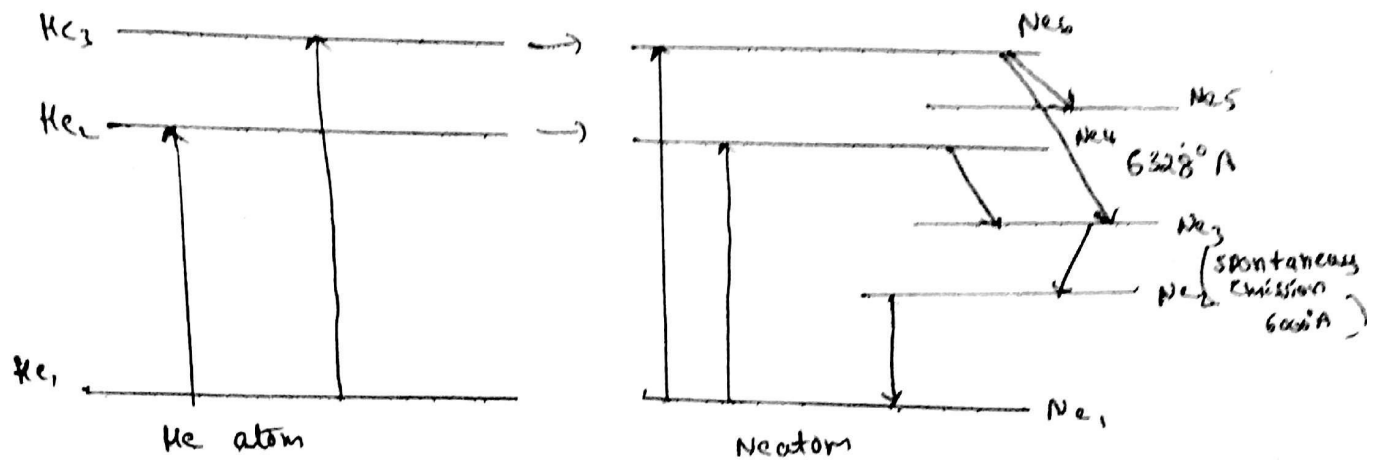
Optical cavity: Arrangement of reflector

Construction:- The He-Ne laser consists of a quartz tube with a diameter of about 1.5 cm and 80 cm length which has three electrodes to connect R.F oscillator. This tube is filled with a mixture of Neon under a pressure of 0.1 mm of Hg and Helium under a pressure of 1 mm of Hg. At one end of the tube a perfect reflector is arranged while on the other hand end a partial reflector. The active material is excited due to energy discharge through the gas mixture by means of radio frequency oscillator with a frequency of several MHz.



Working Principle:- In Helium (He) atoms three energy levels are present, they are named as He_1 , He_2 & He_3 whereas in Neon (Ne) atom six active energy levels, they are named as Ne_1 , Ne_2 , Ne_3 , Ne_4 , Ne_5 & Ne_6 . Ne_4 and He_2 have same energy and life time and similarly Ne_6 and He_3 . When the electric discharge is passed through the He-Ne gas mixture,

then the electrons are accelerated towards the positive electrode. During their passage they collide with Helium and Neon atoms, but only He atoms are suitable to excite to the upper states labeled He_2 and He_3 . These are metastable states in Helium atoms. Thus the atoms remain in these levels for a sufficiently long time. Now these atoms interact with Neon atoms, which are in the ground state. The interaction excites the Neon atoms to their metastables labelled Ne_4 and Ne_6 while the He atoms return to their ground state.



As the energy exchange continues between He and Ne atoms, the population of Neon atoms in the excited state Ne_4 and Ne_6 increases more and more. At a stage population inversion will be achieved in the metastable state Ne_4 and Ne_6 .

- Neon atoms de-excite from Ne_6 to Ne_5 . During this transition electromagnetic radiation of wavelength of 3390 Å will be emitted.
- Many other Neon atoms de-excite from Ne_6 to Ne_3 . During this transition electromagnetic radiation of wavelength 6328 Å will be emitted. ~~that is the~~

Electromagnetic fields

Introduction: Electromagnetic field theory is the study of fields produced by electric charges at rest or in motion.

- * Electrostatic fields are produced by static electric charges, while magnetostatic fields are produced by electric charges moving with uniform velocity also known as direct current.
- * Both electric and magnetic fields are basically independent.
- * If the charges (Electric) move with uniform velocity, these fields are time-invariant or stationary.
- * If the charges are in non-uniform velocity, we have the time varying electric and magnetic fields.
- * A time varying E.F produces a time varying M.F and in turn that a varying M.F produces a varying E.F.
- * The Combined field of such mutually induced E.F and M.F's is called the "Electromagnetic field".
- * The E.M field possesses energy, mass and momentum and can be converted into other

forms of matter and energy.

- * A time varying $E \cdot F$ not only produces a time varying $M \cdot F$ in the region but also in the surround region where the $E \cdot F$ takes place.
- * By a changing $M \cdot F$ produces a changing $E \cdot F$ in the surrounding region.
- * Thus the changing $E \cdot M$ induces a field in the surrounding space and consequently the $E \cdot M$ radiation is propagated outwards in space.

* As this action continues, a wave of $E \cdot M$ energy is generated and transmitted.

* Maxwell's Equations are the fundamental laws of $E \cdot M$ behaviour of matter and explain the properties of the $E \cdot M$ field.

* The Conservation of electric charge and its relation to electric current leads to the dynamic connection bet. electricity & magnetism and how two can be unified into one theory classical electrodynamics described by Maxwell's Eqs.

Del operator: $E \cdot M$ theory deals with the field vectors, $E \cdot F$ E and $M \cdot F$ H .

In general vector fields may be functions of space and time.

The differential operator can be treated in the same way as an ordinary quantity. The differential vector operator ∇ is called del or nabla is a vector space-function operator and is defined through the spatial derivatives with respect to the space coordinates.

In Cartesian Coordinates: $\nabla = \bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z}$

where $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along x, y, z axes resp.

Gradient: Multiplication of a vector with a scalar quantity, the product of which was found to be a vector quantity.

If $\bar{F} = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$ is a scalar quantity and is multiplied by vector ∇ then

$$\nabla F = \bar{i} \frac{\partial F}{\partial x} + \bar{j} \frac{\partial F}{\partial y} + \bar{k} \frac{\partial F}{\partial z}$$

which is a vector quantity. This (vector $\times$ scalar = vector) operation is called gradient abbreviated as grad.

$$\text{Grad } F = \bar{i} \frac{\partial F}{\partial x} + \bar{j} \frac{\partial F}{\partial y} + \bar{k} \frac{\partial F}{\partial z}$$

The gradient of any scalar function is defined as a vector whose magnitude is equal to the max. rate of change of that function w.r. to space variables and has the direction of that change.

Second product of multiplication

is the scalar of dot product.

If F_x, F_y, F_z be the components of a vector F along the coordinate axes, so $F = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$ is a vector function multiplied by another vector ∇ , their dot or scalar product is given by

$$\nabla \cdot F = \left(\bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z} \right) \cdot (F_x \bar{i} + F_y \bar{j} + F_z \bar{k})$$

$$\nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Curl: Multiplication is cross or vector product. If a vector $F = F_x \bar{i} + F_y \bar{j} + F_z \bar{k}$ is multiplied by another vector ∇ , then

$$\begin{aligned} \nabla \times F &= \left(\bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z} \right) \times (F_x \bar{i} + F_y \bar{j} + F_z \bar{k}) \\ &= \bar{i} \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + \bar{j} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \\ &\quad + \bar{k} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \end{aligned}$$

$$\therefore \nabla \times F = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

which is a vector quantity and is called Curl of a vector function.

Line integral: The integral of a point function along a curve is called line integral. If F is a vector field the line integral of F is: $\int_a^b F \cdot dl$

Surface integral: Let S be any surface divided into infinitesimal elements each of which may be considered as a vector $ds = \vec{n} ds$ where $\vec{n}$ is a unit vector normal to the surface of the element ds . If $\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$ be a vector field function then the surface integral may be written as: $\oint_S \vec{F} \cdot d\vec{s}$.

Volume integral: These are also called space integrals. Let $dV = dx dy dz$ denote the element of volume in any region. If $\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$ be a vector field function then the volume integral may be written as: $\oint_V F dV$.

Gauss Theorem: This is an imp. theorem associated with the names of Green and Gauss and it is popularly known as "Divergence theorem". It permits us to express certain integrals by means of surface integrals. The divergence theorem relates to a closed volume V in space and surface S that bounds it.

of the divergence of a vector field $\vec{F}$ over a volume V is equal to the surface integral of the normal component of the vector over the surface bounding the volume. Mathematically

$$\oint_V \text{div } \vec{F} dV = \oint_S \vec{F} \cdot \vec{n} dS$$

Stokes's Theorem: This theorem permits transformation of certain integral calculated over the surface into line integral. It can be stated as the line integral of the tangential component of a vector $\vec{F}$ around a closed path L is equal to the surface integral of the normal component of $\vec{F}$ over the surface S enclosed by the path L . Mathematically $\oint_L \vec{F} \cdot d\vec{l} = \oint_S (\nabla \times \vec{F}) \cdot d\vec{S}$.

Fundamental Laws of Electromagnetism:

Gauss law of Electrostatics: Gauss law is the converse of Coulomb's law. From Coulomb's law we can calculate E.F intensity for a given charge while Gauss law enables us to determine the charge by knowing either E.F intensity (E) or electric flux density (D).

Introduction:- Maxwell's equation form foundation of electro magnetic theory. Maxwell's expressed four fundamental equations known as "Maxwell's equation". The first two are known as "steady state equation" while the last two are called "time varying equation".

Light and other electromagnetic waves can be described by two vectors one the amplitude of the electric field strength E and the other the amplitude of the mag magnetic field strength H . Both these vectors are functions of space and time; they oscillate at right angle to each other as well as to the direction of propagation.

It states that the electric flux ϕ through a closed surface is equal to $1/\epsilon$ times the net charge q enclosed by the surface.

(or)

It states that the surface integral of the normal component of the electric field E over any closed surface equals $1/\epsilon$ times the net charge within that volume.

$$\therefore \phi = \oint E \cdot ds = \frac{q}{\epsilon} \rightarrow (1) \quad (\because \epsilon \text{ permittivity of dielectric medium})$$

This is the integral form of Gauss law of electrostatics.

Using divergence operation it can be $\phi = \nabla \cdot E$

If there are many charges and if these charges are distributed throughout a volume V and the charge distribution can be represented

then eqn (1) can be written as $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon} \int_V \rho \, dv = \frac{\rho}{\epsilon}$

$$\therefore \text{Hence } \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \Rightarrow \boxed{\nabla \cdot \epsilon \mathbf{E} = \rho} \rightarrow (2)$$

This is the differential form of Gauss' law.

Deduction of Maxwell's first equation:-

An external electric field causes polarization of electric charges in dielectric medium. The magnitude of the polarization can be described by the electric displacement, $\mathbf{D}$.

The displacement $\mathbf{D}$ can be related to field strength $\mathbf{E}$ by

$$\mathbf{D} = \epsilon \mathbf{E} \rightarrow (3)$$

Sub (3) eqn in eqn (2) we get $\boxed{\nabla \cdot \mathbf{D} = \rho} \rightarrow (4)$

Equation (4) is called Maxwell's first equation in differential form.

If the charge density (ρ) in the region is zero (i.e. in free space)

$$\nabla \cdot \mathbf{D} = 0$$

Let us consider Gauss' law $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon} \int_V \rho \, dv$

$$\text{i.e. } \oint_S \epsilon \mathbf{E} \cdot d\mathbf{S} = \int_V \rho \, dv$$

But $\epsilon \mathbf{E} = \mathbf{D}$ from eqn (3) and $\rho = \nabla \cdot \mathbf{D}$ from eqn (4)

$$\therefore \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{D} \, dv$$

This Gauss' divergence theorem relates the surface integral of a vector function to the volume integral of the divergence of the same vector function. Substituting for $\nabla \cdot \mathbf{D}$ from equation (4),

This is Maxwell's first equation in integral form.

In free space $\rho = 0 \quad \therefore \oint_S \mathbf{B} \cdot d\mathbf{s} = 0$

min min ~ min

Let us consider a magnet inside a closed surface/volume, the number of lines of force of force leaving the closed surface will be equal to the number line of force entering the surface i.e. the net outward bound magnetic flux from any closed sub surface is zero.

i.e. $\oint \mathbf{B} \cdot d\mathbf{s} = 0 \rightarrow (1)$

Deduction of Maxwell's Second equation:-

By Gauss' divergence theorem $\oint_S \mathbf{B} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{B} dV \rightarrow (2)$

where 'S' is the surface bounding volume V.

From eqn's (1) & (2) $\int_V (\nabla \cdot \mathbf{B}) dV = 0$ (or) $\int_S \mathbf{B} \cdot d\mathbf{s} = 0 \rightarrow (3)$

eqn (3) is known as Maxwell's second equation in integral form.

From eqn (3) $\int_V (\nabla \cdot \mathbf{B}) dV = 0$

This equation holds good for any arbitrary volume.

$\therefore \nabla \cdot \mathbf{B} = 0$

This is Maxwell's second equation in differential form.

Electromagnetic Induction:-

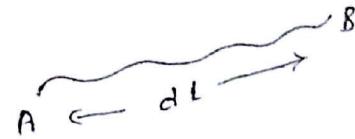
A bar magnet thrust through a loop of wire causes an electromotive force (e.m.f) to occur in the wire. The magnitude of this force is proportional to the rate of change of the magnetic flux ϕ

$$\therefore \text{e.m.f} = -\frac{d\phi}{dt} \text{---} \rightarrow (1)$$

eqn (1) is Faraday's law of induced electricity.

Let us consider work done on a charge.

moving it from a point A to a point B, along a fractional displacement dL , then



$$W = \int_A^B E \cdot dL$$

which is a line integral. If the work is done along a closed path,

$$\text{e.m.f} = \oint_C E \cdot dL \text{---} \rightarrow (2)$$

Let us consider the magnetic flux ϕ , of the induction B through an element dS , $\phi = \int_S B \cdot dS \text{---} \rightarrow (3)$

Sub 3rd eqn in 1st eqn we get

$$\text{e.m.f} = -\frac{d\phi}{dt} = -\frac{d}{dt} \left[\int_S B \cdot dS \right]$$

$$\therefore \text{e.m.f} = - \int_S \frac{dB}{dt} \cdot dS \text{---} \rightarrow (4)$$

Equate the eqn's (2) & (4) $\therefore \boxed{\oint_C E \cdot dL = - \int_S \frac{dB}{dt} \cdot dS} \rightarrow (5)$

eqn (5) is known as Maxwell's third equation in integral form.

∴ From eqn's (5) & (6)

$$\nabla \times E = -\frac{dB}{dt} \rightarrow (7)$$

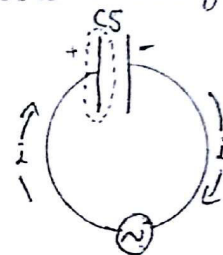
This is Maxwell's third equation in differential form.

From eqn (7) we can say that whenever a magnetic field changes with time, it generates an electric field.

Ampere's Circuit Law:-

According to Ampere's law, steady current carrying conductor generates magnetic field around it.

$$\oint B \cdot dl = \mu_0 i$$



Displacement Current:-

Let us consider a capacitor without dielectric. As charge flows in, it accumulates on one of the plates until the capacitor is fully charged to the value of the source. Let us consider a closed surface CS, surrounding one of the plates. While the capacitor is being charged, current i goes in but nothing comes out. For reasons of continuity, Maxwell thought that as much real current goes in, as much displacement current must go out across the closed surface.

Displacement current exists whenever the electric field changes. As soon as E becomes constant it ceases to exist. The displacement current density J , varies with E as a function of time, $J = \epsilon_0 \frac{\partial E}{\partial t}$ or since in free space $\epsilon_0 E = D$

$$\therefore J = \frac{\partial D}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t}$$

If the two plates of the capacitor are separated by a dielectric.

the electric field causes the dielectric to get polarized. Although the polarization charges never break away from the dielectric, to some extent, it is like a current. Hence the polarization contribution $\frac{\partial P}{\partial t}$ has to be added to the total displacement current density.

$$\therefore \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t}$$

Deduction of Maxwell's fourth equation:-

Ampere's Circuit law states $\oint B \cdot d\ell = \mu_0 i$

Replacing B by $\mu_0 H$ $\oint_L H \cdot dL = i$

But $i = \int_S J \cdot ds$

Hence $\oint_L H \cdot dL = \int_S J \cdot ds$

Accounting for displacement current density also

$$\oint_L H \cdot dL = \int_S J \cdot ds + \int_S \frac{\partial D}{\partial t} \cdot ds$$

This is Maxwell's fourth equation in integral form.

For steady current the last term vanishes.

According to Stoke's theorem line integral may be transformed into a surface integral. $\oint_L H \cdot dL = \int_S (\nabla \times H) \cdot ds$

$$\text{Hence } \int_S (\nabla \times H) \cdot ds = \int_S J \cdot ds + \int_S \frac{\partial D}{\partial t} \cdot ds \quad (\text{or})$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

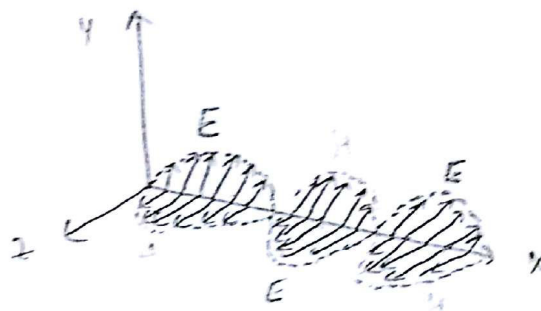
This is Maxwell's fourth equation in differential form.

Varying electric field induces Varying magnetic field and vice versa

- (a) The electric and magnetic fields are perpendicular to each other and both are mutually perpendicular to the direction of propagation of the wave.

The speed of electro magnetic wave, the electric and magnetic fields oscillate in phase with each other

- (b) The speed of electromagnetic wave is given by $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ which is nothing but the speed of light



Light is electromagnetic wave.

Ques.

1. Deduce Maxwell's first and second equations from Gauss theorem in electrostatics and Gauss theorem in magnetostatics respectively.
2. Deduce Maxwell's third and fourth equations from Faraday's law of electro magnetic induction and Ampere's circuit law respectively.
3. Write Maxwell's equations in both differential form and integral form. Discuss the significance of Maxwell's equations.

Maxwell's Electromagnetic Equations

In 1864, James Clark Maxwell brought together various laws of electrostatics and magnetostatics. Maxwell gave four equations by which electromagnetic field can be described. These set of four equations are known as Maxwell's equations. These equations describe how electric and magnetic fields propagate, interact and how they are influenced by objects.

Differential form

$$1. \nabla \cdot \mathbf{D} = \rho \quad \text{or} \quad \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$2. \nabla \cdot \mathbf{B} = 0 \quad \text{or} \quad \nabla \cdot \mathbf{H} = 0$$

$$3. \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{or} \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

$$4. \nabla \times \mathbf{H} = \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \quad \text{or} \quad \nabla \times \mathbf{H} = \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Propagation of Electromagnetic waves in Dielectric medium

Dielectric medium is one which offers infinite resistance to the current and hence its conductivity is zero, i.e., the current density $\mathbf{J} = 0$. In homogeneous isotropic medium there is no free distribution of charge, i.e., the charge density $\rho = 0$. Also $\sigma = 0$, $\mathbf{D} = \epsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$, where ϵ is the permittivity and μ is the permeability of the medium.

$\therefore$ Maxwell's equations for a dielectric medium become

$$\nabla \cdot \mathbf{E} = 0 \quad \rightarrow (1)$$

$$\nabla \cdot \mathbf{H} = 0 \quad \rightarrow (2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \rightarrow (3)$$

$$\nabla \times \mathbf{H} = \epsilon \mu \frac{\partial \mathbf{E}}{\partial t} \quad \rightarrow (4)$$

We can obtain the equation of propagation of a wave in

Integral form

$$(1) \oint \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\epsilon_0}$$

$$(2) \oint \mathbf{H} \cdot d\mathbf{s} = 0$$

$$(3) \oint \mathbf{E} \cdot d\mathbf{l} = -\mu_0 \frac{d}{dt} \int \mathbf{H} \cdot d\mathbf{s}$$

$$(4) \oint \mathbf{H} \cdot d\mathbf{l} = \int \mathbf{J} \cdot d\mathbf{s} + \epsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{s}$$

$$\nabla \times \nabla \times B = \nabla \times \left(\mu \epsilon \frac{\partial E}{\partial t} \right)$$

$$= \mu \epsilon \left(\nabla \times \frac{\partial E}{\partial t} \right)$$

$$= \mu \epsilon \frac{\partial (\nabla \times E)}{\partial t} = \mu \epsilon \frac{\partial \left(-\frac{\partial B}{\partial t} \right)}{\partial t} \quad \left[\because \nabla \times E = -\frac{\partial B}{\partial t} \right]$$

Thus $\nabla \times \nabla \times B = \mu \epsilon \left(-\frac{\partial^2 B}{\partial t^2} \right) \rightarrow (5)$

We know that $\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B$

$$\therefore \nabla \times \nabla \times B = -\nabla^2 B \quad \left[\because (\nabla \cdot B) = 0 \right]$$

$$\therefore -\nabla^2 B = \mu \epsilon \left(-\frac{\partial^2 B}{\partial t^2} \right)$$

$$\therefore \nabla^2 B = \mu \epsilon \frac{\partial^2 B}{\partial t^2} \rightarrow (6)$$

Similarly taking curl of eqn (3) we get

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2} \rightarrow (7)$$

Eqn's (6) & (7) represents the relation between the space and time variation of magnetic field and electric field E.

$\therefore$ The general form of differential equation of wave motion is represented by $\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \rightarrow (8)$

where v is the velocity of wave & ψ is amplitude. Compare eqn's

(6), (7) & (8) then we get $\mu \epsilon = \frac{1}{v^2} \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu \epsilon}}} \rightarrow (9)$

This is the expression for electromagnetic wave.

The velocity of EM wave in free space, is obtained by substituting μ_0 and ϵ_0 in the place of μ and ϵ respectively in the eqn.

$$\therefore v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 \text{ m/s.}$$

Thus the EM wave propagates with the velocity equal to the velocity of light

fields the conduction current density is due to the flux density D changing with time. Physically displacement current is not a current in the sense that there is no flow of physical quantity like charge.

This is an apparent current representing the rate at which flow of charge takes from electrode to electrode in the external circuit is called displacement current.

Conduction Current density J_c ; Displacement Current density J_D

$$\begin{aligned}\text{Ampere's law } \nabla \times B &= \mu_0 \left(J + \frac{\partial D}{\partial t} \right) \\ &= \mu_0 (J_c + J_D)\end{aligned}$$

The concept of displacement current is found very useful in explaining the magnetic fields in free space due to time varying electric fields.

Expression for displacement current in terms of displacement

$$\text{current density } I_D = \int J_D \, ds = \int \frac{\partial D}{\partial t} \, ds$$

Planck's Quantum theory of black body radiation:-

A body that absorbs all the wavelengths at low temperature and emits all wavelengths of radiation at high temperature is known as "black body".

The radiation emitted by a black body varies with temp. The intensity of radiation corresponding to different wavelengths is measured at different temperatures and plotted as shown. The area under the plot indicates the total radiation (E), the power emitted per unit area.

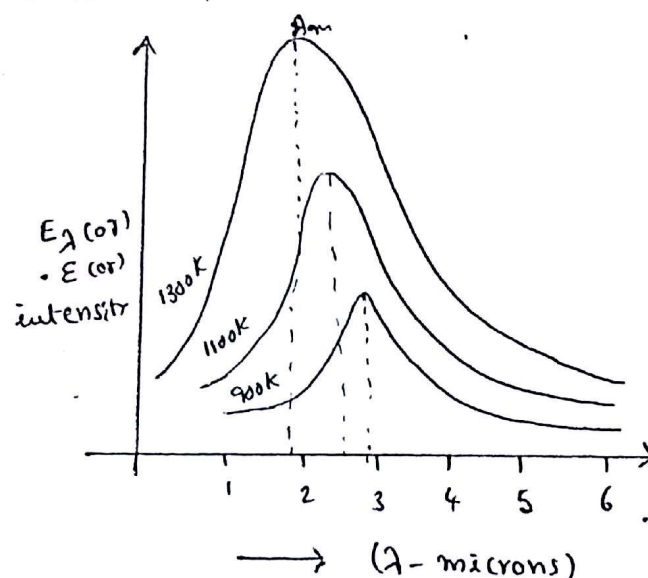


Fig: Energy Spectrum of black body

From this study it is observed that

- At a given temperature the radiation is not uniformly distributed in random radiation spectrum.
- At a given temperature, the intensity of radiation is maximum at particular wavelength λ_m .
- With increase in temperature λ_m decreases.

Stefan - Boltzmann's law:- According to this law, the area is directly proportional to fourth power of the temperature of the body.

$$E \propto T^4$$

Louis de-Broglie in 1924 extended the wave particle paradoxism of optics all the fundamental entities of physics Such as electrons, protons, neutrons, atoms and molecules etc. [de-Broglie put a bold suggestion that the correspondence between wave and particle should not be confined only to electromagnetic radiation, but it should also be valid for material particles, i.e. like radiation, matter also has a dual characteristic. In his thesis he wrote that there was an intimate connection between waves and corpuscles not only in the case of radiation but also in the case of matter.

A moving particle has always got a wave associated with it and the particle is controlled by the wave in a manner similar to that in which a photon is controlled by waves. His suggestion was based on the fact that nature loves symmetry, if radiation like light can act like wave some time and like a particle at other times, then the material particles should act as wave at some other time.

According to de-Broglie's hypothesis, a moving particle is associated with a wave which is known as de-Broglie wave. The wavelength of the matter wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p},$$

where m is the mass of the material particle, v its velocity and p is its momentum.

Expression for de-Broglie wavelength:-

Considering the Planck's theory of radiation the energy of a photon is given by $E = h\nu = \frac{hc}{\lambda} \rightarrow (1)$

where c is the velocity of light in vacuum and λ is its wavelength.

According to Einstein mass-energy relation $E = mc^2 \rightarrow (2)$

from eq (1) & (2) we get $mc^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{h}{mc} \rightarrow (3)$

where $mc = p$ momentum associated with photon.

Consider the case of a material particle of mass m moving with a velocity v . i.e. momentum mv , then the wavelength associated with this particle is given by $\lambda = \frac{h}{mv} = \frac{h}{p} \rightarrow (4)$

If E is the K.E. of the material particle, then

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \frac{mv^2}{1} = \frac{p^2}{2m} \quad (\because p=mv)$$

$$\Rightarrow p = \sqrt{2mE}$$

$$\therefore \text{de-Broglie wave length } \lambda = \frac{h}{\sqrt{2mE}} \rightarrow (5)$$

(b) When a charged particle carrying a charge q is accelerated by a potential difference V volts, then its K.E., E is given by

$$E = qV$$

$\therefore$ de-Broglie wavelength associated with this particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}} \rightarrow (6)$$

(c) When a material particle is in thermal equilibrium at a temperature T , then $E = \frac{3}{2} kT$

where $k = \text{Boltzmann's Constant} = 1.38 \times 10^{-23} \text{ J/K}$

$\therefore$ de-Broglie wavelength of a material particle at temperature T is given by

$$\lambda = \frac{h}{\sqrt{2m(\frac{3}{2}kT)}} = \frac{h}{\sqrt{3mkT}} \rightarrow (7)$$

(d) de-Broglie wavelength associated with electrons:

Consider the case of an electron of rest mass m_0 and charge e which is accelerated by a potential V volt from rest to velocity v , then

$$\frac{1}{2}m_0v^2 = eV \quad (\text{or}) \quad v = \sqrt{\frac{2eV}{m_0}}$$

$$\therefore \lambda = \frac{h}{m_0v} = \frac{h\sqrt{m_0}}{m_0\sqrt{2eV}} = \frac{h}{\sqrt{2eVm_0}}$$

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-19} \times V \times 9.1 \times 10^{-31}}} = \frac{12.26}{\sqrt{V}} \text{ \AA}$$

then $\lambda = \frac{12.26}{\sqrt{100}} = 1.226 \text{ \AA}$

properties of matter waves:-

1. Lighter is the particle, greater is the wavelength associated with it.
2. Smaller is the velocity of the particle, greater is the wavelength associated with it.
3. when $v=0$ then $\lambda=\infty$, wave becomes indeterminate & if $v=\infty$ then $\lambda=0$. This shows that matter waves are generated by the motion of particles.

4. The matter waves ~~can~~ travel faster than light.

Proof: we know that $E=h\nu$ and $E=mc^2$
 $\therefore h\nu = mc^2 \Rightarrow \nu = \frac{mc^2}{h}$

The wave velocity (ω) is given by

$$\omega = \nu \lambda = \frac{mc^2}{h} \cdot \lambda = \frac{mc^2}{h} \cdot \frac{h}{mv} = \frac{c^2}{v}$$

As the particle velocity v cannot exceed velocity of light c , ω is greater than velocity of light. We can understand result by considering the wave velocity also known as phase velocity & the group velocity.

Group velocity:- A group of waves, each wave having the wavelength given above is associated with particle. This group as a whole must travel with the particle velocity v . Hence, the group velocity of the matter waves $v_{gr} = v$

The particle is guided by the group of matter waves, these waves are sometimes called pilot waves.

Phase velocity:- Each wave of the group of matter waves travel with a velocity known as the phase velocity of the wave.

$$\therefore v_{ph} = \frac{\omega}{k} = \frac{2\pi\nu}{2\pi/\lambda} = \lambda\nu = \frac{h\nu}{mv} \rightarrow (1)$$

$$\therefore E = h\nu = mc^2 \quad \therefore v_{ph} = \frac{c^2}{v}$$

Schrodinger's equation is a differential equation for de-Broglie waves associated with a particle and thus describes the motion of the particle. The variable quantity associated with the moving particle is a mathematical function ψ called the wave function ψ . For steady state systems, ψ is a function of spatial co-ordinates only.

A one dimensional equation for the steady waves associated with a particle may be written as

$$\psi = A \sin \frac{2\pi x}{\lambda} \rightarrow (1)$$

where λ is the wavelength, the function ψ represents the amplitude of the wave for any given value of x . A is the maximum amplitude.

Differentiate the above ^{eqn} (1) with respect to x

$$\frac{d\psi}{dx} = A \frac{2\pi}{\lambda} \cos \frac{2\pi x}{\lambda}$$

once again differentiate above eqn with respect to x

$$\frac{d^2\psi}{dx^2} = -A \frac{4\pi^2}{\lambda^2} \sin \frac{2\pi x}{\lambda} = -\frac{4\pi^2}{\lambda^2} \psi \rightarrow (2)$$

the

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2} \psi = 0 \rightarrow (2)$$

The wave length associated with a particle of mass m moving with velocity v is given by $\lambda = \frac{h}{mv} \Rightarrow \frac{1}{\lambda^2} = \frac{h^2}{m^2 v^2} \rightarrow (3)$

3rd can substitute in eqn 2nd

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2 m^2 v^2}{h^2} \psi = 0 \rightarrow (4)$$

The total Energy E of the particle is the sum of the kinetic energy K and potential Energy V

$$\therefore E = K + V \quad \& \quad K = \frac{1}{2} m v^2$$

$$\therefore E - V = \frac{1}{2} m v^2 \Rightarrow 2m(E - V) = m^2 v^2 \rightarrow (5)$$

Substitute the eqn 5 in eqn (4)

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m(E - V)}{h^2} \psi = 0$$

For the same particle moving in 3-D space, the equation becomes

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

$$\boxed{\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0}$$

$$\left[\text{Here } \nabla^2 \text{ is the Laplacian operator} \right]$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

For a free particle $V=0$.

The Schrodinger's equation for a free particle is

$$\boxed{\nabla^2 \psi + \frac{8\pi^2 m E}{h^2} \psi = 0}$$

* PHYSICAL SIGNIFICANCE OF THE WAVE FUNCTION:-

MA A satisfactory interpretation of the wave function ψ associated with a moving particle was given by Born in 1926. He postulated that the square of the magnitude of the wave function $|\psi|^2$ evaluated at a particular point represents the probability of finding the particle at that point. $|\psi|^2$ is called the probability density and ψ is the probability amplitude.

The probability P of finding the particle within any element of volume $dx dy dz$ is given by $P = \psi^* \psi dx dy dz$ (ψ is complex)

Since the probability of finding the particle somewhere is unity, ψ is such function that satisfies the condition

$$\iiint |\psi|^2 dx dy dz = 1$$

If ψ satisfying the above condition then ψ is called normalized function.

Limitations on wave function ψ :-

There are certain limitations to take ψ as a solution for the Schrodinger's wave equation they are

- (i) ψ must be single valued i.e. for each set of values of x, y and z , ψ must have only one value.
- (ii) ψ must be continuous in all regions except where potential energy is infinite.
- (iii) ψ is analytical i.e., it possess continuous first order derivative
- (iv) ψ vanishes at the boundaries

Particle in One dimensional Box:-

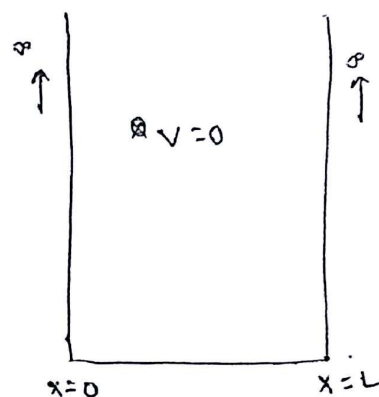
Consider an electron of mass m in an infinitely deep 1-D potential box with width of L units in which potential is constant and zero. The motion of the electron is constrained by the walls of the box.

The potential energy V of the particle is zero inside the box, but rises to infinity on the out side

$\therefore$ Boundary conditions

$$V(x) = 0 \quad 0 < x < L$$

$$= \infty \quad x \leq 0 \text{ \& } x \geq L$$



The motion of the electron in one dimensional box can be described by the Schrodinger's wave equation

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0 \quad \rightarrow (1)$$

Inside the box, the potential $V=0$.

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0 \Rightarrow \frac{d^2\psi}{dx^2} + K^2 \psi = 0 \rightarrow (2) \quad \left[\because K^2 = \frac{2mE}{\hbar^2} \right] \rightarrow A$$

$\therefore$ Solution of above equation can be written as

$$\psi(x) = A \sin kx + B \cos kx \quad \rightarrow (3)$$

where A, B and K are unknown quantities. And to be calculated by the boundary conditions.

Applying the first boundary condition in eqn (3)

$$\therefore \text{at } x=0 \quad 0 = A \sin 0 + B \cos 0 \Rightarrow B=0. \rightarrow (4)$$

when $x=0$ then the wave function $\psi=0$ i.e. $(\psi)^2=0$
when $x=L$ $\psi=0$ i.e. $(\psi)^2=0$

4th eqn Substitute in 3rd eqn
 $\psi(x) = A \sin kx \rightarrow (5)$

Applying 2nd boundary condition to 5th eqn

$x=L$
 $0 = A \sin kL$

$A \neq 0$ why because particle is present in the box and it cannot come out from the box, because the potential is infinite outside the box.

$\therefore \sin kL = 0 \Rightarrow kL = n\pi$

$\therefore k = \frac{n\pi}{L} \rightarrow (6)$ where $n=1,2,3, \dots$

6th eqn Substitute in 5th then we can get

$\psi(x) = A \sin \frac{n\pi}{L} x \rightarrow (7)$

To calculate the unknown constant A , it is necessary to consider the normalization condition i.e. the probability to find an electron in one-dimensional box is maximum and equals to unity

i.e. $\int_0^L |\psi(x)|^2 dx = 1 \rightarrow (8)$

7th eqn Substitute in 8th eqn

$\int_0^L A^2 \sin^2 \frac{n\pi x}{L} dx = 1$

$\Rightarrow A^2 \int_0^L \left(\frac{1 - \cos \frac{2n\pi x}{L}}{2} \right) dx = 1$ $\left[\because \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \right]$

$\Rightarrow \frac{A^2}{2} \left[x - \frac{\sin \frac{2n\pi x}{L}}{\frac{2n\pi}{L}} \right]_0^L = 1$ $\therefore \left[\int \cos nx dx = \frac{\sin nx}{n} \right]$

$\Rightarrow \frac{A^2}{2} [(L-0) - 0 + 0] = 1$

$\frac{A^2 L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}} \rightarrow (9)$

9th eqn Substitute in eqn 7 $\psi(x) = \sqrt{\frac{2}{L}} \sin \left(\frac{n\pi x}{L} \right)$

in 3-D $\psi(x,y,z) = \sqrt{\frac{8}{L^3}} \sin \left(\frac{n\pi x}{L} \right) \sin \left(\frac{m\pi y}{L} \right) \sin \left(\frac{p\pi z}{L} \right)$

From equations A & G

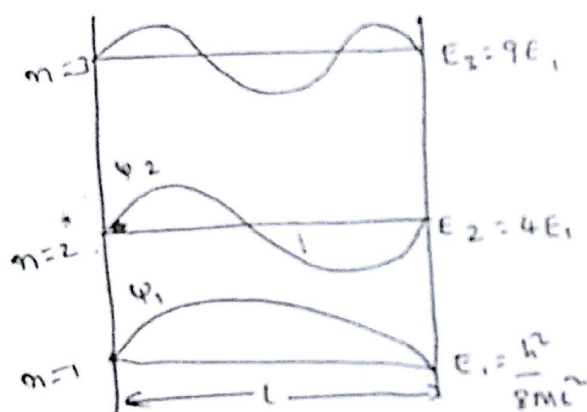
$$\frac{2mE}{h^2} = \left(\frac{n\pi}{L}\right)^2 = \frac{n^2 \pi^2}{L^2}$$

$$\Rightarrow \frac{2mE}{h^2} \hat{=} \frac{n^2 \pi^2}{L^2}$$

$$\Rightarrow \boxed{E_n = \frac{n^2 h^2}{8mL^2}} \rightarrow (k) \text{ in 3-D } E = \frac{(n_1^2 + n_2^2 + n_3^2) h^2}{8mL^2}$$

where $n=1, 2, 3, \dots$ n is called quantum number.

Special cases. The wave functions and the corresponding energy levels of the particles in a potential well



Case i) For $n=1$ the most probability of the particle is at $x = L/2$

Case ii) For $n=2$ the most probability of the particle is at $x = L/4$ & $3L/4$

Case iii) For $n=3$ the most probability of the particle is at $x = L/6, 3L/6, 5L/6$

SCHRODINGER'S Time dependent wave equation:-

It is the differential equation of the de-Broglie waves associated with particles and describes the motion of particles.

Let us assume that a particle having wave function ψ associated with it is moving freely in positive x direction, therefore

$$\psi = Ae^{-i\omega(t - \frac{x}{v})}$$

Replacing ω in above equation by $2\pi\nu$ and $v = \frac{\omega}{k}$ we get

$$\psi = Ae^{-2\pi i(\nu t - \frac{x}{\lambda})}$$

$$\therefore \text{Since } E = h\nu = 2\pi\hbar\nu ; \lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$

$\therefore$ Thus for a free particle wave function becomes

Above eqn is mathematical description of the wave equivalent of a free particle of total energy E and momentum p moving in x direction

Partially differentiating above equation w.r.t t we get

$$\frac{\partial \psi}{\partial t} = -\left(\frac{iE}{\hbar}\right) A e^{-\frac{i}{\hbar}(Et - Px)} \Rightarrow \frac{\partial \psi}{\partial t} = -\left(\frac{iE}{\hbar}\right) \psi \rightarrow (2)$$

multiply and divide by i and substitute $i^2 = -1$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \rightarrow (3)$$

Similarly partially differentiating equation (1) w.r.t x we get

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} P A e^{-\frac{i}{\hbar}(Et - Px)} = \frac{iP}{\hbar} \psi \Rightarrow P\psi = \hbar \frac{\partial \psi}{\partial x} \rightarrow (4)$$

$\therefore$ Total energy of the particle is

$$E = K + P.E = \left(\frac{p^2}{2m}\right) + V \quad E = K + V$$

$$\Rightarrow E\psi = \left(\frac{p^2}{2m}\right)\psi + V\psi \rightarrow (5) \quad E\psi = K\psi + V\psi$$

Substitute the equation (3) & (4) in eqn (5) we get

$$i\hbar \frac{\partial \psi}{\partial t} = -\hbar^2 \left(\frac{\partial^2 \psi}{\partial x^2}\right) \frac{1}{2m} + V\psi$$

This is Schrodinger's time dependent wave equation in one dimension.

In 3D

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}\right) + V\psi$$

Operator Version of Schrodinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi$$

$$\hat{H}\psi = E\psi$$

where $\hat{H}$ is the Hamiltonian operator

Introduction:- The valence electrons of isolated metallic atoms are bound to the atoms, whereas in a metallic crystal they are not bound to any particular atom; they freely move in the space between the atoms. So they are called free electrons. These free electrons participate in electrical conduction; hence they are called conduction electrons.

Physical Properties of metal:-

- (1) metals obey Ohm's law i.e., the current through a metal is proportional to the applied field.
- (2) Due to the existence of free electrons, metals possess high electrical and thermal conductivities.
- (3) The resistivity is proportional to the fifth power of absolute temp i.e. $\rho \propto T^5$ at low temperatures.
At high temperature resistivity is proportional to absolute Temp.
 $\therefore \rho \propto T$
- (4) In metals, the ratio of thermal conductivity to electrical conductivity is proportional to absolute temperature. This is known as Wiedmann-Franz law.
- (5) Near absolute zero of temperature, the resistivity of some metals drops to zero; hence they show superconductivity. The resistance of a metal increases with temperature.

The electron theory of solids aim to explain the structures and properties of solids through their electronic structure. The theory has been developed in three main stages.

- (1) The Classical Free electron theory
- (2) The Quantum free electron theory
- (3) The zone theory.

(Drude - Lorentz theory of metals):

Drude and Lorentz developed this theory in 1900. According to this theory, the metals containing free electrons obey the law of classical mechanics.

Postulate:- (or assumptions).

- (1) The valence electrons of metallic atoms, ^{are} free to move in the space between ions from one place to another place within the metallic specimen similar to gaseous molecules. So that these electrons are called free electron gas. These electrons also participate in electrical conduction; hence they are called conduction electrons.
- (2) There is no interaction between these conduction electrons.
- (3) The interaction of free electrons with ion cores is negligible.
- (4) ~~The~~ The free electrons move in a completely uniform potential field due to ions fixed in the lattice.

Electrical Conductivity:-

Let us now derive an expression for electrical conductivity in terms of number of free electrons and their mobility.

The current is due to the motion of the conduction electrons under the influence of the electric field. The field E exerts a force $-eE$ on the electron. The accelerated electron collides with the rest of the medium and its velocity drops to zero due to collision and again accelerated due to the field. So that there is a frictionless (or) opposing force acting on the electron due to its collision.

If v is the velocity of the electron and τ is the time between two consecutive collisions, the frictionless force can be written as $-m \frac{v}{\tau}$. where m is the effective mass of the electron.

$$\therefore \text{From Newton's law } m \frac{dv}{dt} = -eE - m \frac{v}{\tau} \rightarrow (1)$$

$$\text{Under steady state condition } \frac{dv}{dt} = 0 \therefore v = -\frac{e\tau}{m} E \rightarrow (2)$$

$\therefore$ This is the Steady State velocity of the electron.

The amount of velocity gained by the electron in unit electric field is known as drift velocity (v_d).

$$\therefore v_d = \frac{-e\tau}{m} E \rightarrow (3)$$

(- neglect b'coz -ve velocity does not exist)

If n is the number of conduction electrons per unit volume then the charge per unit volume is $-ne$.

The amount of charge crossing a unit area per unit time is given called current density J .

$$\therefore J = (-ne)v_d = -ne\left(\frac{-e\tau}{m}\right)E = \frac{ne^2\tau}{m} E$$

From ohm's law $J = \sigma E$ where σ is the conductivity

$$\therefore \sigma = \frac{ne^2\tau}{m} \rightarrow (4)$$

From above equation increase of electron concentration n , the conductivity increases. (inside periodic)

As m increases the motion of electrons becomes slow and hence the conductivity σ decreases.

(where τ is the mean free time-lifetime which is actually the time between two consecutive collisions) no
mean free path (λ):- The average distance travelled by an electron between two successive collisions in the presence of applied field is known as mean free path. $\lambda = \bar{c}\tau$ ($\bar{c}$ is root mean square velocity.)

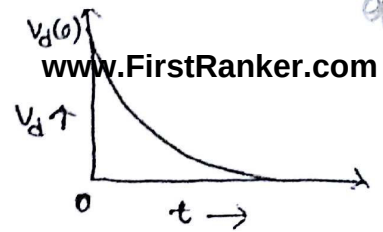
Relaxation time:- (The mean free life time τ is called the relaxation time.)

When an electric field is applied on an electron then it drifts in opposite direction to the field applied. After removal of the electric field the drift velocity decays exponentially.

This can be related mathematically as $v_d = v_0 \exp\left(\frac{-t}{\tau_r}\right)$

where v_0 is the initial velocity of the electron (before the applied of electrical field).

$$\text{If } t = \tau_r \text{ then } v_d = v_0 \exp\left(\frac{-\tau_r}{\tau_r}\right)$$



location : The duration of time required for an electron to decay its drift velocity to $\frac{1}{e}$ times of its initial velocity.

mobility of the electron (μ):-

The steady state drift velocity per unit electric field is called mobility of the electron (μ).

In addition to thermal motion, electrons drift due to the applied field. The magnitude of the drift velocity per unit field is defined as the mobility of electrons.

$$\mu = \frac{\langle v_d \rangle}{E} = \frac{e\tau}{m} \rightarrow (5) \quad [\because v_d = \frac{e\tau}{m} E]$$

$$\text{from equation (4)} \quad \sigma = ne \frac{e\tau}{m} \Rightarrow \frac{e\tau}{m} = \frac{\sigma}{ne}$$

$$\Rightarrow \sigma = ne\mu \rightarrow (6)$$

$\therefore$ The resistivity of the material of the conductor is

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu} \quad \boxed{\rho = \frac{m}{ne^2\tau}}$$

from (6) $\therefore$ The electrical conductivity σ depends on ~~two factors~~ Charge density n and their mobility μ . These two factors depend upon temperature. In metals n is constant and μ decreases slightly with temperature and hence with increase of temperature, the conductivity decreases.

Merits of free electron theory:-

- (1) It verifies Ohm's law
- (2) It explains the electrical and thermal conductivity of metals.
- (3) It explains increase of temperature resistance with temperature.

(4) It explains optical properties of metals

Resistivity: A measure of material's ability to oppose the flow of an electric current. The resistivity of a material is $\rho = \frac{RA}{l}$ where R is the resistance of a uniform specimen of the material having a length l and a cross section A .

1. The phenomena such as photo electric effect, Compton effect and the black body radiation couldn't be explained by classical free electron theory.
2. Electrical conductivity of semiconductor or insulators couldn't be explained using this model.
3. The classical theory of free electrons failed to explain the concept of specific heat of metals.
4. Calculation of mean free path (λ).

To calculate mean free path of an electron, τ relaxation time. $\tau = \frac{m}{ne^2\rho}$

for copper $\rho = 1.69 \times 10^{-8} \Omega\text{-m}$; charge density $n = 8.5 \times 10^{28} \text{ gm/m}^3$ for copper is

$$\tau = \frac{9.1 \times 10^{-31}}{(8.5 \times 10^{28})(1.6 \times 10^{-19})^2 (1.69 \times 10^{-8})}$$

$$\therefore \tau = 2.4 \times 10^{-14} \text{ s}$$

mean free path of an electron $\lambda = \bar{c} \tau$

$$\lambda = 1.16 \times 10^5 \times 2.4 \times 10^{-14} \quad (\bar{c} = 1.16 \times 10^5 \text{ m/s})$$

$$= 2.8 \text{ nm.}$$

The experimental value is 10 times greater than above value. Hence the classical free electron theory failed to explain the mean free path of the electrons.

2. **Wiedemann-Franz Law:** It states that $\frac{k}{\sigma T} = \text{Const}$ where k is Thermal conductivity. The constant is called Lorentz Number L . Derivation based on Drude's model yields

$$L = \frac{3}{2} \left[\frac{k_B}{e} \right]^2 \rightarrow B$$

The theoretical value of L can be calculated by the values of k_B and e in above ^(B) equation. L can also be calculated from equation A by substituting the measured values of k and σ at a given Temp. Some discrepancy is found between the calculated and measured values of L .

According to Quantum theory, at absolute zero of temperature, the free electrons occupy different energy levels continuously without any vacancy in between filled states. The electrons obey Pauli's exclusion principle.

The highest filled level, which separates the filled and empty levels at 0K is known as the Fermi level and the energy corresponding to this level is called Fermi energy (E_F).

The probability $F(E)$ of an electron occupying an energy level E is given by $F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$ where k_B = Boltzmann Const.

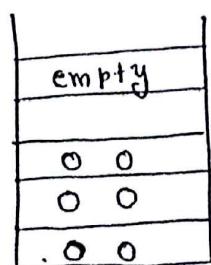
$F(E)$ is 0 if $E > E_F$

Case i) If $E > E_F$ then $F(E) = 0$ ($T = 0\text{K}$)

It indicates that energy levels above the Fermi level are empty.

Case ii) If $E < E_F$ then $F(E) = 1$ ($T = 0\text{K}$)

It indicates that the energy levels are below the Fermi level are full with electrons.



$E_F \rightarrow$ Fermi level

Case iii) At any temperature other than 0K .

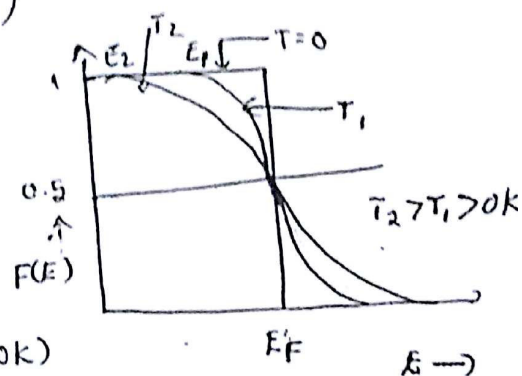
If $E = E_F$.

$$F(E) = \frac{1}{2}$$

Hence the Fermi level is that state at

which the probability of electron occupation is $\frac{1}{2}$ at any temperature above 0K .

From the graph the probability to find an electron decreases below the Fermi level and increases above the Fermi level as temperature increases. And there exists a two fold symmetry in the probability curves above the Fermi level.



Sommerfeld in 1928 applied the principles of Quantum mechanics to the classical free electron theory.

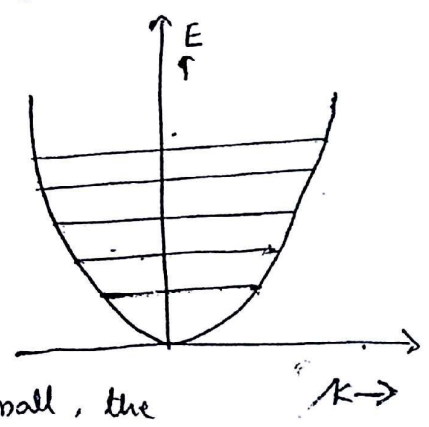
According to classical theory the free electrons in metals move randomly such that the velocity is uniform in all the directions. But, according to quantum theory the free electrons occupy different energy levels present in the metal. Hence different free electrons possess different energy values, hence they possess different velocities.

we assume that electrons move in a solid metal similar to gas molecules. Taking zero for the constant value of the potential inside the metal specimen, we solve the Schrodinger time independent wave equation to get the possible energy values E_n of the electrons. The problem is similar to that of a particle in a potential box. The solution yields

$$E_n = \frac{n^2 h^2}{8mL^2} \quad \text{or} \quad = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$

where $n^2 = n_x^2 + n_y^2 + n_z^2$ and n_x, n_y, n_z are positive integers.

$\therefore k = \frac{n\pi}{L}$ Say
then $E = \frac{\hbar^2 k^2}{2m}$



If L is large. say 1 cm the adjacent permitted energy levels differ by 10^{-19} eV. since the separation is too small, the energy levels can be considered to form a quasi-continuous band.

when an external electric field E is applied, the force exerted on the electron is $-eE$. since force is also rate of change of momentum.

$$-eE = \frac{dp}{dt}$$

$$\therefore \text{momentum } p = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar k$$

$$\therefore -eE = \hbar \left(\frac{dk}{dt} \right) \quad \text{or} \quad dk = -\frac{eE}{\hbar} dt$$

This means that displacement of k space means displacement of k space in time dt on application of external field. Because of collisions with imperfections, displacement of k space becomes steady ($= \Delta k$) and dt is then the average collision time τ .

$$\therefore \Delta k = -\frac{eE\tau}{\hbar} \rightarrow (1)$$

$p = mv = \hbar k$, incremental velocity Δv is given by

$$m \Delta v = \hbar \Delta k$$

$$\Rightarrow \Delta v = \frac{\hbar}{m} \Delta k \rightarrow (2)$$

Substitute (1) in 2nd eqn then we get

$$\Delta v = -\frac{eE\tau}{m} \rightarrow (3)$$

If the number of electrons per unit volume is n , then the current density J is given by

$$J = n(-e)\Delta v = \frac{n e^2 \tau}{m} E = \sigma E$$

From ohm's law $J = \sigma E$

$\therefore$ electrical conductivity

$$\sigma = \frac{n e^2 \tau}{m}$$

Success of Quantum Free electron theory:-

1) Specific heat: According to classical free electron theory all the conduction electrons are capable of absorbing the heat energy of per maxwell Boltzmann statistics which results in large value of specific heat.

According to quantum free electron theory, it is only those electron those that are occupying energy levels close to E_F , which are capable of absorbing the heat energy to get excited to higher energy levels.

are capable of receiving the thermal energy input, thus the specific heat value becomes very small for the metal.

c) It Explains temperature dependency of electrical conductivity

*) Density of Energy States:-

The number of electrons per unit volume in a given energy range of interest is called carrier concentration. This is given by summing the product of the density of states $Z(E)$ and the occupancy probability $F(E)$:

$$\text{that is } n_c = \int Z(E) F(E) dE \rightarrow (1)$$

The number of energy states with a particular value of E depends on how many combinations of quantum numbers result in the same value of n (degeneracy).

The number of energy states having energy values between E and $E+dE$ can be

$$Z(E) dE = \frac{\pi}{2} \tilde{n}^2 d\tilde{n} \rightarrow (2)$$

$$\text{we know that } E = \frac{\tilde{n}^2 h^2}{8mL^2}$$

$$\Rightarrow \tilde{n}^2 = \frac{8mL^2 E}{h^2} \rightarrow (3) \Rightarrow n = \left[\frac{8mL^2 E}{h^2} \right]^{\frac{1}{2}}$$

$\therefore$ Differentiating above equation.

$$2\tilde{n} d\tilde{n} = \frac{8mL^2}{h^2} dE$$

$$d\tilde{n} = \left(\frac{1}{2\tilde{n}} \right) \frac{8mL^2}{h^2} dE = \left(\frac{8mL^2}{h^2} \right)^{\frac{1}{2}} \left(\frac{1}{2} \right) \left(\frac{h^2}{8mL^2 E} \right)^{\frac{1}{2}} dE$$

$$\therefore d\tilde{n} = \left(\frac{1}{2} \right) \left(\frac{8mL^2}{h^2} \right)^{\frac{1}{2}} \frac{dE}{E^{\frac{1}{2}}} \rightarrow (4)$$

Substitute the value of $\tilde{n}$ and $d\tilde{n}$ from (3) & (4) equations in equation (2) then we can get

$$Z(E) dE = \frac{\pi}{4} \left(\frac{8mL^3}{h^3} \right)^{3/2} E^{1/2} dE$$

According to Pauli's exclusion principle two electrons of opposite spin can occupy each state and hence the number of energy states available for electron occupancy is given

$$Z(E) dE = 2 \times \frac{\pi}{4} \left(\frac{8mL^3}{h^3} \right)^{3/2} E^{1/2} dE$$

$$= \frac{4\pi}{h^3} (2m)^{3/2} L^3 E^{1/2} dE$$

Density of energy states is given by number of energy states per unit volume.

i.e. Density of states $Z(E) dE = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE \rightarrow (5)$

According to Fermi Dirac distribution

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \rightarrow (6)$$

(6) & (5) equations Substitute in equation (1)

n. Carrier Concentration

$$n_c = \int Z(E) F(E) dE$$

$$n_c = \frac{4\pi}{h^3} (2m)^{3/2} \int E^{1/2} \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} dE$$

This is the expression for density of Charge Carriers at any temperature.

Electron Scattering and Resistance:-

The main factors affecting the electrical resistivity of solids are (a) temperature and (b) defects. The resistivity change is generally given by the Matthiessen's rule.

Introduction:- It is assumed in quantum free electron theory of metals that the free electrons in a metal express a constant potential and is free to move in the metal. This theory explains successfully most of the phenomena of solids, but it could not explain why some solids are good conductors and some other are insulators and semiconductors. It can be understood successfully using the band theory of solids.

Bloch Theorem:-

metals and alloys are crystalline

$\oplus \oplus \oplus \oplus$

$\oplus \oplus \oplus \oplus$

$\oplus \oplus \oplus \oplus$

$\oplus \oplus \oplus \oplus$

in Nature. When conduction electrons are released, they move inside periodic positive ion cores. Hence instead of considering uniform constant potential as we have done in free electron theory, ^{according} we have to consider ~~the~~ the potential is minimum at the positive ion sites and maximum between the two ions.

The Schrodinger equation for an electron moving in one dimensional periodic potential is

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0 \quad (\hbar = \frac{h}{2\pi})$$

V is a potential at a distance x from the origin which is assumed to vary periodically in space with an interval equal to the lattice constant a .

$$V(x) = V(x+a)$$

Bloch has shown that the one dimensional solution of the Schrodinger wave equation is of the form

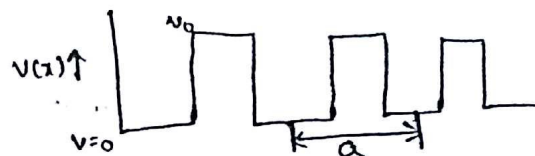
$$\psi_k(x) = \exp(ikx) U_k(x) \quad \text{--- (1)}$$

in 3-D

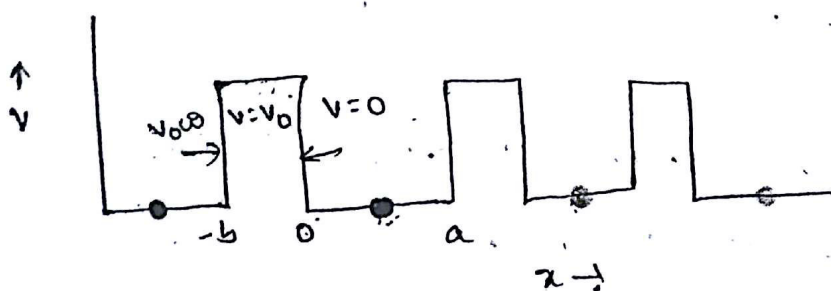
$$\psi_k(r) = \exp(ikr) U_k(r)$$

So the solutions are plane waves modulated by the functions $u_k(x)$ which has the same periodicity as the lattice. This theorem is known as Bloch theorem.

Bloch theorem is a mathematical statement regarding the form of the one electron wave functions for a perfectly periodic potentials. Above equations are called Bloch functions (Floquet's theorem).



Kronig-Penny model :- According to this theory the electrons move in a periodic potential produced by the positive ion cores. The potential of the electron varies periodically with periodicity of ion core and the potential energy of the electron is zero near nucleus of the positive ion core and maximum where it is lying between the adjacent nuclei which are separated by the interatomic spacing a .



Schrodinger equation for one dimensional periodic potential field denoted by $V(x)$ can be written as

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

If $V=0$ $\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2} \psi = 0 \rightarrow (1)$ for $0 < x < a$

$$\frac{\partial}{\partial x} \frac{\partial \psi}{\partial x} = \frac{2mE}{\hbar^2} \psi$$

Substitute in equation (1)

$$\frac{d^2 \psi}{dx^2} + \alpha^2 \psi = 0$$

The solution of this equation have the form by Bloch's.

$$\psi(x) = U_k(x) \exp i k x$$

where $U_k(x)$ is periodic with the periodicity of the lattice

$$\text{That is } U_k(x+a) = U_k(x)$$

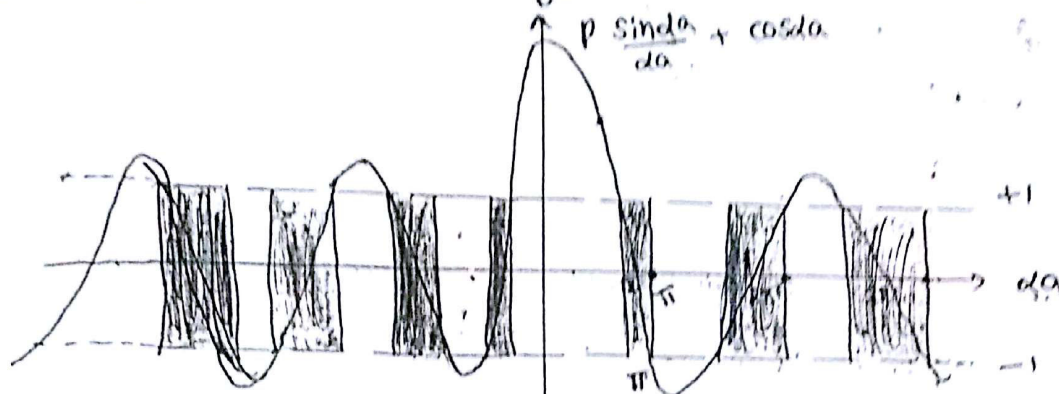
The form of $U_k(x)$ depends on the exact nature of the potential field. Further ψ and $\frac{d\psi}{dx}$ should be continuous through the crystal. Also in order to simplify the computations, an assumption is made regarding the potential barrier.

As V_0 increases the width of the barrier a decreases so that the product $V_0 a$ remains constant. It turns out that solutions are possible only for energies given by the relation

$$\cos k a = P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos k a$$

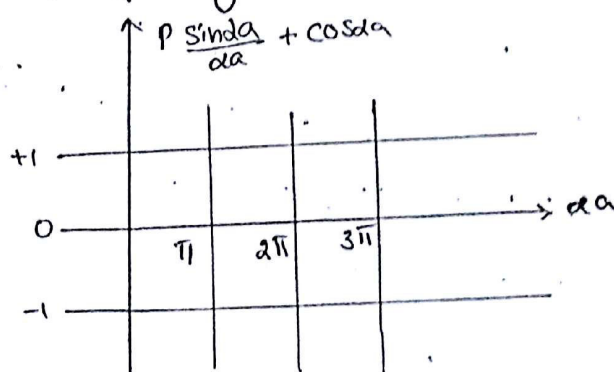
$$\text{where } P = \frac{4\pi^2 m a V_0}{\hbar^2} ; \alpha = \frac{\sqrt{2mE}}{\hbar} \text{ and } k = \frac{2\pi}{a}$$

P is known as scattering power of potential barrier and $V_0 a$ is called barrier strength. with an increase of P , an electron is bound more strongly to a potential well, when $(P \rightarrow 0, \text{ the electrons are free})$ The right hand side of equation can take values between ± 1 , But the left-hand side exceeds this value. Hence only certain range of values of α are allowed. This means that energy E is restricted to lie within certain ranges

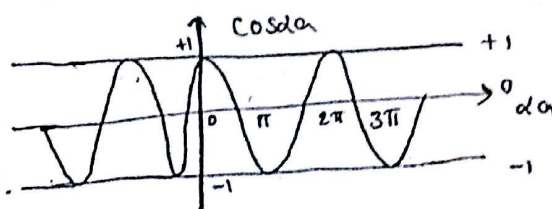


The permitted values of energy are shown as shaded portions. This gives rise to the concept of ranges of permitted values of α for a given ion lattice spacing a .

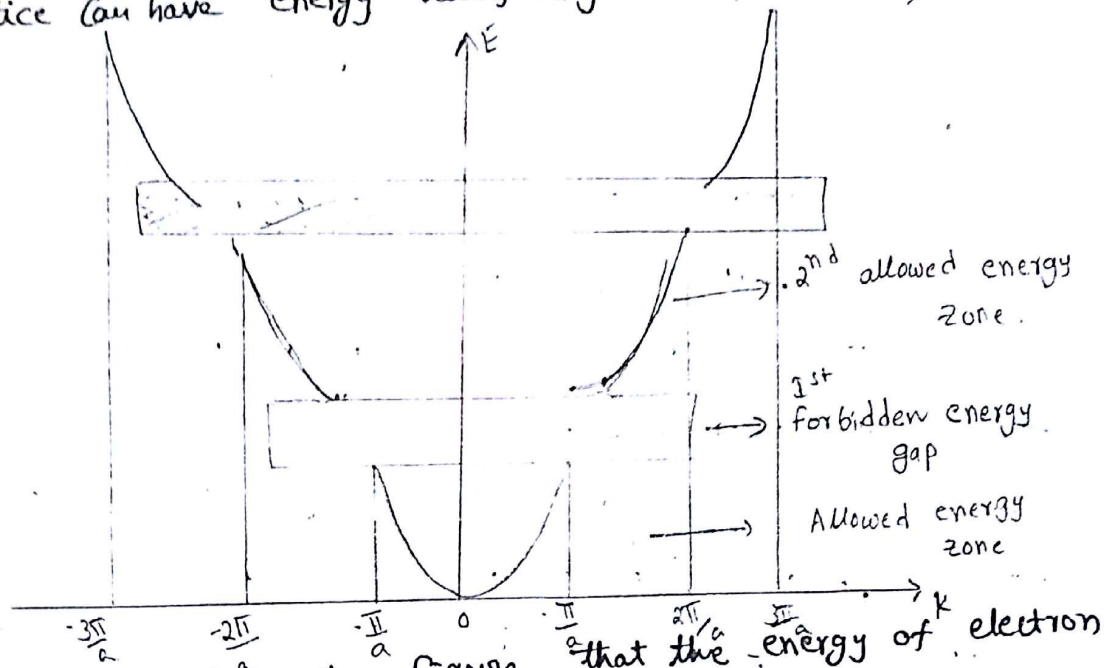
- (i) The energy spectrum of the electron consists of a number of allowed and forbidden energy bands.
- (ii) The width of the allowed energy band increases with increase of energy values i.e. increasing the value of αa . This is because the first term of equation decreases with increase of αa .
- (iii) With increasing p i.e. with increasing potential barrier, the width of an allowed band decreases.
- (iv) In the limit $p \rightarrow \infty$ the allowed band reduces to one single energy level corresponding to the discrete energy level of an isolated atom.



- (v) In the when $p \rightarrow 0$ then the left hand side of Equation will not cross ± 1 line. Hence, all the energies are allowed to the electrons.



k-k diagram:- The energy of electron in a periodic potential of lattice can have energy values only in the allowed regions or zones.



It is clear from the figure that the energy of electron is continuously increasing $k=0$ to π/a . The right hand side of the equation becomes ± 1 for values of $k = \pm \frac{n\pi}{a}$ and hence discontinuity appears in the $E-k$ graph at a $k = \pm \frac{n\pi}{a}$.

It can be seen that the energy spectrum of electron is consisting allowed regions and forbidden regions. The allowed region extended from $-\frac{\pi}{a}$ to $\frac{\pi}{a}$. This is known as first Brillouin zone. After a discontinuity in energy, called forbidden gap, the another allowed region extended from $-\frac{2\pi}{a}$ to $-\frac{\pi}{a}$ and $\frac{\pi}{a}$ to $\frac{2\pi}{a}$. This is known as second Brillouin zone.

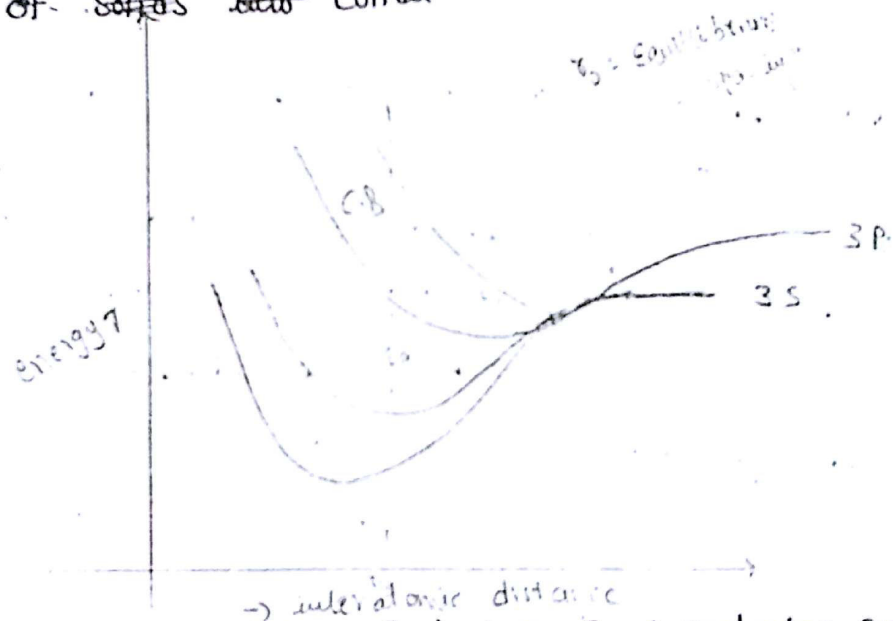
Origin of 'Energy Gap':

An isolated atom possesses discrete energies of different electrons. Suppose two isolated atoms are brought to very close proximity, then the electrons in the orbits of two atoms interact with each other. So, that in the combined system, the energies of electrons will not be in the same level but changes and the energies will be slightly lower and larger than the origin value. So, at the place of each energy level, a closely spaced two energy levels exists.

and if these atoms electrons interact and give N number of closely spaced energy levels in the place of discrete energy levels, it is known as bands of allowed energies. Between the bands of allowed energies, there are empty ^{energy} zones, called forbidden band of energies. Kronig-Penny model supports the existence of these bands of energy.

The formation of energy bands has been explained taking Sodium metal. When isolated Sodium atoms are brought together to form a solid, then the energy levels of the valence electrons spread into bands. The 3s and 3p orbitals electrons are shown in fig. These bands are seen to overlap strongly at the interatomic spacing of Sodium.

Classification of Solids into Condu



Classification of Solids into Conductors, Semi Conductors and

insulators :- [According to band theory, the electrons in a solid can possess bands of energies called allowed bands of energies and these electrons may not possess some other bands of energies called forbidden bands of energies. The allowed bands of energies and forbidden bands of energies are present alternatively one after another for the electrons of a solid. The to

The top most fully filled band at absolute zero of temperature is known as the valence band and next allowed band is called the conduction band. These two bands are separated by forbidden band.

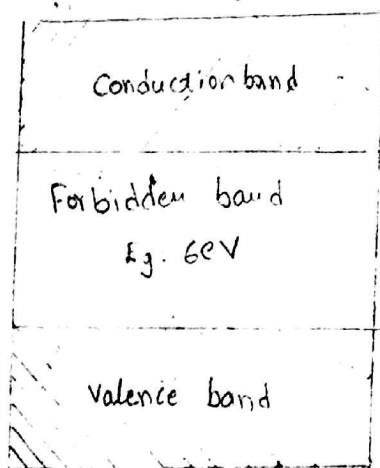
On the basis of magnitude of forbidden band the solids are classified into insulators, semiconductors and conductors.

(i) Insulators:- In case of insulators, the forbidden energy band is very wide. Due to this fact electrons cannot jump from valence band to conduction band under normal conditions. In insulators the valence electrons are bound very tightly to their parent atoms.
Ex: for glass energy band gap 10 eV at 0K.

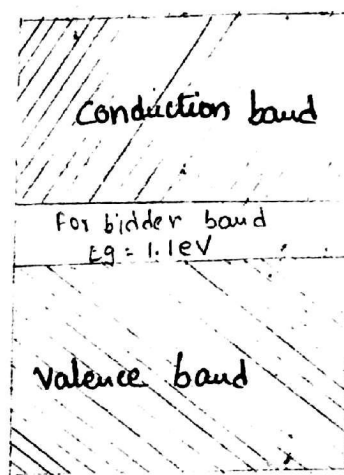
(ii) Semiconductors:- In semiconductors, the forbidden band is very small.
Ex: Ge (Germanium) and Silicon.

In Germanium the forbidden band is of the order of 0.7 eV while in case of silicon, the forbidden band is of the order of 1.1 eV.

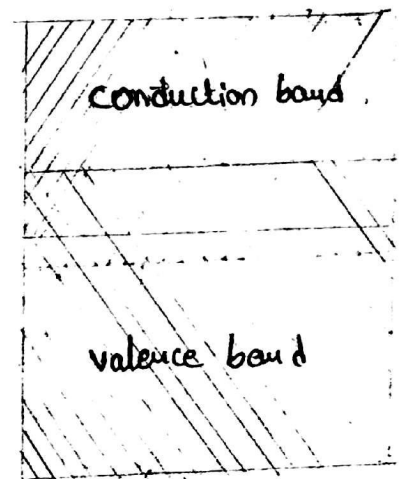
A semiconductor material is one whose electrical properties lie between insulators and conductors. At 0K there are no electrons in conduction band and valence band is completely filled. When a small amount of energy is supplied, the electrons can easily jump from valence band to conduction band. In semiconductors, the conductivity is of the order of $10^2 \Omega m$.



Insulators



Semiconductors



Conductors

1) Conductors In case of conductors, there are no forbidden band and both valence band and conduction bands overlap each other. In conduction band, plenty of free electrons are available for the process of electrical conduction. The electrons from valence band freely enter into conduction band. The most important point in conductor is that due to the absence of forbidden band, there is no structure to establish holes. The total current in conductors is simply a flow of electrons.

Concept effective mass of an electron.

When an electron in a periodic potential of lattice is accelerated by an electric field or magnetic field, then the mass of the electron is called effective mass (m^*).

Let us consider an electron of charge 'e' and mass 'm' moving inside a crystal lattice of electric field E .

Let, eE be the force experienced by the electron in the applied field E .

According to Newton's 2nd law force $F = ma \Rightarrow$

$$\therefore m^*a = eE \Rightarrow a = \frac{eE}{m^*} \rightarrow (1)$$

According to Sommerfeld's quantum free electron theory, free electron motion in the metal is considered not as a moving particles but as passages of waves in a periodic lattice. As a result a moving electron is considered a wave packet and the velocity of electron treated as group velocity v_g of the wave packet.

$\therefore$ The group velocity can be written as

$$v_g = \frac{d\omega}{dk} = 2\pi \frac{dv}{dk} = \frac{2\pi}{h} \frac{dE}{dk} = \frac{1}{h} \frac{dE}{dk} \rightarrow (2)$$

$$\text{Acceleration } a = \frac{dv_g}{dt} = \frac{1}{h} \frac{d^2E}{dk^2} \frac{dk}{dt} = \frac{1}{h} \left(\frac{d^2E}{dk^2} \right) \frac{dk}{dt} \rightarrow (3)$$

having with velocity v

$$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda}$$

$$\therefore p = \hbar k$$

$$\text{and } \frac{dp}{dt} = F \Rightarrow \frac{d(\hbar k)}{dx} = F \Rightarrow \hbar \frac{dk}{dx} = F$$

$$\Rightarrow \frac{dk}{dx} = \frac{F}{\hbar} \rightarrow (5)$$

(5) $\Rightarrow$ (4)

$$a = \frac{1}{\hbar^2} \left(\frac{d^2 E}{dk^2} \right) \cdot F$$

$$\Rightarrow F = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2} \right)} \cdot a \rightarrow (6)$$

Comparing: equation (5) & (6)

$$m^* = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2} \right)}$$

Valence band: A band which is occupied by the valence electrons. The valence band may be partially or completely filled up depending on the nature of the material.

Conduction band: The next higher permitted band is the conduction band. The energy levels occupying this band may also be defined as the lowest unfilled energy band. This band may be empty or partially filled. In conductor band the electrons can move freely.

Forbidden band: Both the bands conduction band and valence band are separated by a region or gap known as forbidden band.

Introduction:- On the basis of Conduction mechanism, solids are classified into three Categories, Conductors, Semiconductors and insulators. In Conductors large no of free electrons are available which are useful for the electrical Conduction mechanism. These materials have very low resistivity and very high conductivity.

In insulators no free electrons are available, hence these are not useful for the electrical Conduction mechanism. These materials have high resistivity and zero conductivity.

The Semiconductors have intermediate property of conductors and insulators. These materials behave as insulators at low temperature and as conductors at high temperature. These materials have two types of Charge carriers 'electrons' and 'hole'.

Definitions:-

Intrinsic Semiconductor:- A Semiconductor which does not have any kind of impurities, behaves as insulator at OK and behaves as a conductor at higher temperature is known as intrinsic Semiconductor.

Extrinsic Semiconductor:- A Semiconductor which has either the donor impurities or the acceptor impurities is known as extrinsic semiconductor.

Electron-hole pair generation:- When a suitable form of energy is supplied to a semiconductor then electrons take transition from valence band to Conduction band. Hence, a free electron in Conduction band and a free hole in valence band is formed. This phenomenon is known as electron-hole pair generation.

The process of combining a conduction electron with a valence hole is known as recombination. Some energy, which is equal to energy gap of a semiconductor is released.

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Life time:- The duration of time elapses between the processes of electron-hole pair generation and recombination. Can be defined as life time.

Carrier Concentration:- The number of charge carriers per unit volume is defined as carrier concentration.

Units: no. of electrons / m^3 .

Intrinsic Semiconductor	Extrinsic Semiconductor
1. It is Chemically pure, structurally perfect in a compound it is stoichiometric material.	1. It is an impure metal, obtained by adding a group II or group IV element as dopant into an intrinsic semiconductor or structurally imperfect or non stoichiometric, its compound
2. At normal temperature, conductivity is low	2. At normal temperature, the conductivity is high
3. Number of electrons is equal to no. of holes	3. Current carriers are essentially majority carriers at normal temp.
4. Carrier generation is due to breaking of covalent bonds	4. carrier generation is due to ionization of impurity atoms (majority carriers) and breaking of covalent bonds (minority carriers)
5. Fermi level lies at the centre of the forbidden energy gap at all temperature.	5. At 0K the Fermi level lies close to the conduction band in n-type & close to the valence band in p-type semi
6. Hall coefficient is negative due to higher mobility of electrons	6. Hall coefficient is negative for n-type semiconductor and positive for p-type.

Intrinsic Semiconductors are those in which impurities are not present and therefore called pure Semiconductors. Fermi level exists exactly at mid way of the Energy gap. In these Semiconductors few crystal defects may be present. When a semiconductor is taken at 0K then it behaves as an insulator and conduction occurs at higher temperatures due to thermal excitation of electrons from the valence band to the conduction band. 8.

Ex:- Germanium, Silicon etc.

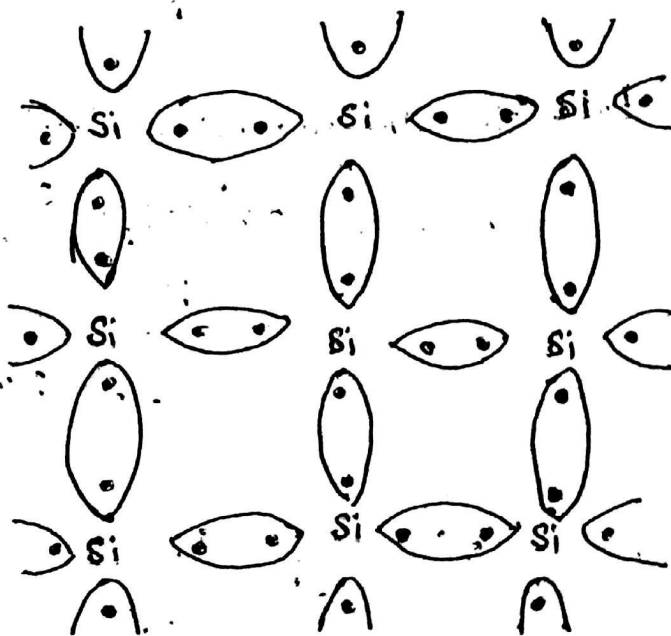
In order to get insight view of an intrinsic Semiconductor, let us consider Silicon, which has four valence electrons. In order to gain stability it has to make four covalent bonds. In this regard each silicon atom makes four covalent bonds with four other silicon atoms.

* The electrons which are participating in the covalent bonds are known as valence electrons.

* If some energy is supplied then covalent bonds break, electrons will come out and move freely, resulting in the

formation of vacant sites in the covalent bonds. These are known as positive charge carriers named as holes.

* The electrons which come out from the valence bonds move freely without any constraints and have more energy than the electrons in the valence band.



The number of electrons will be equal to the number of vacant sites or holes in the valence band.

Carrier Concentrations -

In intrinsic Semiconductors, as the temperature is increased electron-hole pairs will be generated. Hence, the electron concentration, n_i in the conduction band will be equal to the hole concentration, p_i in the valence band.

Let E_F be the Fermi level of the intrinsic Semiconductor in equilibrium, then the number of electrons per unit volume in the conduction band

$$n_i = 2 \left(\frac{2m_e^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right)$$

and the number of holes per unit volume in the

valence band

$$p_i = 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

but in an intrinsic Semiconductor $n_i = p_i$

hence

$$2 \left(\frac{2m_e^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right)$$

$$= 2 \left(\frac{2m_h^* \pi k_B T}{h^2} \right)^{3/2} \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

$$\left(\frac{m_e^*}{m_h^*} \right)^{3/2} \exp \left(\frac{E_F - E_C}{k_B T} \right) = \exp \left(\frac{E_V - E_F}{k_B T} \right)$$

$$\Rightarrow \exp \left(\frac{E_F - E_C - E_V + E_F}{k_B T} \right) = \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$2E_F - E_C - E_V = k_B T \ln \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \log \left(\frac{m_h^*}{m_e^*} \right)$$

If the effective masses $m_e^* = m_h^*$ then

$$E_F = \frac{E_V + E_C}{2} \quad (\because \ln 1 = 0)$$

From above equation intrinsic fermi level (E_F) lies midway of E_C and E_V , which happens to be the centre of the band gap at any temperature.

If $m_h > m_e$ E_F linearly increases towards E_C . The material properties E_C, E_V, m_h and m_e determine the value of E_F , which for most semiconductors like Si and Ge lie at the centre of the band gap.

Combining n_i and p_i values

$$n_i \cdot p_i = 4 \left[\frac{2m_e^* kT}{h^2} \right]^{3/2} \left[\frac{2m_h^* kT}{h^2} \right]^{3/2} \exp\left(\frac{E_F - E_C}{kT}\right) \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$n_i^2 = 4 \left(\frac{2\pi k}{h^2} \right)^3 (m_e^* m_h^*)^{3/2} T^3 \exp\left(\frac{E_V - E_C}{kT}\right) \quad (\because n_i = p_i)$$

$$\therefore n_i^2 = A T^3 \exp\left(-\frac{E_g}{kT}\right)$$

where $A = 4 \left(\frac{2\pi k}{h^2} \right)^3 (m_h^* m_e^*)^{3/2}$, a constant value of a semiconductor.

and $E_g = E_C - E_V$ is energy gap of the semiconductor.

It is to be noted that A and E_g are constants for a given semiconductor and n_i is called the intrinsic concentration which indicates thermally generated electrons and holes. It is a strong function of temperature T .

$$\therefore n_i = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/4} (m_e^* m_h^*)^{3/4} \exp\left(-\frac{E_g}{2kT}\right)$$

INTRINSIC SEMICONDUCTOR - Carrier Concentration -

Equation for conductivity -

We know that for a Semiconducting material, electrical conductivity σ is given by

$$\sigma = (ne\mu_e + p\mu_h) \rightarrow (1)$$

where μ_e and μ_h are the mobilities of electrons and holes respectively. Mobility is the velocity acquired by the electron or hole under unit potential gradient

Hence $\sigma_i = n_i e (\mu_e + \mu_h)$ ($n = p = n_i$)

We know that $n_i = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2kT}\right)$

$$\therefore \sigma = (\mu_e + \mu_h) 2e \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} \exp\left(\frac{-E_g}{2kT}\right)$$

$\therefore$ The electrical conductivity depends upon the negative exponential of the forbidden energy gap between the valence and conduction band and on the mobilities of both holes and electrons. The mobilities are determined by the interaction of the electron or hole with lattice waves or phonons. In such a case μ_e and μ_h are both proportional to $T^{-3/2}$

$\therefore$ The electrical conductivity of an intrinsic Semiconductor can be written as

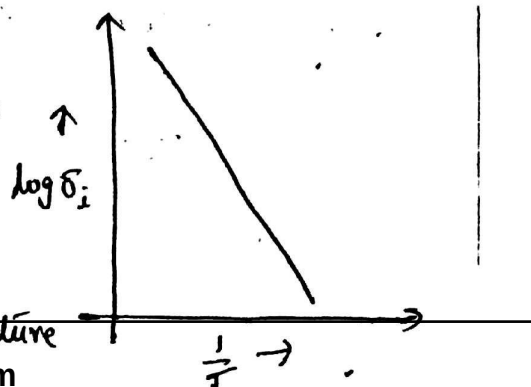
$$\sigma_i = A \exp\left(\frac{-E_g}{2kT}\right)$$

where A is Constant.

Taking logarithms on both sides

$$\log \sigma_i = \log A - \frac{E_g}{2kT}$$

From the graph we understand that conductivity increases with temperature



The electrical conductivity of an intrinsic Semiconductor

$$\sigma_i = A \exp\left(\frac{-E_g}{2k_B T}\right)$$

Resistivity of the intrinsic Semiconductor

$$\rho_i = \frac{1}{A} \exp\left(\frac{E_g}{2k_B T}\right)$$

$$\frac{\rho_i a}{L} = \frac{1}{A} \exp\left(\frac{E_g}{2k_B T}\right) \quad \left(\because \rho = \frac{R_a}{L}\right)$$

$$\text{or } R_i = \frac{L}{aA} \exp\left(\frac{E_g}{2k_B T}\right) = C \exp\left(\frac{E_g}{2k_B T}\right) \quad \left[\because C = \frac{L}{aA}\right]$$

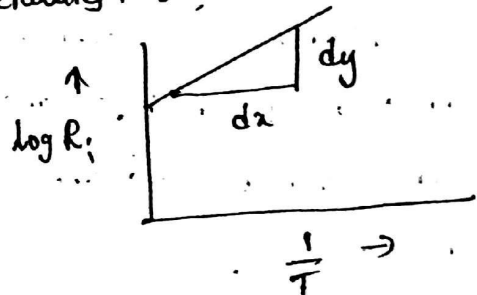
Taking logarithms on both sides

$$\log R_i = \log C + \frac{E_g}{2k_B T}$$

From above equation measuring the resistance of intrinsic Semiconductors at different temperatures we can measure the forbidden energy gap E_g .

The Slope of the Curve

gives the value of $\frac{E_g}{2k}$



$$\therefore \text{From the graph } \frac{dy}{dx} = \frac{E_g}{2k}$$

$$\therefore E_g = 2k \left(\frac{dy}{dx}\right)$$

Extrinsic Semiconductors :-

To get appreciable current density through the semiconductor by adding suitable impurities into the intrinsic semiconductors. Thus they are called extrinsic semiconductors.

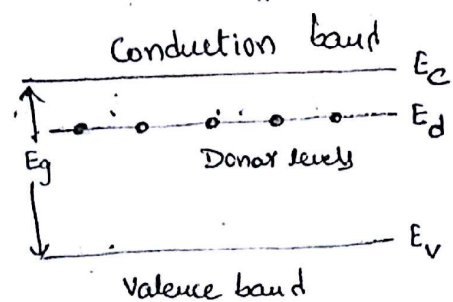
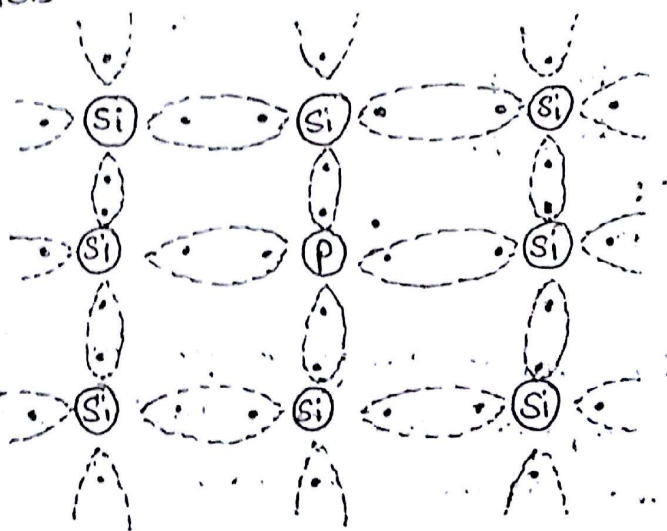
The extrinsic semiconductors are those in which impurities of large quantity are present. Usually, the impurities can be either (i) group elements or (ii) group elements. Based on the impurities

Present in the extrinsic semiconductors are classified into two categories

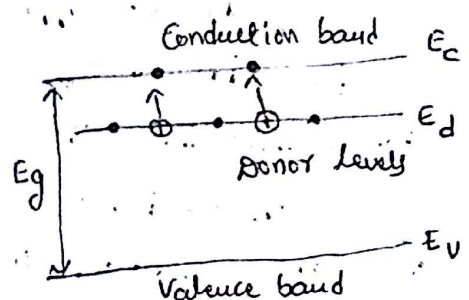
i) n-type semiconductors:-

In order for silicon crystal to conduct electricity, we need to introduce an impurity atom such as, arsenic, antimony or phosphorus into the crystalline structure. These atoms have five outer electrons in their outermost covalent bond to share with other atoms and are commonly called pentavalent impurities. They allow four of the five electrons to bond with its neighbouring silicon atoms leaving one free electron to move about when an electrical voltage is applied. As each impurity atom donates one electron, pentavalent atoms are generally known as donors.

Antimony (Sb) is frequently used as a semiconductor material as it has 51 electrons arranged in 5 shells around the nucleus. The resulting semiconductor material has an excess of current-carrying electrons, each with a negative charge is therefore referred to as n-type material. The electrons are called majority carriers and the resultant holes are called minority carriers.



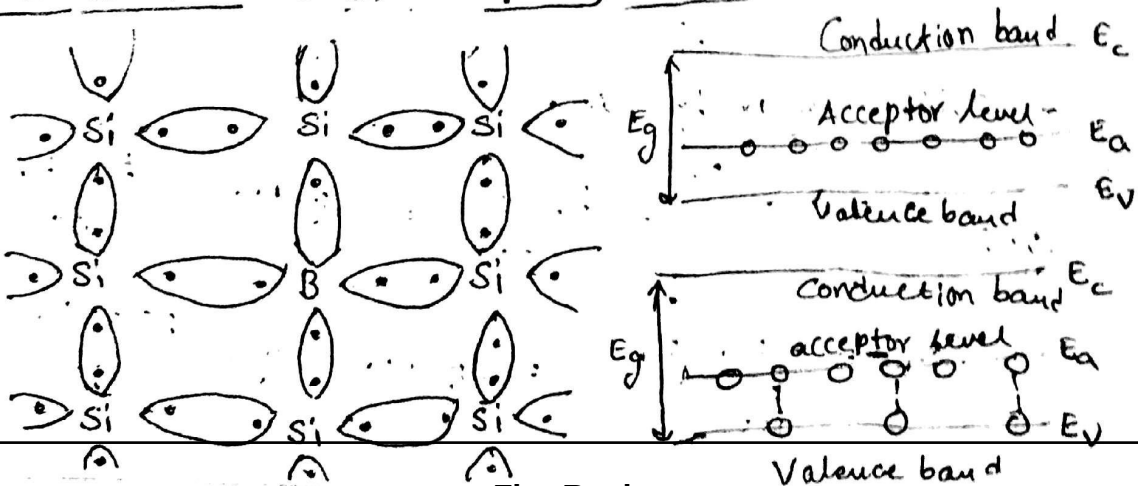
AT $T = 0\text{K}$



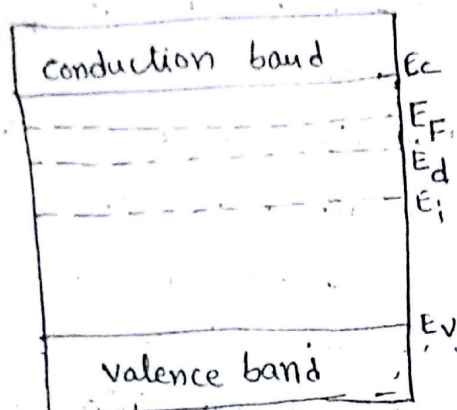
AT $T > 0\text{K}$

atoms, its trivalent (Group III) impurity atoms such as B (or Al, Ga, In) atoms are doped in an intrinsic semiconductor Si or Ge then the impurity atoms occupy some of the Silicon atom sites because the size of the impurity atoms and Si atom is almost the same.

With three valence electrons each boron atom makes three covalent bonds with the surrounding three silicon atoms. To attain sp^3 therefore, a complete connection is not possible, giving the semiconductor material an abundance of positively charged carriers known as holes in the structure of the crystal. As there is a hole an adjoining free electron is attracted to it and will try to move into the hole to fill it. However, the electron filling the hole leaves another hole behind it as it moves. This in turn attracts another electron which in turn creates another hole behind, and so forth, giving the appearance that the holes are moving as a positive charge through the crystal structure. As each impurity atom generates a hole, trivalent impurities are generally known as acceptors & they are continually accepting extra electrons. Boron is frequently used as trivalent additive as it has only 5 electrons arranged in 2 shells around the nucleus. Addition of boron causes conduction to consist mainly of positive charge carriers results in a p-type material, and the positive holes are called majority carriers while the free electrons are called minority carriers.



Carrier concentration in n-type semiconductor



N_d is the donor concentration i.e. the number of donor atoms per unit volume of the material and E_d is the donor energy level. At very low temperatures all donor levels are filled with electrons. With increase of temperature, more and

more donor atoms get ionised and the density of electrons in the conduction band increases.

Density of electrons in conduction band is given by

$$n = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{kT} \right) \rightarrow (1)$$

If we assume that E_F lie more than a few kT above the donor level then the density of ionised donors is given by

$$N_d [1 - F(E_d)] \approx N_d \exp \left(\frac{E_d - E_F}{kT} \right) \rightarrow (2)$$

At very low temperature, when electron-hole pairs are not generated due to breaking of covalent bonds, the number of electrons in the conduction must be equal to the number of ionised donors.

$$\text{i.e. } 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{kT} \right) = N_d \exp \left(\frac{E_d - E_F}{kT} \right)$$

→ Taking logarithm and

$$\log 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2} + \frac{E_F - E_c}{kT} = \log N_d + \frac{E_d - E_F}{kT}$$

$$\Rightarrow \left(\frac{E_F + E_c - E_d + E_F}{kT} \right) = \log N_d - \log 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$$



$$(E_d + E_c) = \frac{K T}{2} \log \left[\frac{N_d}{2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2}} \right] \quad \left[= \log \left(\frac{a}{b} \right) \right]$$

$$\Rightarrow E_F = \frac{E_d + E_c}{2} + \frac{K T}{2} \log \frac{N_d}{2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2}} \quad \text{--- (3)}$$

At 0K $E_F = \frac{E_d + E_c}{2}$

i.e. at 0K, Fermi level lies exactly at the middle of the donor level E_d and the bottom of the conduction band E_c .

Density of electrons in the conduction band

From equation (1) density of electrons in the conduction band is given by

$$n = 2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2} \exp \left(\frac{E_F - E_c}{K T} \right) \quad \text{--- (4)}$$

Substitute the E_F value in $\exp \left(\frac{E_F - E_c}{K T} \right)$

$$\begin{aligned} \exp \left(\frac{E_F - E_c}{K T} \right) &= \exp \left\{ \left(\frac{E_d + E_c}{2 K T} \right) + \frac{1}{2} \log \frac{N_d}{2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2}} \right\} \\ &= \exp \left\{ \left(\frac{E_d + E_c}{2 K T} \right) + \log \frac{N_d^{1/2}}{\left[2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2} \right]^{1/2}} \right\} \end{aligned}$$

$$= \left[\exp \left(\frac{E_d + E_c}{2 K T} \right) \right] \cdot \frac{N_d^{1/2}}{\left[2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2} \right]^{1/2}} \quad \left[\begin{aligned} &\because \exp(a+b) \\ &= \exp(a) \exp(b) \end{aligned} \right]$$

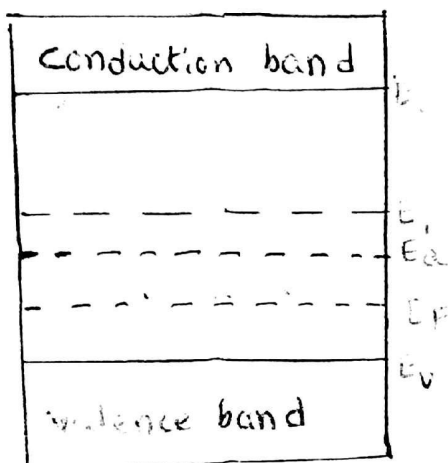
Hence, n & Substitute the above value in equation (4):

$$n = 2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2} \cdot \frac{N_d^{1/2}}{\left[2 \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/2} \right]^{1/2}} \exp \left(\frac{E_d + E_c}{2 K T} \right)$$

$$\therefore n = (2 N_d)^{1/2} \left(\frac{2 \pi m_e^* k T}{h^2} \right)^{3/4} \exp \left(\frac{E_d - E_c}{2 K T} \right)$$

The density of electrons in the conduction band is proportional to the square root of the donor concentration at low temperatures. At high temperatures intrinsic behaviour predominates and donor concentration becomes insignificant.

Carrier Concentration in p-type semiconductor



N_a is the acceptor concentration i.e. the number of acceptor atoms per unit volume of the material and E_a is the acceptor energy level. At very low temperatures all the acceptor levels are empty. With increase of temperature acceptor atoms get ionised

i.e. the electrons move from valence band and occupy the vacant sites in the acceptor energy level thereby leaving holes in the valence band.

Density of holes in the valence band

$$p = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_v - E_F}{kT} \right) \quad (1)$$

$\therefore E_F$ lies below acceptor level, the density of ionised acceptors is given by

$$N_a F(E_a) = N_a \exp \left(\frac{E_F - E_a}{kT} \right)$$

$\therefore$ the density of holes in the valence band is equal to the density of ionised acceptors.

$$2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_v - E_F}{kT} \right) = N_a \exp \left(\frac{E_F - E_a}{kT} \right)$$

$$\text{i.e. } \exp \left(\frac{E_v + E_a - 2E_F}{kT} \right) = \frac{N_a}{2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}}$$

Taking logarithm

$$\left(\frac{E_v + E_a - 2E_F}{kT} \right) = \log \frac{N_a}{2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}}$$

$$\text{i.e. } E_F = \frac{E_v + E_a}{2} - \frac{kT}{2} \log \frac{N_a}{2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}}$$

At 0K, Fermi level lies exactly at the middle of the acceptor level and the top of the valence band. Increase of temperature E_F decreases slightly as per eqn(3). As the temperature is increased, more and more acceptor atoms are ionised. For a particular temp all the acceptor atoms are ionised. Further increase in temp results in generation of electron-hole pairs due to breaking of covalent bonds and the material tends to behave in intrinsic manner. The Fermi level gradually moves towards the intrinsic Fermi level E_i .

Density of holes in the valence band :-

Density of holes in the valence band is

$$p = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \exp \left(\frac{E_v - E_F}{kT} \right) \rightarrow (4)$$

$$\begin{aligned} \exp \left(\frac{E_v - E_F}{kT} \right) &= \exp \left\{ \frac{E_v}{kT} - \frac{E_v + E_a}{2kT} + \frac{1}{2} \log \frac{N_a}{2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2}} \right\} \\ &= \exp \left\{ \left(\frac{E_v - E_a}{2kT} \right) + \log \frac{N_a^{1/2}}{\left[2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \right]^{1/2}} \right\} \end{aligned}$$

$$\exp \left(\frac{E_v - E_F}{kT} \right) = \exp \left(\frac{E_v - E_a}{2kT} \right) \cdot \frac{N_a^{1/2}}{\left[2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \right]^{1/2}} \rightarrow (5)$$

Substitute eqn (5) in equation (4)

$$p = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \left[\exp \left(\frac{E_v - E_a}{2kT} \right) \right] \frac{N_a^{1/2}}{\left[2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{3/2} \right]^{1/2}}$$

∴ The density of holes in the valence band is proportional to the square root of the acceptor concentration at moderately low temperatures. As we have at higher temperature intrinsic behaviour predominates and the contribution due to acceptor atoms becomes insignificant.

Electrical Conduction in Extrinsic Semiconductors:-

The thermal equilibrium may be disturbed by an external agent and the chaotic* motion of charge carriers may acquire a directional movement leading to flow of current in the material. Electric field, temperature and concentration gradients are examples of such disturbing agents.

Drift Current:- Under the action of an electric field, the charge carriers in the semiconductor material drift and produce drift current. The two types of charge carriers, namely electrons and holes produce two drift current components.

The electron drift in the conduction band produces a component J_e given by

$$J_e(\text{drift}) = n e \mu_e E$$

* Chaotic: Complete disorder and confusion

The hole drift in the valence band causes a component J_h given by $J_h(\text{drift}) = p e \mu_h E$

Although electrons and holes flow in opposite directions, the direction of conventional current flow due to both the carriers is in the same direction. Hence their contributions add to each other.

The total drift density is given by

$$J(\text{drift}) = J_e(\text{drift}) + J_h(\text{drift})$$

$$J(\text{drift}) = e (n \mu_e + p \mu_h) E$$

two variables

formal

(i) The Carriers Concentration

(ii) The Electric field

~~minimum current~~ In a volume of Solid, there may occur situation, where, there would be more charged particles on one side of an arbitrary Surface of than on the other side.

A directed movement of Charge Carriers constitute an electric current. Diffusion takes place due to the existence of a non-uniform Concentration of Carriers.

According to Fick's ~~law~~ law the diffusion flux F i.e. the particle current is proportional to and in a direction opposite to the Concentration gradient of the particles.

Mathematically $\therefore F \propto -\frac{dn}{dx}$

$$\therefore F = -D \frac{dn}{dx}$$

where D is diffusion Current

If n and p are electron and hole Concentration, then the flux densities of electrons and hole J_e and J_h can be written

as $J_e = -D_n \frac{dn}{dx}$ and $J_h = -D_p \frac{dp}{dx}$

where D_n and D_p are diffusion constants of the electrons and holes respectively.

$\therefore$ Diffusion Current densities

$$J_n(\text{diff}) = e D_n \frac{dn}{dx}$$

$$J_p(\text{diff}) = -e D_p \frac{dp}{dx}$$

where $+e$ and $-e$ are the charges of the hole and electron respectively.

Einstein's Equation

Einstein showed the direct relation between the mobility (μ) and diffusion (D) of a semiconductor. Suppose a semiconductor is in equilibrium at some temperature T K, with no applied electric field. The free electrons and holes distribution is uniform and there is no net current flow in any direction. Any disturbance in the carrier concentration leads to diffusion current in the material. As the material is at some temperature T K, it possesses electrical resistance due to thermal vibrations of ions. The product of resistance and diffusion current results in voltage and hence electric field in the material. Therefore, in equilibrium the drift and diffusion currents due to excess concentrations are equal. So, for electrons

$$(\Delta n) e \mu_n E = D_n e \frac{\partial(\Delta n)}{\partial x} \rightarrow (1)$$

The force (F) on excess electrons (Δn) to restore equilibrium is given by the product of Δn and electric force eE on each electron.

$$\therefore F = (\Delta n) e E \rightarrow (2)$$

$$\therefore \text{From (1) \& (2)} \quad (\Delta n) e E = \frac{D_n}{\mu_n} e \frac{\partial(\Delta n)}{\partial x} = F \rightarrow (3)$$

At temperature T K, the force on excess electron to maintain equilibrium depends on thermal energy of excess electrons times concentration gradient $\frac{\partial(\Delta n)}{\partial x}$

$$\therefore F = k_B T \frac{\partial(\Delta n)}{\partial x} \rightarrow (4)$$

$$\text{From equations (3) \& (4)} \quad \frac{D_n}{\mu_n} e \frac{\partial(\Delta n)}{\partial x} = k_B T \frac{\partial(\Delta n)}{\partial x}$$

$$\Rightarrow \frac{D_n}{\mu_n} = \frac{k_B T}{e} \quad (\text{or}) \quad D_n = \mu_n \frac{k_B T}{e} \rightarrow (5)$$

$$\text{For holes} \quad D_p = \mu_p \frac{k_B T}{e} \rightarrow (6)$$

balances the Lorentz force. After some time of the applied voltage, both the forces become equal in magnitude and act in opposite direction, then the potential difference between the top and bottom faces is equal to 'Hall voltage' and that can be measured.

At equilibrium, the Lorentz force ~~F_L~~ ^{on a} on a Carrier

$$F_L = Bev_d \rightarrow (1)$$

and the Hall force $F_H = eE_H \rightarrow (2)$ ~~eE_H~~

E_H is the Hall electric field due to accumulated charge.

At equilibrium $F_H = F_L$

$$eE_H = Bev_d$$

$$\therefore E_H = BV_d \rightarrow (3)$$

If 'd' is the distance between the upper and lower surfaces of the slab. then the Hall field

$$E_H = \frac{V_H}{d} \rightarrow (4)$$

In n-type material $J_x = -nev_d$

$$\Rightarrow V_d = -\frac{J_x}{ne} \rightarrow (5)$$

where 'n' is free electron concentration.

Substitute eqn (5) in (3)

$$E_H = -\frac{BJ_x}{ne} \rightarrow (6)$$

For a given semiconductor, the Hall field E_H is proportional to the current density J_x and the intensity of magnetic field 'B' in the material.

$$E_H \propto J_x B \Rightarrow E_H = R_H J_x B \rightarrow (7)$$

where R_H = Hall coefficient.

From equations (6) & (7) $R_H J_x B = -\frac{BJ_x}{ne}$

$$R_H = -\frac{1}{ne}$$