VSM COLLEGE OF ENGINEERING
EEE DEPARTMENT

LECTURE NOTES

ON

ELECTRICAL CIRCUIT ANALYSIS-I
I B.Tech II semester

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SYLLABUS

UNIT-I
Introduction to Electrical Circuits
Passive components and their V-I relations. Sources (dependent and independent) - Kirchoff's laws, Network reduction techniques(series, parallel, series - parallel, star-to-delta and delta-to-star transformation). source transformation technique, nodal analysis and mesh analysis.

UNIT-II
Single Phase A.C Systems
Periodic waveforms (determination of rms, average value and form factor). Concept of phase angle and phase difference.
Complex and polar forms of representations, steady state analysis of R, L and C circuits.Power Factor and its significance – Real, Reactive power and apparent Power.

UNIT-III
Resonance
Locus diagrams for various combination of R, L and C. Resonance, concept of band width and Quality factor.

UNIT-IV
Magnetic Circuit
Basic definition of MMF, flux and reluctance. Analogy between electrical and magnetic circuits.
Faraday's laws of electromagnetic induction Concept of self and mutual inductance.
Dot convention-coefficient of coupling and composite magnetic circuit.
Analysis of series and parallel magnetic circuits.

UNIT-V
Network topology
Definitions of Graph and Tree. Basic cutset and tieset matrices for planar networks. Loop and nodal methods of analysis of networks with dependent and independent voltage and current sources. Duality and Dual networks.

UNIT-VI
Network theorems (DC & AC Excitations)
Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum Power Transfer theorem, Reciprocity theorem, Millman's theorem and compensation theorem.

UNIT-I
Introduction to Electrical Circuits

Introduction

Today's engineering graduates are no longer employed solely to work on the technical design aspects of engineering problems.

Their efforts now extend beyond the creation of better computers and communication systems etc…..

To contribute to the solution of engineering problems an engineer must acquire many skills, one of which is a knowledge of electric and electronic circuits analysis.

They take a fundamental understanding of various scientific principles, combine this with practical knowledge often expressed in mathematical terms and with little creativity arrive at a solution.

1.1Electric Circuit:

Electric circuit can be defined as an interconnection between components or electrical devices for the purpose of communicating or transferring energy from one point to another.

The components of electric circuit are always referred to as **circuit elements**

1.2 Circuit element definition:

It is important to differentiate between the physical device itself and the mathematical model which we will use to analyze its behavior in a circuit. The model is only an approximation.

Expression use the circuit element to refer to the mathematical model.

All simple circuit elements that we will consider can be classified according to the relationship of the current through the element to the voltage across the element.

Dependant sources are used a great deal in electronics to model both DC and AC behavior of transistors, especially in amplifier circuits.

1.3 The derived unit commonly used in electric circuit theory

Quantity	Unit	Symbol
electric charge	coulomb	C
electric potential	volt	V
resistance	ohm	Ω
conductance	siemens	S
inductance	henry	H
capacitance	farad	F
frequency	hertz	Hz
force	newton	N
energy, work	joule	J
power	watt	W
magnetic flux	weber	Wb
magnetic fiux density	tesla	T

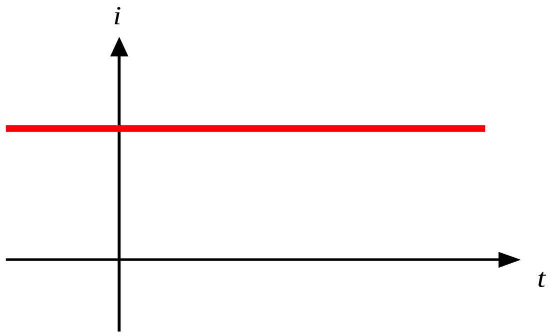
1.4 ELECTRIC CHARGE:

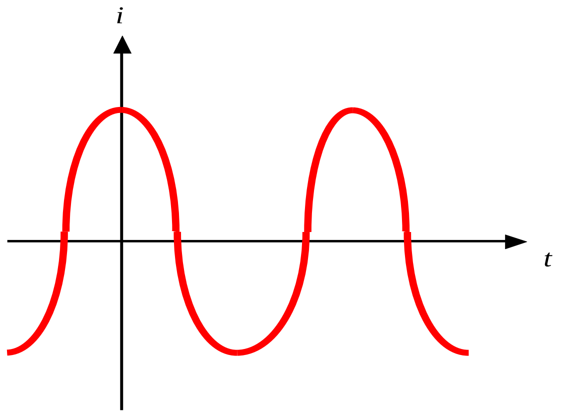
- Electric charge is an electrical property of the atomic particles of which matter consists measured In coulombs (C).
- Electric charge create electric field of force.
- The charge Q on one electron is negative and equal in magnitude $1.602 \times 10^{-19} C$ which is called as electronic charge.

1.5 CURRENT :

Current is defined as the movement of charge in a specified direction.
Electric current i = dq/dt.
An Ampere = Coulomb per Second

Types Of Current





Direct current **Alternating current**

A)Example

A conductor has a constant current of 5 A. How many electrons pass a fixed point on the conductor in one minute**?**

Solution

Total no. of charges, pass in 1 min is given by

5 A = (5 C/s)(60 s/min) = 300 C/min

$$\frac{300 \text{ C/min}}{1.602x10^{-19} \text{ C/electron}} = 1.87x10^{21} \text{ electrons}/\text{min}$$

1.5 Voltage:

- Voltage is the electric pressure or force that causes current.
- It is a potential energy difference between two points.
- It is also known as an electromotive force (EMF).
- A Volt = Joule per Second

1.6 Resistance :

Resistance is the opposition a material offers to current

Resistance is determined by

Type of material (resistivity)
Temperature of material
Cross-sectional area
Length of material

- **Resistance Relationships**

Resistance =($resistivity * length)/area$

$R=\rho L/A$

ohm = Volt per Ampere

1.7 POWER:

Power is the rate of using energy or doing work.

- **Work (*W*): consists of a force moving through a distance.**
- **Energy (*W*): is the capacity to do work.**
- **Joule (J) : is the base unit for both energy and work.**
- **A watt = Joule per second.**
- **Power = 200 Watts**

1.8 Active Element And Passive Element

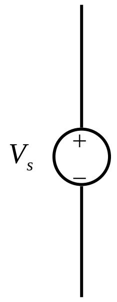
Active Element– elements capable of generating electrical Energy.
- Current source
- Voltage source

Passive Element– elements not capable of generating electrical energy.

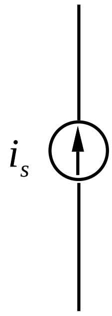
- **Resistor** (dissipates energy)
- **Capacitor and Inductor** (can store or release energy)

1.9 Independent Sources

Voltage Source maintains a specified voltage between its terminals but has no control on the current passing through it. The symbol of the independent voltage source is a plus-minus sign enclosed by a circle

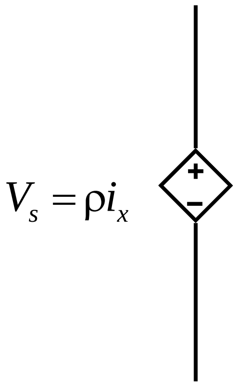


Current Source maintains a specified current through its terminals but has no control on the voltage across its terminals. The symbol of the independent current source is an arrow enclosed by a circle.

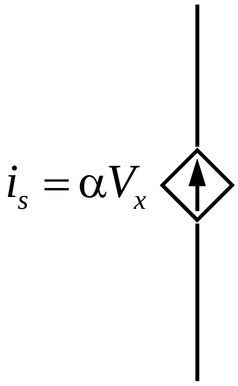


1.10 Dependent Source

- The voltage source has a specified voltage between its terminals but it is dependable on some other variable defined somewhere in the circuit.
- The symbol for the dependent voltage source is a plus-minus sign enclosed by a diamond shape.



- This kind of current source has a specified current between its terminals but it is dependent on some other variable defined somewhere in the circuit.
- The symbol for the dependent current source is an arrow enclosed by a diamond shape.



1.11 Controlled sources

Current controlled current source : Current ratio= $\alpha = \frac{i_2}{i_1}$

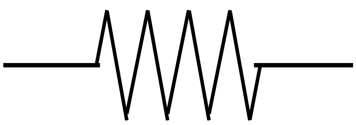
Voltage controlled current source : Transconductance = $g_m = \frac{i_2}{v_1}$

Voltage controlled voltage source : Voltage ratio= $\mu = \frac{v_2}{v_1}$

Current controlled voltage source: Transresistance= $r_m = \frac{v_2}{i_1}$

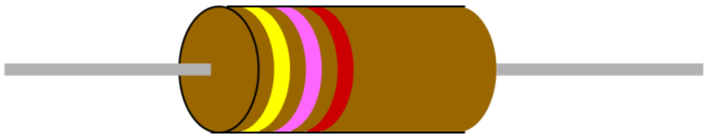
1.12 Resistor:

Resistor is passive element that dissipates electrical energy.
Linear resistor is the resistor that obeys Ohm's law.



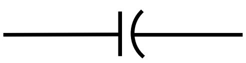
UNIT: Ohm (Ω)

1.13 Resistor colour code:



Color	Color Name	1st Digit 1st Stripe	2nd Digit 2nd Stripe	Multiplier 3rd Stripe	Tolerance 4th Stripe
	Black	0	0	x1	-
	Brown	1	1	x10	-
	Red	2	2	x100	-
	Orange	3	3	x1,000	-
	Yellow	4	4	x10,000	-
	Green	5	5	x100,000	-
	Blue	6	6	x1,000,000	-
	Violet	7	7	-	-
	Gray	8	8	-	-
	White	9	9	-	-
	Gold	-	-	-	5%
	Silver	-	-	-	10%

1.14 Capacitor:



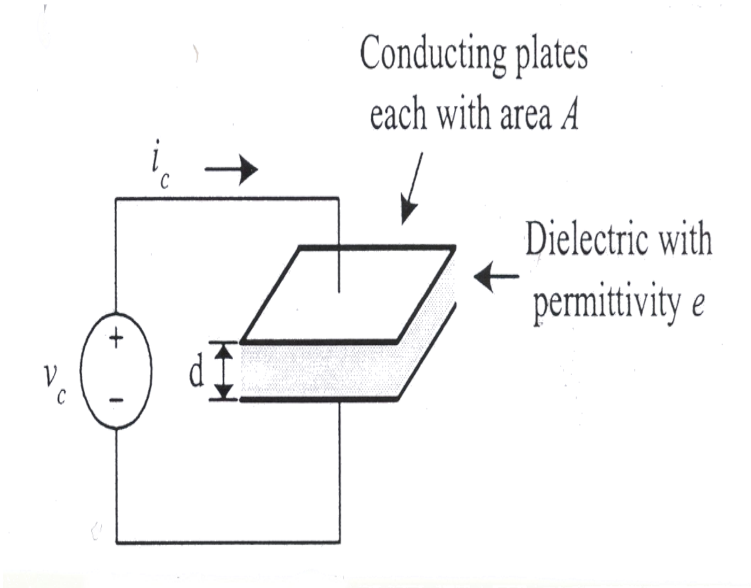
UNIT: Farad (F)

- ❖ Electrical component that consists of two conductors separated by an insulator or dielectric material.

Its behavior based on phenomenon associated with electric fields, which the source is voltage.

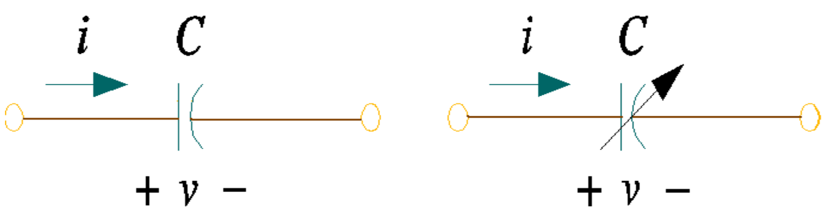
- ❖ A time-varying electric fields produce a current flow in the space occupied by the fields.
- ❖ Capacitance is the circuit parameter which relates the displacement current to the voltage.

A capacitor with an applied voltage:



Plates – aluminum foil
Dielectric – air/ceramic/paper/mica

Circuit symbols for capacitors



(a) Fixed capacitor (b) Variable capacitor

Circuit parameters

- ❖ The amount of charge stored, q = CV.
- ❖ C is capacitance in Farad, ratio of the charge on one plate to the voltage difference between the plates.
- ❖ But it does not depend on q or V but capacitor's physical dimensions i.e.,

ε=permeability of dielectric in Wb/Am
A = surface area of plates in m^2
d = distance between the plates m

$$C = \frac{\varepsilon A}{d}$$

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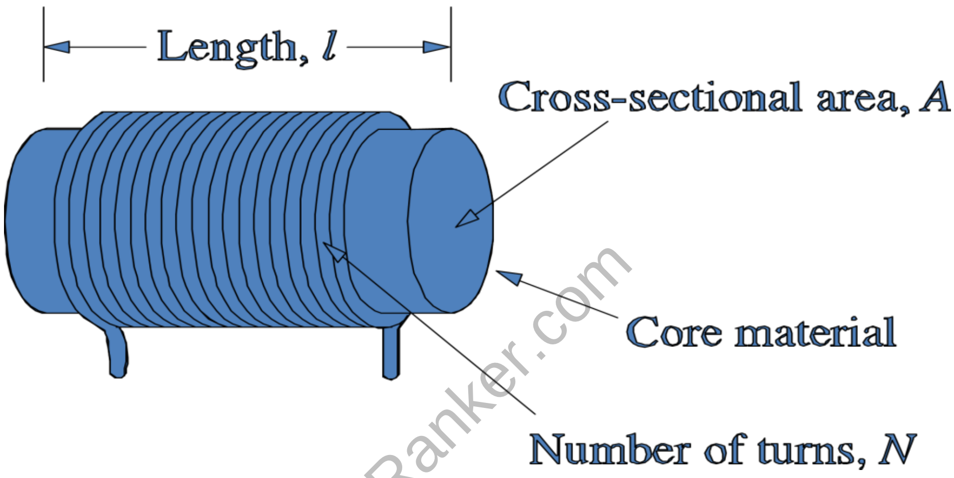
1.15 Inductor:

UNIT: Henry (H)

- ❖ Electrical component that opposes any change in electrical current.
- ❖ Composed of a coil or wire wound around a non-magnetic core/magnetic core.
- ❖ Its behavior based on phenomenon associated with magnetic fields, which the source is current.
- ❖ A time-varying magnetic fields induce voltage in any conductor linked by the fields.
- ❖ Inductance is the circuit parameter which relates the induced voltage to the current.

Typical form of an inductor



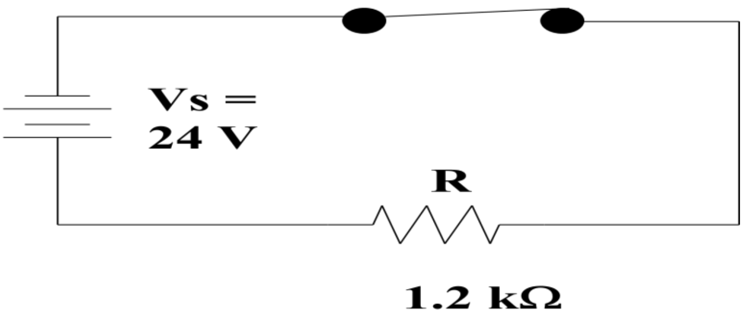
1.16 OHM'S LAW:

- ➢ Georg Simon Ohm (1787-1854) formulated the relationships among voltage, current, and resistance as follows:
- ➢ The current in a circuit is directly proportional to the applied voltage and inversely proportional to the resistance of the circuit.

$$V = IR$$

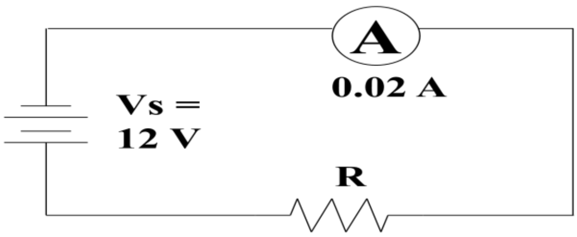
Examples:

Calculating Current



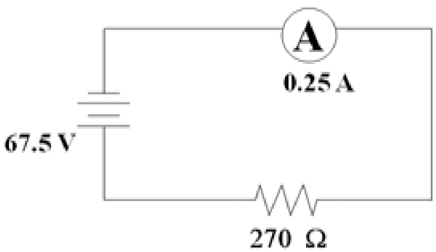
$$I = \frac{V}{R} = \frac{24}{1.2*1000} = 20mA$$

Calculating Resistance



$$R = \frac{V}{I} = \frac{12}{0.02} = 600ohm$$

Calculating Power



$P=V*I=67.5*0.25=16.9kW$
$P=I^2R=0.25^2*270=16.9KW$
$P=V^2/R=67.5^2/270=16.9KW$

1.17 KIRCHHOFF'S Law:

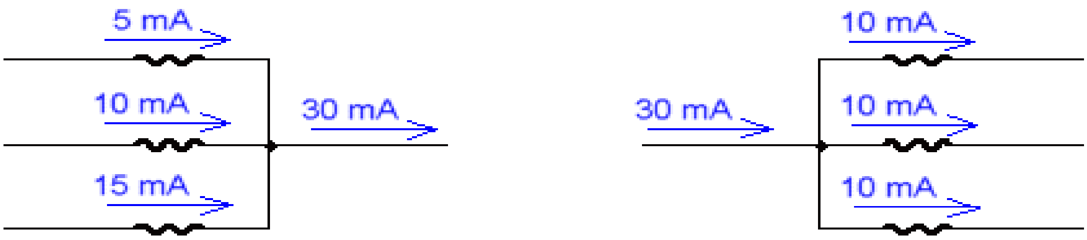
- ➢ Gustav Robert Kirchhoff (1824 – 1887)
- ➢ Models relationship between:
  - ❖ circuit element and currents (KCL)
  - ❖ circuit element and voltages (KVL)
  - ❖ He introduces two laws:
  - ❖ Kirchhoff Current Law (KCL)
  - ❖ Kirchhoff Voltage Law (KVL)

Kirchhoff's Current Law (KCL):

- ❖ Current entering node = current exiting
- ❖ Convention: +i is exit.ing, -i is entering
- ❖ For any circuit node:

$$\sum i = 0$$

Kirchhoff's Current Law (KCL):



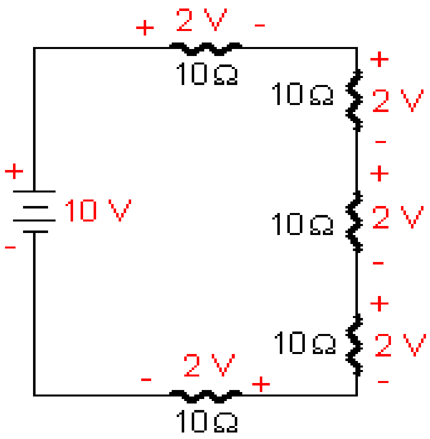
Kirchhoff's Current Law (KCL) states that the algebraic sum of current entering a node must be equal to that of leaving the same node.

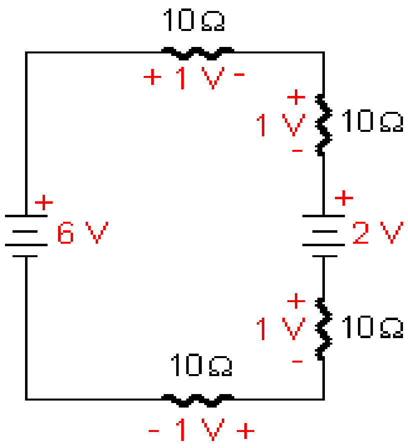
Kirchhoff's Voltage Law (KVL)

- ❖ **voltage increases = voltage decreases**
- ❖ **Convention: hit minus (-) side first, write negative**
- ❖ **For any circuit loop:**

$$\sum v = 0$$

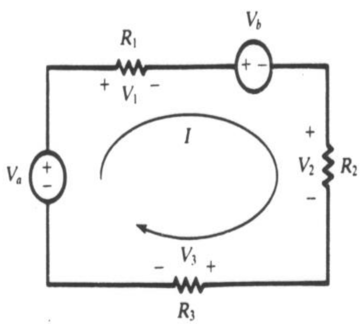
Kirchhoff's Voltage Law (KVL)





Kirchhoff's Voltage Law states that the algebraic sum of voltage drop in a loop must be equal to that of voltage rise in the same loop.
Stated it in a different way is that the algebraic sum of all voltages around a loop must be zero.

<u>Example</u>
Applying the KVL equation for the circuit of the figure below



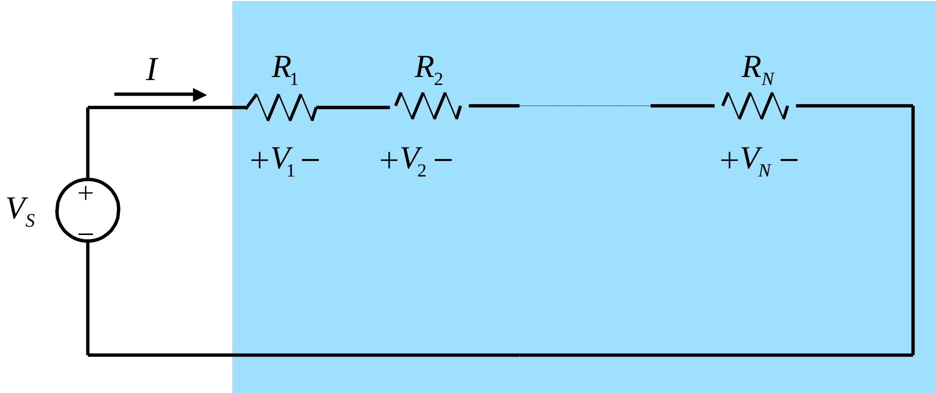
$v_a - v_1 - v_b - v_2 - v_3 = 0$

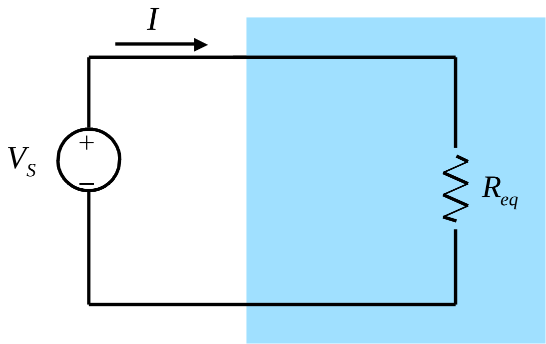
$V_1 = IR_1 \quad v_2 = IR_2 \quad v_3 = IR_3$

$v_a - v_b = I(R_1 + R_2 + R_3)$

$$I = \frac{v_a - v_b}{R_1 + R_2 + R_3}$$

1.18 Series And Parallel Circuit:





- The equivalent resistance for any number of resistors in series connection is the sum of each individual resistor.

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots + R_N$$

Current in Series Circuit:
Current in series circuit is the same as in each circuit element.

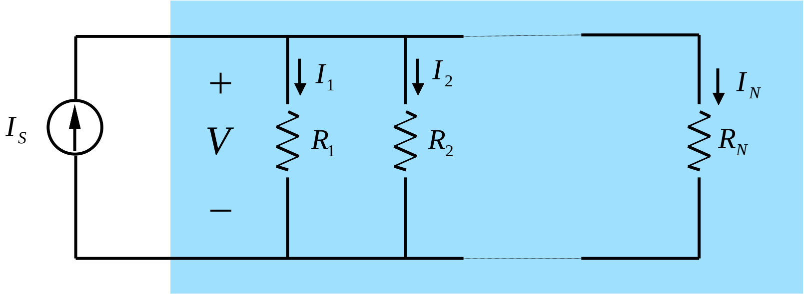
$$I = I_1 = I_2 = I_N$$

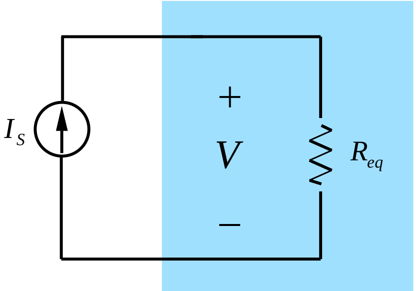
Voltage In Series Circuit:

Voltage (V_T) in series circuit is the total voltage of each element circuit.

$$V_T = V_1 + V_2 + .. + V_N$$

Resistor below is arranged in parallel connection:





The equivalent resistance for any number of resistors in parallel connection is obtained by taking the reciprocal of the sum of the reciprocal of each single resistor in the circuit.

Equivalent resistance:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

$$R_{eq} = \frac{1}{1/R_1 + 1/R_2 + ... + 1/R_N}$$

1.19For the circuit which have two resistors in parallel connection:

$$R_{eq} = R_1 \| R_2$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

Current In Parallel Circuit

Current in series circuit is equal to the total current for each element circuit

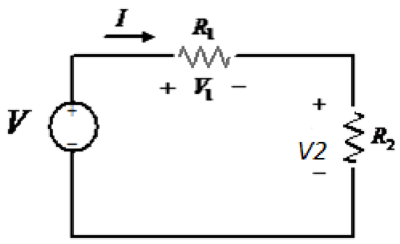
$$I = I_1 + I_2 + .. + I_N$$

Voltage In Parallel Circuit

Voltage (V_T) in series circuit is the same as for each element circuit

$$V_T = V_1 = V_2 = V_N$$

1.20 Voltage Divider:



Whenever voltage has to be divided among resistors in series use voltage divider rule principle.

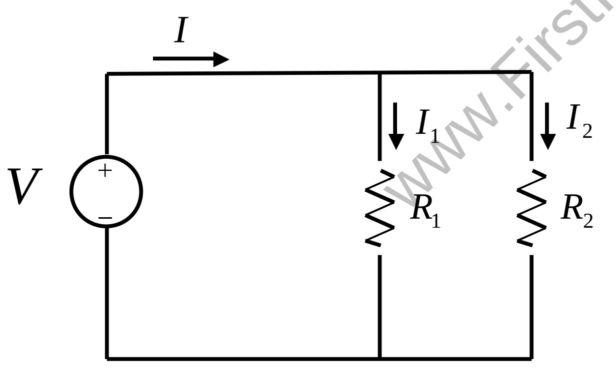
By applying Ohm's Law:

$$I = \frac{V}{R_1 + R_2} \qquad V_2 = R_2 I$$

Voltage at resistor R2:

$$V_2 = R_2\left(\frac{V}{R_1 + R_2}\right) = V\left(\frac{R_2}{R_1 + R_2}\right)$$

1.21Current Divider:



Whenever current has to be divided among resistors in parallel, use current divider rule principle.

By applying Ohm's Law:

$$V = I_1 R_1 = I_2 R_2 = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

So, to find current, I_1 and I_2 :

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I$$
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I$$

1.21 Mesh Analysis

- Mesh – the smallest loop around a subset of components in a circuit
- Technique to find voltage drops around a loop using the currents that flow within the loop, Kirchoff's Voltage Law, and Ohm's Law.
- multiple meshes are defined so that every component in the circuit belongs to one or more meshes

Steps in Mesh Analysis:

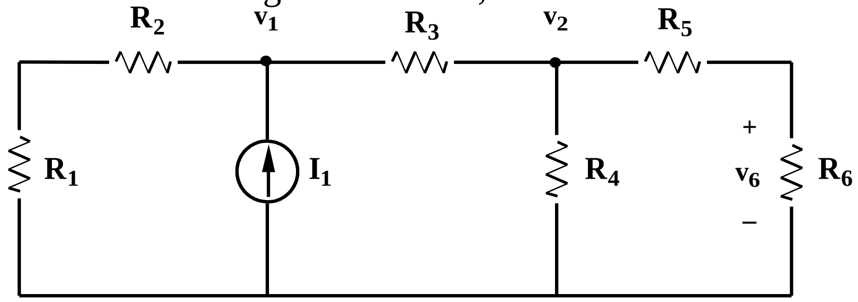
1. Identify all of the meshes in the circuit
2. Label the currents flowing in each mesh
3. Label the voltage across each component in the circuit
4. Write the voltage loop equations using Kirchoff's Voltage Law.
5. Use Ohm's Law to relate the voltage drops across each component to the sum of the currents flowing through them.
6. Solve for the mesh currents
7. Once the mesh currents are known, calculate the voltage across all of the components.

1.22 Nodal Analysis

- **Technique to find currents at a node using Ohm's Law and the potential differences betweens nodes**.

Steps in Nodal Analysis:

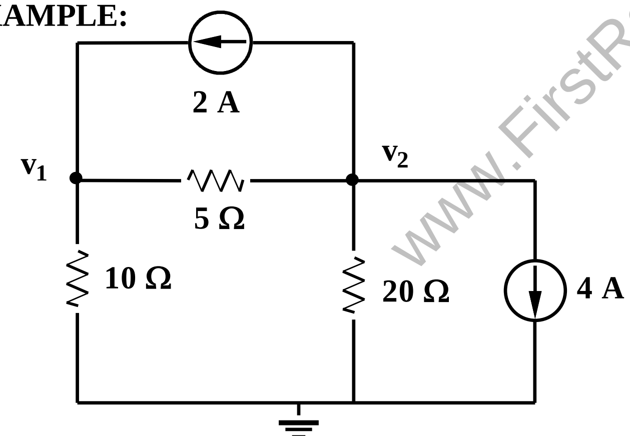
1. Pick one node as a reference node
2. Label the voltage at the other nodes
3. Label the currents flowing through each of the components in the circuit
4. Use Kirchoff's Current Law
5. Use Ohm's Law to relate the voltages at each node to the currents flowing in and out of them.
6. Solve for the node voltage
7. Once the node voltages are known, calculate the currents.



$$\frac{V_1}{R_1+R_2}+\frac{V_1-V_2}{R_3}=I_1$$

$$\frac{V_2-V_1}{R_3}+\frac{V_2}{R_4}+\frac{V_2}{R_5+R_6}=0$$

EXAMPLE:



ind V_1 and V_2 ?

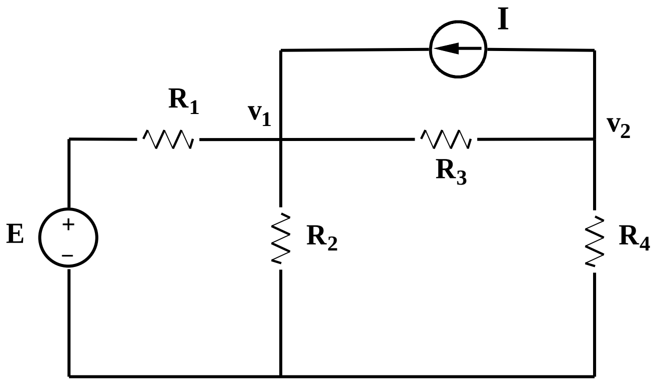
At v_1:

$$\frac{V_1}{10}+\frac{V_1-V_2}{5}=2$$

At v_2:

$$\frac{V_2-V_1}{5}+\frac{V_2}{20}=-6$$

Example:



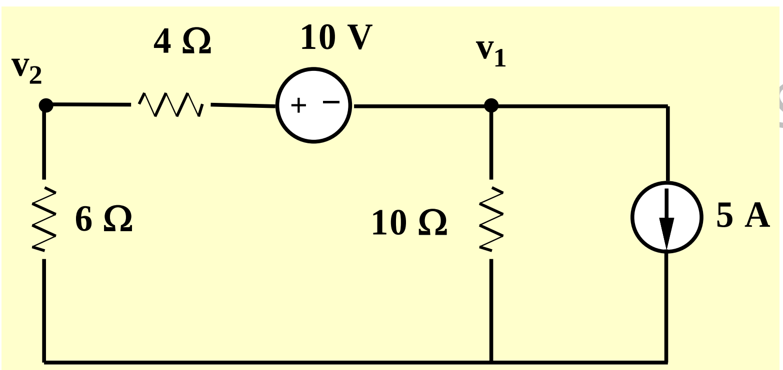
At V1:

$$\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} = I$$

At V2:

$$\frac{V_2}{R_4} + \frac{V_2 - V_1}{R_3} = -I$$

EXAMPLE:



What do we do first?

At v1:

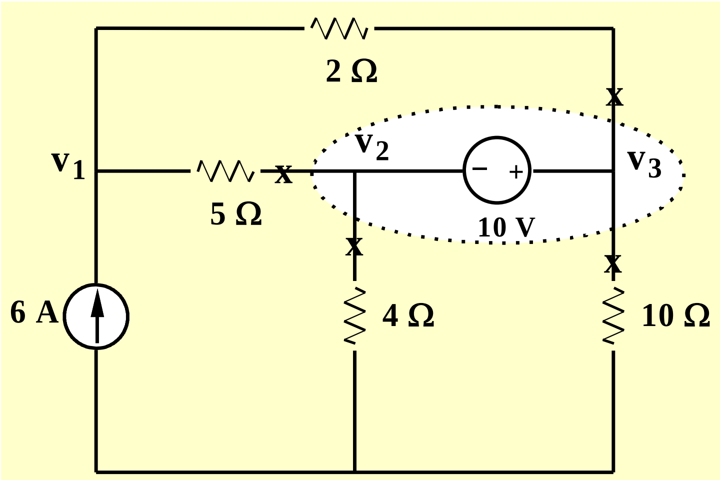
$$\frac{V_1}{10} + \frac{V_1 + 10 - V_2}{4} = -5$$

At v2:

$$\frac{V_2}{6} + \frac{V_2 - 10 - V_1}{4} = 0$$

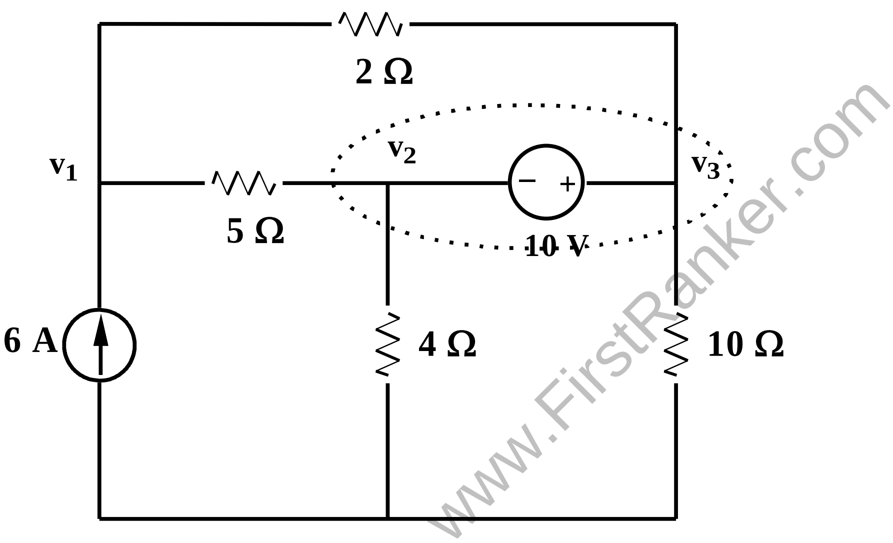
Ans: $V_1 = 30$ V, $V_2 = 12$ V, $I_1 = 2$ A

EXAMPLE:



When a voltage source appears between two nodes, an easy way to handle this is to form a super node.

The Super Node encircles the voltage source and the tips of the branches connected to the nodes.



Constraint Equation

$V_2 - V_3 = -10$

At V_1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2} = 6$$

At super node

$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0$$

$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0$$

$-14V_1 + 9V_2 + 12V_3 = 0$

$V_2 - V_3 = -10$

Solving gives:

$V_1 = 30$ V, $V_2 = 14.29$ V, $V_3 = 24.29$ V

UNIT-II
Single Phase A.C Systems

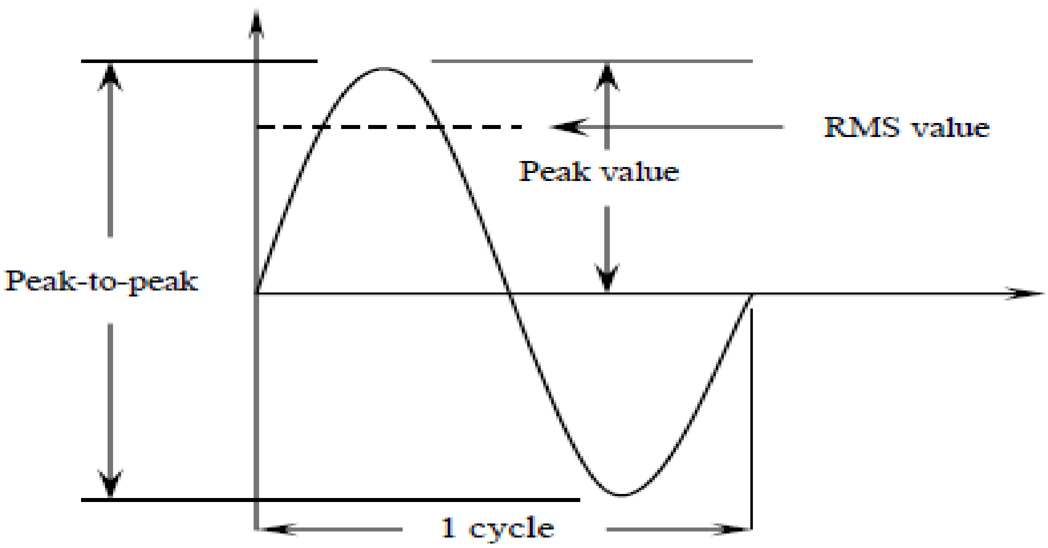
2.1 INTRODUCTION:

- The majority of electrical power in the world is generated, distributed and consumed in the form of 50 Hz or 60-Hz sinusoidal alternating current (AC) and voltage.
- It is used for household and industrial applications such as television sets, computers, microwave ovens, electric stoves, to the large motors used in the industry.
- AC has several advantages over DC. The major advantage of AC is the fact that it can be transformed, however, DC cannot.

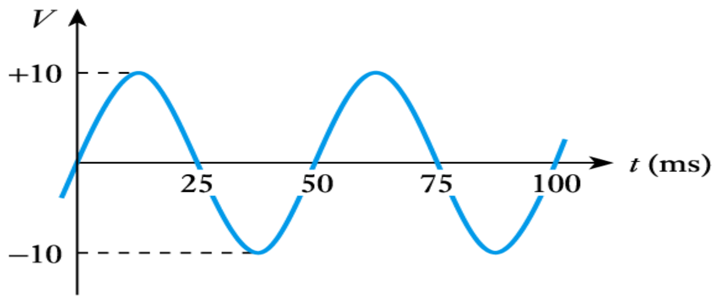
- A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of kV) is that less current is required to produce the same amount of power. Less current permits smaller wires to be used for transmission.
- AC unlike DC flows first in one direction then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform. Sine waves are the signal whose shape neither is nor altered by a linear circuit, therefore, it is ideal as a test signal.
- In discussing AC signal, it is necessary to express the current and voltage in terms of maximum or peak values, peak-to-peak values, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage.

The correspondence mathematical form of sinusoidal AC signal is

$$v(t) = V_P \cos(\omega t + \theta)$$



Example – Determine the equation of the following voltage signal.



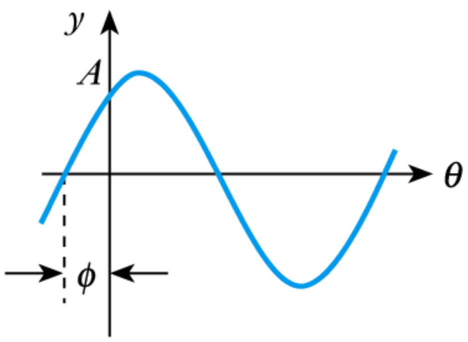
from diagram:

Period is 50 ms = 0.05 s
Thus f = 1/T =1/0.05 = 20 Hz
Peak voltage is 10 V
Therefore

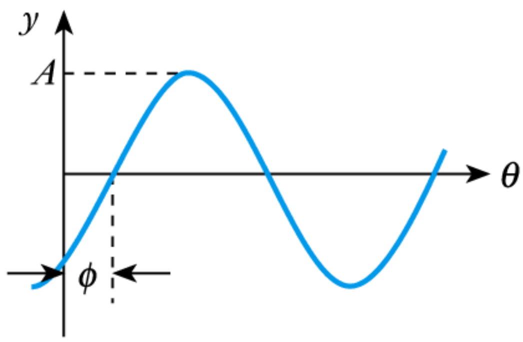
$$v = V_p \sin 2\pi f t$$
$$= 10 \sin 2\pi 20 t$$
$$= 10 \sin 126 t$$

2.2 Phase angles

- **the expressions given above assume the angle of the sine wave is zero at t = 0**
- **if this is not the case the expression is modified by adding the angle at t = 0**



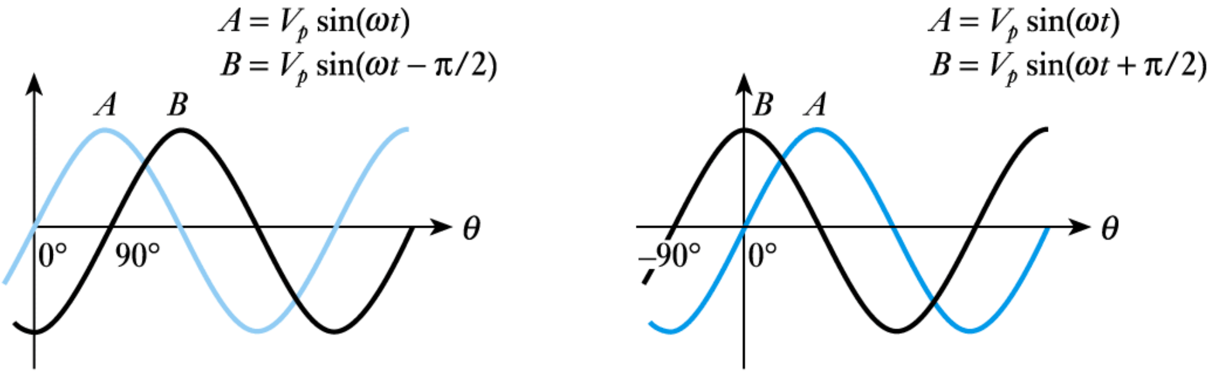
(a) $y = A \sin(\omega t + \phi)$



(b) $y = A \sin(\omega t - \phi)$

2.3 Phase difference

- two waveforms of the same frequency may have a constant phase difference
- we say that one is phase-shifted with respect to the other



(a) *B* lags *A* by 90° (b) *B* leads *A* by 90°

2.4 Average value of a sine wave

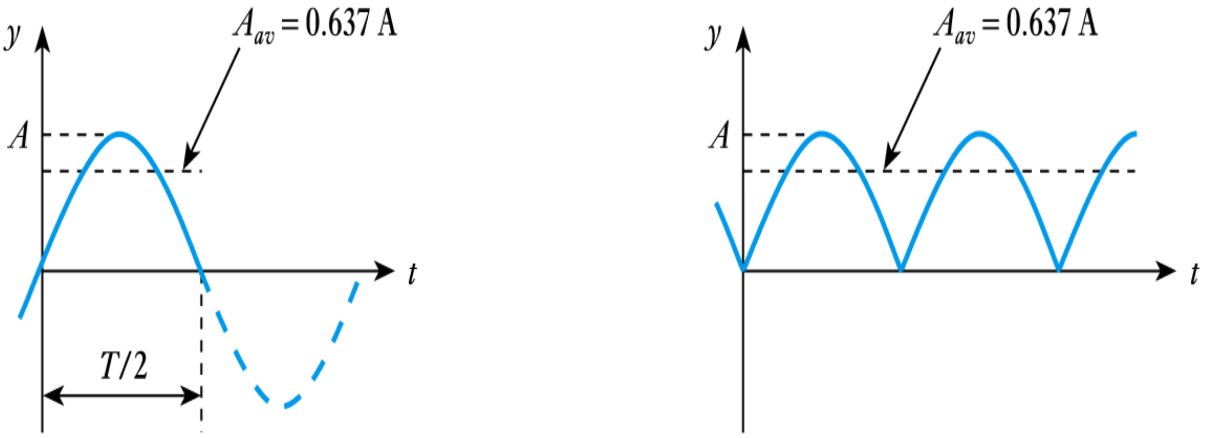
- average value over one (or more) cycles is clearly zero
- however, it is often useful to know the average magnitude of the waveform independent of its polarity
- we can think of this asthe average value over half a cycle…
- … or as the average value of the rectified signal

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_p \sin\theta \, d\theta$$

$$= \frac{V_p}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{2V_p}{\pi} = 0.637 \times V_p$$

2.5 Average value of a sine wave



(a) Average value over half a cycle of a sine wave

(b) Average value of a rectified sine wave

2.6 r.m.s. value of a sine wave

- the instantaneous power (p) in a resistor is given by
- therefore the average power is given by

$$P_{av} = \frac{[\text{average (or mean) of } v^2]}{R} = \frac{\overline{v^2}}{R}$$

- where $\overline{v^2}$ is the **mean-square voltage**
- While the mean-square voltage is useful, more often we use the square root of this quantity, namely the root-mean-square voltage V_{rms}
- where $V_{rms} = \sqrt{\overline{v^2}}$
- we can also define $I_{rms} = \sqrt{\overline{i^2}}$

  it is relatively easy to show that

$$V_{rms} = \frac{1}{\sqrt{2}} \times V_p = 0.707 \times V_p \quad I_{rms} = \frac{1}{\sqrt{2}} \times I_p = 0.707 \times I_p$$

Voltage and Current Values for a Sine Wave

The *default* sine wave ac measurement is V_{rms} .

- r.m.s. values are useful because their relationship to average power is similar to the corresponding DC values

$$P_{av} = \frac{V_{rms}^{2}}{R}$$

$$P_{av} = V_{rms} I_{rms}$$

$$P_{av} = I_{rms}^{2} R$$

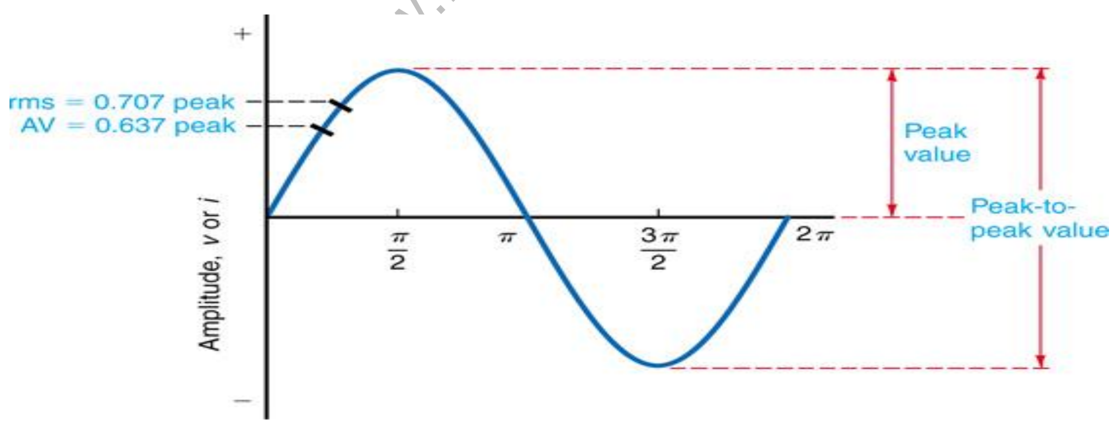
- **Form factor**

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}}$$

- for a ***sine wave*** this gives

$$\text{Form factor} = \frac{0.707 V_p}{0.637 V_p} = 1.11$$

- **Voltage and Current Values for a Sine Wave:**

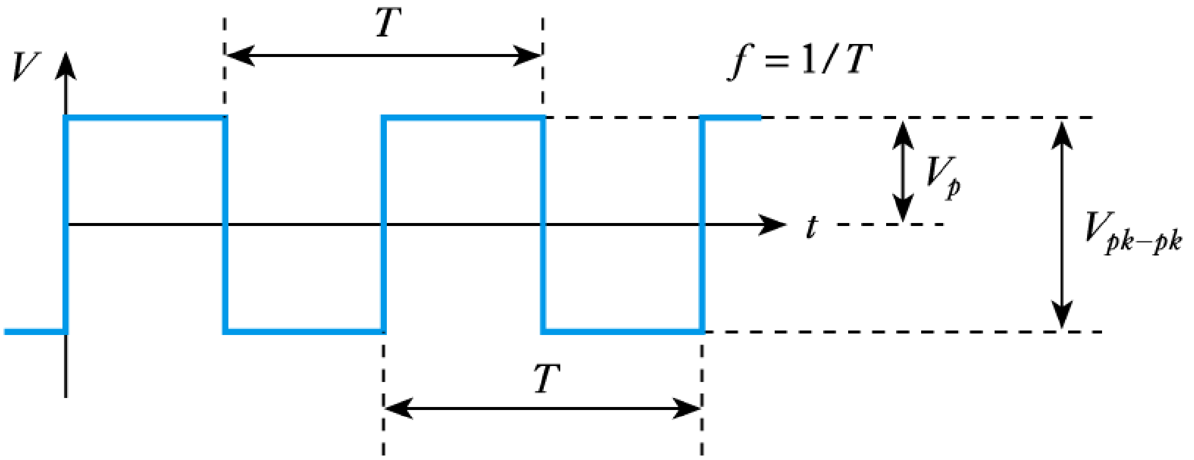


Definitions of important amplitude values for a sine wave of voltage or current

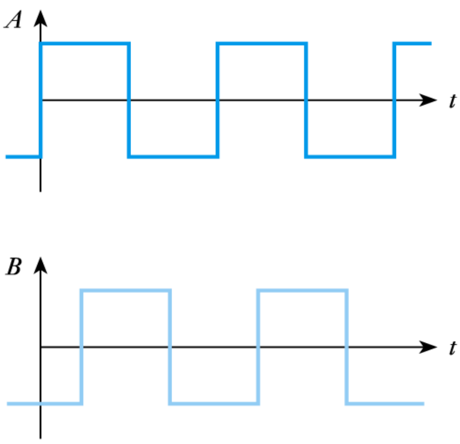
Voltage and Current Values for a Sine Waves:

- ❖ The average value is 0.637 × peak value.
- ❖ The rms value is 0.707 × peak value.
- ❖ The peak value is 1.414 × rms value.
- ❖ The peak-to-peak value is 2.828 × rms value.

2.8 Square Waves:



Phase angle we can divide the period into 360° or 2π radians useful in defining phase relationship between signals in the waveforms shown the time delay of one with respect to the other



Average and r.m.s. values

- ❖ the average value of a symmetrical waveform is its average value over the positive half-cycle

$$V_{av} = V_p$$

- ❖ thus the average value of a symmetrical square wave is equal to its peak value

similarly, since the instantaneous value of a square wave is either its peak positive or peak negative value, the square of this is the peak value

$$V_{rms} = V_p$$

Form factor and **peak factor**

from the earlier definitions, for a square wave

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}} = \frac{V_p}{V_p} = 1.0$$

$$\text{Peak factor} = \frac{\text{peak value}}{\text{r.m.s. value}} = \frac{V_p}{V_p} = 1.0$$

Root-mean squared value of

a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

Compare to the average power expression

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt$$

Apply v(t) to a resistor

$$P_{avg} = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt = \frac{1}{T}\int_{t_o}^{t_o+T}\left[\frac{v^2(t)}{R}\right]dt = \frac{1}{RT}\int_{t_o}^{t_o+T} v^2(t)dt$$

$$P_{avg} = \frac{V_{rms}^2}{R}$$

rms is based on a power concept, describing the equivalent voltage that will produce a given average power to a resistor

Root-mean squared value of a periodic waveform with period T

$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v^2(t)dt$$

For the sinusoidal case

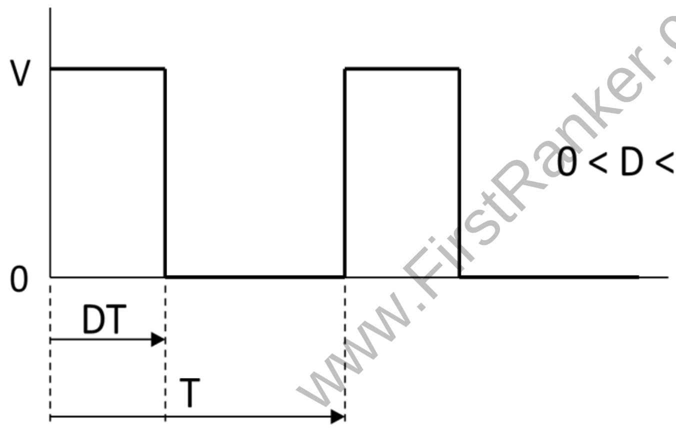
$$V_{rms}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} V^2 \sin^2(\omega_o t + \delta)dt$$

$$V_{rms}^2 = \frac{V^2}{2T}\int_{t_o}^{t_o+T} [1 - \cos 2(\omega_o t + \delta)]dt = \frac{V^2}{2T}\left[t - \frac{\sin 2(\omega_o t + \delta)}{2\omega_o}\right]_{t_o}^{t_o+T}$$

$$V_{rms}^2 = \frac{V^2}{2}, \qquad V_{rms} = \frac{V}{\sqrt{2}}$$

2.9 RMS of some common periodic waveforms:

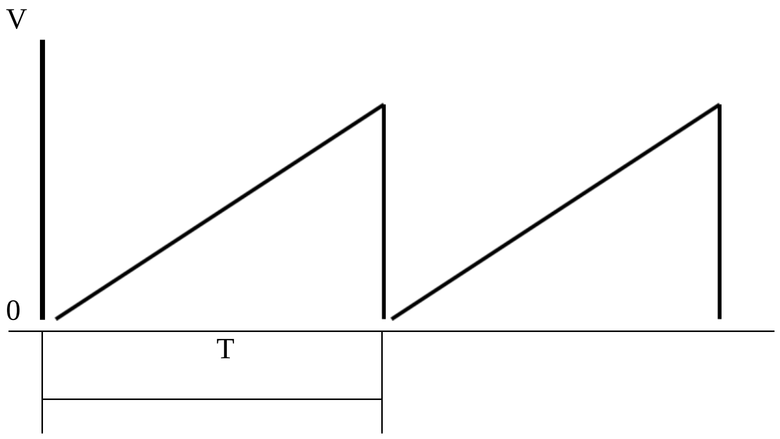
Duty cycle controller: By inspection, this is the average value of the squared waveform



$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2(t)dt = \frac{1}{T}\int_0^{DT} V^2 dt = \frac{V^2}{T} \bullet DT = DV^2$$

$$V_{rms} = V\sqrt{D}$$

2.10 Saw tooth :

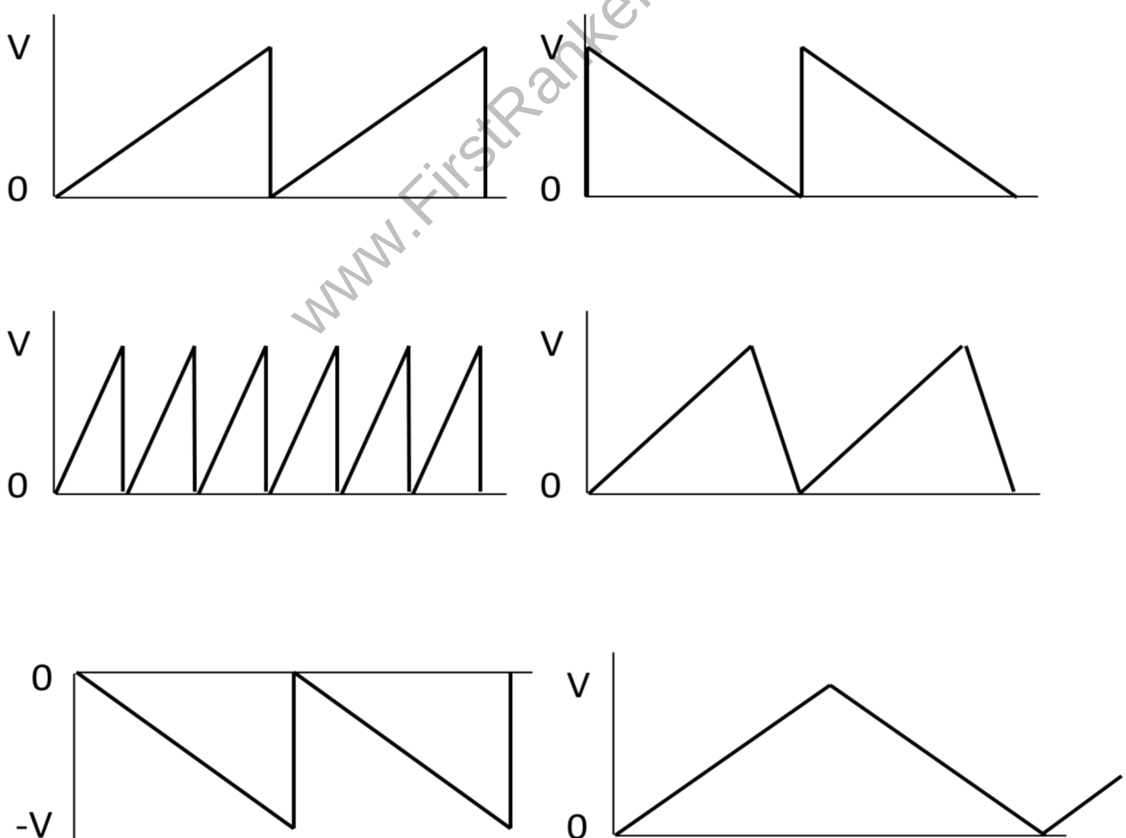


$$V_{rms}^2 = \frac{1}{T}\int_0^T \left[\frac{V}{T}t\right]^2 dt = \frac{V^2}{T^3}\int_0^T t^2 dt = \frac{V^2}{3T^3}t^3\Big|_0^T$$

$$V_{rms} = \frac{V}{\sqrt{3}}$$

2.11 RMS of common periodic waveforms,

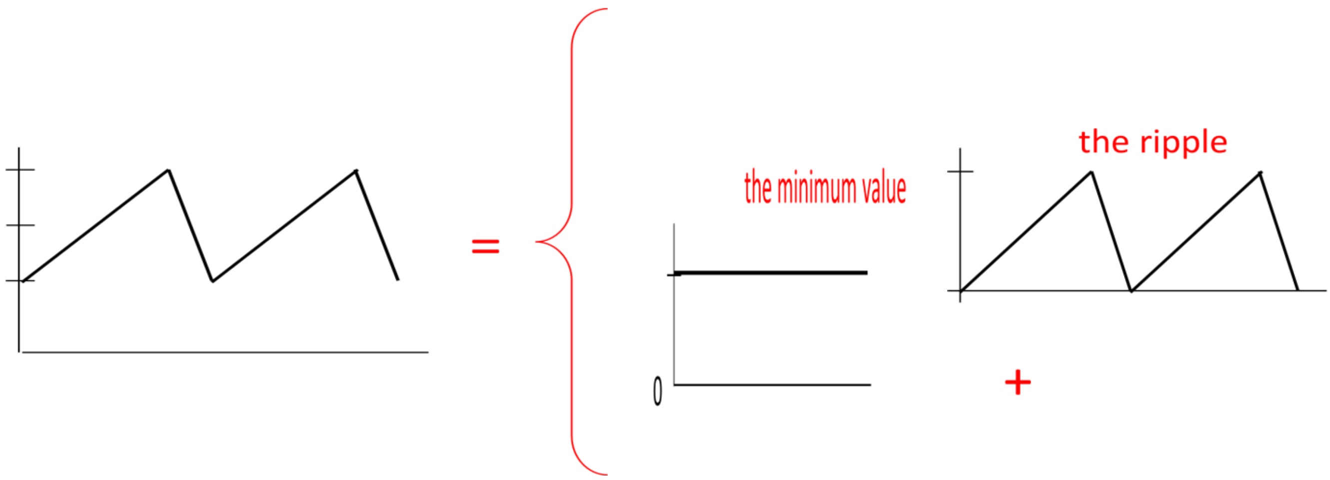
Using the power concept, it is easy to reason that the following waveforms would all produce the same average power to a resistor, and thus their rms values are identical and equal to the previous example



$$V_{rms} = \frac{V}{\sqrt{3}}$$

EXAMPLE:

Now, consider a useful example, based upon a waveform that is often seen in DC-DC converter currents. Decompose the waveform into its ripple, plus its minimum value.



$$I_{avg} = \frac{(I_{max} + I_{min})}{2}$$

$$I_{rms}^2 = Avg\{(i_\Delta(t) + I_{min})^2\}$$

$$I_{rms}^2 = Avg\{i_\Delta^2(t) + 2i_\Delta(t) \bullet I_{min} + I_{min}^2\}$$

$$I_{rms}^2 = Avg\{i_\Delta^2(t)\} + 2I_{min} \bullet Avg\{i_\Delta(t)\} + I_{min}^2$$

$$I_{rms}^2 = \frac{(I_{max} - I_{min})^2}{3} + 2I_{min} \bullet \frac{(I_{max} - I_{min})}{2} + I_{min}^2$$

define $I_{PP} = I_{max} - I_{min}$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{min} I_{PP} + I_{min}^2$$

$$I_{min} = I_{avg} - \frac{I_{PP}}{2}$$

Recognize that

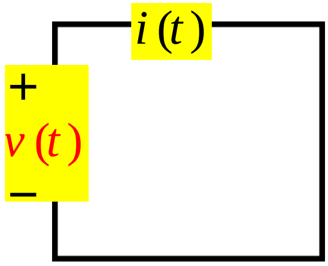
$$I_{rms}^2 = \frac{I_{PP}^2}{3} + \left(I_{avg} - \frac{I_{PP}}{2}\right) I_{PP} + \left(I_{avg} - \frac{I_{PP}}{2}\right)^2$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} + I_{avg} I_{PP} - \frac{I_{PP}^2}{2} + I_{avg}^2 - I_{avg} I_{PP} + \frac{I_{PP}^2}{4}$$

$$I_{rms}^2 = \frac{I_{PP}^2}{3} - \frac{I_{PP}^2}{4} + I_{avg}^2$$

$$I_{rms}^2 = I_{avg}^2 + \frac{I_{PP}^2}{12} \quad \Longrightarrow \quad I_{avg} = \frac{(I_{max} + I_{min})}{2}$$

2.12Instantaneous power p(t) flowing into the box



$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$p(t) = v(t) \bullet i(t)$$

$$p(t) = v(t) i(t) = \{V_m \cos(\omega t + \theta_v - \theta_i)\}\{I_m \cos(\omega t)\}$$

$$= V_m I_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$$

Since $= \cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$

Therefore $p(t) = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$

Power Calculations Summery

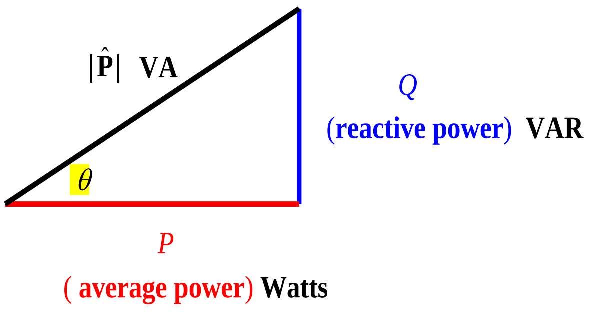
$$\hat{\mathbf{P}} = P + jQ = \frac{VI}{2}\cos(\theta_v - \theta_i) + j\,\frac{VI}{2}\sin(\theta_v - \theta_i)$$

$$\hat{\mathbf{P}} = \frac{1}{2}\,\boldsymbol{V}\boldsymbol{I}^*$$

$$= \frac{1}{2}\boldsymbol{Z}\,|\boldsymbol{I}|^2$$

$$= \frac{1}{2}\frac{|\boldsymbol{V}|^2}{\boldsymbol{Z}^*}$$

2.13 Relation Between Active Power And Reactive Power(Power Traiangle):



$$\hat{P} = P + jQ \qquad = |\hat{P}|\, e^{j\theta}$$

$$\theta = \underbrace{\theta_v - \theta_i}_{\text{power factor angle}}$$

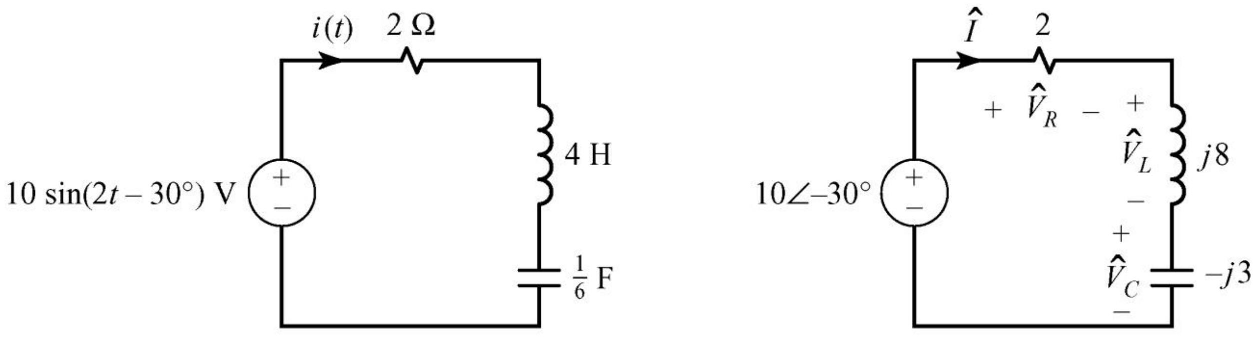
$$|\hat{P}| = \sqrt{P^2 + Q^2}$$ Is called **apparent power (VA)**

In any circuit, conservation of complex power is achieved

$$\sum_{\text{all circuit elements}} \hat{P}_i = 0$$

This implies that in any circuit, conservation of average power and Conservation of reactive power are achieved

Ex: Determine the average and reactive power delivered by the source.



(a) (b)

The phasor current leaving the source is

$$\hat{I} = \frac{10\angle -30^\circ}{2 + j8 - j3} = 1.86\angle -98.2^\circ$$

$$\hat{I} = 1.86\angle -98.2^\circ$$

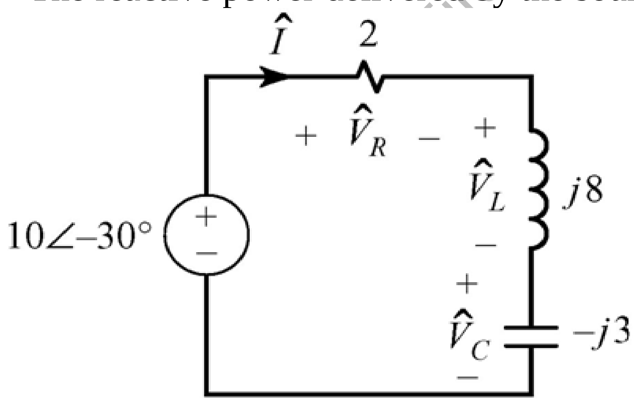
The average power delivered by the source is:

$$P_{AV,source} = \frac{1}{2}\text{Re}\left[(10\angle -30)(1.86\angle -98.2)^*\right]$$

$$= \frac{1}{2}\text{Re}\left[(10\angle -30)(1.86\angle 98.2)\right]$$

$$= 9.28\cos(-30 + 98.2) = 3.45\text{W}$$

The reactive power delivered by the source is:



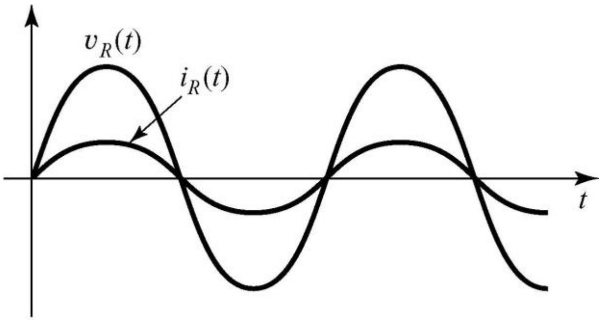
$$= 9.28\sin(-30 + 98.2) = 8.62VA$$

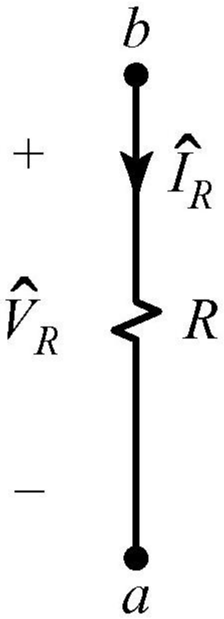
$$Q_{source} = \frac{1}{2}\text{Im}\left[(10\angle -30)(1.86\angle -98.2)^*\right]$$

And the complex power delivered by the source is

$$\hat{P}_{source} = P_{AV,source} + jQ_{source}$$

2.14 Power Relations for the Resistor



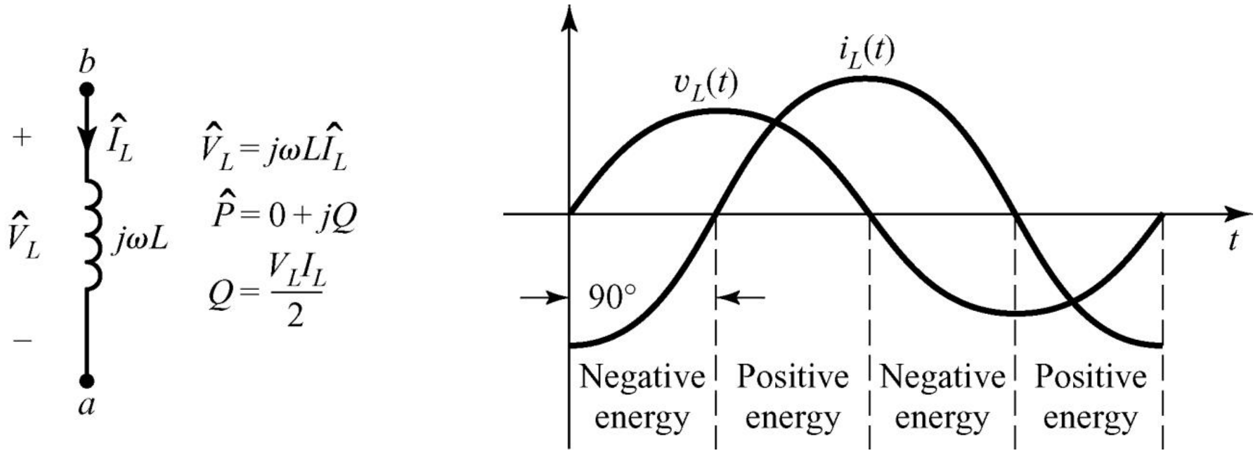


$$\hat{V}_R = R\hat{I}_R$$

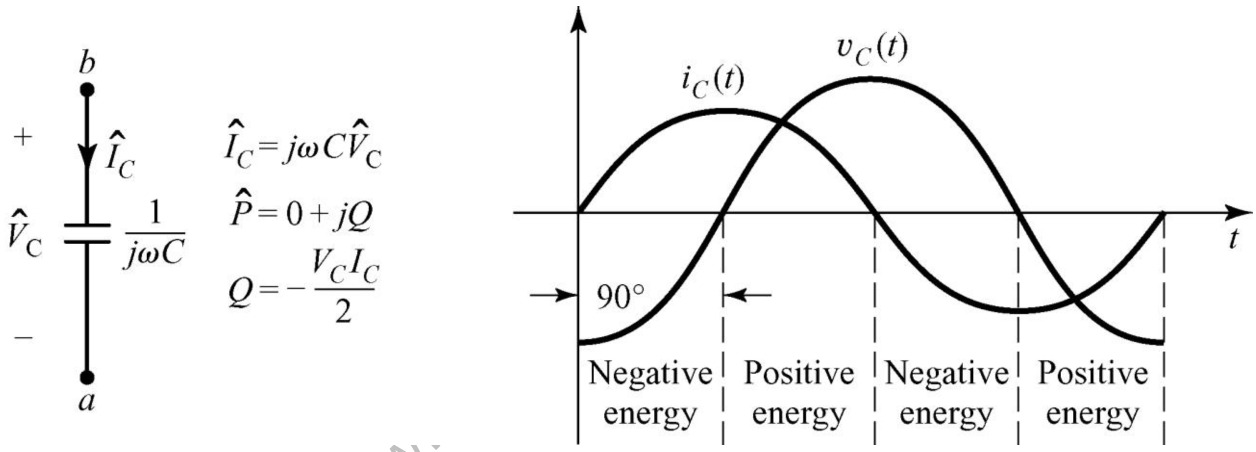
$$\hat{P} = P_{\text{AV}} + j0$$

$$P_{\text{AV}} = \frac{V_R^2}{2R} = \frac{1}{2} I_R^2 R$$

2.15 Power Relations for the Inductor

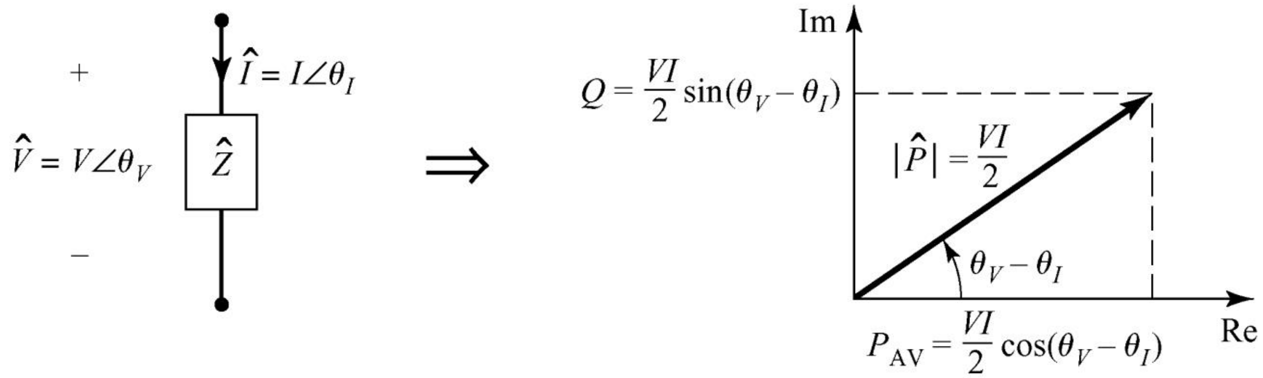


2.16 Power Relations for the Capacitor



Lagging power factor implies that current **lags** voltage hence an inductive load
Leading power factor implies that current **leads** voltage hence a capacitive load

2.17 Power Factor

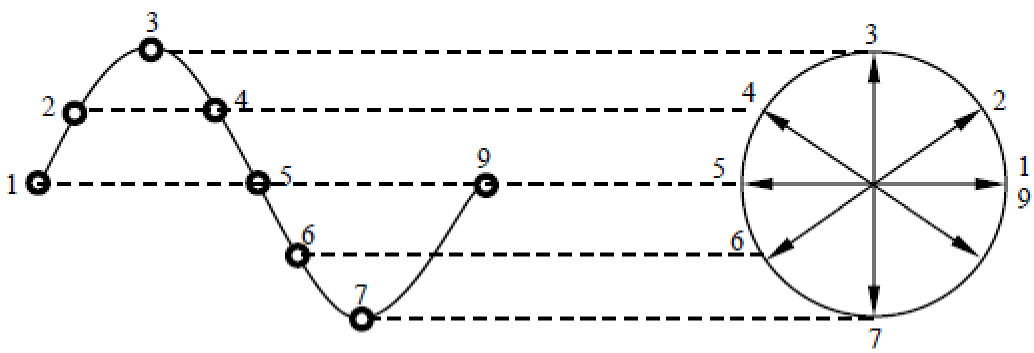


2.18 Phasors

- We have learnt from the previous section how to define and express in a single equation the magnitude, frequency, and phase shift of a sinusoidal signal.
- Any linear circuit that contains resistors, capacitors, and inductors do not alter the shape of this signal, nor its frequency.
- However, the linear circuit does change the amplitude of the signal (amplification or attenuation) and shift its phase (causing the output signal to lead or lag the input).
- The amplitude and phase are the two important quantities that determine the way the circuit affects the signal.

- Accordingly, signal can be expressed as a linear combination of complex sinusoids.
- Phase and magnitude defines a phasor (vector) or complex number. The phasor is similar to vector that has been studied in mathematics.

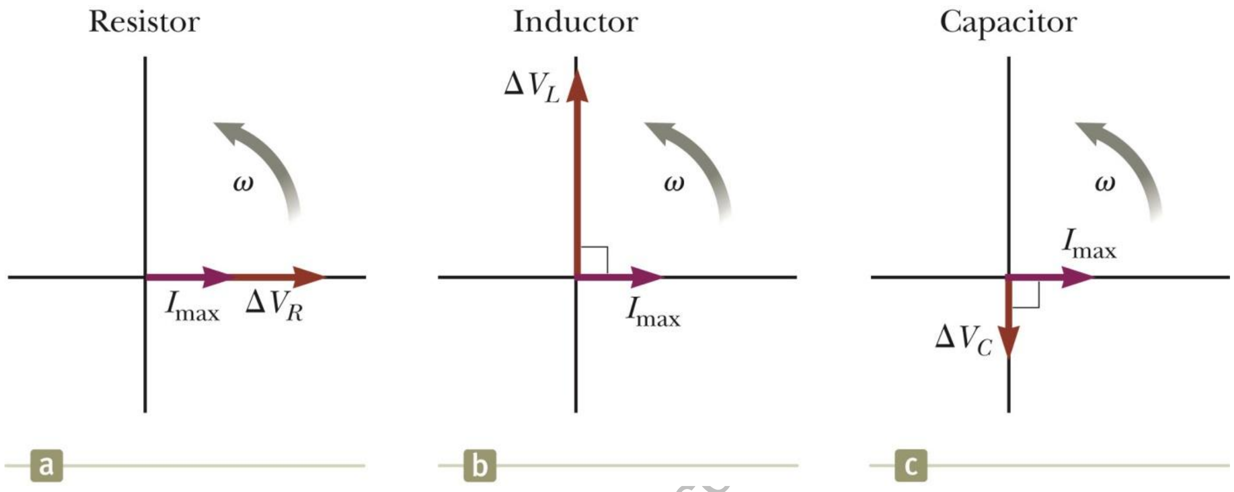
  Figure shows how AC sinusoidal quantities are represented by the position of a rotating vector. As the vector rotates it generates an angle. The location of the vector on the plane surface is determined by the magnitude (length) of the vector and



(a) (b)

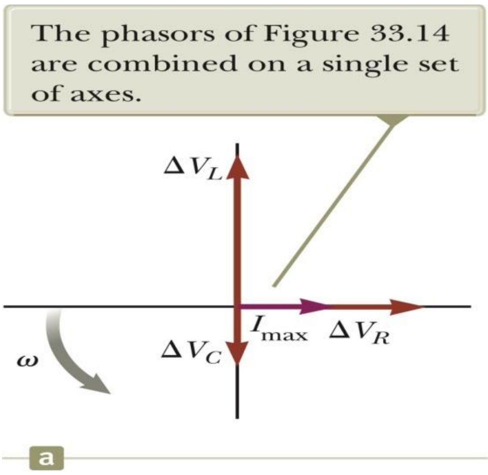
- Representing sinusoidal signals by phasors is useful since circuit analysis laws such as KVL and KCL and familiar algebraic circuit analysis tools, such as series and parallel equivalence, voltage and current division are applicable in the phasor domain, which have been studied in DC circuits can be applied.
- We do not need new analysis techniques to handle circuits in the phasor domain. The only difference is that circuit responses are phasors (complex numbers) rather than DC signals (real numbers).

Phasor Diagrams:



- To account for the different phases of the voltage drops, vector techniques are used.
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown.

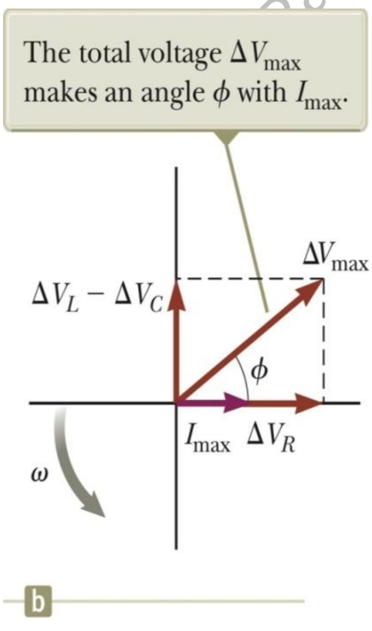
2.19Resulting Phasor Diagram:



- ❖ The individual phasor diagrams can be combined.
- ❖ Here a single phasor I_{max} is used to represent the current in each element.
- ❖ In series, the current is the same in each element.

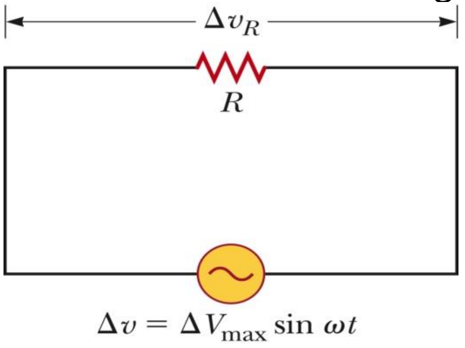
Vector Addition of the Phasor Diagram

- ❖ Vector addition is used to combine the voltage phasors.
- ❖ ΔV_L and ΔV_C are in opposite directions, so they can be combined.
- ❖ Their resultant is perpendicular to ΔV_R.
- ❖ The resultant of all the individual voltages across the individual elements is Δv_{max}.
- ❖ This resultant makes an angle of φ with the current phasor I_{max}.



2.20 Resistors in an AC Circuit:

- **Consider a circuit consisting of an AC source and a resistor**

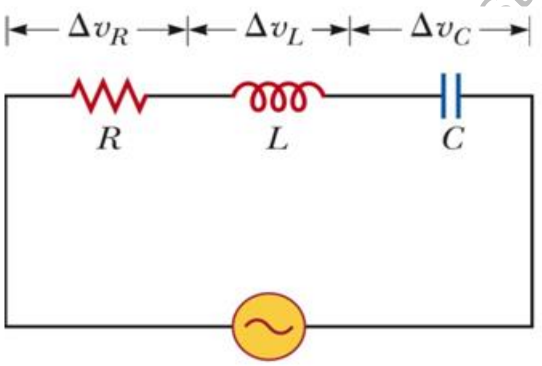


- The AC source is symbolized by
- Δv_R = DV_{max}= V_{max} sin *wt*
- Δv_R is the instantaneous voltage across the resistor.
- The instantaneous current in the resistor is

$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- The instantaneous voltage across the resistor is also given as
  $\Delta v_R = I_{max} R \sin \omega t$

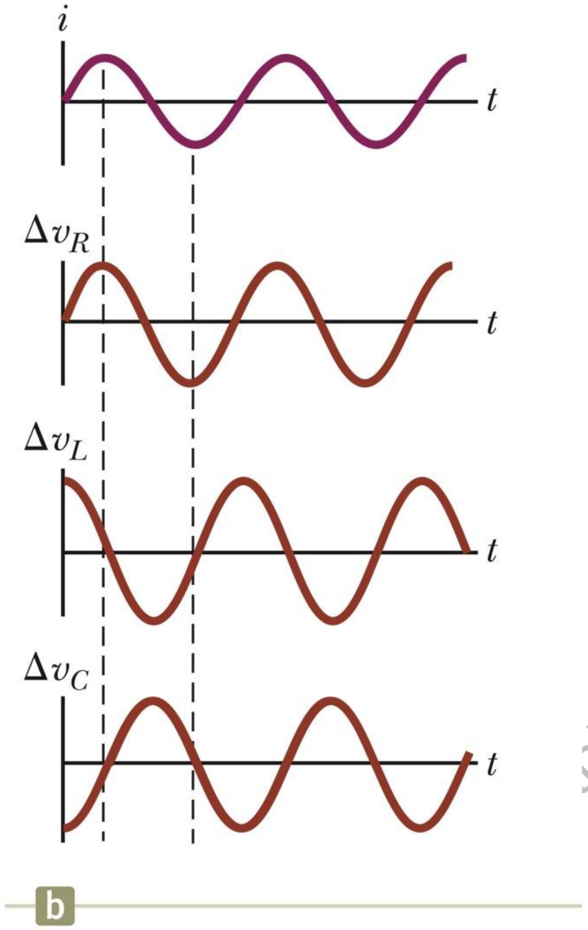
2.21 The *RLC* Series Circuit:



- The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$.
- The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$.
- φ is the phase angle between the current and the applied voltage.
- Since the elements are in series, the current at all points in the circuit has the same amplitude and phase

2.22 I and *v* Phase Relationships – Graphical View

- The instantaneous voltage across the resistor is in phase with the current.
- The instantaneous voltage across the inductor leads the current by 90°.
- The instantaneous voltage across the capacitor lags the current by 90°.



- **The instantaneous voltage across each of the three circuit elements can be expressed as**

$$\Delta v_R = I_{max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{max}R$.
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{max}X_L$.
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{max}X_C$.
- The sum of these voltages must equal the voltage from the AC source.
- Because of the different phase relationships with the current, they cannot be added directly.

Total Voltage in RLC Circuits

From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + (I_{max} X_L - I_{max} X_C)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

The current in an *RLC* circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\Delta V_{max}}{Z}$$

2.23 Impedance :

- Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where
- Impedance has units of ohms

$$Z \equiv \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

2.24 Phase Angle:

- The right triangle in the phasor diagram can be used to find the phase angle, φ.

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- The phase angle can be positive or negative and determines the nature of the circuit.

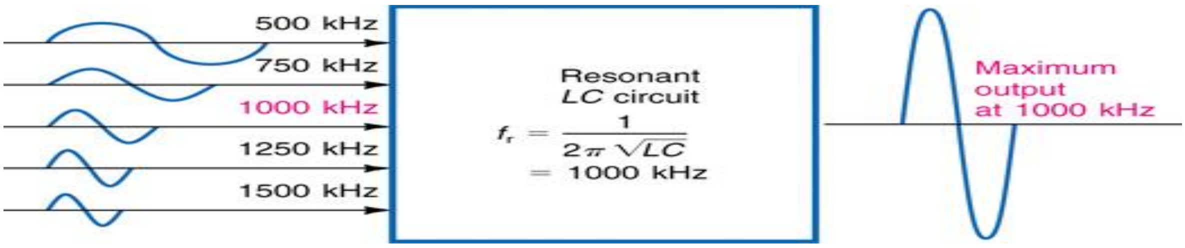
2.25 Determining the Nature of the Circuit:

- If f is positive
  - $X_L > X_C$ (which occurs at high frequencies)
  - The current lags the applied voltage.
  - The circuit is more inductive than capacitive.
- If f is negative
  - $X_L < X_C$ (which occurs at low frequencies)
  - The current leads the applied voltage.
  - The circuit is more capacitive than inductive.
- If f is zero
  - $X_L = X_C$
  - The circuit is purely resistive

UNIT-III
Resonance

3.1 INTRODUCTION:

- The most common application of resonance in rf circuits is called tuning.
- In Fig. below, the LC circuit is resonant at 1000 kHz.
- The result is maximum output at 1000 kHz, compared with lower or higher frequencies.



If an electriacl circuit offers impedance which is purely resistive then it is said to be uder resonance and frequency of the circuit at which it happens is called as resonant frequency. While studying the resonance of electrical circuits we unedrstand terms like resonant frequency, bandwidth, cut-off frequencies and quality factor.

3.2 Resonant frequency:

It is the frequency at which maximum response occurs or net impedance is purely resistive or minimum impedance is offered by circuit.(fr)

3.3 Bandwidth:

It is the range of frequencies within which signal can be esily transmitted with out any overlap of other signals. It is also given as difference between Higher cutt-off frequency and lower cutt-off frequency.(Bw = fh - fl)

3.4 Cutt-off frequencies:

It is the frequency at which response of the circuit is the 70.7% of maximum value or 0.707 of maximum value. This can be happen at two frequencies called as lower cutt-off frequencies < fr and higher cutt-off frequencies < fr.

3.5 Quality factor:

Quality factor is the measurement of quality of the energy storing elements , which in turn indictaes life time of energ storing elements.

Q = 2Π * energy stored in the element

Energy dissipated in one cycle.

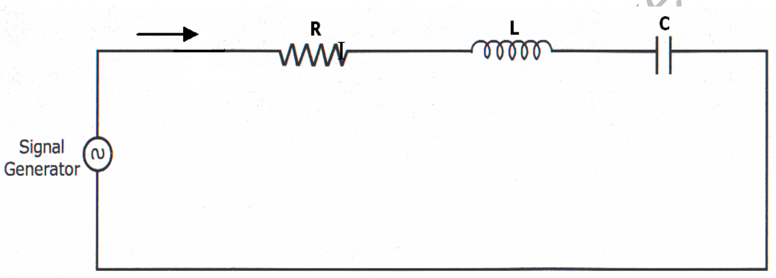
3.6 Types of resonance:

Depending on types of circuit resonance is defined. The are

1. **Series resonance:** series is related to series RLC circuit. In an series RLC circuit resonance occurs when voltage across L and C are same in magnitude and 180 degrees out of phase.

2. **Parallel resonance:** series is related to Parallel RLC circuit. In an Parallel RLC circuit resonance occurs when current flowing through L and C are same in magnitude and 180 degrees out of phase.

3.7 Series Resonance:



Let us consider series RLC circuit as shown,

Here , Z = R+ j(XL –XC)

Where, XL = 2ΠfL

XC = 1/ 2ΠfC

To say that circuit is under resonance , Z = R

This happens only when, XL = XC i.e imaginary part of total impedance is zero.(XL –XC = 0)

XL = XC

2ΠfL = 1/ 2ΠfC

wL = 1/ wC (w= 2Πf , angular frequency rad/sec)

.

3.8 BANDWIDTH :

Let f1 , f2 --- lower and higher cut-off frequencies

At f1, I = V /

And also at f2, I = V /

This is possible only when ,

At f1 , 1/ w1 C – w1. L = R ----1

f2, w2 L – 1/w2. C = R ----2

equate 1 and 2

1/ w1 C – w1. L = w2 L – 1/w2. C

1/ w1 C – w1. L = w2 L – 1/w2. C

w1.w2 = 1/ LC

$w1.w2 = wr^2$

now add two equations,

1/ w1 C – w1. L + w2 L – 1/w2. C =2R

(w2-w1)L + (w2-w1)/w1w2C = 2R

By sloving above equation, f2 – f1 = R / 2ΠL

Lower cut off frequency (f_1) = fr-R/4∏L

Upper cut off frequency (f_2) = fr+R/4

3.9 Quality factor: For inductor.

$$Q = 2\Pi * \frac{\text{energy stored in the element}}{\text{Energy dissipated in one cycle.}}$$

$$Q = 2\Pi * \frac{\frac{1}{2} LI^2}{I^{2.} R.t}$$

$$Q = 2\Pi * \frac{\frac{1}{2} LI^2}{I^{2.} R.1/f}$$

$$Q = 2\Pi L / R = XL / R$$

For capacitor.

$$Q = 2\Pi * \frac{\text{energy stored in the element}}{\text{Energy dissipated in one cycle.}}$$

$$Q = 2\Pi * \frac{\frac{1}{2} CV^2}{I^{2.} R.t}$$

$$Q = 2\Pi * \frac{\frac{1}{2} CV^2}{(V/R)^{2.} R.1/f}$$

$$Q = 1 / 2\Pi fC R = XC / R$$

3.10 MAGNIFICATION:

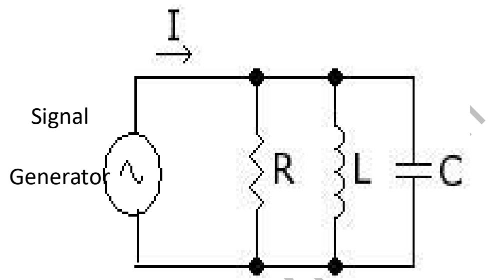
Magnification is defined ratio voltage across energy storing elements and input voltage under resonance.

VL / Vi = IXL / IR = XL / R =Q

VC / Vi = IXC / IR = XC / R =Q

To say that life of the circuit is high the magnification must be high.

3.11 Parallel Resonance:



Let us consider parallel RLC circuit as shown,

Here, Y =1/ R+ j(1/XL –1/XC) = G+ j(BL – BC)

Where, BL = 1 / 2ΠfL

BC = 2ΠfC

To say that circuit is under resonance , Y = G

This happens only when, BL = BC i.e imaginary part of total impedance is zero. (BL –BC = 0)

BL = BC

$1 / 2\Pi fL = 2\Pi fC$

$1 / wL = wC$ ($w= 2\Pi f$, angular frequency rad/sec)

$w^2 = 1/\sqrt{LC}$

$fr = 1/ 2\Pi\sqrt{LC}$ --- resonant frequency.

3.12 BANDWIDTH :

Let f1 , f2 --- lower and higher cut-off frequencies

At f1, $V = I /\sqrt{2}G$

And also at f2, $V = I /\sqrt{2}G$

This is possible only when ,

At f1 , $w1\ C – 1/w1.\ L = G$ ----1

f2, $1 / w2\ L – w2.\ C = G$ ----2

equate 1 and 2

$1w1\ C – 1 / w1.\ L = 1 / w2\ L – w2.\ C$

$1/ w1\ C – w1.\ L = w2\ L – 1/w2.\ C$

$w1.w2 = 1/ LC$

$w1.w2 = wr^2$

now add two equations,

$w1\ C – 1 / w1.\ L + 1 / w2\ L – w2.\ C =2G$

By sloving above equation, $f2 – f1 = 1 / 2\Pi RC$

Lower cut off frequency $(f_1) =fr-1/4\Pi RC$

Upper cut off frequency $(f_2) = fr+1/4\Pi RC$

3.13 Quality factor: For inductor.

$$Q = 2\Pi * \frac{\text{energy stored in the element}}{\text{Energy dissipated in one cycle.}}$$

$$Q = 2\Pi * \frac{\frac{1}{2} L(V/XL)^2}{(V/\sqrt{2}R)^{2.} .t}$$

$$Q = 2\Pi * \frac{\frac{1}{2} LI^2}{I^{2.} R.1/f}$$

$$Q = R / XL$$

For capacitor.

$$Q = 2\Pi * \frac{\text{energy stored in the element}}{\text{Energy dissipated in one cycle.}}$$

$$Q = 2\Pi * \frac{\frac{1}{2} CV^2}{I^{2.} R.t}$$

$$Q = 2\Pi * \frac{\frac{1}{2} CV^2}{(V/\sqrt{2})^{2.} R.1/f}$$

$$Q = R / BC = XC.R$$

3.14 MAGNIFICATION:

Magnification is defined ratio voltage across energy storing elements and input voltage under resonance.

IL / I = V/BL / V/R = R / BL

IC / Ii = V / BC / V / R = R / BC =Q

To say that life of the circuit is high the magnification must be high.

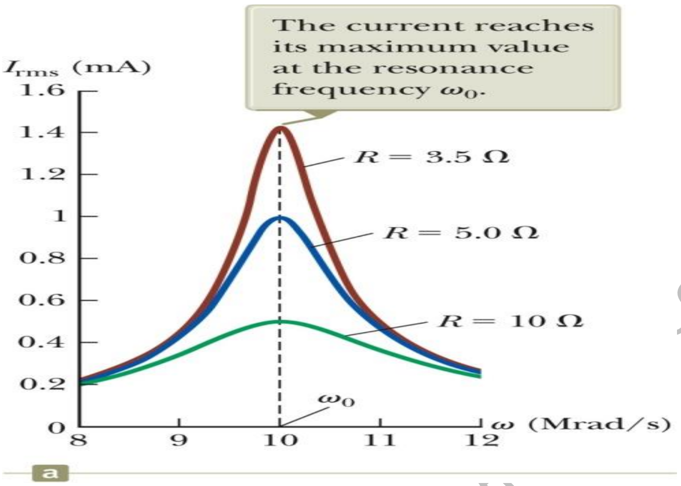
3.15 Comparison between series resonance and parallel resonance circuits

Specifications	Series resonance circuit(Acceptors)	Parallel resonance circuit(Rejectors)
Impedance at resonance	Minimum	Maximum
Current at Comparison between series resonance and parallel resonance circuits resonance	Maximum	Minimum
Effective impedance	R	L/CR
Resonant frequency	$1/(2*\pi*(LC)^{0.5})$	$(1/2*\pi)*\{(1/LC)- R^2/L^2$
It magnifies	Voltage	Current
It is known as	Acceptor circuit	Rejector circuit
Power Factor	Unity	Unity

Resonance in an AC Circuit:

- *Resonance* occurs at the frequency ω_o where the current has its maximum value.
  - To achieve maximum current, the impedance must have a minimum value.
  - This occurs when $X_L = X_C$
  - Solving for the frequency gives
- The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.
- The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.
- At the resonance frequency, the current is in phase with the applied voltage.

Resonance occurs at the same frequency regardless of the value of *R*.



- As *R* decreases, the curve becomes narrower and taller.
- Theoretically, if *R* = 0 the current would be infinite at resonance.
- Real circuits always have some resistance.

Quality Factor

- The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor, Q.
- $Q = \omega_o / \Delta\omega = (\omega_o L) / R$
  - $\Delta\omega$ is the width of the curve, measured between the two values of ω for which P_{avg} has half its maximum value.
  - These points are called the *half-power points.*
  - A high-*Q* circuit responds only to a narrow range of frequencies.
  - Narrow peak
- A low-*Q* circuit can detect a much broader range of frequencies.
- A radio's receiving circuit is an important application of a resonant circuit