FLUID MECHANICS

Syllabus:

UNIT I Introduction : Dimensions and units – Physical properties of fluids - specific gravity, viscosity, surface tension, vapour pressure and their influences on fluid motion, pressure at a point, Pascal's law, Hydrostatic law -atmospheric, gauge and vacuum pressuresmeasurement
of pressure. Pressure gauges, Manometers: Differential and Micro Manometers.

UNTI – II Hydrostatics: Hydrostatic forces on submerged plane, Horizontal, Vertical, inclined and curved surfaces – Center of pressure.
Fluid Kinematics: Description of fluid flow, Stream line, path line and streak line and stream tube. Classification of flows: Steady, unsteady, uniform, non-uniform, laminar, turbulent, rotational and irrotational flows – Equation of continuity for one, two , three dimensional flows – stream and velocity potential functions, flow net analysis.

UNIT – III Fluid Dynamics: Surface and body forces – Euler's and Bernoulli's equations for flow along a stream line - Momentum equation and its application – forces on pipe bend.

UNIT – IV Laminar Flow And Turbulent Flows: Reynold's experiment – Characteristics of Laminar & Turbulent flows, Shear and velocity distributions, Laws of Fluid friction, Hagen-Poiseulle Formula, Flow between parallel plates, Flow through long tubes, hydrodynamically smooth and rough flows.
Closed Conduit Flow: Darcy-Weisbach equation, Minor losses – pipes in series – pipes in parallel – Total energy line and hydraulic gradient line, variation of friction factor with Reynold's number – Moody's Chart, Pipe network problems, Hazen-Williams formula, Hard-Cross Method

UNIT – V Measurement of Flow: Pitot tube, Venturi meter and Orifice meter – classification of orifices, small orifice and large orifice, flow over rectangular, triangular, trapezoidal and Stepped notches - –Broad crested weirs.

UNIT – VI Boundary Layer Theory: Boundary layer (BL) – concepts, Prandtl contribution, Characteristics of boundary layer along a thin flat plate, Vonkarman momentum integral equation, laminar and turbulent Boundary layers(no deviations)- BL in transition, separation of BL, Control of BL, flow around submerged objects-Drag and Lift- Magnus effect.

Introduction:-

Hydraulics:

Hydraulics may be defined as follows:

"It is that branch of Engineering - science, which deals with water (at rest or in motion)"

(or)

" It is that branch of Engineering science which is based on experimental observation of water flow."

Fluid mechanics:

Fluid mechanics may be defined as that branch of Engineering - science which deals with the behaviour of fluid under the conditions of rest and motion.

The fluid mechanics may be divided into three parts.

- Statics
- kinematics
- Dynamics.

- Statics: The study of incompressible fluids under static conditions is called hydrostatics and that dealing with the compressible static gases is termed as aerostatics.
- kinematics: It deals with the velocities, accelerations and the patterns of flow only. Forces or energy causing velocity and acceleration are not dealt under this heading.
- Dynamics: It deals with the relations between velocities, accelerations of fluid with the forces or energy causing them.

The matter can be classified on the basis of the spacing between the molecules of the matter as follows:

- Solid state
- Fluid state
  i, Liquid state and ii, Gaseous state

In solids, the molecules are very closely spaced whereas in liquids the spacing between the different molecules is relatively large and in gases the spacing between the molecules is still large.

Inter molecular cohesive forces are large in solids, smaller in liquids and extremely small in gases, and on account of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movement. So that the gases fill the container completely in which they are placed.

Physical properties of the Fluid:

* 1. Density:

i, Mass density: The density also known as (mass density or specific mass) of a liquid may be defined as the mass per unit volume $\left(\frac{m}{V}\right)$ at a standard temperature and pressure. It is usually denoted by ρ (rho)

Its unit are kg/m^3 i.e., $\boxed{\rho = \frac{m}{V}}$

ii, Weight density: The weight density also known as specific weight is defined as the weight per unit volume at the standard temperature and pressure. It is usually denoted by ω. $\boxed{\omega = \rho g}$

For the purpose of all calculations relating to Hydraulics and hydraulics machines, the specific weight of water is taken as follows:

In S.I units : $w = 9.81\ kN/m^3$ (or $9.81 \times 10^{-6}\ N/mm^3$)

In M.K.S units : $w = 1000\ kgf/m^3$

iii, Specific Volume: It is defined as volume per unit mass of fluid. It is denoted by v.

Mathematically,

$$v = \frac{V}{m} = \frac{1}{\rho}$$

Kinematic Viscosity: Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by ν (called nu)

Mathematically, $\nu = \frac{\text{Viscosity}}{\text{density}} = \frac{\mu}{\rho}$

$$\nu = \frac{\mu}{\rho}$$

Units:

In S.I units : m^2/sec.

In MKS units : m^2/sec

In C.G.S units the kinematic viscosity is also known as stoke

$= cm^2/sec$

One stoke $= 10^{-4}\ m^2/sec$

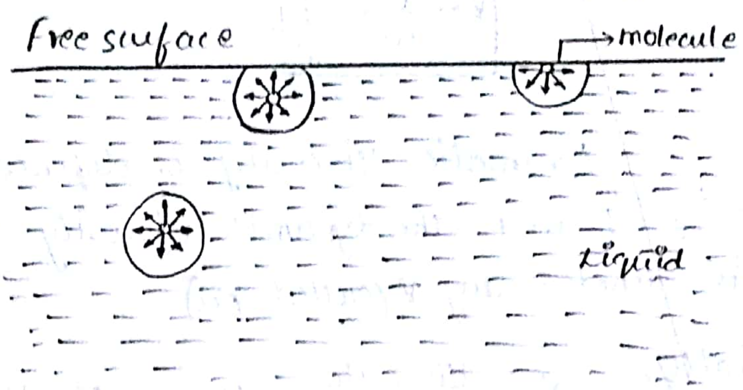
4. Surface Tension:

Cohesion: It means inter molecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one assemblage of particles. Surface tension is due to cohesion between particles at the free surface.

Adhesion: It means attraction between the molecule of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick another body.

Capillary action is due to both cohesion and adhesion.

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by another molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of molecules below it. Consequently there is a net inward force on the molecule. The force is normal to the liquid surface.



At the free surface a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface.(the layer acts as a membrane).

Some important examples of phenomenon of surface tension are as follows:

i, Rain drop A falling raindrop becomes spherical due to cohesion and surface tension.

ii, Rise of sap in a tree.

iii, Bird can drink water from ponds.

iv, Capillary rise and capillary siphoning.

v, collection of dustparticles on water surface.

vi, Break up of liquid jets.

<u>Dimensional formula for surface tension:</u>

The dimensional formula for surface tension is given by:

$\left[\frac{E}{L}\right]$ or $\left[\frac{M}{T^2}\right]$

 It is also expressed in N/m. The value of surface tension
depends upon the following factors:

i, Nature of liquid
ii, Nature of surrounding matter (E.g., solid, liquid or gas)
iii, Kinetic energy (and hence the temperature of the liquid molecules)

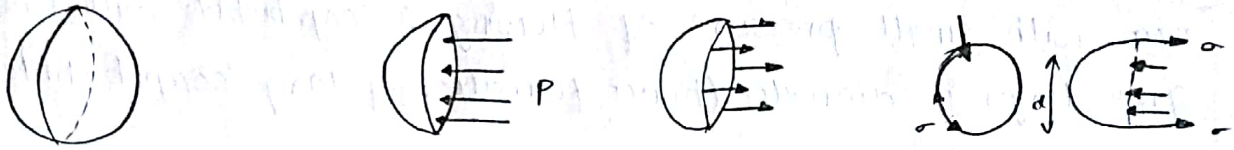
Water – air ---- 0.073 N/m at 20°C.
Water – air ---- 0.058 N/m at 100°C
Mercury – air --- 0.1 N/m length.

→ **Pressure inside a water droplet, soap bubble and a liquid jet:**

1. **Water Droplet:**

Let p = pressure inside the droplet above outside pressure
(i.e., $\Delta p = p - 0 = p$ above atm. pressure)

d = diameter of droplet ; σ = surface tension of liquid.



(a) Water droplet (b) pressure forces (c) surface tension (d) free body diagram

Fig: Pressure inside a water droplet

From the free body diagram,
i, Pressure force = $p \times \frac{\pi}{4} d^2$, and
ii, Surface tension force acting around the circumference $= \sigma \times \pi d$

Under equilibrium conditions these two forces will be equal & opposite. i.e.,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\frac{pd}{4} = \sigma$$

$$\boxed{p = \frac{4\sigma}{d}}$$

With an increase in size of the droplet the pressure intensity decreases.

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Soap (or Hollow) bubble :

Soap bubble have two surfaces on which surface tension σ acts.

Free body diagram

Fig : pressure inside soap bubble

From the free body diagram, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

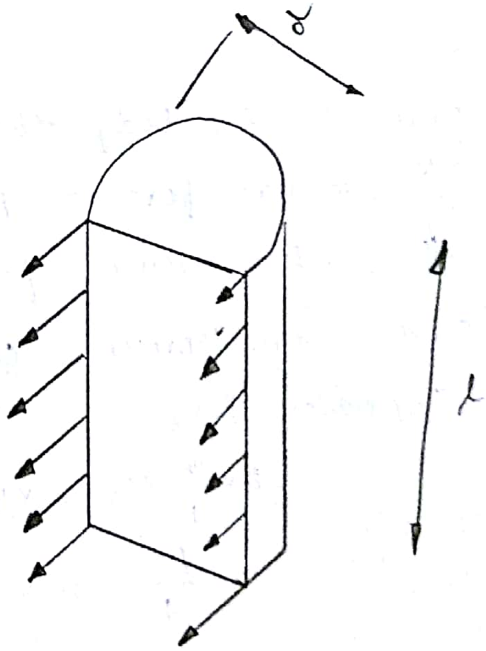
$$\frac{pd}{4} = 2\sigma$$

$$\boxed{p = \frac{8\sigma}{d}}$$

Since the soap solution has a high value of surface tension σ, even with small pressure of blowing a soap bubble will tend to grow larger in diameter (hence formation of large soap bubbles).

3. **Liquid Jet :**

Let us consider a cylindrical liquid jet of diameter d and length l.



Pressure force = $p \times l \times d$.

Surface tension force = $\sigma \times 2l$.

Equating the two forces, we have,

$$p \times l \times d = \sigma \times 2l$$

$$pd = 2\sigma$$

$$\boxed{p = \frac{2\sigma}{d}}$$

Vapour Pressure :

All liquids have a tendency to evaporate or vaporize (i.e., to change from liquid to gaseous state). Molecules are continuously projected from free surface to the atmosphere. These ejected molecules are in a gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid (P_v). If the surface above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When the equilibrium condition is reached, the vapour pressure is called saturation vapour pressure (P_{vs}).

The following points are worth nothing :

i, If the pressure on the liquid surface is lower than or equal to the saturation vapour pressure, boiling takes place.

ii, Vapour pressure increases with the rise in temperature.

iii, Mercury has a very low vapour pressure and hence, it is an excellent fluid to be used in a barometer.

Absolute and Gauge Pressures :

* Atmospheric pressure :

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure. The atmospheric pressure is also known as Barometric pressure.

The atmospheric pressure at sea level (above absolute zero) is called standard atmospheric pressure.

* Gauge Pressure :

It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken a datum the atmospheric pressure on the scale is marked as zero.

Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is below the local atmospheric pressure, then the gauge is designated as "vacuum gauge" and the recorded value indicates the amount by which the pressure of the liquid is below local atmospheric pressure. i.e., negative pressure.

Vacuum pressure is defined as the pressure below the atmospheric pressure.

Absolute pressure:

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum.

Any pressure measured above the absolute zero of pressure is termed as an "absolute pressure".

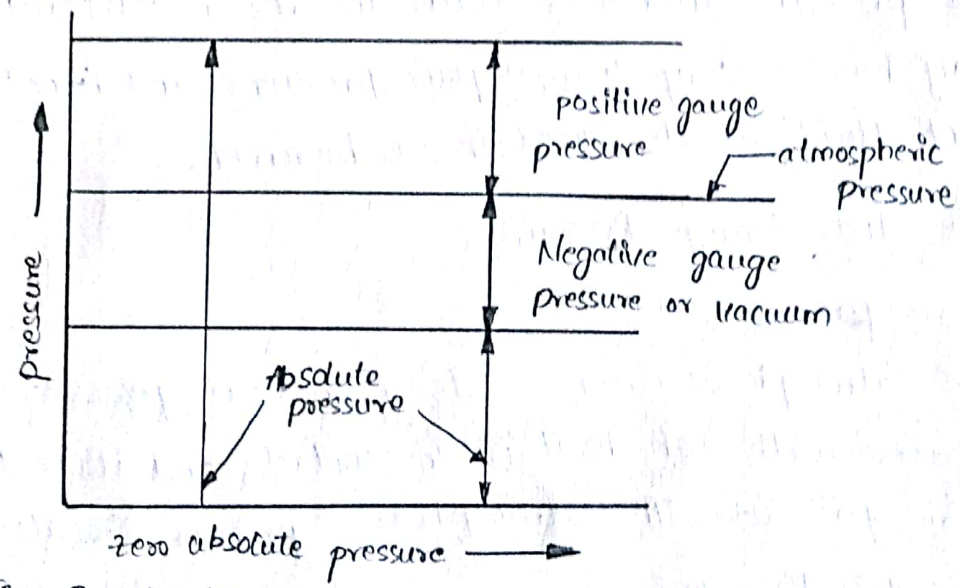


Fig: Relationship between pressures.

Mathematically,

1. Absolute pressure = Atmospheric pressure + gauge pressure

$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

2. Vacuum Pressure = Atmospheric pressure − Absolute pressure

Units for Pressure:

The fundamental S.I unit of pressure is newton per square meter (N/m^2). This is also known as pascal.

Low pressures are often expressed in terms of mm of mercury. This is an abbreviated way of saying that the pressure is such that will support a liquid column of stated height.

Standard atmospheric pressure has the following equivalent values:

101.3 kN/m^2 (or) 101.3 kPa ; 10.3m of water, 760mm of mercury 1013 mb (millibar) ; $\simeq$ 1bar $\simeq$ 100kPa = 10^5 N/m^2.

→ Measurement of Pressure:

The pressure of a fluid may be measured by the manometers.

* 1. Manometers: Manometers are defined as the devices used for measuring the pressures at a point in a fluid by balancing the column of fluid by the same or another column of liquid. These are classified as follows:

- (a) Simple manometers:
  - i, Piezometer
  - ii, U-tube manometer
  - iii, Single column manometer.
- (b) Differential manometers:

(a) Simple manometers:

A simple manometer is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other remains open to atmosphere.

i. piezometer:

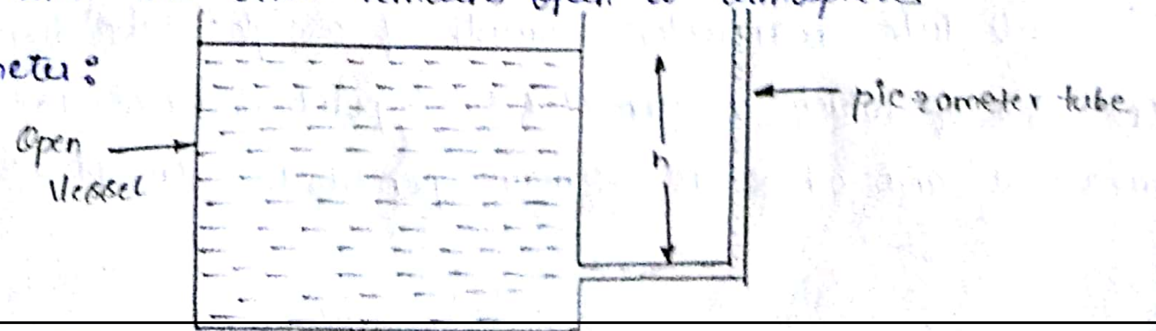
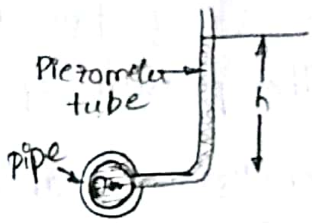


fig: piezometer tube filled to open vessel

A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids. It consists of a glass tube interested in the wall of a vessel or of a pipe, containing liquid whose pressured is to be measured. The tube extends vertically upwards to such a height that liquid can freely rise in it without over flowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it. Thus if w is the specific weight of the liquid, then the pressure at point A (p) is given by

$$p = wh$$



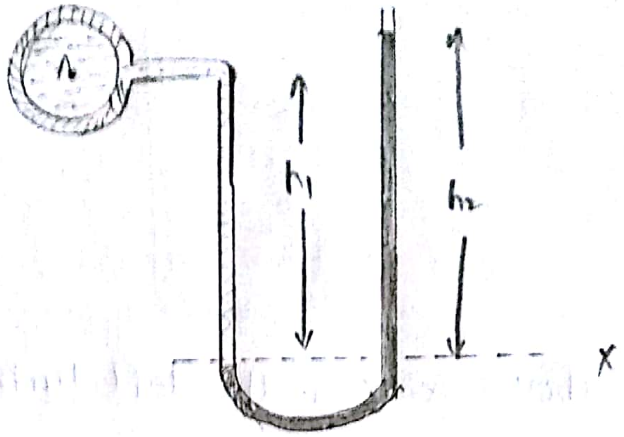
Piezometer measures gauge pressure only (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is not suitable for measuring negative pressure; as in such case the air will enter in pipe through the tube.

2. U-tube manometer:

Piezometers cannot be employed when large pressure in the lighter liquids due are to be measured, since this would require very long tubes, which cannot be handled conveniently. Further more gas pressures cannot be measured by the piezometers because a gas forms no free atmospheric surface. These limitations can be overcome by the use of U-tube manometers.

U-tube manometers consists of a glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remain open to the atmosphere.

i. For positive pressure :



Let A be the point at which pressure is to be measured. X – X is the datum line as shown in figure.

Let h_1 = Height of the light liquid in the left limb above the datum line,

h_2 = Height of the heavy liquid in the right limb above the datum line.

h = Pressure in pipe, expressed in terms of head.

S_1 = Specific gravity of the light liquid.

S_2 = Specific gravity of the heavy liquid.

The pressures in the left and right limb above the datum line X – X are equal (as the pressures at two points at the same level in a continuous homogeneous liquid are equal.)

Pressure head above X – X in the left limb = $h + h_1 S_1$

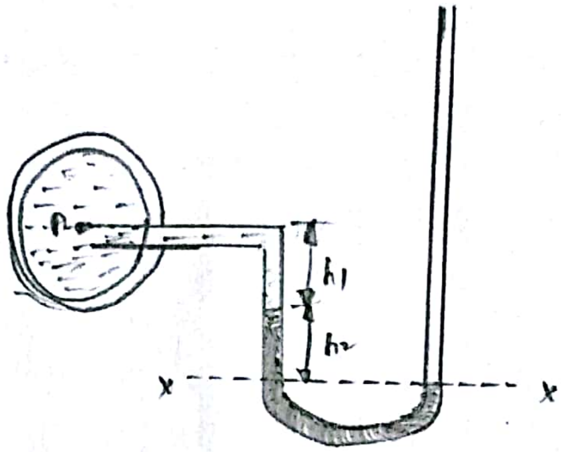
Pressure head above X – X in the right limb = $h_2 S_2$.

Equating these two pressures,

$$h + h_1 S_1 = h_2 S_2$$

(or) $\boxed{h = h_2 S_2 - h_1 S_1}$

ii, For negative Pressure:



Pressure head above x–x in the left limb

$$= h + h_1 S_1 + h_2 S_2$$

Pressure head above x–x in the right limb = 0.

Equating these two pressures,

$$h + h_1 S_1 + h_2 S_2 = 0$$

$$\boxed{h = -(h_1 S_1 + h_2 S_2)}$$

Problems:

1. Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of $6 m^3$ and weight of 44 N

Sol: Volume of the liquid = ~~44 N~~. $6 m^3$

Weight of the liquid = 44 N.

Specific weight ω:

$$\omega = \frac{\text{weight of liquid}}{\text{Volume of liquid}}$$

$$= \frac{44}{6}$$

$$= 7.333 \ KN/m^3$$

Specific mass (or) density ρ:

$$\rho = \frac{w}{g} = \frac{7.333 \times 1000}{9.81} = 747.5 \ kg/m^3$$

Specific volume, $V = \frac{1}{\rho} = \frac{1}{747.5}$

$= 0.00134 \ m^3/kg$

specific Gravity s:

$$s = \frac{w_{liquid}}{w_{water}} = \frac{7.333}{9.81} = 0.747$$

2. If the surface tension at air-water interface is 0.069 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.009 mm?

Sol: Given $\sigma = 0.069$ N/m

$d = 0.009$ mm

An air bubble has only one surface, hence

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}}$$

$= 30667 \ N/m^2$

$= 30.667 \ kN/m^2$

$= 30.667$ kPa

3. If the surface tension at soap-air interface is 0.09 N/m. Calculate the internal pressure in a soap bubble of 28mm diameter.

Sol: Given $\sigma = 0.09$ N/m, $d = 28$ mm $= 28 \times 10^{-3}$ m

In soap bubble there are two interfaces,

Hence $p = \frac{8\sigma}{d} = \frac{8 \times 0.09}{28 \times 10^{-3}} = 25.71 \ N/m^2$

above (atmospheric pressure)

2. Specific Gravity:

Specific gravity is the ratio of the specific weight of the liquid to the specific weight of standard fluid. It is dimensionless and has no units.

- It is represented by S.

For liquids, the standard fluid is pure water at 4°c.

$$\therefore \text{Specific Gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}}$$

$$S = \frac{\omega_{liquid}}{\omega_{water}}$$

3. Viscosity:

Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow. It is primarily due to cohesion and molecular momentum, exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

An ideal fluid has no viscosity. There is no fluid which can be classified as perfectly ideal fluid. However, the fluids which with very little viscosity are sometimes considered as ideal fluids.

Viscosity of fluid is due to cohesion and interaction between particles.

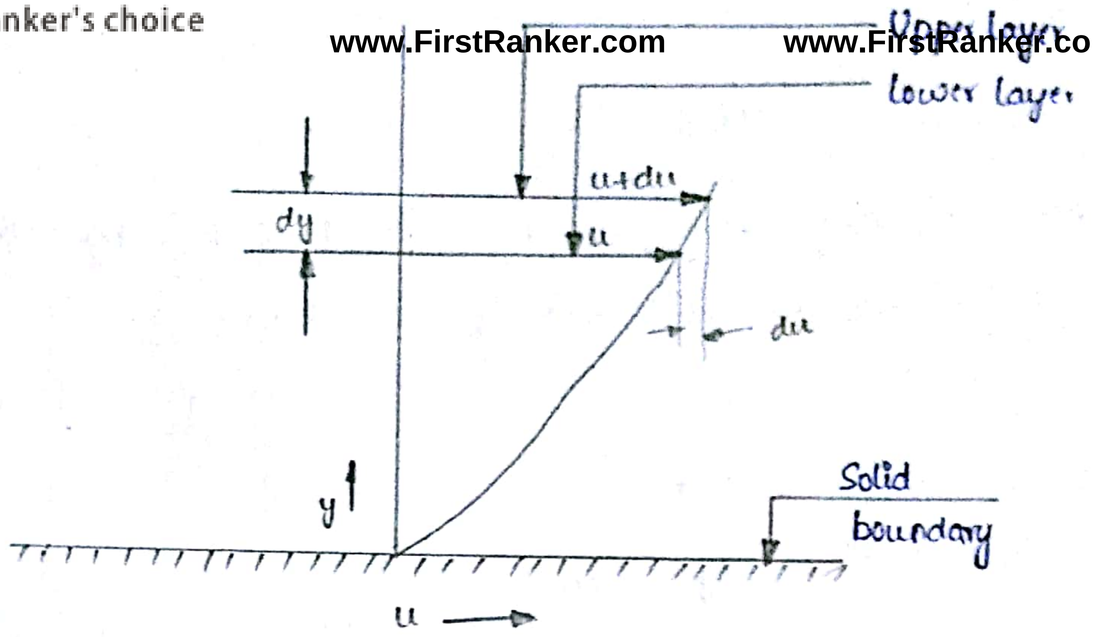


Fig: Velocity variation near a solid boundary.

When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du, the viscosity together with relative velocity causes a sheer stress acting between the fluid layers. The top layers cause a sheer stress on the adjacent top layer.

This sheer stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ (called Tau)

Mathematically,

$$\tau \propto \frac{du}{dy}$$

(or)

$$\boxed{\tau = \mu \cdot \frac{du}{dy}}$$

where μ = constant of proportionality and is known as co-efficient of dynamic viscosity (or) simply viscosity.

$\frac{du}{dy}$ = Rate of sheer stress or rate of sheer deformation or velocity gradient.

We have

$$\mu = \frac{\tau}{du/dy}.$$

Thus viscosity may also be defined as the shear stress required to produce unit rate of shear strain.

Units:

In S.I units : $N \cdot s/m^2$

In M.K.S units : $kgf \cdot sec/m^2$

The units of viscosity in C.G.S unit is called poise.

$$\text{poise} = \frac{\text{dyne-sec}}{cm^2}$$

$$\text{one poise} = \frac{1}{10} N.s/m^2$$

Kinematics of Fluid Flow:

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

Description of Fluid motion:

The motion of fluid particles may be described by two methods.

1. Lagrangian Method.
2. Eulerian Method.

In the Lagrangian method, a single fluid particle is followed during its motion and its velocity, acceleration, density etc are described.

In case of Eulerian method, the velocity, acceleration, pressure and density etc are described at a flow field.

The Eulerian method is commonly used in Fluid Mechanics.

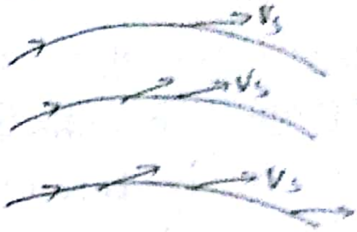
Description of the flow pattern (or) Types of flow:

Whenever a fluid in motion its innumerable particles move along certain lines depending upon the condition of flow. Although flow lines are of several types, yet some important flow pattern may described as

1. Stream line
2. Path line
3. Streak line
4. Stream tube

1. Stream line:

A stream line is an imaginary line drawn in a flow field such that a tangent drawn at any point on this line represent the direction of the velocity vector.



From the deflection, it follows that there can be no flow across a stream line.

Considering a particle moving along a Streamline for a very short distance ds having its components dx, dy and dz along the three mutually perpendicular co-ordinate axes. Let the components of the velocity vector V_s along x, y and z directions be u, v & w respectively. The time taken by a fluid particle to move a distance ds along the stream line with velocity V_s is $t = ds/V_s$

which is same as $t = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

Hence differential equation of the stream line may be written as:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Path Line:

A path line is the locus of a fluid particle as it moves along. In other words, a pathline is a current traced by a single fluid particle during its motion.

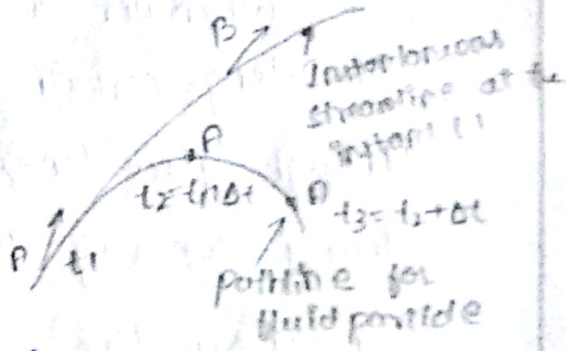


Fig. shows a streamline at time t_1 indicating the velocity vectors for particles A and B. At times t_2 and t_3, the particle A is shown to occupy the successive positions. The line connecting these various positions of A represents its path line.

Streak Line:

When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path followed by the dye or smoke is called streak line. Thus, a streakline connects all particles passing through a given point.

In steady flow, the streamlines remain fixed with respect to the co-ordinate axes. Streamlines in steady flow also represent the pathlines and streaklines. In unsteady flow, a fluid particle will not, in general, remain on the same streamline (except for unsteady uniform flow), hence streamlines and pathlines do not coincide in unsteady nonuniform flow.

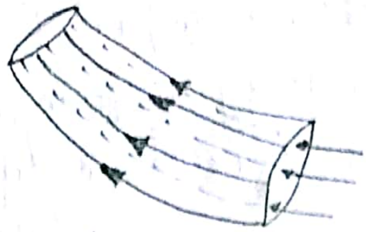
Instantaneous Stream Line:

In fluid motion which is independent of time, the position of streamline is fixed in space and a fluid particle following a streamline will continue to do so. In case of time-dependent flow, a fluid particle follows a stream line for only a short interval of time, before changing over to another Streamline. The streamlines in such cases are not fixed in space, but change with time. The position of a stream line at a given instant of time is known as instantaneous stream line. For different instants of time, we shall have different instantaneous stream-lines in the same space. The stream line, path line and streak lines are one and the same, if the flow is steady.

Stream Tube:

If streamlines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate. Such a surface bounded by streamline is a sort of tube, and known as a stream tube.

From the definition of streamline, it is evident no fluid can across the bounding surface tof the stream tube. This implies that the quantity(mass) of fluid entering the streamtube, at one end must be the same as the quantity leaving it at the other end. The stream tube is generally assumed to be a small cross-sectional area so that the velocity over it could be considered uniform.



The concept of stream tube can be extended, and the entire flow region may be composed of innumerable stream tubes of small cross-section. The stream tubes may be of any shape regular (or) irregular. The solid boundaries of flow represent the surface containing the stream lines.

Types of Fluid Flow:

The fluid flow is classified as:

i, Steady & unsteady flows:
ii, Uniform & non-uniform flows:
iii, Laminar & turbulent flows:
iv, Compressible & incompressible flows.
v, Rotational & irrotational flows.
vi, One, two and three dimensional flows.

i, Steady & unsteady Flows:

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.

Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Where (x_0, y_0, z_0) is a fixed point in fluid field.

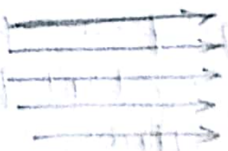
Unsteady flow is that type of flow, in which the velocity, pressure and density at a point changes with respect to time. Thus, mathematically, for unsteady flow.

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0.$$

$$\left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \text{etc.}$$

→ **Uniform and Non-uniform Flows:**

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e., length of direction of the flow). Mathematically, for uniform flow



$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where, ∂V = Change of velocity.
∂s = Length of flow in the direction's.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus mathematically, for non-uniform flow,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

→ **Laminar & Turbulent Flows:** Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths (or) stream line and all the stream-lines are straight and parallel. Thus the particles move laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of the fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

Where D = Diameter of pipe.
V = Mean velocity of flow in pipe.
ν = kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 & 4000, the flow may be laminar (or) turbulent.

→ **Compressible & Incompressible Flows:**

Compressible flow is that type in which density of fluid changes from point to point (or) in other words density (ρ) is not constant for the fluid. Thus, mathematically, for compressible fluid,

$$\rho \neq \text{constant}$$

Incompressible flow is that type in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible,

Mathematically, for incompressible flow. $\rho = \text{constant}$

→ **Rotational & Irrotational Flows:**

Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

One, Two and Three - Dimensional Flows:

One-dimensional Flow: It is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence, Mathematically, for one-dimensional flow

$$u = f(x)$$
$$v = 0$$
$$w = 0$$

where u, v and w are velocity compone in x, y & z directions respectively.

Two-dimensional Flow: It is that type of flow in which the velocity is a function of time and two rectangular space-co-ordinates say x and y. For a steady two dimensional flow the velocity is a function of two space co-ordinate only. The variation of velocity in the third direction is negligible. Thus, mathematically for 2-dimensional flow

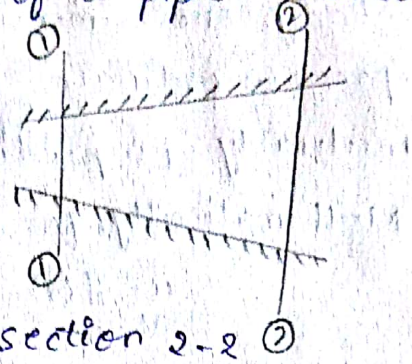
$$u = f_1(x, y)$$
$$v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional Flow: It is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, Mathematically, for three-dimensional flow.

$$u = f_1(x, y, z)$$
$$v = f_2(x, y, z)$$
$$w = f_3(x, y, z)$$

Continuity Equation:

The eqn. based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure.



Let V_1 = average velocity at cross-section 1-1

ρ_1 = Density at section-1-1

A_1 = Area of pipe at section 1-1

V_2, ρ_2, A_2 are corresponding values at section 2-2

The rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$.

According to law of conservation of mass.

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

The above equation is applicable to the compressible as well as incompressible fluids and is called continuity eqn. If the fluid is incompressible, then $\rho_1 = \rho_2$ and the above continuity equation reduces to

$$\boxed{A_1 V_1 = A_2 V_2}$$

1. The direction of a pipe at the sections 1 & 2 are 10cms and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s Determine also the velocity at section 2.

Sol: Given Data:

At section 1, $D_1 = 10cm = 0.1m$

$V_1 = 5m/s$

$A_1 = \frac{\pi}{4}(D_1)^2 = \frac{\pi}{4}(0.1)^2$

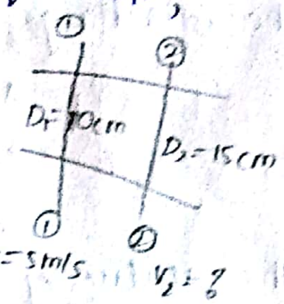
$= 0.007854\ m^2$.

At section 2, $D_2 = 15\,cm = 0.15\,m$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

i, Discharge through pipe is given by the equation,

$$Q = A_1 V_1$$



We know that equation (continuity)

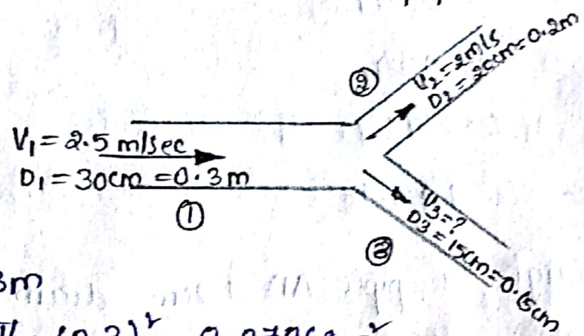
$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

$$= \frac{0.03927}{0.01767}$$

$$= 2.22\ m/s$$

2. A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15cm respt. If the average velo-city in the 30cm diameter pipe is 2.5m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s.



$D_1 = 30\,cm = 0.3\,m$

$A_1 = \pi/4\, D_1^2 = \pi/4\,(0.3)^2 = 0.07068\ m^2$

$V_1 = 2.5\ m/s$

$D_2 = 20\ cm = 0.2\,m$

$A_2 = \pi/4\, D_2^2 = \pi/4\,(0.2)^2 = 0.0314\ m^2$

$V_2 = 2\ m/s$

$D_3 = 15\,cm = 0.15\,m$

$A_3 = \pi/4\, D_3^2 = \pi/4\,(0.15)^2 = 0.01767\ m^2$

Find i, Discharge in pipe 1 (or) Q_1

ii, Velocity in pipe diameter 15cm (or) V_3.

Let Q_1, Q_2 & Q_3 are discharges in pipe 1, 2 & 3 respt.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

i, The discharge in pipe 1 is given by

$$Q_1 = A_1 V_1$$
$$= 0.07068 \times 2.5$$
$$= 0.1767 \ m^3/sec$$

ii, The value of V_3.

$$Q_2 = A_2 V_2$$
$$= 0.0314 \times 2$$
$$= 0.0628 \ m^3/sec$$

Substitute the values of Q_1 and Q_2 in continuity equation

$$Q_1 = Q_2 + Q_3$$
$$0.1767 = 0.0628 + (0.01767 \times V_3)$$
$$0.1767 - 0.0628 = 0.01767 \ V_3$$
$$V_3 = \frac{0.1767 - 0.0628}{0.01767}$$
$$V_3 = \frac{0.1139}{0.01767}$$
$$V_3 = 6.44 \ m/s$$

3. Water flows through a pipe AB 1.2m diameter at 3m/s and then passes through a pipe BC 1.5m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC the velocity in CD and the diameter of CE.

Given that, diameter of pipe AB, $D_{AB} = 1.2m$

Velocity of flow through AB, $V_{AB} = 3.0 \ m/s$

Diameter of pipe, BC $D_{BC} = 1.5m$

Diameter of branched pipe CD, $D_{CD} = 0.8m$

Velocity of flow in pipe CE, $V_{CE} = 2.5 \ m/s$

Let the flow rate in pipe AB = Q m³/s

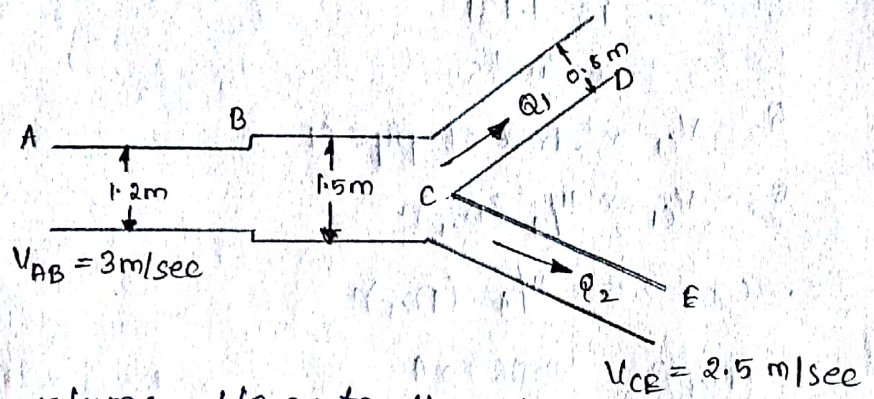
Velocity of flow in pipe BC = V_{BC} m/s.

Velocity of flow in pipe CD = V_{CD} m/s,

Diameter of pipe CE = D_{CE}

Then flow rate through CD = Q/3

flow rate through CE = $Q - Q/3 = \frac{2Q}{3}$.



i, Now volume flow rate through AB,

$$Q = V_{AB} \times \text{Area of } AB$$

$$= 3.0 \times \frac{\pi}{4} (D_{AB})^2$$

$$= 3.0 \times \frac{\pi}{4} (1.2)^2$$

$$= 3.393 \text{ m}^3/s$$

ii, Applying continuity equation to pipe AB and pipe BC,

$V_{AB} \times$ Area of pipe AB = $V_{BC} \times$ Area of pipe BC

$$3.0 \times \frac{\pi}{4} (D_{AB})^2 = V_{BC} \times \frac{\pi}{4} (D_{BC})^2$$

$$3.0 (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = \frac{3.0 (1.2)^2}{(1.5)^2}$$

$$= 1.92 \text{ m/s}$$

iii, The flow rate through pipe CD

$$= Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131 \text{ m}^3/s$$

$Q_1 = V_{CD} \times$ Area of pipe CD

$1.131 = V_{CD} \times \pi/4\ (0.8)^2$

$1.131 = 0.5026\ V_{CD}$

$$V_{CD} = \frac{1.131}{0.5026} = 2.25\ m/s$$

iv. Flow rate through CE,

$Q_2 = Q - Q_1$

$= 3.393 - 1.131$

$= 2.262\ m^3/s$

$Q_2 = V_{CE} \times$ area of pipe CE

$Q_2 = V_{CE} \times \pi/4\ (D_{CE})^2$

$2.262 = 2.5 \times \pi/4\ (D_{CE})^2$

$$(D_{CE})^2 = \frac{2.262 \times 4}{2.5\pi}$$

$$D_{CE} = \sqrt{\frac{2.262 \times 4}{2.5\pi}} = \sqrt{1.152} = 1.0735\ m$$

∴ Diameter of pipe CE = 1.0735 m

4. A 25 cm diameter pipe carries oil of sp. gr. 0.9 at a velocity of 3 m/s. At another section the diameter is 20 cm. Find the velocity at this section and also mass rate of flow of oil.

Sol: Given data,

at section 1, $D_1 = 25\ cm = 0.25\ m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$

$V_1 = 3\ m/sec$

at section 2, $D_2 = 20\ cm = 0.2\ m$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$

$V_2 = ?$

Mass rate of flow of oil = ?

Applying continuity eqn. at sections 1 and 2,

$A_1 V_1 = A_2 V_2$

 $0.049 \times 3 = 0.0314 \times V_2$

$V_2 = \frac{0.049 \times 3.0}{0.0314}$

$V_2 = 4.68$ m/s

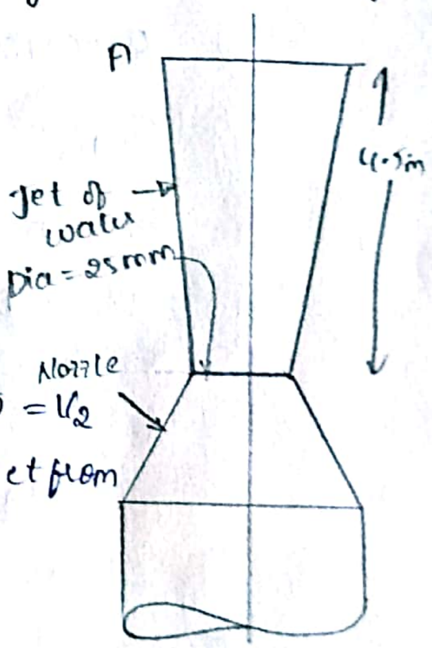
Mass rate of flow of oil = Mass density x a
$= \rho A_1 V_1$

Specific gravity of oil = $\frac{\text{Density of oil}}{\text{Density of water}}$

Density of oil = sp.gr of oil x Density of water
$= 0.9 \times 1000$
$= 900$ kg/m3

∴ Mass rate of flow $= 900 \times 0.049 \times 3.0$
$= 132.23$ kg/s

5. A jet of water from a 25mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, that will be the diameter at a point 4.5m above the nozzle, if the velocity with which the jet leaves the nozzle is 12m/s.



Sol: Given that

Diameter of nozzle, $D_1 = 25$mm $= 0.025$m

Velocity of jet at nozzle, $V_1 = 12$ m/s

Height of point A, $h = 4.5$m

Let the velocity of jet at a height 4.5 $= V_2$

consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy)

Initial velocity, $u = V_1 = 12$ m/s

Final velocity $V = V_2$

Value of $g = -9.81$ m/s2 and $h = 4.5$ m

using $V^2 - u^2 = 2gh$

$V_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$

$V_2^2 = 12^2 - (2 \times 9 \cdot$ [illegible]$)$

$V_2^2 = 144 - 86 \cdot 27$

$V_2 = 144 - 86 \cdot 27$

$V_2 = \sqrt{144 - 86 \cdot 27}$

$V_2 = 7 \cdot 16 \text{ m/s}$

Now applying continuity eqn. to outlet nozzle and at point A_1

we get $A_1 V_1 = A_2 V_2$

$$A_2 = \frac{A_1 V_1}{V_2}$$

$$A_2 = \frac{\pi/4 \; D_1^2 \times V_1}{V_2}$$

$$A_2 = \frac{\pi \times (0.025)^2 \times 12}{4 \cdot 7 \cdot 16} = 0 \cdot 0007896$$

Let D_2 = diameter of a jet at point A.

Then $A_2 = \pi/4 \; D_2^2$

$0.0007896 = \pi/4 \; D_2^2$

$$D_2 = \sqrt{\frac{0 \cdot 0007896 \times 4}{\pi}}$$

$D_2 = 0 \cdot 0317 \text{ m}$

$D_2 = 31 \cdot 7 \text{ mm}$

Velocity and Acceleration:

Let V is the resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y, z directions. The velocity components are functions of space-co-ordinates and time. Mathematically velocity components are given as

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

Resultant velocity, $V = ui + vj + wk$

$$= \sqrt{u^2 + v^2 + w^2}$$

Let a_x, a_y, a_z are the total acceleration in x, y and z directions respectively. Then by the chain rule of differentiation, we have

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

But $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$, $\frac{dz}{dt} = w$

$$a_x = \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$
$$a_y = \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$
$$a_z = \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For steady flow, $\frac{\partial V}{\partial t} = 0$

Where V = resultant velocity.

$$\frac{\partial u}{\partial t} = 0, \quad \frac{\partial v}{\partial t} = 0 \quad \text{and} \quad \frac{\partial w}{\partial t} = 0$$

Hence accelerations in x, y, and z directions becomes

$$a_x = \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$$
$$a_y = \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$$
$$a_z = \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$$

Acceleration vector $= a_x i + a_y j + a_z k$

$= \sqrt{a_x^2 + a_y^2 + a_z^2}$

→ **Local Acceleration and convective Acceleration:**

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow fluid. The expressions $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ is known as local acceleration.

convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particle in a fluid flow. The expressions other than $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$, $\frac{\partial w}{\partial t}$ in the above (acceleration) equation are known as convective acceleration.

6. The velocity vector in a fluid flow is given

$$V = 4x^3 i - 10x^2 y j + 2t k.$$

Find the velocity and acceleration of a fluid particle at (2,1,3) at time t=1

Sol: Given data $V = 4x^3 i - 10x^2 y j + 2t k$

This is compared with $V = ui + vj + wk$.

velocity components are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2,1,3) we have $x=2$, $y=1$, $z=3$ at time $t=1$.

Hence velocity components at (2,1,3) are

$u = 4(2)^3 = 4 \times 8 = 32$ units.

$v = -10(2)^2 (1) = -10 \times 4 = -40$ units

$w = 2(1) = 2$ units.

Velocity vector at (2,1,3) $= 32i - 40j + 2k$

Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$= \sqrt{(32)^2 + (-40)^2 + (2)^2}$

$= \sqrt{1024 + 1600 + 4}$

$= 51.26$ units.

We know that acceleration is

$$a_x = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

From the velocity components.

$$\frac{\partial u}{\partial x} = 12x^2, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial u}{\partial z} = 0, \quad \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy, \quad \frac{\partial v}{\partial y} = -10x^2, \quad \frac{\partial v}{\partial z} = 0, \quad \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0, \quad \frac{\partial w}{\partial y} = 0, \quad \frac{\partial w}{\partial z} = 0, \quad \frac{\partial w}{\partial t} = 2$$

Substituting the values, the acceleration components at (2,1,3) at time $t=1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t(0) + 0$$
$$= 48x^5$$
$$= 48(2)^5$$
$$= 48\times 32 = 1536 \text{ units}$$

$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$
$$= -80x^4y + 100x^4y$$
$$= 20x^4y = 20(2)^4(1) = 20\times 16 = 320 \text{ units}$$

$$a_z = 4x^3(0) + (-10x^2y)(0) + 2t(0) + 2$$
$$= 2 \text{ units}$$

Acceleration vector, $A = 1536i + 320j + 2k$

Resultant of acceleration $A = \sqrt{1536^2 + 320^2 + 2^2}$

$$= \sqrt{2359296 + 102400 + 4}$$

$$= 1568.9 \text{ units}$$

7. The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation.

i) $u = x^2+y^2+z^2$; $v = xy^2-yz^2+xy$

ii) $v = 2y^2$; $w = 2xyz$

Sol: The continuity eqn. for incompressible fluid,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

i, $u=x^2+y^2+z^2$ $\quad \frac{\partial u}{\partial x}=2x$

$v= xy^2-yz^2+xy$ $\quad \frac{\partial v}{\partial y}=2xy-z^2+x$

substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation

$$2x+2xy-z^2+x+\frac{\partial w}{\partial z}=0$$

$$3x+2xy-z^2+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial w}{\partial z}=-3x-2xy+z^2$$

$$\partial w=(-3x-2xy+z^2)\,\partial z$$

Integrating on both sides

$$\int \partial w=\int(-3x-2xy+z^2)\,dz$$

$$w=[-3xz-2xyz+z^3/3] + \text{constant of integration}$$

Where constant of integration cannot be a function of z. But it can be function of x and y that is $f(x,y)$

$$w=[-3xz-2xyz+z^3/3]+f(x,y)$$

Case (ii): $v=2y^2$, $w=2xyz$

$$\frac{\partial v}{\partial y}=4y, \quad \frac{\partial w}{\partial z}=2xy$$

substituting values of $\frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}$ in continuity eqn.

$$\frac{\partial u}{\partial x}+4y+2xy=0$$

$$\frac{\partial u}{\partial x}=-4y-2xy$$

$$\partial u=(-4y-2xy)\,\partial x$$

 Integrating on both sides,

$$\int du = \int (-4y - 2xy)\, dx$$

$$u = -4xy - 2 \cdot \frac{x^2}{2} \cdot y$$

$$= -4xy - x^2 y + \text{const. of Integration}$$

$$u = -4xy - x^2 y + f(y, z)$$

8. A flow field is given by

$$V = x^2 y i + y^2 z j - (2xyz + yz^2) k$$

P.T it is a case of possible steady incompressible fluid flow. Calculate the velocity and acceleration at the point (2,1,3)

Sol: Given that

$$V = x^2 y i + y^2 z j - (2xyz + yz^2) k$$

This eqn. is compared with $V = ui + vj + wk$

Then the velocity components are,

$$u = x^2 y \qquad \frac{\partial u}{\partial x} = 2xy.$$

$$v = y^2 z \qquad \frac{\partial v}{\partial y} = 2yz.$$

$$w = -(2xyz + yz^2) \qquad \frac{\partial w}{\partial z} = -(2xy + 2yz)$$

substituting the values of $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}$ in continuity eqn.

We know that the continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2xy + 2yz - (2xy + 2yz) = 0$$

$$0 = 0$$

Hence, the velocity field, $V = x^2 y i + y^2 z j - (2xyz + z^2 y) k$ is a case of possible steady incompressible fluid flow.

Velocity at (2,1,3) $x = 2, y = 1, z = 3$

$$V = (2)^2 (1)\, i + (1)^2 (3)\, j - [(2 \times 2 \times 1 \times 3) + (1)(3)^2]\, k$$

$$= 4i + 3j - (12 + 9) k$$

$$= 4i + 3j - 21 k$$

Resultant of velocity $= \sqrt{(4)^2+(3)^2+(?)^2}$

$= \sqrt{16+9+441}$

$= \sqrt{466}$

$= 21.587$ units.

Acceleration at (2,1,3):

$$a_x = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$u = x^2y \quad \frac{\partial u}{\partial x} = 2xy \quad \frac{\partial u}{\partial y} = x^2 \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = 0$

$v = y^2z \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 2yz \quad \frac{\partial v}{\partial z} = y^2 \quad \frac{\partial v}{\partial t} = 0$

$w = -(2xyz + z^2y) \quad \frac{\partial w}{\partial x} = -(2yz) \quad \frac{\partial w}{\partial y} = -(+2xz + z^2)$

$\frac{\partial w}{\partial z} = -(2xy + 2zy) \quad \frac{\partial w}{\partial t} = 0$

$a_x = x^2y\,(2xy) + y^2z\,(x^2) - (2xyz + z^2y)(0) + 0$

$= 2x^3y^2 + x^2y^2z$

$= 2(2)^3(1)^2 + (2)^2(1)^2(3)$

$= 16 + 12 = 28$ units

$a_y = x^2y(0) + y^2z\,(2yz) - (2xyz + z^2y)\,y^2 + 0$

$= -2xy^3z + 2y^3z^2 - y^3z^2$

$= -[2(2)\times(1)^3(3)] + [2(1)^3(3)^2] - [(1)^3(3)^2]$

$= 18 - 12 - 9$

$= 18 - 21$

$= -3$ units.

$a_z = x^2y\,(-2yz) + y^2z\,(-2xz - z^2) + (2xyz + z^2y)(2xy + 2zy) + 0$

$= -2x^2y^2z - 2xy^2z^2 - y^2z^3 + 4x^2y^2z + 4xy^2z^2 + 2xy^2z^2 + 2y^2z^3$

$= 2x^2y^2z + 4xy^2z^2 - y^2z^3$

$= [2(2)^2(1)^2(3)] + [4(2)(1)^2(3)^2] - [(1)^2(3)^3] = 24 + \quad = 105$

Acceleration = $28i - 3j + 105k$

Resultant acceleration $= \sqrt{(28)^2 + (-3)^2 + (105)^2}$

$= \sqrt{784 + 9 + 11025}$

$= \sqrt{11818}$

$= 108.71$ units/m²

→ **Velocity Potential Function and Stream Function:**

Velocity Potential Function: It is defined as a scalar function of space and time such that its negative derivative with respect to any direction given the fluid velocity in that direction. It is defined by ϕ (phi). Mathematically, the velocity potential is defined as $\phi = f(x, y, z)$ for steady flow such that

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\} \quad \text{——— (1)}$$

Where u, v, w are components of velocity in x, y and z directions respectively.

The continuity eqn. for an incompressible steady flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{——— (2)}$$

Substituting the values of u, v, w in above continuity eqn.

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{——— (3)}$$

This eqn. is known as Laplace Equation.

For two-dimension case the laplace Equation reduces to

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad \text{——— (4)}$$

Properties of the potential Function:

We know the rotational components are given by

$$\left.\begin{aligned} \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \end{aligned}\right\} \longrightarrow (5)$$

Substituting the values of u, v, w from (1) in eqn (5)

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial x}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial}{\partial z}\left(\frac{-\partial \phi}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{-\partial \phi}{\partial z}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial z \partial x} + \frac{\partial^2 \phi}{\partial x \partial z}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial}{\partial y}\left(\frac{-\partial \phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{-\partial \phi}{\partial y}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial y \partial z} + \frac{\partial^2 \phi}{\partial z \partial y}\right]$$

If ϕ is a continuous function, then

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \phi}{\partial y \partial x} \; ; \; \frac{\partial^2 \phi}{\partial z \partial x} = \frac{\partial^2 \phi}{\partial x \partial z} \; ; \; \frac{\partial^2 \phi}{\partial y \partial z} = \frac{\partial^2 \phi}{\partial z \partial y}.$$

$$\therefore \omega_z = \omega_y = \omega_x = 0$$

When rotational components are zero, the flow is called irrotational. Hence the properties of the potential function are;

1. If velocity potential (ϕ) exists, the flow should be irrotational.
2. If velocity potential (ϕ) satisfies, the Laplace equation it represents the possible steady incompressible irrotational flow.

→ **Stream Function:**

It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (Psi) and defined only for two-dimensional

(i) Mathematically, for steady flow it is defined as
$\psi = f(x, y)$ such that

$$\left.\begin{aligned}\frac{\partial \psi}{\partial x} &= v \\ \frac{\partial \psi}{\partial y} &= -u\end{aligned}\right\} \quad \text{------ (1)}$$

The continuity equation for two-dimensional flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{------ (2)}$$

substituting the values of u and v from ① to ②

$$\frac{\partial}{\partial x}\left(\frac{-\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

$$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x} = 0.$$

Hence existence of ψ means a possible case of fluid flow. The flow may be rotational (or) irrotational. The rotational component ω_z is given by.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

substituting the values of u and v from ① in above eqn.

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

For irrotational flow, $\omega_z = 0$. Hence above equation becomes as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Which is Laplace Equation for ψ.

The properties of stream function are:

1. If stream function (ψ) exists, it is a possible case of fluid flow which may be rotational (or) irrotational.
2. If stream function (ψ) satisfies the Laplace equation, it is a possible case of an irrotational flow.

→ **Equipotential Line:**

A line along which the velocity potential ϕ is constant, is called equipotential line.

For equipotential line ϕ = constant

$$\partial\phi = 0$$

But $\phi = f(x,y)$ for steady flow

$$\partial\phi = \frac{\partial\phi}{\partial x} dx + \frac{\partial\phi}{\partial y} dy$$

$$= -u \cdot dx - v \cdot dy$$

$$= -(udx + vdy)$$

For equipotential line, $\partial\phi = 0$.

$$-(udx + vdy) = 0$$

$$udx + vdy = 0$$

$$udx = -vdy$$

$$\boxed{\frac{dy}{dx} = \frac{-u}{v}} \quad \text{——— (1)}$$

But $\frac{dy}{dx}$ = slope of equipotential line.

→ **Line of constant Stream Function:**

$$\psi = \text{constant}$$

$$d\psi = 0$$

But $d\psi = \frac{\partial\psi}{\partial x} dx + \frac{\partial\psi}{\partial y} dy$

$$= v\,dx - u\,dy$$

For a line of constant stream function.

$$d\psi = 0$$

$$vdx - udy = 0$$

$$vdx = udy$$

$$\frac{dy}{dx} = \frac{v}{u} \quad \text{——— (2)}$$

But $\frac{dy}{dx}$ is slope of stream line.

For eqn. (1) and (2) it is clear that the product of the slope of the equipotential line and the slope of the stream line at the point of intersection is equal to −1. Thus the equipotential lines are orthogonal to the stream lines at all points of intersection.

→ **Flow Net:**

A grid obtained by drawing a series of equipotential lines and streamlines is called a flow net. The flow net is an important tool in analysing two-dimensional irrotational flow problems.

→ **Relation between stream function and velocity potential function:**

We know that $u = -\frac{\partial \phi}{\partial x}$ and $v = -\frac{\partial \phi}{\partial y}$

and $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$

Thus, we have $u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$

$$v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\therefore \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \; ; \; \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$$

9. The velocity potential function (ϕ) is given by an expression,

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$$

i, Find the velocity components in x and y direction.

ii, Show that ϕ represents a possible case of flow.

Sol: Given $\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$

The partial derivatives of ϕ w.r.to x and y are

$$\frac{\partial \phi}{\partial x} = -\frac{y^3}{3} - 2x + \frac{3x^2 y}{3} \quad \text{——(1)}$$

$$\frac{\partial \phi}{\partial y} = -\frac{3xy^2}{3} + \frac{x^3}{3} + 2y \quad \text{——(2)}$$

(i) The velocity components (u and v) are given by:

$$u = -\frac{\partial \phi}{\partial x}$$

$$u = -\left[-\frac{y^3}{3} - 2x + \frac{3x^2 y}{3}\right]$$

$$u = \frac{y^3}{3} + 2x - x^2 y$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$= -\left[-\frac{3xy^2}{3} + \frac{x^3}{3} + 2y\right]$$

$$= \frac{3xy^2}{3} - \frac{x^3}{3} - 2y$$

$$v = xy^2 - \frac{x^3}{3} - 2y$$

ii, The given value of ϕ, will represent a possible case of flow if it satisfies the Laplace equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

From eqn's (1) & (2) we have

$$\frac{\partial \phi}{\partial x} = -\frac{y^3}{3} - 2x + x^2 y$$

$$\frac{\partial^2 \phi}{\partial x^2} = -2 + 2xy$$

and $$\frac{\partial \phi}{\partial y} = -xy^2 + \frac{x^3}{3} + 2y$$

$$\frac{\partial^2 \phi}{\partial y^2} = -2xy + 2$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (-2 + 2xy) + (-2xy + 2) = 0$$

Laplace equation is satisfied and hence ϕ represent a possible case of flow.

10. The velocity potential function is given by $\phi = 5(x^2 - y^2)$. Calculate the velocity components at the point (4, 5)

Sol: $\phi = 5(x^2 - y^2)$

$$\frac{\partial \phi}{\partial x} = 10x$$

$$\frac{\partial \phi}{\partial y} = -10y$$

But velocity components are given by eqn

$$u = -\frac{\partial\phi}{\partial x} = -10x$$

$$v = -\frac{\partial\phi}{\partial y} = -(-10y) = 10y$$

The velocity components at the point (4,5) i.e., at x=4 & y=5.

$u = -10 \times 4 = -40$ units

$v = 10 \times 5 = 50$ units.

11. A stream function is given by $\psi = 5x - 6y$. Calculate the velocity components and also magnitude and direction of the resultant velocity at any point.

Sol:
$$\psi = 5x - 6y$$

$$\frac{\partial\psi}{\partial x} = 5 \quad \text{and} \quad \frac{\partial\psi}{\partial y} = -6$$

But the velocity components u and v in terms of stream function are given by equation as.

$$u = -\frac{\partial\psi}{\partial y} = -(-6) = 6 \text{ units/sec.}$$

$$v = \frac{\partial\psi}{\partial x} = 5 \text{ units/sec.}$$

Resultant velocity $= \sqrt{u^2+v^2}$

$= \sqrt{6^2+5^2}$

$= \sqrt{61}$

$= 7.81$ units/sec

Direction is given by, $\tan\theta = \frac{u}{v} = \frac{5}{6} = 0.833$

$\theta = \tan^{-1}(0.833)$

$\theta = 39^\circ 48'$

12. If for two-dimensional potential flow, the velocity potential is given by $\phi = x(2y-1)$

determine the velocity at the point P(4,5). Determine also the value of stream function ψ at point P.

 Given: $\phi = x(2y-1)$

i, The velocity components in the direction of x and y are

$$u = -\frac{\partial\phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y-1)] = -[2y-1] = 1-2y$$

$$v = -\frac{\partial\phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y-1)] = -[2x] = -2x$$

At point p(4,5) i.e., at $x=4$ & $y=5$

$$u = 1-2(5) = 1-10 = -9 \text{ units/sec.}$$

$$v = -2(4) = -8 \text{ units/sec}$$

velocity at $P = -9i - 8j$

Resultant velocity at $P = \sqrt{9^2+8^2} = \sqrt{81+64}$

$= 12.04$ units/sec

ii, Value of stream function at p.

WKT $\frac{\partial\psi}{\partial y} = -u = -(1-2y) = 2y-1$ ——— (i)

$\frac{\partial\psi}{\partial x} = v = -2x$ ——— (ii)

Integrating equation i, with respect to y.

$$\int d\psi = \int (2y-1)\,dy$$

$\psi = 2 \cdot y^2/2 - y +$ constant of integration.

The constant of integration is not a function of y but it can be a function of x.

Let the value of constant of integration is K. Then

$$\psi = y^2 - y + K \text{ ——— iii,}$$

Differentiating the above eqn. w.r.to x.

$$\frac{\partial\psi}{\partial x} = \frac{\partial K}{\partial x}.$$

But $\frac{\partial\psi}{\partial x} = -2x$ from eqn ii,

Equating the value of $\frac{\partial\psi}{\partial x}$, we get $\frac{\partial K}{\partial x} = -2x$.

Integrating this equation,

$$K = \int -2x \cdot dx = -\frac{2x^2}{2} = -x^2$$

substituting the valu[e in] eq (iii),

$\psi = y^2 - y - x^2$

∴ stream function ψ at $P(4,5) = (5)^2 - 5 - (4)^2$

$= 25 - 5 - 16$

$= 4$ units

13. The stream function for a two-dimensional flow is given by $\psi = 2xy$, calculate velocity at the point $P(2,3)$. Find the velocity potential function.

Sol: Given $\psi = 2xy$

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y$$

At the point $P(2,3)$ we get

$u = -2 \times 2 = 4$ units/sec.

$v = 2 \times 3 = 6$ units/sec.

Resultant velocity at $P = \sqrt{4^2 + 6^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 7.21$ units/sec.

Velocity potential function ϕ

$$\frac{\partial\phi}{\partial x} = -u = -(-2x) = 2x \longrightarrow (i), \qquad \frac{\partial\phi}{\partial y} = -v = -2y \longrightarrow (ii),$$

Integrating eqn (i) we get $\int d\phi = \int 2x\,dx$

$$\phi = \frac{2x^2}{2} + c = x^2 + c \longrightarrow (iii),$$

$c \rightarrow$ constant independent of x but can be a function of y

differentiating eqn (iii), w.r.to 'y' $\frac{\partial\phi}{\partial y} = \frac{\partial c}{\partial y}$

But from eqn (ii), $\frac{\partial\phi}{\partial y} = -2y \Rightarrow \frac{\partial c}{\partial y} = -2y$

Integrating the eqn, we get $c = \int -2y \cdot dy \Rightarrow c = \frac{-2y^2}{2} = -y^2$

substituting the value of c in eqn (iii), $\phi = x^2 - y^2$

14. Sketch the stream lines represented by $\psi = x^2+y^2$. Also find out the velocity and its direction at point (1, 2).

Sol: Sketch of stream lines:

$$\psi = x^2+y^2$$

Let $\psi = 1, 2, 3$ and so on

$$1 = x^2+y^2$$
$$2 = x^2+y^2$$
$$3 = x^2+y^2$$

Each eqn. is a eqn. of circle. Thus we shall get concentric circles of differential diameters as shown in figure.

Given, $\psi = x^2+y^2$

The velocity components u and are given by

$$u = -\frac{\partial\psi}{\partial y} = -\frac{\partial}{\partial y}(x^2+y^2) = -2y$$

$$v = \frac{\partial\psi}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2) = 2x$$

At the point (1,2) the velocity components are

$$u = -2(2) = -4 \text{ units/sec.}$$
$$v = 2(1) = 2 \text{ units/sec}$$

Resultant velocity $= \sqrt{u^2+v^2}$

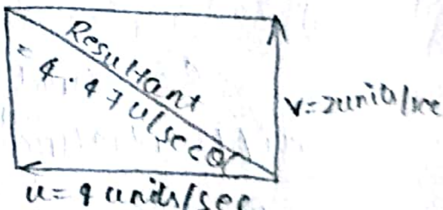
$= \sqrt{(-4)^2+(2)^2}$

$= \sqrt{20}$

$= 4.47$ units/sec

$$\tan\theta = \frac{u}{v} = \frac{2}{+4} = \frac{+1}{2}$$

$$\theta = \tan^{-1}\left(\frac{+1}{2}\right) = 26^\circ 34'$$



Resultant velocity makes an angle 26°34' with x-axis.

15. The velocity components in a two-dimensional flow field for an incompressible fluid are as follows:

$$u = \frac{y^3}{3} + 2x - x^2y \text{ and } v = xy^2 - 2y - \frac{x^3}{3}.$$

Obtain an expression for the stream function ψ.

Sol: Given: $u = \frac{y^3}{3} + 2x - x^2 y$

$v = xy^2 - 2y - \frac{x^3}{3}$

The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - \frac{x^3}{3} \quad \text{—— (i)}$$

$$\frac{\partial \psi}{\partial y} = -u = -\frac{y^3}{3} - 2x + x^2 y \quad \text{—— (ii)}$$

Integrating equation (i) w.r.to 'x'

$$\psi = \int \left(xy^2 - 2y - \frac{x^3}{3}\right) dx$$

$$= \frac{x^2}{2} \cdot y^2 - 2xy - \frac{x^4}{12}$$

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} + k \quad \text{—— (iii)}$$

Where k = constant of integration, which is independent of x but (dependent of y) function of y

differentiating eqn (iii) w.r.to y

$$\frac{\partial \psi}{\partial y} = \frac{x^2}{2}(2y) - 2x + \frac{\partial k}{\partial y}$$

$$= x^2 y - 2x + \frac{\partial k}{\partial y}$$

but from equation (ii) $\frac{\partial \psi}{\partial y} = -\frac{y^3}{3} - 2x + x^2 y$

$$\therefore \quad -\frac{y^3}{3} - 2x + x^2 y = x^2 y - 2x + \frac{\partial k}{\partial y}$$

$$\frac{\partial k}{\partial y} = -\frac{y^3}{3} \Rightarrow k = -\frac{y^4}{12}$$

substituting the value of k in eqn (ii)

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12}$$

16. In a two-dimensional incompressible flow, the fluid velocity components are given by $u = x - 4y$ & $v = -y - 4x$

S.T velocity potential exists and determine is form. Find also the stream function.

Sol: Given $u = x - 4y$ and $v = -y - 4x$

$$\frac{\partial u}{\partial x} = 1 \quad \text{and} \quad \frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence flow is continued and velocity potential exists

Let ϕ = velocity potential

Velocity components in terms of velocity potential is given by.

$$\frac{\partial \phi}{\partial x} = -u = -x + 4y \text{ —— i,}$$

$$\frac{\partial \phi}{\partial y} = -v = y + 4x \text{ —— ii,}$$

Integrating eqn i, we get

$$\int d\phi = \int(-x + 4y)\,dx$$

$$\phi = -\frac{x^2}{2} + 4xy + c \text{ —— iii,}$$

where c = constant of integration which is independent of x.

The constant can be function of y.

differentiating the eqn iii, w.r.to y.

$$\frac{\partial \phi}{\partial y} = 4x + \frac{\partial c}{\partial y}$$

but from eqn ii, $\frac{\partial \phi}{\partial y} = y + 4x$

$$y + 4x = 4x + \frac{\partial c}{\partial y}$$

$$\frac{\partial c}{\partial y} = y$$

integrating the above eqn, $c = \frac{y^2}{2} + c$

where c_1 = constant of integration which is independent of x and y

$$\therefore c = y^2/2$$

substituting the value of c in eqn iii,

$$\phi = -\frac{x^2}{2} + 4xy + \frac{y^2}{2}$$

$$\Rightarrow \boxed{\phi = \frac{y^2}{2} - \frac{x^2}{2} + 4xy} \text{ velocity potential.}$$

Stream function:

Velocity components in terms of stream function are given by

$$\frac{\partial\psi}{\partial x} = v = -y - 4x \quad \text{——— (iv)}$$

$$\frac{\partial\psi}{\partial y} = -u = -x + 4y \quad \text{——— (v)}$$

Integrating eqn (iv) we get

$$\int d\psi = \int (-y - 4x)\, dx$$

$$\psi = -yx - \frac{4x^2}{2} + K$$

$$\psi = -yx - 2x^2 + K \quad \text{——— (vi)}$$

Where K = constant of integration which is independent of x. But it is a function of y.

differentiating equation (iii) w.r.t. y

$$\frac{\partial\psi}{\partial y} = -x + \frac{\partial K}{\partial y}$$

But from eqn (v) $\frac{\partial\psi}{\partial y} = -x + 4y$

$$-x + 4y = -x + \frac{\partial K}{\partial y}$$

$$\frac{\partial K}{\partial y} = 4y$$

$$\Rightarrow K = \frac{4y^2}{2} + K_1$$

Where K_1 = constant of integration which independent of x and y

$$K = 2y^2$$

Substituting the value of K in equation (vi),

$$\psi = -yx - 2x^2 + 2y^2$$

$\psi = 2y^2 - 2x^2 - yx$ stream function.

Dynamics of Fluid flow

UNIT-II

→ The study of the fluid motion the forces and energies that are involved in the flow are required to be considered. This aspect of fluid motion is known as dynamics of fluid flow.

→ The various forces acting on the fluid mass may be classified as
i, body (or) volume forces.
ii, Surface forces.
iii, Line forces.

* i, Body or volume Forces: The body (or) volume forces are the forces which are proportional to the volume of the body.
Eg: weight, centrifugal force, magnetic force, electromotive force etc

* ii, Surface forces: The surface forces are the forces which are proportional to surface area.
Eg: pressure force, shear (or) tangential force, force of compressibility, force due to turbulence etc.

* iii, Line forces: These are forces which are proportional to length.
Eg: Surface tension.

→ **Equation of motion:**

Newton's second law of motion states that the resultant force on any fluid element must equal to the product of the mass and the acceleration of the element and the acceleration vector has the direction of the resultant force vector.

$$\Sigma F = Ma$$

Where ΣF = the resultant external force acting on the fluid element of mass M.

a = total acceleration.

→ **Forces acting on Fluid in Motion:**

The various forces that may influence the motion of a fluid are due to Gravity, pressure, viscosity, turbulence and compressibility

- For steady rotational flow of a fluid Bernoulli's equation can be derived for the points lying on the same stream line.
- Stream lines, streak lines & path lines are all identical in case of Steady flow.
- Navier-Strokes equ. is useful in the analysis of viscous flow.

Euler's equation of motion can be integrated when it is assumed that the fluid is incompressible.

- In irrotational flow of an ideal fluid a velocity potential exists.

If stream function $\psi = 2xy$ then the velocity at a point $(1,2)$ is equal to $\sqrt{20}$.

- A equipotential line has no velocity component tangent to it.
- The continuity equation fullfilled by the flow of any fluid, real or ideal, laminar or turbulent.
- A source in two-dimensional flow is a line from which fluid is imagined to flow uniformly in all directions.

- The gravity force (F_g) is due to the weight of the fluid
- The pressure force (F_p) is exerted on the fluid mass if there exists a pressure gradient between the two points in the direction of flow.
- The viscous force (F_v) is due to the viscosity of the flowing fluid and thus exists in the case of all real fluids.
- The turbulent force (F_t) is due to the turbulence of the flow. In the turbulent flow the fluid particles move from one layer to the other and therefore, there is a continuous momentum transfer between adjacent layers, which results in developing additional stresses (called Reynolds stresses) for the flowing fluid.
- The compressibility force F_c is due to the elastic property of the fluid and it is important only either for compressible fluids or in the case of flowing fluids in which the elastic properties of fluids are significant.

→ **Equations of Motion:**

According to Newton's second law of motion

$$F_x = m a_x$$

In above eqn, the net force,

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

i) If the force due to compressibility F_c is negligible, the resulting net force,

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and eqn. of motion are called Reynold's equation of motion:

ii) For flow, where (F_t) is negligible, the resulting equations of motion are known as Navier-Stokes Equation

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x$$

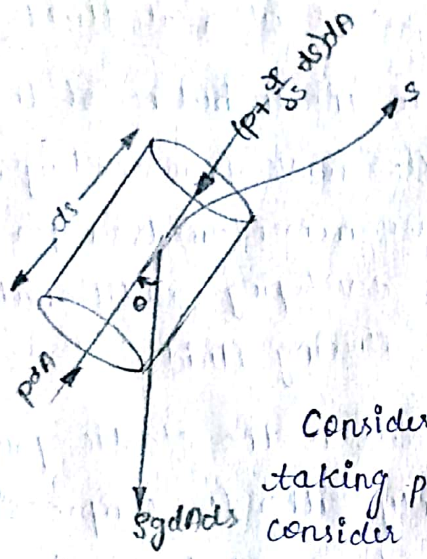
iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and eqns of motion are known as Euler's Equation of motion.

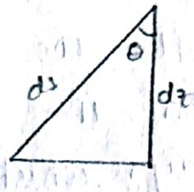
$$F_x = (F_g)_x + (F_p)_x$$

Euler's Equation of Motion:

This is eqn. of motion in which the forces due to gravity and pressure are taken into consideration.

This is derived by considering the motion of a fluid element along a stream-line as:





Consider a stream-line in which flow is taking place in s-direction as shown in fig. Consider a cylindrical element of cross-section dA and length ds. The forces acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow.
2. Pressure force $(p+\frac{\partial p}{\partial s}\cdot ds)dA$ opposite to the direction flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction s.

$$pdA - \left(p+\frac{\partial p}{\partial s}\cdot ds\right)dA - \rho g\, dA\, ds\cos\theta = \rho\, dA ds \times a_s \quad \text{——(1)}$$

Where, a_s is the acceleration in the direction of s.

Now $a_s = \frac{dV}{dt}$, where V is function of s and t.

$$= \frac{\partial V}{\partial s}\cdot\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$= V\cdot\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \qquad \left\{\because \frac{ds}{dt} = V\right\}$$

If the flow is steady, $\frac{\partial V}{\partial t} = 0$.

$$\therefore a_s = V\cdot\frac{dV}{ds}$$

[illegible] the value [illegible] equation (1)

$$pdA - \left(p + \frac{\partial p}{\partial s} \cdot ds\right) dA - \rho g \cdot dA\, ds \cos\theta = \rho\, dA\, ds \times V \cdot \frac{\partial V}{\partial s}$$

$$pdA - pdA - \frac{\partial p}{\partial s} ds\, dA - \rho g \cdot dA \cdot ds \cos\theta = \rho\, dA \cdot ds \times V \cdot \frac{\partial V}{\partial s}$$

$$-\frac{\partial p}{\partial s} ds \cdot dA - \rho g \cdot dA\, ds \cos\theta = \rho\, dA \cdot ds \times V \cdot \frac{\partial V}{\partial s}$$

dividing on both sides by $\rho\, ds\, dA$,

$$\frac{-1}{\rho} \cdot \frac{\partial p}{\partial s} - g\cos\theta = V \cdot \frac{\partial V}{\partial s}$$

$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial s} + g\cos\theta + V \cdot \frac{\partial V}{\partial s} = 0$$

But we know that $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho} \cdot \frac{\partial p}{\partial s} + g \cdot \frac{dz}{ds} + V \cdot \frac{\partial V}{\partial s} = 0$$

$$\Rightarrow \frac{dp}{\rho} + g\,dz + V \cdot dV = 0$$

This equation is called Euler's equation of motion.

Bernoulli's Equation From Euler's equation:

Bernoulli's eqn. is obtained by integrating the Euler's eqn. of motion as

$$\int \frac{dp}{\rho} + \int g \cdot dz + \int V \cdot dV = \text{constant}.$$

If flow is incompressible, ρ is constant.

$$\frac{p}{\rho} + gz + \frac{V^2}{2} = \text{constant}$$

dividing with g.

$$\frac{p}{\rho g} + z + \frac{V^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$$

This is Bernoulli's equation, in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid (or) pressure Head.

$\frac{V^2}{2g}$ = Kinetic energy per unit weight of fluid (or) kinetic Head.

z = Potential energy per unit weight of fluid (or) Potential Head.

Assumptions:

The following are the assumptions made in the derivation of Bernoulli's theorem.

1. The fluid is ideal i.e., viscosity in zero.
2. The flow is steady.
3. The flow is incompressible.
4. The flow is irrotational.

1. Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above datum line.

Sol: Given, diameter of pipe = 5 cm = 0.5 m

Pressure, $P = 29.43\ N/cm^2$

$= 29.43 \times 10^4\ N/m^2$

Velocity, $V = 2.0\ m/sec$

Datum head, $z = 5m$

Total head = Pressure head + Velocity head + Datum head

Pressure head $= \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30\ m$.

Velocity head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204\ m$

Datum head $= z = 5m$

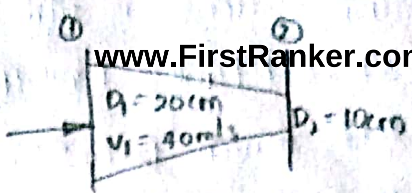
Total head $= \frac{P}{\rho g} + \frac{V^2}{2g} + z$.

$= 30 + 0.204 + 5$

$= 35.204\ m$

2. A pipe through which water is flowing is having diameters, 20cm and 10cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at section 1 and 2 and also rate of discharge.

Sol: Given Data:



$D_1 = 20cm = 0.2 cm$

$\therefore$ Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.2)^2$

$= 0.0314 \ m^2$

$V_1 = 4.0 \ m/sec$

$D_2 = 0.1 \ m$

$A_2 = \pi/4 \ (0.1)^2 = 0.00785 \ m^2$

(i) Velocity head at section 1

$= \frac{V_1^2}{2g}$

$= \frac{4.0 \times 4.0}{2 \times 9.81}$

$= 0.815$ ~~cm~~ m.

(ii) Velocity head at section 2 $= \frac{V_2^2}{2g}$

To find V_2, apply continuity equation at 1 and 2.

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$= \frac{0.0314 \times 4.0}{0.00785}$

$= 16.0 \ m/sec$

$\therefore$ Velocity heat at section $2 = \frac{V_2^2}{2g}$

$= \frac{16 \times 16}{2 \times 9.81}$

$= 3.047 \ m$

(iii) Rate of discharge $= A_1 V_1$ (or) $A_2 V_2$

$= 0.0314 \times 4.0$

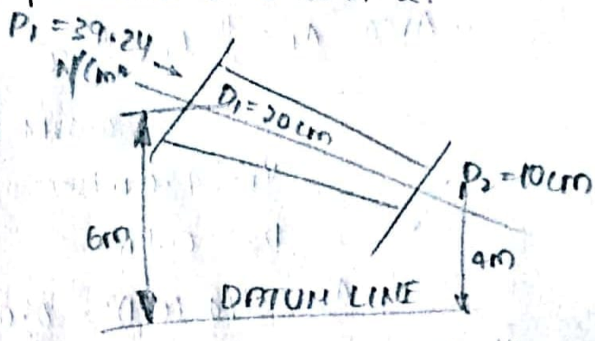
$= 0.1256 \ m^3/sec$

$= 125.6$ litres / sec.

3. The water is flowing through a pipe having diameters 20 cm and 10 cm at section 1 and 2 respectively. The rate of flow through pipe is 35 litres/sec. The section 1 is 6m above datum

and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm², Find the intensity of pressure at section 2.

Sol: Given Data:



At Section 1, $D_1 = 20cm = 0.2m$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2$$
$$= 0.0314 \, m^2$$

$$P_1 = 39.24 \, N/cm^2$$
$$= 39.24 \times 10^4 \times N/m^2$$
$$z_1 = 6m$$

At Section 2, $D_2 = 0.10cm$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.10)^2$$
$$= 0.00785 \, m^2$$

$$z_2 = 4m$$
$$P_2 = ?$$

Rate of flow, $q = 35$ lit/sec

$$= \frac{35}{1000} m^3/sec = 0.035 \, m^3/sec$$

$$Q = A_1V_1 = A_2V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \, m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \, m/sec$$

Applying Bernoulli's eqn. at section 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4$$

$$40 + 0.063 + 6 = \frac{P_2}{9810} + 1.012 + 4$$

$$46.063 = \frac{P_2}{9810} + 5.012$$

$\frac{P_2}{9810} = 46 \cdot 068 - 5 \cdot 017$

$\frac{P_2}{9810} = 41 \cdot 051$

$P_2 = 41 \cdot 051 \times 9810 \ N/m^2$

$= \frac{41 \cdot 051 \times 9810}{10^4} \ N/cm^2$

$= 40 \cdot 27 \ N/cm^2$

Intensity of pressure at section 2 = $40 \cdot 27 \ N/cm^2$

4. Water is flowing through a pipe having diameter 300mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is $24 \cdot 525 \ N/cm^2$ and the pressure at the upper end is $9 \cdot 81 \ N/cm^2$. Determine the difference in datum head if the rate of flow through pipe is 40 lit/sec.

Sol: Given,

Section 1, $D_1 = 300mm = 0 \cdot 3m$

$P_1 = 24 \cdot 525 \ N/cm^2$

$= 24 \cdot 525 \times 10^4 \ N/m^2$

Section 2, $D_2 = 200mm = 0 \cdot 2m$

$P_2 = 9 \cdot 81 \ N/cm^2$

$= 9 \cdot 81 \times 10^4 \ N/m^2$

Rate of flow = 40 lit/sec.

$Q = \frac{40}{1000} = 0 \cdot 04 \ m^3/sec$

Now, $A_1V_1 = A_2V_2$ = rate of flow = $0 \cdot 04$

$V_1 = \frac{0 \cdot 04}{A_1} = \frac{0 \cdot 04}{\pi/4 \, D_1^2}$

$= \frac{0 \cdot 04}{\pi/4 \, (0 \cdot 3)^2}$

$= 0 \cdot 5658 \ m/sec$

$\simeq 0 \cdot 566 \ m/sec$

$V_2 = \frac{0 \cdot 04}{A_2} = \frac{0 \cdot 04}{\pi/4 \, D_2^2}$

$= \frac{0.04}{\frac{\pi}{4}(0.2)^2}$

$= 1.274$ m/sec

Applying Bernoulli's equation at (1) and (2):

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{0.566 \times 0.566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2.$$

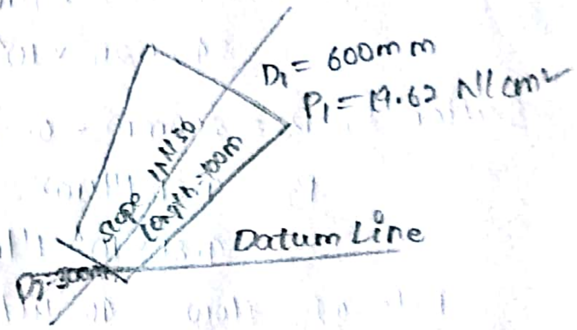
$$25 + 0.32 + z_1 = 10 + 1.623 + z_2$$

$$25.32 + z_1 = 11.623 + z_2$$

$$z_2 - z_1 = 25.32 - 11.623 = 13.697 \simeq 13.70 \text{ m}$$

Difference in datum head $= z_2 - z_1 = 13.70$ m.

5. The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm²



Sol: Given that;

length of pipe, $L = 100$m

Diameter at the upper end,

$D_1 = 600\text{mm} = 0.6\text{m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.6)^2 = 0.2827 \text{ m}^2$

$P_1 =$ Pressure at upper end $= 19.62$ N/cm².

$= 19.62 \times 10^4$ N/m²

Diameter at lower end, $D_2 = 300 \text{ mm} = 0.3$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2$

$= \frac{\pi}{4}(0.3)^2 = 0.07068$ m.

$Q =$ rate of flow $= 50$ litres/s $= \frac{50}{1000} = 0.05 \text{ m}^3/\text{sec}$.

Let the datum line is passing through the centre of lower end.

Then $z_2 = 0$

As slope is 1 in 30 means ... $\frac{1}{30} \times 100 = \frac{10}{3}$ m.

Also, we know $Q = A_1V_1 = A_2V_2$

$$V_1 = \frac{Q}{A} = \frac{0.05}{0.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}.$$

$$V_2 = \frac{Q}{A} = \frac{0.5}{0.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}.$$

Applying Bernoulli's equation at section (1) and (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(0.177)^2}{2 \times 9.81} + \frac{10}{3} = \frac{P_2}{\rho g} + \frac{(0.707)^2}{2 \times 9.81} + z_2$$

$$20 + 0.001596 + 3.334 = \frac{P_2}{\rho g} + 0.0254$$

$$23.335 - 0.0254 = \frac{P_2}{1000 \times 9.81}$$

$$P_2 = 23.3 \times 9810$$

$$= 228573 \text{ N/m}^2$$

$$= 22.857 \text{ N/cm}^2$$

Bernoulli's Equation for Real Fluid:

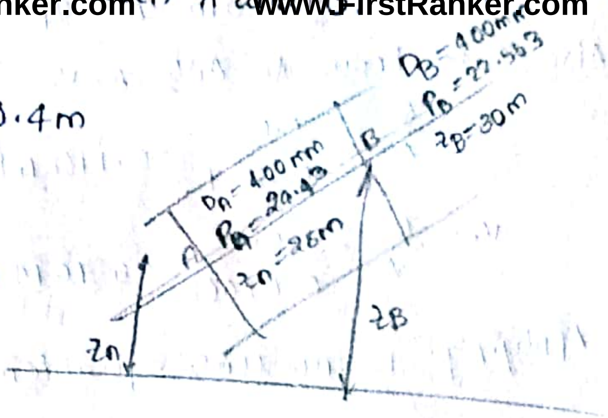
The Bernoulli's eqn. was derived on the assumption that fluid is non-viscous and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's eqn. these losses have to be taken into consideration. Thus, the Bernoulli's eqn. for real fluids between point 1 and 2 is given as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

Where, h_L = loss of energy between point 1 and 2.

6. A pipe of diameter 400mm carries water at a velocity of 25m/s. The pressure at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively. While the datum heat at A and B are

28m and 30m. Find the loss of head between A and B.



Sol: Given

Diameter of pipe, $D = 400mm = 0.4m$

velocity, $V = 25 m/sec$

At point A: $P_A = 29.43 \; N/cm^2$

$= 29.43 \times 10^4 \; N/m^2$

$z_A = 28m$

$V_A = 25 m/sec$

Total energy at point A $= \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 28$$

$= 30 + 31.85 + 28$

$E_A = 89.85m$

At point B: $P_B = 22.563 \; N/cm^2 = 22.563 \times 10^4 \; N/m^2$

$z_B = 30 m$

$V_B = 25 m/sec$

Total Energy at B, $E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 30$$

$= 23 + 31.85 + 30$

$= 84.85m$

Loss of head between A and B

$= E_A - E_B$

$= 89.85 - 84.85$

$= \underline{5m}$

7. A conical tube of length 2m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5m/s while at the lower end it is 2m/s. The pressure head at the smaller end is 2.5m of liquid. The loss of head in the tube is $\frac{0.35(V_1 - V_2)^2}{2g}$, where V_1 is the velocity at the smaller end and V_2

to the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Sol: Let the smaller end is represented by (1) and lower end by (2)

Given: Length of tube, $L = 2.0m$

$V_1 = 5 m/s$

Pressure head $\frac{P}{\rho g} = 2.5m$ of liquid

$V_2 = 2 m/s$

Loss of head $= h_L = \frac{0.35(V_1-V_2)^2}{2g}$

$= \frac{0.35(5-2)^2}{2\times 9.81}$

$= \frac{0.35\times 9}{2\times 9.81}$

$= 0.16m$

∴ Pressure at lower end $\frac{P_2}{\rho g} = ?$

Applying Bernoulli's theorem at sections (1) and (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$2.5 + \frac{(5)^2}{2\times 9.81} + 2.0 = \frac{P_2}{\rho g} + \frac{(2)^2}{2\times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{P_2}{\rho g} + 0.203 + 0.16$$

$$\frac{P_2}{\rho g} = (2.5+1.27+2.0) - (0.203+0.16)$$

$= 5.77 - 0.363$

$= 5.407m$ of fluid.

8. A pipe line carrying oil of specific gravity 0.87 changes in diameter from 200mm diameter at a position A to 500 mm diameter at a position B which is 4 m at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 l/s. determine the loss of head and direction of flow.

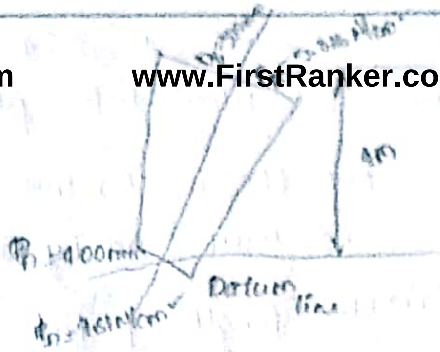
Sol: Discharge, $Q = 200$ lit/sec

$= 0.2 \ m^3/sec$

Specific gravity of oil = 0.87

∴ density of oil, $\rho = 0.87 \times 1000$

$= 870\ kg/m^3$



At Section A, $D_A = 200mm = 0.2m$

Area, $A_A = \frac{\pi}{4}(D_A)^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$

$P_A = 9.81\ N/cm^2 = 9.81 \times 10^4\ N/m^2$

If datum line is passing through A, then $z_A = 0$

$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369\ m/sec$

At Section B, $D_B = 500mm = 0.5m$

Area, $A_B = \frac{\pi}{4}D_B^2 = \frac{\pi}{4}(0.5)^2 = 0.1963\ m^2$

$P_B = 5.886\ N/cm^2$

$= 5.886 \times 10^4\ N/m^2$

$z_B = 4.0\ m$

$V_B = \frac{Q}{A_B} = \frac{0.2}{0.1963} = 1.018\ m/sec$

Total Energy at A is given by

$E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$

$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0$

$E_A = 11.49 + 2.067 = 13.557m$

Total Energy at B is given by

$E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$

$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{1.018}{2 \times 9.81} + 4$

$E_B = 6.896 + 0.052 + 4.0 = 10.948m$

i, Direction of flow : As E_A is more than E_B and hence flow is taking place from A to B

ii, Loss of head $= h_L = E_A - E_B = 13.557 - 10.948$

$= 2.609m$

The momentum Equation:

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass "m" is given by the Newton's second law of motion.

$$F = m \times a$$

Where a = acceleration acting in the same direction as force F.

But $a = \frac{dV}{dt}$

Substitute the value of a in above equation

$$F = m \cdot \frac{dV}{dt}$$

$$F = \frac{d(mV)}{dt}$$

{∵ m is constant and can be taken inside the differential}

$$F = \frac{d(mV)}{dt}$$

This equation is known as the momentum principle.

$$F\,dt = d(mV)$$

Which is known as impulse-momentum equation and states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d(mV) in the direction of the force.

→ **Force exerted by a flowing fluid on a pipe-Bend:**

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe-bend.

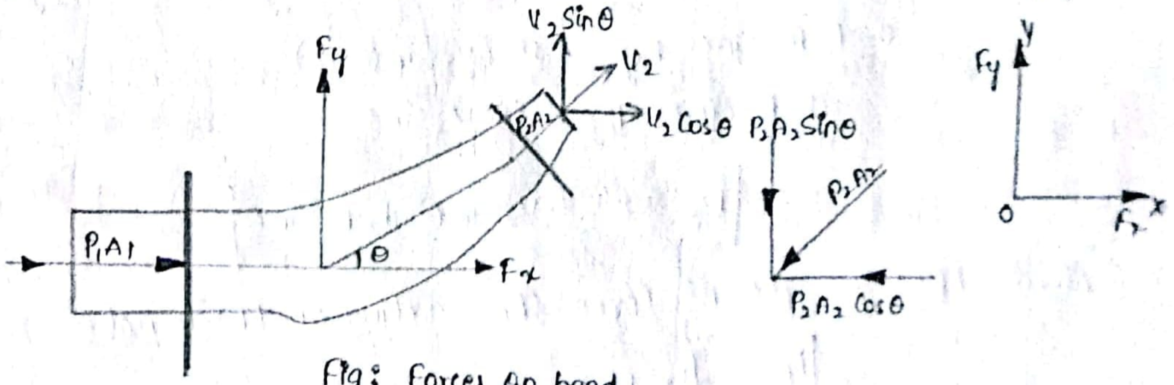


Fig: Forces on bend

Consider two sections (1) and (2) as shown in fig.

Let V_1 = Velocity of flow at section (1)

P_1 = Pressure intensity at section (1).

A_1 = Area of cross-section of pipe at section (1)

V_2, P_2, A_2 = Corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the following fluid on the bend in x and y directions respectively. Then the force exerted by the bend on the fluid in the direction of x and y will be equal to F_x and F_y but in the opposite directions.

Hence components of the force exerted by bend on the fluid in the x-direction $= -F_x$.

and in the direction of $y = -F_y$.

The other external forces acting on the fluid are P_1A_1 and P_2A_2 on the section (1) and (2) respectively. Then momentum eqn. in x-direction is given by

Net force acting on the fluid in the direction of x } = { Rate of change of momentum in x-direction.

$P_1A_1 - P_2A_2\cos\theta - F_x$ = (Mass per sec) × (change of velocity)

$= \rho Q$ (Final velocity in the direction of x − Initial velocity in the direction of x)

$$P_1A_1 - P_2A_2\cos\theta - F_x = \rho Q(V_2\cos\theta - V_1)$$

$$F_x = P_1A_1 - P_2A_2\cos\theta - \rho Q(V_2\cos\theta - V_1)$$

$$\boxed{F_x = P_1A_1 - P_2A_2\cos\theta + \rho Q(V_1 - V_2\cos\theta)}$$

Similarly, the momentum equation in y-direction gives

$$0 - P_2A_2\sin\theta - F_y = \rho Q(V_2\sin\theta - 0)$$

$$-P_2A_2\sin\theta - F_y = \rho Q V_2\sin\theta$$

$$\boxed{F_y = \rho Q(-V_2\sin\theta) - P_2A_2\sin\theta}$$

Now the resultant force (F_R) acting on the bend

$$\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$$

...nd the angle made by the resultant force with horizontal direction is given by.

$$\tan\theta = \frac{F_y}{F_x}$$

1. A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600mm and 300mm respectively. Find the force exerted by water on the bend if the intensity of the pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/sec.

Sol: Given data :

Angle of bend, $\theta = 45°$

Diameter of at inlet $D_1 = 600mm = 0.6m$

$\therefore$ Area, $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.6)^2 = 0.2827\ m^2$

Diameter at outlet, $D_2 = 300mm = 0.3m$

$\therefore$ Area, $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

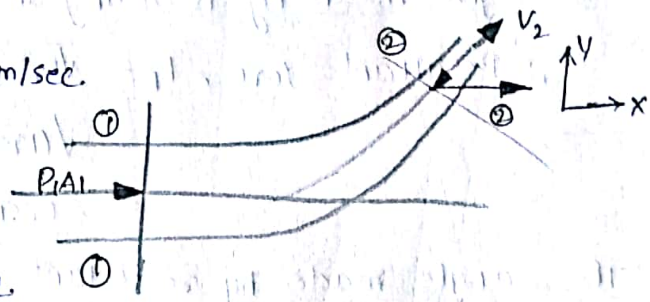
Pressure at inlet, $P_1 = 8.829\ N/cm^2 = 8.829\times10^4\ N/m^2$

$Q = 600\ lit/sec = 0.6\ m^3/sec$

$V_1 = \frac{Q}{A_1} = \frac{0.6}{0.2827} = 2.122\ m/sec.$

$V_2 = \frac{Q}{A_2} = \frac{0.6}{0.07068}$

$= 8.488\ m/sec.$



Applying Bernoulli's eqn. at sections (1) and (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{8.829\times10^4}{1000\times9.81} + \frac{2.122^2}{2\times9.81} = \frac{P_2}{1000\times9.81} + \frac{8.488^2}{2\times9.81}$$

$$9 + 0.2295 = \frac{P_2}{980} + 3.672$$

$\frac{P_2}{980} = 9 + 0.2295 - 3.672$

$\frac{P_2}{980} = 5.5575$ m of water

$P_2 = 5.5575 \times 980$

$P_2 = 5.45 \times 10^4$ N/m².

Forces on the bend in x and y directions are given by equations,

$F_x = \rho Q(V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$

$= \{1000 \times 0.6 [2.122 - 8.488 \cos 45°]\} + (8.829 \times 10^4 \times 0.2827)$
$- (5.45 \times 10^4 \times 0.07068 \times \cos 45°)$

$F_x = -2327.9 + 24959.6 - 2720.3$

$= 24959.6 - 5048.2$

$= 19911.4$ N

$F_y = \rho Q(-V_2 \sin\theta) - P_2 A_2 \sin\theta$

$= +1000 \times 0.6(-8.488 \sin 45°) - (5.45 \times 10^4 \times 0.07068 \times \sin 45°)$

$= -3601.1 - 2721.1$

$= -6322.2$ N

-ve sign mean F_y is acting in the downward direction.

∴ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(19911.4)^2 + (-6322.2)^2}$

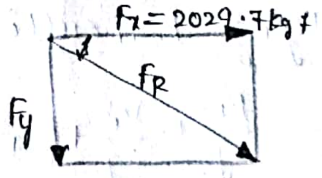
$= 20890.9$ N

The angle made by resultant force with x-axis is given by

$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$

$\theta = \tan^{-1}(0.3175)$

$\theta = 17° 36'$



2. 250 lit/sec of water is flowing in a pipe having a diameter of 300mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm²

Sol: Given data:

Pressure, $P_1 = P_2 = 39.24$ N/cm² $= 39.24 \times 10^4$ N/m²

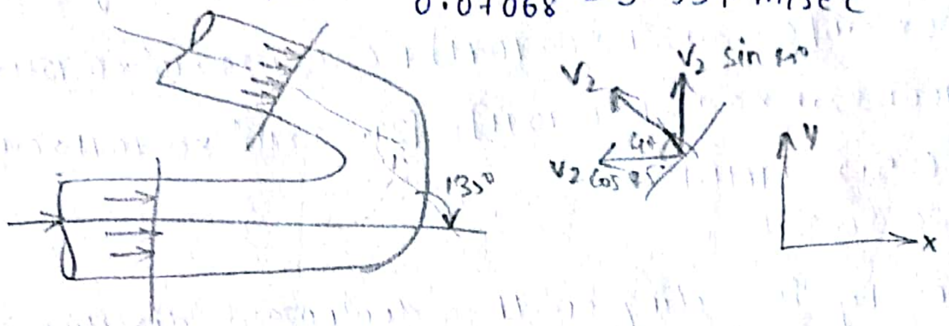
Discharge Q = 250 litres/s = 0.25 m^3/sec

Diameter of bend at inlet and outlet, $D_1 = D_2 = 300mm = 0.3m$.

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \, m^2$

Velocity of water at (1) and (2)

$$V = V_1 = V_2 = \frac{Q}{Area} = \frac{0.25}{0.07068} = 3.537 \, m/sec$$



Force along x-axis:

$$F_x = \rho Q [V_{1x} - V_{2x}] + P_{1x} A_1 + P_{2x} A_2$$

Where, V_{1x} = Initial velocity in the direction of $x = 3.537$ m/sec

V_{2x} = Final velocity in the direction of $x = -V_2 \cos 45°$

$= -3.537 \cos 45° = -3.537 \times 0.7071$

P_{1x} = Pressure at section (1) in x-direction

$= 39.24 \times 10^4 \, N/m^2$

P_{2x} = Pressure at section (2) in x-direction.

$= P_2 \cos 45°$

$= 39.24 \times 10^4 \times 0.7071$

Substituting all the values in the equation of F_x.

$\therefore F_x = 1000 \times 2.5 [3.537 + (3.537 \times 0.7071)]$
$+ (39.24 \times 10^4 \times 0.07068) + (39.24 \times 10^4 \times 0.07068 \times 0.7071)$

$F_x = 1000 \times 2.5 [3.537 + (3.537 \times 0.7071)]$
$+ 39.24 \times 10^4 \times 0.07068 [1 + 0.7071]$

$= 1509.4 + 47346$

$F = 48855.4 \, N$

Force along y-axis:

$$F_y = \rho Q [V_{1y} - V_{2y}] + (P_1 A_1)_y + (P_2 A_2)_y$$

Where V_{1y} = Initial velocity in y-direction = 0

V_{2y} = Final Velocity in y-direction $= V_2 \sin 45^0$
$= 3.537 \times \sin 45^0 = 3.537 \times 0.7071$

$(P_1 A_1)_y$ = Pressure force at section (1) in y-direction $= -P_2 \sin 45^0 A_2$
$= -39.24 \times 10^4 \times 0.7071 \times 0.07068$

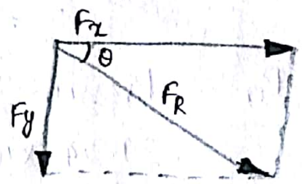
Substituting all the above values in the equation of F_y.

$$F_y = 1000 \times 2.5[0 - 3.537 \times 0.7071] + 0 - (39.24 \times 10^4 \times 0.7071 \times 0.07068)$$
$$= -[1000 \times 2.5 \times 3.537 \times 0.7071] - [39.24 \times 10^4 \times 0.7071 \times 0.07068]$$
$$= -625.2 - 19611.1$$
$$= -20236.3 \text{ N}$$

-ve sign means F_y is acting on the downward direction.

∴ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$
$$= \sqrt{48855.4^2 + 20236.3^2}$$
$$= 52880.6 \text{ N}$$

The direction of the resultant force F_R, with the x-axis is given as



$$\tan\theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$
$$\theta = \tan^{-1}(0.4142)$$
$$\theta = 22^\circ 30'$$

3. A 300mm diameter pipe carries water under a head of 20m with a velocity of 3.5 m/s. If the axis of pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.

Sol: Diameter of pipe, $D = D_1 = D_2 = 300\text{mm} = 0.3\text{m}$

∴ Area, $A_1 = A_2 = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

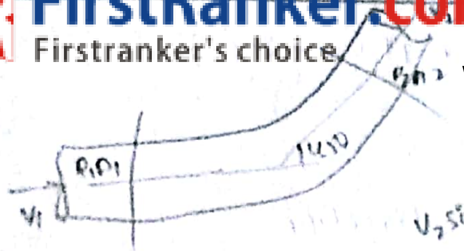
Velocity, $V = V_1 = V_2 = 3.5$ m/sec

$\theta = 45^0$ discharge $Q = AV = 0.07068 \times 3.5$
$= 0.2475$ m³/s

Pressure head = 20m of water

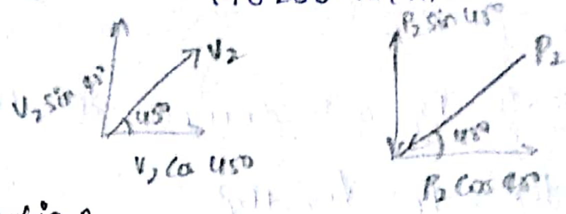
$$\frac{P}{\rho g} = 20\text{m of water}$$

Pressure intensity, $P = P_1 = P_2 = 20 \times \rho g$



$= 20 \times 1000 \times 9.81$

$= 196200 \ N/m^2$



Force along x-direction:

$$F_x = \rho Q[V_{1x} - V_{2x}] + (P_1A_1)_x + (P_2A_2)_x$$

V_{1x} = Initial velocity in x-direction = 3.5 m/sec

V_{2x} = Final velocity in x-direction = $V_2 \cos 45° = 3.5 \times 0.7071$

$(P_1A_1)_x$ = pressure force at section (1) in x-direction.

$= 196200 \times 0.07068$

$(P_2A_2)_x$ = Pressure force at section (2) in x-direction.

$= -P_2 \cos 45° A_2$

$= -196200 \times 0.7071 \times 0.07068$

Substituting all the above values in the equation of F_x

$F_x = 1000 \times 0.2475 [3.5 - (3.5 \times 0.7071)] + (196200 \times 0.07068)$
$- (196200 \times 0.7071 \times 0.07068)$

$= 253.68 + 13871.34 - 9808.04$

$= 4316.98 \ N$

Force along y-direction:

$$F_y = \rho Q[V_{1y} - V_{2y}] + (P_1A_1)_y + (P_2A_2)_y$$

Where, V_{1y} = Initial velocity in y-direction = 0

V_{2y} = Final velocity in y-direction = $V_2 \sin 45° = 3.5 \times 0.7071$

$(P_1A_1)_y$ = Pressure force at section (1) in y-direction = 0

$(P_2A_2)_y$ = Pressure force at section (2) in y-direction

$= -P_2 \sin 45° \times A_2$

$= -196200 \times 0.7071 \times 0.07068$

Substituting all the above values in equation of F_y.

$F_y = 1000 \times 0.2475 [0 - 3.5 \times 0.7071) + 0 - 196200 \times 0.7071 \times 0.07068$

$= ([illegible] \times 3.5 \times 0.7071) - (196200 \times 0.7071 \times 0.07068)$

$= -612.44 - 9808$

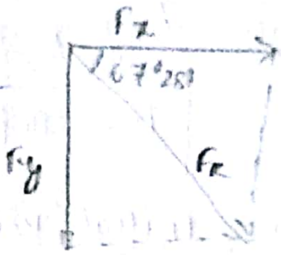
$= -10420.44\ N$

−ve sign indicate F_y acts downward direction.

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{4316.96^2 + 10420.44^2}$

$= 11279\ N$



The angle made by F_R with x-axis

$\tan\theta = \frac{F_y}{F_x} \Rightarrow \tan\theta = \frac{10420.44}{4316.96} = 2.411$

$\theta = \tan^{-1}(2.411)$

$= 67°28'$

Applications of Impulse – Momentum Equation:

The impulse momentum eqn. is used in the following types of Engineering problems:

1. To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude (or) both.

• Problem of this types are:

i) Pipe bend
ii) Reducers
iii) Moving vanes
iv) Jet propulsion etc.

2. To determine the characteristic of flow when there is an abrupt change of flow section.

• Problems of this type are:

Sudden enlargement in pipe
Hydraulic jump in a channel etc.

UNIT –IV

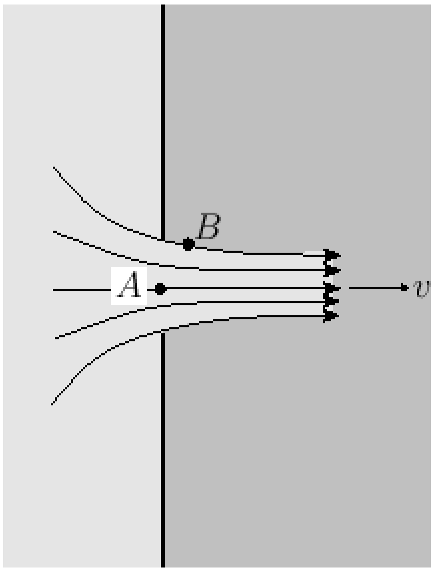
INTRODUCTION:

Accurate measurement of flow rate of liquids and gases is an essential requirement for maintaining the quality of industrial processes. In fact, most of the industrial control loops control the flow rates of incoming liquids or gases in order to achieve the control objective. As a result, accurate measurement of flow rate is very important. Needless to say that there could be diverse requirements of flow measurement, depending upon the situation. It could be volumetric or mass flow rate, the medium could be gas or liquid, the measurement could be intrusive or nonintrusive, and so on. As a result there are different types of flow measuring techniques that are used in industries. The common types of flowmeters that find industrial applications can be listed as below: (a) Obstruction type (differential pressure or variable area) (b) Inferential (turbine type), (c) Electromagnetic, (d) Positive displacement (integrating), (e) fluid dynamic (vortex shedding), (f) Anemometer, (g) ultrasonic and (h) Mass flowmeter (Coriolis). In this lesson, we would learn about the construction and principle of operation few types of flowmeters.

Basic Principle It is well know that flow can be of two types: viscous and turbulent. Whether a flow is viscous or turbulent can be decided by the Reynold's number RD. If RD > 2000, the flow is turbulent. In the present case we will assume that the flow is turbulent, that is the normal case for practical situations. We consider the fluid flow through a closed channel of variable cross section.

Flow Through an Orifice

Consider the situation, illustrated in Figure 4.3, in which a horizontal jet of fluid emerges from an orifice in the side of a container. As shown in the figure, the jet narrows over a short distance beyond the orifice that is comparable with the jet diameter to form what is generally known as a *vena contracta*--that is, a ``contracted vein." The jet is bound to narrow in this manner because of the curvature of the lines of flow as they pass through the orifice. The narrowing of the jet implies the existence of a transverse pressure gradient. In other words, the pressure at A, on the axis of the jet, is higher than the atmospheric pressure that acts at B. The pressure excess at A suggests that the fluid on the axis is still accelerating longitudinally as it leaves the orifice. Only in the vena contracta does the flow velocity becomes uniform, and the pressure atmospheric, all the way across the jet.



Outflow through an orifice.

Let S be the cross-sectional area of the orifice, and $\mathcal{C}\,S$ that of the vena contracta. Here, $\mathcal{C}$ is known as the *contraction coefficient*. Let us apply Bernoulli's theorem to a streamline that starts on the surface of the fluid within the container, and ends in the vena contracta. Suppose that the surface of the fluid lies a height h above the orifice. Let us assume that the fluid close to the surface is essentially at rest (which implies that the outflow through the orifice is not sufficiently strong to cause the surface level to drop at a significant rate.) Let v be the uniform fluid velocity in the vena contracta. Of course, the pressure is atmospheric both at the surface of the fluid and in the vena contracta. It follows that

$$g\,h = \frac{1}{2}\,v^2, \tag{4.14}$$

or

$$v = (2\,g\,h)^{1/2}. \tag{4.15}$$

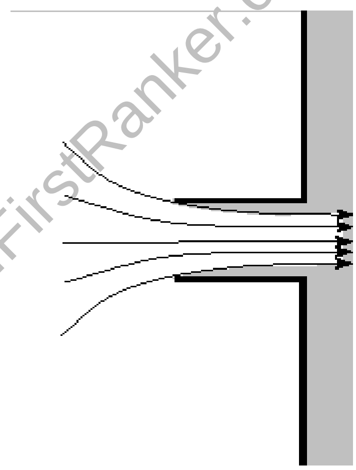
In other words, the efflux velocity of the fluid from the orifice is the same as that it would have acquired by falling a height h under gravity. This result is known as *Torricelli's law*, after Evangelista Torricelli (1608-1647). Finally, the discharge rate of fluid flowing through the orifice is

$$Q = \mathcal{C}\,S\,v = \mathcal{C}\,S\,(2\,g\,h)^{1/2}. \tag{4.16}$$

Let $p = p_0 + \rho\, g\, h$ be the hydrostatic pressure at the level of the orifice when the orifice is closed. Here, p_0 is atmospheric pressure. The fluid experiences a thrust $S\,p$ from the section of the wall directly opposite the orifice, and a thrust $-S\,p$ from the section of the wall closing the orifice. Let us suppose, as a first approximation, that the hydrostatic pressure remains unaltered when the orifice is opened. In this situation, the fluid experiences a thrust $S\,p$ from the section of the wall directly opposite the orifice, and a thrust $-S\,p_0$ from the orifice. The net thrust, $S\,(p-p_0) = S\,\rho\,g\,h$, is responsible for accelerating the jet. Now, the jet's rate of momentum outflow is $\rho\,v\,\mathcal{C}\,S \times v$. Momentum conservation yields

$$S\,\rho\,g\,h = \mathcal{C}\,S\,\rho\,v^2 = 2\mathcal{C}\,S\,\rho\,g\,h, \tag{4.17}$$

where use has been made of Equation ,Thus, we conclude that the contraction coefficient takes the value $1/2$.



A Borda mouthpiece.

In reality, Bernoulli's theorem suggests that when the orifice is opened the pressure on the walls in the neighborhood of the orifice will fall below the hydrostatic value,

$$S\,(p - p_0)$$

which implies that the accelerating thrust is actually greater than . Consequently, $\mathcal{C} > 1/2$.

Obviously, $\mathcal{C}$ cannot exceed unity, so we conclude that, in general, $1/2 < \mathcal{C} < 1$. For instance, if the orifice is a circular hole punched in a thin plate then the contraction coefficient is observed to take the value 0.62 (Batchelor 2000).

Suppose, however, that we fit a small cylindrical nozzle projecting inward from the orifice, as shown in Figure .In this case, the original assumption that the pressure on the walls in the neighborhood of the orifice is hydrostatic is essentially correct. This follows because the region where the lines of flow are converging on the orifice is far removed from the walls, and the velocity of the fluid in contact with the walls is negligible. Thus, the contraction coefficient is exactly $1/2$. This arrangement is known as a *Borda mouthpiece*, after Jean-Charles Borda (1733-1799).

Triangular Notch:

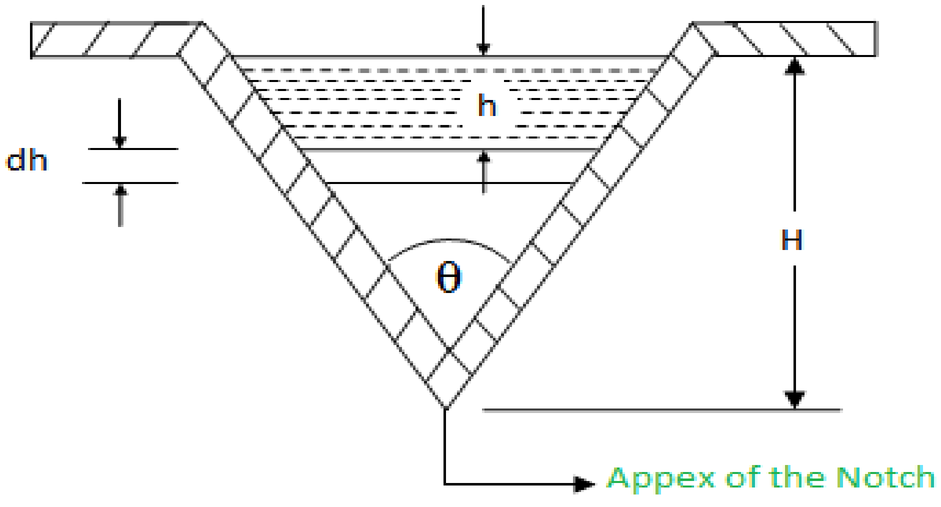


Fig : Triangular Notch

A triangular notch is also called a **V-notch**. Consider a triangular notch, in one side of the tank, over which water is flowing as shown in figure.

Let,

- H = Height of the liquid above the apex of the notch
- θ = Angle of the notch
- C_d = Coefficient of discharge

From the geometry of the figure, we find that the width of the notch at the water surface,

$$= 2H \tan\frac{\theta}{2}$$

$\therefore$ Area of the strip = $2(H-h)\tan\frac{\theta}{2}.dh$

We know that the theoretical velocity of water through the strip = $\sqrt{2gh}$

and discharge over the notch,

$$dq = C_d \times Area\ of\ strip \times Theoretical\ velocity$$

$$\Rightarrow dq = C_d \times 2(H-h)\tan\frac{\theta}{2}.dh\sqrt{2gh}$$

The total discharge over the whole notch may be found out only by integrating the above equation within the limits 0 and H.

$$Q = \int_0^H C_d \times 2(H-h)\tan\frac{\theta}{2}.dh\sqrt{2gh}$$

$$\Rightarrow Q = 2C_d\sqrt{2g} \times \tan\frac{\theta}{2}\int_0^H (H-h)\sqrt{h}dh$$

$$\Rightarrow Q = 2C_d\sqrt{2g} \times \tan\frac{\theta}{2}\int_0^H (Hh^{\frac{1}{2}} - h^{\frac{3}{2}})dh$$

$$\therefore Q = \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2} \times H^{\frac{5}{2}}$$

A triangular notch gives more accurate results for low discharges than rectangular notch and the same triangular notch can measure a wide range of flows accurately.

Example:

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Example - Discharge through a triangular notch
Problem

A right-angled **V-notch** was used to measure the discharge of a centrifugal pump. If the depth of water at V-notch is 200mm, calculate the discharge over the notch in liters per minute. Assume coefficient of discharge as 0.62.

Workings

Given,

- $\theta = 90^\circ$
- $H = 200\,mm = 0.2\,m$
- $C_d = 0.62$

We know that the discharge over the triangular notch,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{\frac{5}{2}}$$

$$\Rightarrow Q = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \tan 45^\circ \times (0.2)^{\frac{5}{2}}$$

$$\Rightarrow Q = 1.465 \times 0.018 = 0.026 m^3/s$$

$$\therefore Q = 26\ liter\,s/s = 1560\ liter\,s/min$$

Notch

A notch may be defined as an opening in one side of a tank or a reservoir, like a large orifice, with the upstream liquid level below the top edge of the opening.

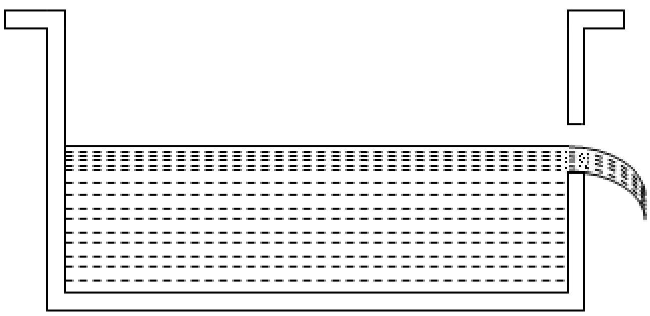


Fig : A Notch

Since the top edge of the notch above the liquid level serves no purpose, therefore a notch may have only the bottom edge and sides.

The bottom edge, over which the liquid flows, is known as **sill** or **crest** of the notch and the sheet of liquid flowing over a notch (or a weir) is known as **nappe** or **vein**. A notch is, usually made of a metallic plate and is used to measure the discharge of liquids.

Types Of Notches

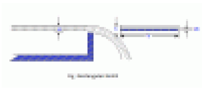
There are many types of notches, depending upon their shapes. But the following are important from the subject point of view.

- Rectangular notch
- Triangular notch
- Trapezoidal notch
- Stepped notch

Section Pages

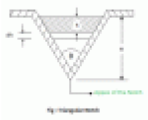
Rectangular notch

Discharge over a rectangular notch



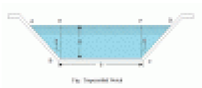
Triangular Notch

Discharge over a Triangular Notch



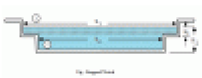
Trapezoidal Notch

Discharge over a Trapezoidal Notch



Stepped Notch

Discharge over a Stepped Notch



Rectangular notch
Discharge over a rectangular notch

View other versions (2)

Theory

Consider a rectangular notch in one side of a tank over which water is flowing as shown in figure.

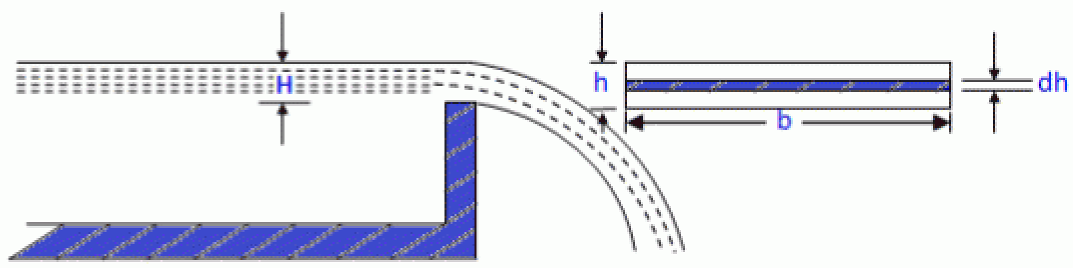


Fig : Rectangular Notch

Let,

- H = Height of water above sill of notch
- b = Width or length of the notch
- C_d = Coefficient of discharge

Let us consider a horizontal strip of water of thickness *dh* at a depth of *h* from the water level as shown in figure.

∴ Area of the strip

$$= b.dh$$

We know know that the theoretical velocity of water through the strip,

$$= \sqrt{2gh}$$

Discharge through the strip,

$$dq = C_d \times Area\ of\ strip\ \times\ Theoretical\ velocity$$

$$\Rightarrow dq = C_d.bdh\sqrt{2gh}$$

The total discharge over the whole notch, may be found out by integrating the above equation within the limits 0 and H.

$$Q = \int_0^H C_d.b.dh\sqrt{2gh}$$

$$\Rightarrow Q = C_d.b\sqrt{2g}\int_0^H h^{\frac{1}{2}}.dh$$

$$\therefore Q = \frac{2}{3}C_d.b\sqrt{2g}(H)^{\frac{3}{2}}$$

Example:

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[metric]

Example - Discharge over a rectangular notch
Problem

A rectangular notch 0.5m wide has constant head of 400 mm. Find the discharge over the notch in liters per second, if the coefficient of discharge for the notch is 0.62.

Workings

Given,

- b = 0.5 m
- H = 400 mm = 0.4 m
- C_d = 0.62

We know that discharge over the rectangular notch,

$$Q = \frac{2}{3}C_d.b\sqrt{2g}(H)^{\frac{3}{2}}\ m^3/s$$

$$\Rightarrow Q = \frac{2}{3} \times 0.62 \times 0.5\sqrt{2 \times 9.81}(0.4)^{\frac{3}{2}}\ m^3/s$$

$$\Rightarrow Q = 0.915 \times 0.253 = 0.231\ m^3/s = 231\ liters/s$$

Discharge over a Trapezoidal Notch

Overview

A trapezoidal notch is a combination of a rectangular notch and two triangular notches as shown in figure. It is, thus obvious that the discharge over such a notch will be the sum of the discharge over the rectangular and triangular notches.

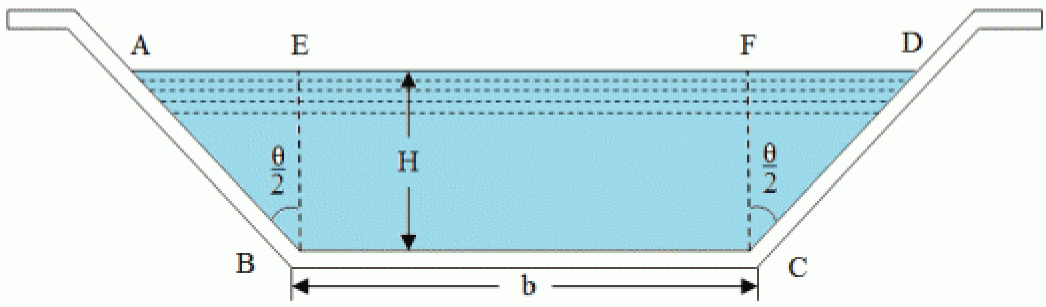

Fig : Trapezoidal Notch

Consider a trapezoidal notch *ABCD* as shown in figure. For the purpose of analysis, split up the notch into a rectangular notch *BCFE* and two triangular notches *ABE* and *DCF*. The discharge over these two triangular notches is equivalent to the discharge over a single triangular notch of angle θ.

Let,

- H = Height of the liquid above the sill of the notch
- C_{d1} = Coefficient of discharge for the rectangular portion
- C_{d2} = Coefficient of discharge for the triangular portion
- b = Breadth of the rectangular portion of the notch
- $\frac{\theta}{2}$ = Angle, which the sides make with the vertical

∴ Discharge over the trapezoidal notch,

Q = Discharge over the rectangular notch + Discharge over the triangular notch

$$\therefore Q = \frac{2}{3} C_d . b \sqrt{2g} (H)^{\frac{3}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} \times H^{\frac{5}{2}}$$

Example: 1

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Example - Discharge over the trapezoidal notch

Problem

A trapezoidal notch notch of 1.2m wide at the top and 450mm at the bottom is 300mm high. Find the discharge through the notch, if the head of water is 225mm. Take coefficient of discharge as 0.6.

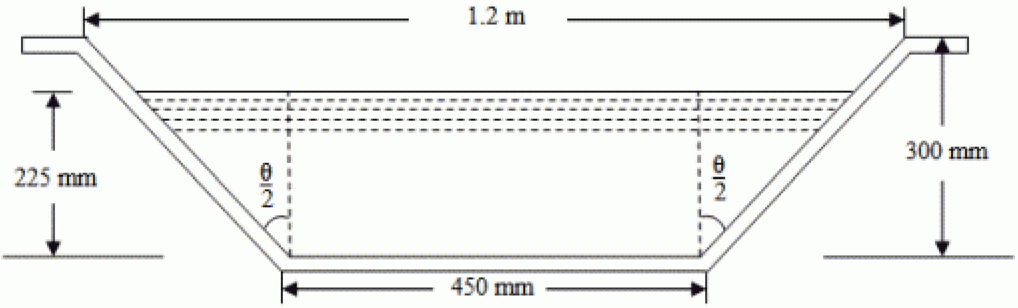


Fig : Discharge over a trapezoidal notch

Workings

Given,

- Width of the notch = 1.2m
- b = 450mm = 0.45m
- Height of the notch = 300mm = 0.3m
- H = 225mm = 0.225m
- C_d = 0.6

From the geometry of the notch, we get,

$$\tan\frac{\theta}{2} = \frac{1200 - 450}{2} \times \frac{1}{300} = \frac{750}{600} = 1.25$$

and the discharge over trapezoidal notch,

$$Q = \frac{2}{3}C_d.b\sqrt{2g}(H)^{\frac{3}{2}} + \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2} \times H^{\frac{5}{2}}$$

$= \frac{2}{3} \times 0.6 \times 0.45\sqrt{2 \times 9.81} \times (0.225)^{\frac{3}{2}} + \frac{8}{15} \times 0.6\sqrt{2 \times 9.81} \times 1.25 \times (0.225)^{\frac{5}{2}}\ m^3/s$

$= 0.085 + 0.043 = 0.128\ m^3/s = 128\ liter\,s/s$

Reynold observed that loss of head is approximately proportional to the square of velocity. More exactly the loss of head, $h_f \propto V^n$, where n varies from 1·75 to 2·0.

LOSS OF ENERGY IN PIPES:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as

Energy losses

1. Major Energy losses

This is due to friction and it is calculated by the following formulae:

(a) Darcy – Weisbach Formula

(b) Chezy's formula

2. Minor Energy losses

This is due to

(a) Sudden expansion of pipe.

(b) Sudden contraction of pipe.

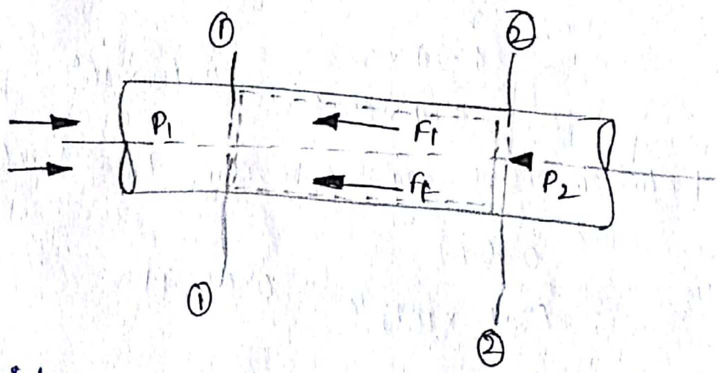
(c) Bend in pipe.

(d) Pipe fittings etc

(e) An obstruction in pipe.

LOSS OF ENERGY (OR HEAD) DUE TO FRICTION :

(a) Darcy – Weisbach Formula:



Consider a uniform horizontal pipe, having steady flow as shown in figure.

Let 1-1 and 2-2 are two sections of pipe

Let P_1 = Pressure intensity at section 1-1.

V_1 = Velocity of flow at section 1-1.

L = Length of the pipe between sections 1-1 & 2-2

d = diameter of the pipe.

f' = frictional resistance per unit wetted area per unit velocity.

The loss of head (or energy) in pipes due to friction is calculated from Darcy-weisbach equation.

$$h_f = \frac{4fLV^2}{2gd}$$

Where, h_f = loss of head due to friction.

f = co-efficient of friction which is a function of Reynold-number.

$= \frac{16}{R_e}$ for $R_e < 2000$ viscous flow

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = Length of the pipe.

V = mean velocity of flow.

d = diameter of the pipe.

(b) Chezy's Formula for loss of head due to friction in pipes

We know the equation,

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

Where h_f = loss of head due to friction.

P = Wetted perimeter of pipe.

A = area of cross-section of pipe

L = length of pipe.

V = mean velocity of flow.

Now the ratio of $\frac{A}{P} = \left(\frac{\text{Area of flow}}{\text{perimeter (wetted)}}\right)$ is called hydraulic mean depth (or) hydraulic radius and is denoted by m.

Hydraulic mean depth, $m = \frac{A}{P} = \frac{\pi/4\, d^2}{\pi d} = \frac{d}{4}$

substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (1)

$$h_f = \frac{f'}{\rho g} \cdot \frac{1}{m} \cdot L \times V^2$$

$$V^2 = h_f \times \frac{\rho g}{f'} \cdot m \cdot \frac{1}{L}$$

The forces acting on the fluid between sections 1-1 and 2-2 are;

1. Pressure force at section 1-1 = $P_1 \times A$

where A = Area of pipe

2. Pressure force at section 2-2 = $P_2 \times A$
3. Frictional force F_1

Resolving all the forces in the horizontal direction.

$$P_1 A_1 - P_2 A_2 - F_1 = 0$$

$$(P_1 - P_2) A = F_1$$

$$P_1 - P_2 = \frac{F_1}{A}$$

$$P_1 - P_2 = \frac{f' P L V^2}{A}$$

But from eqn (i) $P_1 - P_2 = \rho g h_f$

$$\rho g h_f = \frac{f' P L V^2}{A} \rightarrow \boxed{h_f = \frac{f'}{\rho g} \frac{P}{A} L V^2} \quad \text{——— (3)}$$

In eqn. (3) $\frac{P}{A} = \frac{\text{Wetted Perimeter}}{\text{area}}$

$$= \frac{\pi d}{\frac{\pi}{4} d^2} = 4/d$$

Substitute the value of $\frac{P}{A}$ in eqn (3)

$$\boxed{h_f = \frac{f'}{\rho g} \cdot \frac{4}{d} \cdot L V^2} \quad \text{——— (4)}$$

Putting $\frac{f'}{\rho g} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (4) becomes as $h_f = \frac{f}{2g} \cdot \frac{4}{d} L V^2$

$$\boxed{h_f = \frac{4 f L V^2}{2 g d}}$$

The above eqn. is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes. Sometimes the Darcy-Weisbach eqn. is written as

$$\boxed{h_f = \frac{f \cdot L \cdot V^2}{2 g d}}$$

Then f is known as friction factor.

- Friction factor f is not constant. It depends on roughness condition of pipe surface and Reynolds number of the flow.

h_f = loss of head due to friction.

and P_2, V_2 = values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's eqn. at sections 1-1 and 2-2.

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal.

$V_1 = V_2$ as diameter of pipe is same at 1-1 and 2-2.

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\boxed{h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}} \quad \text{——— (1)}$$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now,

frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity².

$$F_1 = f' \times \pi d L \times V^2$$

$$= f' P L V^2$$

[∵ Wetted area $= \pi d L$; Velocity $V = V_1 = V_2$; P = perimeter $= \pi d$]

$$\boxed{\therefore F_1 = f' P L V^2} \quad \text{——— (2)}$$

4. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300mm at the rate of 500 lit/sec. Find the head lost due to friction and power required to maintain the flow for a length of 1000m. Take ν = 0.29 stokes.

Sol: Given:

Specific gravity of oil, $S = 0.7$

diameter of pipe, $d = 300\text{mm} = 0.3\text{m}$

discharge, $Q = 500$ litres/sec.

length of pipe, $L = 1000\text{m}$

Area $= \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5\times4}{\pi\times0.3^2} = 7.073\ m/s$

$\therefore$ Reynold number, $R_e = \frac{V\times d}{\nu}$

$= \frac{7.073\times0.3}{0.29\times10^{-4}}$

$= 7.316\times(10)^4$

Coefficient of friction, $f = \frac{0.79}{Re^{1/4}} = \frac{0.79}{(7.316\times10^4)^{1/4}}$

$= 0.0048.$

$\therefore$ Head lost due to friction,

$h_f = \frac{4fLV^2}{d\times2g} = \frac{4\times0.0048\times1000\times7.073^2}{0.3\times2\times9.81}$

$= 163.16\ m$

Power required $= \frac{Sg\cdot Q\cdot h_f}{1000}$ kW

4. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 lit/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take $\nu = 0.29$ stokes.

Sol: Given:

Specific gravity of oil, $s = 0.7$

diameter of pipe, $d = 300mm = 0.3m$

discharge, $Q = 500$ lit/sec $= 0.5\ m^3/sec$

Length of pipe, $L = 1000m$

Velocity, $V = \frac{Q}{Area} = \frac{0.5}{\pi/4\,d^2} = \frac{0.5\times4}{\pi\times0.3^2}$

$= 7.073\ m/s.$

$\therefore$ Reynold number, $R_e = \frac{V\times d}{\nu} = \frac{7.073\times0.3}{0.29\times10^{-4}}$

$= 7.316\times(10)^4$

Coefficient of friction, $f = \frac{0.79}{Re^{1/4}}$

$= \frac{0.79}{(7.316\times10^4)^{1/4}}$

$= 0.0048$

Head lost due to friction, $h_f = \frac{4fLV^2}{d \times 2g}$

$= \frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81}$

$= 163.18\ m$

Power required $= \frac{\rho g \cdot Q \cdot h_f}{1000}$ kW

where ρ = density of oil $= 0.7 \times 1000 = 700\ kg/m^3$

$\therefore$ Power required $= \frac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$

$= \underline{560.28\ kW}$

5. Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500m apart is 4m of formula, $h_f = \frac{4fLV^2}{d \times 2g}$.

Sol: Given,

Diameter of pipe, $d = 200mm = 0.2m$

Length of pipe, $L = 500\ m$

Difference of pressure head, $h_f = 4m$ of water

$f = 0.009$

$$h_f = \frac{4fLV^2}{d \times 2g}$$

$$4 = \frac{4 \times 0.009 \times 500 \times V^2}{0.2 \times 2 \times 9.81}$$

$$V^2 = \frac{4 \times 0.2 \times 2 \times 9.81}{4 \times 0.009 \times 500}$$

$$V^2 = 0.872$$

$$V = \sqrt{0.872} = 0.9338 \simeq 0.934\ m/s$$

$$V = 0.934\ m/s$$

$\therefore$ Discharge, Q = Area × velocity

$= \frac{\pi}{4} d^2 \times V$

$= \frac{\pi}{4} (0.2)^2 \times 0.934$

$= 0.0293\ m^3/s$

$= 29.3\ lit/s.$

6. Water is flowing through a pipe of diameter 200mm with a velocity of 3m/s. Find the head lost due to friction for a length of 5m if the co-efficient of friction is given by $f = 0.002 + \frac{0.09}{Re^{0.3}}$ where Re = Reynold number. The kinematic viscosity of water = 0.01 stoke.

Sol: Given:

Diameter of pipe, d = 200mm = 0.2 m

Velocity, V = 3 m/sec

length of the pipe, L = 5m

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01 \times 10^{-4}\, m^2/sec$.

∴ Reynold number, $Re = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{0.01 \times 10^{-4}}$

$= 6 \times 10^5$

Value of $f = 0.002 + \frac{0.09}{Re^{0.3}}$

$= 0.002 + \frac{0.09}{(6 \times 10^5)^{0.3}}$

$= 0.002 + \frac{0.09}{54.13}$

$= 0.002 + 0.00166$

$= 0.00366$

∴ Head lost due to friction,

$h_f = \frac{4fLV^2}{2gd}$

$= \frac{4 \times 0.02166 \times 5.0 \times 3^2}{2 \times 9.81 \times 0.2}$

$= 0.1678$ m of water

7. An oil of specific gravity 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200mm at the rate of 600 lit/s. Find the head lost due to friction for a 500m length of pipe. Find the power required to maintain this flow.

Sol: Given: Specific gravity of oil, S = 0.9

viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10}\, N\text{-}s/m^2$

diameter of pipe, d = 200mm = 0.2m

Discharge Q = 60 litres/s = 0.06 m³/sec.

Length of pipe, L = 500m

Density, $\rho = 0.9 \times 1000 = 900\ kg/m^3$

$\therefore$ Reynold number, $R_e = \frac{\rho V d}{\mu} = \frac{900 \times V \times 0.2}{0.06/10}$

where, $V = \frac{Q}{Area} = \frac{0.6}{\frac{\pi}{4}(0.2)^2} = 1.909\ m/s \simeq 1.91\ m/s$

$$R_e = \frac{900 \times 1.91 \times 0.2}{0.06/10}$$

$$= 57300$$

R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}}$$

$$= \frac{0.079}{(57300)^{0.25}} = 0.0051$$

Head lost due to friction, $h_f = \frac{4fLV^2}{2gd}$

$$= \frac{4 \times 0.0051 \times 500 \times 1.91^2}{2 \times 9.81 \times 0.2}$$

$$= 9.48\ m \text{ of water}$$

$\therefore$ Power required $= \frac{\rho g Q h_f}{1000}$

$$= \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000}$$

$$= 5.02\ kW$$

MINOR ENERGY (HEAD) LOSSES:

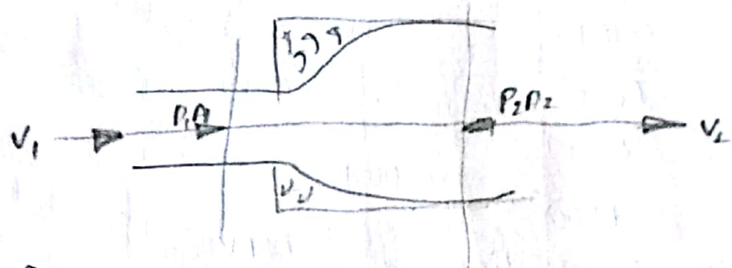
The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases:

1. Loss of head due to sudden enlargement.
2. Loss of head due to sudden contraction.
3. Loss of head at the entrance of a pipe.

4. loss of head at the exit of a pipe.
5. loss of head due to an obstruction in a pipe.
6. loss of head due to bend in the pipe.
7. loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in with the loss of head due to friction.

→ Loss of head due to sudden enlargement:



Consider a liquid flowing through a pipe which has sudden enlargement as shown in fig. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

Let P_1 = Pressure intensity at section (1) – (1)
V_1 = Velocity of flow at section 1-1
A_1 = area of pipe at section 1-1
P_2, V_2, A_2 = Corresponding values at section 2-2.

Due to sudden change of diameter of pipe from D_1 to D_2 the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown. The loss of head (or energy) takes place due to the formation of the eddies.

Let P' = Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$.
h_e = loss of head due to sudden enlargement.

Applying Bernoulli's eqn. to section 1-1 & 2-2

$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$

$$h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \longrightarrow (1)$$

Consider the control volume of liquid between section 1-1 and 2-2, Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = P_1 A_1 + P'(A_2 - A_1) - P_2 A_2$$

But experimentally it is found that $P_1 = P'$

$$F_x = P_1 A_1 + P_1 (A_2 - A_1) - P_2 A_2$$

$$F_x = P_1 A_1 + P_1 A_2 - P_1 A_1 - P_2 A_2$$

$$F_x = P_1 A_2 - P_2 A_2$$

$$F_x = (P_1 - P_2) A_2 \quad \text{——— (2)}$$

Momentum of liquid/sec at section 1-1 = mass × velocity

$= \rho A_1 \times V_1 V_1$

$= \rho A_1 V_1^2$

Momentum of liquid/sec at section 2-2 $= \rho A_2 V_2 \times V_2$

$= \rho A_2 V_2^2$

Change of momentum/sec $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

But from continuity equation, we have

$$A_1 V_1 = A_2 V_2 \text{ (or) } A_1 = \frac{A_2 V_2}{V_1}$$

∴ Change of momentum/sec $= \rho A_2 V_2^2 - \rho \frac{A_2 V_2}{V_1} V_1^2$

$= \rho A_2 V_2^2 - \rho A_2 V_2 V_1$

$= \rho A_2 [V_2^2 - V_2 V_1] \quad \text{——— (3)}$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating (2) & (3) eqn.s

$$(P_1 - P_2) A_2 = \rho A_2 [V_2^2 - V_2 V_1]$$

$$P_1 - P_2 = \rho (V_2^2 - V_2 V_1)$$

 $\frac{P_1-P_2}{\rho} = V_2^2 - V_2V_1$

Dividing by "g" on both sides, we have

$$\frac{P_1-P_2}{\rho g} = \frac{V_2^2 - V_2V_1}{g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_2V_1}{g}$$

Substituting the value of $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ in equation (1)

$$h_e = \frac{V_2^2 - V_2V_1}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

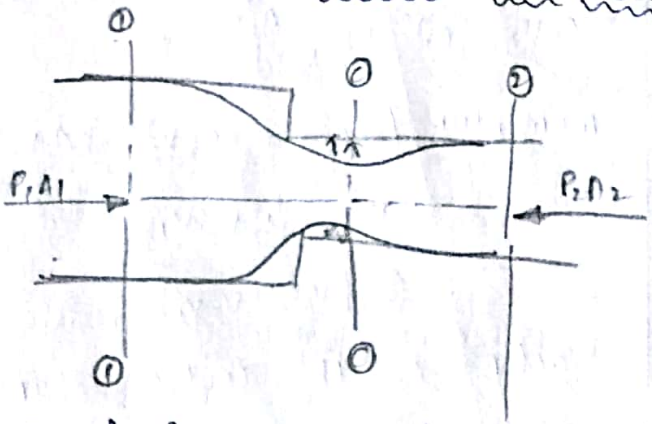
$$= \frac{V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 - 2V_1V_2 + V_1^2}{2g_2}$$

$$= \frac{(V_2-V_1)^2}{2g}$$

$$\therefore \boxed{h_e = \frac{(V_2-V_1)^2}{2g}}$$

→ Loss of head due to sudden contraction:



Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in fig. Consider two sections 1-1 and 2-2 before and after contraction. As the liquids flows from large pipe to smaller pipe, the area of flow goes on

decreasing and becomes min. at section c-c as shown in fig.3
This section c-c called Vena-contracta. After section c-c a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

Let A_c = Area of flow at section c-c
V_c = Velocity of flow at section c-c
A_2 = Area of flow at section 2-2
V_2 = Velocity of flow at section 2-2
h_c = loss of head due to sudden contraction.

Now, h_c = actually loss of head due to enlargement from section c-c to section 2-2 and is given by

$$h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \quad \text{--- i,}$$

From continuity equation, we have

$$A_c V_c = A_2 V_2 \Rightarrow \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c} \quad \left[\because V_c = \frac{A_c}{A_2}\right]$$

substituting the value of $\frac{V_c}{V_2}$ in eqn. i,

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= k \cdot \frac{V_2^2}{2g} \quad \text{where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Then h_c becomes as $$h_c = \frac{kV_2^2}{2g} = 0.375 \frac{V_2^2}{2g}$$

If the value of C_c is not given then the head loss due to sudden contraction is taken as $0.5 \frac{V_2^2}{2g}$

$$h_c = 0.5 \frac{V_2^2}{2g}$$

Loss of head at the entrance of pipe:

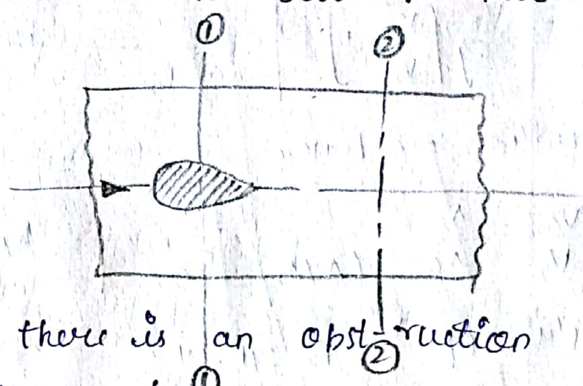
This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at entrance is taken

$= 0.5 \frac{V^2}{2g}$ where V = velocity of liquid in pipe.

This loss is denoted by h_i.

$$h_i = 0.5 \frac{V^2}{2g}$$

Loss of head at the end of pipe:



Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in fig.

Consider a pipe of area of cross-section A having an obstruction.

Let a = Maximum area of obstruction,
A = Area of pipe.
V = Velocity of liquid in pipe.

Then $(A-a)$ = Area of flow of liquid at section 1-1

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

Let V_c = Velocity of liquid at vena-contracta.

Then loss of head due to obstruction } = { loss of head due to enlargement from vena-contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \quad \text{——— (i)}$$

From continuity equation, we have

$$\boxed{a_c V_c = A \times V} \quad \text{——— (ii)}$$

Where a_c = area of cross-section at vena-contracta.

If C_c = coefficient of contraction.

$$C_c = \frac{\text{area at vena-contracta}}{(A-a)}$$

$$C_c = \frac{a_c}{(A-a)}$$

$$\boxed{a_c = C_c \times (A-a)}$$

Substituting the value of a_c in eqn. (ii)

$$C_c \times (A-a) V_c = A \times V$$

$$\boxed{V_c = \frac{A \times V}{C_c (A-a)}}$$

Substituting the value of V_c in equation (i), we get

Head loss due to obstruction

$$= \frac{(V_c - V)^2}{2g} = \left(\frac{AV}{C_c(A-a)} - V\right)^2 / 2g$$

$$= \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

→ **Loss of Head due to Bend in pipe:**

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{KV^2}{2g}$$

where, h_b = loss of head due to bend
V = velocity of flow.
K = coefficient of bend.

The value of K depends on
i, Angle of bend
ii, Radius of curvature of bend.
iii, Diameter of pipe.

Loss of Head in various Pipe Fittings:

The loss of head in the various pipe fittings such as valves, coupling etc is expressed as

$$= \frac{KV^2}{2g}$$

where V = velocity of flow
k = coefficient of pipe fitting.

1) Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 litres/sec

Sol: Diameter of small pipe, D_1 = 200mm = 0.2m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141\ m^2$

Diameter of large pipe, D_2 = 400mm = 0.4m

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.4)^2 = 0.12564\ m^2$

Discharge, Q = 250 litres/sec
= 0.25 m^3/sec

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96$ m/s

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99$ m/s

Loss of head due to sudden expansion is given by,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$= 1.816 \text{ m of water}$$

2) At a sudden enlargement of water main from 240mm to 480mm diameter the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Sol: Given

Diameter of smaller pipe, $D_1 = 240mm = 0.24m$

area, $A_1 = \frac{\pi}{4}(0.24)^2 =$

Diameter of larger pipe, $D_2 = 480mm = 0.48m$

area, $A_2 = \frac{\pi}{4}(0.48)^2$

Rise of hydraulic gradient, $\left[z_2 + \frac{P_2}{\rho g}\right] - \left[z_1 + \frac{P_1}{\rho g}\right] = 100mm = \frac{1}{100} m.$

Let the rate of flow $= Q$

Applying Bernoulli's equation theorem to both sections.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \quad \text{(1)}$$

But head loss due to enlargement is given by,

$$h_e = \frac{(V_1 - V_2)^2}{2g} \quad \text{(2)}$$

From continuity equation we have,

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_1}{A_1} = \frac{\frac{\pi}{4} D_2^2 V_2}{\frac{\pi}{4} D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$= \left(\frac{0.48}{0.24}\right)^2 V_2$$

$$V_1 = 4V_2$$

substituting the value of V_1 in eqn. (2)

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the values of V_1 and h_e in eqn(1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

$$\frac{6V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

$$\frac{3V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

But hydraulic gradient rise, $\left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right] = \frac{1}{100}$

$$\frac{3V_2^2}{2g} = \frac{1}{100}$$

$$V_2^2 = \frac{2g}{300}$$

$$V_2 = \sqrt{\frac{2 \times 9.81}{300}} = 0.1808 \simeq 0.181 \text{ m/s}$$

Discharge, $Q = A_2 V_2$

$= \frac{\pi}{4}(0.48)^2 (0.181)$

$= 0.03275 \text{ m}^3/\text{sec}$

$= 32.75$ litres/sec

3) The rate of flow of water through a horizontal pipe is $0.25 \text{ m}^3/\text{s}$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine.

(i) loss of head due to sudden enlargement.
(ii) Pressure intensity in the large pipe.
(iii) power lost due to enlargement.

Sol: Given;

Discharge, $Q = 0.25 \text{ m}^3/\text{s}$

Diameter of smaller pipe, $D_1 = 200\text{mm} = 0.2\text{m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141 \, m^2$

Diameter of large pipe, $D_2 = 400mm = 0.4 \, m$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.4)^2 = 0.12566 \, m^2$

pressure in smaller pipe, $P_1 = 11.772 \, N/cm^2$

$= 11.772 \times 10^4 \, N/m^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 \, m/s.$

velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12566} = 1.99 \, m/s.$

(i) Loss of head due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$= 1.816 \, m$$

(ii) Let the pressure intensity in larger pipe $= P_2$.

The applying Bernoulli's eqn. before and after sudden elargement,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$\frac{P_2}{\rho g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$\frac{P_2}{1000 \times 9.81} = 12.0 + 3.229 - 0.2018 - 1.8160$$

$$\frac{P_2}{9810} = 15.229 - 2.0178 = 13.21 \, m \text{ of water}$$

$$P_2 = 13.21 \times 9810 = 12.96 \times 10^4 \, N/m^2 \Rightarrow 12.96 \, N/cm^2$$

(iii) power lost due to sudden enlargement,

$$P = \frac{\rho g Q h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = 4.453 \, kW$$

... pipe of diameter 500mm is suddenly contracted to a diameter of 250mm. The pressure intensities in the larger and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine the rate of flow of water.

Sol: Given:

Diameter of large pipe, $D_1 = 500mm = 0.5m$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.5)^2 = 0.1963\ m^2$

Diameter of smaller pipe, $D_2 = 250mm = 0.25\ m$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.25)^2 = 0.04908\ m^2$

Pressure in large pipe, $P_1 = 13.734\ N/cm^2$
$= 13.734 \times 10^4\ N/m^2$

Pressure in smaller pipe, $P_2 = 11.772\ N/cm^2$
$= 11.772 \times 10^4\ N/m^2$.

Head lost due to contraction, $h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$

$$= \frac{V_2^2}{2g}\left[\frac{1}{0.62} - 1\right]^2$$

$$\boxed{h_c = \frac{V_2^2}{2g}\, 0.375}$$

From continuity eqn, we have

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1}$$

$$V_1 = \frac{\frac{\pi}{4} D_2^2 V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$V_1 = \left(\frac{0.25}{0.5}\right)^2 V_2$$

$$\boxed{V_1 = \frac{1}{4} V_2}$$

Applying Bernoulli's eqn. before and after contraction,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{\rho g} + z_2 + h_c$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c \quad \text{(1)}$$

substitute the values of h_c and V_1 in eqn. (1)

$$\frac{13.734 \times 10^4}{1000 \times 9.81} + \frac{(1/4\, V_2)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{2 \times 9.81} + \frac{V_2^2}{2g} + 0.375\frac{V_2^2}{2g}$$

$$14.0 + \frac{V_2^2}{16 \times 2 \times 9.81} = 12.0 + 1.375 \times \frac{V_2^2}{2 \times 9.81}$$

$$14 - 12 = \frac{V_2^2}{2 \times 9.81}\left[1.375 - \frac{1}{16}\right]$$

$$2 = 1.3125 \frac{V_2^2}{2 \times 9.81}$$

$$V_2^2 = \frac{2 \times 2 \times 9.81}{1.3125}$$

$$V_2 = \sqrt{\frac{2 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s}$$

(i) loss of due to sudden contraction.

$$h_c = 0.375 \frac{V_2^2}{2g}$$

$$= 0.375 \times \frac{(5.467)^2}{2 \times 9.81}$$

$$= 0.571 \text{ m}$$

(ii) Rate of flow of water $= A_2 V_2$

$$= \frac{\pi}{4}(0.5)^2 (0.571)$$

$$= 0.04908 \times 0.571$$

$$= 0.2683 \text{ m}^3/\text{s}$$

$$= 268.3 \text{ litres/sec}$$

6. If in the above problem, the rate of flow of water is 300 lit/sec, other data remaining the same, find the value of co-efficient of contraction.

Sol: Given:

$D_1 = 0.5$ m $\quad P_1 = 13.734 \times 10^4$ N/m² $\quad Q = 300$ lit/s

$D_2 = 0.25$ m $\quad P_2 = 11.772 \times 10^4$ N/m² $\quad = 0.3$ m³/s

$$V_1 = \frac{V_2}{4} \quad \text{where} \quad V_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi}{4}(0.5)^2} = 1.528 \text{ m/s.}$$

$$V_2 = 4V_1$$

$= 4 \times 1.528 = 6.112\ m/s$

From Bernoulli's eqn. we have,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$$\frac{13.734 \times 10^4}{1000 \times 9.81} + \frac{1.528^2}{2 \times 9.81} = \frac{11.772}{1000 \times 9.81} + \frac{6.112^2}{2 \times 9.81} + h_c$$

$$14.0 + 0.119 = 12.0 + 1.904 + h_c$$

$$14.119 = 13.904 + h_c$$

$$h_c = 14.119 - 13.904 = 0.215$$

We know that loss of head due to sudden contraction,

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$0.215 = \frac{(6.112)^2}{2 \times 9.81}\left[\frac{1}{C_c} - 1\right]^2$$

$$\left[\frac{1}{C_c} - 1\right]^2 = \frac{0.215 \times 2 \times 9.81}{(6.112)^2}$$

$$\left[\frac{1}{C_c} - 1\right]^2 = 0.1129$$

$$\frac{1}{C_c} - 1 = \sqrt{0.1129} = 0.336$$

$$\frac{1}{C_c} = 1 + 0.336$$

$$\frac{1}{C_c} = 1.336 \Rightarrow C_c = \frac{1}{1.336} = 0.748$$

Coefficient of contraction, $C_c = 0.748$.

7) A 150mm diameter pipe reduces in diameter abruptly to 100mm diameter. If the pipe carries water at 30 lit/sec, calculate the pressure loss across the contraction. Take the coefficient of contraction as 0.6.

Sol: Diameter of larger pipe, $D_1 = 150mm = 0.15\ m$

Diameter of smaller pipe, $D_2 = 100\ mm = 0.1\ m$

$\therefore$ area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$

area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ m^2$

Discharge, $Q = 30$ lit/sec

$= 0.03\ m^3/sec$

coefficient of contraction, $C_c = 0.6$

From continuity eqn. $A_1V_1 = A_2V_2 = Q$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82 \text{ m/sec}$$

Applying Bernoulli's eqn. before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c \text{ ——— (i)} \qquad (\because z_1 = z_2)$$

h_c = loss of head due to sudden contraction.

$$= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= \frac{(3.82)^2}{2 \times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

substituting all the values in eqn (i)

$$\frac{P_1}{\rho g} + \frac{(1.697)^2}{2 \times 9.81} = \frac{P_2}{\rho g} \times \frac{(3.82)^2}{2 \times 9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271 \text{ m}$$

$$P_1 - P_2 = 0.9271 \times 9.81 \times 2 \text{ N/m}^2$$

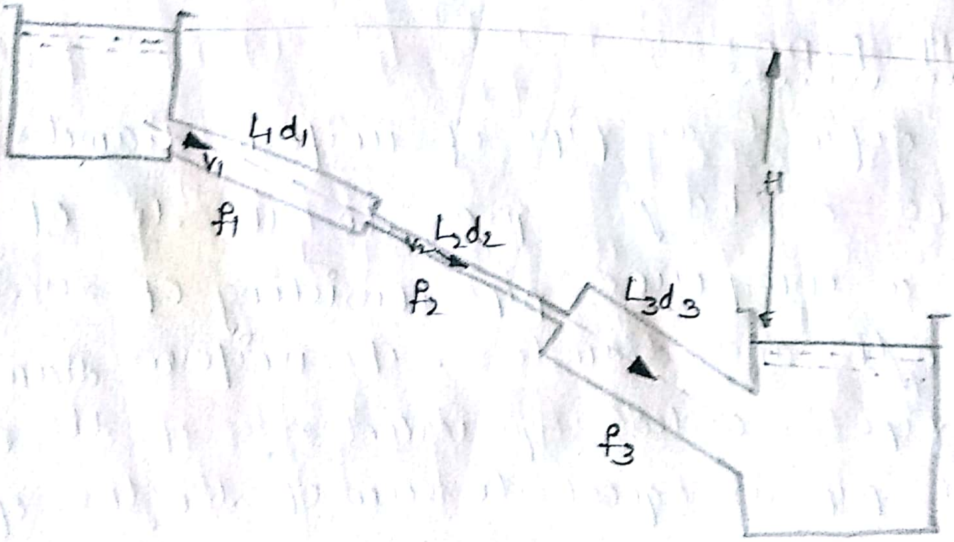
$$= 0.909 \times 10^4 \text{ N/m}^2$$

$$= 0.909 \text{ N/cm}^2$$

Pressure loss across contraction, $(P_1 - P_2) = 0.909$ N/cm².

FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES :

pipes in series (or) compound pipes, is defined as the pipe of different lengths and different diameters connected end to end (in series) to form a pipe line.



Let L_1, L_2, L_3 = length of pipes 1, 2, and 3 respectively

d_1, d_2, d_3 = diameter of pipes 1, 2, and 3 respectively.

V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = coefficient of frictions for pipes 1, 2, 3.

H = difference of water level in the two tanks.

The discharge passing through each pipe is same.

$$Q = A_1V_1 = A_2V_2 = A_3V_3.$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g} \quad \text{—— (i)}$$

If minor losses are neglected then eqn. (i) becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \quad \text{—— (ii)}$$

If the co-efficient of friction is same for all pipes

$f_1 = f_2 = f_3 = f$ then eqn. (ii) becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

→ **EQUIVALENT PIPE:**

This is defined as pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1, d_1 = dia of pipe 1

L_2 = length of pipe 2, d_2 = dia of pipe 2

L_3 = length of pipe 3, d_3 = dia of pipe 3.

H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe.

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglected minor losses.

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3$

$$Q = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}d_3^2$$

$$V_1 = \frac{4Q}{\pi d_1^2}; \quad V_2 = \frac{4Q}{\pi d_2^2}; \quad V_3 = \frac{4Q}{\pi d_3^2}$$

substituting these values in eqn. (i)

$$H = \frac{4fL_1\left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$H = \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

Head loss in the equivalent pipe, $H = \frac{4fLV^2}{d \times 2g}$

[Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$H = \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16Q^2f}{\pi^2 2g}\left[\frac{L}{d^5}\right]$$

...d loss in compound pipe and in equivalent pipe is same hence. equating eqn-s i, and ii,

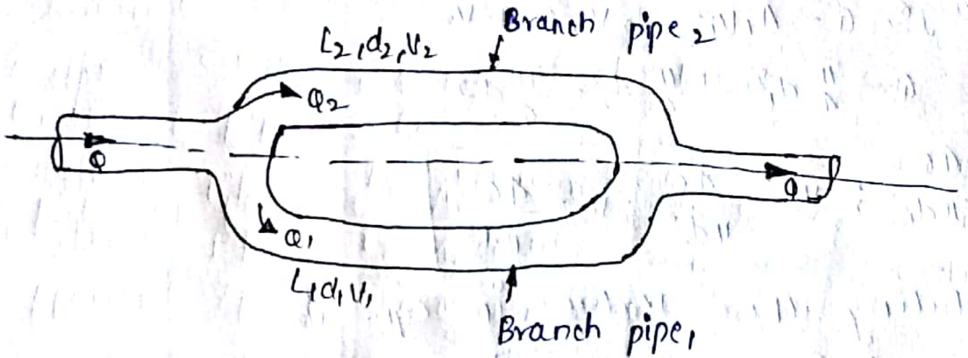
$$\frac{4 \times 16 f Q^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left(\frac{L}{d^5}\right)$$

$$\boxed{\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5} = \frac{L}{d^5}} \quad \text{——— iii,}$$

The eqn. iii, is known as Dupuit's eqn. In this eqn. $L = L_1 + L_2 + L_3$ In this equation $L = L_1 + L_2 + L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, i.e., value of d obtained.

→ FLOW THROUGH PARALLEL PIPES:

Consider a main pipe which divides into two (or) more branches as shown in fig. below.



and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from fig.

$$\boxed{Q = Q_1 + Q_2}$$

In this arrangement, the loss of head for each branch pipe is same.

∴ loss of head for branch pipe 1 = loss of head for branch pipe 2

$$\frac{4f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4f_2 L_2 V_2^2}{d_2 \times 2g}$$

If $f_1 = f_2$ then

$$\boxed{\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}}$$

1. The difference in water surface levels in two levels which are connected by three pipes in series of lengths 300m, 170m and 210m and of diameters 300mm, 200mm and 400mm respectively, is 12m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively, considering
   i, minor losses also.
   ii, Neglecting minor losses.

Sol: Given:

difference of water level, $H = 12m$

length of pipe 1, $L_1 = 300m$ diameter, $d_1 = 0.3m$

length of pipe 2, $L_2 = 170m$ diameter, $d_2 = 0.2m$

length of pipe 3, $L_3 = 210m$ diameter $d_3 = 0.4m$

co-efficient of friction, $f_1 = 0.005$

$f_2 = 0.0052$

$f_3 = 0.0048$

i, Considering minor losses:

Let V_1, V_2, V_3 are the velocities in the 1st, 2nd & 3rd pipe respectively.

From continuity eqn., we have

$$A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} d_1^2 V_1}{\frac{\pi}{4} d_2^2} = \left(\frac{d_1}{d_2}\right)^2 V_1 = \left(\frac{0.3}{0.2}\right)^2 V_1$$

$$= 2.25 V_1$$

$$V_3 = \frac{A_1 V_1}{A_3} = \frac{\frac{\pi}{4} d_1^2 V_1}{\frac{\pi}{4} d_3^2} = \left(\frac{d_1}{d_3}\right)^2 V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1$$

$$= 0.5625 V_1$$

We know the equation,

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g} + \frac{V_3^2}{2g}$$

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5(2.25V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0052 \times 170 \times (2.25V_1)^2}{0.2 \times 2g} + \frac{(2.25V_1 - 0.05625V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0048 \times 210 \times (0.5625V_1)^2}{0.4 \times 2g} + \frac{(0.5625V_1)^2}{2g}$$

$$12 = \frac{V_1^2}{2g}[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$12 = \frac{V_1^2}{2g}(118.87)$$

$$V_1^2 = \frac{12 \times 2 \times 9.81}{118.87}$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.87}} = 1.407 \text{ m/s}$$

$\therefore$ Rate of flow, Q = Area $\times$ velocity

$= A_1V_1$

$= \frac{\pi}{4}d_1^2 \times V_1$

$= \frac{\pi}{4}(0.3)^2 \times 1.407$

$= 0.09945 \text{ m}^3/\text{sec}$

$= 99.45$ lit/sec

ii, Neglecting minor losses:

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

$$12 = \frac{V_1^2}{2g}\left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170(2.25)^2}{0.2} + \frac{4 \times 0.0048 \times 210(0.5625)^2}{0.4}\right]$$

$$12 = \frac{V_1^2}{2g}[20 + 89.505 + 3.189]$$

$$12 = \frac{V_1^2}{2g}(112.694)$$

$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{112.694}} \Rightarrow \sqrt{\frac{12 \times 2 \times 9.81}{112.694}} = 1.446 \text{ m/s}$

$\therefore$ Rate of flow, $Q = A_1 V_1$

$$= \frac{\pi}{4} d_1^2 V_1$$

$$= \frac{\pi}{4} (0.3)^2 \times 1.446$$

$$= 0.1021 \text{ m}^3/\text{s}$$

$$= 102.1 \text{ lit/sec}$$

2. A three pipes of 400mm, 200mm and 300mm diameters have lengths of 400m, 200m and 300m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16m. If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.

Sol: Given:

Difference of water levels, $H = 16$m

Length and dia of pipe 1, $L_1 = 400$m

$d_1 = 400\text{mm} = 0.4$m

Length and dia of pipe 2, $L_2 = 200$m

$d_2 = 200\text{ mm} = 0.2$m

Length and dia of pipe 3, $L_3 = 300$m

$d_3 = 300\text{mm} = 0.3$m

$f_1 = f_2 = f_3 = 0.005$

i, Discharge through the compound pipe first neglecting minor losses,

Let V_1, V_2, V_3 are the velocities in the 1st, 2nd & 3rd pipes respectively.

From continuity eqn. we have

$$\boxed{A_1 V_1 = A_2 V_2 = A_3 V_3}$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\pi/4 \, d_1^2 V_1}{\pi/4 \, d_2^2} = \left(\frac{d_1}{d_2}\right)^2 V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

$\frac{V_3}{V_1}$ ... $\frac{A_1 V_1}{A_3} = \frac{\pi/4\, d_1^2 V_1}{\pi/4\, d_3^2} = \left(\frac{d_1}{d_3}\right)^2 V_1 = \left(\frac{0.4}{0.3}\right)^2 V_1 = 1.77 V_1$

We know the eqn.

$$H = \frac{4f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4f_3 L_3 V_3^2}{d_3 \times 2g}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2g} + \frac{4 \times 0.005 \times 200 (4V_1)^2}{0.2 \times 2g} + \frac{4 \times 0.005 \times 300 \times (1.77 V_1)^2}{0.3 \times 2g}$$

$$16 = \frac{V_1^2}{2 \times 9.81}\left[\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 1.77^2}{0.3}\right]$$

$$16 = \frac{V_1^2}{2 \times 9.81}\left[20 + 320 + 63.14\right]$$

$$16 = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$$V_1^2 = \frac{16 \times 2 \times 9.81}{403.14} \Rightarrow V = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

Discharge through the compound pipe

$$Q = A_1 V_1$$
$$= \frac{\pi}{4} d_1^2 \times V_1$$
$$= \frac{\pi}{4} (0.4)^2 \times 0.882$$
$$= 0.1108 \text{ m}^3/\text{s} = 110.8 \text{ lit/s}$$

ii, Discharge through the compound pipe considering minor losses also.

Minor losses are:

(a) at inlet, $h_i = \frac{0.5 V_1^2}{2g}$

(b) Between first pipe and second pipe due to contraction,

$$h_c = \frac{0.5 V_2^2}{2g} = \frac{0.5 (4V_1)^2}{2g}$$
$$= \frac{0.5 \times 16 V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(c) Between second pipe and 3rd pipe due to sudden enlargement.

$$h_c = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_1 - 1.77V_1)^2}{2g}$$

$$= (2.23)^2 \times \frac{V_1^2}{2g}$$

$$= 4.973 \frac{V_1^2}{2g}.$$

(d) At the outlet of 3rd pipe,

$$h_o = \frac{V_3^2}{2g} = \frac{(1.77V_1)^2}{2g} = 1.77^2 \frac{V_1^2}{2g} = 3.1329 \frac{V_1^2}{2g}$$

The major losses are:

$$= \frac{4f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4f_3 L_3 V_3^2}{d_3 \times 2g}$$

$$= \frac{4 \times 0.005 \times 400 V_1^2}{0.4 \times 2g} + \frac{4 \times 0.005 \times 200 (4V_1)^2}{0.2 \times 2g} + \frac{4 \times 0.005 \times 300 \times (1.77 V_1)^2}{0.3 \times 2g}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ Sum of minor losses and major losses

$$= \left[\frac{0.5V_1^2}{2g} + \frac{8V_1^2}{2g} + \frac{4.973 V_1^2}{2g} + \frac{3.1329 V_1^2}{2g}\right] + 403.14 \frac{V_1^2}{2g}$$

$$= 419.746 \frac{V_1^2}{2g}$$

But total loss must be equal to H

$$419.746 \frac{V_1^2}{2g} = H$$

$$V_1^2 = \frac{H \times 2g}{419.746} \Rightarrow V_1 = \sqrt{\frac{H \times 2g}{419.746}}$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864 \text{ m/s}$$

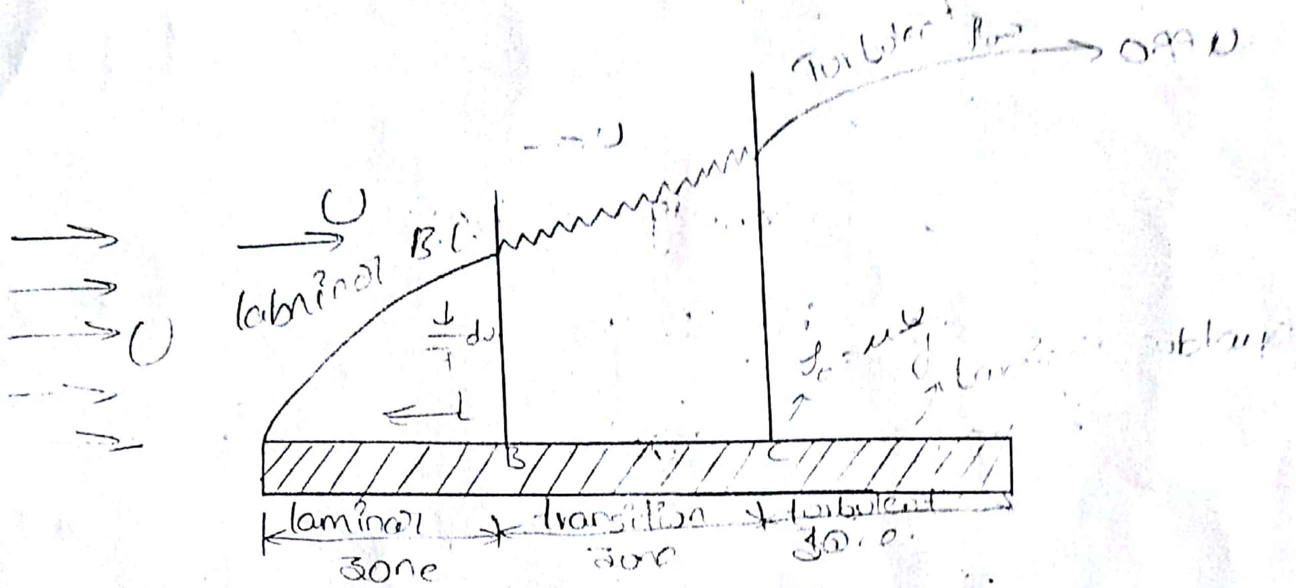
$\therefore$ Discharge, $Q = A_1 V_1$

$$= \frac{\pi}{4} (0.4)^2 \times 0.864$$

$$= 0.1085 \text{ m}^3/\text{s}$$

$$= 108.5 \text{ lit/sec}$$

UNIT-II Boundary Layer Theorem.



we have shear stress $\tau = \mu \frac{du}{dy}$.

Boundry layer thickness:-

The ⊥ler distance b/w the boundry layer to top of the flow.

δ_{lam} – laminar flow thickness

δ_{tub} – turbulent flow thickness

δ_{sb} – ~~laminar~~ sublayer thickness boundry layer thickness

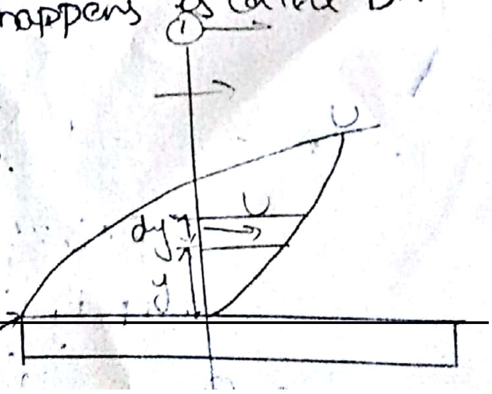
Displacement Thickness:- The distance ⊥ler to boundary due to which ~~flo~~ the free flow happens is called D.T.

It is denoted by δ^*

mass flow through the strip.

$\rho \times V \times A$.

$= \rho \times u \times b \times dy$. →(1)



without plate

$$\text{mass}/\text{sec} = \rho \times U \times b \times dy \rightarrow ②$$

loss of mass flow/sec

$$\delta^* = ② - ①$$

$$= \rho U \times b \times dy - \rho u\, b \times dy$$

$$= \rho b dy (U - u)$$

$$\rho \times b \times dy \int_0^\delta (U-u) = \rho U \delta^* b$$

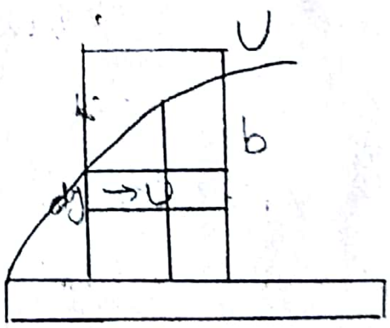
$$\delta^* = \int_0^\delta \frac{(U-u)}{U} dy$$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$ loss in mass volume.

Momentum thickness (θ).

= mass × velocity → loss in momentum

$$= (\rho \times U \times b \times dy) U = \rho \times U^2 b dy$$



By plate:

$$(\rho \times u \times b \times dy) u = \rho \times u^2 b dy$$

$$\text{loss of mass} = \rho b dy (U-u) u$$

$$= (\rho \times U \times \theta \times b) U$$

$$= \rho b dy \int_0^\infty (U-u) u$$

$$= \int_0^\infty \left(\frac{U-u}{U^2}\right) u\,dy.$$

$$= \int_0^\infty \left(\frac{Uu-u^2}{U^2}\right) dy.$$

$$= u \int_0^\infty \left(\frac{U-u}{U^2}\right) dy.$$

$$= \frac{u}{U}\left(\int_0^\delta 1-\frac{u}{U}\right) dy.$$

Energy thickness (δ^{**}).

$K.E = \frac{1}{2} mv^2$ → loss in K.E.

$$= \frac{1}{2}(\rho u b\,dy) u^2$$

$$= \frac{1}{2}\rho b\,dy\,(U^3 - u^3).$$

$$= \frac{1}{2}(\rho U \delta^{**} b) U^2.$$

$$\frac{1}{2}(\rho \times U^3 \delta^{**} b) = \frac{1}{2} \rho b\,dy\,(U^3 - u^3).$$

$$\delta^{**} = \int_0^\delta \left(\frac{U^3-u^3}{U^3}\right) dy.$$

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right) dy.$$

~~Rate~~ $F_D = \tau_0 \times b \times \Delta x.$

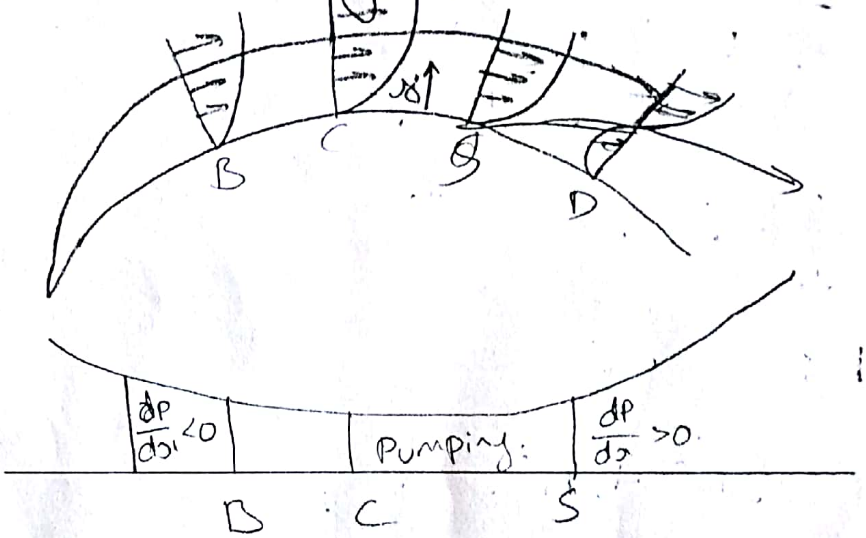
Total rate of change of momentum is equal to BL is equal to drag force. (F_D)

$M_{BC} - M_{AD} - M_{DC}$

$= M_{AD} + \frac{d}{dx}(M_{AD})\Delta x - M_{AD} - M_{DC} = FD. = \tau_0 \times b \times \Delta x.$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}.$$

Seperation of boundry layer.



$\frac{\partial u}{\partial y} > 0$ $\frac{\partial u}{\partial y} < 0$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} < 0.$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} > 0.$$

Methods of B.L

1) Motion of a solid body:-

2. 2) Acceleration of fluid in the boundry layer

3) Suction of fluid from boundry.

4) Stream line of body shapes.

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\Rightarrow \delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right) dy$$

$$= \int_0^\delta 1\,dy - \int_0^\delta 2\frac{y}{\delta} dy + \int_0^\delta \frac{y^2}{\delta^2} dy$$

$$= y\Big]_0^\delta - \frac{2}{\delta}\frac{y^2}{2}\Big]_0^\delta + \frac{1}{\delta^2}\frac{y^3}{3}\Big]_0^\delta$$

$$= 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \left(1 - 2\frac{y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= 2\int_0^\delta \frac{y}{\delta} - 2\int \frac{y^2}{\delta^2} \cdot 1\,dy -$$

$$= 2\frac{y^2}{2\delta} - 2\frac{y^3}{3\delta^2} - 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^3$$

$$= \frac{\delta}{3}$$

Energy thickness :-

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right) dy.$$

$$= \int_0^\delta 2\cdot\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \left(1-\left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right)\right)^2 dy.$$

$$= 2\cdot\frac{y^2}{2\delta}\Big]_0^\delta - \frac{y^3}{3\delta}\Big]_0^\delta \left(1\,dy\Big]_0^\delta - \left(2\frac{y^2}{2\delta}\right)_0^\delta - \frac{y^3}{3\delta}\right)_0^\delta$$

$$= \frac{y^2}{\delta}$$

$$= \delta - \frac{\delta^3}{3\delta}\left(\delta - \frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta}\right.$$

$$= \delta.$$

$$= \delta^2 - \delta^2 - \frac{\delta^3}{3} - \frac{\delta^3}{3} - \delta^3 - \frac{\delta^4}{9}$$

$$= \int_0^\delta \frac{u}{U} - \frac{u^2}{U^2} dy.$$

$$= \int_0^\delta \left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right) dy - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)^2 dy.$$

$$= \delta/3 - \int_0^\delta 4\frac{y^2}{\delta^2} - 4\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\Big) dy.$$

$$= 4\cdot\frac{y^3}{3\delta^2} - 4\cdot\frac{y^4}{4\delta^3} + \frac{y^5}{5\delta^4}\Big)_0^\delta$$

$$= \frac{4}{3}\frac{\delta^3}{\delta^2} - \frac{\delta^4}{\delta^3} + \frac{\delta^5}{5\delta^4}.$$

$$= \frac{4}{3}\delta - \delta + \delta = \frac{4}{3}\delta - \delta + \frac{\delta}{5} + \frac{\delta}{3}.$$

$$= \frac{4}{3}\delta + \frac{\delta}{3} = \frac{20\delta - 15\delta + 3\delta + 5\delta}{15} = \frac{3\delta}{15}.$$

$$\frac{\tau}{\rho y} \quad \frac{\tau}{\rho U^2} = \frac{\partial \theta}{\partial x}.$$

$$\tau = \mu \frac{du}{dy}.$$

$$F_D = \tau \times \Delta x \times b.$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}.$$

$$F_D = \frac{C_D \times \rho A U^2}{2}.$$

For the velocity profile for ~~the~~ laminar boundary layer given us. $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Find an expression for boundary layer thickness δ shearstress & τ and co-efficient of drag C_D in terms of Reynold's number.

we have,

$$\frac{\tau}{\rho U} = \frac{d}{dx}\left(\int_0^\delta \left(1 - \frac{u}{U}\right)\frac{u}{U}\, dy\right).$$

$$= \frac{\partial}{\partial x}\left(\frac{3\delta}{15}\right)dy.$$

$$= \frac{3}{15}\frac{\partial}{\partial x}(\delta).$$

$$\tau = \frac{3\rho U^2}{15}\frac{\partial}{\partial x}(\delta).$$

$$\frac{u}{U} = 2.\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2.$$

let $u = U\left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right).$

$$\tau = \mu \frac{du}{dy}.$$

$$= \frac{2U}{\delta^2} \times (\delta - y).$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{\delta^2}(\delta - 0)$$

$$= \frac{2U}{\delta} \quad max.$$

$$\tau = \mu \frac{2U}{\delta}.$$

$$\tau = \frac{3\rho U^2}{15} \cdot \frac{\partial}{\partial x}(\delta).$$

$$\mu \frac{\partial U}{\partial y} = \frac{3\rho U^2}{15}.$$

$$\mu \cdot \frac{2U}{\delta} = \frac{3\rho U^2}{15} \frac{\partial}{\partial x}(\delta).$$

$$\mu \cdot \frac{2U}{\delta} = \frac{3\rho U^2}{15} \cdot \frac{\partial}{\partial x}(\delta)$$

$$2\mu = \frac{3}{15} \rho U \times \frac{\partial}{\partial x}(\delta).$$

$$\frac{\partial}{\partial x}(\delta) = 2\mu \frac{15}{3\rho U}.$$

$$= \frac{10\mu}{\rho U}.$$

$$\delta \, \partial \delta = \frac{10\mu}{\rho U} \partial x.$$

$$\frac{\delta^2}{2} = \frac{10\mu}{\rho U} x + C.$$

$$(\because C = 0).$$

$$\delta^2 = \frac{20\mu}{\rho U} x$$

$$\delta^2 = \frac{20x}{Re} \quad \left(\because Re = \frac{\rho U}{\mu}\right).$$

$$\therefore \delta = \sqrt{\frac{20x}{Re}}.$$

Page 617 & 622.

δ

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

$$F_D = \int_0^L \tau_0 \times \Delta x \times b.$$

Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Given distribution.

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Displacement thickness

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy.$$

$$= \int_0^\delta 1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] dy.$$

$$= \int_0^\delta \left(1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right) dy$$

$$= \int_0^\delta 1\,dy - 2\int_0^\delta \frac{y}{\delta}\,dy + \int_0^\delta \frac{y^2}{\delta^2}\,dy.$$

$$= y - 2\frac{y^2}{2\delta} + \frac{y^3}{3\delta^2}\Big]_0^y$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2}$$

$$= \delta - \delta + \frac{\delta}{3}$$

$$\therefore = \frac{\delta}{3}.$$

Momentum thickness:

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy.$$

$$= \int_0^\delta \frac{u}{U} - \frac{u^2}{U^2}$$

$$= \int_0^\delta 2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2 \left(1-\left(2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right)\right) dy.$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$\Rightarrow \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right) dy$$

$$\Rightarrow \left[\frac{2y^2}{2\delta} - 5\cdot\frac{y^3}{3\delta^2} + 4\cdot\frac{y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$\Rightarrow \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$\Rightarrow \frac{15\delta - 25\delta + 15\delta - 3\delta}{15}$$

$$\Rightarrow \frac{2\delta}{15}$$

Energy thickness:

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right) dy.$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1-\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]^2\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1-\left(\frac{4y^2}{\delta^2} - 4\cdot\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right)\right) dy.$$

$$\Rightarrow \int_0^\delta \left(\frac{2y}{\delta} - \frac{8y^3}{\delta^3} + \frac{8y^4}{\delta^4} - \frac{2y^5}{\delta^5} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right) dy$$

$$= \frac{\partial}{\partial x}\int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \frac{\partial}{\partial x}\left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$\Rightarrow \frac{\partial}{\partial x}\left[\delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}\right]$$

$$\Rightarrow \frac{\partial}{\partial x}\left[\frac{15\delta - 25\delta + 15\delta - 3\delta}{15}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{30\delta - 28\delta}{15}\right] = \frac{\partial}{\partial x}\left(\frac{2\delta}{15}\right)$$

$$= \frac{2}{15}\frac{\partial}{\partial x}(\delta)$$

$$\tau_0 = \rho U^2 \times \frac{2}{15}\frac{\partial}{\partial x}(\delta)$$

$$= \frac{2}{15}\rho U^2 \frac{\partial[\delta]}{\partial x}$$

The shear stress at the boundary in laminar flow is also given by Newton's law of viscosity as

$$\tau_0 = \mu\left(\frac{dU}{dy}\right)_{y=0} \rightarrow (2)$$

$$u = U\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]$$

$$\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$$

$$\left(\frac{du}{dy}\right)_{y=0} = U\left[\frac{2}{\delta} - \frac{2\times(0)}{\delta^2}\right] = \frac{2U}{\delta}$$

Subs the value in eqn (2), we get:

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta} \rightarrow (3)$$

Equating the two values of τ_0 given by eqn (3) to

$$\left(\tau_0 = \rho U^2 \times \frac{2}{15}\frac{\partial}{\partial x}(\delta)\right) = \frac{2}{15}\rho U^2 \frac{\partial(\delta)}{\partial x}$$

we get

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta} \rightarrow$$

$$\frac{2}{15}\rho U^2 \frac{\partial}{\partial x}(\delta) = \frac{2\mu U}{\delta}$$

$$\delta \frac{\partial}{\partial x}[\delta] = \frac{15\mu U}{\rho U^2} = \frac{15\mu}{\rho U}$$

$$\delta \partial[\delta] = \frac{15\mu}{\rho U}\partial x.$$

As the boundary layer thickness (δ) is a function of x only. Hence partial derivative can be changed to total derivative.

$$\delta d[\delta] = \frac{15\mu}{\rho U} dx.$$ $\left[\because \frac{\mu}{\rho U} \text{ is constant}\right]$

On integration, we get $\frac{\delta^2}{2} = \frac{15\mu}{\rho U}x + C.$

$x = 0, \delta = 0$ and hence $C = 0$.

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U}.$$

$$\delta = \sqrt{\frac{2 \times 15\mu x}{\rho U}}$$

$$= \sqrt{\frac{30\mu x}{\rho U}} = 5.48\sqrt{\frac{\mu x}{\rho U}}.$$

$$= 5.48\sqrt{\frac{\mu x \times x}{\rho U \times x}}$$

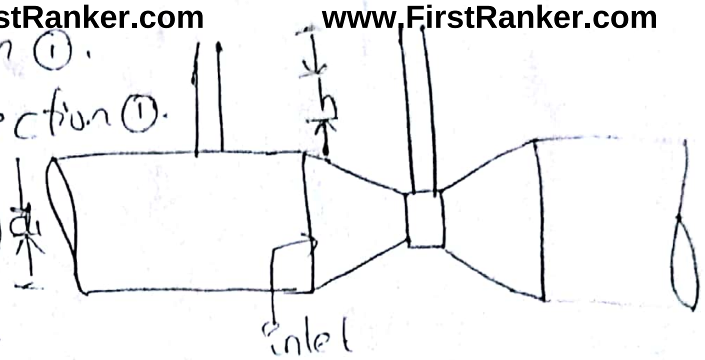
$$= 5.48 \frac{x}{\sqrt{Re_x}}.$$

Shear stress τ_0 in terms of Reynold's number.

From eqn (3), we have $\tau_0 = \frac{2\mu U}{\delta}$.

Sub. the value of δ from eqn $5.48\frac{x}{\sqrt{Re_x}}$, we get

$$\tau_0 = \frac{2\mu U}{5.48\frac{x}{\sqrt{Re_x}}} = \frac{2\mu U\sqrt{Re_x}}{5.48x} = 0.365\frac{\mu U}{x}\sqrt{Re_x}$$



let a_1 = area of section ①.

d_1 = diameter of section ①.

v_1 = velocity of fluid at section ①.

P_1 = Pressure of fluid at the inlet or section ①.

P_2, v_2, a_2, d_2 are the corresponding values at section

Apply Bernoulli's theorem

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2.$$

As pipe is horizontal.

$$z_1 = z_2.$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}.$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}.$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}. \rightarrow ①.$$

By continuity eqn $a_1 v_1 = a_2 v_2$.

$$v_1 = \frac{a_2 v_2}{a_1}.$$

$$h = \frac{v_2^2}{2g} - \left(\frac{\frac{a_2 v_2}{a_1}}{2g}\right)^2.$$

$$h = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right].$$

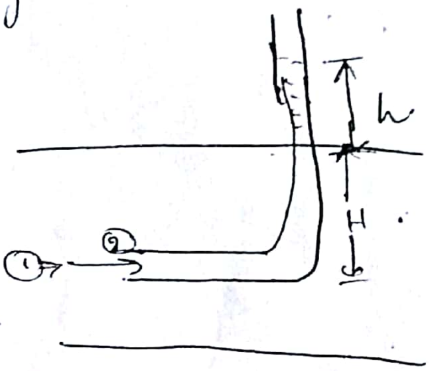
$$\Rightarrow v_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right). \Rightarrow v_2 = \sqrt{2gh}\cdot\frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

$Q = a_2 v_2$

$\Rightarrow a_2 \cdot \sqrt{2gh} \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$

$\Rightarrow \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$

$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$



$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{\rho g} + Z_2$

$Z_1 = Z_2$

$V_2 = 0$

$\frac{P_1}{\rho g} = H$

$\frac{P_2}{\rho g} = (h + H)$

$H + \frac{V_1^2}{2g} = (H + h) + 0$

$\frac{V_1^2}{2g} = h$

$\Rightarrow V_1^2 = 2gh$

$V_1 = \sqrt{2gh}$

$V_{act} = C_v \sqrt{2gh}$

where C_v – coefficient of pitot tube.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy.$$

$$= \int_0^\delta \left(1 - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)\right) dy.$$

$$= y - 2\frac{y^2}{2\delta} + \frac{y^3}{3\delta^2}\Big]_0^\delta.$$

$$= \delta - \delta + \frac{\delta}{3}$$

$$= \frac{\delta}{3}.$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy.$$

$$\Rightarrow \int_0^\delta 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \left(1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right] dy.$$

$$\Rightarrow \int_0^\delta 2\cdot\frac{y}{\delta} - 4\cdot\left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\Big]$$

$$\Rightarrow \int_0^\delta \left(2\cdot\frac{y}{\delta} - 5\left(\frac{y}{\delta}\right)^2 + 4\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right].$$

$$\Rightarrow 2\frac{y^2}{2\delta} - 5\cdot\frac{y^3}{3\delta^2} + 4\cdot\frac{y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\Big]_0^\delta$$

$$= \frac{\delta^2}{\delta} - \frac{5}{3}\cdot\frac{\delta^3}{\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}.$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}.$$

$$\Rightarrow \frac{15\delta - 25\delta + 15\delta - 3\delta}{15}.$$

$$\Rightarrow \frac{2\delta}{15}.$$

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left(1-\left(\frac{u}{U}\right)^2\right)dy.$$

$$\Rightarrow \int_0^{\delta} 2\left(\frac{y}{\delta}\right)\left(\frac{y}{\delta}\right)^2\left(1-\left(2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right)^2\right)dy$$

$$\Rightarrow \int_0^{\delta} 2\left(\frac{y}{\delta}\right)\left(\frac{y}{\delta}\right)^2\left(1-\left(4\frac{y^2}{\delta^2}-4\left(\frac{y}{\delta}\right)^3+\left(\frac{y}{\delta}\right)^4\right)\right)dy.$$

$$\Rightarrow \int_0^{\delta} 2\left(\frac{y}{\delta}\right)-8\left(\frac{y}{\delta}\right)^3+8\left(\frac{y}{\delta}\right)^4-2\left(\frac{y}{\delta}\right)^5-\left(\frac{y}{\delta}\right)^2+$$

$$4\left(\frac{y}{\delta}\right)^4-4\left(\frac{y}{\delta}\right)^5+\left(\frac{y}{\delta}\right)^6\Big)dy.$$

$$\Rightarrow 2\left(\frac{y^2}{2\delta}\right)-8\left(\frac{y^4}{4\delta^3}\right)+8\left(\frac{y^5}{5\delta^4}\right)-2\cdot\left(\frac{y^6}{6\delta^5}\right)-\frac{y^3}{3\delta^2}+$$

$$4\left(\frac{y^5}{5\delta^4}\right)-4\left(\frac{y^6}{6\delta^5}\right)+\frac{y^7}{7\delta^6}\Big]_0^{\delta}.$$

$$\Rightarrow \frac{\delta^2}{\delta}-2\cdot\frac{\delta^4}{\delta^3}+\frac{8}{5}\cdot\frac{\delta^5}{\delta^4}-\frac{2}{6}\cdot\frac{\delta^6}{\delta^5}-\frac{\delta^3}{3\delta^2}+$$

$$\frac{4}{5}\frac{\delta^5}{\delta^4}-\frac{4}{6}\frac{\delta^6}{\delta^5}+\frac{\delta^7}{7\delta^6}.$$

$$\Rightarrow \delta-2\delta+\frac{8}{5}\delta-\frac{1}{3}\delta-\frac{1}{3}\delta+\frac{4}{5}\delta-\frac{2}{3}\delta+\frac{1}{7}\delta.$$

$$\frac{22\,\delta}{105}$$

Dimension Analysis:- We have 4 System of Units which (1)
are universally used & recognise. They are

1. F.P.S UNIT

The System of unit based on the foots, pounds & Seconds. Length in foots, Mass in pounds & time in Seconds.

2. C.G.S Unit:- The system of units based on Centimeters, grams & seconds. Length in centimeters, mass in grams, time in Seconds.

3. M.K.S unit:- The System of Units based on meters, kilograms, and second. Length in meters, mass in kilograms time in Seconds.

4. S.I Unit:- There are system of International units which is world most widely used modern form a M.K.S System.

System	Prefer	Mult.
kilo	K	10^3
Mega	M	10^6
Giga	G	10^9
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
nano	n	10^{-9}

* Force - Which ever causes & tense to cause motion is a force - Newtons
* Power - Rate of doing work - Watts.
* Velocity - Rate of change of displacement - m^3
* Acceleration:- Rate of change of Velocity - m/sec^2
* Dimension:- Five important physical qualities are involved in study of fluid.