STRENGTH OF MATERIALS-I

SYLLABUS:

UNIT – I: Simple Stresses And Strains And Strain Energy: Elasticity and plasticity – Types of stresses and strains – Hooke's law – stress – strain diagram for mild steel – Workingstress – Factor of safety – Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them – Bars of varying section – composite bars –Temperature stresses. **Strain Energy** – Resilience – Gradual, sudden, impact and shock loadings – simple applications.

UNIT – II: Shear Force And Bending Moment: Definition of beam – Types of beams –Concept of shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, u.d.l., uniformly varying loads and combination of these loads – Point of contraflexure – Relation between S.F., B.M and rate of loading at a section of a beam

UNIT – III: Flexural Stresses: Theory of simple bending – Assumptions – Derivation of bending equation: M/I = f/y = E/R, Neutral axis – Determination bending stresses – section modulus of rectangular and circular sections (Solid and Hollow), I, T, Angle and Channel sections – Design of simple beam sections.

UNIT –IV: Shear Stresses: Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections, built up beams, shear centre.

UNIT – V: Deflection Of Beams: Bending into a circular arc – slope, deflection and radius of curvature – Differential equation for the elastic line of a beam – Double integration and Macaulay's methods – Determination of slope and deflection for cantilever and simply supported beams subjected to point loads, **-** U.D.L. Uniformly varying load. Mohr's theorems – Moment area method – application to simple cases including overhanging beams.

UNIT – VI: Thin And Thick Cylinders: Thin seamless cylindrical shells – Derivation of formula for longitudinal and circumferential stresses – hoop, longitudinal and Volumetric strains – changes in diameter, and volume of thin cylinders – Thin spherical shells.
Thick Cylinders: Introduction Lame's theory for thick cylinders – Derivation of Lame's formulae – distribution of hoop and radial stresses across thickness – design of thick cylinders – compound cylinders – Necessary difference of radii for shrinkage – Thick spherical shells.

U-I Simple Stresses and Strains and Strain energy

Load: External force applied on a body. Units are N or KN

Types of loads:

Dead load: Dead loads are static forces that are relatively constant for an extended time. They can be in tension or compression.

Live load: Live loads are usually unstable or moving loads.

Snow load: Snow load is the downward force acting on a body by the weight of snow.

Earthquake load: It is the total force that an earthquake exerts on a given structure.

Tension load: Load applied on both sides of an object to pull in opposite direction.

Compression load: Load applied to crush a material in the direction of its action.

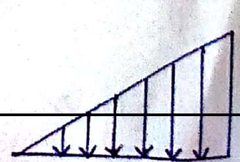
Shear load: Load applied to produce a sliding failure on material parallel to the direction of the force.

Point load: Force applied at a single, specific point. ↓

Uniformly distributed load: Load applied or spread on some length of a beam. It is expressed by intensity.

Ex: N/m 

Uniformly varying load: Magnitude remains uniform throughout the length



...uals:

① Elastic material:

* A member which allows itself to be deformed, but will offer a resistance to the deformation is said to be elastic.

* When load is applied on a body, it changes its shape and after removing the load, the body comes to its original shape. This property is known as elasticity.

② Plastic material:

* A member which allows itself to be deformed without any resistance is said to be plastic.

* When load is applied on a body, the deformation occurs and after removing the load, the body doesn't obtain its original shape. This property is called plasticity.

③ Brittle material:

* These materials does not elongate on placing a load.

* They bear the load upto its capacity and then fails.

④ Rigid materials:

* A member which does not allow itself to be deformed at all is said to be rigid.

Stress: The internal resistance offered by a body against the deformation due to load applied.

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} \qquad \boxed{\sigma = \frac{P}{A}}$$

Units are N/mm^2, $\frac{KN}{mm^2}$, $\frac{N}{m^2}$

Types of stresses:

① **Tensile stress:** When the resistance offered by a section of a member is against an increase in length, the section is said to offer a tensile stress.

② **Compressive stress:** When the resistance offered by a section of a member is against the decrease in length, the section is said to offer a compressive stress.

③ **Shear stress:** Force that causes deformation of a material by slippage along the plane parallel to the imposed stress.

Strain:

The ratio of change in length to the total length.

$$\boxed{\varepsilon = \frac{\delta L}{L}}$$

Types of strains:

① **Tensile strain:** The ratio of increase in length to the original length.

$$\text{Tensile strain} = \frac{\text{increase in length}}{\text{Original length}}$$

② **Compressive strain:** The ratio of decrease in length to the original length.

$$\text{Compressive strain} = \frac{\text{Decrease in length}}{\text{Original length}}$$

③ **Shear strain:** The ratio of transverse displacement (dL) to the distance from the lower face.

$$\text{Shear strain} = \frac{\text{Transverse Displacement}}{\text{Distance from the lower face}}$$

Hooke's law:

When a material is loaded such that the intensity of stress within a certain limit, the ratio of the intensity of stress to the corresponding strain is constant.

$$\frac{\text{Intensity of stress}}{\text{strain}} = \text{constant}$$

$$\frac{\sigma}{\varepsilon} = E$$

$$\frac{\frac{P}{A}}{\frac{\delta L}{L}} = E \Rightarrow \frac{P}{A} = E \cdot \frac{\delta L}{L} \Rightarrow \boxed{\sigma = E \cdot \varepsilon}$$

Volumetric strain (e_v):

The ratio of the change in volume to the original volume is called volumetric strain. It is usually denoted by e_v.

$$e_v = \frac{\text{change in volume}}{\text{original volume.}}$$

Modulus of rigidity (C or G):

The ratio of shear stress to shear strain is called modulus of rigidity. It is denoted by C or G.

$$\text{Modulus of rigidity} = \frac{\text{shear stress}}{\text{shear strain}}$$

$$C = \frac{\tau}{\phi}$$

Bulk modulus (K):

The ratio of stress to volumetric strain is defined as Bulk modulus (K).

$$K = \frac{\text{stress}}{\text{volumetric strain}} = \frac{\sigma_n}{e_v}$$

Poisson's ratio: (μ or $\frac{1}{m}$)

The ratio of lateral strain to longitudinal strain is defined as Poisson's ratio.

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Lateral strain: The ratio $\frac{\delta b}{b}$ (width) or $\frac{\delta d}{d}$ (depth) is called lateral strain.

Longitudinal strain: The ratio of $\frac{\delta L}{L}$ is called longitudinal strain.

* A square rod of cross-section 20mm x 20mm subjected to a load of 50 KN (Tensile). Find the change in length of 500 mm length of rod. Take $E = 1.8 \times 10^8$ KN/mm^2

Sol Given:

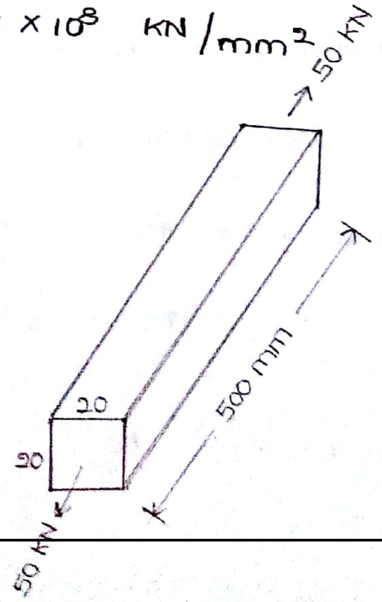
Cross-section Area, $A = 20\,mm \times 20\,mm$

Load, $P = 50\ KN = 50 \times 10^3\ N$

Length, $L = 500\ mm$

$E = 1.8 \times 10^8\ KN/mm^2$

$= 1.8 \times 10^8 \times 10^3\ \frac{N}{mm^2}$



Stress, $\sigma = \frac{P}{A} = \frac{50 \times 10^3}{20 \times 20}$

$$\boxed{\sigma = 125 \ \frac{N}{mm^2}}$$

Strain, $\varepsilon = \frac{\delta L}{L} = \frac{\delta L}{500}$

$$\sigma = E \cdot \varepsilon$$

$$\Rightarrow 125 = 1.8 \times 10^8 \times 10^3 \times \frac{\delta L}{500}$$

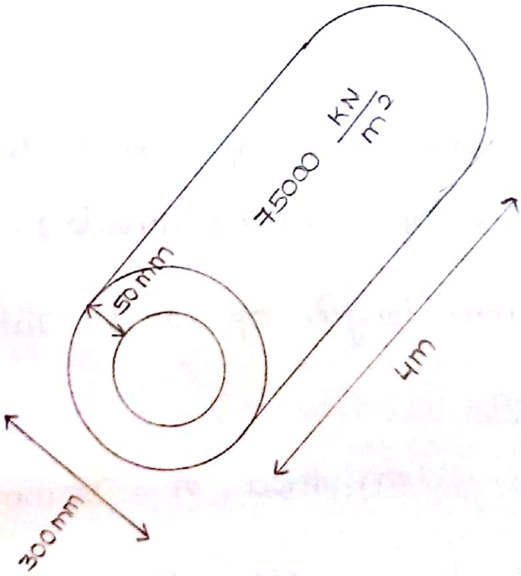
$$\boxed{\delta L = 3.47 \times 10^{-7} \ mm}$$

* A hollow cast iron and cylinder 4m long 300 mm outer diameter and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 $\frac{KN}{m^2}$. Assume young's modulus for cast iron as 1.5×10^8 KN/m^2. Find.

(i) Magnitude of the load (ii) Longitudinal strain

(iii) Total decrease in length.

Sol Given:



Length, L = 4 m = 4000 mm

Outer diameter, D = 300 mm

Thickness, t = 50 mm

Inner diameter, d = 200 mm

Stress, $\sigma = 75000 \ \frac{KN}{m^2}$

$$= 75000 \times 10^3 \ \frac{N}{10^6 \ mm^2} = 75000 \times 10^{-3} \ \frac{N}{mm^2}$$

$E = 1.5 \times 10^8 \ KN/m^2 = 1.5 \times 10^8 \times 10^3 \ \frac{N}{10^6 \ mm^2}$

$$= 1.5 \times 10^5 \ \frac{N}{mm^2}$$

Area, $A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(300^2 - 200^2) = 39269.9 \ mm^2$

Stress, $\sigma = \frac{P}{A}$

$$75000 \times 10^{-3} = \frac{P}{39269.9}$$

$$P = 2945242.5 \ N$$

$$P = 2945.24 \ KN$$

(ii) $\varepsilon = ?$

$$\sigma = E \cdot \varepsilon$$

$$75000 \times 10^{-3} = 1.5 \times 10^5 \times \varepsilon$$

$$\Rightarrow \varepsilon = 5 \times 10^{-4}$$

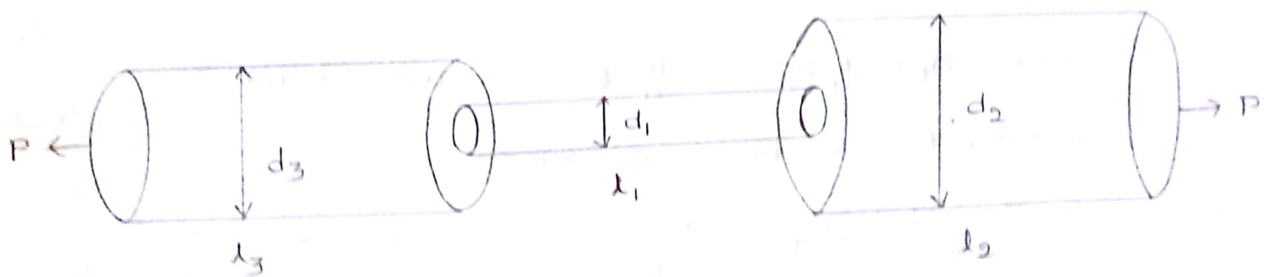
(iii) $\delta l = ?$

Strain, $\varepsilon = \frac{\delta l}{l}$

$$5 \times 10^{-4} = \frac{\delta l}{4000}$$

$$\delta l = 2 \ mm$$

Bars of varying cross-sections



Bar-1: Load = P

Length = l_1

Dia = d_1

Area, $A_1 = \frac{\pi}{4} d_1^2$

Stress, $\sigma = \frac{P}{A_1}$

Strain, $\varepsilon = \frac{\delta l_1}{l_1}$

$$\delta L_1 = \frac{P l_1}{A_1 E_1} \qquad \left(\delta l_1 = \frac{\sigma}{E} l_1 = \frac{P l_1}{A_1 E_1} \right)$$

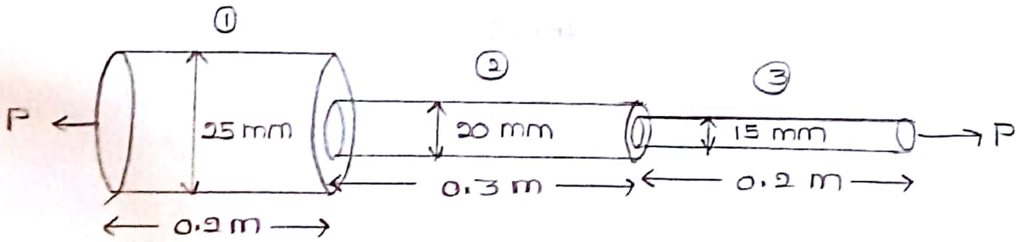
$$\delta L_1 = \frac{4 P l_1}{\pi d_1^2 E_1}, \quad \delta L_2 = \frac{4 P l_2}{\pi d_2^2 E_2}, \quad \delta L_3 = \frac{4 P l_3}{\pi d_3^2 E_3}$$

$$\delta L = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{4P}{\pi} \left(\frac{l_1}{d_1^2 E_1} + \frac{l_2}{d_2^2 E_2} + \frac{l_3}{d_3^2 E_3} \right) \qquad (E_1 = E_2 = E_3 = E)$$

$$= \frac{4P}{\pi E} \left(\frac{l_1}{d_1^2} + \frac{l_2}{d_2^2} + \frac{l_3}{d_3^2} \right)$$

* A bar of steel is 0.7 m long (i) 0.2 m it is 25 mm in dia (ii) 0.3 m it is 20 mm in dia (iii) Remaining 0.2 m it is 15 mm dia. Find the change in length and stresses in each bar, if it is supported tensile load of 100 KN. Take $E = 0.21 \frac{MN}{mm^2}$



Sol **Given:** Dia, $d_1 = 25$ mm $\qquad E = 0.21 \frac{MN}{mm^2} = 0.21 \times 10^6 \frac{N}{mm^2}$

Length, $l_1 = 0.2$ m $= 0.2 \times 10^3$ mm

Area, $A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (25)^2 = 490.87$ mm²

Load, $P = 100$ KN $= 100 \times 10^3$ N

Change in length, $\delta l_1 = \frac{P l_1}{A_1 E} = \frac{100 \times 10^3 \times 0.2 \times 10^3}{490.87 \times 0.21 \times 10^6}$

$\delta l_1 = 0.194$ mm

Stress, $\sigma_1 = \frac{P}{A_1} = \frac{100 \times 10^3}{490.87} = 203.71 \frac{N}{mm^2}$

<u>Given</u>: Dia, $d_2 = 20$ mm

Length, $L_2 = 0.3\text{ m} = 0.3 \times 10^3$ mm

$$\text{Area, } A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(20)^2 = 314.15 \text{ mm}^2$$

$$\text{Stress, } \sigma_2 = \frac{P}{A_2} = \frac{100 \times 10^3}{314.15} = 318.31 \frac{N}{mm^2}$$

$$\text{Change in length, } \delta l_2 = \frac{Pl_2}{A_2 E} = \frac{100 \times 10^3 \times 0.3 \times 10^3}{314.15 \times 0.21 \times 10^6}$$

$$\delta l_2 = 0.454 \text{ mm}$$

(iii) <u>Given</u>: Dia, $d_3 = 15$ mm

Length, $l_3 = 0.2\text{ m} = 0.2 \times 10^3$ mm

$$\text{Area, } A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4}(15)^2 = 176.71 \text{ mm}^2$$

$$\text{Stress, } \sigma_3 = \frac{P}{A_3} = \frac{100 \times 10^3}{176.71} = 565.89 \frac{N}{mm^2}$$

$$\text{Change in length, } \delta l_3 = \frac{Pl_3}{A_3 E} = \frac{100 \times 10^3 \times 0.2 \times 10^3}{176.71 \times 0.21 \times 10^6}$$

$$\delta l_3 = 0.538 \text{ mm}$$

$\therefore$ Total change in length, $\delta L = \delta l_1 + \delta l_2 + \delta l_3$

$$= 0.194 + 0.454 + 0.538$$

$$\delta L = 1.186 \text{ mm}$$

* A steel wire 2m long 3 mm in dia is extended 0.75 mm when a weight (W) is suspended from a wire. If the same weight is suspended from a brass wire 2.5 m long 2 mm in dia. It is elongated by 4.64 mm. Determine the modulus of velocity of brass if the tough steel be $2 \times 10^5 \frac{N}{mm^2}$.

Steel wire - 1

Length, l_1 = 2 m = 2000 mm

Dia, d_1 = 3 mm

Elongation, δl_1 = 0.75 mm

$E_1 = 2 \times 10^5 \frac{N}{mm^2}$

Load, W = ?

$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(3)^2$

$= 7.06\ mm^2$

$\delta l_1 = \frac{P l_1}{A_1 E_1}$

$0.75 = \frac{P \times 2000}{7.06 \times 2 \times 10^5}$

$P = 529.5\ N$

Brass wire - 2

length, l_2 = 2.5 m = 2.5×10^3 mm

Dia, d_2 = 2 mm

$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(2)^2 = 3.14\ mm^2$

W = 529.5 N

$\delta l_2 = 4.64$ mm

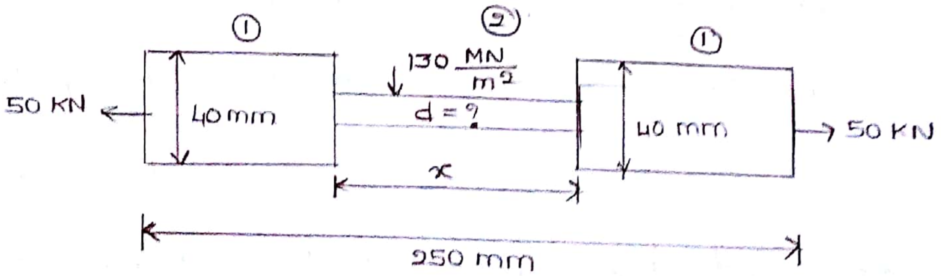
$\delta l_2 = \frac{P l_2}{A_2 E_2}$

$\Rightarrow 4.64 = \frac{529.5 \times 2.5 \times 10^3}{3.14 \times E_2}$

$E_2 = 90856.98$

$= 0.9085 \times 10^5 \frac{N}{mm^2}$

* The bar shown in figure is subjected to tensile load of 50 KN. Find the diameter of the middle portion, If the stress is limited to 130 $\frac{MN}{m^2}$. Find also the length of the middle portion, If the total expansion of the bar is 0.15 mm. Take E = 200 $\frac{GN}{m^2}$.



Sol Given: Load, P = 50 KN = 50×10^3 N

Stress ②, $\sigma_2 = 130 \frac{MN}{m^2} = 130 \times \frac{10^6\ N}{10^6\ mm^2} = 130 \frac{N}{mm^2}$

Young's modulus, $E = 200 \frac{GN}{m^2}$

$= 200 \times \frac{10^9\ N}{10^6\ mm^2} = 200 \times 10^3 \frac{N}{mm^2}$

in length, $\delta L = 0.15$ mm

Let diameter of middle portion be 'd'.

$$\text{Stress} = \frac{P}{A}$$

$$130 = \frac{50 \times 10^3}{A} \Rightarrow A = 384.61\ mm^2$$

$$\frac{\pi}{4} d^2 = 384.61$$

$$d^2 = 489.7$$

$$d = 22.12\ mm$$

Let the length of middle portion be 'x'.

Bar ①

$$\delta l_1 = \frac{P l_1}{A_1 E}$$

$$= \frac{50 \times 10^3 \times (250 - x)}{1256.63 \times 200 \times 10^3}$$

$$= 39.78 \times \frac{(250-x)}{2 \times 10^5}$$

Bar ②

$$\delta l_2 = \frac{P l_2}{A_2 E}$$

$$= \frac{50 \times 10^3 \times x}{384.61 \times 200 \times 10^3}$$

$$= 130.1 \times \frac{x}{2 \times 10^5}$$

$$\delta L = \delta l_1 + \delta l_2$$

$$0.15 = 39.78 \times \frac{(250-x)}{2 \times 10^5} + 130.1 \times \frac{x}{2 \times 10^5}$$

$$0.15 = \frac{1}{2 \times 10^5} \left(39.78 (250 - x) + 130.1 x \right)$$

$$30000 = 9945 - 39.78 x + 130.1 x$$

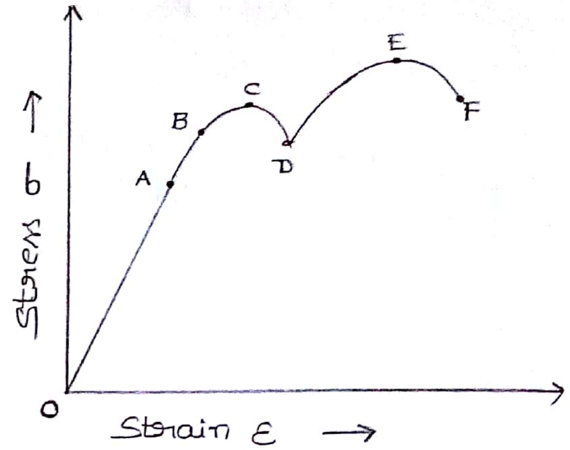
$$3000 = 9945 + 90.22 x.$$

$$90.22 x = 30000 - 9945$$

$$90.22 x = 20055$$

$$x = 222.28\ mm$$

Stress, strain relationship for mild steel



* Figure shows a stress - strain diagram obtained from a mild steel specimen subjected to a tensile stress.
* The plot from o to A is a straight line. This stress corresponding to the point A is called limit of proportionality.
* In this range of extension, stress is proportional to strain i.e., Hooke's law is applicable.
* In this range, Stress = Young's modulus × Strain.

$$\sigma = E \cdot \varepsilon.$$

* If the specimen is extended beyond the limit of proportionality upto the condition shown at B, the material still remains elastic.
* But in the range A to B, the relation between stress and strain is not linear.
* The stress at B is called elastic limit.
* If the specimen is extended beyond the elastic limit, plastic deformation takes place in the range B to C.
* This strain increases with almost constant stress. All the conditions shown at c, there is considerable extension corresponding to decrease in load.

This stress is called upper yield point.

* At the condition shown at D, the material again offers resistance to greater extension.

* The stress at D is called lower yield point.

As the load is increased, the extension increases and the condition shown at E, necking of the specimen is developed.

* This stress corresponding to E is called the ultimate tensile stress.

* As the extension is increased, the load required decreases and the specimen breaks at the condition shown at F.

* The stress at F is called stress of failure.

* The following observations are made during a tensile test on mild steel specimen 40 mm in dia and 200 mm length. Elongation with 40 KN load is 0.0304 mm, yield load 160 KN, maximum load 242 KN, length of specimen after the load 249 mm.

Determine (a) young's modulus of elasticity

(b) yield point stress. (c) Ultimate stress.

(d) Percentage elongation.

Sol (a) Young's modulus

Length of specimen, $l = 200$ mm

Dia of specimen, $d = 40$ mm

Load, $P = 40$ KN $= 40 \times 10^3$ N

Elongation, $\delta L = 0.0304$ mm

[Ultim]ate load = Maximum load = 242 KN = 242×10^3 N

yield load = 160 KN = 160×10^3 N

Final length = 249 mm.

$$\text{Stress}, \sigma = \frac{P}{A} = \frac{40 \times 10^3}{\frac{\pi}{4}(40)^2} = 31.83 \frac{N}{mm^2}$$

$$\text{Strain}, \varepsilon = \frac{\delta l}{l} = \frac{0.0304}{200}$$

$$\text{Young's modulus}, E = \frac{\sigma}{\varepsilon} = \frac{31.83}{\frac{0.0304}{200}} = 209407.89$$

$$= 2.09 \times 10^5 \frac{N}{mm^2}$$

(ii) yield point stress = ?

$$\text{Stress}, \sigma = \frac{P}{A} = \frac{\text{yield load}}{\text{Area}} = \frac{160 \times 10^3}{\frac{\pi}{4}(40)^2}$$

$$= 127.32 \frac{N}{mm^2}.$$

(iii) ultimate stress = ?

$$\text{stress}, \sigma = \frac{P}{A} = \frac{\text{ultimate load}}{\text{Area}} = \frac{242 \times 10^3}{\frac{\pi}{4}(40)^2}$$

$$= 192.57 \frac{N}{mm^2}$$

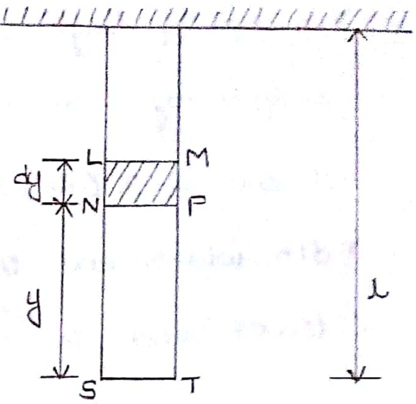
(iv) Percentage elongation = ?

$$\%\ \text{elongation} = \frac{\text{final length} - \text{original length}}{\text{original length}} \times 100.$$

$$= \frac{249 - 200}{200} \times 100$$

$$= 24.5\%$$

Stress and elongation produced in a bar due to self-weight:



* A bar of length 'l' (m) and area 'A' (m^2) is rigidly fixed at one end as shown in the figure.
* Let ρ (kg/m³) be the density of material.
* Consider a small strip of bar (shaded) LMNP of thickness 'dy' and at a distance 'y' from free ~~ba~~ end.
* Now the force acting down at NP = weight of bar NPTS

$$= V \times \gamma$$

$$= A \times l \times \rho g$$

$$= A \times y \times \rho \times 9.81$$

Stress at section NP is given by, $\sigma = \frac{P}{A} = \frac{A \times y \times \rho \times 9.81}{A}$

$$\sigma = 9.81 \rho y$$

At lower end ($y=0$), $\sigma = 0$

At upper end ($y=l$), $\sigma = 9.81 \rho l$

Strain in a small strip, $\varepsilon = \frac{\sigma}{E} = \frac{9.81 \rho y}{E}$

Change in length in a small strip, $\delta L = \frac{\sigma}{E} \times l$

$$= \frac{9.81 \rho y}{E} \times dy.$$

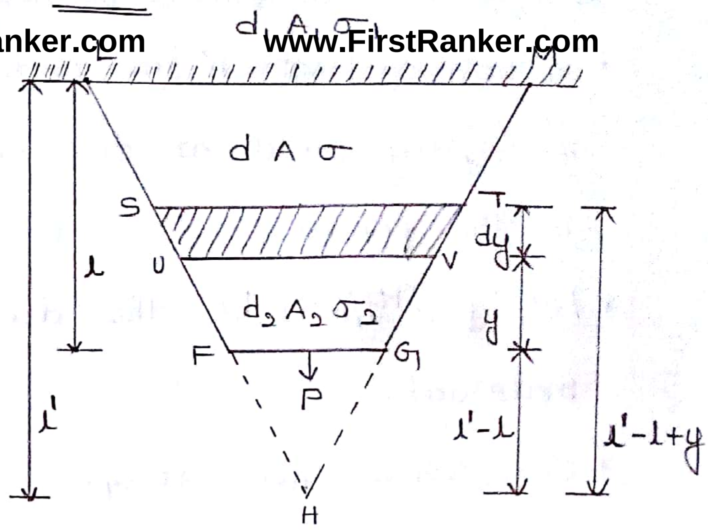
$\therefore$ Total extension of the bar,

$$\delta L = \int_0^l \frac{9.81 \rho y}{E} dy = \frac{9.81 \rho}{E} \int_0^l y \cdot dy = \frac{9.81 \rho}{E} \left[\frac{y^2}{2}\right]_0^l$$

$$= \frac{9.81 \rho}{E} \left(\frac{l^2}{2} - 0\right)$$

$$\delta L = \frac{9.81 \rho l^2}{2E}$$

[illegible] of tapering rod:



* A rod of length (l) tapers uniformly from diameter (d_1) to a diameter (d_2).
* Its wider end is fixed and lower end is subjected to an axial tensile load.
* MG and LF are produced to meet at H.
* Consider a small length 'dy' at a distance 'y' from the lower end.
* Let A_1, A_2 and A_3 be the cross-sectional area of the top, bottom and UV respectively.

Area, $A_1 = \frac{\pi}{4} d_1^2$, $A_2 = \frac{\pi}{4} d_2^2$, $A = \frac{\pi}{4} d^2$

Stress, $\sigma = \frac{P}{A} \Rightarrow P = \sigma \times A$

$$P = \sigma_1 A_1 = \sigma_2 A_2 = \sigma A.$$

$$\sigma = \frac{\sigma_2 A_2}{A}$$

Extension of a small strip, $\delta L = \frac{\sigma}{E} \cdot l$

$$= \frac{\sigma_2 A_2}{AE} dy.$$

$$= \frac{\sigma_2}{E} \left(\frac{d_2}{d}\right)^2 dy$$

$$= \frac{\sigma_2}{E} \left(\frac{l'-l}{l'-l+y}\right)^2 dy$$

∴ Total elongation of the bar,

$$\delta L = \int_0^l \frac{\sigma_2}{E} \frac{(l'-l)^2}{(l'-l+y)^2} dy = \frac{\sigma_2 (l'-l)^2}{E} \int_0^l \frac{1}{(l'-l+y)^2} dy$$

$$= \frac{\sigma_2 (l'-l)^2}{E} \left(\frac{-1}{l'-l+y}\right)_0^l$$

$$= \frac{-\sigma_2 (l'-l)^2}{E}\left(\frac{1}{l'-l+l} - \frac{1}{l'-l+0}\right)$$

$$= \frac{-\sigma_2}{E}(l'-l)^2\left(\frac{1}{l'} - \frac{1}{l'-l}\right)$$

$$= \frac{-\sigma_2}{E}(l'-l)^2\left(\frac{l'-l-l'}{l'(l'-l)}\right) = \frac{-\sigma_2}{E}(l'-l)^2\left(\frac{-l}{l'(l'-l)}\right)$$

$$= \frac{\sigma_2}{E}\,\frac{(l'-l)\,l}{l'}$$

From figure, $\frac{l'-l}{l'} = \frac{d_2}{d_1}$

$$\delta L = \frac{\sigma_2 l}{E}\cdot\frac{d_2}{d_1} = \frac{P}{A_2}\cdot\frac{l d_2}{E d_1} = \frac{P}{\frac{\pi}{4}d_2^2}\cdot\frac{l d_2}{E d_1} = \frac{4Pl}{\pi E d_1 d_2}$$

$$\delta L = \frac{4Pl}{\pi E d_1 d_2}$$

Composite bars:

Frequently ties consists of two materials fastened together to prevent unevens straining of the two materials.

If an axial load P is applied to the bar then

$$P = P_1 + P_2$$

$$\Rightarrow P = \sigma_1 A_1 + \sigma_2 A_2$$

where σ_1, σ_2 are the stresses induced and.

A_1, A_2 are the cross-sectional area of the materials.

These strains produced are also equal.

$$\epsilon_1 = \epsilon_2$$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

where E_1 and E_2 are the young's modulus of two materials.

* A copper rod of 40 mm dia. is surrounded tightly by a cast iron tube of 80 mm external dia. The ends been fastened together when put to a compressive load of 30 KN. What will be the shared by each. Also determine the amount by which the compound wall shorten if it is 2m long.

Take E for cast iron = 175 $\frac{GN}{m^2}$

E for copper rod = 75 $\frac{GN}{m^2}$

Sol Copper rod

Diameter, $d_c = 40$ mm

Load, $P = 30$ KN $= 30 \times 10^3$ N

Area, $A_c = \frac{\pi}{4}(40)^2$

$= 1256.63$ mm^2.

$E_c = 75 \frac{GN}{m^2} = 75 \times \frac{10^9 N}{10^6 mm^2}$

$= 75 \times 10^3 \frac{N}{mm^2}$

Cast iron

External dia, $D_{ci} = 80$ mm

Internal dia, $d_{ci} = 40$ mm

Area, $A_{ci} = \frac{\pi}{4}(D_{ci}^2 - d_{ci}^2)$

$= \frac{\pi}{4}(80^2 - 40^2)$

$A_{ci} = 3769.9$ mm^2

$E_{ci} = 175 \times 10^3 \frac{N}{mm^2}$

Load on bar = Load shared by copper rod + Load shared by cast iron tube

$P = P_c + P_{ci}$

$P = \sigma_1 A_1 + \sigma_2 A_2 \longrightarrow (1)$

Strains in two materials are equal i.e.,

$e_c = e_{ci}$

$\frac{\sigma_c}{E_c} = \frac{\sigma_{ci}}{E_{ci}}$

$\sigma_{ci} = \frac{\sigma_c}{E_c} \times E_{ci}$

$= \frac{175 \times 10^3}{75 \times 10^3} \times \sigma_c = 2.33 \sigma_c$

Sub. this value in eqn ①

$$30 \times 10^3 = \sigma_c (1256.63) + 2.33\, \sigma_c (3769.9)$$

$$= \sigma_c (1256.63 + 8783.86)$$

$$= 10040.49\, \sigma_c$$

$$\sigma_c = 2.98 \ \frac{N}{mm^2}$$

$$\sigma_{ci} = 2.33\, \sigma_c = 2.33 \times 2.98 = 6.94 \ \frac{N}{mm^2}$$

Load shared by copper rod, $P_c = \sigma_c A_c = 2.98 \times 1256.63$

$= 3744.71\ N$

$P_c = 3.744\ KN$

Load shared by cast iron, $P_{ci} = \sigma_{ci} A_{ci} = 6.94 \times 3769.9$

$= 26163.17\ N$

$P_{ci} = 26.163\ KN.$

Thermal stresses (or) temperature stresses

* Thermal stresses are the stresses induced in a body due to change in temperature.
* Thermal stresses are set up in a body when the temperature of the body is raised or lowered, the body is not allowed to expand or contract freely.

∴ Extension of the rod, $\delta L = l \alpha t$

Strain, $\varepsilon = \frac{\delta l}{l} = \frac{l \alpha t}{l} = \alpha t$

Stress, $\sigma = E \cdot \varepsilon = E \alpha t$

rod of steel 20 m long at a temperature of 20°c. Find the free expansion of the rod when the temperature is raised to 65°c. Find the temperature stress produced.

(i) When the expansion of the rod is prevented.

(ii) When the rod permitted to expand by 5.8 mm.

Take $\alpha = 12 \times 10^{-6}$ /°c and $E = 2 \times 10^5 \frac{N}{mm^2}$

Sol Given: Length, $l = 20$ m $= 20 \times 10^3$ mm

Initial temperature, $t_1 = 20°c$

Final temperature, $t_2 = 65°c$

$\alpha = 12 \times 10^{-6}$ /°c.

$E = 2 \times 10^5 \frac{N}{mm^2}$

Change in temperature, $t = t_2 - t_1 = 65 - 20$

$t = 45°c$

(i) Fully prevented.

$\delta L = l \alpha t = 20 \times 10^3 \times 12 \times 10^{-6} \times 45 = 10.8$ mm

$$\varepsilon = \frac{\delta l}{L} = \frac{10.8}{20 \times 10^3}$$

Temperature stress, $\sigma = E \cdot \varepsilon = 2 \times 10^5 \times \frac{10.8}{20 \times 10^3}$

$$\sigma = 108 \frac{N}{mm^2}$$

(ii) Expansion is prevented upto 5.8 mm.

Elongation, $\delta L = 10.8 - 5.8$

$= 5$ mm

Temperature strain, $\varepsilon = \frac{\delta l}{l} = \frac{5}{20 \times 10^3}$

Temperature stress, $\sigma = E \cdot \varepsilon = 2 \times 10^5 \times \frac{5}{20 \times 10^3}$

$$\sigma = 50 \frac{N}{mm^2}$$

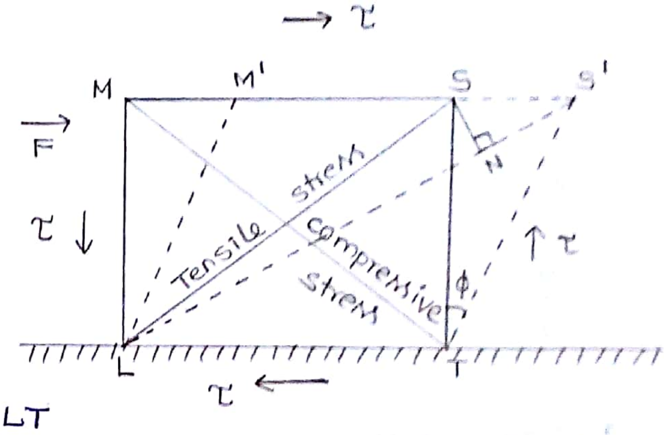
Relations between the elastic modulus:

Young's modulus, $E = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}}$

Modulus of rigidity, $G = \dfrac{\text{shear stress}}{\text{shear strain}} = \dfrac{\tau}{\phi}$

Bulk modulus of elasticity, $K = \dfrac{\sigma_n}{e_v}$

Relation between E and G:



* Refer to figure LMST is a solid cube subjected to shear force.
* Let τ be the shear stress produced in the faces MS and LT due to shear force.
* Due to this shearing load, the cube is distorted to LM'S'T.

$\therefore$ Shear strain, $\phi = \dfrac{SS'}{ST}$

$\tan\phi = \dfrac{SS'}{ST}$ (Since $\tan\phi$ is small $\tan\phi = \phi$)

$\Rightarrow \phi = \dfrac{SS'}{ST}$

* Modulus of rigidity, $G = \dfrac{\tau}{\phi}$

$$\Rightarrow \phi = \frac{\tau}{G}$$

$\therefore \dfrac{SS'}{ST} = \dfrac{\tau}{G}$

* On the diagonal LS', draw a perpendicular SN from S.

Now diagonal strain, $= \dfrac{NS'}{LN}$

$NS' = SS' \cos 45^0 = \dfrac{SS'}{\sqrt{2}}$

$LN \simeq LS$

$\cos 45^\circ = \frac{ST}{LS}$

$$LS = \frac{ST}{\cos 45^\circ} = ST \times \sqrt{2}$$

$$\text{Diagonal strain} = \frac{NS'}{LN} = \frac{SS'}{\sqrt{2}} \times \frac{1}{\sqrt{2}ST} = \frac{SS'}{2ST} = \frac{\tau}{2G}$$

$$\varepsilon_x = \frac{\sigma_n}{2G}$$

σ_n is the normal stress due to shear stress.

The net strain in the direction of diagonal,

$$LS = \frac{\sigma_n}{E} + \frac{1}{m} \cdot \frac{\sigma_n}{E}$$

$$\varepsilon_x = \frac{\sigma_n}{E}\left(1 + \frac{1}{m}\right)$$

$$\frac{\sigma_n}{2G} = \frac{\sigma_n}{E}\left(1 + \frac{1}{m}\right) \Rightarrow \boxed{E = 2G\left(1 + \frac{1}{m}\right)}$$

Relation between E and K:

* If the solid cube is subjected to σ_n on all the faces.

$\therefore$ Compressive strain in each axis

i.e., $\varepsilon_x = \frac{\sigma_x}{E} - \mu \cdot \frac{\sigma_y}{E} \cdot \mu \cdot \frac{\sigma_z}{E}$

$$= \frac{\sigma_n}{E} - \mu \cdot \frac{\sigma_n}{E} - \mu \cdot \frac{\sigma_n}{E}$$

$$\varepsilon_x = \frac{\sigma_n}{E}(1 - 2\mu)$$

$\therefore$ Volumetric strain, $e_v = 3 \times \varepsilon_x$

$$= \frac{3\sigma_n}{E}(1 - 2\mu)$$

Bulk modulus, $K = \frac{\sigma_n}{e_v} \Rightarrow e_v = \frac{\sigma_n}{K}$

$$\therefore \frac{\sigma_n}{K} = \frac{3\sigma_n}{E}(1 - 2\mu)$$

$$\Rightarrow \boxed{E = 3K(1 - 2\mu)}$$

$$E = 2G\left(1+\frac{1}{m}\right)$$

$$\Rightarrow \frac{E}{2G} = 1+\frac{1}{m} \quad \Rightarrow \frac{E}{2G} - 1 = \frac{1}{m}$$

$$\frac{E-2G}{2G} = \frac{1}{m}$$

$$\Rightarrow m = \frac{2G}{E-2G}$$

from eqn, $E = 3K\left(1-\frac{2}{m}\right) = 3K\left(1 - \frac{2(E-2G)}{2G}\right)$

$$= 3K\left(\frac{2G - 2E + 4G}{2G}\right)$$

$$= 3K\left(\frac{6G - 2E}{2G}\right)$$

$$= 3K\left(\frac{3G-E}{G}\right)$$

$$= \frac{9KG - 3KE}{G}$$

$$E + \frac{3KE}{G} = \frac{9KG}{G}$$

$$\Rightarrow EG + 3KE = 9KG$$

$$\boxed{E = \frac{9KG}{G+3K}}$$

* The following data relate to a bar subjected to tensile stress diameter of the bar 30 mm, tensile load is 54 KN, length of bar, L = 300 mm. Extension of the bar $\delta L = 0.112$ mm. Change in dia $\delta d = 0.00366$.

Calculate (i) Poisson's ratio. (ii) The values of three moduli.

Sol Given: Length, $l = 300$ mm

Load, $P = 54$ KN $= 54 \times 10^3$ N

Dia, $d = 30$ mm

Area, $A = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2$

$$\sigma = \frac{P}{A} = \frac{54 \times 10^3}{706.85} = 76.39 \ \frac{N}{mm^2}$$

$\delta L = 0.112$ mm

$\delta d = 0.00366$ mm

(i) Poisson's ratio, $\mu = \dfrac{\text{Lateral strain}}{\text{Longitudinal strain.}}$

$$\text{Lateral strain} = \frac{\delta d}{d} = \frac{0.00366}{30} = 1.22 \times 10^{-4}$$

$$\text{Longitudinal strain} = \frac{\delta L}{L} = \frac{0.112}{300} = 3.73 \times 10^{-4}$$

$$\mu = \frac{1.22 \times 10^{-4}}{3.73 \times 10^{-4}} = 0.32.$$

(ii) Young's modulus, $E = \dfrac{\sigma}{\epsilon} = \dfrac{76.39}{3.73 \times 10^{-4}}$

$$E = 2.04 \times 10^5 \ \frac{N}{mm^2}$$

$$E = 2G(1+\mu)$$

$$\Rightarrow 2.04 \times 10^5 = 2G(1 + 0.32)$$

$$2G = \frac{2.04 \times 10^5}{1.32.}$$

$$G = 0.77 \times 10^5 \ \frac{N}{mm^2}$$

$$E = 3K(1 - 2\mu)$$

$$\Rightarrow 2.04 \times 10^5 = 3K(1 - 2(0.32))$$

$$3K = \frac{2.04 \times 10^5}{0.36}$$

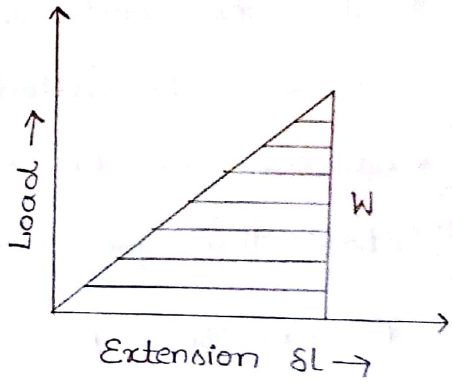
$$K = 1.88 \times 10^5 \ \frac{N}{mm^2}$$

Strain Energy

* When an elastic body is loaded it undergoes deformation and it is released of the load it regains its original shape.
* For the time loaded energy is stored in it. This same energy is given up released by the loading. This energy is called strain energy.
* This strain energy stored by the body within the elastic limit, when loaded externally is called resilience.
* And the maximum energy which a body stores upto elastic limit is called proof resilience.
* Proof resilience for unit volume is called modulus of resilience.

Strain energy in simple tension & Compression

* Let us take the case of a bar of cross-sectional area 'A' and length 'L' subjected to a load W.



* Suppose this load extends the bar by an amount δL and produces maximum stress (σ).
* The workdone by W and hence this strain energy (U) stored in the material is equal to the area in figure.

$$U = \text{Workdone} = \frac{1}{2} \times W \times \delta L$$

$$= \frac{1}{2} \times \sigma A \times \frac{\sigma}{E} \times l = \frac{\sigma^2}{2E}(A \times l)$$

$$U = \frac{\sigma^2}{2E} V$$

Stresses due to different types of loads:

A body may be subjected to following types of loads.

① Gradually applied load:

* A body is to be acted upon by a gradually applied load, if the load increases from zero and reaches its final value step-wise.

* Let W be the load applied gradually on a body and δL, σ be the corresponding change in length and maximum stress induced.

$\therefore$ stress $= \frac{W}{A}$.

Energy due to external load $= \frac{1}{2} \times W \times \delta L$

$$= \frac{1}{2} \times \sigma A \times \frac{\sigma l}{E}$$

$$= \frac{\sigma^2 A l}{2E}$$

② Suddenly applied load:

* When the load is applied all of a sudden and not step-wise is called suddenly applied load.

* Let the load W is applied all of a sudden and maximum stress thus produced be σ_{su}.

The extension be δL

Stress, $\sigma_{su} = \frac{2W}{A}$.

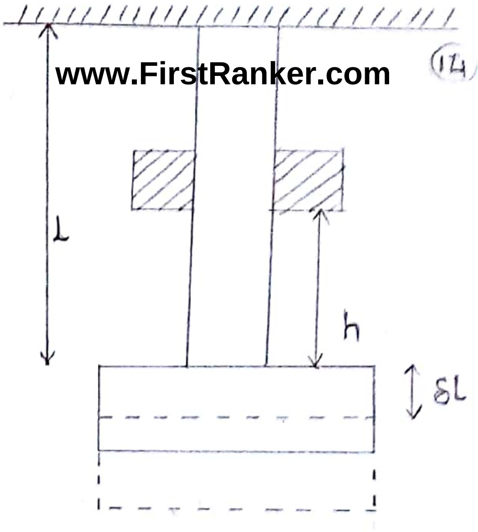
③ Impact load:

* The load which falls from a height or strike the body with certain momentum is called falling or impact load.

* Refer to figure,

Consider a weight W falling through a height 'h' on a

...r fixed on the rod which of length 'l' and as a cross-sectional area 'A'.



* Let the extension be δL and stress σ_i respectively.

$$\text{Energy stored} = \frac{1}{2} \times W \times \delta L$$

$$= \frac{1}{2} \times \sigma_i A \times \frac{\sigma_i l}{E}$$

$$= \frac{\sigma_i^2 A l}{2E}$$

$$\text{Energy workdone on the bar} = W(h + \delta L) = W\left(h + \frac{\sigma_i l}{E}\right)$$

$\therefore$ Energy = Workdone.

$$\frac{\sigma_i^2 A l}{2E} = W\left(h + \frac{\sigma_i l}{E}\right) \Rightarrow \left(\frac{Al}{2E}\right)\sigma_i^2 - \left(\frac{Wl}{E}\right)\sigma_i - Wh = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\sigma_i = \frac{\frac{Wl}{E} \pm \sqrt{\left(\frac{Wl}{E}\right)^2 - 4\left(\frac{Al}{2E}\right)(-Wh)}}{2\left(\frac{Al}{2E}\right)}$$

$$= \frac{\frac{Wl}{E} \pm \sqrt{\frac{W^2 l^2}{E^2} + \frac{2AlWh}{E}}}{\frac{Al}{E}}$$

$$= \frac{\frac{Wl}{E} \pm \frac{Wl}{E}\sqrt{1 + \frac{2AlWh}{E} \times \frac{E^2}{W^2 l^2}}}{\frac{Al}{E}}$$

$$= \frac{\frac{Wl}{E}\left(1 \pm \sqrt{1 + \frac{2AEh}{Wl}}\right)}{\frac{Al}{E}}$$

$$\sigma_i = \frac{W}{A}\left(1 \pm \sqrt{1 + \frac{2AEh}{Wl}}\right)$$

...orm metal bar has a cross-sectional area of $7 cm^2$ and length 1.5 m with an elastic limit of $160 \frac{MN}{m^2}$. What will be its proof resilience. Determine also the maximum value of an applied load which may be suddenly applied without exceeding the elastic limit. Calculate the value of gradually applied load which will produce the same extension as that produced by the suddenly applied above. Take $E = 200 \frac{GN}{m^2}$.

Given: Area, $A = 7 cm^2 = 7 \times 10^2 \ mm^2$

Length, $L = 1.5 m = 1.5 \times 10^3 \ mm$

$$\sigma_p = 160 \frac{MN}{m^2} = 160 \times \frac{10^6 N}{10^6 mm^2} = 160 \frac{N}{mm^2}.$$

$$E = 200 \frac{GN}{m^3} = 200 \times \frac{10^9 N}{10^6 mm^2} = 200 \times 10^3 \frac{N}{mm^2}$$

Proof resilience, $U_p = \frac{\sigma_p^2}{2E} (A \times L)$

$$= \frac{160^2}{2 \times 200 \times 10^3} (7 \times 10^2 \times 1.5 \times 10^3)$$

$$= 67200 \text{ Joules.}$$

Suddenly applied load, $\sigma = \frac{2W}{A}$

$$W = \frac{\sigma A}{2} = \frac{160 \times 7 \times 10^2}{2}$$

$$\Rightarrow W = 56000 \ N$$

Gradually applied load, $\sigma = \frac{W}{A}$

$$\Rightarrow W = \sigma A = 160 \times 7 \times 10^2$$

$$W = 112000 \ N.$$

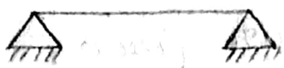
UNIT-2: Shear Force & Bending Moment

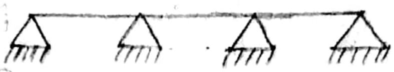
Beam:

* It is one of the structural member subjected to loads perpendicular to the axis of member.

Types of beams:

① Cantilever beam. 

② Simply supported beam. 

③ Continuous beam 

④ Propped cantilever. beam. 

⑤ Fixed beam 

Types of Supports:

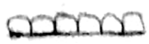
① Fixed support 

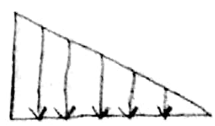
② Hinged support 

③ Roller support 

Types of loads:

Point load. ↓ W = 50 KN

Uniformly Distributed load  W = 50 KN/m

Uniformly varying load 

Couple (or) Moment 

Shear force:

The algebraic sum of all the vertical forces either left of the section or right of the section.

Bending moment:

The algebraic sum of all the moments either left of the section or right of the section.

Sign conversion:

For shear force,

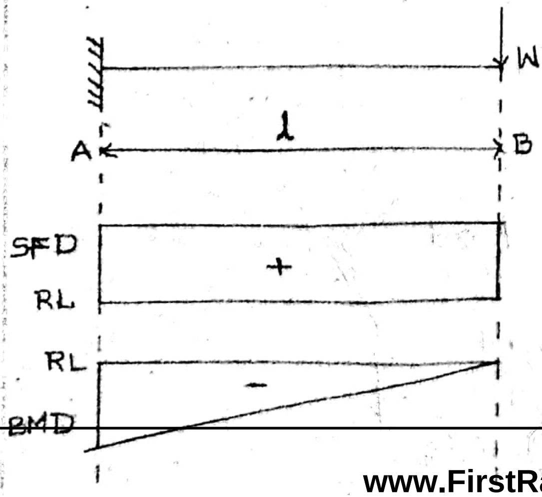
	LoS		RoS
	$\uparrow$	+	$\downarrow$
	$\downarrow$	-	$\uparrow$

For bending moment,

	LoS		RoS
	$\circlearrowright$	+	$\circlearrowleft$
	$\circlearrowleft$	-	$\circlearrowright$

Cantilever Beam:

Cantilever beam subjected to point load at the end.



Reaction: $\Sigma V = 0$

$R_A - W = 0$

$R_A = W$

AB portion (0 to L)

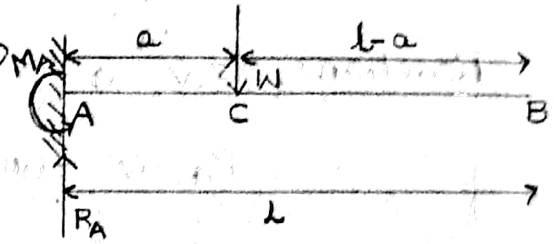
SF: $F_x = W$

BM: $M_x = -Wx$

$M_{x=0} = 0$ $\quad M_{x=l} = -Wl$

iii) Cantilever beam with point load (W) at a distance (a) from the fixed end:

Let us divide the beam into two portions i.e., BC and CA.



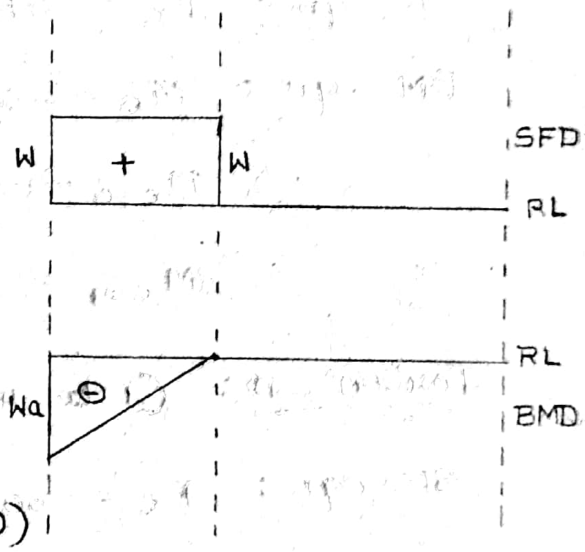
Reaction: $\Sigma V = 0 \Rightarrow R_A - W = 0$

$$\boxed{R_A = W}$$

BC-portion: $(0 \text{ to } l-a)$

SF equation. $F_x = 0$

BM equation $M_x = 0$



CA- portion: $(l-a \text{ to } l)$

SF eqn. $F_x = W$

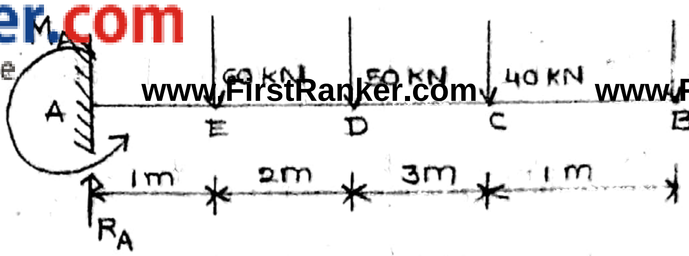
BM eqn. $M_x = -W(x-(l-a))$

$= -W(x-l+a)$

$M_{x=l-a} = -W(l-a-l+a) = 0$

$M_{x=l} = -W(l-l+a) = -Wa$

* Draw shear force and bending moment of diagrams of the cantilever beam as shown in figure.



Reaction: $\Sigma V = 0$

$R_A - 60 - 50 - 40 - 30$

$\Rightarrow R_A = 180$ KN

Portion BC: (0 to 1 m)

SF eqn : $F_x = 30$ KN

BM eqn : $M_x = -30x$ KN-m.

$M_{x=0} = -30(0) = 0$ KN-m

$M_{x=1} = -30(1) = -30$ KN-m

Portion CD: (1 to 4 m)

SF eqn : $F_x = 30 + 40 = 70$ KN

BM eqn : $M_x = -30x - 40(x-1)$ KN-m.

$M_{x=1} = -30(1) - 40(1-1) = -30$ KN-m

$M_{x=4} = -30(4) - 40(4-1) = -240$ KN-m

Portion DE: (4 to 6 m)

SF eqn : $F_x = 30 + 40 + 50 = 120$ KN.

BM eqn : $M_x = -30x - 40(x-1) - 50(x-4)$ KN-m

$M_{x=4} = -30(4) - 40(4-1) - 50(4-4)$

$= -240$ KN-m

$M_{x=6} = 30(6) - 40(6-1) - 50(6-4)$

$= -480$ kN-m.

Portion EA: (6 to 7m)

SF eqn: $F_x = 30 + 40 + 50 + 60 = 180$ KN.

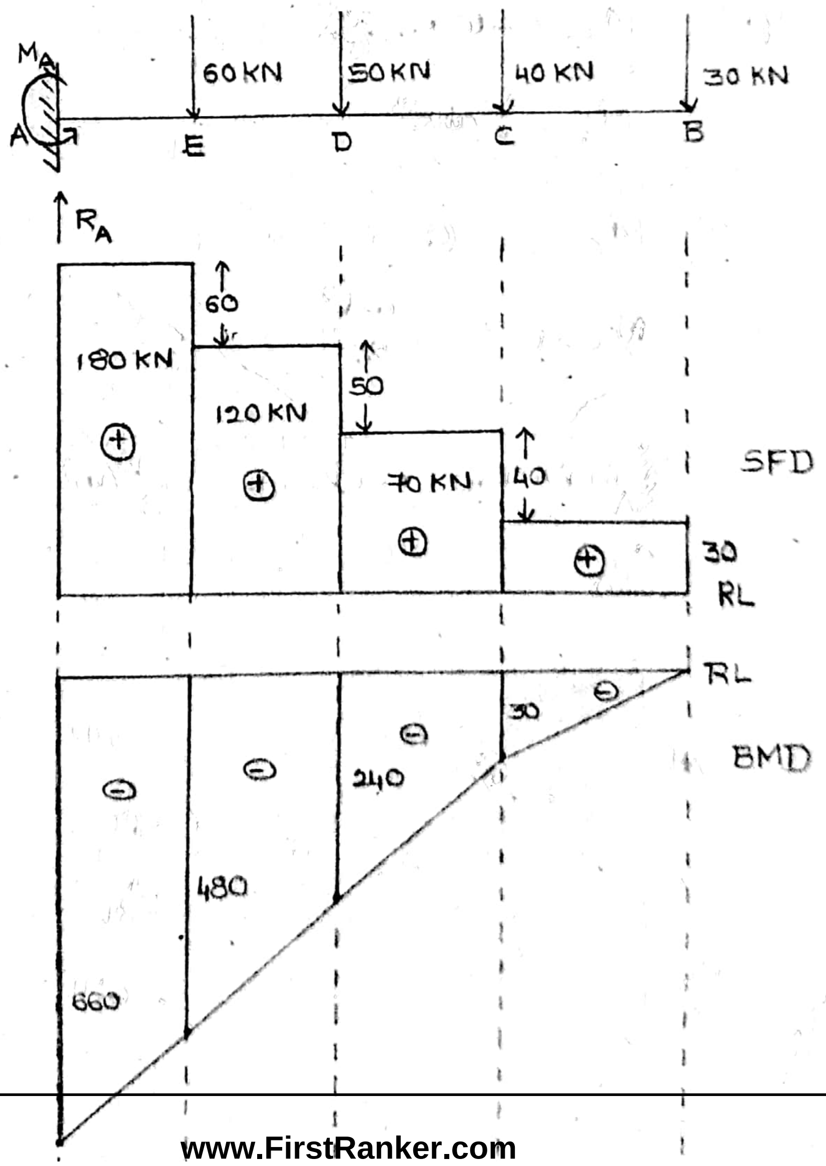
BM eqn: $M_x = -30x - 40(x-1) - 50(x-4) - 60(x-6)$ kN-m

$M_{x=6} = -30(6) - 40(6-1) - 50(6-4) - 60(6-6)$

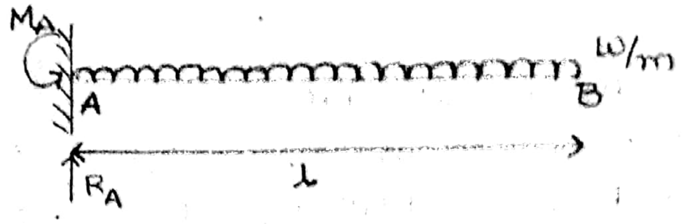
$= -480$ kN-m.

$M_{x=7} = -30(7) - 40(7-1) - 50(7-4) - 60(7-6)$

$= -660$ kN-m



Cantilever beam with UDL throughout the length:



Reaction: $\Sigma V = 0 \Rightarrow R_A - wl = 0$

$$\boxed{R_A = wl}$$

Portion AB: (0 to l)

SF eqn: $F_x = wx$

$F_{x=0} = w(0) = 0$

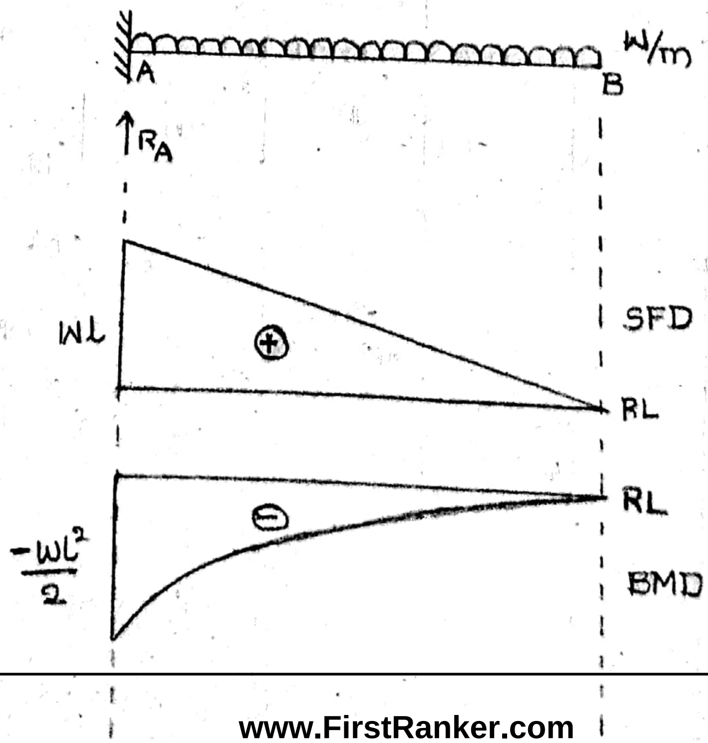
$F_{x=l} = w(l) = wl.$

BM eqn: $M_x = -wx \cdot \frac{x}{2}$

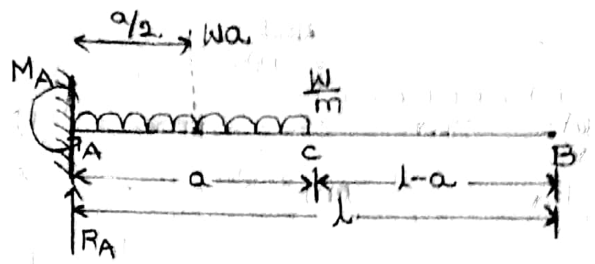
$M_x = -\frac{wx^2}{2}$

$M_{x=0} = 0$

$M_{x=l} = \frac{-wl^2}{2}.$



Cantilever beam with UDL at a distance (a) from fixed support:



Reaction: $\Sigma V = 0 \Rightarrow R_A - wa = 0$

$$\boxed{R_A = wa}$$

Portion BC: (0 to l-a)

SF eqn : $F_x = 0$

BM eqn : $M_x = 0$

Portion CA: (l-a to l).

SF eqn : $F_x = w(x-l+a)$

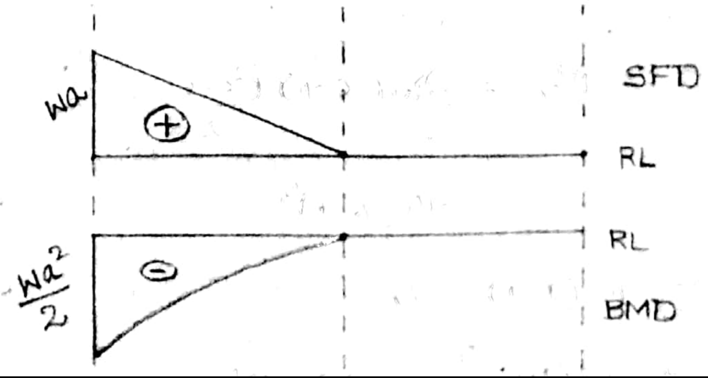
BM eqn : $M_x = -w(x-l+a)\dfrac{(x-l+a)}{2}$

$F_{x=l-a} = w(l-a-l+a) = 0$

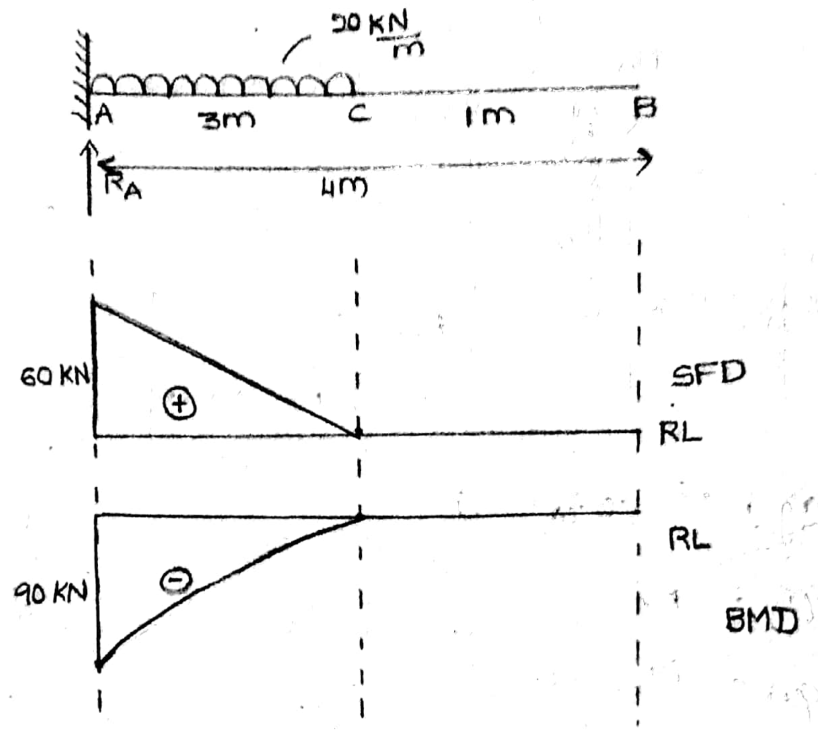
$F_{x=l} = w(l-l+a) = wa.$

$M_{x=l-a} = -w(l-a-l+a)\dfrac{(l-a-l+a)}{2} = 0$

$M_{x=l} = -w(l-l+a)\dfrac{(l-l+a)}{2} = -\dfrac{wa^2}{2}$



* Draw shear force and bending moment diagrams for cantilever beam as shown in fig.



<u>Reaction:</u> $\Sigma V = 0 \Rightarrow R_A - (20 \times 3) = 0$

$$\boxed{R_A = 60\ KN.}$$

<u>Portion BC:</u> (0 to 1m)

SF eqn : $F_x = 0$

BM eqn : $M_x = 0$

<u>Portion CA:</u> (1m to 4m)

SF eqn : $F_x = 20(x-1)$.

$F_{x=1} = 20(1-1) = 0$

$F_{x=4} = 20(4-1) = 60\ KN$

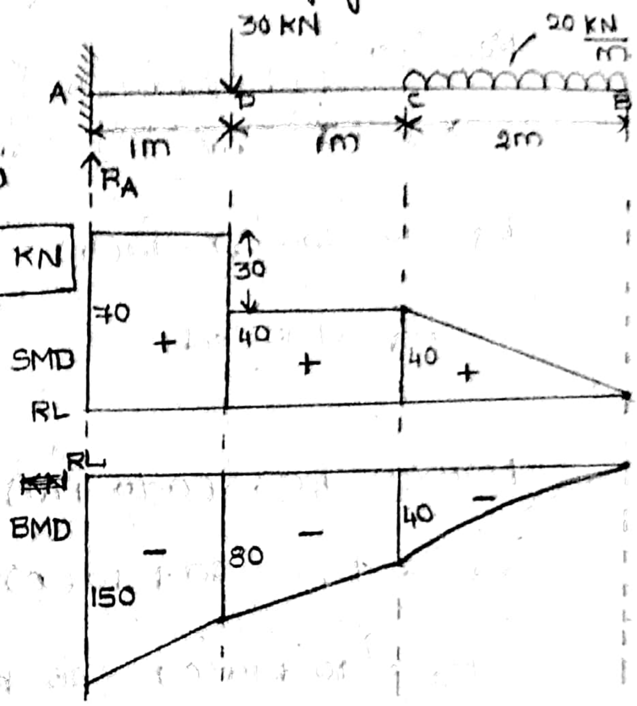
BM eqn : $M_x = -20(x-1)\dfrac{(x-1)}{2}$

$= -10(x-1)^2$.

$M_{x=1} = -10(1-1)^2 = 0$

$M_{x=4} = -10(4-1)^2 = -90\ KN$

* Draw shear force and bending moment diagrams for cantilever beam as shown in figure.



Reaction: $\Sigma V = 0$

$R_A - (20\times 2) - 30 = 0$

$\boxed{R_A = 70\ KN}$

Portion BC: (0 – 2m)

SF. eqn : $F_x = 20 \times x$ ~~= 40 KN~~

BM eqn : $M_x = -20 \cdot x \cdot \frac{x}{2}$

$= -10x^2$

$M_{x=0} = -10(0) = 0$ $F_{x=0} = 20(0) = 0$

$M_{x=2} = -10(2)^2 = -40\ KN$ $F_{x=2} = 20(2) = 40\ KN.$

Portion CD: (2 to 3m)

SF. eqn: $F_x = 20(2) = 40\ KN.$

BM eqn : $M_x = -40(x-1)$

$M_{x=2} = -40(2-1) = -40\ KN$

$M_{x=3} = -40(3-1) = -80\ KN.$

Portion DA: (3 to 4m)

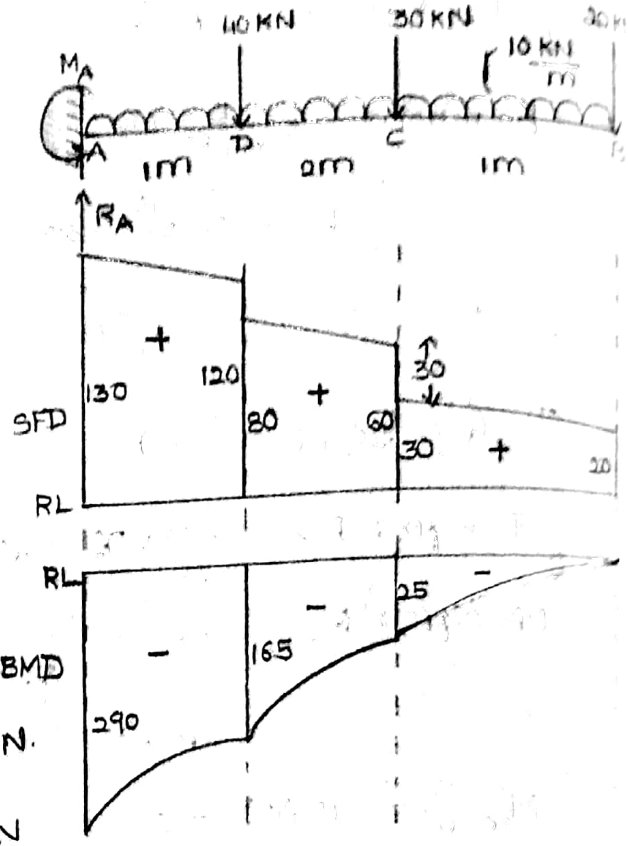
SF. eqn : $F_x = (20\times 2) + 30 = 70\ KN.$

BM. eqn : $M_x = \cancel{-70}\ -30(x-3) - 40(x-1)$

$M_{x=3} = -30(3-3) - 40(3-1) = -80\ KN$

$M_{x=4} = -30(4-3) - 40(4-1) = -150\ KN$

Draw S.F and B.M diagrams for cantilever beam as show in fig.



Reaction:

$\Sigma V = 0$

$R_A - 40 - 30 - 20 - 10(4) = 0$

$R_A = 130$ KN

Portion BC: (0 to 1m)

S.F: $F_x = 20 + 10(x)$

$F_0 = 20 + 10(0) = 20$ KN.

$F_1 = 20 + 10(1) = 30$ KN

BM: $M_x = -20x - 10x \cdot \frac{x}{2} = -20x - 5x^2$

$M_0 = 0$

$M_1 = -20(1) - 5(1)^2 = -25$ KN/m.

Portion CD: (1 to 3m)

SF: $F_x = 30 + 20 + 10x$

$F_1 = 30 + 20 + 10(1) = 60$ KN

$F_3 = 30 + 20 + 10(3) = 80$ KN

BM: $M_x = -20x + 30(x-1) + 10 \cdot x \cdot \frac{x}{2}$

$= -20x - 30(x-1) - 5x^2$

$M_1 = -20(1) - 30(1-1) - 5(1)^2 = -25$ KN/m

$M_{x=3} = -20(3) - 30(3-1) - 5(3)^2 = -165$ KN/m

Portion DA: (3 to 4 m).

SF: $F_x = 20 + 30 + 40 + 10x = 90 + 10x$

$F_{x=3} = 90 + 10(3) = 120$ KN

$F_{x=4} = 90 + 10(4) = 130$ KN

BM: $M_x = -20x - 30(x-1) - 40(x-3) - 10x \cdot \frac{x}{2}$

$M_{x=3} = -20(3) - 30(3-1) - 40(3-3) - 10 \cdot 3 \cdot \frac{3}{2}$

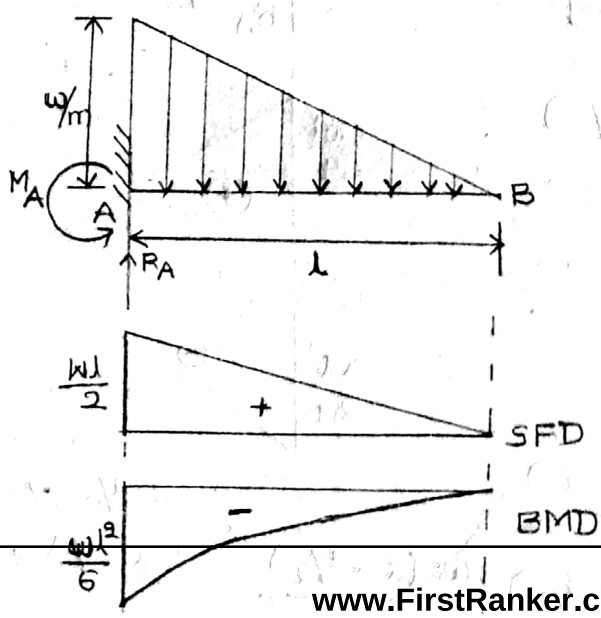
$= -165$ KN/m

$M_{x=4} = -20(4) - 30(3) - 40(1) - 10 \times \frac{4^2}{2}$

$= -290$ KN/m.

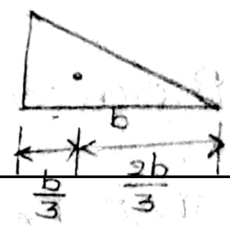
3.17 Cantilever beam with UVL:

Consider a cantilever beam of span 'l' having a uniformly varying load. 0 at the free end and maximum ordinate w/m at the fixed end as shown in figure.



Area $= \frac{1}{2} \times l \times w$

$= \frac{wl}{2}$



Reaction: $\Sigma V = 0$

$\Rightarrow R_A - \frac{\omega l}{2} = 0$

$$\boxed{R_A = \frac{\omega l}{2}}$$

Area $= \frac{1}{2} bh$

$= \frac{1}{2} \omega l$

$l \rightarrow \omega$

$x \rightarrow ?$

Area $= \frac{1}{2} x \cdot \frac{\omega x}{l}$

$= \frac{\omega x^2}{2l}$

Portion BA: (0 to L)

Shear force, $F_x = \frac{\omega x^2}{2l}$

$F_{x=0} = 0$

$F_{x=l} = \frac{\omega l^2}{2l} = \frac{\omega l}{2}$

Bending moment, $M_x = -\frac{\omega x^2}{2l} \times \frac{x}{3}$

$M_{x=0} = 0$

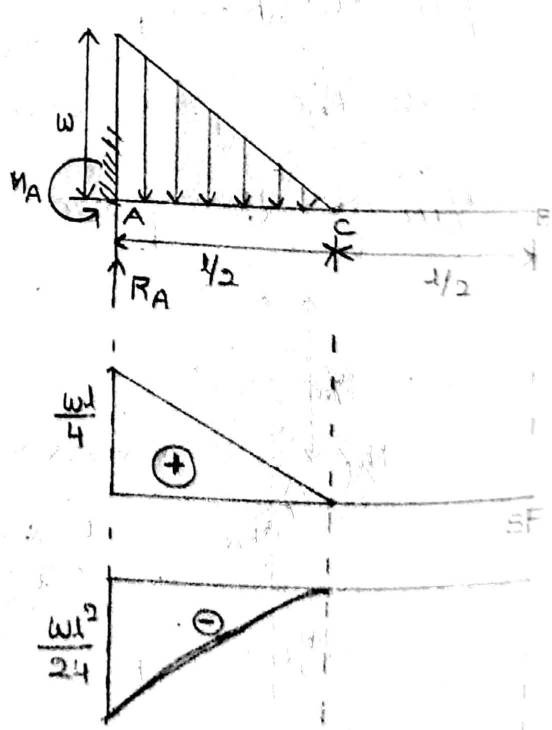
$M_{x=l} = -\frac{\omega l^2}{2l} \times \frac{l}{3} = \frac{-\omega l^2}{6}$

Cantilever beam with half UVL:

Reaction: $\Sigma V = 0$

$R_A - \frac{1}{2}\omega \cdot \frac{l}{2} = 0$

$$\boxed{R_A = \frac{\omega l}{4}}$$



Portion BC: (0 to $l/2$)

SF: $F_x = 0$

BM: $M_x = 0$

Portion CA: ($l/2$ to L)

$l/2 \rightarrow \omega$

$(x - l/2) \rightarrow ?$ $\frac{\omega(x-}{l/2}$

SF: $F_x = \frac{1}{2}(x - l/2) \frac{\omega(x - l/2)}{l/2}$

$F_{x=l/2} = 0$

$$F_{x=l} = \frac{1}{2} \times \left(\frac{l}{2}\right) \frac{\omega \left(\frac{l}{2}\right)}{\left(\frac{l}{2}\right)} = \frac{\omega l}{4}$$

BM: $$M_x = -\frac{1}{2}(x - l/2)\,\frac{\omega(x - l/2)}{l/2} \times \frac{(x - l/2)}{3}$$

$$M_{x=l/2} = 0$$

$$M_{x=l} = -\frac{1}{2} \times \frac{l}{2} \times \frac{\omega \cdot l/2}{l/2} \times \frac{l/2}{3}$$

$$= -\frac{\omega l^2}{24}$$

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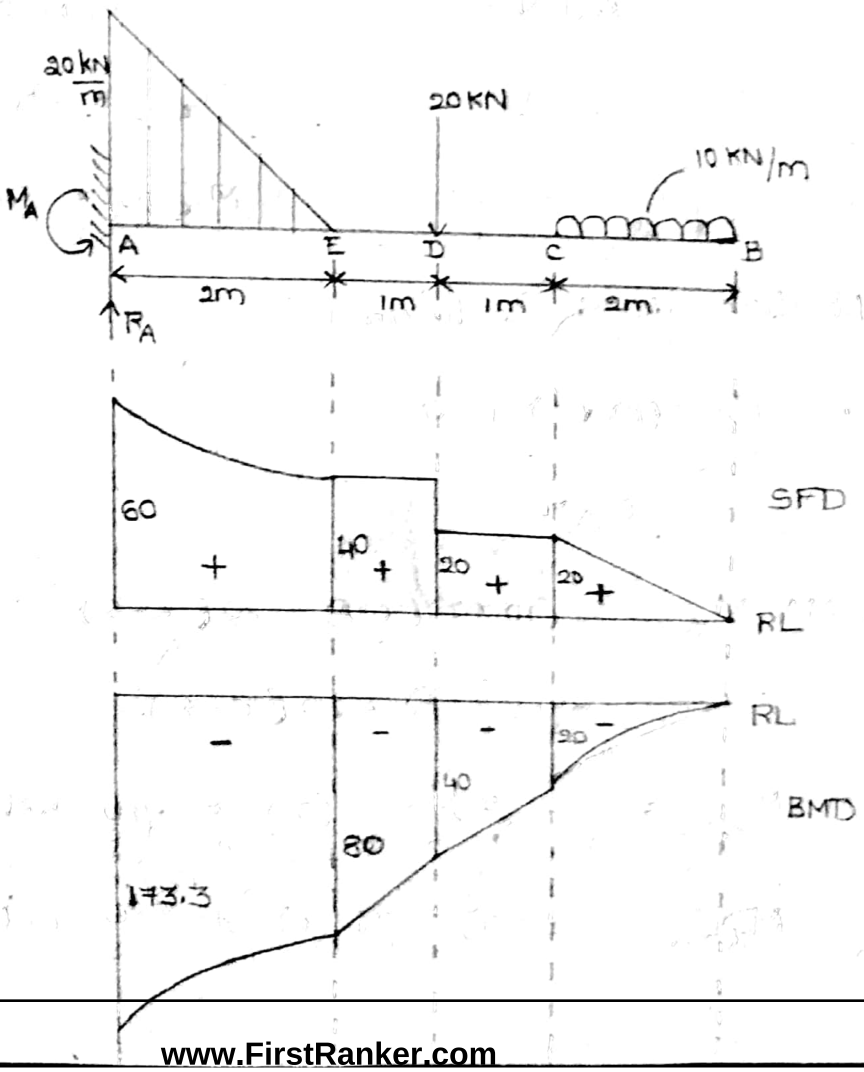
* Draw SFD and BMD for the following cantilever beam.



Reactions: $\Sigma V = 0$

$$R_A - 20 - (10 \times 2) - \left(\frac{1}{2} \times 2 \times 20\right) = 0$$

$$R_A = 60 \text{ KN}$$

Portion BC : (0 to 2m)

SF: $F_x = 10x.$

$F_{x=0} = 0$

$F_{x=2} = 10(2) = 20 \text{ KN}$

BM: $M_x = -10x \cdot \frac{x}{2}$

$= -5x^2$

$M_{x=0} = 0$

$M_{x=2} = -20 \text{ KN-m}$

Portion CD: (2 to 3m)

SF: $F_x = 10(2)$

$= 20 \text{ KN}.$

BM: $M_x = -10 \times 2 \times (x-1)$

$= -20(x-1) \text{ KN-m}$

$M_{x=2} = -20 \text{ KN-m}$

$M_{x=3} = -40 \text{ KN-m}.$

Portion DE: (3 to 4m)

SF: $F_x = (10 \times 2) + 20$

$= 40 \text{ KN}$

BM: $M_x = -(10 \times 2)(x-1) - 20(x-3)$

$= -20(x-1) - 20(x-3)$

$M_{x=3} = -20(2) - 20(0) = -40 \text{ KN-m}$

$M_{x=4} = -20(3) - 20(1) = -80 \text{ KN-m}$

Portion EA: (4 to 6 m)

SF: $F_x = (10\times2) + 20 + \frac{1}{2}(x-4)\frac{(x-4)20}{2}$

$= 40 + 5(x-4)^2$

$F_{x=4} = 40 + 5(0) = 40$ KN.

$F_{x=6} = 40 + 5(4) = 60$ KN

BM: $M_x = -(10\times2)(x-1) - 20(x-3) - \cancel{5(x}$

$\frac{1}{2}(x-4)\frac{20(x-4)}{2}\cdot\frac{(x-4)}{3}$

$= -20(x-1) - 20(x-3) - \frac{5(x-4)^3}{3}$

$M_{x=4} = -20(3) - 20(1) - \frac{5(0)^3}{3} = -80$ KN-m.

$M_{x=6} = -20(5) - 20(3) - \frac{5(2)^3}{3} = -173.3$ KN-m

* Draw SFD and BMD for the following cantilever beam.

Reaction: $\Sigma V = 0$

$R_A - 20 - (20\times2) = 0$

$R_A = 40$ KN

Portion BC: (0 to 2m)

SF: $F_x = 0$

BM: $M_x = -30$ KN-m

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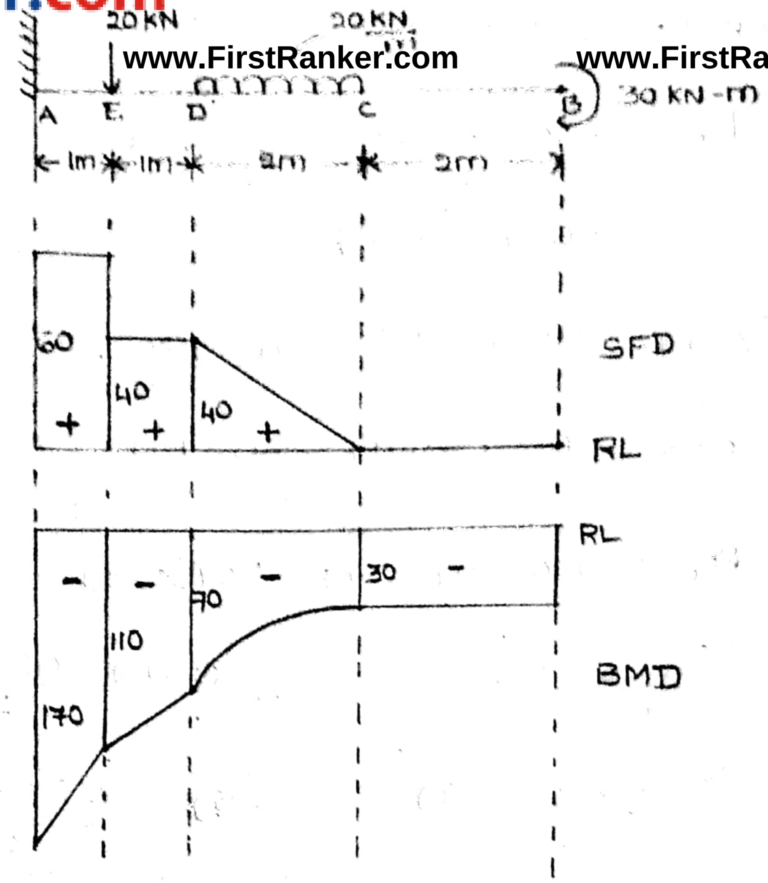


Portion CB: (2 to 4 m)

SF: $F_x = 20(x-2)$

$F_{x=2} = 20(0) = 0$

$F_{x=4} = 20(2) = 40$ KN

BM: $M_x = -30 - 20(x-2)\dfrac{(x-2)}{2}$

$= -30 - 10(x-2)^2$

$M_{x=2} = -30 - 10(0) = -30$ KN-m

$M_{x=4} = -30 - 10(2)^2 = -70$ KN-m

Portion DE: (4 to 5 m)

SF: $F_x = 20 \times 2$

$= 40$ KN

BM: $M_x = -30 - (20 \times 2)(x-3)$

$= -30 - 40(x-3)$

$M_{x=4} = -30 - 40(1) = -70$ KN-m

$M_{x=5} = -30 - 40(2) = -110$ KN-m

Portion EA: (5 to 6m)

SF: $F_x = (20 \times 2) + 20$

$F_x = 60$ KN

BM: $M_x = -30 - (20 \times 2)(x-3) - 20(x-5)$

$= -30 - 40(x-3) - 20(x-5)$

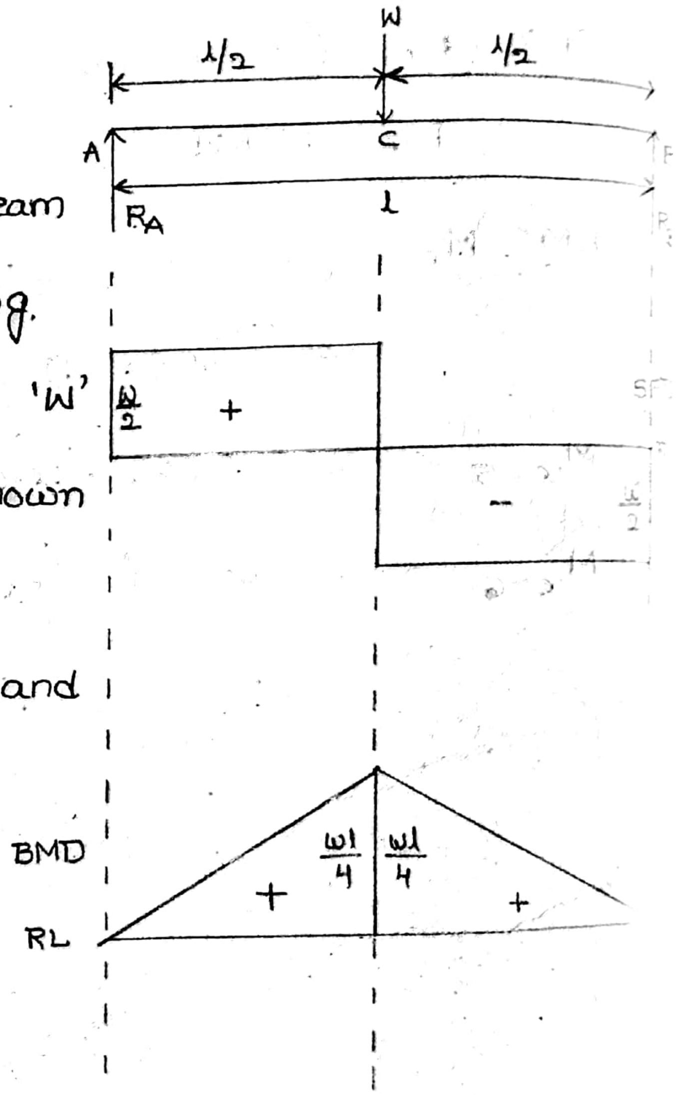
$M_{x=5} = -30 - 40(2) - 20(0) = -110$ KN-m.

$M_{x=6} = -30 - 40(3) - 20(1) = -170$ KN-m

Simply supported beam:

The beam which supports freely is called Simply supported beam.

Simply supported beam with central point load:



Consider a Simply supported beam of span 'L' carrying a concentrated load 'W' at the centre as shown in fig.

Let reactions at A and B be R_A and R_B respectively.

Reaction:

$\Sigma V = 0$

$\Rightarrow R_A + R_B = W$

$\Sigma M_A = 0$

$\Rightarrow R_B \times L - \omega \times \frac{L}{2} = 0$

$\boxed{R_B = \frac{\omega}{2}}$

$R_A = \omega - \frac{\omega}{2}$

$\boxed{R_A = \frac{\omega}{2}}$

18. Portion AC (0 to $l/2$):

SF: $F_x = R_A$

$= \frac{W}{2}$

BM: $M_x = \frac{W}{2} x$

$M_{x=0} = 0$

$M_{x=l/2} = \frac{W}{2} \times \frac{l}{2}$

$= \frac{Wl}{4}$

Portion BC (0 to $l/2$):

SF: $F_x = R_B$

$= \frac{-W}{2}$

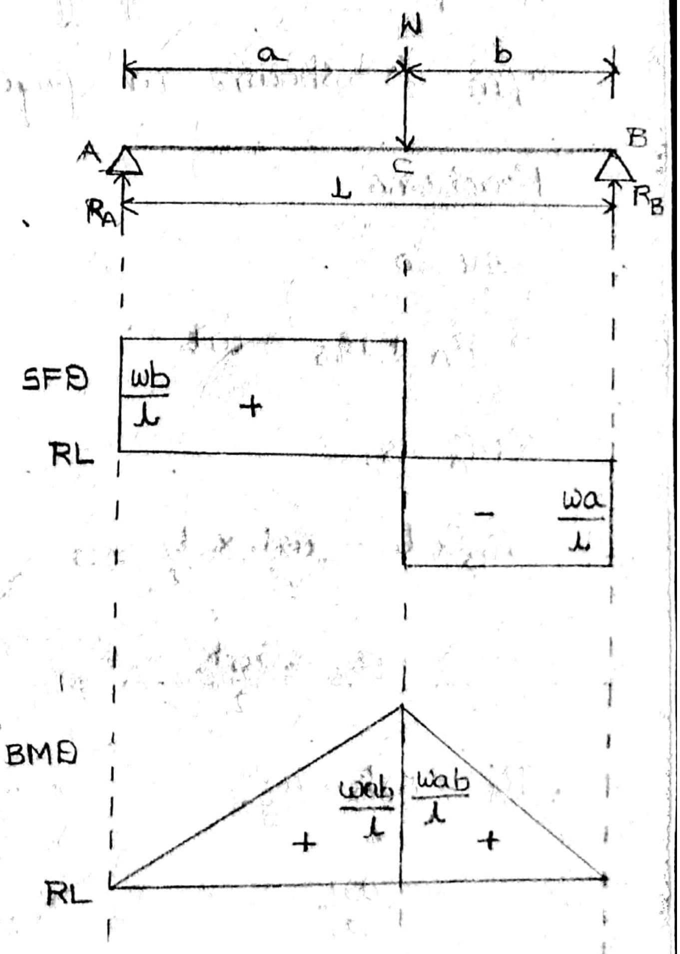
BM: $M_x = \frac{W}{2} x$

$M_{x=0} = 0$

$M_{x=l/2} = \frac{W}{2} \times \frac{l}{2}$

$= \frac{Wl}{4}$

Simply supported beam with unsymmetrical point load:



Reactions: $\Sigma V = 0$

$R_A + R_B = W$

$\Sigma M_A = 0$

$\Rightarrow R_B \times l - W \times a = 0$

$R_B = \frac{Wa}{l}$

$R_A = W - \frac{Wa}{l}$

$= \frac{Wl - Wa}{l}$

$= \frac{W(l-a)}{l}$

$R_A = \frac{Wb}{l}$

Portion AC : (0 to a)

SF: $F_x = R_A = \frac{wb}{L}$

BM: $M_x = R_A x$

$M_{x=0} = 0$

$M_{x=a} = \frac{wb}{L} \times a$

$= \frac{wab}{L}$

Portion BC (0 to b)

SF: $F_x = -R_B = \frac{-wa}{L}$

BM: $M_x = R_B x = \frac{wax}{L}$

$M_{x=0} = 0$

$M_{x=b} = \frac{wab}{L}$

Simply supported beam with UDL:

Consider a simply supported beam of span 'L' carrying a UDL of intensity w/m as shown in figure.

Reactions:

$\Sigma V = 0$

$\Rightarrow R_A + R_B = wL$

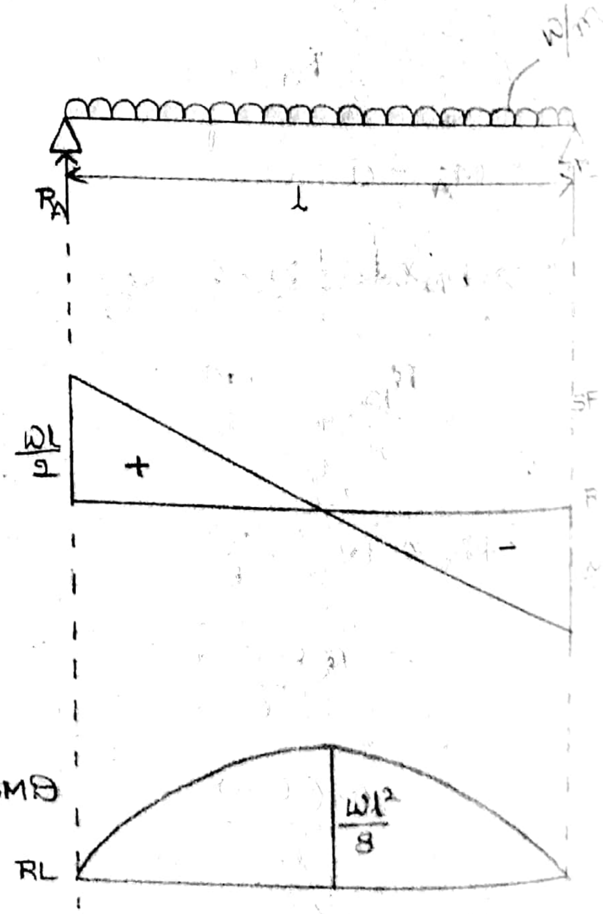
$\Sigma M_A = 0$

$R_B \times L - wL \times \frac{L}{2} = 0$

$R_B = \frac{wL}{2}$

$R_A = wL - \frac{wL}{2}$

$= \frac{wL}{2}$



Portion AB (0 to l)

SF: $F_x = R_A - wx$

$= \frac{wl}{2} - wx$

$F_{x=0} = \frac{wl}{2}$

$F_{x=l} = \frac{wl}{2} - wl$

$= \frac{-wl}{2}$

BM: $M_x = R_A x - wx \cdot \frac{x}{2}$

$= \frac{wl}{2} x - \frac{wx^2}{2}$

$M_{x=0} = 0$

$M_{x=l} = \frac{wl^2}{2} - \frac{wl^2}{2}$

$= 0$

$M_{x=l/2} = \frac{wl^2}{4} - \frac{wl^2}{8}$

$= \frac{wl^2}{8}$

Simply supported beam with half UDL:

Reactions:

$\Sigma V = 0$

$R_A + R_B = \frac{wl}{2}$

$\Sigma M_A = 0$

$R_B \times l - \frac{wl}{2} \times \frac{l/2}{2} = 0$

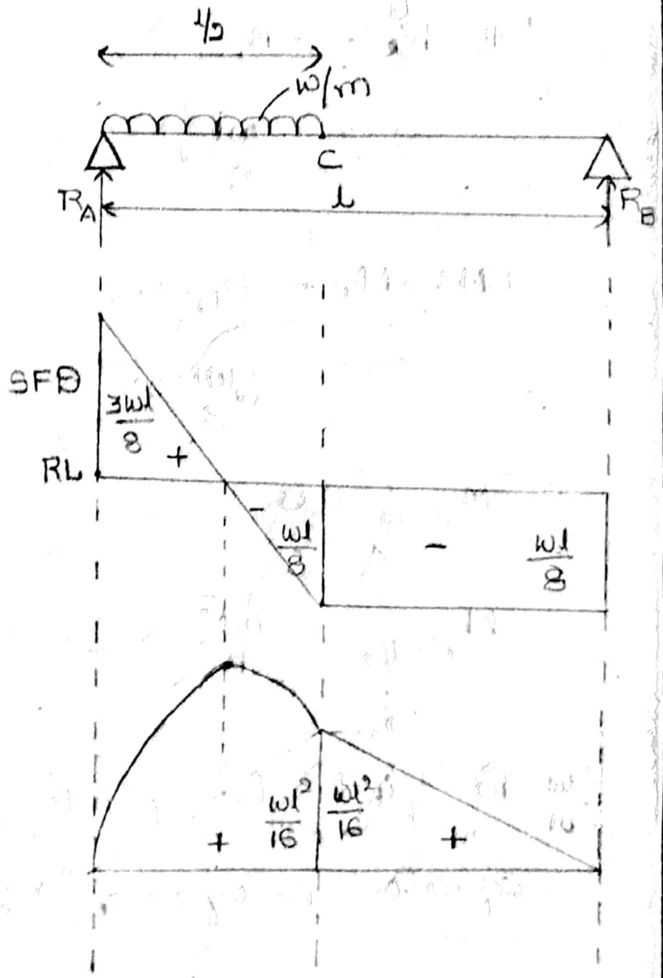
$R_B = w \times \frac{l}{2} \times \frac{l}{4}$

$R_B = \frac{wl}{8}$

$R_A = \frac{wl}{2} - \frac{wl}{8}$

$= \frac{4wl - wl}{8}$

$R_A = \frac{3wl}{8}$



Portion AC : (0 to l/2)

SF: $F_x = R_A - wx$

$= \frac{3wl}{8} - wx$

$F_{x=0} = \frac{3wl}{8}$

$F_{x=l/2} = \frac{3wl}{8} - \frac{wl}{2}$

$= \frac{3wl - 4wl}{8}$

$= \frac{-wl}{8}$

BM:

$M_x = R_A \times x - wx \cdot \frac{x}{2}$

$= \frac{3wxl}{8} - \frac{wx^2}{2}$

$M_{x=0} = \frac{3wl^2}{8} = 0$

$M_{x=l/2} = \frac{3wl^2}{8\times 2} - \frac{wl^2}{8}$

$= \frac{3wl^2 - 2wl^2}{16}$

$= \frac{wl^2}{16}$

Portion BC : (0 to l/2)

SF: $F_x = -R_B$

$= -\frac{wl}{8}$

BM: $M_x = R_B \cdot x$

$= \frac{wl}{8} x$

$M_{x=0} = 0$

$M_{x=l/2} = \frac{wl^2}{16}$

To find the point of zero shear force. equate F_x eqn in AC portion to 0.

$\therefore F_x = \frac{3wl}{8} - wx = 0$

$$\frac{3l}{8} - x = 0$$

$$x = \frac{3l}{8}$$

$$M_{x=\frac{3l}{8}} = \frac{3\omega \cdot \left(\frac{3l}{8}\right) l}{8} - \frac{\omega\left(\frac{3l}{8}\right)^2}{2}$$

$$= \frac{9\omega l^2}{64} - \frac{9\omega l^2}{128}$$

$$= \frac{18\omega l^2 - 9\omega l^2}{128}$$

$$= \frac{9\omega l^2}{128}$$

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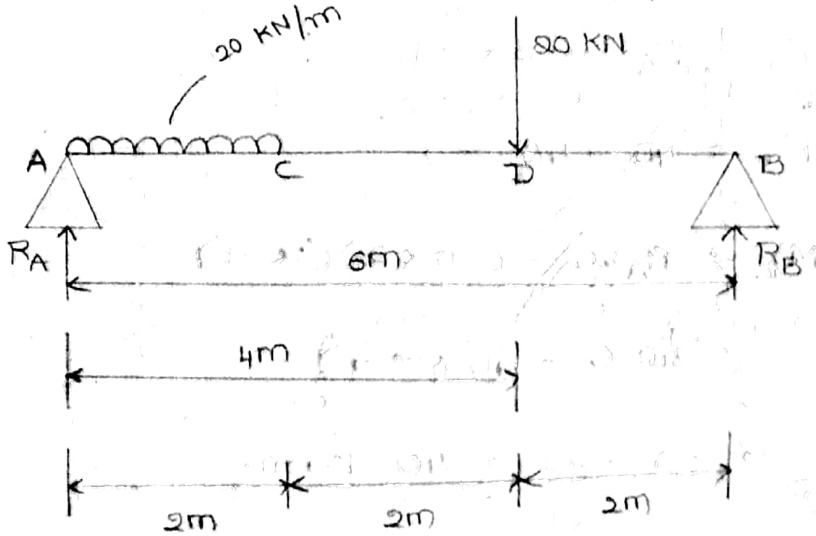
* A simply supported beam of span 6m carrying. A load of 20 KN at a distance 4m from left support and also a UDL of 20 KN/m over in a distance of 2m from left support. Draw shear force and bending moment diagrams.



Reactions:

$\Sigma V = 0$

$\Rightarrow R_A + R_B - 20 - (20 \times 2) = 0$

$R_A + R_B = 60 \text{ KN}$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 6 - 20 \times 4 - 20 \times 2 \times 1 = 0$

$6R_B = 120$

$R_B = 20 \text{ KN}$

$R_A = 60 - 20 = 40 \text{ KN}$

__Portion AC:__ (0 to 2m)

$F_x = R_A - 20x$ $\quad$ $M_x = R_A \cdot x - 20 \cdot x \cdot \frac{x}{2}$

$= 40 - 20x$ $\quad$ $= 40x - 10x^2$

$F_{x=0} = 40$ $\quad$ $M_{x=0} = 0$

$F_{x=2} = 0$ $\quad$ $M_{x=2} = 80 - 40 = 40$

__Portion CD:__ (2 to 4m)

__SF:__ $F_x = R_A - (20 \times 2)$

$F_x = 40 - 40 = 0$

__BM:__ $M_x = R_A x - (20 \times 2)(x-1)$

$= 40x - 40(x-1)$

$M_{x=2} = 80 - 40 = 40 \text{ KN-m}$

$M_{x=4} = 160 - 120 = 40 \text{ KN-m}$

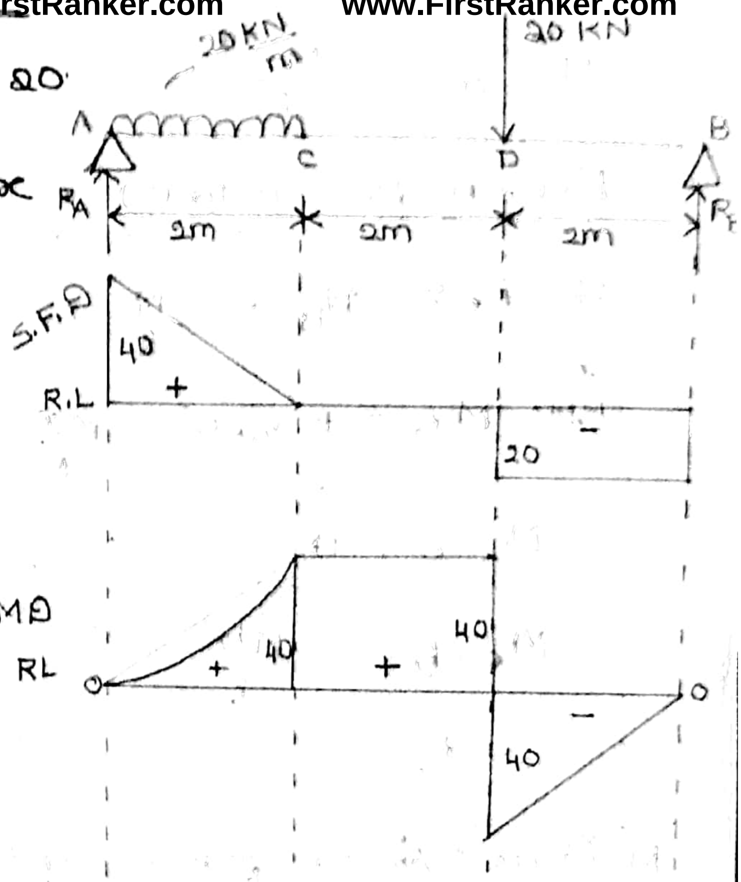
Portion BD: (0 to 2m)

SF: $F_x = -R_B = -20$

$M_x = -R_B \cdot x = -20x$

$M_{x=0} = 0$

$M_{x=2} = -40$ KN-m



* Simply supported beam with couple (γ):

Reactions:

$\Sigma V = 0$

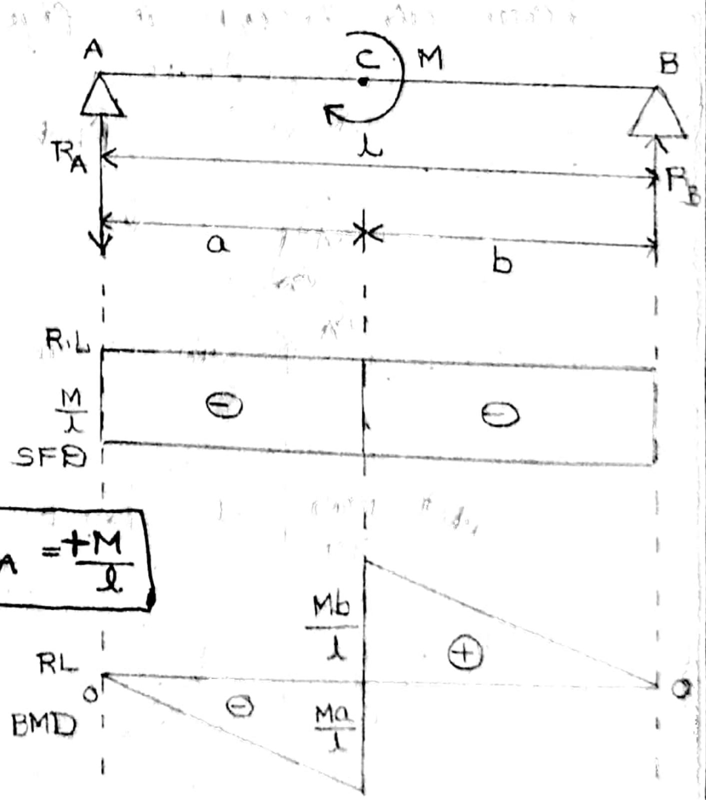
$\Rightarrow R_A + R_B = 0$

$\Sigma M_A = 0$

$\Rightarrow R_B \times l + M = 0$

$$\boxed{R_B = \frac{+M}{l}}$$

$R_A + \frac{M}{l} = 0 \Rightarrow \boxed{R_A = \frac{+M}{l}}$



Portion AC: (0 to a)

SF: $F_x = -R_A = \frac{-M}{l}$

BM: $M_x = -R_A \cdot x = \frac{-Mx}{l}$

$M_{x=0} = 0$

$$M_{x=a} = \frac{-Ma}{l}$$

Portion BC : (0 to b)

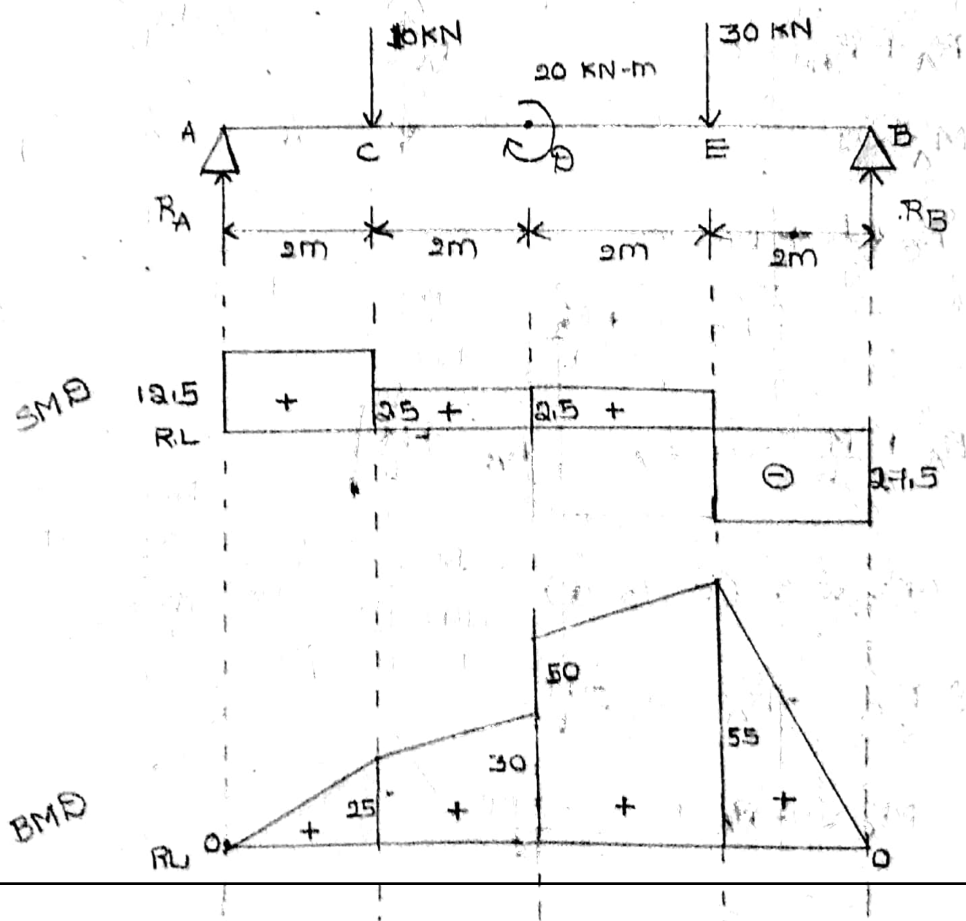
SF: $F_x = -R_B = \frac{-M}{l}$

BM: $M_x = +R_B \cdot x = \frac{Mx}{l}$

$$M_{x=0} = 0$$

$$M_{x=b} = \frac{Mb}{l}$$

* Draw shear force and bending moment diagrams for the following simple supported beam as shown in figure.



3) Reactions:

$\Sigma V = 0$

$\Rightarrow R_A + R_B - 10 - 30 = 0$

$R_A + R_B = 40$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 8 - 30 \times 6 - 20 - 10 \times 2 = 0$

$8R_B = 180 + 20 + 20$

$R_B = 27.5$ $\quad R_A = 12.5$

$R_A = 40 - 27.5 = 12.5$

Portion AC (0 to 2)

SF: $F_x = R_A = 12.5$ KN

BM: $M_x = R_A \cdot x = 12.5x$

$M_{x=0} = 0$ $\quad M_{x=2} = 12.5(2) = 25$

Portion CD (2 to 4m)

SF: $F_x = R_A - 10 = 12.5 - 10 = 2.5$ KN

BM: $M_x = R_A \cdot x - 10(x-2) = 12.5x - 10(x-2)$

$M_{x=2} = 12.5(2) - 10(2-2) = 25$ KN-m

$M_{x=4} = 12.5(4) - 10(4-2) = 30$ KN-m

Portion BE (0 to 2m)

SF: $F_x = -R_B = -27.5$

BM: $M_x = R_B \cdot x = 27.5x$

$M_{x=0} = 0$

$M_{x=2} = 27.5(2) = 55 \text{ KN-m}$

Portion EB (2 to 4m)

SF: $F_x = -R_B + 30 = -27.5 + 30 = 2.5 \text{ KN}$

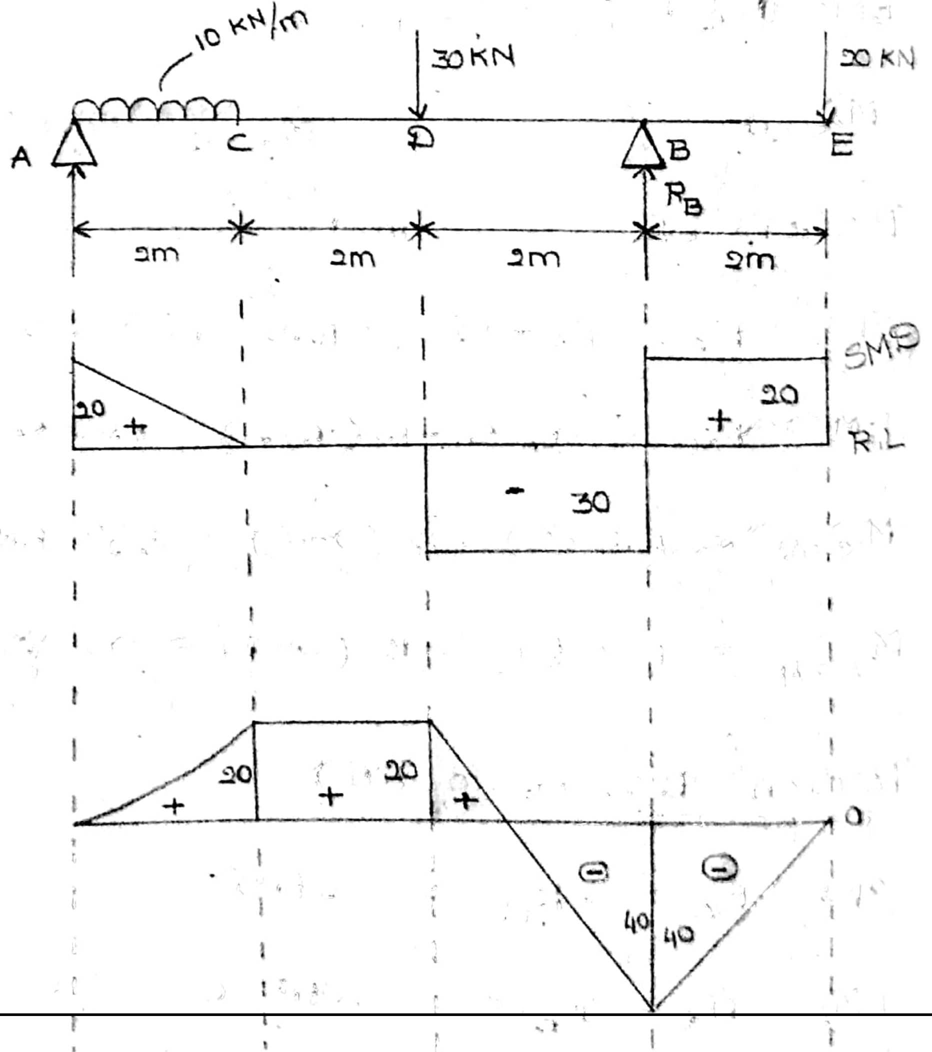
BM: $M_x = R_B \cdot x - 30(x-2)$

$= 27.5x - 30(x-2)$

$M_{x=2} = 27.5(2) - 30(0) = 55 \text{ KN-m}$

$M_{x=4} = 27.5(4) - 30(4-2) = 50 \text{ KN-m}$

* Draw shear force and bending moment diagram for the following figure.



Reactions:

$\Sigma V = 0$

$\Rightarrow R_A + R_B - 30 - 20 - (10 \times 2) = 0$

$R_A + R_B = 70$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 6 - (20 \times 8) - (30 \times 4) - (10 \times 2) \times 1 = 0$

$6 R_B = 300$

$R_B = 50$ KN

$R_A = 70 - 50 = 20$ KN.

Portion AC: (0 to 2m)

SF: $F_x = R_A - 10x = 20 - 10x$

$F_{x=0} = 20$ KN.

$F_{x=2} = 20 - 10(2) = 0$

BM: $M_x = R_A \cdot x - 10x \cdot \frac{x}{2} = 20x - 5x^2$

$M_{x=0} = 0$

$M_{x=2} = 20(2) - 5(2)^2 = 20$ KN-m

Portion CB: (2 to 4m)

SF: $F_x = R_A - (10 \times 2) = 20 - 20 = 0$

BM: $M_x = R_A \cdot x - (10 \times 2) \times (x-1) = 20x - 20(x-1)$

$M_{x=2} = 20(2) - 20(2-1) = 20$ KN-m

$M_{x=4} = 20(4) - 20(4-1) = 20$ KN-m

Portion BE: (0 to 2m)

SF: $F_x = 20$ KN

BM: $M_x = -20x$

$M_{x=0} = 0$

$M_{x=2} = -40$ KN-m

Portion BD: (2 to 4m)

SF: $F_x = 20 - R_B = 20 - 50 = -30$ KN

BM: $M_x = -20x + R_B(x-2)$

$= -20x + 50(x-2)$

$M_{x=2} = -20(2) + 50(2-2) = -40$ KN-m

$M_{x=4} = -20(4) + 50(4-2) = 20$ KN-m

Note:

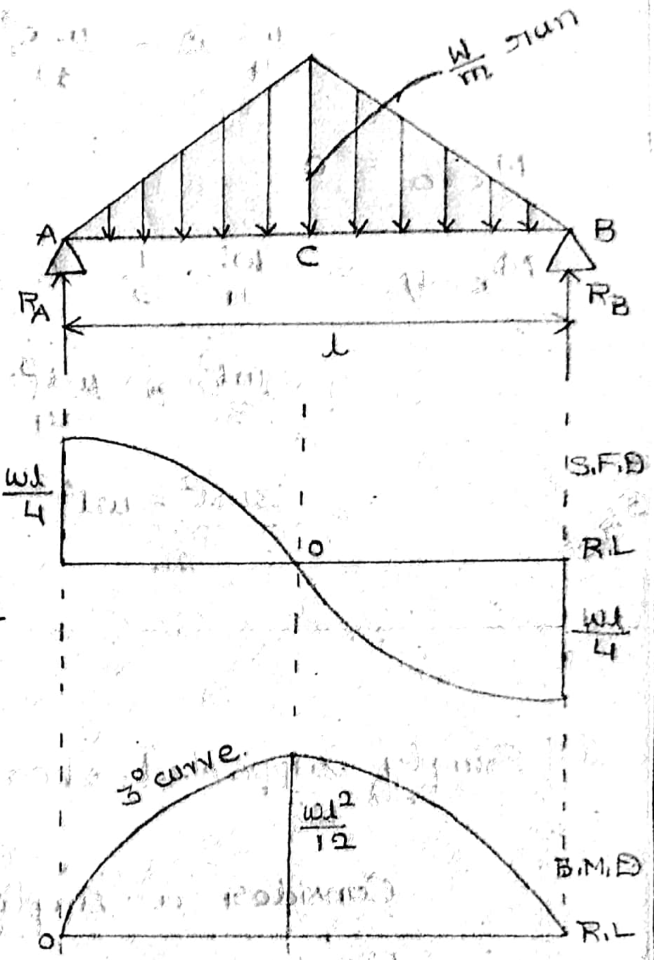
Point of contraflexure:

The point where the bending moment changes its sign from positive to negative or negative to positive is called as a point of contraflexure.

Simply supported beam with triangular or

ⓐ UVL load:

Consider a simply supported beam of span 'L' carrying a UVL having zero intensity at the supports and w/m run at the centre as shown in figure.



Reactions:

$\Sigma V = 0$

$R_A + R_B = \frac{1}{2} \times w \times l$

$R_A + R_B = \frac{wl}{2}$

$\Sigma M_A = 0 \Rightarrow -R_B \times l + \left(\frac{1}{2} \times w \times l\right)\frac{l}{2} = 0$

$R_B = \frac{wl}{4}$

$R_A = \frac{wl}{2} - \frac{wl}{4} = \frac{2wl - wl}{4} = \frac{wl}{4}$

$R_A = \frac{wl}{4}$

Portion AC $(0 \text{ to } l/2)$

$l/2 \rightarrow w$

$x \rightarrow ?$

SF: $F_x = R_A - \left(\frac{1}{2} \times x \times \frac{2wx}{l}\right)$

$F_x = \frac{wl}{4} - \frac{wx^2}{l}$

$F_{x=0} = \frac{wl}{4}$ $F_{x=l/2} = \frac{wl}{4} - \frac{wl^2}{4l} = 0$

BM: $M_x = R_A \cdot x - \left(\frac{1}{2} \times x \times \frac{2wx}{l}\right)\frac{x}{3}$

$$= \frac{wl}{4}x - \frac{wx^3}{3l}$$

$$M_{x=0} = 0$$

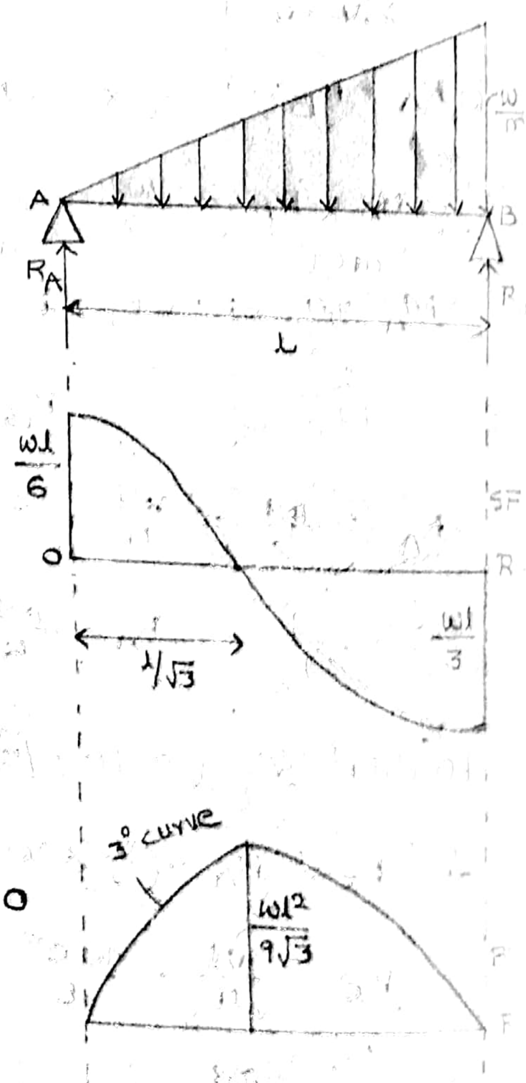
$$M_{x=l/2} = \frac{wl}{4} \cdot \frac{l}{2} - \frac{w}{3l} \cdot \frac{l^3}{8}$$

$$= \frac{wl^2}{8} - \frac{wl^2}{24}$$

$$= \frac{3wl^2 - wl^2}{24} = \frac{2wl^2}{24} = \frac{wl^2}{12}$$

ⓑ **Simply supported beam with UVL:**

Consider a simply supported beam of span 'l' carrying a UVL as shown in figure.



Reactions:

$$\Sigma V = 0$$

$$R_A + R_B = \frac{1}{2} \times w \times l$$

$$R_A + R_B = \frac{wl}{2}$$

$$\Sigma M_A = 0$$

$$\Rightarrow R_B \times l - \left(\frac{1}{2} \times w \times l\right)\frac{2l}{3} = 0$$

$$\Rightarrow R_B = \frac{wl}{3}$$

$$R_A = \frac{wl}{2} - \frac{wl}{3} = \frac{3wl - 2wl}{6} = \frac{wl}{6}$$

$$R_A = \frac{wl}{6}$$

Portion AB (0 to l)

SF: $F_x = R_B - \left(\frac{1}{2} \times x \times \frac{wx}{l}\right)$

$$= \frac{wl}{6} - \frac{wx^2}{2l}$$

$$F_{x=0} = \frac{wl}{6}$$

$$F_{x=l} = \frac{wl}{6} - \frac{wl^2}{2l} = \frac{wl}{6} - \frac{wl}{2} = \frac{wl - 3wl}{6}$$

$$= \frac{-2wl}{6}$$

$$= \frac{-wl}{3}$$

To find point of zero S.F.

Equate F_x eqn to zero.

$$\frac{wl}{6} - \frac{wx^2}{2l} = 0$$

$$\frac{wx^2}{2l} = \frac{wl}{6}$$

$$x^2 = \frac{l^2}{3} \Rightarrow x = \frac{l}{\sqrt{3}}$$

BM: $M_x = R_A . x - \left(\frac{1}{2} \times \frac{wx}{l} \times x\right) \frac{x}{3}$

$$= \frac{wl}{6} x - \frac{wx^3}{6l}$$

$$M_{x=0} = 0$$

$$M_{x=l} = \frac{wl^2}{6} - \frac{wl^3}{6l} = \frac{wl^2}{6} - \frac{wl^2}{6} = 0$$

$$M_{x=\frac{l}{\sqrt{3}}} = \frac{wl}{6}\cdot\frac{l}{\sqrt{3}} - \frac{w}{6l}\cdot\frac{l^3}{(\sqrt{3})^3}$$

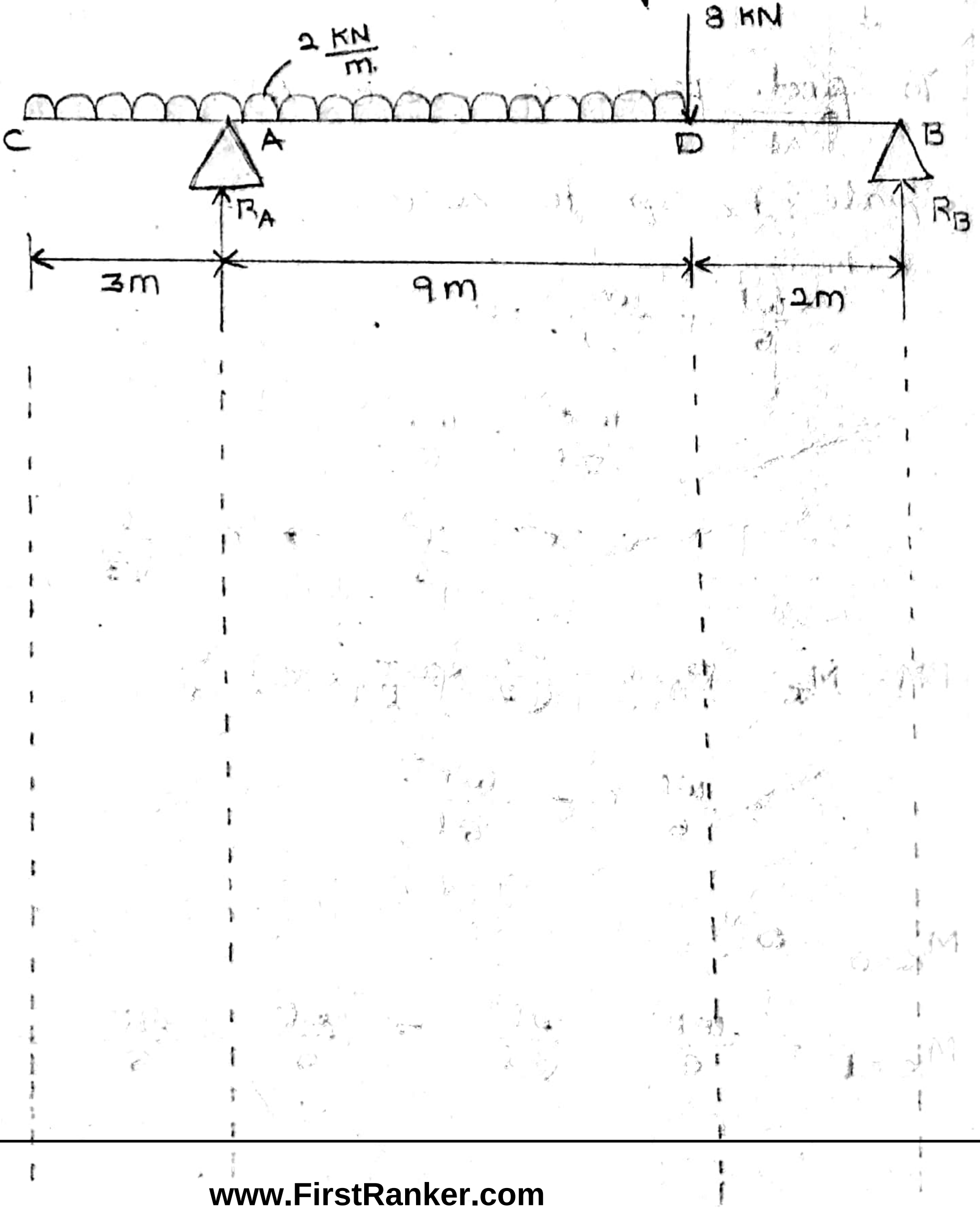
$$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{6\cdot 3\sqrt{3}}$$

$$= \frac{3wl^2 - wl^2}{(6\sqrt{3})\cdot 3}$$

$$= \frac{2wl^2}{18\sqrt{3}}$$

$$= \frac{wl^2}{9\sqrt{3}}$$

* Draw SFD and BMD for the simply supported beam as shown in fig.



Reactions: $\Sigma V = 0$

$$R_A + R_B = 8 + (2 \times 12)$$

$$R_A + R_B = 32$$

$\Sigma M_B = 0$

$$\Rightarrow R_A \times 11 - 8 \times 2 - (2 \times 12)(6+2) = 0$$

$$R_A \times 11 - 16 - 192 = 0$$

$$R_A = 18.9 \text{ KN}$$

$$R_B = 32 - 18.9 = 13.1 \text{ KN}$$

Portion CA (0 to 3m)

SF: $F_x = -2x$

$F_{x=0} = 0$

$F_{x=3} = -6$ KN

BM: $M_x = -2x \cdot \frac{x}{2} = -x^2$

$M_{x=0} = 0$

$M_{x=3} = -9$ KN-m

Portion AD (3 to 12m)

SF: $F_x = R_A - 2x = 18.9 - 2x$

$F_{x=3} = 18.9 - 2(3) = 12.9$ KN.

$F_{x=12} = 18.9 - 2(12) = -5.1$ KN

BM: $M_x = R_A(x-3) - 2x \cdot \frac{x}{2}$

$$= 18.9(x-3) - x^2$$

$M_{x=3} = 0 - 3^2 = -9$ KN-m

$M_{x=12} = 18.9(9) - (12)^2 = 170.1 - 144$

$= 26.1$ KN-m

Portion BA (0 to 2m)

SF: $F_x = -R_B = -13.1$ KN

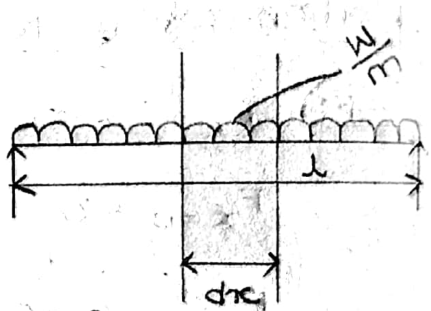
BM: $M_x = R_B \cdot x = 13.1\,x$.

$M_{x=0} = 0$

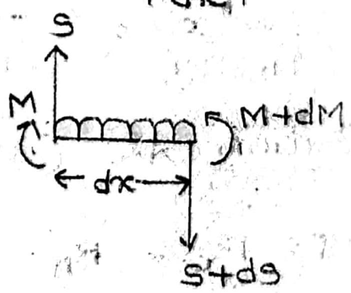
$M_{x=2} = 13.1\,(2) = 26.2$ KN-m

6/9/17

Relation between the load SF and BM:



* Consider a short length 'dx' of a beam at a distance 'x' from origin.
* Let the load over this short length be $\frac{w}{m}$ run acting vertically downwards.



* Then this shear force over this short length will increase from S to S+dS, while the bending moment increases from M to M+dM.

Reaction: $\Sigma V = 0$

$$S = S + dS + w\,dx$$

$$w\,dx = -dS$$

$$W = -\frac{dS}{dx}$$

Taking moments about P.

$\Sigma M_P = 0$

$\Rightarrow S\,dx + M - (w\,dx)\dfrac{dx}{2} - (M + dM) = 0$

$S\,dx + M - \dfrac{w\,dx^2}{2} - M - dM = 0$

Neglecting the terms containing higher powers of dx.

$$S\,dx = dM$$

$$S = \frac{dM}{dx}$$

① Rate of change of SF at any section represent the rate of loading at the section.

② The rate of change of BM at any section represent the SF at the section.

* Draw SF and BM diagrams for the simply supported beam.

Ans <u>Reactions</u>: $\Sigma V = 0$

$$R_A + R_B = 20 + \left(\tfrac{1}{2} \times 10 \times 2\right)$$

$$R_A + R_B = 30$$

$\Sigma M_A = 0$

$\Rightarrow R_B \times 6 - 30 - \left(\tfrac{1}{2} \times 10 \times 2\right)\left(1 + 1 + \tfrac{2}{3}\right) - (20 \times 1) = 0$

$$6R_B = 30 + 10 \times \frac{8}{3} + 20$$

$$R_B = 12.77$$

$R_A = 17.23$

Portion AC (0 to 1m)

SF: $F_x = R_A = 17.23$ KN

BM: $M_x = R_A x = 17.23x$

$M_{x=0} = 0$ $\quad$ $M_{x=1} = 17.23$ KN-m

Portion CA (1 to 2m)

SF: $F_x = R_A - 20 = 17.23 - 20 = -2.77$.

BM: $M_x = R_A . x - 20(x-1)$

$= 17.23x - 20(x-1)$

$M_{x=1} = 17.23 - 0 = 17.23$ KN-m

$M_{x=2} = 17.23(2) - 20(1) = 14.46$ KN-m.

Portion BF (0 to 1m)

SF: $F_x = -R_B = -12.77$ KN

BM: $M_x = R_B . x = 12.77x$.

$M_{x=0} = 0$

$M_{x=1} = 12.77(1) = 12.77$ KN-m.

Portion FE (1 to 2m)

SF: $F_x = -R_B = -12.77$ KN.

BM: $M_x = R_B x - 30 = 12.77x - 30$

$M_{x=1} = 12.77(1) - 30 = -17.23$ KN-m

$M_{x=2} = 12.77(2) - 30 = -4.46$ KN-m

Portion ED (2 to 4m)

SF: $F_x = -R_B - \left[\frac{1}{2}(x-2)\,5(x-2)\right]$

$= -12.77 - \frac{5}{2}(x-2)^2$

$F_{x=2} = -12.77$ KN

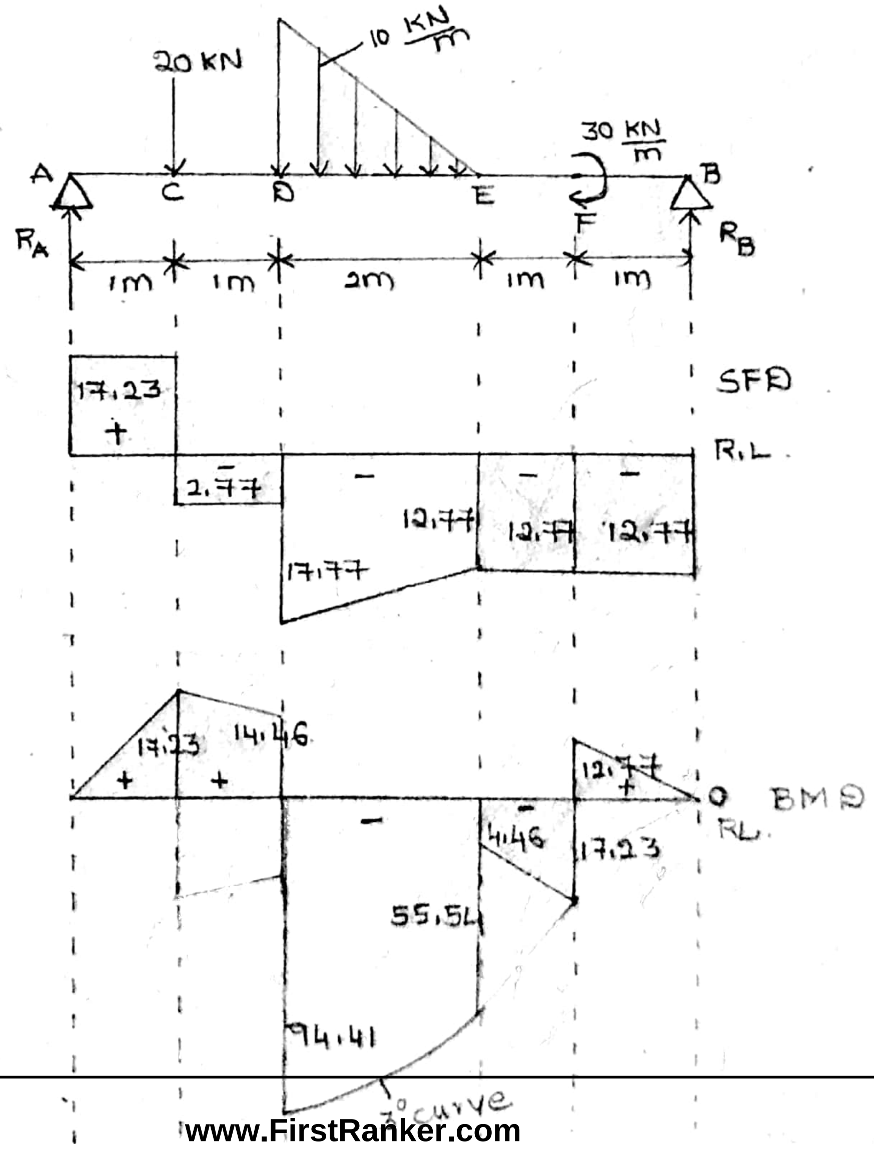
$F_{x=4} = -12.77 - \frac{5}{2} \times 4 = -17.77$ KN.

BM: $M_x = -R_B x - \left(\frac{1}{2}(x-2)\,5(x-2)\right)\frac{2(x-2)}{3} - 30$

$= -12.77x - \frac{5}{3}(x-2)^3 - 30$

$M_{x=2} = -12.77(2) - 0 - 30 = -55.54$ KN-m.

$M_{x=4} = -12.77(4) - \frac{5}{3} \times 8 - 30 = -94.41$ KN-m



UNIT-4 Shear Stresses

Shear eqn:

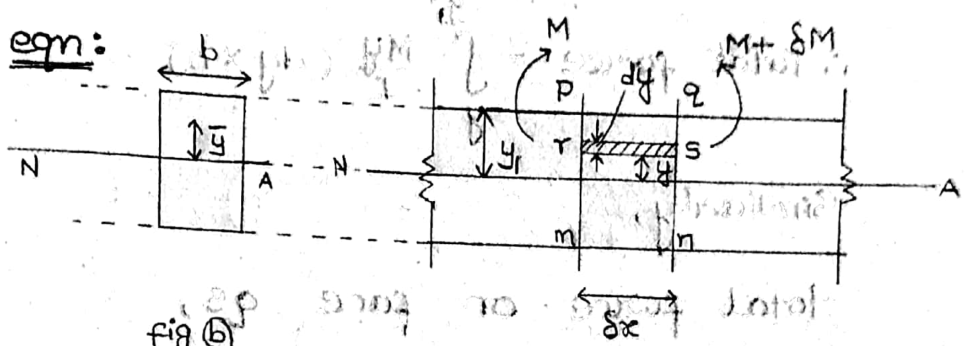


fig ⓑ fig ⓐ

* Let us consider a short slice of the beam of length δx, with a variation of bending moment over its length from M to M+δM.
* Take a layer of rs at a distance 'y' from the neutral axis.
* Let width of this layer be 'b'.
* Therefore area of this layer $= \delta x \times b$
* Between the successive faces of this slice there will be an excess force, since this stress at 'pr' will be less than that of the face 'qs'.
* This excess force will be balanced by the shearing stress acting along the layer rs.
* ∴ Total force on this face pr,

$P = \sigma \times A$ $\qquad\qquad$ $\frac{M}{I} = \frac{\sigma}{y}$

$= \frac{My}{I} \times (dy \times b)$ $\qquad$ $\Rightarrow \sigma = \frac{My}{I}$

$\therefore$ Total force $= \int_{y}^{y_1} \frac{My}{I} (dy \times b)$

Similarly,

total force on face qs,

$$P = \int_{y}^{y_1} \frac{(M + \delta M) y}{I} (dy \times b)$$

$\therefore$ Excess force between qs and pr

$$= \int_{y}^{y_1} \frac{\delta M}{I} . y . b . dy$$

But $\int_{y}^{y_1} y b . dy$ is the moment of area of the face pr about the neutral axis.

$$\int_{y}^{y_1} y b . dy = A\bar{y}$$

$$= \frac{\delta M}{I} . A\bar{y}$$

This excess force is balanced by a shear stress acting along rs.

The force due to this stress equal to shear stress x Area.

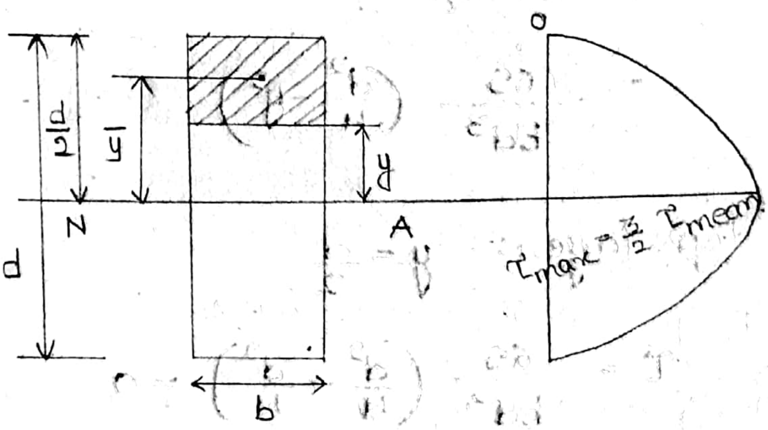
$= \tau \times (b \times dx)$

$$\tau \times b \times dx = \frac{\delta M}{I} A\bar{y}$$

$$\tau = \left(\frac{\delta M}{dx}\right)\frac{A\bar{y}}{Ib}$$

$$\tau = \frac{S}{Ib}(A\bar{y})$$

→ **Shear stress variation in rectangular beams:**



Let 'b' be the width and 'd' be the depth of the rectangular section.

Let τ be the shear stress at a layer at a distance of 'y' from neutral axis where a particular section is subjected to shear force 'S'.

Shear stress, $\tau = \frac{S}{Ib}(A\bar{y})$

Here $I = \frac{bd^3}{12}$

$b = b$

$A = b\left(\frac{d}{2} - y\right)$

$\bar{y} = y + \frac{\left(\frac{d}{2} - y\right)}{2}$

$$= \frac{2y + \frac{d}{2} - y}{2}$$

$$= \frac{1}{2}\left(y + \frac{d}{2}\right)$$

$$\tau = \frac{S}{Ib}(A\bar{y})$$

$$= \frac{\overset{6}{\cancel{12}}S}{bd^3 \times \cancel{b}} \cdot \cancel{b}\left(\frac{d}{2} - y\right)\frac{1}{\cancel{2}}\left(y + \frac{d}{2}\right)$$

$$= \frac{6S}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

At top layer: $y = \frac{d}{2}$

$$\tau = \frac{6S}{bd^3}\left(\frac{d^2}{4} - \frac{d^2}{4}\right) = 0$$

$$\tau = 0$$

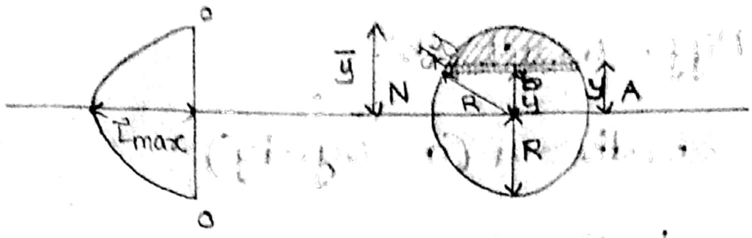
At neutral axis: $y = 0$

$$\tau_{max} = \frac{6S}{bd^3}\left(\frac{d^2}{4}\right)$$

$$\tau_{max} = \frac{3}{2}\left(\frac{S}{bd}\right)$$

$$\boxed{\tau_{max} = \frac{3}{2} \cdot \tau_{mean}}$$

Shear stress distribution for circular section:



A solid circular section of radius 'R' is shown in figure.

Consider an elementary strip of thickness 'dy' at a distance 'y' from neutral axis. Let the width of the strip be 'b'.

$\therefore$ Shear stress, $\tau = \frac{S}{Ib}(A\bar{y})$

$$I = \frac{\pi d^4}{64} = \frac{\pi (2R)^4}{64}$$

$$= \frac{\pi \times 16 \times R^4}{64}$$

$$= \frac{\pi}{4} R^4$$

From diagram,

$$R^2 = y^2 + \left(\frac{b}{2}\right)^2$$

$$\frac{b}{2} = \sqrt{R^2 - y^2}$$

$$b = 2\sqrt{R^2 - y^2}$$

$$b = 2\sqrt{R^2 - y^2}$$

Area of elementary strip $= b.dy$.

Moment of this strip $= (b.dy)y$.

Total moment of this area, $A\bar{y} = \int_y^R (b.dy)y$

$$A\bar{y} = \int_y^R b.y\,dy$$

$b = 2\sqrt{R^2 - y^2}$

$b^2 = 4(R^2 - y^2)$

Diff. w.r.t 'b'.

$2b.db = 4(0 - 2y.dy)$

$b.db = -4y.dy$

$y\,dy = \frac{-1}{4} b.db$

Let $y = R \Rightarrow b = 2\sqrt{R^2 - R^2} = 0$

$y = y \Rightarrow b = 2\sqrt{R^2 - y^2} = b$

$$A\bar{y} = \int_y^R b.y\,dy$$

$$= \int_b^0 b.\left(\frac{-1}{4} b.db\right)$$

$$= \frac{-1}{4}\int_b^0 b^2.db.$$

$$= \frac{-1}{4}\left[\frac{b^3}{3}\right]_b^0$$

$$= \frac{-1}{4}\left(0 - \frac{b^3}{3}\right)$$

$$= \frac{b^3}{12}$$

$\therefore$ Shear stress, $\tau = \frac{S}{Ib}(A\bar{y})$.

$= \frac{\pi R^4}{4}$

$$= \frac{S}{\frac{\pi R^4}{4} b} \times \frac{b^3}{12} \qquad = \frac{S}{\pi R^4} \times \frac{b^2}{3}$$

$$\frac{S}{3\pi R^4} \; 4(R^2-y^2)$$

$$= \frac{4}{3} \frac{S}{\pi R^4} (R^2-y^2)$$

At top layer $(y=R)$

$$\tau = \frac{4}{3} \frac{S}{\pi R^4} (R^2-R^2)$$

$$\tau = 0$$

At the neutral axis $(y=0)$

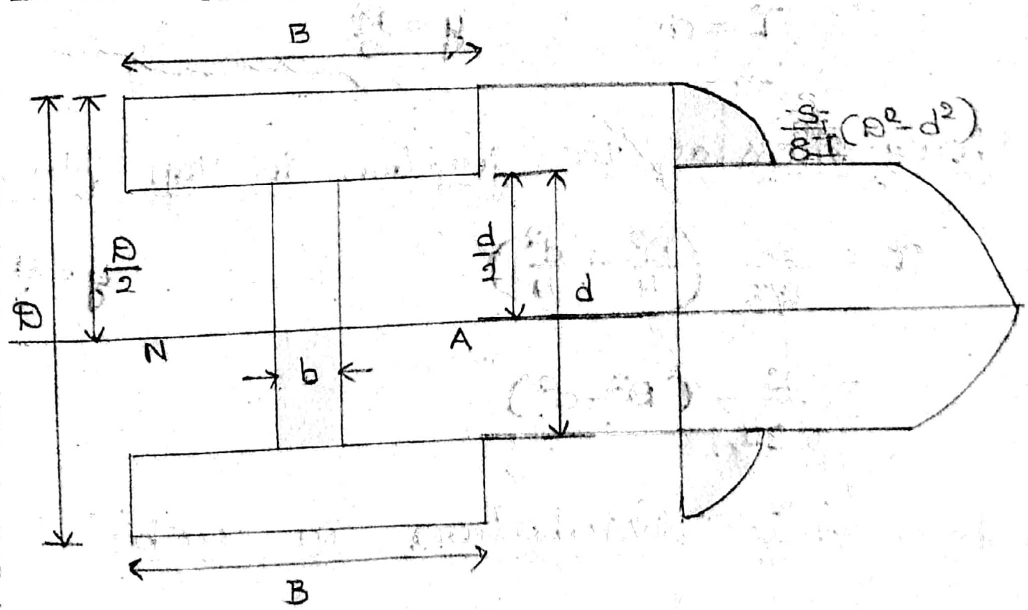
$$\tau = \frac{4}{3} \frac{S}{\pi R^4} (R^2-0)$$

$$\tau = \frac{4}{3} \frac{S}{\pi R^4} \cdot R^2$$

$$= \frac{4}{3} \frac{S}{\pi R^2}$$

$$\boxed{\tau_{max} = \frac{4}{3} \tau_{mean}}$$

→ Stress distribution for I - section :



Shear stress in top flange:

$$\tau = \frac{S}{Ib}(A\bar{y})$$

Here $b = B$.

$$A = B\left(\frac{D}{2} - y\right)$$

$$\bar{y} = y + \frac{\left(\frac{D}{2} - y\right)}{2}.$$

$$= \frac{2y + \frac{D}{2} - y}{2}$$

$$= \frac{1}{2}\left(y + \frac{D}{2}\right)$$

$$\tau = \frac{S}{IB} B\left(\frac{D}{2} - y\right)\frac{1}{2}\left(y + \frac{D}{2}\right)$$

$$= \frac{S}{IB} \cdot \frac{B}{2}\left(\frac{D^2}{4} - y^2\right)$$

$$= \frac{S}{2I}\left(\frac{D^2}{4} - y^2\right)$$

At top fibre:

$\tau = 0$ $\qquad y = \frac{D}{2}$

Shear stress at the junction in top flange:

$$\tau = \frac{S}{2I}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) \qquad y = \frac{d}{2}$$

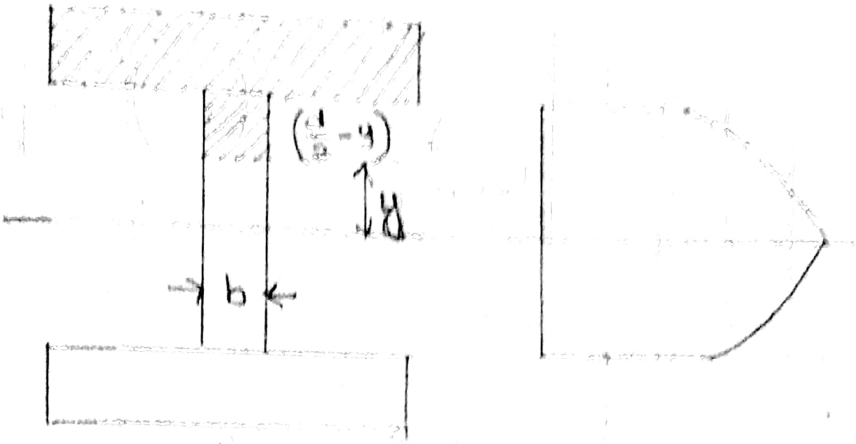
$$= \frac{S}{8I}(D^2 - d^2)$$

shear stress distribution in web:

$$\tau = \frac{S}{IB}(A\bar{y})$$

$b = b$

$A\bar{y} = B\left(\frac{D}{2} - \frac{d}{2}\right)\left(\frac{D}{2} - \frac{\left(\frac{D}{2} - \frac{d}{2}\right)}{2}\right) + b\left(\frac{d}{2} - y\right)\left(y + \frac{\left(\frac{d}{2} - y\right)}{2}\right)$



$$A\bar{y} = B\left(\frac{D-d}{2}\right) \times \frac{1}{2}\left(D - \left(\frac{D}{2} - \frac{d}{2}\right)\right) + b\left(\frac{d}{2} - y\right)\left(\frac{2y + \frac{d}{2} - y}{2}\right)$$

$$= \frac{B}{4}(D-d)\left(\frac{D}{2} + \frac{d}{2}\right) + \frac{b}{2}\left(\frac{d}{2} - y\right)\left(\frac{d}{2} + y\right)$$

$$= \frac{B}{8}(D-d)(D+d) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau = \frac{S}{Ib}(A\bar{y})$$

$$\tau = \frac{S}{Ib}\left\{\frac{B}{8}(D^2-d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right\} \qquad 0 \text{ to } \frac{d}{2}$$

At neutral axis $\quad y = 0$

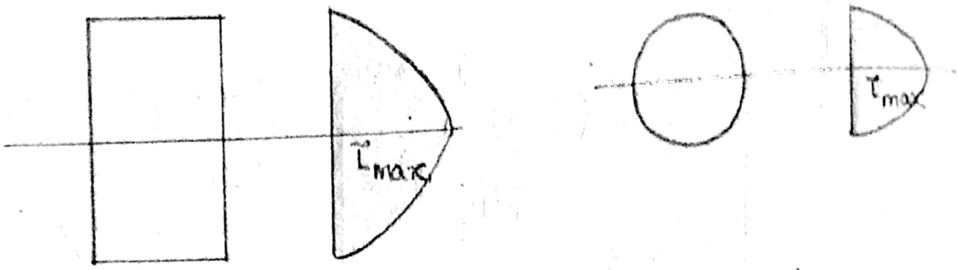
$$\tau_{max} = \frac{S}{Ib}\left\{\frac{B}{8}(D^2-d^2) + \frac{bd^2}{8}\right\}$$

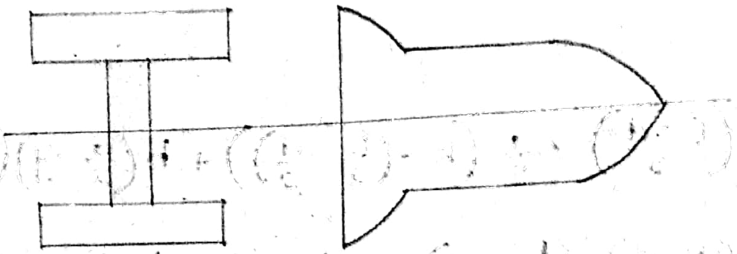
At junction: $\quad y = \frac{d}{2}$

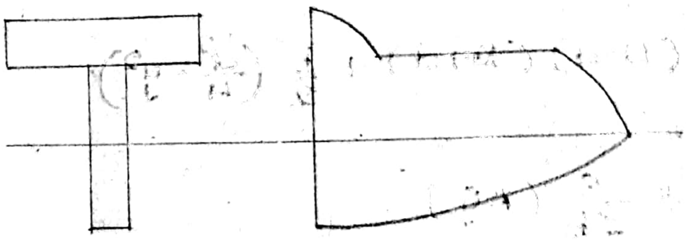
$$\tau = \frac{S}{Ib}\left\{\frac{B}{8}(D^2-d^2)\right\}$$

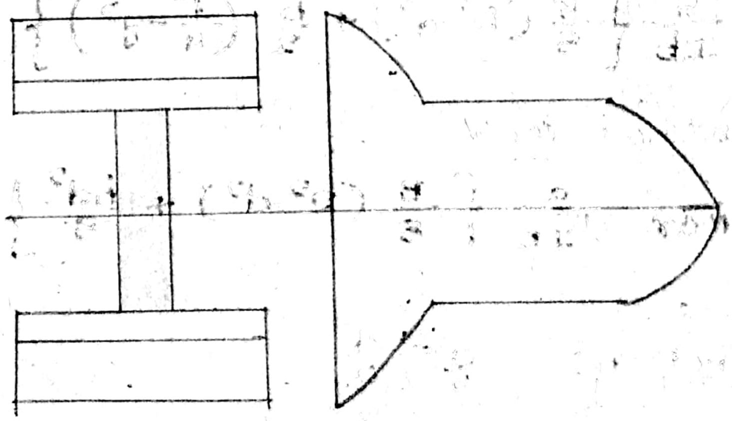
$$\tau = \frac{S}{8Ib} \cdot B(D^2-d^2)$$

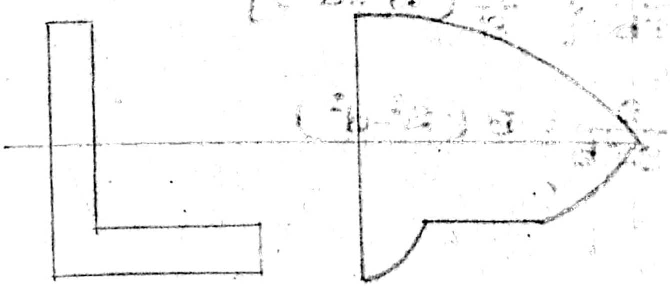
Shear stress distribution across different sections:







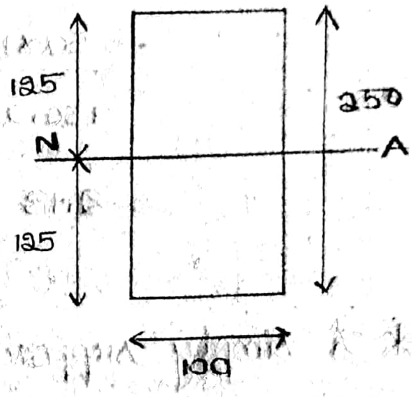




21/9

* A rectangular beam 100 mm wide and 250 mm deep is subjected to a shear force of 50 KN. Determine average shear stress, maximum shear stress, shear stress at 65 mm from the neutral axis.

Ans. Given:



Rectangular beam,

Width, $b = 100$ mm

depth, $d = 250$ mm

Shear force, $S = 50$ KN

$= 50 \times 10^3$ N

(i) Average shear stress $= \frac{S}{bd} = \frac{50 \times 10^3}{100 \times 250}$

$= 2 \frac{N}{mm^2}$

(ii) Maximum shear stress,

$\tau_{max} = \frac{3}{2} \tau_{mean}$

$= \frac{3}{2} \times 2$

$\tau_{max} = 3 \frac{N}{mm^2}$

(iii) Shear stress at 65 mm from the neutral axis

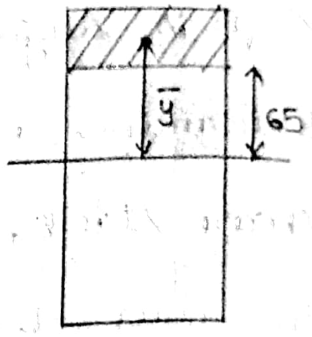
$\tau = \frac{S}{Ib}(A\bar{y})$

$I = \frac{bd^3}{3} = \frac{100 \times 250^3}{3} = 130.2 \times 10^6 \ mm^4$

$b = 100 \ mm$

$A = 100 \times 60 = 6000 \ mm^2$

$\bar{y} = 65 + \frac{60}{3} = 95 \ mm$



$\tau = \frac{S}{Ib}(A\bar{y})$

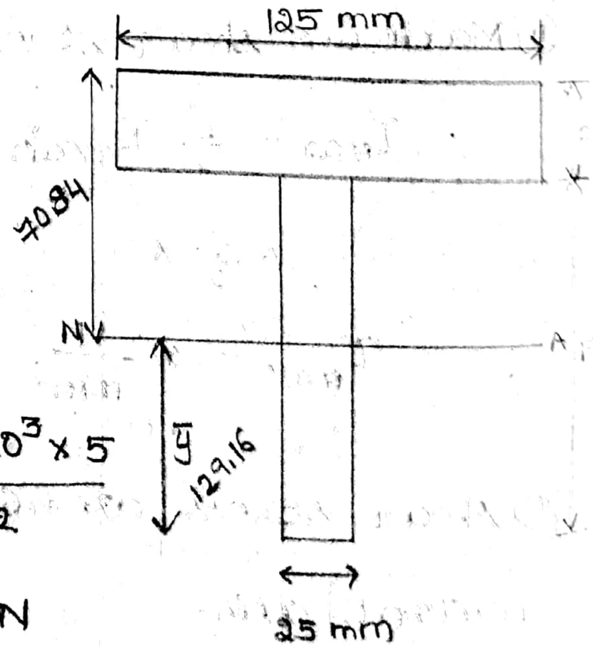
$= \frac{50 \times 10^3}{130.2 \times 10^6 \times 100}(6000 \times 95)$

$= 2.18 \ \frac{N}{mm^2}$

* A simply supported beam carries a UDL of intensity $2.5 \ \frac{KN}{m}$ over entire span of 5m. The cross-section of the beam is a T-section having the dimensions as shown in figure. Calculate max. shear stress of the section of the beam.

Ans Given: $UDL = 2.5 \ \frac{KN}{m}$

length, $l = 5 \ m$



shear force $S_1 = \frac{Wl}{2}$

$= \frac{2.5 \times 10^3 \times 5}{2}$

$= 6250 \ N$

$= 6.25 \ KN$

Let $\bar{y}$ be the centroid of the section from

the neutral axis as shown in fig.

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(25 \times 175)\left(\frac{175}{2}\right) + (125 \times 25)\left(175 + \frac{25}{2}\right)}{(25 \times 175) + (125 \times 25)}$$

$$= \frac{382812.5 + 585937.5}{4375 + 3125}$$

$$= 129.16 \text{ mm}$$

Moment of inertia, $I = I_1 + I_2$

$$I_1 = \frac{bd^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{25 \times 175^3}{12} + 4375 \left(\frac{175}{2} - 129.16\right)^2$$

$$= 18.75 \times 10^6 \text{ mm}^4$$

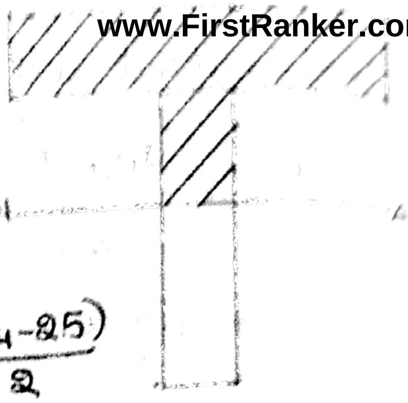
$$I_2 = \frac{125 \times 25^3}{12} + 3125 \left[\left(175 + \frac{25}{2}\right) - 129.16\right]^2$$

$$= 10.79 \times 10^6 \text{ mm}^4$$

$$I = 18.75 \times 10^6 + 10.79 \times 10^6$$

$$I = 29.54 \times 10^6 \text{ mm}^4$$

$\frac{S}{Ib}(A\bar{y})$



$b = 25$ mm

$A\bar{y} = (125 \times 25)\left(70.84 - \frac{25}{2}\right)$

$+ (25 \times (70.84 - 25))\left(\frac{70.84 - 25}{2}\right)$

$= 182312.5 + 26266.32$

$= 208578.82 \text{ mm}^3$

$$\tau = \frac{6250}{29.54 \times 10^6 \times 25} \times 208578.82$$

$$\tau = 1.765 \frac{N}{mm^2}$$

* A beam of channel section 120 mm x 60 mm as uniform thickness of 15 mm. Draw diagram showing the distribution of shear stress where shear force is 50 KN. Find the ratio between max. shear stress and mean shear stress.

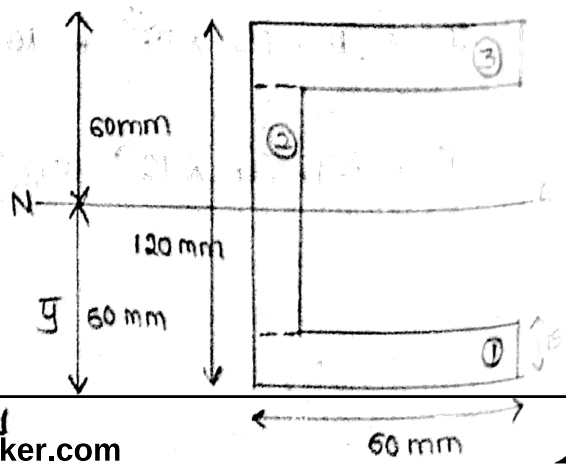
Ans Given:

Depth, d = 120 mm

Breadth, b = 60 mm

Thickness, = 15 mm

Shear force, S = 50 KN



Let $\bar{y}$ be the centroid of the section from the bottom of the section.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 60 \times 15 = 900$

$A_2 = 15 \times (120 - 30) = 1350$

$A_3 = 60 \times 15 = 900$

$y_1 = \frac{15}{2} = 7.5$

$y_2 = 15 + \frac{90}{2} = 60$

$y_3 = 15 + 90 + \frac{15}{2} = 112.5$

$$\bar{y} = \frac{900 \times (7.5) + (1350 \times 60) + (900 \times 112.5)}{900 + 1350 + 900}$$

$\bar{y} = 60$ mm.

Moment of inertia, $I = I_1 + I_2 + I_3$

$$I_1 = \frac{bd^3}{12} + A_1 (y_1 - \bar{y})^2$$

$$= \frac{60 \times 15^3}{12} + 900 \left(\frac{15}{2} - 60\right)^2$$

$= 2.49 \times 10^6$ mm^4

$$I_2 = \frac{15 \times 90^3}{12} + 1350 (60 - 60)^2$$

$= 0.91 \times 10^6$ mm^4

$$I_3 = \frac{60 \times 15^3}{12} + 900\,(112.5 - 60)^2$$

$$= 2.49 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3$$

$$I = (2.49 \times 10^6) + (0.91 \times 10^6) + (2.49 \times 10^6)$$

$$I = 5.89 \times 10^6 \text{ mm}^4$$

Max. shear stress, $\tau = \frac{S}{Ib}(A\bar{y})$

Average shear stress, $\tau_{mean} = \frac{S}{A}$

$$= \frac{50 \times 10^3}{900 + 1350 + 900}$$

$$= 15.87 \frac{N}{mm^2}.$$

∴ Max. shear stress, $\tau_{max} = \frac{S}{Ib}(A\bar{y})$

$b = 15$ mm

$$A\bar{y} = (60 \times 15)\left(60 - \frac{15}{2}\right) + (15 \times 45)\left(\frac{45}{2}\right)$$

$$= 47250 + 15187.5$$

$$= 62437.5$$

$$\tau_{max} = \frac{50 \times 10^3}{5.89 \times 10^6 \times 15} \times 62437.5$$

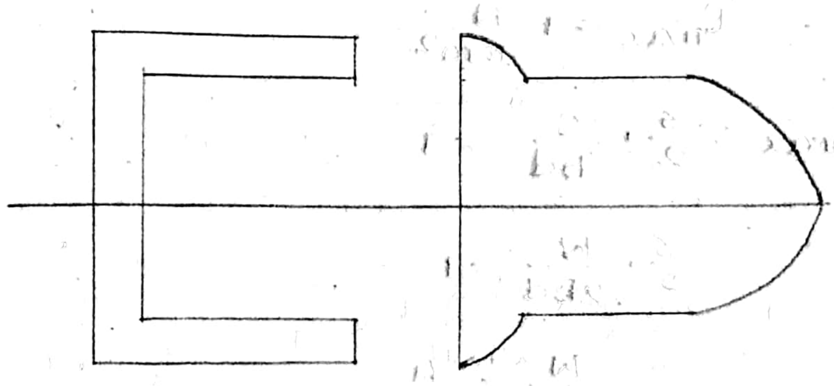
$$\tau_{max} = 35.33 \frac{N}{mm^2}$$

Ratio between max. shear stress and mean shear stress.

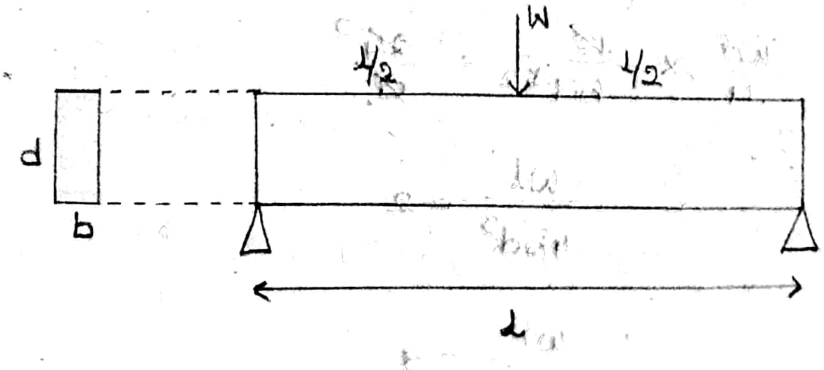
$$\tau_{max} : \tau_{mean}$$

$$\cancel{35:7} \quad 35.33 : 15.87$$

$$\frac{\tau_{max}}{\tau_{mean}} = \frac{35.33}{15.87} = 2.22$$



* A timber beam of rectangular section is simply supported at the ends and carries a point load at the centre of the beam. The maximum bending stress is 12 N/mm^2 and maximum shear stress 1 N/mm^2. Find the ratio of the span to depth.



Ans. Max. bending stress, $\sigma = 12 \frac{N}{mm^2}$

Max. shear stress, $\tau_{max} = 1 \frac{N}{mm^2}$

$$\frac{l}{d} = ?$$

Let breadth of the beam $= b$

depth $= d$

$$\tau_{max} = 1 \frac{N}{mm^2}$$

$$\tau_{max} = \frac{3}{2} \cdot \frac{S}{bd} = 1$$

$$\frac{3}{2} \cdot \frac{W}{2bd} = 1$$

$$\frac{W}{bd} = \frac{4}{3}$$

$$\text{Max. BM} = \frac{Wl}{4}$$

$$I = \frac{bd^3}{12}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\Rightarrow \frac{\frac{Wl}{4}}{\frac{bd^3}{12}} = \frac{12}{\frac{d}{2}}$$

$$\Rightarrow \frac{Wl}{4} \times \frac{\cancel{12}}{bd^{\cancel{3}2}} = \frac{\cancel{24}^{2}}{\cancel{d}}$$

$$\frac{Wl}{4bd^2} = 2$$

$$\frac{Wl}{bd^2} = 8$$

$$\left(\frac{W}{bd}\right)\left(\frac{l}{d}\right) = 8$$

$$\frac{4}{3} \times \frac{l}{d} = 8$$

$$\frac{l}{d} = \frac{24}{4} = 6$$

$$\frac{l}{d} = 6$$

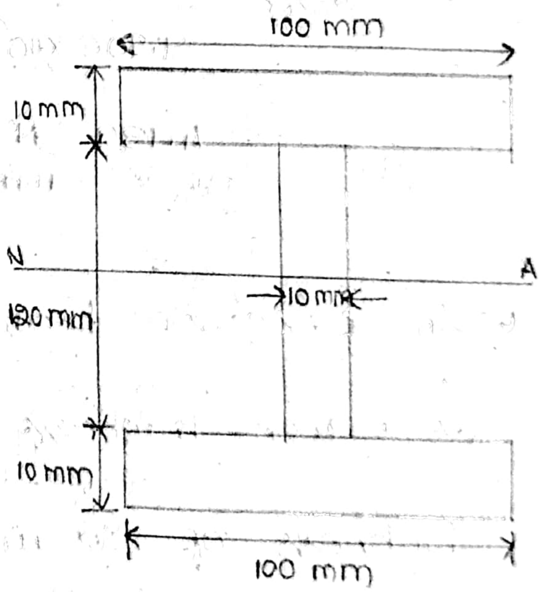
* An I - section with rectangular ends as the following dimensions.

Flange dimensions = 10 cm x 1 cm

Web dimensions = 12 cm x 1 cm.

If this section is subjected to a bending moment of 5 KN-m and shear force of 5 KN. Find the maximum tensile and shear stress.

Sol Given:



B.M , M = 5 KN-m

S.F , S = 5 KN

Moment of inertia,

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{100 \times 140^3}{12} - \frac{90 \times 120^3}{12}$$

$$= 9.906 \times 10^6 \text{ mm}^4$$

(i) Max. tensile stress

$$\frac{M}{I} = \frac{\sigma_t}{y_t}$$

$$\Rightarrow \frac{5 \times 10^6}{9.906 \times 10^6} = \frac{\sigma_t}{70}$$

$$\sigma_t = 35.33 \ \frac{N}{mm^2}$$

(ii) Max. shear stress

$$\tau_{max} = \frac{S}{Ib}(A\bar{y})$$

$b = 10$ mm.

$$A\bar{y} = (100 \times 10)\left(60 + \frac{10}{2}\right) + (10 \times 60)\left(\frac{60}{2}\right)$$

$$= 65000 + 18000$$

$$= 83000 \ mm^3.$$

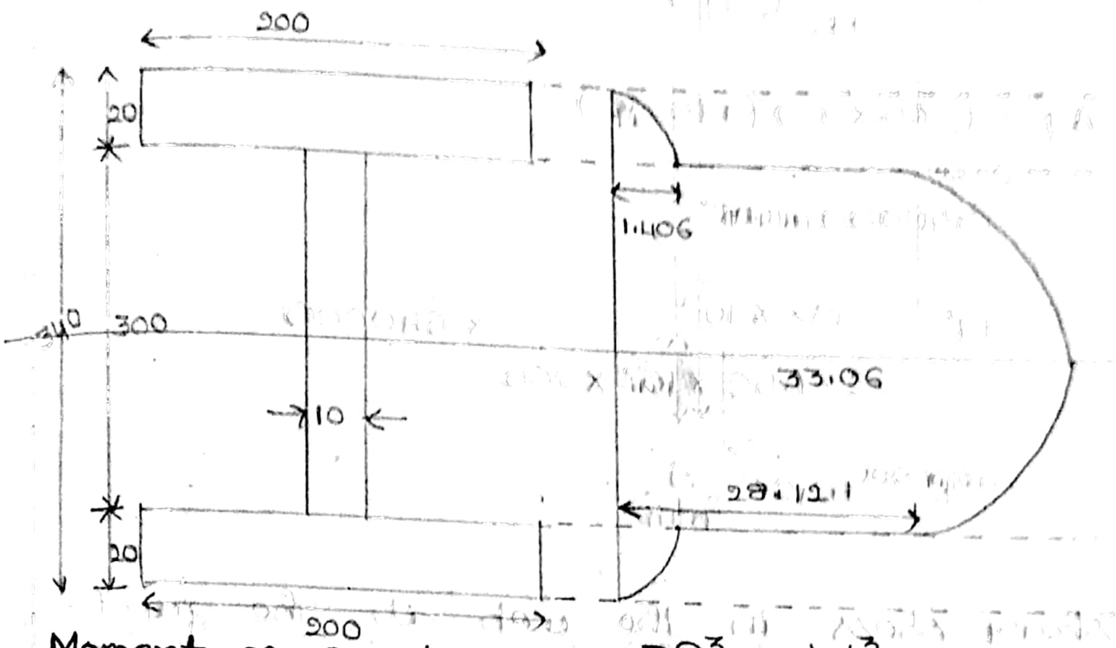
$$\tau_{max} = \frac{5 \times 10^3}{9.906 \times 10^6 \times 10}(83000)$$

$$= 4.189 \ \frac{N}{mm^2}$$

* An I-section beam 340 mm × 200 mm (width) as a web thickness of 10 mm and flange thickness of 20 mm. It carries a shear force of 100 KN. Sketch the shear stress distribution.

Given

S.F, $S = 100$ KN



Moment of inertia, $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

$$= \frac{200 \times 340^3}{12} - \frac{190 \times 300^3}{12}$$

$$= 227.56 \times 10^6 \text{ mm}^4$$

<u>Shear stress at neutral axis</u>

$$\tau = \frac{S}{Ib}(A\bar{y})$$

$$A\bar{y} = (200 \times 20)(170 - 10) + (10 \times 150)\left(\frac{150}{2}\right)$$

$$= 752500 \text{ mm}^3$$

$$\tau = \frac{100 \times 10^3}{227.56 \times 10^6 \times 10} \times 752500$$

$$\tau = 33.06 \frac{N}{mm^2}$$

Shear stress in the flange at the junction

$$\tau = \frac{S}{Ib}(A\bar{y})$$

$$A\bar{y} = (800 \times 20)(170-10)$$

$$= 640000 \ mm^3$$

$$\tau = \frac{100 \times 10^3}{227.56 \times 10^6 \times 200} \times 640000$$

$$\tau = 1.406 \ \frac{N}{mm^2}$$

Shear stress in the web at the junction

$$\tau = \frac{S}{Ib}(A\bar{y})$$

$$A\bar{y} = (200 \times 20)(160)$$

$$= 640000 \ mm^3$$

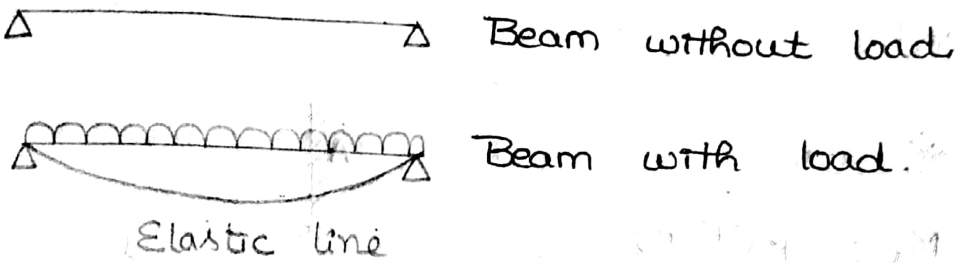
$$\tau = \frac{100 \times 10^3}{227.56 \times 10^6 \times 10} \times 640000$$

$$= 28.12 \ \frac{N}{mm^2}$$

DEFLECTIONS OF BEAMS

Beam deflection:

When a load is placed on a beam, the beam tends to deflect or sag as shown in figure.



Deflection plays an significant role in the design of structures and machines.

Under load the neutral axis becomes a curved line and is called elastic line.

Methods

1. Double integration.
2. Maculays method
3. Moment area method.

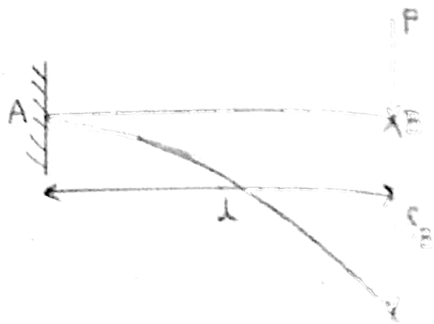
Double Integration method

Formula: $EI \dfrac{d^2y}{dx^2} = M_x$

Cantilever beams :

Case-1

Cantilever beam with concentrated load at end.



BM eqn:

$$M_x = -P(l-x)$$

$$EI \frac{d^2y}{dx^2} = -P(l-x)$$

Integrate once.

$$EI \frac{dy}{dx} = -P\left(lx - \frac{x^2}{2}\right) + C_1 \rightarrow (1)$$

Integrate again,

$$EI \cdot y = -P\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) + C_1 x + C_2 \rightarrow (2)$$

Boundary conditions:

At A, $x=0$ $y=0$

$x=0$ $\dfrac{dy}{dx}=0$

from (1) $EI(0) = -P(0-0) + C_1$

$$\boxed{C_1 = 0}$$

from ②, $EI(0) = -P(0-0) + 0 + C_2$

$$\boxed{C_2 = 0}$$

from ①

$$EI.\frac{dy}{dx} = -P\left(lx - \frac{x^2}{2}\right) \longrightarrow ③$$

$$EI.y = -P\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) \longrightarrow ④$$

At $x = l$, $y = y_B$

from ④,

$$EI.y_B = -P\left(\frac{l \times l^2}{2} - \frac{l^3}{6}\right)$$

$$EI.y_B = -P\left(\frac{3l^3 - l^3}{6}\right)$$

$$EI y_B = -P\,\frac{2l^3}{6}$$

$$y_B = \frac{-Pl^3}{3EI}$$

At $x = l$, $\frac{dy}{dx} = \theta_B$

from ③,

$$EI.\theta_B = -P\left(l \times l - \frac{l^2}{2}\right)$$

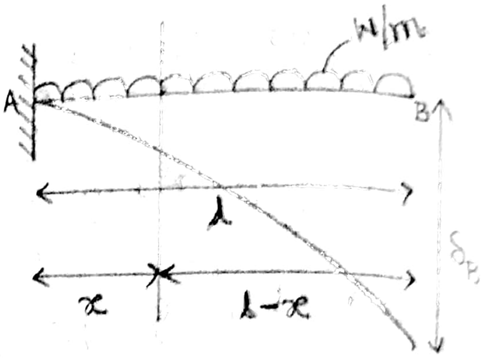
$$EI.\theta_B = -P\left(\frac{l^2}{2}\right)$$

$$\theta_B = \frac{-Pl^2}{2EI}.$$

Here -ve sign indicates anti clockwise direction.

Cantilever beam with UDL:

Consider a section at a distance 'x' from the fixed end as shown in figure.



$$EI.\frac{d^2y}{dx^2} = M_x.$$

$$M_x = -w(l-x)\frac{(l-x)}{2}.$$

$$= -\frac{w}{2}(l-x)^2.$$

$$EI.\frac{d^2y}{dx^2} = -\frac{w}{2}(l-x)^2.$$

Integrate once.

$$EI.\frac{dy}{dx} = -\frac{w}{2}\frac{(l-x)^3}{3}(0-1) + C_1$$

$$EI.\frac{dy}{dx} = \frac{w}{6}(l-x)^3 + C_1 \longrightarrow (1)$$

Integrate again.

$$EI.y = \frac{w}{6}\frac{(l-x)^4(0-1)}{4} + C_1x + C_2.$$

$$EI.y = -\frac{w}{24}(l-x)^4 + C_1x + C_2 \longrightarrow (2)$$

Boundary conditions:

At $x=0$, $\frac{dy}{dx} = 0$

from ①

$$E.I(0) = \frac{w}{6}(l-0)^3 + c_1$$

$$0 = \frac{wl^3}{6} + c_1$$

$$c_1 = \frac{-wl^3}{6}$$

At $x=0$, $y=0$

from ② $\quad EI(0) = \frac{-w}{24}(l-0)^4 - \frac{wl^3}{6}(0) + C_2$

$$0 = \frac{-wl^4}{24} - 0 + C_2$$

$$C_2 = \frac{wl^4}{24}$$

eqn ① $\Rightarrow$ $EI.\frac{dy}{dx} = \frac{w}{6}(l-x)^3 - \frac{wl^3}{6} \longrightarrow ③$

eqn ② $\Rightarrow$ $EI.\,y = \frac{-w}{24}(l-x)^4 - \frac{wl^3}{6}x + \frac{wl^4}{24} \longrightarrow ④$

Slope at B:

At $x=l$, $\frac{dy}{dx} = \theta_B$

eqn ③ $\Rightarrow$ $EI.\theta_B = \frac{w}{6}(l-l)^3 - \frac{wl^3}{6}$

$$\boxed{\theta_B = \frac{-wl^3}{6EI}}$$

Deflection at B:

At $x=l$, $y = \delta_B$

eqn ④ $\Rightarrow$ $EI.\delta_B = \frac{-w}{24}(l-l)^4 - \frac{wl^3}{6}.l + \frac{wl^4}{24}$

$$\delta_B = \frac{-wl^4}{6} + \frac{wl^4}{24}$$

$$\delta_B = \frac{1}{EI}\left(\frac{-4wl^4 + wl^4}{24}\right)$$

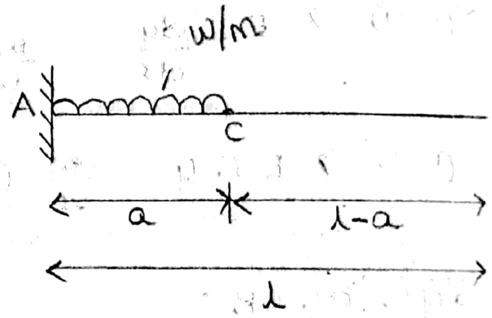
$$= \frac{1}{EI}\left(\frac{-3wl^4}{24}\right)$$

$$\boxed{\delta_B = \frac{-wl^4}{8EI}}$$

-ve sign indicates downward deflection.

Cantilever beam of length 'l' carrying UDL for a distance 'a' from the fixed end:

Consider a cantilever beam of length 'l' carrying UDL for a distance 'a' from the fixed end.



Consider a section of distance 'x' from fixed end.

$$EI.\frac{d^2y}{dx^2} = M_x$$

BM: $M_x = -w(a-x)\dfrac{(a-x)}{2}$

$$= \frac{-w(a-x)^2}{2}$$

$$EI.\frac{d^2y}{dx^2} = \frac{-w}{2}(a-x)^2$$

Integrate once

$$EI.\frac{dy}{dx} = \frac{-w}{2}\frac{(a-x)^3}{3}(0-1) + c_1$$

$$EI.\frac{dy}{dx} = \frac{w}{6}(a-x)^3 + c_1 \longrightarrow (1)$$

Integrate again.

$$EI.y = \frac{w}{6}\frac{(a-x)^4}{4}(0-1) + c_1 x + c_2 \longrightarrow (2)$$

Boundary conditions:

At $x=0$, $\frac{dy}{dx}=0$

eqn (1) $\Rightarrow$ $EI.(0) = \frac{w}{6}(a-0)^3 + c_1$

$$c_1 = \frac{-wa^3}{6}$$

At $x=0$, $y=0$

eqn (2) $\Rightarrow$ $EI(0) = -\frac{w}{6}\frac{(a-0)^4}{4} - \frac{wa^3}{6}.(0) + c_2$

$$c_2 = +\frac{wa^4}{24} + \frac{wa^4}{6}$$

$$c_2 = \frac{wa^4 + 4wa^4}{24}$$

$$c_2 = \frac{5wa^4}{24}$$

$$c_2 = \frac{+wa^4}{24}$$

eqn (1) $\Rightarrow$ $EI.\frac{dy}{dx} = \frac{w}{6}(a-x)^3 - \frac{wa^3}{6} \longrightarrow (3)$

eqn (2) $\Rightarrow$ $EI.y = \frac{+w}{24}(a-x)^4 - \frac{wa^3}{6}x + \frac{wa^4}{24} \longrightarrow (4)$

Slope at C:

$$x = a, \quad \frac{dy}{dx} = \theta_c$$

$$\text{eqn (3)} \Rightarrow EI.\theta_c = \frac{w}{6}(a-a)^3 - \frac{wa^3}{6}$$

$$\theta_c = \frac{-wa^3}{6EI}$$

Deflection at C:

$$x = a, \quad y = \delta_c.$$

$$\text{eqn (4)} \Rightarrow EI.\delta_c = \frac{-w}{24}(a-a)^4 - \frac{wa^3}{6}.a + \frac{wa^4}{24}$$

$$= \frac{-wa^4}{6} + \frac{wa^4}{24}$$

$$\delta_c = \frac{wa^4}{EI}\left(\frac{-4+1}{24}\right)$$

$$\delta_c = \frac{-wa^4}{8EI}.$$

Slope at B:

Since portion BC is not loaded, it doesn't bend and remains straight

$$\theta_B = \theta_c = \frac{-wa^3}{6EI}$$

Deflection at B:

$$y_B = y_C + BB'$$

$$\tan\theta_c = \frac{BB'}{C'B}$$

$AB' = \tan\theta_c \times (l-a)$

$AB' = \theta_c \times (l-a)$

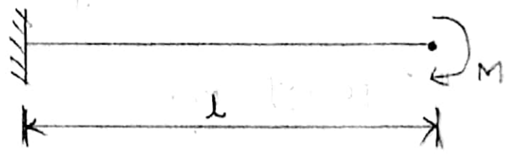
$$= \frac{-wa^3}{6EI}(l-a)$$

$$y_B = y_c + AB'$$

$$= \frac{-wa^4}{8EI} - \frac{wa^3}{6EI}(l-a)$$

Cantilever beam with a moment applied at free end:

Consider a section at a distance 'x' from fixed end.



$$EI.\frac{d^2y}{dx^2} = M_x$$

BM: $M_x = -M$

$$EI.\frac{d^2y}{dx^2} = -M$$

Integrate once.

$$EI.\frac{dy}{dx} = -Mx + c_1 \longrightarrow (1)$$

Again integrate

$$EI.y = \frac{-Mx^2}{2} + c_1x + c_2 \longrightarrow (2)$$

Boundary conditions:

At $x=0$, $\frac{dy}{dx}=0$

eqn ① $\Rightarrow$ $EI(0) = -M(0) + c_1$

$c_1 = 0$

eqn ② At $x=0$, $y=0$

eqn ② $\Rightarrow$ $EI.(0) = -M(0) + c_1(0) + c_2$

$c_2 = 0$

~~Slope at B:~~

eqn ① $\Rightarrow$ $EI\frac{dy}{dx} = -Mx \longrightarrow$ ③

eqn ② $\Rightarrow$ $EI.y = -M\frac{x^2}{2} \longrightarrow$ ④

Slope at B:

$x=l$, $\frac{dy}{dx} = \theta_B$

③ $\Rightarrow$ $EI.\theta_B = -Ml$

$$\boxed{\theta_B = \frac{-Ml}{EI}}$$

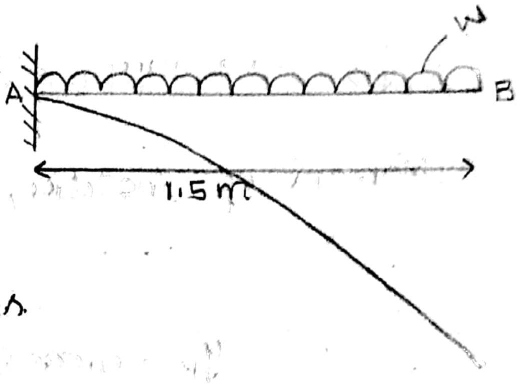
Deflection at B:

$x=l$, $y=\delta_B$

④ $\Rightarrow$ $EI\,\delta_B = \frac{-Ml^2}{2}$

$$\boxed{\delta_B = \frac{-Ml^2}{2EI}}$$

* A cantilever 1.5 m long carries a UDL over the entire length. Find the deflection at the free end. If the slope at the free end is 1.5°



Sol Given: Length, $l = 1.5$ m.

Slope, $\theta_B = 1.5°$

$$= 1.5 \times \frac{\pi}{180} \text{ radians}$$

$$\frac{wl^3}{6EI} = 1.5 \times \frac{\pi}{180}$$

$$\frac{wl^3}{EI} = 0.157$$

Deflection at B, $y_B = \frac{wl^4}{8EI}$

$$= \frac{1}{8}\left(\frac{wl^3}{EI}\right) l$$

$$= \frac{1}{8} \times 0.157 \times 1.5$$

$$= 0.029 \text{ m}$$

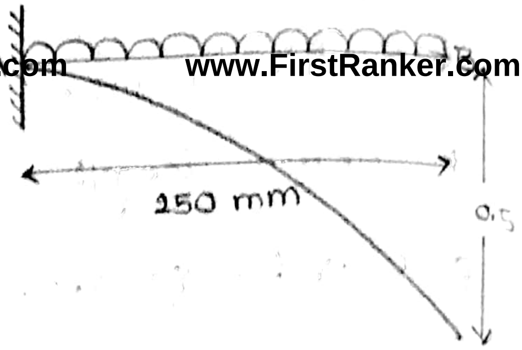
* A 250 mm long cantilever of rectangular section 40 mm wide and 30 mm deep carries a UDL. Calculate the value of W. If the maximum deflection in the cantilever is not to exceed 0.5 mm. Take $E = 70 \frac{GN}{m^2}$.

Sol Given: Length, $l = 250$ mm $= 0.25$ m

b $= 0.04$ m

Depth, $d = 30$ mm

$= 0.03$ m.



$E = 70 \frac{GN}{m^2}$

$= 70 \times 10^9 \frac{N}{m^2}$

$y_B = 0.5$ mm $= 0.0005$ m.

Moment of inertia, $I = \frac{bd^3}{12} = \frac{0.04 \times 0.03^3}{12}$

$= 9 \times 10^{-8}$ m^4

$y_B = 0.0005$

$$\frac{wl^4}{8EI} = 0.0005$$

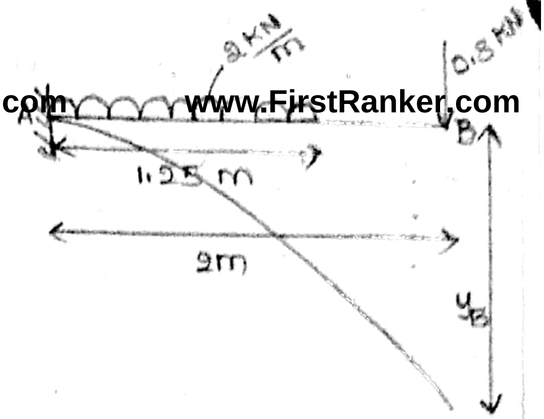
$$\Rightarrow \frac{w \times (0.25)^4}{8 \times 70 \times 10^9 \times 9 \times 10^{-8}} = 0.0005$$

$w = 6451.2 \frac{N}{m}$

$w = 6.451 \frac{kN}{m}$

* A cantilever 2m long is of rectangular section 100 mm wide and 200 mm deep. It carries a uniformly distributed load (UDL) of $2 \frac{kN}{m}$ for a length of 1.25 m. from the fixed end, a point load 0.8 kN at the free end. Find the deflection at the free end. Take $E = 10 \frac{GN}{m^2}$

Given:



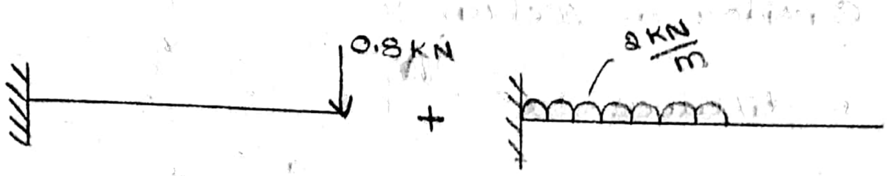
Length, $l = 2m$.

Width, $b = 100\ mm = 0.1\ m$

Depth, $d = 200\ mm = 0.2\ m$

$$E = 10\ \frac{GN}{m^2} = 10 \times 10^9\ \frac{N}{m^2}.$$

$$I = \frac{bd^3}{12} = \frac{0.1 \times (0.2)^3}{12} = 6.66 \times 10^{-5}\ m^4.$$



Maximum deflection due to point load:

$$y_{B_1} = \frac{Wl^3}{3EI}$$

$$= \frac{0.8 \times 10^3 \times 2^3}{3 \times 10 \times 10^9 \times 6.66 \times 10^{-5}}$$

$$= 3.2 \times 10^{-3}\ m.$$

$$= 3.2\ mm.$$

Maximum deflection due to UDL:

$$y_{B_2} = \frac{wa^4}{8EI} + \frac{wa^3}{6EI}(l-a)$$

$$= \frac{2 \times 10^3 \times (1.25)^4}{8 \times 10^{10} \times 6.66 \times 10^{-5}} + \frac{2 \times 10^3 \times (1.25)^3}{6 \times 10^{10} \times 6.66 \times 10^{-5}}(2-1.25)$$

$$= (9.16 \times 10^{-4}) + (7.33 \times 10^{-4})$$

$$= 16.49 \times 10^{-4}\ m.$$

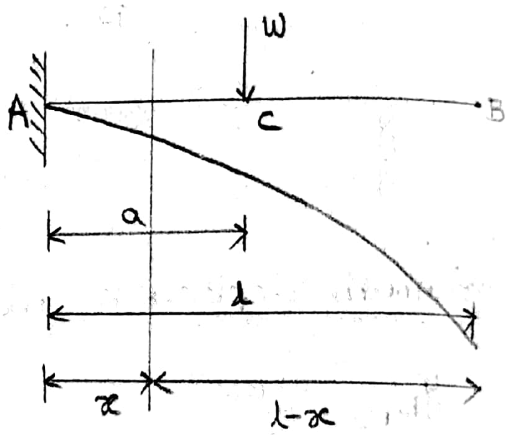
$$= 1.649\ mm$$

Total deflection $y_B = y_{B_1} + y_{B_2}$

$$= 3.2 + 1.649$$

$$= 4.849 \text{ mm}$$

Cantilever beam with a point load at a distance 'a' from fixed end:



Consider a section at a distance 'x' from fixed end.

BM: $M_x = -W(a-x)$

$$EI.\frac{d^2y}{dx^2} = -W(a-x)$$

Integrate once.

$$EI.\frac{dy}{dx} = -W\left(ax - \frac{x^2}{2}\right) + C_1 \longrightarrow ①$$

Again integrate.

$$EI.y = -W\left(a\frac{x^2}{2} - \frac{x^3}{6}\right) + C_1x + C_2 \longrightarrow ②$$

Boundary conditions:

At $x=0$, $\frac{dy}{dx} = 0$

$① \Rightarrow EI(0) = -W(0-0) + C_1$

$$C_1 = 0$$

At $x=0$, $y=0$

$$② \Rightarrow EI(0) = -w(0-0) + c_1(0) + c_2$$

$$c_2 = 0$$

$$EI.\frac{dy}{dx} = -w\left(ax - \frac{x^2}{2}\right) \longrightarrow ③$$

$$EI.y = -w\left(\frac{ax^2}{2} - \frac{x^3}{6}\right) \longrightarrow ④$$

Slope at c:

At $x=a$, $\frac{dy}{dx} = \theta_c$

$$③ \Rightarrow E.I.\theta_c = -w\left(a^2 - \frac{a^2}{2}\right)$$

$$= \frac{-wa^2}{2}$$

$$\boxed{\theta_c = \frac{-wa^2}{2EI}}$$

Deflection at c:

At $x=a$, $y=y_c$

$$④ \Rightarrow EI.y_c = -w\left(\frac{a^3}{2} - \frac{a^3}{6}\right)$$

$$= -w\left(\frac{3a^3 - a^3}{6}\right)$$

$$= \frac{-wa^3}{3}$$

$$\boxed{y_c = \frac{-wa^3}{3EI}}$$

Slope at B:

Since there is no load in portion BC,

$$\theta_C = \theta_B = \frac{-wa^2}{2EI}$$

Deflection at B:

$$y_B = y_C + BB'$$

$$\tan\theta_C = \frac{BB'}{C'B}$$

$$BB' = \tan\theta_C \cdot C'B$$

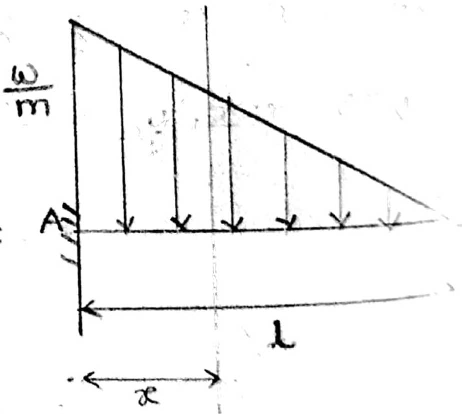
$$= \theta_C\,(l-a)$$

$$= \frac{-wa^2}{2EI}(l-a)$$

$$y_B = \frac{-wa^3}{3EI} - \frac{wa^2}{2EI}(l-a)$$

7.10 Cantilever beam with UVL:

Consider a section at a distance 'x' from fixed end.



BM:

$$M_x = -\frac{1}{2}(l-x)\,\frac{w(l-x)}{l} \times \frac{(l-x)}{3}$$

$w \rightarrow l$

$? \rightarrow (l-x)$

$$= \frac{-w(l-x)^3}{6l}$$

$$EI \frac{d^2y}{dx^2} = -\frac{w(l-x)^3}{6l}$$

Integrate once.

$$EI. \frac{dy}{dx} = \frac{-w}{6l} \frac{(l-x)^4}{4} (0-1) + c_1$$

$$EI. \frac{dy}{dx} = \frac{w}{24l} (l-x)^4 + c_1 \longrightarrow (1)$$

Again integrate.

$$EI. y = \frac{w}{24l} \frac{(l-x)^5}{5} . (0-1) + c_1 x + c_2$$

$$EI. y = \frac{-w}{120l} (l-x)^5 + c_1 x + c_2 \longrightarrow (2)$$

Boundary conditions:

At $x = 0$, $\frac{dy}{dx} = 0$

$(1) \Rightarrow \quad EI.(0) = \frac{w}{24l} (l-0)^4 + c_1$

$$c_1 = \frac{-wl^4}{24l}$$

$$c_1 = -\frac{wl^3}{24}$$

At $x = 0$, $y = 0$

$(2) \Rightarrow \quad EI(0) = \frac{-w}{120l} (l-0)^5 + c_1(0) + c_2$

$$c_2 = \frac{wl^5}{120l}$$

$$c_2 = \frac{wl^4}{120}$$

$(1) \Rightarrow EI.\frac{dy}{dx} = \frac{w}{24l}(l-x)^4 - \frac{wl^3}{24} \rightarrow (3)$

$(2) \Rightarrow EI.y = \frac{-w}{120l}(l-x)^5 - \frac{wl^3}{24}x + \frac{wl^4}{120} \rightarrow (4)$

Slope at B.

At $x = l$, $\frac{dy}{dx} = \theta_B$

$(3) \Rightarrow EI.\theta_B = \frac{w}{24l}(0) - \frac{wl^3}{24}$

$$\boxed{\theta_B = \frac{-wl^3}{24EI}}$$

Deflection at B

At $x = l$, $y = y_B$

$(4) \Rightarrow EI.y_B = \frac{-w}{120l}(0) - \frac{wl^3}{24}(l) + \frac{wl^4}{120}$

$$EI.y_B = \frac{-5wl^4 + wl^4}{120}$$

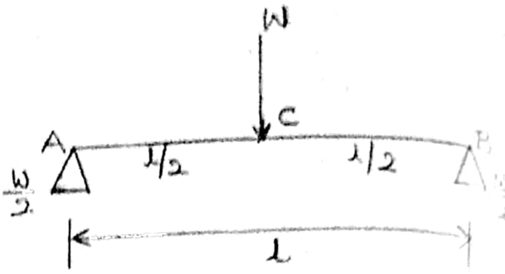
$$= \frac{-4wl^4}{120}$$

$$= \frac{-wl^4}{30}$$

$$\boxed{y_B = \frac{-wl^4}{30EI}}$$

Simply supported beams:

Simply supported beam with centre point load:



Consider a section at a distance 'x' from A.

BM: $M_x = \frac{W}{2} x$

$$EI.\frac{d^2y}{dx^2} = \frac{W}{2}x$$

Integrate once.

$$EI.\frac{dy}{dx} = \frac{W}{2}.\frac{x^2}{2} + c_1$$

$$EI.\frac{dy}{dx} = \frac{W}{4}.x^2 + c_1 \longrightarrow (1)$$

Again integrate.

$$EI.y = \frac{W}{4}.\frac{x^3}{3} + c_1x + c_2$$

$$EI.y = \frac{W}{12}x^3 + c_1x + c_2 \longrightarrow (2)$$

Boundary conditions:

At $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$.

$(1) \Rightarrow EI.(0) = \frac{W}{4} \times \frac{l^2}{4} + c_1$

$$c_1 = \frac{-Wl^2}{16}$$

At $x=0$, $y=0$

$$② \Rightarrow EI(0) = \frac{w}{12}(0) + c_1(0) + c_2$$

$$c_2 = 0$$

$$EI\frac{dy}{dx} = \frac{w}{4}x^2 - \frac{wl^2}{16} \longrightarrow ③$$

$$EI.\, y = \frac{w}{12}x^3 - \frac{wl^2}{16}x \longrightarrow ④$$

Slope at A:

At $x=0$, $\frac{dy}{dx} = \theta_A$.

$$EI.\,\theta_A = \frac{w}{4}(0) - \frac{wl^2}{16}$$

$$\boxed{\theta_A = \frac{-wl^2}{16EI}}$$

Max Deflection at C:

At $x = \frac{l}{2}$, $y = y_c$.

$$EI.\, y_c = \frac{w}{12} \times \frac{l^3}{8} - \frac{wl^2}{16} \times \frac{l}{2}$$

$$= \frac{wl^3}{96} - \frac{wl^3}{32}$$

$$= \frac{wl^3 - 3wl^3}{96}$$

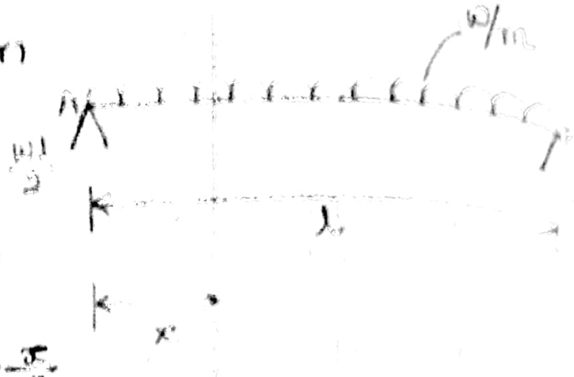
$$= \frac{-2wl^3}{96}$$

$$= \frac{-wl^3}{48}$$

$$\boxed{y_c = \frac{-wl^3}{48EI}}$$

Simply supported beam with UDL:

Consider a section at a distance 'x' from A.



BM: $M_x = \frac{wl}{2}x - wx \cdot \frac{x}{2}$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

Integrate once

$$EI \cdot \frac{dy}{dx} = \frac{wl}{2} \cdot \frac{x^2}{2} - \frac{w}{2} \cdot \frac{x^3}{3}$$

$$EI \cdot \frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + c_1 \longrightarrow (1)$$

Again integrate

$$EI \cdot y = \frac{wl}{4} \cdot \frac{x^3}{3} - \frac{w}{6} \cdot \frac{x^4}{4} + c_1 x + c_2$$

$$EI \cdot y = \frac{wlx^3}{12} - \frac{wx^4}{24} + c_1 x + c_2 \longrightarrow (2)$$

Boundary conditions:

At $x = \frac{l}{2}$, $\frac{dy}{dx} = 0$

$$(3) \Rightarrow EI \cdot (0) = \frac{wl}{4} \cdot \frac{l^2}{4} - \frac{w}{6} \cdot \frac{l^3}{8} + c_1$$

$$c_1 = \frac{wl^3}{48} - \frac{wl^3}{16}$$

$$c_1 = \frac{wl^3 - 3wl^3}{48} = \frac{-2wl^3}{48}$$

$$c_1 = \frac{-wl^3}{24}$$

At $x=0$, $y=0$

$$\Rightarrow EI(0) = \frac{wl}{12}(0) - \frac{w}{24}(0) + c_1(0) + c_2.$$

$$c_2 = 0$$

$$EI.\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} - \frac{wl^3}{24} \longrightarrow (3)$$

$$EI.y = \frac{wlx^3}{12} - \frac{wx^4}{24} - \frac{wl^3}{24}x \longrightarrow (4)$$

Slope at A:

At $x=0$, $\frac{dy}{dx} = \theta_A$

$$(3) \Rightarrow EI.\theta_A = 0-0-\frac{wl^3}{24}$$

$$\boxed{\theta_A = \frac{-wl^3}{24EI}}$$

Max. deflection at C:

At $x = \frac{l}{2}$, $y = y_c$

$$(4) \Rightarrow EI.y_c = \frac{wl}{12}.\frac{l^3}{8} - \frac{w}{24}\frac{l^4}{16} - \frac{wl^3}{24}.\frac{l}{2}$$

$$= \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}$$

$$= \frac{4wl^4 - wl^4 - 8wl^4}{384}$$

$$= \frac{-5wl^4}{384}$$

$$\boxed{y_c = \frac{-5wl^4}{384EI}}$$

Macaulay's method:

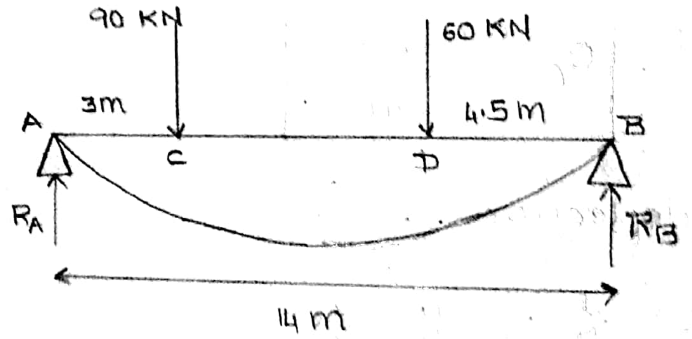
* A steel girder of uniform section 14 m long is simply supported at its ends carries loads of 90 KN and 60 KN at two points 3m and 4.5 m from the ends respectively.

① Calculate the deflection at the girder at the point under two loads.

② The max deflection.

Take $I = 64 \times 10^{-4}\ m^4$ and $E = 210 \times 10^{-6}\ KN/m^2$

Ans



Reactions:

$\Sigma V = 0 \Rightarrow R_A + R_B = 90 + 60 = 150\ KN$

$\Sigma M_A = 0 \Rightarrow R_B \times 14 - 60 \times (3+6.5) - 90(3) = 0$

$R_B = 60\ KN$

$R_A = 150 - 60$

$R_A = 90\ KN$

BM: $M_x = 90x - 90(x-3) - 60(x-9.5)$

$EI \frac{d^2y}{dx^2} = 90x - 90(x-3) - 60(x-9.5)$

Integrate once

$$EI.\frac{dy}{dx} = 90.\frac{x^2}{2} - 90\frac{(x-3)^2}{2} - 60\frac{(x-9.5)^2}{2} + C_1$$

Again integrate.

$$EI.y = 45.\frac{x^3}{3} - 45\frac{(x-3)^3}{3} - 30.\frac{(x-9.5)^3}{3} + C_1 x + C_2$$

At $x=0,\ y=0$.

$$EI.(0) = \frac{45}{3}(0) - 0 - 0 + C_1(0) + C_2$$

$$C_2 = 0$$

At $x = 14\ m,\ y=0$

$$EI.(0) = \frac{45}{3}(14)^3 - \frac{45}{3}(11)^3 - 10\,(4.5)^3 + C_1(14)$$

$$0 =$$

$$C_1 = -1448.84$$

$$EI.\frac{dy}{dx} = 45x^2 - 45(x-3)^2 - 30(x-9.5)^2 - 1448.84$$

$$EI.y = 15x^3 - 15(x-3)^3 - 10(x-9.5)^2 - 1448.84x$$

(i) y_c

At $x=3,\ y=y_c$

$$EIy_c = 15(3)^3 - 0 - 0 - (1448.84)3$$

$$y_c = \frac{-3941.52}{EI}$$

$$y_c = \frac{-3941.52}{64 \times 10^{-4} \times 210 \times}$$

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$$y_c = -2.932 \times 10^{6}$$

y_B

At $x = 9.5$, $y = y_B$.

$$EI.\,y_B = 15(9.5)^3 - 15(6.5)^3 - 0 - 1448.84 \times 9.5$$

$$y_B = \frac{-5022.7}{EI}$$

$$y_B = \frac{-5022.7}{64 \times 10^{-4} \times 210 \times 10^{-3}}$$

$$y_B = -3.73 \times 10^{6}$$

(ii) Max. deflection = ?

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 45x^2 - 45(x-3)^2 - \cancel{30(x-9.5)^2} - 1448.84 = 0$$

$$45x^2 - 45(x^2 + 9 - 6x) - 1448.84 = 0$$

$$270x - 405 - 1448.84 = 0$$

$$270x = 1853.84$$

$$x = 6.86 \text{ m.}$$

$\therefore$ Max. deflection . $x = 6.86$ m, $y = y_{max}$.

Lecture No. 6

-Thick Cylinders-

6-1 Difference in treatment between thin and thick cylinders - basic assumptions:

The theoretical treatment of thin cylinders assumes that the hoop stress is constant across the thickness of the cylinder wall (Fig. 6.1), and also that there is no pressure gradient across the wall. Neither of these assumptions can be used for thick cylinders for which the variation of hoop and radial stresses is shown in (Fig. 6.2), their values being given by the Lame equations: -

$$\sigma_H = A + \frac{B}{r^2} \qquad \text{...6.1}$$

$$\sigma_r = A - \frac{B}{r^2} \qquad \text{...6.2}$$

Where: -

σ_H= Hoop stress ($\frac{N}{m^2} = Pa$).

σ_r= Radial stress ($\frac{N}{m^2} = Pa$).

r= Radius (m). A and B are Constants.

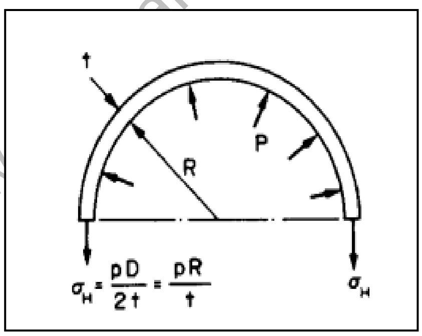

Figure 6.1: - Thin cylinder subjected to internal pressure.

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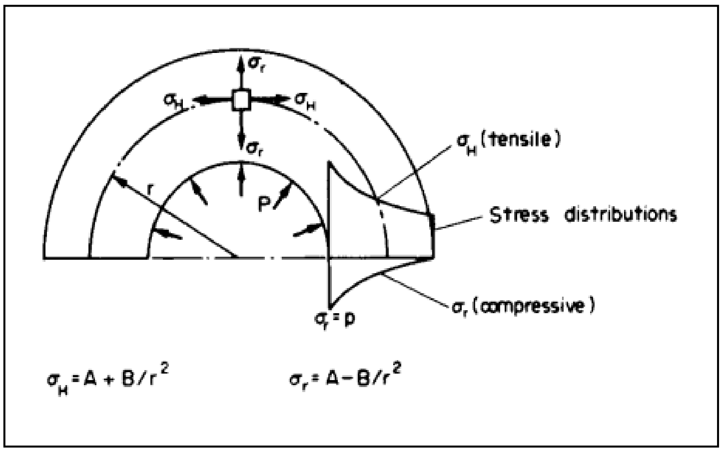


Figure 6.2: - Thick cylinder subjected to internal pressure.

6-2 Thick cylinder- internal pressure only: -

Consider now the thick cylinder shown in (Fig. 6.3) subjected to an internal pressure P, the external pressure being zero.

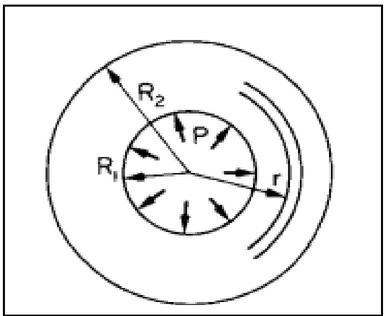


Figure 6.3: - Cylinder cross section.

The two known conditions of stress which enable the Lame constants A and B to be determined are:

At $r = R_1$, $\sigma_r = -P$ and at $r = R_2$, $\sigma_r = 0$

Note: -The internal pressure is considered as a negative radial stress since it will produce a radial compression (i.e. thinning) of the cylinder walls and the normal stress convention takes compression as negative.

Substituting the above conditions in eqn. (6.2),

$$\sigma_r = A - \frac{B}{r^2}$$

$$-P = A - \frac{B}{{R_1}^2} \text{ and } 0 = A - \frac{B}{{R_2}^2}$$

Then $A = \frac{P{R_1}^2}{({R_2}^2 - {R_1}^2)}$ and $B = \frac{P{R_1}^2{R_2}^2}{({R_2}^2 - {R_1}^2)}$

Substituting A and B in equations 6.1 and 6.2,

$$\boxed{\sigma_r = \frac{P{R_1}^2}{({R_2}^2 - {R_1}^2)}\left[1 - \frac{{R_2}^2}{r^2}\right]} \qquad \text{...6.3}$$

$$\boxed{\sigma_H = \frac{P{R_1}^2}{({R_2}^2 - {R_1}^2)}\left[1 + \frac{{R_2}^2}{r^2}\right]} \qquad \text{...6.4}$$

6-3 Longitudinal stress: -

Consider now the cross-section of a thick cylinder with closed ends subjected to an internal pressure P_1 and an external pressure P_2, (Fig. 6.4).

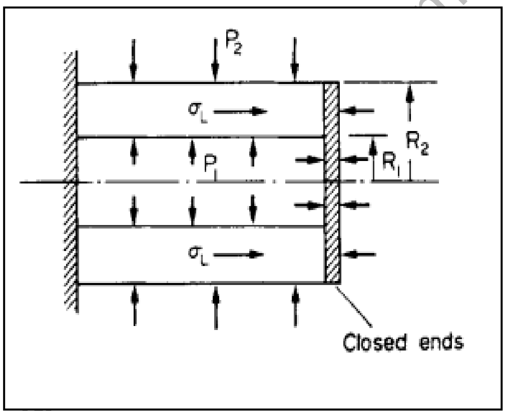


Figure 6.4: - Cylinder longitudinal section.

For horizontal equilibrium:

$${P_1}^2 * \pi {R_1}^2 - {P_2}^2 * \pi {R_2}^2 = \sigma_L * \pi\left[{R_2}^2 - {R_1}^2\right]$$

Where σ_L is the longitudinal stress set up in the cylinder walls, Longitudinal stress,

$$\boxed{\sigma_L = \frac{P_1 {R_1}^2 - P_2 {R_2}^2}{({R_2}^2 - {R_1}^2)}} \quad \ldots .6.5$$

But for P_2 =0 (no external pressure),

$$\sigma_L = \frac{P_1 {R_1}^2}{({R_2}^2 - {R_1}^2)}$$ = A, constant of the Lame equations. ….6.6

6-4 Maximum shear stress: -

It has been stated in section 6.1 that the stresses on an element at any point in the cylinder wall are principal stresses.

It follows, therefore, that the maximum shear stress at any point will be given by equation of Tresca theory as,

$$\frac{\sigma_y}{2} = \tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \quad \ldots .6.7$$

$$\tau_{max} = \frac{\sigma_H - \sigma_r}{2} \quad \ldots .6.8$$

$$\tau_{max} = \frac{1}{2}\left[\left(A + \frac{B}{r^2}\right) - \left(A - \frac{B}{r^2}\right)\right] \quad \ldots .6.9$$

$$\boxed{\tau_{max} = \frac{B}{r^2}} \quad \ldots .6.10$$

6-5 Change of diameter: -

It has been shown that the diametral strain on a cylinder equals the hoop or circumferential strain.

Change of diameter = diametral strain x original diameter.

= circumferential strain x original diameter.

With the principal stress system of hoop, radial and longitudinal stresses, all assumed tensile, the circumferential strain is given by

$$\boxed{\epsilon_H = \frac{1}{E}(\sigma_H - \upsilon\sigma_r - \upsilon\sigma_L)} \qquad \text{....6.11}$$

$$\delta D = \frac{D}{E}(\sigma_H - \upsilon\sigma_r - \upsilon\sigma_L) \qquad \text{....6.12}$$

Similarly, the change of length of the cylinder is given by,

$$\boxed{\delta L = \frac{L}{E}(\sigma_L - \upsilon\sigma_r - \upsilon\sigma_H)} \qquad \text{....6.13}$$

6-6 Comparison with thin cylinder theory: -

In order to determine the limits of D/t ratio within which it is safe to use the simple thin cylinder theory, it is necessary to compare the values of stress given by both thin and thick cylinder theory for given pressures and D/t values. Since the maximum hoop stress is normally the limiting factor, it is this stress which will be considered.

Thus for various D/t ratios the stress values from the two theories may be plotted and compared; this is shown in (Fig. 6.5).

Also indicated in (Fig. 6.5) is the percentage error involved in using the thin cylinder theory.

It will be seen that the error will be held within 5 % if D/t ratios in excess of 15 are used.

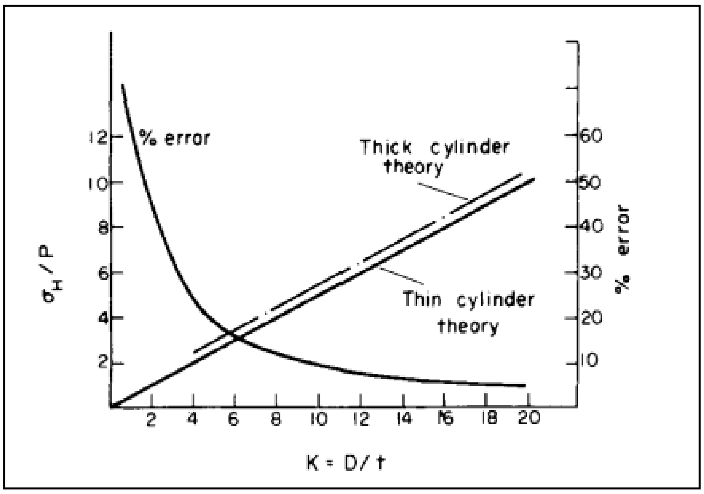


Figure 6.5: - Comparison of thin and thick cylinder theories for various diameter/thickness ratios.

6-7 Compound cylinders:-

From the sketch of the stress distributions in Figure 6.6 it is evident that there is a large variation in hoop stress across the wall of a cylinder subjected to internal pressure. The material of the cylinder is not therefore used to its best advantage. To obtain a more uniform hoop stress distribution, cylinders are often built up by shrinking one tube on to the outside of another. When the outer tube contracts on cooling the inner tube is brought into a state of compression. The outer tube will conversely be brought into a state of tension. If this compound cylinder is now subjected to internal pressure the resultant hoop stresses will be the algebraic sum of those resulting from internal pressure and those resulting from shrinkage as drawn in Fig. 6.6; thus a much smaller total fluctuation of hoop stress is obtained. A similar effect is obtained if a cylinder is wound with wire or steel tape under tension.

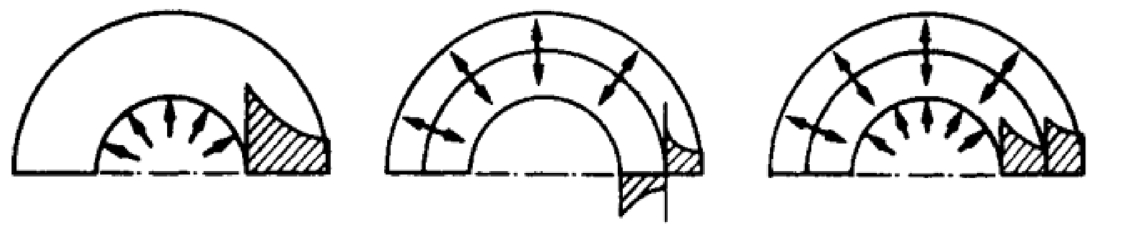


Figure 6.6: - Compound cylinders-combined internal pressure and shrinkage effects.

The method of solution for compound cylinders constructed from similar materials is to break the problem down into three separate effects:

(a) shrinkage pressure only on the inside cylinder.

(b) shrinkage pressure only on the outside cylinder.

(c) internal pressure only on the complete cylinder.

For each of the resulting load conditions there are two known values of radial stress which enable the Lame constants to be determined in each case

condition (a) shrinkage - internal cylinder:

At $r = R_1$, $\sigma_r = 0$

At $r = R_c$, $\sigma_r = -p$ (compressive since it tends to reduce the wall thickness)

condition (b) shrinkage - external cylinder:

At $r = R_2$, $\sigma_r = 0$

At $r = R_c$, $\sigma_r = -p$

condition (c) internal pressure - compound cylinder:

At $r = R_2$, $\sigma_r = 0$

At $r = R_1$, $\sigma_r = -P_1$

Thus for each condition the hoop and radial stresses at any radius can be evaluated and the principle of superposition applied, i.e. the various stresses are then combined algebraically to produce the stresses in the compound cylinder subjected to both shrinkage and internal pressure. In practice this means that the compound cylinder is able to withstand greater internal pressures before failure occurs or, alternatively, that a thinner compound cylinder (with the associated reduction in material cost) may be used to withstand the same internal pressure as the single thick cylinder it replaces.

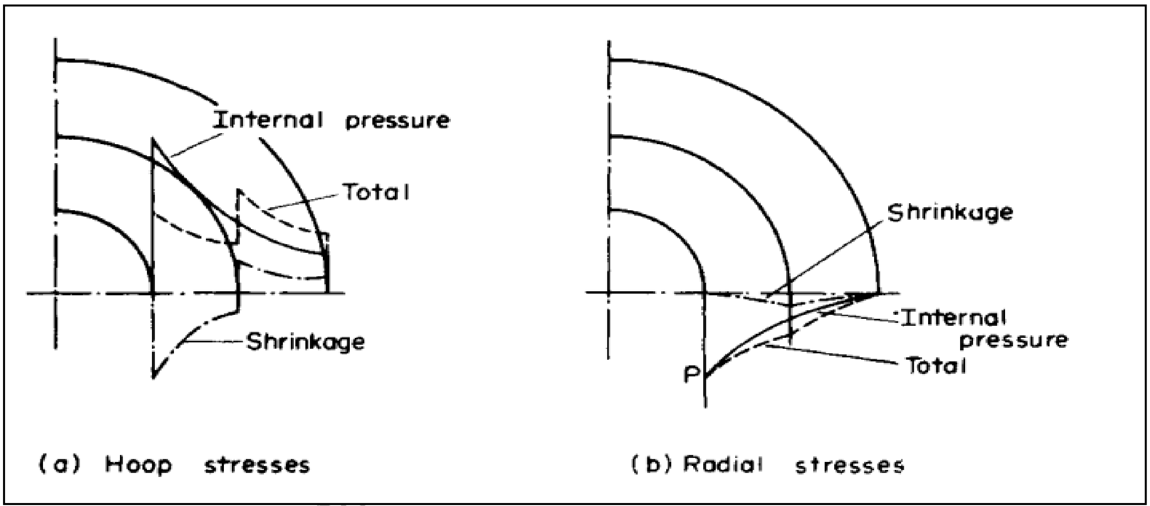
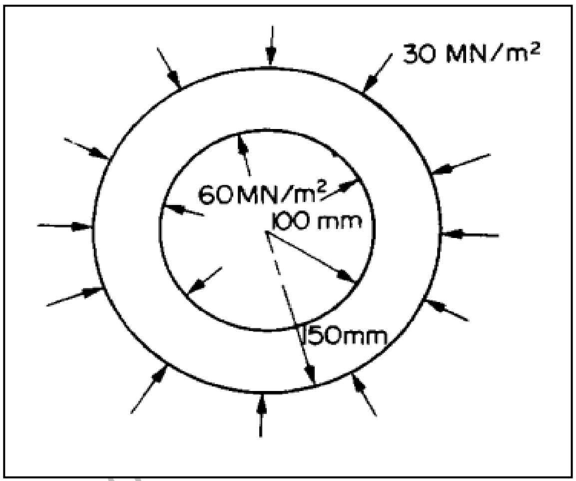


Figure 6.7: - Distribution of hoop and radial stresses through the walls of a compound cylinder.

Example 6-1: - A thick cylinder of 100 mm internal radius and 150 mm external radius is subjected to an internal pressure of 60 MN/m^2 and an external pressure of 30 MN/m^2. Determine the hoop and radial stresses at the inside and outside of the cylinder together with the longitudinal stress if the cylinder is assumed to have closed ends.



Solution: -

At r= 0.1m, σ_r=-60MPa.

r= 0.15 m, σ_r=-30 MPa.

So,

-60=A-100B ….1

-30=A-44.5B ….2

By solving equations 1 and 2,

A=-6 and B=0.54

Therefore at r=0.1m

$$\sigma_H = A + \frac{B}{r^2} = -6 + \frac{0.54}{(0.1)^2} = 48\text{MPa}.$$

At r=0.15m,

$$\sigma_H = A + \frac{B}{r^2} = -6 + \frac{0.54}{(0.15)^2} = 18\text{MPa}$$

$$\sigma_L = \frac{P_1 {R_1}^2 - P_2 {R_2}^2}{({R_2}^2 - {R_1}^2)} = \frac{60(0.1)^2 - 30(0.15)^2}{(0.15^2 - 0.1^2)} = \text{-6MPa i.e. compression.}$$

Example 6-2: - An external pressure of 10 MN/m^2 is applied to a thick cylinder of internal diameter 160 mm and external diameter 320 mm. If the maximum hoop stress permitted on the inside wall of the cylinder is limited to 30 MN/m^2, what maximum internal pressure can be applied assuming the cylinder has closed ends? What will be the change in outside diameter when this pressure is applied? E = 207 GN/m^2, v = 0.29.

Solution: -

At r =0.08m, σ_r=-P, $\frac{1}{r^2}$=156

At r = 0.16 m, σ_r= -10, $\frac{1}{r^2}$=39

And at r =0.08m, σ_H= 30MPa

- 10 = A - 39B ….(1) 30 = A + 156B ….(2)

Subtracting (1) from (2), A=-2 and B=0.205

Therefore, at r = 0.08, σ_r = - p = A -156B= -2-156*0.205 = -34MPa.

i.e. the allowable internal pressure is 34 MN/m^2.

The change in diameter is given by

$$\delta D = \frac{D}{E}(\sigma_H - \upsilon\sigma_r - \upsilon\sigma_L) \quad \text{…. (3)}$$

But σ_r = - 10 MN/m^2, $\sigma_H = A + \frac{B}{r^2}$=-2+39*0.205=6 MN/m^2

And finally, $\sigma_L = \frac{P_1{R_1}^2 - P_2{R_2}^2}{({R_2}^2 - {R_1}^2)} = \frac{(34*0.08^2 - 10*0.16^2)}{(0.16^2 - 0.08^2)} =$ -1.98 MPa i.e compressive.

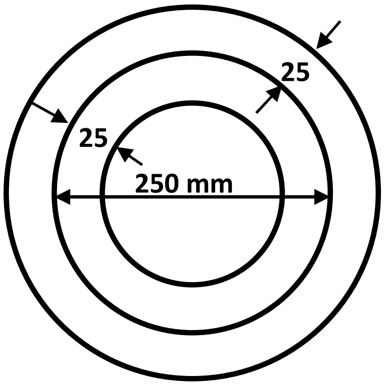
Substitute σ_r, σ_H and σ_L in eqn. 3,

$$\delta D = \frac{0.32}{207*10^9}[6 - 0.29(-10) - 0.29(-1.98)]10^6 = 14.7\mu m$$

Example 6-3: - A compound cylinder is formed by shrinking a tube of 250 mm internal diameter and 25 mm wall thickness onto another tube of 250 mm external diameter and 25 mm wall thickness, both tubes being made of the same material. The stress set up at the junction owing to shrinkage is 10 MN/m^2. The compound tube is then subjected to an internal pressure of 80 MN/m^2. Compare the hoop stress distribution now obtained with that of a single cylinder of 300 mm external diameter and 50 mm thickness subjected to the same internal pressure.

A solution is obtained as described before by considering the effects of shrinkage and internal pressure separately and combining the results algebraically.

Shrinkage only - outer tube,



At r = 0.15, $\sigma_r = 0$ and at r = 0.125, σ_r = -10 MN/m^2

$$0 = A - \frac{B}{(0.15^2)} = A - 44.5B$$

$$-10 = A - \frac{B}{(0.125^2)} = A - 64B$$

$$\therefore B = 0.514, \qquad A = 22.85$$

hoop stress at 0.15 m radius$= A + 44.5B = 45.7\ MPa.$

hoop stress at 0.125 m radius$= A + 64B = 55.75MPa.$

Shrinkage only- inner tube,

At r = 0.10, $\sigma_r = 0$ and at r = 0.125, σ_r = - 10 MN/m^2

$$0 = A - \frac{B}{(0.1^2)} = A - 100B$$

$$-10 = A - \frac{B}{(0.125^2)} = A - 64B$$

$\therefore B = -0.278, \qquad A = -27.8$

hoop stress at 0.125 m radius$= A + 64B = -45.6\ MPa.$

hoop stress at 0.10 m radius$= A + 100B = -55.6\ MPa.$

Considering internal pressure only (on complete cylinder)

At r = 0.15, $\sigma_r = 0$ and at r = 0.10, σ_r = -80

$$0 = A - \frac{B}{(0.15^2)} = A - 44.5B$$

$$-80 = A - \frac{B}{(0.1^2)} = A - 100B$$

$\therefore B = 1.44, \qquad A = 64.2$

At r = 0.15 m, σ_H = A + 44.5B = 128.4 MN/m^2

r= 0.125m, σ_H = A + 64B = 156.4 MN/m^2

r= 0.1m , σ_H = A + 100B = 208.2 MN/m^2

The resultant stresses for combined shrinkage and internal pressure are then:

outer tube: r = 0.15 σ_H = 128.4 +45.7 = 174.1 MN/m^2 .

$r = 0.125 \qquad \sigma_H = 156.4 + 55.75 = 212.15\ MN/m^2$.

inner tube: $r = 0.125 \qquad \sigma_H = 156.4 - 45.6 = 110.8\ MN/m^2$.

$r = 0.1 \qquad \sigma_H = 208.2 - 55.6 = 152.6\ MN/m^2$.

……………………….END…………………………

UNIT-VI

Thin Cylinders Subjected to Internal Pressure:

When a thin – walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder materials, namely

• Circumferential or hoop stress

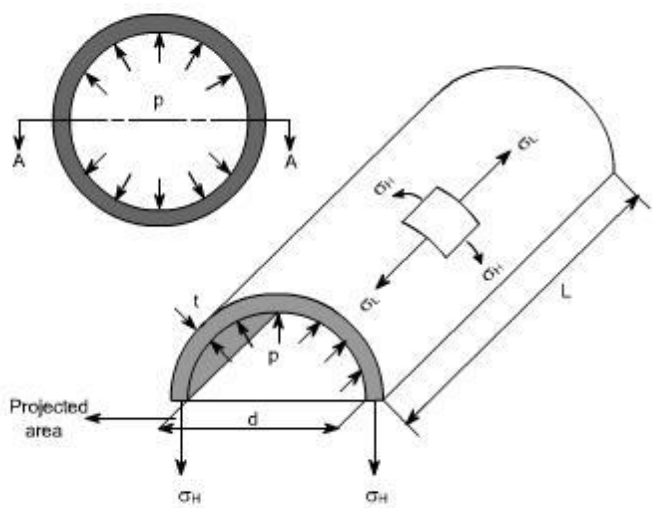
• The radial stress

• Longitudinal stress

now let us define these stresses and determine the expressions for them

Hoop or circumferential stress:

This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.



In the figure we have shown a one half of the cylinder. This cylinder is subjected to an internal pressure p.

i.e. p = internal pressure

d = inside diametre

L = Length of the cylinder

t = thickness of the wall

Total force on one half of the cylinder owing to the internal pressure 'p'

= p x Projected Area

= p x d x L

= **p .d. L** ------- (1)

The total resisting force owing to hoop stresses σH set up in the cylinder walls

= **2 .** σH **.L.t** ---------(2)

Because σH .L.t. is the force in the one wall of the half cylinder.

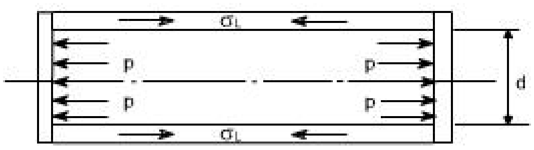
the equations (1) & (2) we get

2 . σH **. L . t** = **p . d . L**

σH = **(p . d) / 2t**

Longitudinal Stress:

Consider now again the same figure and the vessel could be considered to have closed ends and contains a fluid under a gage pressure p.Then the walls of the cylinder will have a longitudinal stress as well as a ciccumferential stress.

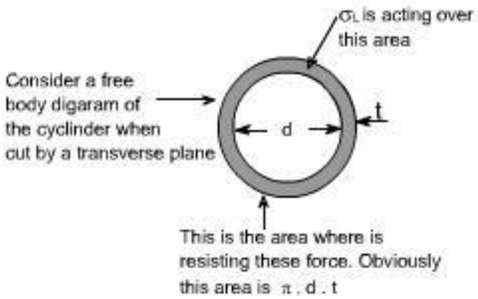


Total force on the end of the cylinder owing to internal pressure

= pressure x area

= p x Π /4 x d2

Area of metal resisting this force =dt



Hence the longitudnal stresses

$$\text{Set up} = \frac{\text{force}}{\text{area}} = \frac{[p \times \pi d^2/4]}{\pi dt}$$

$$= \frac{pd}{4t} \quad \text{or} \quad \sigma_L = \frac{pd}{4t}$$

or alternatively from equilibrium conditions

$$\sigma_L.(\pi dt) = p.\frac{\pi d^2}{4}$$

$$\text{Thus } \boxed{\sigma_L = \frac{pd}{4t}}$$

Change in Dimensions :

The change in length of the cylinder may be determined from the longitudinal strain.

Since whenever the cylinder will elongate in axial direction or longitudinal direction, this will also get decreased in diametre or the lateral strain will also take place. Therefore we will have to also take into consideration the lateral strain.as we know that the poisson's ratio (ν) is

$$\nu = \frac{-\text{lateral strain}}{\text{longitudnal strain}}$$

where the -ve sign emphasized that the change is negative

Let E = Young's modulus of elasticity

Resultant Strain in longitudnal direction $= \frac{\sigma_L}{E} - \nu \frac{\sigma_H}{E} = \frac{1}{E}(\sigma_L - \nu\sigma_H)$

recalling

$$\sigma_L = \frac{pd}{4t} \qquad \sigma_H = \frac{pd}{2t}$$

$\in_1$ (longitudnal strain) $= \frac{pd}{4Et}[1-2\nu]$

or

Change in Length = Longitudalstrain x original Length

$= \in_1 . L$

Similarly the hoop Strain $\in_2 = \frac{1}{E}(\sigma_H - \nu\sigma_L) = \frac{1}{E}\left[\frac{pd}{2t} - \nu\frac{pd}{4t}\right]$

$$\in_2 = \frac{pd}{4Et}[2-\nu]$$

Infact $\in_2$ is the hoop strain if we just go by the definition then

$$\in_2 = \frac{\text{Change in diametre}}{\text{Original diametre}} = \frac{\delta d}{d}$$

where d = original diameter.

if we are interested to find out the change in diametre then

Changein diametre $= \in_2$ .Original diametre

i.e $\delta d = \in_2 .d$ substituting the value of $\in_2$ we get

$$\delta d = \frac{p.d}{4.t.E}[2-\nu].d$$

$$= \frac{p.d^2}{4.t.E}[2-\nu]$$

i.e $\boxed{\delta d = \frac{p.d^2}{4.t.E}[2-\nu]}$

Volumetric Strain or Change in the Internal Volume:

When the thin cylinder is subjected to the internal pressure as we have already calculated that there is a change in the cylinder dimensions i.e, longitudinal strain and hoop strains come into picture. As a result of which there will be change in capacity of the cylinder or there is a change in the volume of the cylinder hence it becomes imperative to determine the change in volume or the volumetric strain.

The capacity of a cylinder is defined as

V = Area X Length

= d2/4 x L

Let there be a change in dimensions occurs, when the thin cylinder is subjected to an internal pressure.

(i) The diameter **d** changes to **d + d**

(ii) The length **L** changes to **L + L**

Therefore, the change in volume = Final volume Original volume

$$= \frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L$$

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\frac{\pi}{4}[d+\delta d]^2.(L+\delta L) - \frac{\pi}{4}d^2.L}{\frac{\pi}{4}d^2.L}$$

$$\epsilon_v = \frac{\{[d+\delta d]^2.(L+\delta L) - d^2.L\}}{d^2.L} = \frac{\{(d^2+\delta d^2+2d.\delta d).(L+\delta L) - d^2.L\}}{d^2.L}$$

simplifying and neglecting the products and squares of small quantities, i.e. δd & δL

hence

$$= \frac{2d.\delta d.L + \delta L.d^2}{d^2 L} = \frac{\delta L}{L} + 2.\frac{\delta d}{d}$$

By definition $\frac{\delta L}{L}$ = Longitudnal strain

$\frac{\delta d}{d}$ = hoop strain, Thus

$\boxed{\textbf{Volumetric strain = longitudnal strain + 2 x hoop strain}}$

on substituting the value of longitudnal and hoop strains we get

$$\epsilon_1 = \frac{pd}{4tE}[1-2\nu] \quad \& \quad \epsilon_2 = \frac{pd}{4tE}[1-2\nu]$$

$$\text{or Volumetric} = \epsilon_1 + 2\epsilon_2 = \frac{pd}{4tE}[1-2\nu] + 2.\left(\frac{pd}{4tE}[1-2\nu]\right)$$

$$= \frac{pd}{4tE}\{1-2\nu+4-2\nu\} = \frac{pd}{4tE}[5-4\nu]$$

$$\text{Volumetric Strain} = \frac{pd}{4tE}[5-4\nu] \qquad \text{or} \quad \boxed{\epsilon_V = \frac{pd}{4tE}[5-4\nu]}$$

Therefore to find but the increase in capacity or volume, multiply the volumetric strain by original volume.

Hence

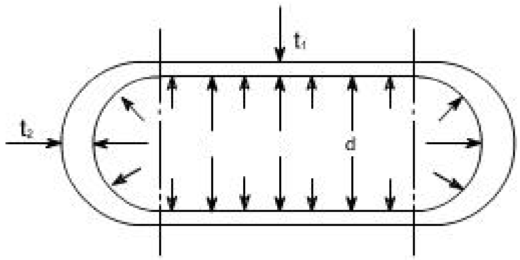
Change in Capacity / Volume or

$$\boxed{\text{Increase in volume} = \frac{pd}{4tE}[5-4\nu]V}$$

Cylindrical Vessel with Hemispherical Ends:

Let us now consider the vessel with hemispherical ends. The wall thickness of the cylindrical and hemispherical portion is different. While the internal diameter of both the portions is assumed to be equal

Let the cylindrical vassal is subjected to an internal pressure p.



For the Cylindrical Portion

hoop or circumferential stress $= \sigma_{HC}$ 'c' here synifies the cylindrical portion.

$$= \frac{pd}{2t_1}$$

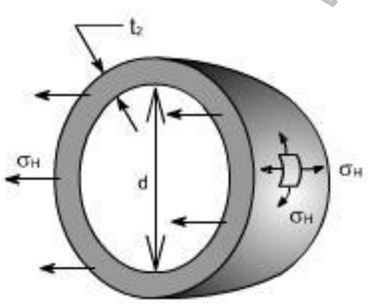
longitudnal stress $= \sigma_{LC}$

$$= \frac{pd}{4t_1}$$

$$\text{hoop or circumferential strain } \epsilon_2 = \frac{\sigma_{HC}}{E} - \nu\frac{\sigma_{LC}}{E} = \frac{pd}{4t_1E}[2-\nu]$$

$$\text{or } \boxed{\epsilon_2 = \frac{pd}{4t_1E}[2-\nu]}$$

For The Hemispherical Ends:



Because of the symmetry of the sphere the stresses set up owing to internal pressure will be two mutually perpendicular hoops or circumferential stresses of equal values. Again the radial stresses are neglected in comparison to the hoop stresses as with this cylinder having thickness to diametre less than1:20.

Consider the equilibrium of the half – sphere

Force on half-sphere owing to internal pressure = pressure x projected Area

= p. Πd2/4

Resisting force = $\sigma_H \cdot \pi . d . t_2$

$$\therefore \quad p.\frac{\pi . d^2}{4} = \sigma_H \cdot \pi d . t_2$$

$$\Rightarrow \sigma_H \text{ (for sphere)} = \frac{pd}{4t_2}$$

similarly the hoop strain $= \frac{1}{E}\left[\sigma_H - \nu . \sigma_H\right] = \frac{\sigma_H}{E}[1-\nu] = \frac{pd}{4t_2E}[1-\nu]$ or $\boxed{\in_{2s} = \frac{\mathbf{pd}}{\mathbf{4t_2E}}[\mathbf{1} - \boldsymbol{\nu}]}$

Fig – shown the (by way of dotted lines) the tendency, for the cylindrical portion and the spherical ends to expand by a different amount under the action of internal pressure. So owing to difference in stress, the two portions (i.e. cylindrical and spherical ends) expand by a different amount. This incompatibly of deformations causes a local bending and sheering stresses in the neighborhood of the joint. Since there must be physical continuity between the ends and the cylindrical portion, for this reason, properly curved ends must be used for pressure vessels.

Thus equating the two strains in order that there shall be no distortion of the junction

$$\frac{pd}{4t_1E}[2-\nu] = \frac{pd}{4t_2E}[1-\nu] \quad \text{or} \quad \frac{t_2}{t_1} = \frac{1-\nu}{2-\nu}$$

-Thick Cylinders-

6-1 Difference in treatment between thin and thick cylinders - basic assumptions:

The theoretical treatment of thin cylinders assumes that the hoop stress is constant across the thickness of the cylinder wall (Fig. 6.1), and also that there is no pressure gradient across the wall. Neither of these assumptions can be used for thick cylinders for which the variation of hoop and radial stresses is shown in (Fig. 6.2), their values being given by the Lame equations: -

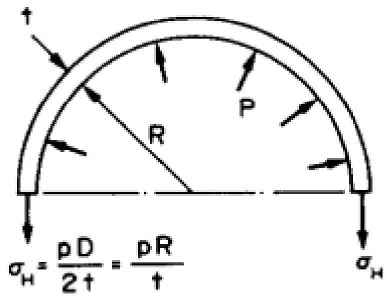


FIG 6.1 Thin cylinder subjected to internal pressure.