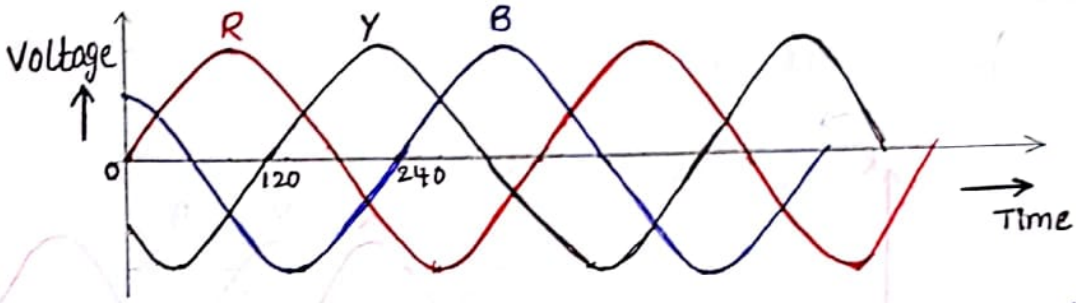
UNIT - I

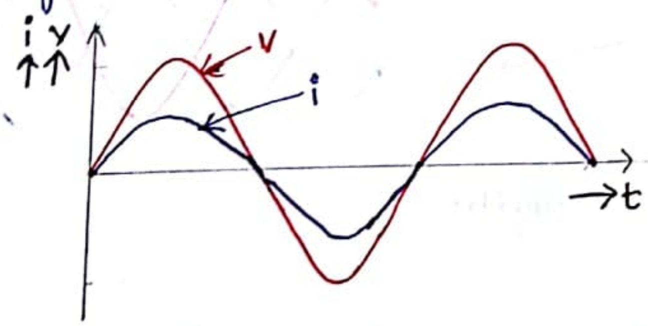
Balanced three phase circuits

A three phase system of voltages & currents is merely a combination of three single phase systems of voltages of which the three voltages differ in phase by 120° from each other in a particular sequence.



The three phase system has many advantages over the single phase system both from the utility point of view as well as from the consumer point of view. some of the advantages are as under.

1. The power in a 1-ϕ circuit is pulsating when the power factor of the circuit is unity the power becomes zero 100 times in a second in a 50Hz supply. Therefore single phase motors have a pulsating torque.



In the three phase the power supplied by each phase is pulsating, the total three phase power supplied to a balanced 3-ϕ circuit every instant of time because of this three phase motors have an absolutely uniform torque.

2. Three phase motors are more easily started than 1-ϕ motors. single phase motors are not self starting whereas three phase motors are.

PHASE SEQUENCE:

The order in which the three phase attain their peak value in a 3-ϕ AC system is known as phase sequence. The sequence depends on the rotation of the field.

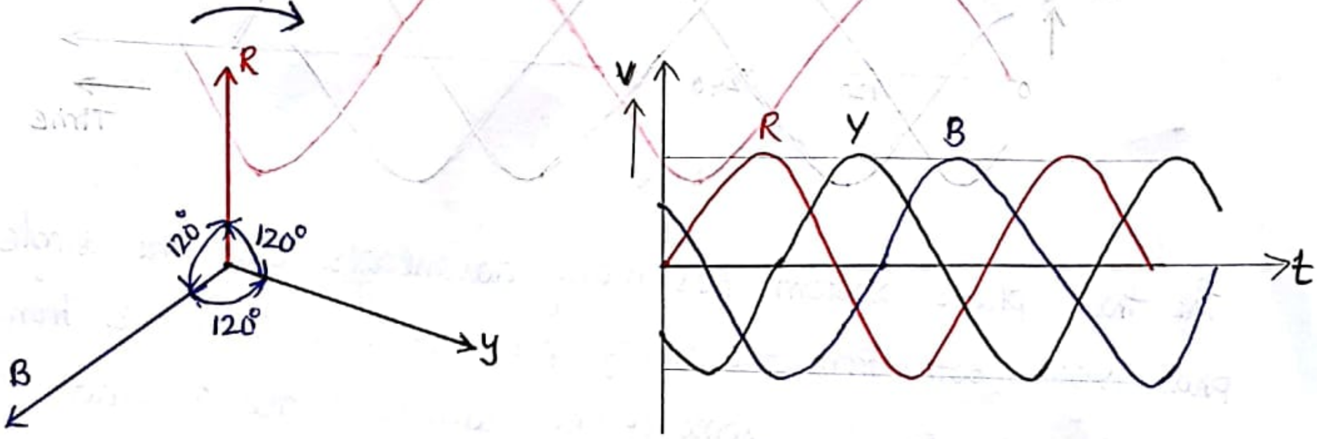


fig (a)

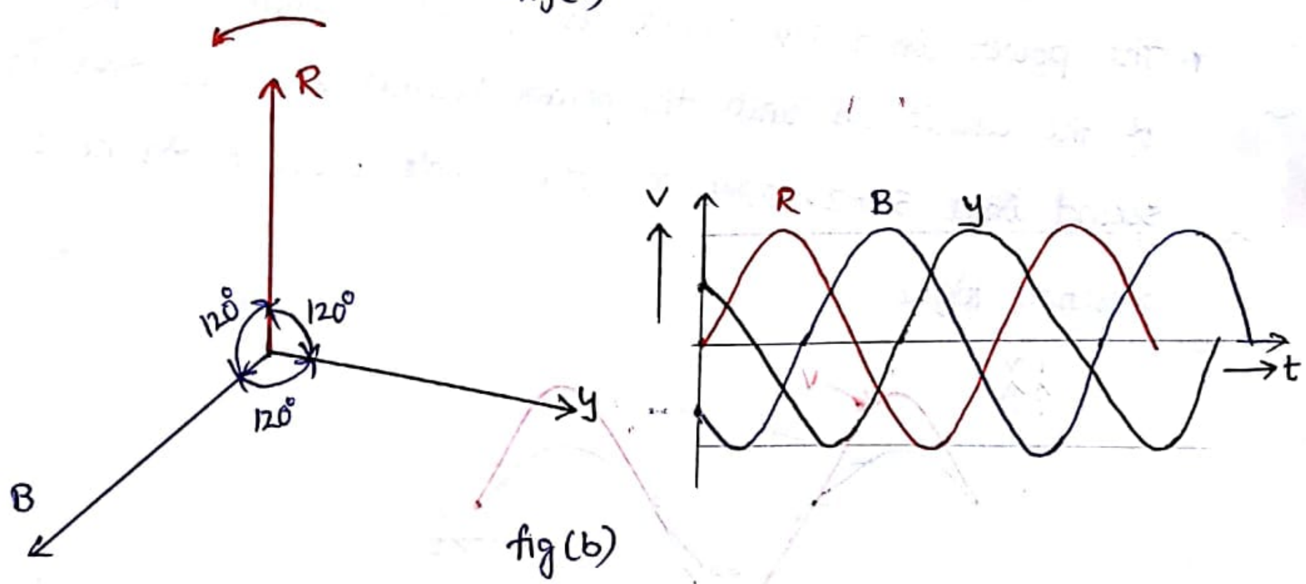


fig (b)

The fig (a) and fig (b) represents the phase sequence RYB and RBY respectively. If the field system is rotated in the clock wise direction then the sequence of the voltages in the three phase are in the order $V_R - V_y - V_B$ briefly we can say that the sequence is RYB. If the field system is rotated in anticlockwise direction then the sequence is RBY and the voltage equations can be written as

$$V_{RR'} = V_m \sin\omega t$$
$$V_{BB'} = V_m \sin(\omega t - 120)$$
$$V_{yy'} = V_m \sin(\omega t - 240)$$

If the sequence is RYB then

$$V_{RR'} = V_m \sin\omega t$$
$$V_{yy'} = V_m \sin(\omega t - 120)$$
$$V_{BB'} = V_m \sin(\omega t - 240)$$

Significance of phase sequence:

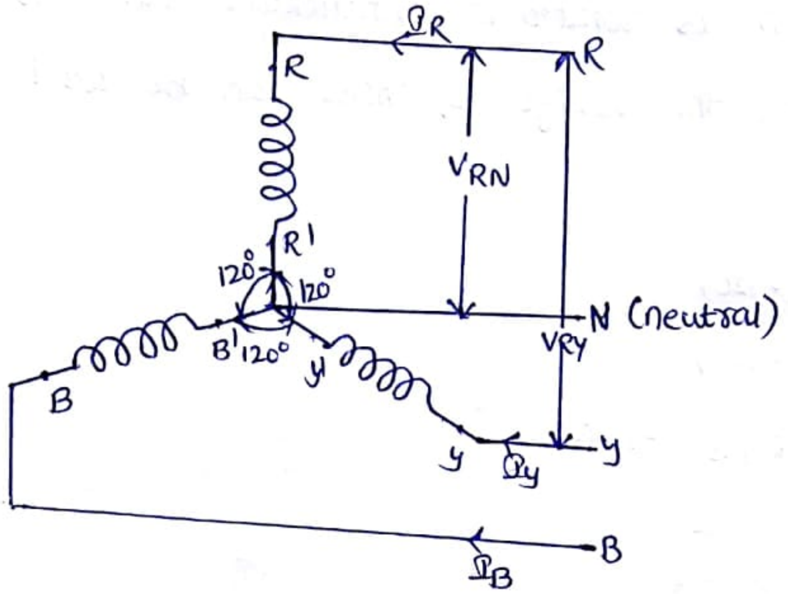
* phase sequence can be reversed by interchanging any pair of lines. In case of Induction motor reversal of sequence results in the reversal direction of motor rotation.

* phase sequence of the voltage applied to the load is determined by the order in which the 3-ϕ lines are connected.

Star and delta connection:

In a three phase system there are three independent phase windings or coils. Each phase or coil has two terminals. The coils are interconnected to form wye (Y) or delta (Δ) connected three phase system to achieve the economy and to reduce the number of conductors and thereby the complexity in the circuit.

Wye or Star connection:



I_R = line current
= phase current
V_{RN} = phase voltage
V_{RY} = line voltage

→ In this connection similar ends of the three phases are joined together within the alternator as shown in figure. The common terminal so formed is referred to as the neutral point N.

→ The terminals R, Y and B are called the line terminals of the source. The voltage b/w any line and the neutral point is called the phase voltage (V_{RN}, V_{YN}, V_{BN}) while the voltage b/w any two lines is called the line voltage (V_{RY}, V_{YB} and V_{BR}).

→ The current flowing in the lines are called line currents. The current flowing through the phases are called phase currents.

* V_{RY} indicates a voltage b/w points R and Y with R being positive with respect to point Y during its positive half cycle. ($V_{RY} = V_{RN} - V_{YN}$)

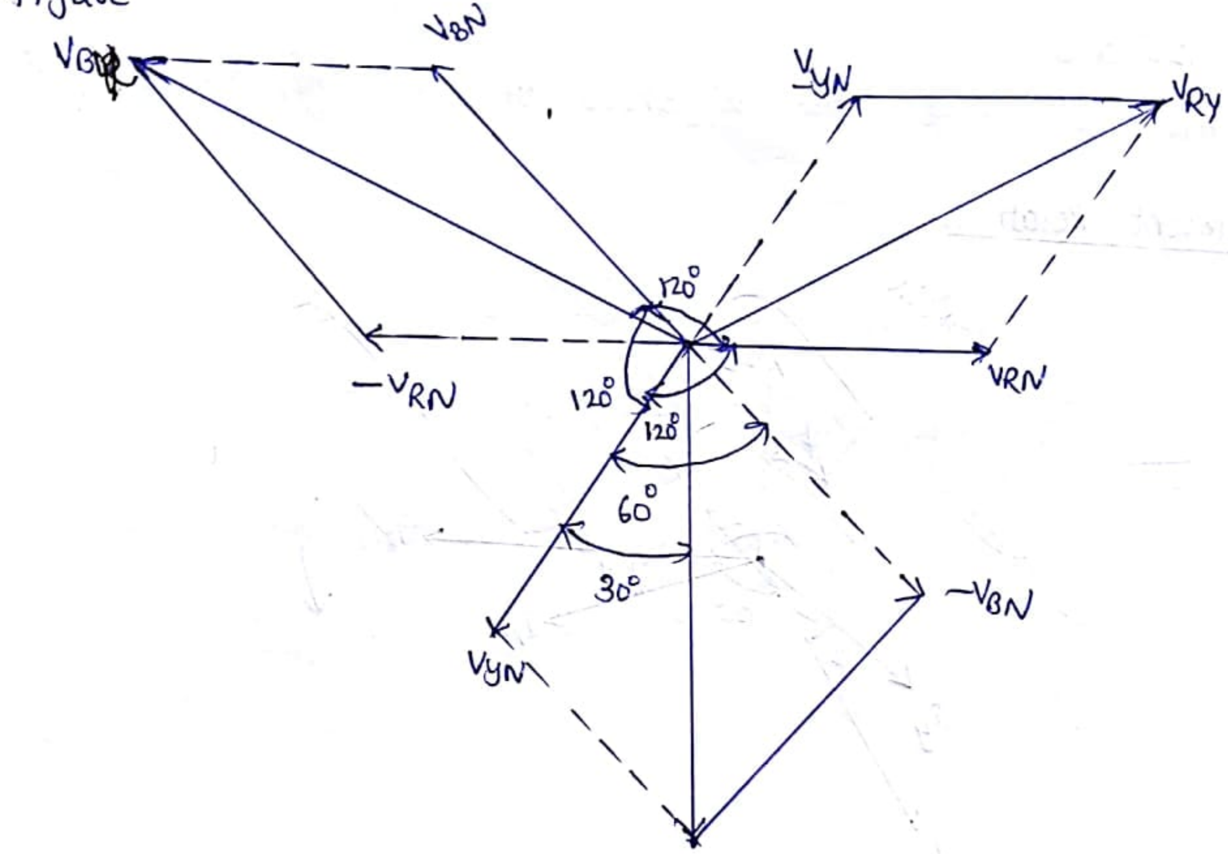
Similarly

$V_{YB} = V_{YN} - V_{BN}$

$V_{BR} = V_{BN} - V_{RN}$

Voltage Relation;

The phasors corresponding to the phase voltages constituting a 3-ϕ system can be represented by a phasor diagram as shown in figure



$V_{YB} = V_{YN} - V_{BN}$

The line voltage V_{RY} is equal to the phasor sum of V_{RN} and V_{NY} which is also equal to the phasor difference of V_{RN} and V_{YN} {$V_{YN} = -V_{NY}$}

To subtract V_{YN} from V_{RN} we reverse the phasor V_{YN} and find its phasor sum with V_{RN} as shown in above figure.

The two phasors V_{RN} and $-V_{YN}$ are equal in length and are 60° apart

$$V_{RN} = -V_{YN} = V_{ph}$$

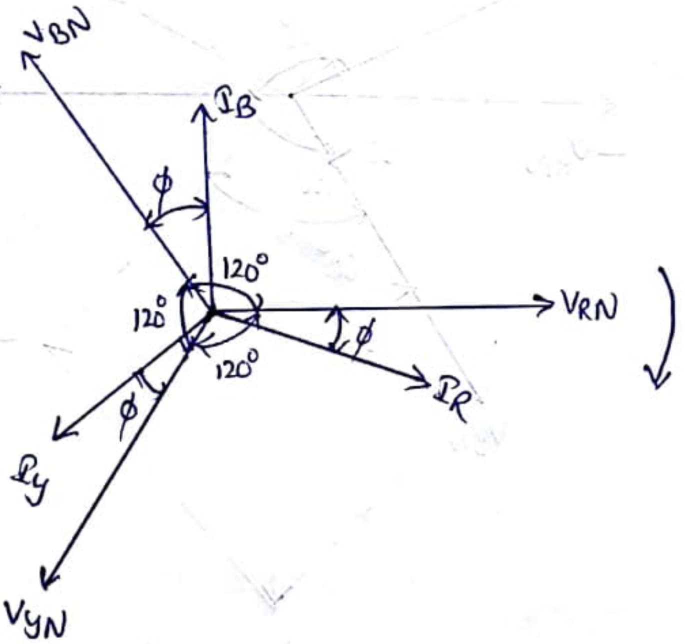
$$V_{RY} = 2V_{ph}\cos 30 = 2V_{ph} \times \sqrt{3}/2 = \sqrt{3}\,V_{ph}$$

similarly $V_{YB} = V_{YN} - V_{BN} = \sqrt{3}\,V_{ph}$, $V_{BR} = V_{BN} - V_{RN} = \sqrt{3}\,V_{ph}$

Hence in a balanced star connected system

* line voltage $= \sqrt{3}\, V_{ph}$
* All line voltages are equal in magnitude and are displaced by 120° and
* All line voltages are 30° ahead of their respective phase voltages.

<u>Current Relations</u>:



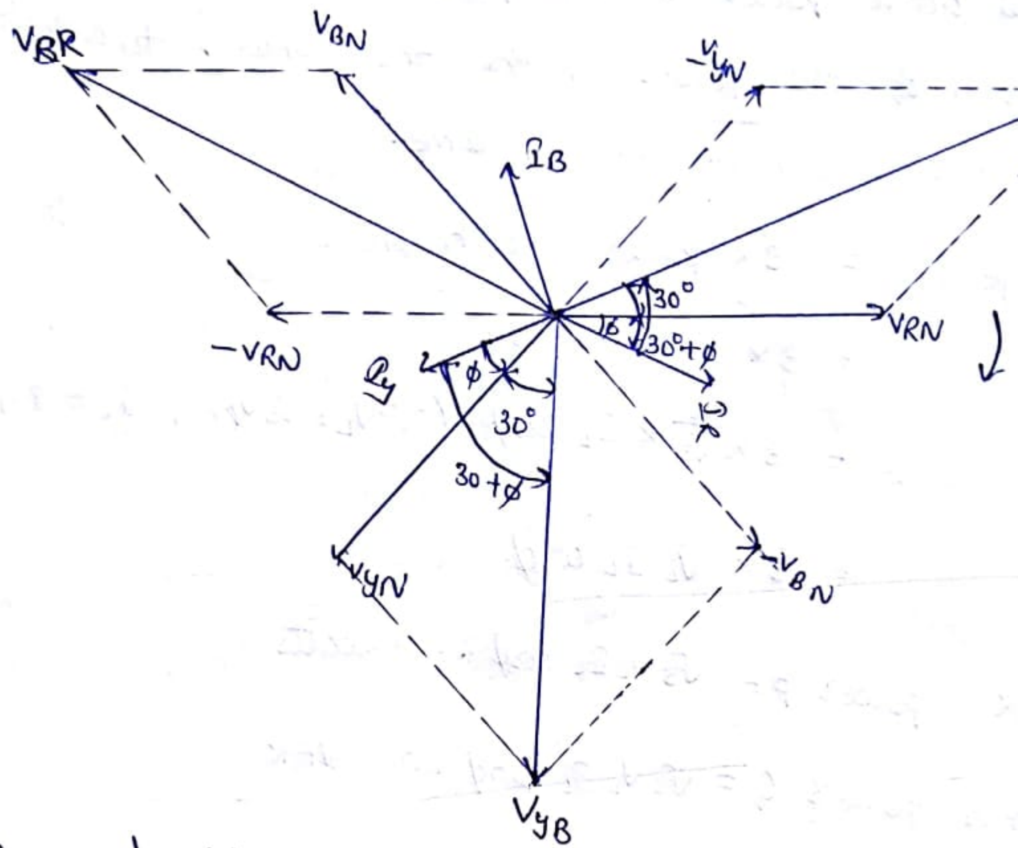
The phase currents are displaced by 120° with respect to each other ϕ is the phase angle between phase voltage and phase current. for a balanced load all the phase currents are equal in magnitude. In the star connection the line conductor is connected in series with its individual phase winding. Therefore the line current is the same as that of phase current.

$$\therefore I_L = I_{ph} = I_R = I_y = I_B$$

where I_L – line current

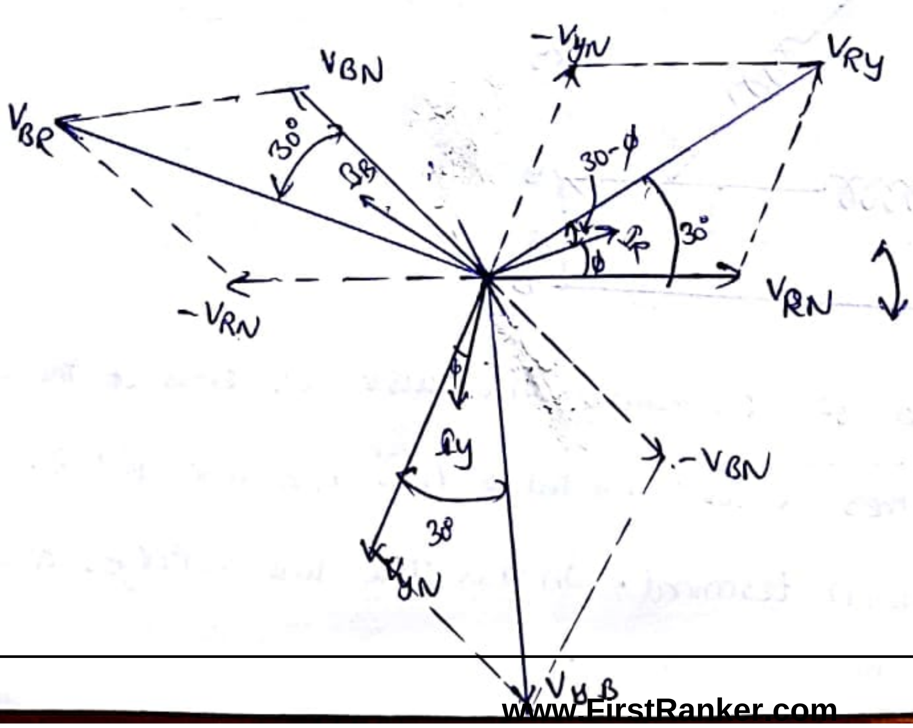
I_{ph} – phase current

Lagging power factor :



The angle b/w line voltage and current is $30+\phi$ ie; the angle b/w V_{YB} and I_y = $30+\phi$ similarly the angle b/w V_{BR} and I_B, V_{RY} and I_R is equal to $30+\phi$.

Leading power factor :



The angle b/w line voltages and current is $30-\phi$

power in the star connected Network:

The total active power or true power in the three phase load is the sum of the powers in the three phases. As a balanced load the power in each load is the same.

$\therefore$ Total power = $3 \times$ power in each phase

$$= 3 \times V_{ph} I_{ph} \cos\phi$$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos\phi \quad (\because V_L = \sqrt{3} V_{ph},\ I_L = I_{ph})$$

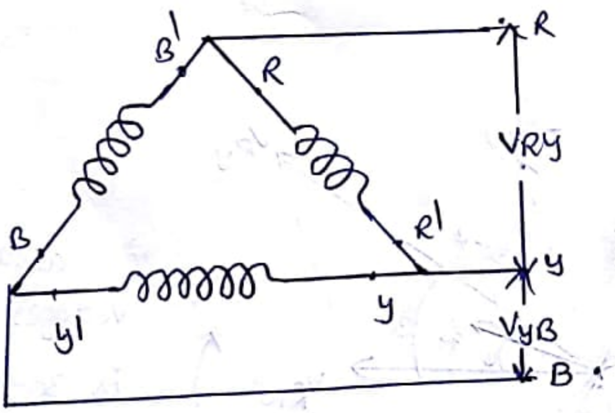
$$= \sqrt{3} \times V_L I_L \cos\phi$$

$\therefore$ Active power $P = \sqrt{3} V_L I_L \cos\phi$ .... watts

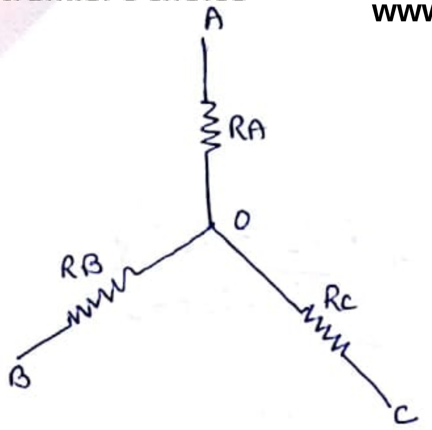
Reactive power $Q = \sqrt{3} V_L I_L \sin\phi$ .... VAR

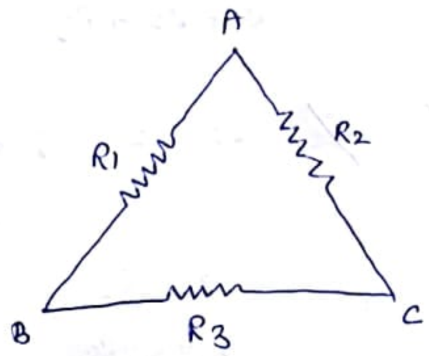
apparant power $= \sqrt{3} V_L I_L$ .... VA

Delta or Mesh connection:



In this method of connection the dissimilar ends of the windings are joined together R' is connected to Y, Y' to B and B' to R. Here there is no common terminal. In this the line voltages are also referred to as phase voltages.





Star connection

$$R_{AB} = R_A + R_B$$

$$R_{BC} = R_B + R_C$$

$$R_{CA} = R_C + R_A$$

Delta connection

$$R_{AB} = R_1 \parallel (R_2+R_3) = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

$$R_{BC} = R_3 \parallel (R_1+R_2) = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3}$$

$$R_{CA} = R_2 \parallel (R_1+R_3) = \frac{R_2(R_1+R_3)}{(R_1+R_2+R_3)}$$

Equate the resistances of star and delta circuits we get

$$R_A+R_B = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} = \frac{R_1R_2+R_1R_3}{R_1+R_2 R_3} \quad \text{--- (1)}$$

$$R_B+R_C = \frac{R_3R_1+R_3R_2}{R_1+R_2+R_3} \quad \text{--- (2)}$$

$$R_C+R_A = \frac{R_1R_2+R_2R_3}{R_1+R_2+R_3} \quad \text{--- (3)}$$

(1) - (2)

$$R_A+\cancel{R_B}-\cancel{R_B}-R_C = \frac{R_1R_2+R_1R_3}{R_1+R_2+R_3} - \frac{R_3R_1+R_3R_2}{R_1+R_2+R_3}$$

$$R_A - R_C = \frac{R_1R_2 - R_2R_3}{R_1 + R_2 + R_3} \quad \text{--- (4)}$$

(4) + (3)

$$R_A - \cancel{R_C} + \cancel{R_C} + R_A = \frac{R_1R_2 - R_2R_3}{R_1 + R_2 + R_3} + \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

$$2R_A = \frac{2R_1R_2}{R_1 + R_2 + R_3}$$

$$\Rightarrow \quad R_A = \frac{R_1R_2}{R_1 + R_2 + R_3}$$

$$R_B = \frac{R_1R_3}{R_1 + R_2 + R_3}$$

$$R_C = \frac{R_2R_3}{R_1 + R_2 + R_3}$$

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2}{R_1+R_2+R_3} \cdot \frac{R_1R_3}{R_1+R_2+R_3} + \frac{R_1R_3}{R_1+R_2+R_3} \cdot \frac{R_2R_3}{R_1+R_2+R_3} + \frac{R_2R_3}{R_1+R_2+R_3} \cdot \frac{R_1R_2}{R_1+R_2+R_3}$$

$$= \frac{R_1^2R_2R_3}{(R_1+R_2+R_3)^2} + \frac{R_1R_2R_3^2}{(R_1+R_2+R_3)^2} + \frac{R_1R_2^2R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1^2 \cancel{+} R_2R_3 + R_1R_2R_3^2 + R_1R_2^2R_3}{(R_1+R_2+R_3)^2} \quad \text{--- (5)}$$

$\frac{⑤}{R_A}$ gives R_3

$\frac{⑤}{R_B}$ gives R_2

$\frac{⑤}{R_C}$ gives R_1

Thus

$$R_1 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_C}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_B}$$

$$R_3 = \frac{R_A R_B + R_B R_C + R_C R_A}{R_A}$$

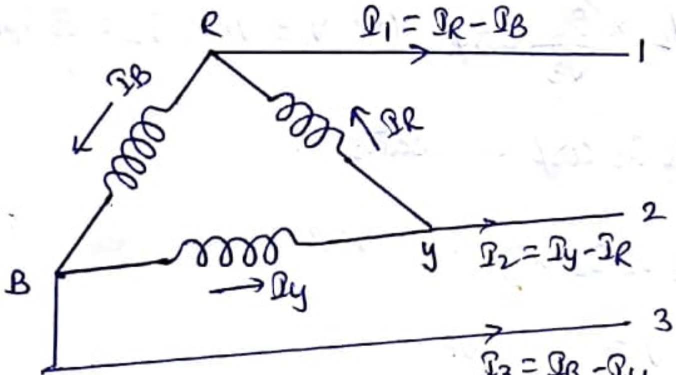
Voltage Relation:

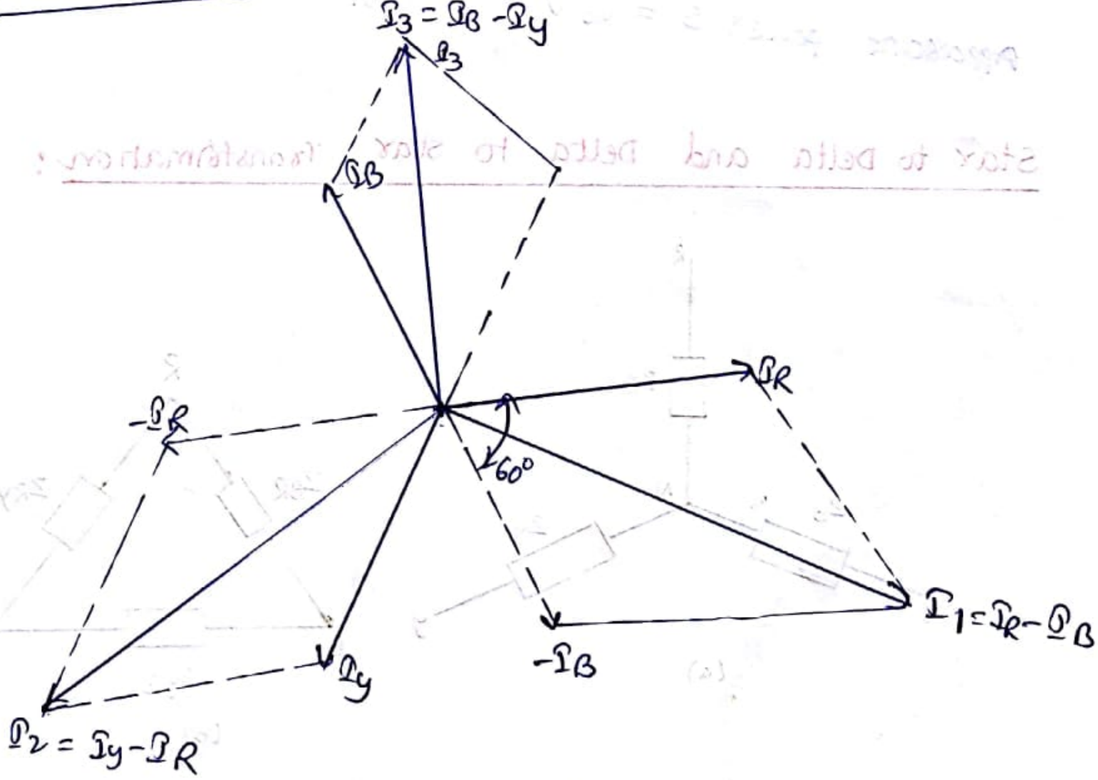
In delta connection only one phase is connected between any two lines Hence the voltage between any two lines (V_L) is equal to the phase voltage V_{ph}

$$V_{RY} = V_{BR} = V_{YB} = V_L = V_{ph}$$

Since the system is balanced all the phase voltages are equal but displaced by 120° from one another.

Current Relation:





$$I_1 = I_R - I_B$$

$$= \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^\circ} = \sqrt{3}\, I_{ph}$$

$I_R = I_B = I_y = I_{ph}$

$I_1 = I_2 = I_3 = \sqrt{3} I_{ph}$

All the line currents are equal in magnitude but displaced by $120°$ from one another and the line currents are $30°$ behind the respective phase currents.

Power in delta-connected system:

Total power = 3x power in each phase

$= 3 \times V_{ph} I_{ph} \cos\phi$

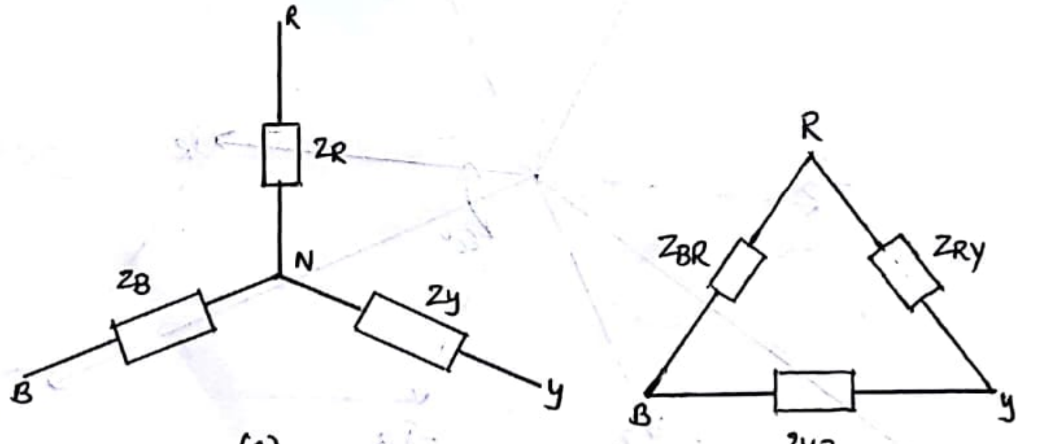
$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos\phi \quad (\because V_L = V_{ph},\ I_L = \sqrt{3} I_{ph})$

$P = \sqrt{3} V_L I_L \cos\phi$ --- watts

Reactive power $Q = \sqrt{3} V_L I_L \sin\phi$ --- VAR

Apparant power $S = \sqrt{3} V_L I_L$ --- VA

Star to Delta and Delta to star Transformation:



(a) (b)

Delta connection of resistances can be replaced by an equivalent star (Y) connection and viceversa. Similar methods can be applied in the case of networks containing general impedances in complex form.

The star connected load can be replaced by an equivalent delta load with branch impedances as shown.

Delta impedances in terms of star impedances are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

Similarly

The delta connected load can be replaced by an equivalent star load with branch impedances as

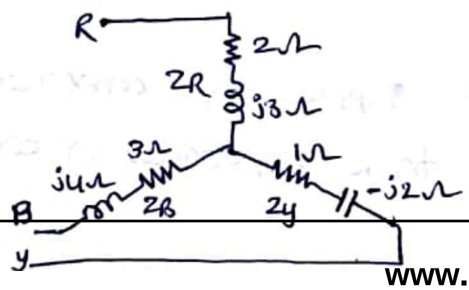
$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

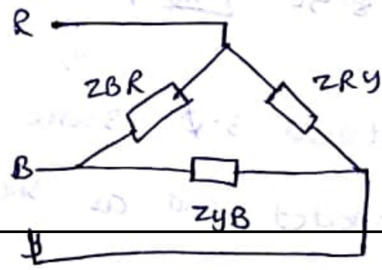
$$Z_Y = \frac{Z_{RY} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{BR} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

problems:

1. A symmetrical 3-ϕ, 3 wire 440V supply is connected to a star connected load as shown in fig. The impedances in each branch are $Z_R = (2+j3)\Omega$, $Z_Y = (1-j2)\Omega$, and $Z_B = (3+j4)\Omega$. find the equivalent delta connected load. the phase sequence is RYB.





$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

given that

$Z_R = 2+j3 = 3.61 \angle 56.3^\circ$

$Z_Y = 1-j2 = 2.23 \angle -63.4^\circ$

$Z_B = 3+j4 = 5 \angle 53.13^\circ$

$$Z_{RY} = \frac{(3.61\angle 56.3)\times(2.23\angle -63.4) + (2.23\angle -63.4)(5\angle 53.13) + (5\angle 53.13)(3.61\angle 56.3)}{5\angle 53.13^\circ}$$

$= 3.82 \angle -5.83^\circ = 3.8 - j0.38 \ \Omega$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$= \frac{(3.61\angle 56.3)(2.23\angle -63.4) + (2.23\angle -63.4)(5\angle 56.13) + (5\angle 53.13)(3.61\angle 56.3)}{3.61\angle 56.3}$$

$= 5.29 \angle -9^\circ = 5.22 - j0.82 \ \Omega$

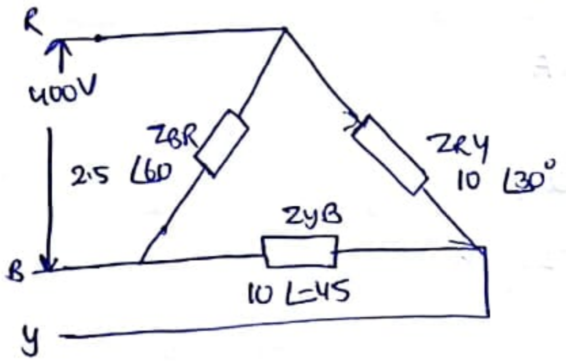
$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

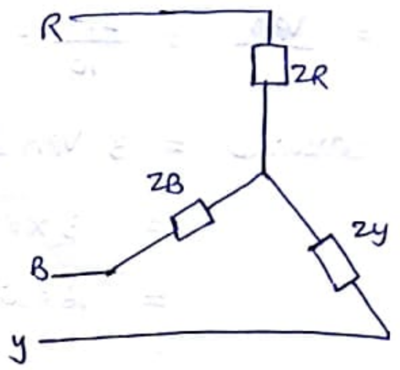
$$= \frac{(3.61\angle 56.3)(2.23\angle -63.4) + (2.23\angle -63.4)(5\angle 56.13) + (5\angle 53.13)(3.61\angle 56.3)}{2.23\angle -63.4}$$

$= 8.56 \angle 110.7 = -3.02 + j8 \ \Omega$

2) A symmetrical $3\text{-}\phi$ 3 wire 400V supply is connected to a delta connected load as shown in figure Impedances in each branch

are $Z_{RY} = 10 \angle 30\ \Omega$, $Z_{YB} = 10 \angle -45\ \Omega$ and $Z_{BR} = 2.5 \angle 60\ \Omega$ find its equivalent star connected load the phase sequence is RYB.





$$Z_R = \frac{Z_{RY}\ Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{(10 \angle 30)(2.5 \angle 60)}{10 \angle 30 + 2.5 \angle 60 + 10 \angle -45} = 1.47 \angle 89.67\ \Omega$$

$$Z_Y = \frac{Z_{RY}\ Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{(10 \angle 30)(10 \angle -45)}{10 \angle 30 + 2.5 \angle 60 + 10 \angle -45} = 5.89 \angle -15.33\ \Omega$$

$$Z_B = \frac{Z_{BR}\ Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{(2.5 \angle 60)(10 \angle -45)}{10 \angle 30 + 2.5 \angle 60 + 10 \angle -45} = 1.47 \angle 14.67\ \Omega$$

3. A Three phase load has a resistance of 10Ω in each phase and is connected in star, Delta against a 400V, 3-ϕ supply compare the power consumed in both the cases.

Star connection ;

$$V_L = 400V \rightarrow V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{10} = 23.1A$$

$$\text{power consumed} = 3\, V_{ph} I_{ph} \cos\phi$$

$$= 3 \times 231 \times 23.1 \times 1$$

$$= 16KW$$

Delta connection:

$$V_L = V_{ph} = 400V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40A$$

$$\text{power consumed} = 3 V_{ph} I_{ph} \cos\phi$$

$$= 3 \times 400 \times 40 \times 1$$

$$= 48KW$$

∴ The power consumed in delta connection is three times that of in star connection.

Balanced Star - Delta and Delta Star conversion:

A balanced star connected load consists of equal impedance in each phase as shown in figure (a)

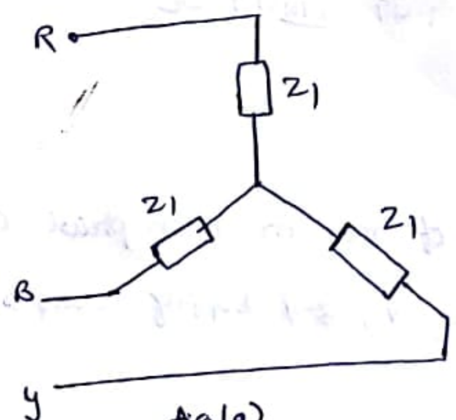


fig (a)

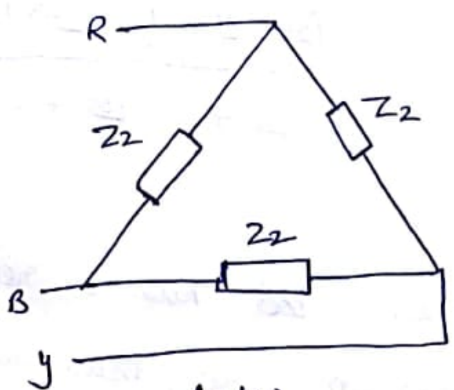


fig (b)

Let the equivalent delta connected load have an impedance of Z_2

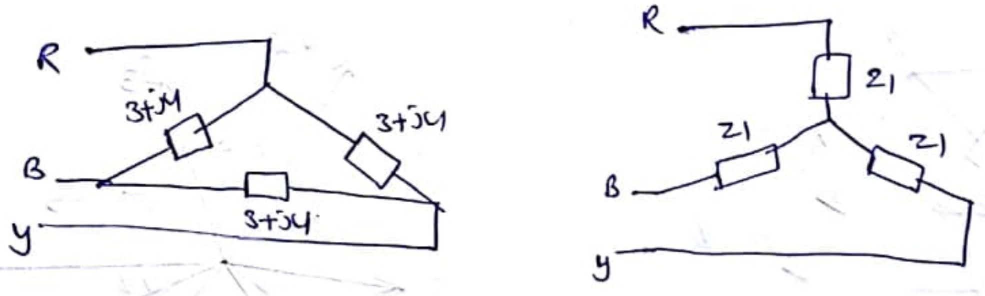
in each phase as shown in fig (b) then delta impedance in terms of star impedances

$$Z_2 = 3Z_1$$

Similarly the star impedances in terms of delta

$$Z_1 = Z_2/3$$

1. Three identical impedances are connected in delta as shown in fig (a) find an equivalent star network such that the line current is the same when connected to the same supply.



$$Z_1 = \frac{Z_2}{3}$$

$$= \frac{3+j4}{3} = 1 + j1.33\,\Omega$$

Analysis of Balanced 3-ϕ circuits:

Balanced three phase system Delta load:

figure shows a 3-ϕ, 3 wire balanced system supplying power to a balanced three phase delta load. The phase sequence is RYB

let us assume Line voltage $V_{RY} = V \angle 0^\circ$ then

$$V_{YB} = V \angle -120^\circ \ V$$

$$V_{BR} = V \angle -240^\circ \ V$$

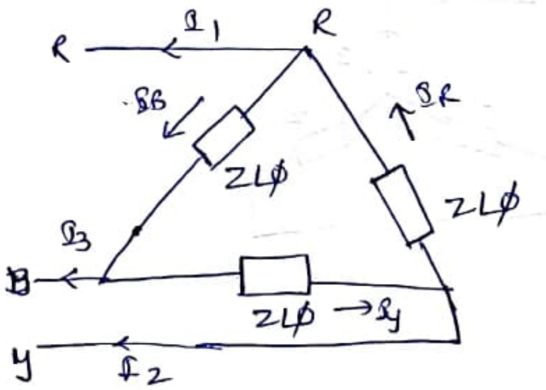
These voltages are represented by phasors in fig (b).

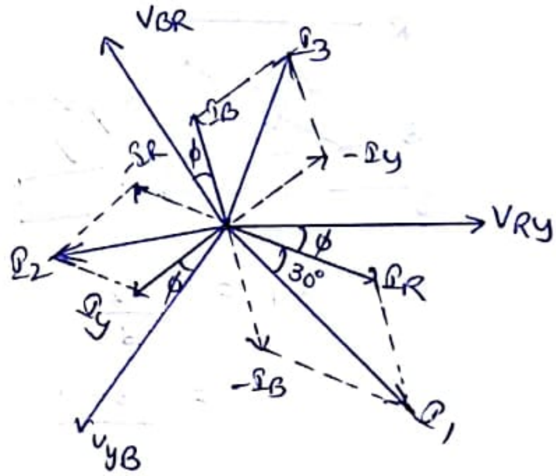
The load is delta connected

$\therefore V_L = V_{ph}$

$I_{ph} = I_L/\sqrt{3}$

The current in phase RY, I_R will lag/lead behind & ahead of the phase voltage V_{RY} by an angle ϕ as dictated by the nature of load impedance. The angle b/w I_Y, V_{YB} and I_B, V_{BR} will be ϕ as the load is balanced.





$$I_R = \frac{V_{RY}\angle 0°}{Z\angle\phi} = \frac{V}{Z}\angle -\phi$$

$$I_Y = \frac{V_{YB}\angle -120}{Z\angle\phi} = \frac{V}{Z}\angle -120-\phi$$

$$I_B = \frac{V_{BR}\angle -240}{Z\angle\phi} = \frac{V}{Z}\angle -240-\phi$$

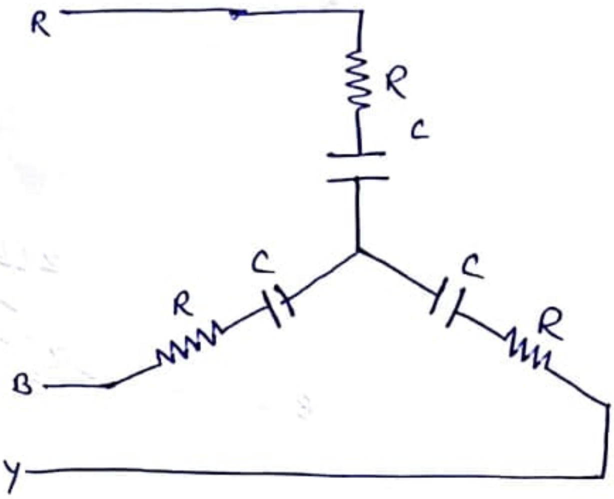
The line currents are $\sqrt{3}$ times the phase currents and are $30°$ behind their respective phase currents.

$$I_1 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-\phi-30°)$$

$$I_2 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-120-\phi-30)$$

→ A balanced 3-ϕ star connected load of 150 KW takes a leading current of 100A with a line voltage of 1100V, 50Hz find the circuit constants of the load per phase.

Given data



power $P = 150\,kW$

current $I = 100A$

line voltage $V = 1100V$

$f = 50Hz$

$$V_{ph} = 1100/\sqrt{3} \text{ --- } V$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{1100/\sqrt{3}}{100} = 6.35\,\Omega$$

power consumed $P = I^2 R$

$$150 \times 10^3 = 100^2 \times R_{ph} \times 3$$

$$R_{ph} = 5\,\Omega$$

$$Z_{ph} = \sqrt{R^2 + X^2}$$

$$\therefore X_{ph} = \sqrt{Z_{ph}^2 - R_{ph}^2} = \sqrt{6.35^2 - 5^2}$$

$$= 3.91\,\Omega$$

given that input current is leading therefore X_{ph} must be capacitive

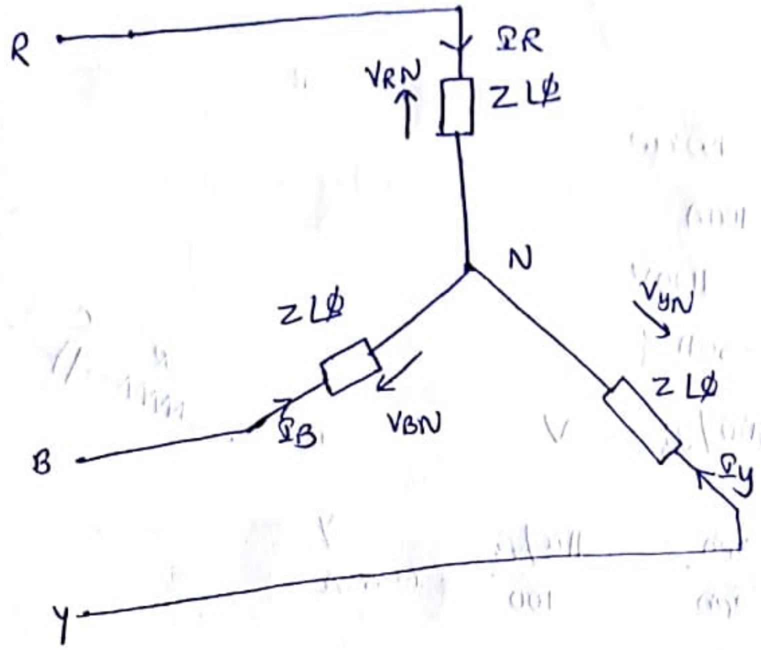
$$X_{ph} = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_{ph}}$$

$$= \frac{1}{2\pi \times 50 \times 3.91}$$

$$= 813.1\,\mu F$$

$$I_3 = \sqrt{3}\left|\frac{V}{Z}\right| \angle(-270-\phi)$$

Balanced 3-ϕ system star connected load:



Let us assume the voltage $V_{RN} = V\angle 0°$

$V_{YN} = V\angle -120$

$V_{BN} = V\angle -240$

then $I_R = \dfrac{V_{RN}}{Z\angle\phi} = \dfrac{V\angle 0}{Z\angle\phi} = \left|\dfrac{V}{Z}\right| \angle -\phi$

$I_y = \dfrac{V_{YN}}{Z\angle\phi} = \dfrac{V\angle -120}{Z\angle\phi} = \left|\dfrac{V}{Z}\right| \angle -120-\phi$

$I_B = \dfrac{V_{BN}}{Z\angle\phi} = \dfrac{V\angle -240}{Z\angle\phi}$

$\therefore$ The currents I_R, I_y and I_B are equal in magnitude and have a 120° phase difference.

1) A star connected 3-ϕ load has a resistance of 8 Ω and a capacitive reactance of 10 Ω in each phase it is fed from a 400V, 3-ϕ balanced supply.

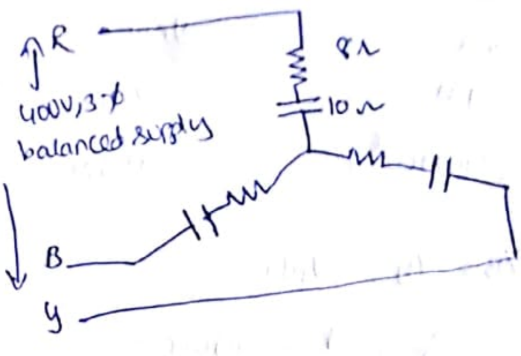
(i) find the line current, total VA, active, reactive powers

(ii) Draw phasor diagram showing phase voltages, line voltages and currents.

$R = 8\Omega$

$X_C = 10\Omega$

$V_L = 400V$



$Z = R - jX_C$

$= 8 - j10 = 12.8 \angle -51.34$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$

$V_{RN} = V_{ph} \angle 0^\circ$

$V_{yN} = V_{ph} \angle -120$

$V_{BN} = V_{ph} \angle -240$

$V_{RN} = 230.94 \angle 0^\circ$, $V_{yN} = 230.94 \angle -120^\circ$

$V_{BN} = 230.94 \angle -240$

$I_R = \frac{V_{RN}}{Z} = \frac{230.94 \angle 0}{12.8 \angle -51.34} = 18.042 \angle 51.34$

$I_y = \frac{V_{yN}}{Z} = \frac{230.94 \angle -120}{12.8 \angle -51.34} = 18.042 \angle -68.66 A$

$I_B = \frac{V_{BN}}{Z} = \frac{230.94 \angle -240}{12.8 \angle -51.34} = 18.042 \angle -188.66$

$S = 3VI = 3 \times 230.94 \times 18.042 = 12499.98 VA$

$P = 3VI\cos\phi = 3 \times 230.94 \times 18.042 \times \cos 51.34 = 7808.63 W$

$Q = 3VI\sin\phi = 3 \times 230.94 \times 18.042 \times \sin 51.34 = 9760.72 VAR$

UNIT - 2

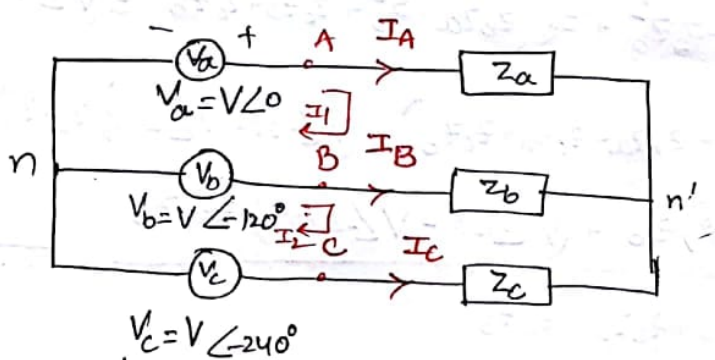
Unbalanced Three phase circuits

Types of unbalanced loads: An unbalance exists in a circuit when the impedance in one (or) more phases differs from the impedances of the other phases. In such cases line (or) phase currents are different and are displaced from one another by unequal angles. Problems on unbalanced three phase loads are difficult to handle because conditions in the three phase are different. However source voltages are assumed to be balanced.

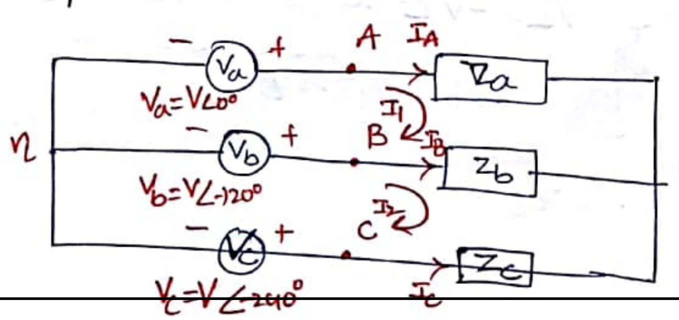
We will now consider different methods to handle unbalanced star-connected and delta connected loads. In practice, we may come across the following unbalanced loads.

① unbalanced delta-connected load
② unbalanced 3-wire star-connected load
③ unbalanced four wire star-connected load.

Loop method:



Solve an unbalanced star connected 3-ϕ wire system shown in Fig using loop method.



Applying Kirchoff's voltage law to loop 1 we get,

$I_1 Z_a + (I_1 - I_2) Z_b + V\angle{-120^\circ} - V\angle{0^\circ} = 0$

$I_1 Z_a + (I_1 - I_2) Z_b + V\angle{-120^\circ} = V\angle{0^\circ}$

$I_1 (Z_a + Z_b) - I_2 Z_b = V\angle 0^\circ - V\angle{-120^\circ}$ —(1)

Applying Kirchoff's voltage law to loop 2 we get,

$(I_2 - I_1) Z_b + I_2 Z_c + V\angle{-240^\circ} - V\angle{-120^\circ} = 0$

$\Rightarrow (I_2 - I_1) Z_b + I_2 Z_c + V\angle{-240^\circ} - V\angle{-120^\circ}$

$-I_1 Z_b + I_2 (Z_b + Z_c) = V\angle{-120^\circ} - V\angle{-240^\circ}$ —(2)

By solving (1) and (2) equations current I_1 and I_2 can be obtained from equation (1)

$$I_1 = \frac{V\angle 0^\circ - V\angle{-120^\circ} + I_2 Z_b}{Z_a + Z_b} \quad —(3)$$

substituting eq (3) in eq (2) we get

$$-\left[\frac{V\angle 0^\circ - V\angle{-120^\circ} + I_2 Z_b}{Z_a + Z_b}\right] Z_b + I_2 (Z_b + Z_c) = V\angle{-120^\circ} - V\angle{-240^\circ}$$

$$\frac{(-V\angle 0^\circ + V\angle{-120^\circ} - I_2 Z_b) Z_b + I_2 (Z_b + Z_c)(Z_a + Z_b)}{(Z_a + Z_b)} = V\angle{-120^\circ} - V\angle{-240^\circ}$$

$$(-V\angle 0^\circ + V\angle{-120^\circ}) Z_b - I_2 Z_b^2 + I_2 (Z_b Z_a + Z_b^2 + Z_c Z_a + Z_c Z_b) = (V\angle{-120^\circ} - V\angle{-240^\circ}) Z_a + Z_b$$

$$I_2 (-\cancel{Z_b^2} + \cancel{Z_b^2} + Z_b Z_a + Z_c Z_a + Z_c Z_b) = (V\angle{-120^\circ} - V\angle{-240^\circ})(Z_a + Z_b) + (V\angle 0^\circ - V\angle{-120^\circ}) Z_b$$

$$I_2 = \frac{(V\angle 0^\circ - V\angle{-120^\circ}) Z_b + (V\angle{-120^\circ} - V\angle{-240^\circ}) Z_a + Z_b}{Z_b Z_a + Z_c Z_a + Z_c Z_b} \quad —(4)$$

substituting eq (4) in eq (3) we get

$$I_1 = \frac{V\angle 0^\circ - V\angle{-120^\circ} + \left[\frac{(V\angle 0^\circ - V\angle{-120^\circ}) Z_b + (V\angle{-120^\circ} - V\angle{-240^\circ}) Z_a + Z_b}{Z_b Z_a + Z_c Z_a + Z_c Z_b}\right] Z_b}{Z_a + Z_b}$$

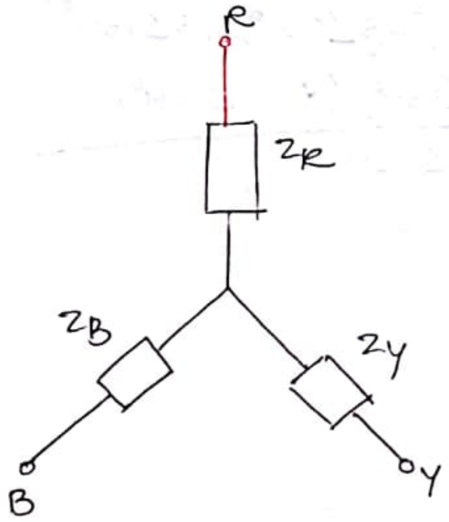
Now, the phase currents I_A, I_B and I_C can be calculated as

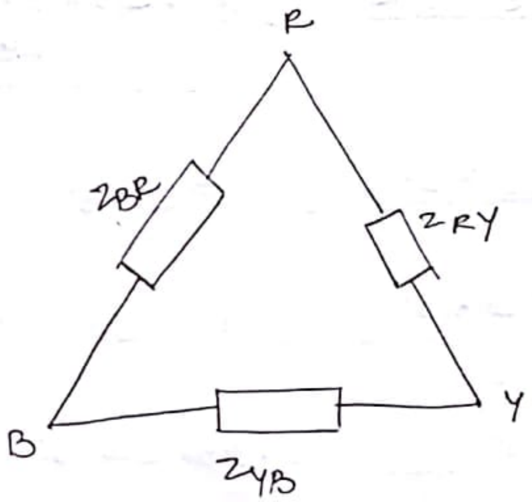
$$I_A = I_1$$
$$I_B = I_2 - I_1$$
$$I_C = -I_2$$

Thus, from the above equations, we can evaluate $V_{An'}$, $V_{Bn'}$ and $V_{Cn'}$ as,

$$V_{An'} = I_A Z_a$$
$$V_{Bn'} = I_B Z_b$$
$$V_{Cn'} = I_C Z_c$$

star delta transformation technique:





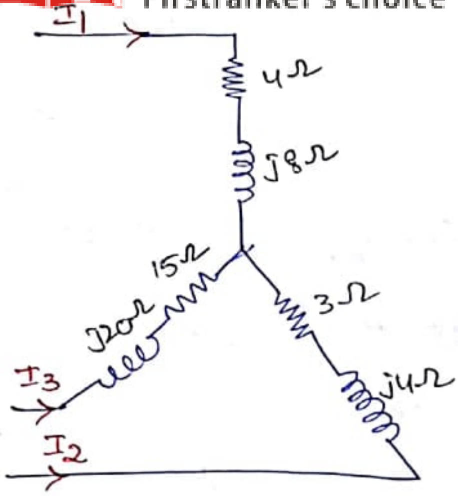
The star load can be replaced by an equivalent delta load.

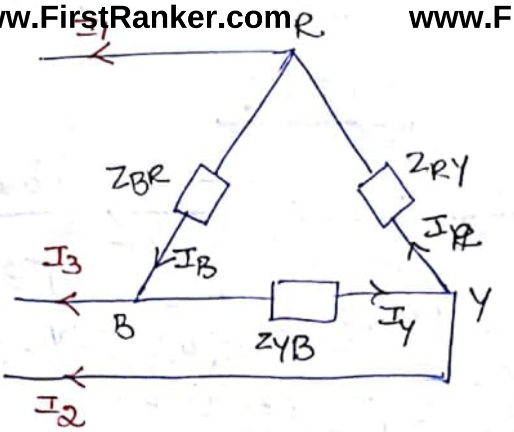
$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

Pb) A 400V, three 3phase supply feeds an unbalanced 3-wire star connected load. The branch impedances of the load are $Z_R = (4+j8)\Omega$, $Z_Y = (3+j4)\Omega$ and $Z_B = (15+j20)\Omega$. Find the line currents and voltage across each phase impedance. Assume RYB phase sequence.





$Z_R = (4+j8)\Omega = 8.944\angle 63.4^\circ\ \Omega$

$Z_Y = (3+j4)\Omega = 5\angle 53.1^\circ\ \Omega$

$Z_B = (15+j20)\Omega = 25\angle 53.1\ \Omega$

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = \frac{(8.94\angle 63.4^\circ)\,5\angle 53.1^\circ + (5\angle 53.1^\circ)(25\angle 53.1^\circ) + (25\angle 53.1^\circ)(8.94\angle 63.4^\circ)}{25\angle 53.1^\circ}$$

$$Z_{RY} = \frac{391.80\angle 113.23^\circ}{25\angle 53.1^\circ} = 15.67\angle 60.13^\circ$$

$$Z_{YB} = \frac{391.80\angle 113.23^\circ}{8.94\angle 63.4^\circ} = 43.83\angle 49.83^\circ$$

$$Z_{BR} = \frac{391.80\angle 113.23^\circ}{5\angle 53.1^\circ} = 78.36\angle 60.13^\circ$$

- Taking V_{RY} as reference $V_{RY} = 400\angle 0^\circ$

$V_{YB} = 400\angle -120^\circ$, $V_{BR} = 400\angle -240^\circ$

$$I_R = \frac{V_{RY}}{Z_{RY}} = \frac{400\angle 0^\circ}{15.67\angle 60.13^\circ} = 25.52\angle -60.13^\circ$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120^\circ}{43.83\angle 49.83} = 9.12\angle -169.83^\circ$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{400\angle -240^\circ}{78.36\angle 60.13^\circ} = 5.10\angle -300.13^\circ$$

- The various line currents in the delta connection are

$I_1 = I_R - I_B = 25.52\angle -60.13^\circ - 5.1\angle -300.13^\circ$

$$\boxed{I_1 = 28.41\angle -69.07^\circ\ A}$$

$I_2 = I_Y - I_R = 9.12\angle -169.83^\circ - 28.41\angle -69.07^\circ$

$= 29.85 \angle 136.58^\circ A$

$I_3 = I_B - I_Y = 5.1 \angle 300.13^\circ - 9.12\angle -169.83^\circ$

$= 13 \angle 27.60^\circ A$

These line currents are also equal to the line (phase) currents of the original star-connected load. The voltage drop across each star-connected load will be as follows.

voltage drop across $Z_R = I_1 Z_R$

$= (28.41 \angle -69.070^\circ)(8.94\angle 63.4^\circ) = 253.89 \angle -5.67^\circ V$

voltage drop across $Z_Y = I_2 Z_Y$

$= (29.85 \angle 136.58^\circ)(5\angle 53.1^\circ) = 149.2 \angle 189.68^\circ V$

voltage drop across $Z_B = I_3 Z_B = (13 \angle 27.60^\circ)(25\angle 53.1^\circ) = 325 \angle 80.70^\circ V$

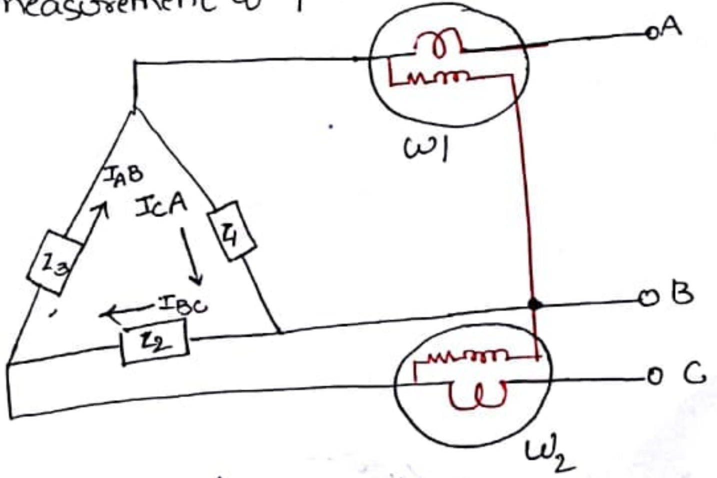
Two wattmeter method for measurement of power

There are two conditions under which the power in a 3 phase delta connected load using two wattmeters can be measured. They are

① when the load is balanced

② when the load is unbalanced.

measurement of power when the load is unbalanced.



The total power flowing into the load at any instant of time is given as,

$$P = E_{AB} I_{AB} + E_{BC} I_{BC} + E_{CA} I_{CA}$$

$$= E_{AB} I_{AB} + E_{BC} I_{BC} - (E_{AB} + E_{BC}) I_{CA}$$

As the sum of line voltages of the system is always zero.

we have $E_{CA} = -E_{AB} - E_{BC}$

Power $P = E_{AB}I_{AB} + E_{BC}I_{BC} - E_{AB}I_{CA} - E_{BC}I_{CA}$

$= E_{AB}(I_{AB} - I_{CA}) + E_{BC}(I_{BC} - I_{CA})$

where $I_{AB} - I_{CA} = I_A$ and

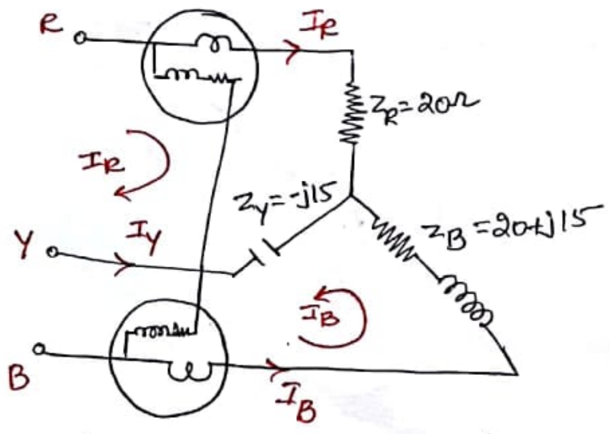
$I_{BC} - I_{CA} = -I_C$

we have $P = E_{AB}I_A - E_{BC}I_C$

$= E_{AB}I_A + E_{CB}I_C$

$\boxed{P = W_1 + W_2}$ $[\because E_{BC} = -E_{CB}]$

(P) An unbalanced star connected load is connected across a 3-φ, 400V balanced supply of sequence RYB as shown in Figure. Two wattmeters are connected to measure the total power supplied as shown in Figure. Find the readings of the wattmeters.



3-φ star connected load

$Z_R = 20\Omega$

$Z_B = 20+j15\Omega$

$Z_Y = -j15\Omega$

$V_{RY} = 400\angle 0° V$ —(1) ; $V_{YB} = 400\angle -120°$ —(2) , $V_{BR} = 400\angle 120° V$ —(3)

Loop equations are given by

$V_{RY} = 20I_R - j15(I_R + I_B) = 20I_R - j15I_R - j15I_B$

$= I_R(20 - j15) - j15I_B$

From Equation ① $V_{RY} = 400\angle 0^\circ$

$$400 = (20 - j15)I_R - j15 I_B \quad \text{— ④}$$

$$V_{YB} = -[(20+j15) I_B + (I_R + I_B)(-j15)]$$

$$= -20 I_B - j15 I_B + j15 I_R + j15 I_B$$

$$= -20 I_B + j15 I_R$$

From Eq ② $V_{YB} = 400\angle -120^\circ V$

$$400\angle -120^\circ V = -20 I_B + j15 I_R$$

$$20 I_B - j15 I_R = -400\angle -120^\circ \quad \text{— ⑤}$$

solving equations ④ and ⑤ i.e.,

$$(20 - j15) I_R - j15 I_B = 400$$

$$20 I_B - j15 I_R = 200 + j346.41$$

$$I_B = \frac{200 + j346.41 + j15 I_R}{20}$$

$$(20 - j15) I_R - j15 \frac{(200 + j346.41 + j15 I_R)}{20} = 400$$

$$(20 - j15) I_R - \frac{1}{20}[j3000 - 5196.15 - 225 I_R] = 400$$

$$(400 - j300) I_R - [j3000 + 5196.15 + 225 I_R] = 8000$$

$$I_R [400 - j300 + 225] - j3000 + 5196.15 = 8000$$

$$I_R (625 - j300) = 2803.85 + j3000$$

$$I_R = 5.923\angle 72.57^\circ$$

substituting the value of I_R in equation ④ we get,

$$400 = (20 - j15) I_R - j15 I_B$$

$$400 = (20 - j15)(5.92\angle 72.57^\circ) - j15 I_B$$

$$400 = (120.18 + j86.36^\circ) - j15 I_B$$

$$I_B = 19.52\angle 72.84^\circ$$

$$j15 I_B = 292.83\angle 162.84^\circ$$

From Equation ① $V_{RY} = 400\angle 0^\circ$

$400 = (20 - j15)I_R - j15 I_B$ — ④

$V_{YB} = -[(20+j15)I_B + (I_R + I_B)(-j15)]$

$= -20I_B - j15I_B + j15I_R + j15I_B$

$= -20I_B + j15I_R$

From Eq ② $V_{YB} = 400\angle -120^\circ V$

$400\angle -120^\circ V = -20I_B + j15I_R$

$20I_B - j15I_R = -400\angle -120^\circ$ — ⑤

solving equations ④ and ⑤ i.e.,

$(20-j15)I_R - j15I_B = 400$

$20I_B - j15I_R = 200 + j346.41$

$$I_B = \frac{200 + j346.41 + j15I_R}{20}$$

$$(20 - j15)I_R - j15\frac{(200 + j346.41 + j15I_R)}{20} = 400$$

$$(20-j15)I_R - \frac{1}{20}[j3000 - 5196.15 - 225I_R] = 400$$

$(400 - j300)I_R - [j3000 + 5196.15 + 225I_R] = 8000$

$I_R[400 - j300 + 225] - j3000 + 5196.15 = 8000$

$I_R(625 - j300) = 2803.85 + j3000$

$I_R = 5.923\angle 72.57^\circ$

substituting the value of I_R in equation ④ we get,

$400 = (20 - j15) I_R - j15I_B$

$400 = (20-j15)(5.92\angle 72.57^\circ) - j15I_B$

$400 = (120.18 + j86.36^\circ) - j15I_B$

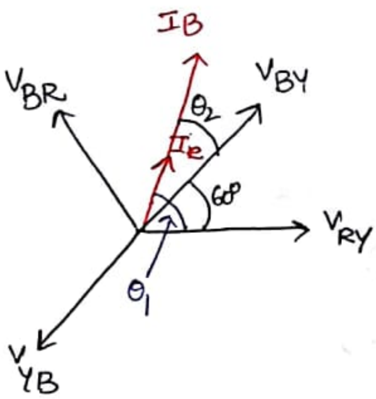
$I_B = 19.52\angle 72.84^\circ$

$j15I_B = 292.83\angle 162.84^\circ$

$I_R = 5.92 \angle 72.57° \ A$

$I_B = 19.52 \angle 72.84° \ A$

The vector diagram for the given network is as shown in Fig



Now the wattmeter readings can be obtained as,

$W_1 = V_{RY} I_R \cos\theta_1$

$= 400 \times 5.923 \cos 72.57°$

$W_1 = 709.67$ watts

$W_2 = V_{BY} I_B \cos\theta_2$

$= 400 \times 19.52 \cos(72.84° - 60°)$

$W_2 = 7612.75$ watts

Total power $= W_1 + W_2 = 709.67 + 7612.75 = 8322.42$ watts.

1) Three impedances of $(7+j4)\Omega$, $(3+j2)\Omega$ and $(9+j2)\Omega$ are connected b/w neutral and the R, Y and B phases. The line voltage is 440V calculate

(i) The line currents

(ii) the current in the neutral wire and

(iii) find the power consumed in each phase and the total power drawn by the ckt

$Z_R = 7+j4\,\Omega$

$Z_Y = 3+j2\,\Omega$

$Z_B = 9+j2\,\Omega$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03V$$

$V_{RN} = 254.03 \angle 0$

$V_{YN} = 254.03 \angle -120$

$V_{BN} = 254.03 \angle -240$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{254.03 \angle 0}{7+j4} = \frac{254.03 \angle 0}{8.06 \angle 29.74} = 31.5 \angle -29.74\ A$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{254.03 \angle -120}{3+j2} = 70.45 \angle -153.69\ A$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{254.03 \angle -240}{9+j2} = 27.55 \angle -252.52\ A$$

$$I_N = -(I_R + I_Y + I_B) = 48.63 \angle 25.02\ A$$

$P_R = I_R^2 \times R_R = (31.5)^2 \times 7 = 6945.75\ W$

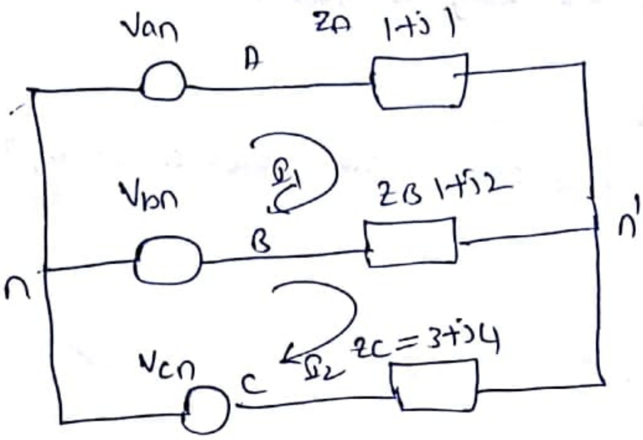
$P_Y = I_Y^2 \times R_Y = 70.45^2 \times 3 = 14889.6\ W$

$P_B = I_B^2 \times R_B = (27.55)^2 \times 9 = 6831.02\ W$

$P = P_R + P_Y + P_B = 28.66\ kW$

2) An unbalanced 3-ϕ 3wire 50Hz 100V supply is given to a load consisting of three impedances $(1+j1)$ $(1+j2)$ and $(3+j4)\Omega$ connected in star as shown in

figure. compute the voltages and currents in the the load using loop current method. phase sequence A B C



$Z_A I_1 + Z_B (I_1 - I_2) = -V_{Bn} + V_{An}$

$(1+j1) I_1 + (1+j2)(I_1 - I_2) = -100\angle 90 + 100\angle 0$

$1I_1 + j1I_1 + I_1 - I_2 + j2I_1 - j2I_2 = 100 - 100j$

$(2+3) I_1 - (1+j2) I_2 = 100 - 100j$

$Z_B (I_2 - I_1) + Z_C I_2 = -V_{Cn} + V_{Bn}$

$(1+j2)(I_2 - I_1) + (3+j4) I_2 = -100\angle -240 + 100\angle 90$

$1I_2 - 1I_1 + j2I_2 - j2I_1 + 3I_2 - j4I_2 = 50 + j13.4$

$(4+j6) I_2 - (1+j2) I_1 = 50 + j13.4$

$(2+j3) I_1 - (1+j2) I_2 = 100 - 100j \quad \text{——} \times (4+j6)$

$(4+j6) I_2 - (1+j2) I_1 = 50 + 13.4j \quad \text{——} \times (1+j2)$

$(-10 + 24j) I_1 - (-8 + j14) I_2 = 1000 + 200j$

$(-8 + 14j) I_2 - (-3 + j4) I_1 = 23.2 + 113.4j$

$I_1 [-10 + 24j + 3 - j4] = 1000 + 200j + 23.2 + j113.4$

$I_1 [-7 + 20j] = 1023.2 + j813.4$

$$I_1 = \frac{1023.2 + j813.4}{-7 + j20}$$

$I_1 = -1.99 - 50.46j$

$(4+j6) I_2 - (1+j2)(50.5\angle -92.26) = 50 + j13.4$

$(4+j6) I_2 = 148.93 - j41.04$

$$I_2 = \frac{148.93 - j41.04}{4+j6} = +6.72 - 20.34j$$

$$I_2 = 21.42 \angle -71.71 \text{ A}$$

$$I_A = I_1 = 50.5 \angle -92.26 \text{ A}$$

$$I_B = I_2 - I_1 = 21.42 \angle -71.71 - 50.5 \angle -92.26 = 31.35 \angle 73.86$$

$$I_C = -I_2 = 21.42 \angle 108 \text{ A}$$

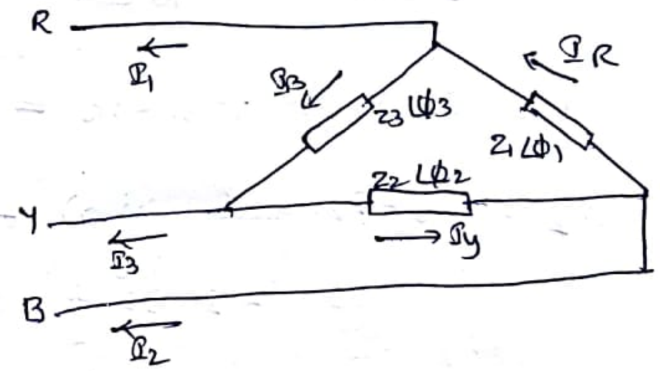
voltages across the loads are

$$V_{An} = I_A Z_A = 50.5 (\angle -92.26 \times (1+j1)) = 70.92 \angle -47.26$$

$$V_{Bn} = I_B Z_B = 31.35 (\angle 73.86 \times (1+j2)) = 70.11 \angle 137.3^\circ \text{ V}$$

$$V_{cn} = I_c Z_c = 21.42 \angle 108.29 \times (3+j4) = 107.1 \angle 161.42 \text{ V}$$

un balanced Delta connected load:



$$V_{RY} = V \angle 0^\circ$$

$$V_{YB} = V \angle -120$$

$$V_{BR} = V \angle -240$$

$$I_R = \frac{V_{RY}}{Z_1 \angle \phi_1}, \quad I_y = \frac{V_{YB}}{Z_2 \angle \phi_2}$$

$$I_B = \frac{V_{BR}}{Z_3 \angle \phi_3}$$

$$I_R = \left|\frac{V}{Z_1}\right| \angle -\phi_1 \text{ A}$$

$$I_y = \left|\frac{V}{Z_2}\right| \angle -120 - \phi_2 \text{ A}$$

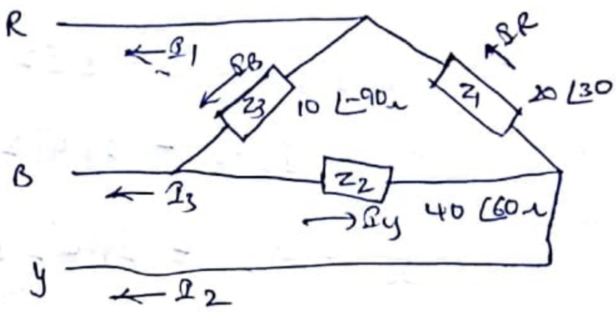
$$I_B = \left|\frac{V}{Z_3}\right| \angle -240 - \phi_3 \text{ A}$$

$$I_1 = I_R - I_B$$

$$I_2 = I_y - I_R$$

$$I_3 = I_B - I_y$$

1) Three impedances $Z_1 = 20\angle 30\ \Omega$, $Z_2 = 40\angle 60\ \Omega$ and $Z_3 = 10\angle -90\ \Omega$ are delta connected to a 400 V 3-ϕ system as shown. Determine the (i) phase currents (ii) line currents and (iii) total power consumed by the load.



$V_{Ry} = 400\angle 0$

$V_{yB} = 400\angle -120$

$V_{BR} = 400\angle -240$

$Z_1 = 20\angle 30 = 17.32 + j10$

$Z_2 = 40\angle 60 = 20 + j34.64$

$Z_3 = 10\angle -90 = 0 - j10\ \Omega$

$$I_R = \frac{V_{Ry}}{Z_1\angle\phi_1} = \frac{400\angle 0}{20\angle 30} = 20\angle -30 = 17.32 - j10\,A$$

$$I_y = \frac{V_{yB}}{Z_2\angle\phi_2} = \frac{400\angle -120}{40\angle 60} = 10\angle -180 = -10 + j0\,A$$

$$I_B = \frac{V_{BR}}{Z_3\angle\phi_3} = \frac{400\angle -240}{10\angle -90} = 40\angle -150 = -34.64 - j20\,A$$

$I_1 = I_R - I_B = 17.32 - j10 - (-34.64 - j20) = (51.96 + j10)\,A$

$I_2 = I_y - I_R = (-10 + j10) - (17.32 - j10)$

$= -27.32 + j10\,A$

$I_3 = I_B - I_y = (-34.64 - j20) - (-10 + j0) = -24.64 - j20\,A$

power in R phase $= I_R^2 \times R_R = 20^2 \times 17.32 = 6928\,W$

power in y phase $= I_y^2 \times R_y = 10^2 \times 20 = 2000\,W$

power in B phase $= I_B^2 \times R_B = 40^2 \times 0 = 0$

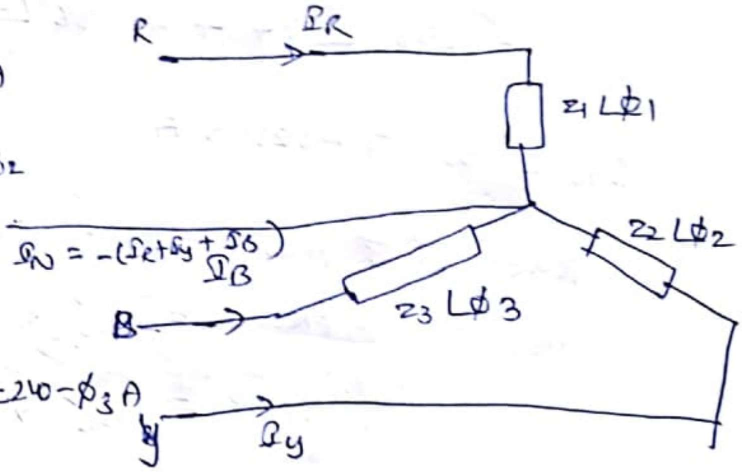
Total power in the load $= 6928 + 2000 = 8928\,W$

Unbalanced four wire star connected load:

$$I_R = \frac{V_{RN}}{Z_1} = \frac{V\angle 0}{Z_1\angle\phi_1} = \frac{V}{Z_1}\angle -\phi_1 \text{ A}$$

$$I_y = \frac{V_{yN}}{Z_2} = \frac{V\angle -120}{Z_2\angle\phi_2} = \frac{V}{Z_2}\angle -120-\phi_2$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{V\angle -240}{Z_3\angle\phi_3} = \frac{V}{Z_3}\angle -240-\phi_3 \text{ A}$$



"The current in the neutral wire is the vector sum of the three line currents".

→ An unbalanced four wire star connected load has a balanced voltage of 400V the loads are

$Z_1 = (4+j8)\,\Omega$, $Z_2 = (3+j4)\,\Omega$, $Z_3 = (15+j20)\,\Omega$

calculate the (i) line currents (ii) current in the neutral wire and (iii) the total power.

$Z_1 = (4+j8)\,\Omega = 8.94\angle 63.40$

$Z_2 = (3+j4)\,\Omega = 5\angle 53.1\,\Omega$

$Z_3 = (15+j20)\,\Omega = 25\angle 53.13\,\Omega$

$V_{RN} = \frac{400}{\sqrt{3}} = 230.94\text{V}$

$V_{yN} = 230.94\angle -120$

$V_{BN} = 230.94\angle -240$

$$I_R = \frac{V_{RN}}{Z_1} = \frac{230.94\angle 0}{8.94\angle 63.4} = 25.83\angle -63.4 \text{ A}$$

$$I_y = \frac{V_{yN}}{Z_2} = \frac{230.94\angle -120}{5\angle 53.1} = 46.188\angle -173.1 \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_3} = \frac{230.94\angle -240}{25\angle 53.13} = 9.23\angle -293.13 \text{ A}$$

$I_N = -(I_R + I_y + I_B)$

$= -(25.83 \angle -63.4 + 46.188 \angle -173.1 + 9.23 \angle -293.13)$

$I_N = 30.67 + j20.15$ A

(iii) power in R phase $= I_R^2 \times R_R = (25.83)^2 \times 4 = 2668.75$ W

" " y phase $= I_y^2 \times R_y = (46.18)^2 \times 3 = 6397.77$ W

" " B phase $= I_B^2 \times R_B = (9.23)^2 \times 15 = 1277.89$ W

Total power absorbed by the load $= 2668.75 + 6397.77 + 1277.89$

$= 10344.41$ W

→ A 400V, 3ϕ supply feeds an unbalanced three wire star connected load. The branch impedances of the load are $Z_R = (4+j8)\Omega$, $Z_y = (3+j4)\Omega$ and $Z_B = (15+j20)\Omega$ find the line currents and voltage across each phase impedance. Assume RyB phase sequence.

$Z_R = 4+j8\ \Omega$

$Z_y = 3+j4\ \Omega$

$Z_B = 15+j20\ \Omega$

$$Z_{Ry} = \frac{Z_R Z_y + Z_y Z_B + Z_B Z_R}{Z_B} = \frac{391.80 \angle 113.23}{25 \angle 53.1} = 15.67 \angle 60.13$$

$$Z_{yB} = \frac{Z_R Z_y + Z_y Z_B + Z_B Z_R}{Z_R} = \frac{391.80 \angle 113.23}{8.94 \angle 63.4} = 43.83 \angle 49.83$$

$$Z_{BR} = \frac{Z_R Z_y + Z_y Z_B + Z_B Z_R}{Z_y} = \frac{391.80 \angle 113.23}{5 \angle 53.1} = 78.36 \angle 60.13$$

$V_{Ry} = 400 \angle 0$

$V_{YB} = 400 \angle -120$, $V_{BR} = 400 \angle -240$

$$I_R = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0}{15.67 \angle 60.13} = 25.52 \angle -60.13$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -120}{43.83 \angle 49.83} = 9.12 \angle -169.83$$

$$I_B = \frac{V_{BR}}{Z_{BR}} = \frac{400 \angle -240}{78.36 \angle 60.13} = 5.10 \angle -300.13$$

$I_1 = I_R - I_B = 25.52 \angle -60.13 - 5.1 \angle -300.13 = 28.41 \angle -69.07$ A

$I_2 = I_Y - I_R = 9.12 \angle -169.83 - 5.52 \angle -60.13 = 29.85 \angle 136.58$ A

$I_3 = I_B - I_Y = 5.1 \angle -300.13 - 9.12 \angle -169.83 = 13 \angle 27.60$ A

voltage drop across $Z_R = I_1 Z_R$

$= 28.41 \angle -69.070 \times 8.94 \angle 63.4$

$= 253.89 \angle -5.67$ V

$Z_Y = I_2 Z_Y$

$= 29.85 \angle 136.58 \times 5 \angle 53.1$

$= 149.2 \angle 189.68$ V

$Z_B = I_3 Z_B$

$= 13 \angle 27.60 \times 25 \angle 53.1$

$= 325 \angle 80.70$ V

→ In a symmetrical 3-ϕ system, phase sequence RYB a capacitive reactance of 8 Ω is across YB and a coil (R+jX) across RY find R and X such that $I_Y = 0$

$$I_C = \frac{V_{YB}}{X_C} = \frac{V \angle -120}{8 \angle -90}$$

$I_Y = I_L - I_C = 0$

$I_L = I_C$

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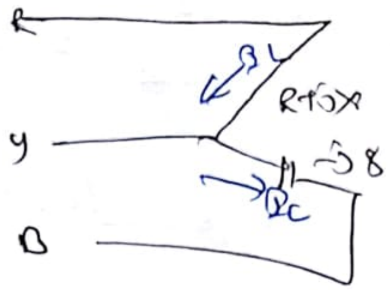
$$\frac{V}{R+jX} = \frac{V}{8}\angle 30$$



$V_{Ry} = V\angle 0$

$V_{yB} = V\angle -120$

$V_{BR} = V\angle 120$

$$I_C = \frac{V_{yB}}{X_C} = \frac{V\angle -120}{8\angle -90} = \frac{V}{8}\angle -30$$

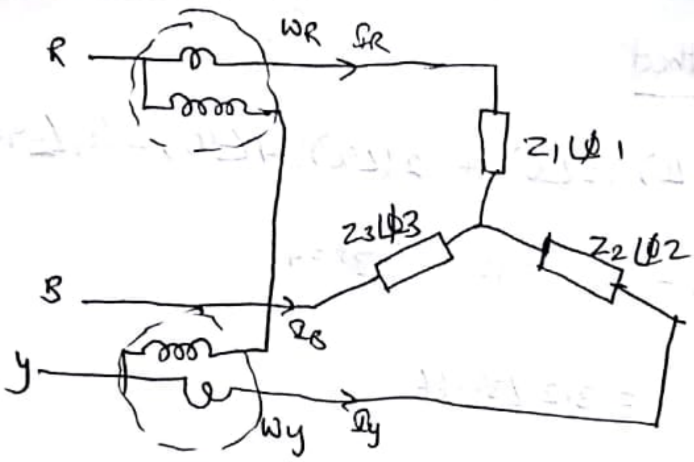
$I_y = I_L - I_C = 0 \Rightarrow I_L = I_C$

$$\frac{V}{R+jX} = \frac{V}{8}\angle 30$$

$(R+jX) = 8\angle 30$

$R = 6.93\Omega, \quad X = 4\Omega$

wattmeter method:



$$W_R = \frac{1}{T}\int_0^T V_{RB} I_R \, dt$$

$$W_y = \frac{1}{T}\int_0^T V_{yB} I_y \, dt$$

$$V_{RB} = V_{RN} - V_{BN}$$

$$V_{yB} = V_{yN} - V_{BN}$$

$$W_R + W_y = \frac{1}{T}\int_0^T V_{RB} I_R + V_{yB} I_y = \frac{1}{T}\int_0^T (V_{RN} - V_{BN}) I_R + (V_{yN} - V_{BN}) I_y$$

$$= \frac{1}{T}\int_0^T V_{RN} I_R - V_{BN} I_R + V_{yN} I_y - V_{BN} I_y$$

$$= \frac{1}{T}\int_0^T V_{RN} I_R + V_{yN} I_y - V_{BN}(I_R + I_y)$$

$$I_R + I_y + I_B = 0$$

$$I_R + I_y = -I_B$$

$$W_R + W_y = \frac{1}{T}\int_0^T V_{RN} I_R + V_{yN} I_y + I_B V_{BN}$$

Powerfactor:

$$\text{power factor} = \frac{\Sigma VI\cos\theta}{\Sigma VI}$$

→ A symmetrical three phase 100V, three wire supply feeds an unbalanced star connected load with impedances of the load as $Z_R = 5\angle 0$ $Z_y = 2\angle 90\,\Omega$ and $Z_B = 4\angle -90\,\Omega$ find the line currents, (ii) voltage across

...pedances and (ii) the displacement neutral voltage.

b) star – delta conversion method:

$$Z_R Z_Y + Z_Y Z_B + Z_B Z_R = (5\angle 0)(2\angle 90) + 2(\angle 90)(4\angle -90) + (4\angle -90)(5\angle 0)$$

$$= 8 - j10 = 12.8\angle -51.34$$

$$Z_{RY} = \frac{12.8\angle -51.34}{4\angle -90} = 3.2\angle 38.66$$

$$Z_{YB} = \frac{12.8\angle -51.34}{5\angle 0} = 2.56\angle -51.34$$

$$Z_{BR} = \frac{12.8\angle -51.34}{2\angle 90} = 6.4\angle -141.34$$

$V_{RY} = 100\angle 0 \quad V_{YB} = 100\angle -120, \; V_{BR} = 100\angle -240$

$$I_R = \frac{100\angle 0}{3.2\angle 38.66} = 31.25\angle -38.66$$

$$I_Y = \frac{V_{YB}}{Z_{YB}} = \frac{100\angle -120}{2.56\angle -51.34} = 39.06\angle -68.66$$

$$I_B = \frac{100\angle -240}{6.4\angle -141.34} = 15.62\angle -98.66$$

$$I_1 = I_R - I_B = 27.06\angle -8.671$$

$$I_2 = I_Y - I_R = 39.06\angle -68.66 - 31.25\angle -38.66 = 19.7\angle 238.85$$

$$I_3 = 26.7\angle 128.33 = I_B - I_Y$$

Voltage across $Z_R = I_1 \times Z_R$

$$= (27.06\angle -8.671)(5\angle 0) = 135.3\angle -8.67$$

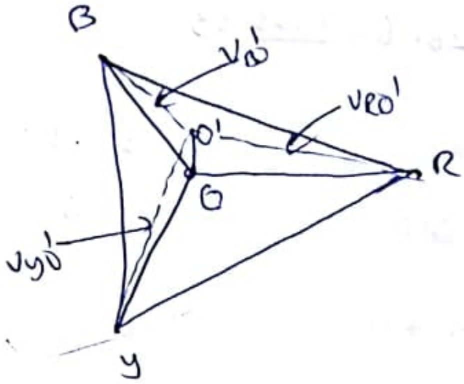
$Z_y = I_2 \times Z_y$

$= 19.7 \angle 238.85 \times 2 \angle 90 = 39.4 \angle 328.85$

$Z_B = I_3 \times Z_B$

$= 26.7 \angle 128.33 \times 4 \angle -90 = 106.8 \angle 38.33$

By apply millimans theorem:



$V_{RO} = \frac{100}{\sqrt{3}} \angle -30$

$V_{yO} = \frac{100}{\sqrt{3}} \angle -150$

$V_{BO} = \frac{100}{\sqrt{3}} \angle -270$

$Y_R = \frac{1}{Z_R} = 0.2 \angle 0$

$Y_y = \frac{1}{Z_y} = 0.5 \angle -90$

$Y_B = \frac{1}{Z_B} = 0.125 \angle 90$

$V_{RO} Y_R + V_{yO} Y_y + V_{BO} Y_B = 26.93 \angle 134.44$

$Y_R + Y_y + Y_B = 0.32 \angle -51.34$

$$V_{O'O} = \frac{V_{RO} Y_R + V_{yO} Y_y + V_{BO} Y_B}{Y_R + Y_y + Y_B} = \frac{26.93 \angle 134.44}{0.32 \angle -51.34}$$

$= 84.15 \angle 185.78$

$V_{RO'} = V_{RO} - V_{O'O} = 135.26 \angle -8.67$

$V_{yO'} = V_{yO} - V_{O'O} = 39.4 \angle 328.8$

$V_{BO'} = V_{BO} - V_{O'O} = 106.73 \angle 38.33$

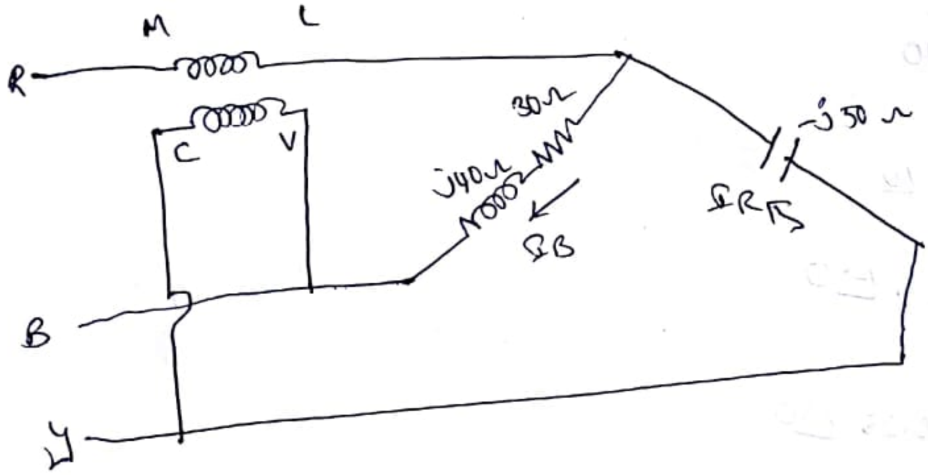
6, 14, 21, 24, 30,

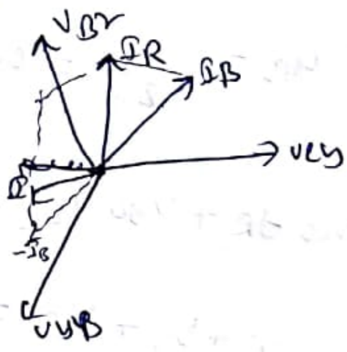
$I_R = \dfrac{V_{RO}}{Z_R} = 20.06 \angle -8.67$

$I_Y = 19.7 \angle 238.8$

$I_B = 26.68 \angle 128.33$

→ find the reading of a wattmeter in the ckt shown in figure Assume a symmetrical 400V supply with RYB phase sequence and draw the vector diagram





$I_R = \dfrac{V_{RY}}{-j50} = \dfrac{400 \angle 0}{50 \angle -90} = 8 \angle 90$

$I_B = \dfrac{V_{BR}}{30+j40} = \dfrac{400 \angle -240}{50 \angle 53.13} = 8 \angle -293.13$

$I_1 = I_R - I_B = 3.2 \angle 168.3$

wattmeter reading $= 400 \times 3.2 \times \cos 71.7$

$= 402\ W$

240 168.3 72

Transient Analysis in DC and AC circuits.

In a network containing energy storage elements with change in excitation the currents and voltages change from one state to another state the behaviour of the voltage (or) current when it is changed from one state to other is called Transient state. The time taken for the circuit to change from one steady state to another steady state is called transient time.

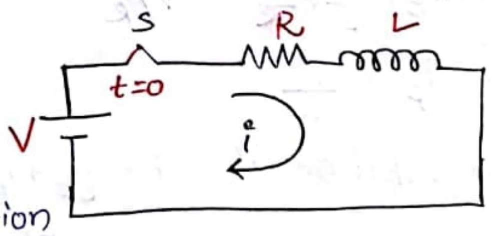
In a circuit the energy storage elements which are independent of the sources, the response depends upon the nature of the circuit is called natural response. The response changes with time gets saturated after some time and is refered to as transient response.

The response depends on nature of source (or) sources acting on a circuit is called forced response.

Consider a differential equation the complete solution consists of two parts
① complementry function.
② particular integral.

The complementry function dies after -- short interval and is refered to as transient response (or) source free response. The particular solution is the steady state response.

First order differential equation: RL circuit with DC excitation



The inductor in the circuit is initially uncharged when switch S is closed we can find the complete solution for the current.

Apply KVL for the given circuit

$$iR + L\frac{di}{dt} = V$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$Di + \frac{Ri}{L} = \frac{V}{L}$$

$$Di + \frac{Ri}{L} - \frac{V}{L} = 0 \Rightarrow \left(D + \frac{R}{L}\right) i = 0$$

$$D + \frac{R}{L} = 0, \quad D = -\frac{R}{L}$$

$$i = i_{CF} + i_P$$

$$x_P = K_2 t$$

$$x_P' = K_2 t + K_3$$

$$i_{CF} = K_1 e^{-\frac{R}{L} t}$$

To Find particular integral

$$i_{PI} = \frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

$$i_P = K_2 t + K_3$$

$$i_P' = K_2$$

$$K_2 + \frac{R}{L}(K_2 t + K_3) = \frac{V}{L}$$

Initial conditions: at $t=0$. At $t=0^-$ i.e., just before closing the switch s, the current in the inductor is zero. At $t=0^+$ just after the switch is closed at $t=0^+$ the current remains zero.

$$\frac{R}{L} K_2 = 0$$

$$K_2 = 0$$

constants

$$K_2 + \frac{R}{L} K_3 = \frac{V}{L}$$

$$K_3 = \frac{V}{L} \times \frac{L}{R} = V/R$$

$$i_P = \frac{V}{R}$$

Complete solution of $i = i_{CF} + i_{PI}$

$$= K_1 e^{-\frac{R}{L} t} + \frac{V}{R}$$

From the initial conditions At $t=0$, $i=0$

$$0 = K_1 e^{-\frac{R}{L} \times 0} + \frac{V}{R} \qquad \boxed{K_1 = -\frac{V}{R}}$$

$$\boxed{i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)}$$

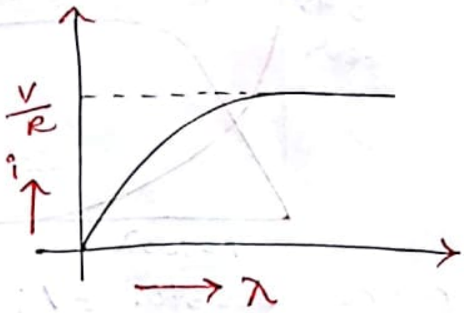
Put $t=0$, then $i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\times 0}\right)$

$= 0$

put $t=\infty$ then $i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\times\infty}\right)$

$i = \frac{V}{R}(1-0)$

$$\boxed{i = \frac{V}{R}}$$



— The time constant of a function $\frac{V}{R}e^{-\frac{R}{L}t}$ is the time at which the exponent of e is unity. where 'e' is the base of natural logarithms. — The term $\frac{L}{R}$ is called the time constant and is dnoted by λ. $\boxed{\lambda = \frac{L}{R}}$

— The transient part of the solution is $i = -\frac{V}{R}e^{-\frac{R}{L}t}$

$i = -\frac{V}{R}e^{-\frac{t}{\tau}}$

If $t/\tau = 1$ then $i(t) = -\frac{V}{R}(e^{-1}) = 0.3678\left(-\frac{V}{R}\right)$

voltage across the resistor $V_R = iR = \frac{V}{\cancel{R}}\left(1 - e^{-\frac{R}{L}t}\right)\times\cancel{R}$

$$\boxed{V_R = V\left(1 - e^{-\frac{R}{L}t}\right)}$$

voltage across the inductor $L\frac{di}{dt} = L\left(\frac{-V}{R}\right)\frac{d}{dt}\left(e^{-\frac{R}{L}t}\right)$

$= L\left(\frac{-V}{R}\right) - \frac{R}{L} \times e^{-\frac{R}{L}t}$

$$\boxed{V_L = Ve^{-\frac{R}{L}t}}$$

For $t=0$,
$V_R = V\left(1 - e^{-\frac{R}{L}\times 0}\right) = 0$

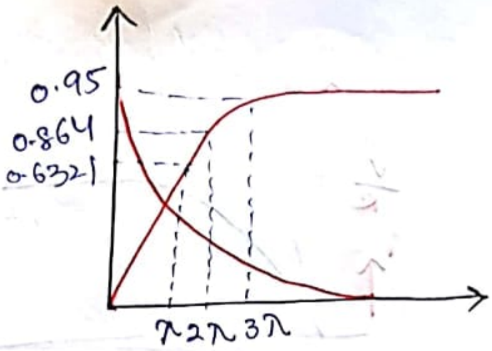
For $t=\lambda$ $V_R = V\left(1 - e^{-\frac{R}{L}\times t}\right)$

$= V\left(1 - e^{-\frac{t}{\tau}}\right) = V\left(1 - e^{-1}\right) = V(1 - 0.3678)$

$= 0.6321V$

$t = 2\lambda$ $= V\left(1 - e^{-\frac{2\cancel{\tau}}{\cancel{\tau}}}\right) = 0.864V$

$t = 3\lambda$ $= V\left(1 - e^{-\frac{3\cancel{\tau}}{\cancel{\tau}}}\right) = 0.95V$

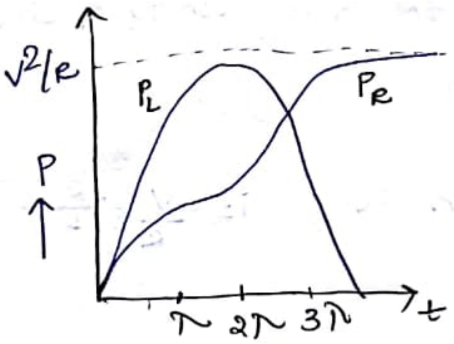


Power in the resistor $= V_R i = \frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right) V\left(1-e^{-\frac{R}{L}t}\right)$

$$= \frac{V^2}{R}\left(1-e^{-\frac{R}{L}t}\right)^2 = \frac{V^2}{R}\left(1^2+e^{-\frac{2R}{L}t}-2e^{-\frac{R}{L}t}\right)$$

Power in the inductor $= V_L i = Ve^{-\frac{R}{L}t} \times \frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)$

$$= \frac{V^2}{R} e^{-\frac{R}{L}t}\left(1-e^{-\frac{R}{L}t}\right)$$



Power in the resistor $= \frac{V^2}{R}\left(1+e^{-\frac{2R}{L}t}-2e^{-\frac{R}{L}t}\right)$ at $t=0$

$$= \frac{V^2}{R}\left(1+e^{-0}-2e^{-0}\right) = \frac{V^2}{R}(2-2)=0$$

at $t=\tau$ $= \frac{V^2}{R}\left(1+e^{-\frac{2\tau}{\tau}}-2e^{-\frac{\tau}{\tau}}\right) = \frac{V^2}{R}\left(1+0.135-2(0.3678)\right)$

$$= \frac{V^2}{R}(0.3994)$$

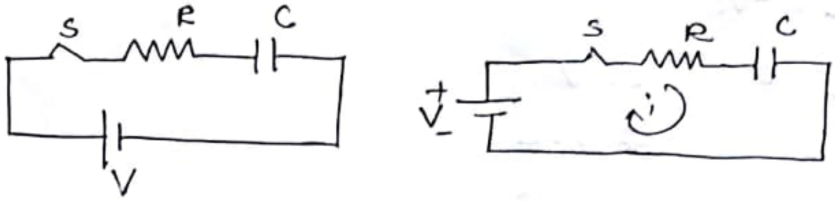
at $t=2\tau$ $= \frac{V^2}{R}\left(1+e^{-2\times\frac{2\tau}{\tau}}-2e^{-\frac{2\tau}{\tau}}\right)$ $= \cancel{V\left(1-e^{-2}\right)=0.864}$

~~at $t=3\tau = V\left(1-e^{-\frac{3\tau}{\tau}}\right)=0.95V$~~

$$= \frac{V^2}{R}\left(1+0.0183-2(0.135)\right) = \frac{V^2}{R}(0.7483)$$

Power in the inductor $= \frac{V^2}{R} e^{-\frac{R}{L}t}\left(1-e^{-\frac{R}{L}t}\right)$

(2) ~~DC~~ RC Circuit with DC excitation and Initial conditions



Initially the capacitor is uncharged when switch is closed at $t=0$ we can determine the complete solution for the current

$$iR + \frac{1}{C}\int i\,dt = V_0$$

Differentiate the above Equation with respect to

$$R\frac{di}{dt} + \frac{1}{C} i = 0$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

Finding the complementary function

$$R\frac{di}{dt} + \frac{1}{C} i = 0$$

$$\left(D + \frac{1}{RC}\right) i = 0 \qquad D = -\frac{1}{RC}$$

$$i_{CF} = k e^{-t/RC}$$

At $t=0$, $i = V/R$, $i = \frac{V}{R} e^{-t/RC}$ At $t = 1\tau$

$\tau = RC$

Voltage across resistor $V_R = \frac{V}{\cancel{R}} e^{-t/RC} \times \cancel{R} = V e^{-t/RC}$

Voltage across Capacitor $V_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC}\,dt$

$$= \frac{1}{C}\frac{V}{R}\int e^{-t/RC}\,dt$$

$$= \frac{V}{\cancel{RC}} \frac{e^{-t/RC}}{-1/\cancel{RC}} + C$$

$$V_C = -V e^{-t/RC} \qquad = -V e^{-t/RC} + C$$

At $t=0$ voltage across the capacitor $=0$

$$V_c = -Ve^{-0/RC}$$

$$V_c = 0 \quad C = V$$

constant

$$V_c = V(1-e^{-t/RC})$$

$$= -Ve^{-t/RC} + V$$

$$\boxed{V_c = V(1-e^{-t/RC})}$$

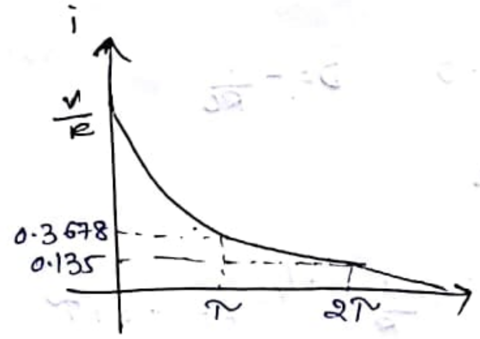
Power in the resistor $P_R = V_R i = Ve^{-t/RC} \times \frac{V}{R}e^{-t/RC}$

$$= \frac{V^2}{R}(e^{-t/RC})^2$$

Power in the capacitor $= V_c \times i = V(1-e^{-t/RC}) \times \frac{V}{R}e^{-t/RC}$

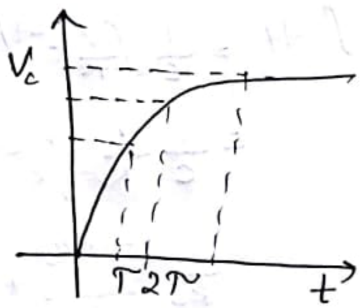
$$= \frac{V^2}{R}(1-e^{-t/RC})e^{-t/RC}$$

$$P_c = \frac{V^2}{R}(e^{-t/RC} - e^{-2t/RC})$$



At $t = 1\tau$ $\quad i(t) = \frac{V}{R}e^{-t/RC} = \frac{V}{R}e^{-\tau/\tau} = \frac{V}{R}(0.3678)$

At $t = 2\tau$ $\quad i(t) = \frac{V}{R}e^{-t/RC} = \frac{V}{R}e^{-2\tau/2\tau} = 0.135\frac{V}{R}$



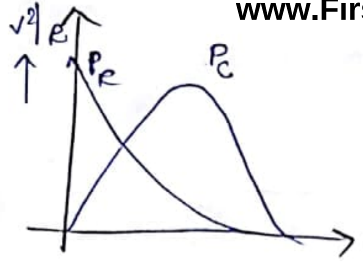
$$V_c = Ve^{-t/RC}$$

at $t=0$

$$V(e^{-0}) = V$$

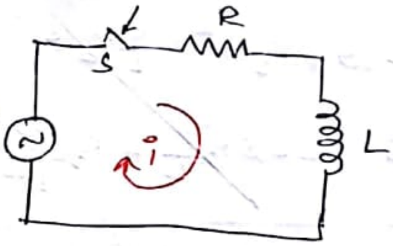
At $t = 1\tau$

$$V_R = Ve^{-\tau/\tau} = 0.3678V \,; \quad V_R = Ve^{-2\tau/\tau} = 0.135V$$



$$P_R = \frac{V^2}{R}\left(e^{-t/RC}\right)^2$$

· First order differential equation with RL ckts with ac excitation.



$$Ri + L\frac{di}{dt} = V\cos(\omega t+\theta)$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta)$$

$$i = i_{CF} + i_{P.I}$$

To find $i_{CF} = \frac{di}{dt} + \frac{R}{L}i = 0$

$$\left(D+\frac{R}{L}\right)i = 0 \qquad D+\frac{R}{L}=0 \qquad D = -\frac{R}{L}$$

$$i_{CF} = ke^{-R/L\,t}$$

$$i_{P.I} = \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}\cos(\omega t+\theta) \quad \text{— ①}$$

Assume $i_{PI} = k_2(\cos(\omega t+\theta)) + k_3\sin(\omega t+\theta)$

$$i_p' = -k_2\sin(\omega t+\theta)\omega + k_3\omega\cos(\omega t+\theta)$$

$$\left.\begin{array}{l}\frac{di}{dt} = i_p' \quad \text{— ②}\\ i_{P.I} = i \quad \text{— ③}\end{array}\right.$$

Substituting the ② & ③ in Eq ①

$$\frac{V}{L}\cos(\omega t+\theta) = -k_2\sin(\omega t+\theta)\omega + k_3\omega\cos(\omega t+\theta) + \frac{R}{L}\left(k_2(\cos(\omega t+\theta)) + k_3\sin(\omega t+\theta)\right)$$

$$\frac{V}{L}\cos(\omega t+\theta) = -k_2\sin(\omega t+\theta)\omega + k_3\omega\cos(\omega t+\theta) + \frac{R}{L}\left[k_2\cos(\omega t+\theta) + k_3\sin(\omega t+\theta)\right]$$

$$\frac{V}{L}\cos(\omega t+\theta) = \sin(\omega t+\theta)\left[-\omega k_2 + k_3\frac{R}{L}\right] + \cos(\omega t+\theta)\left[k_3\omega + \frac{R}{L}k_2\right]$$

$$\frac{V}{L}\cancel{\cos(\omega t+\theta)} = \left(k_3\omega + \frac{R}{L}k_2\right)\cancel{\cos(\omega t+\theta)}$$

$$\frac{V}{L} = k_3\omega + \frac{R}{L}k_2 \quad (4)$$

$$\sin(\omega t+\theta)\left[-\omega k_2 + k_3\frac{R}{L}\right] = 0$$

$$-\omega k_2 + k_3\frac{R}{L} = 0 \quad (5)$$

$$\frac{V}{L} = k_3\omega + \frac{R}{L}k_2$$

Eq (4) $\times\,\omega$ and Eq (5) $\times \frac{R}{L}$

$$\frac{V}{L}\omega = k_3\omega^2 + \cancel{\frac{R}{L}k_2\omega}$$

$$0 = \cancel{-\omega k_2\frac{R}{L}} + k_3\frac{R^2}{L^2}$$

$$\frac{V}{L}\omega = k_3\omega^2 + k_3\frac{R^2}{L^2}$$

$$k_3 = \frac{\frac{V}{L}\omega}{\omega^2 + \frac{R^2}{L^2}}$$

$$\frac{V}{L} = \frac{\frac{V}{L}\omega}{\omega^2+\frac{R^2}{L^2}} \times \omega + \frac{R}{L}k_2$$

$$\frac{V}{L} = \frac{V}{L}\omega^2 \times \frac{L^2}{R^2+\omega^2L^2} + \frac{R}{L}k_2$$

$$\frac{R}{L}k_2 = \frac{V}{L} - \frac{V}{L}\,\frac{\omega^2L^2}{R^2+\omega^2L^2}$$

$$\frac{R}{L}k_2 = \frac{V}{L}\left(1 - \frac{\omega^2L^2}{R^2+\omega^2L^2}\right)$$

$$k_2 = \frac{V}{\cancel{L}} \times \frac{\cancel{L}}{R}\left(1 - \frac{\omega^2L^2}{R^2+\omega^2L^2}\right)$$

$$k_2 = \frac{V}{R}\left(\frac{R^2+\cancel{\omega^2L^2}-\cancel{\omega^2L^2}}{R^2+\omega^2L^2}\right)$$

$$k_2 = \frac{V}{R}\left(\frac{R^2}{R^2+\omega^2L^2}\right)$$

$$\boxed{k_2 = \frac{VR}{R^2+\omega^2L^2}}$$

$$\frac{R}{L}K_3 = K_2\omega$$

$$K_3 = \frac{L}{R} \frac{RV}{R^2+\omega^2L^2} \times \omega = \frac{VL\omega}{R^2+\omega^2L^2}$$

$$i = i_{PI} + i_{CF} = K_1 e^{-\frac{R}{L}t} + \frac{VR}{R^2+\omega^2L^2}\cos(\omega t+\theta) + \frac{VL\omega}{R^2+\omega^2L^2}\sin(\omega t+\theta)$$

Let $M\cos\phi = \frac{RV}{R^2+\omega^2L^2}$, $M\sin\phi = \frac{VL\omega}{R^2+\omega^2L^2}$

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{R^2V^2}{(R^2+\omega^2L^2)^2} + \frac{(VL\omega)^2}{(R^2+\omega^2L^2)^2}$$

$$M^2 = \frac{R^2V^2+V^2L^2\omega^2}{(R^2+\omega^2L^2)^2} \qquad M = \sqrt{\frac{V^2(R^2+\omega^2L^2)}{(R^2+\omega^2L^2)^2}}$$

$$M = \frac{V}{\sqrt{R^2+\omega^2L^2}}$$

$$\frac{M\sin\phi}{M\cos\phi} = \frac{VL\omega}{RV} = \frac{L\omega}{R}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i_P = \frac{V}{\sqrt{R^2+\omega^2L^2}}\left[\cos\left((\omega t+\theta) - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right]$$

If $t=0$; $i(t)=0$

$$i(t) = K_1 e^{-R/L\, t} + \frac{V}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta - \tan^{-1}\frac{\omega L}{R})\right)$$

$$0 = K_1 + \frac{V}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta) - \tan^{-1}\frac{\omega L}{R}\right)$$

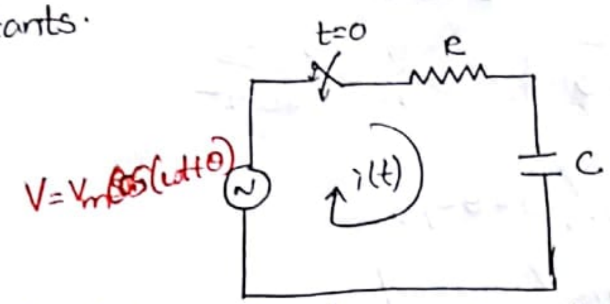
$$K_1 = \frac{-V}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta) - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$\therefore i(t) = \frac{-V}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta) - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)e^{-\frac{R}{L}t} + \frac{V}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t+\theta - \tan^{-1}\right.$$

$$i(t) = \frac{V}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)\left(-e^{-\frac{R}{L}t}+1\right)$$

The final solution is $i(t) = \frac{V}{\sqrt{R^2+\omega^2 L^2}} \cos\left(\omega t+\theta-\tan^{-1}\left(\frac{\omega L}{R}\right)\right)\left(-e^{-\frac{R}{L}t}+1\right)$

First order differential Equation RC circuit with AC Excitation and time constants.



when the switch 'S' is closed at t=0, by applying KVL to the circuit we get,

$$iR + \frac{1}{C}\int i\,dt = V$$

$$iR + \frac{1}{C}\int i\,dt = V_m \cos(\omega t+\theta)$$

$$R\frac{di}{dt} + \frac{i}{C} = -V\omega \sin(\omega t+\theta)$$

$$\left(D+\frac{1}{RC}\right)i = -\frac{V\omega}{R}\sin(\omega t+\theta)$$

The complementry function $i_c = Ce^{-t/RC}$

The particular solution can be obtained by using undetermined coefficients.

$$i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta) \quad \text{— (1)}$$

$$i_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$$

$$\{-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\} + \frac{1}{RC}\{A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\}$$

$$= -\frac{V\omega}{R}\sin(\omega t+\theta)$$

comparing both sides, $-A\omega + \frac{B}{RC} = -\frac{V\omega}{R}$

$$B\omega + \frac{A}{RC} = 0$$

From which

$$A = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}$$

$$B = \frac{-V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

substituting the values of A and B in Eq (1)

$$i_p = \frac{VR}{R^2+\left(\frac{1}{\omega C}\right)^2}\cos(\omega t+\theta) + \frac{-V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}\sin(\omega t+\theta)$$

$$i_p = \frac{VR}{R^2+\omega^2L^2}\cos(\omega t+\theta) + \frac{V\omega L}{R^2+\omega^2L^2}\sin(\omega t+\theta)$$

let $M\cos\phi = \frac{VR}{L^2\omega^2+R^2}$ and $M\sin\phi = \frac{V\omega L}{R^2+\omega^2L^2}$

$$i(t) = k_1 e^{-(r/L)t} + M\cos(\omega t+\theta-\phi)$$

$$i(t) = k_1 e^{-(R/L)t} + M\cos(\omega t+\theta-\phi)$$

$$(M\cos\phi)^2+(M\sin\phi)^2 = \frac{R^2V^2}{(L^2\omega^2+R^2)^2} + \frac{V^2\omega^2L^2}{(R^2+\omega^2L^2)^2}$$

$$M^2 = \frac{R^2V^2+V^2\omega^2L^2}{(R^2+\omega^2L^2)^2} = \frac{V^2}{R^2+\omega^2L^2}$$

$$M = \frac{V}{\sqrt{R^2+\omega^2L^2}}$$

and $\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{V\omega L}{VR} = \frac{\omega L}{R}$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i_p = \frac{V}{\sqrt{R^2+\omega^2L^2}}\left(\cos\left(\omega t+\theta-\tan^{-1}\frac{\omega L}{R}\right)\right)$$

The complete solution is

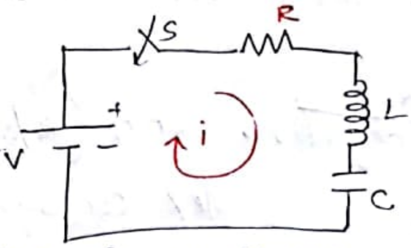
$$\therefore i(t) = k_1 e^{-\left(\frac{R}{L}\right)t} + \frac{V}{\sqrt{R^2+\omega^2L^2}}\left(\cos(\omega t+\theta) - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

If $t=0$, $i(t)=0$

$$\therefore k_1 = \frac{-V}{\sqrt{R^2+\omega^2L^2}}\left(\cos\left(\theta - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)\right)$$

$$\therefore i = e^{-\left(\frac{R}{L}\right)t}\left[\frac{-V}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t+\theta) - \tan^{-1}\left(\frac{\omega L}{R}\right)\right] + \frac{V}{\sqrt{R^2+\omega^2L^2}}\left(\cos\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

DC Response of an R-L-C circuit:



When switch s is closed at t=0 we can determine the complete solution for the current. Application of Kirchhoff's voltage law to the circuit results in the following differential Equation.

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

By differentiating the above Equation, we have

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i \quad \text{--- (1)}$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0 \quad \text{--- (2)}$$

The above equation is a second order linear differential equation, with only complementary function. The particular solution for the above equation is zero. characteristic equation for the above differential Equation is

$$\left[D^2 + \frac{R}{L}D + \frac{1}{LC}\right] = 0 \quad \text{--- (3)}$$

The roots of the eq (3) are

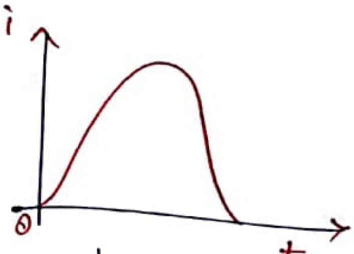
$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$K_1 = -\frac{R}{2L} \text{ and } K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$D_1 = K_1 + K_2 \text{ and } D_2 = K_1 - K_2$$

Here K_2 may be positive, negative (or) zero.

K_2 is positive, when $\left(\frac{R}{2L}\right)^2 > 1/LC$



over damped response

Case (1) overdamped

The roots are real and unequal, and give the over damped response

$$[D-(K_1+K_2)][D-(K_1-K_2)]\,i = 0$$

The solution for the above equation is

$$i = C_1e^{(K_1+K_2)t} + C_2e^{(K_1-K_2)t}$$

— The current curve for the overdamped case is as shown in Fig.

Case ②: under damped

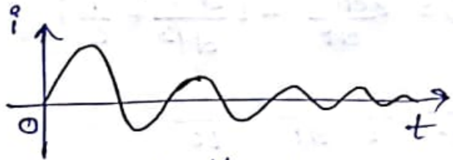
k_2 is negative, when $\left(R/2L\right)^2 < 1/LC$

— The roots are complex conjugate, and give the underdamped response

$$[D-(k_1+jk_2)][D-(k_1-jk_2)]\,i=0$$

— The solution for the above equation is $i = e^{k_1 t}[c_1 \cos k_2 t + c_2 \sin k_2 t]$

— The current curve for the underdamped case is shown in Fig



k_2 is zero, when $\left(R/2L\right)^2 = 1/LC$

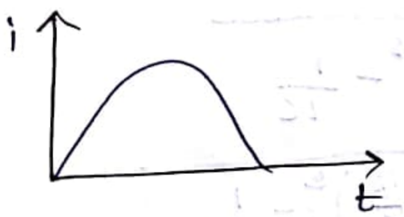
Case ③: Critically damped

— The roots are equal, and give the critically damped response then the eq becomes

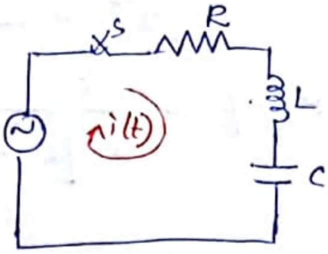
$$(D-k_1)(D-k_1)\,i=0$$

— The solution for the above equation is $i = e^{k_1 t}(c_1 + c_2 t)$

— The current curve for the critically damped case is as shown in Fig.



Sinusoidal response of RLC circuit



Consider a circuit consisting of resistance, Capacitance, inductance in series. Switch 's' is closed at $t=0$. At $t=0$ a sinusoidal voltage $V\cos(\omega t+\theta)$ is applied to the RLC series circuit where V is the amplitude of the wave and θ is the phase angle. Application of Kirchhoff's voltage law to the circuit results in the following differential equation.

$$V\cos(\omega t+\theta) = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

differentiating the above equation we get

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + i/C = -V\omega\sin(\omega t+\theta)$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = -\frac{V\omega}{L}\sin(\omega t+\theta) \quad \text{--- (1)}$$

The particular solution can be obtained by using undetermined coefficients

By assuming $i_p = A\cos(\omega t+\theta) + B\sin(\omega t+\theta)$ --- (2)

$$i_p' = -A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)$$

$$i_p'' = -A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta)$$

substituting i_p, i_p' and i_p'' in equation (1) we have

$$\{-A\omega^2\cos(\omega t+\theta) - B\omega^2\sin(\omega t+\theta)\} + \frac{R}{L}\{-A\omega\sin(\omega t+\theta) + B\omega\cos(\omega t+\theta)\}$$
$$+ \frac{1}{LC}\{A\cos(\omega t+\theta) + B\sin(\omega t+\theta)\} = -\frac{V\omega}{L}\sin(\omega t+\theta)$$

Comparing both sides we have sine coefficients.

$$-B\omega^2 - A\frac{\omega R}{L} + \frac{B}{LC} = -\frac{V\omega}{L}$$

$$A\left(\frac{\omega R}{L}\right) + B\left(\omega^2 - \frac{1}{LC}\right) = \frac{V\omega}{L} \quad \text{--- (1)}$$

putting $M\cos\phi = \frac{VR}{\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$

$$M\sin\phi = \frac{V}{\omega C\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]}$$

To find M and ϕ, we divide one equation by the other

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{1}{\omega CR}$$

Squaring both equations and adding, we get

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2}{\left[R^2+\left(\frac{1}{\omega C}\right)^2\right]} \qquad M = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

The complete solution for the current $i = i_c + i_p$

$$i = c\,e^{-t/RC} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

Since the capacitor does not allow sudden changes in voltages at $t=0$,

$$i = \frac{V}{R}\cos\theta$$

$$\therefore \frac{V}{R}\cos\theta = c + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

$$c = \frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

The complete solution for the current is

$$\therefore i = e^{-t/RC}\left[\frac{V}{R}\cos\theta - \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\theta+\tan^{-1}\frac{1}{\omega CR}\right)\right] + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \cos\left(\omega t+\theta+\tan^{-1}\frac{1}{\omega CR}\right)$$

$$-A\omega^2 + B\frac{\omega R}{L} + \frac{A}{LC} = 0$$

$$A\left(-\omega^2 + \frac{1}{LC}\right) + B\frac{\omega R}{L} = 0$$

$$A\left(\omega^2 - \frac{1}{LC}\right) - B\frac{\omega R}{L} = 0 \quad -(3)$$

$$A\left(\frac{\omega R}{L}\right) + B\left(\omega^2 - \frac{1}{LC}\right) = \frac{V\omega}{L} \quad -(4)$$

By solving (3) (4) equations we get

$$A = \frac{\frac{V\omega^2 R}{L^2}}{\left[\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2\right]}$$

$$B = \frac{\left(\omega^2 - \frac{1}{LC}\right)V\omega}{L\left[\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2\right]}$$

substituting the values of A and B in Eq (2) we get

$$i_p = \frac{\frac{V\omega^2 R}{L^2}}{\left[\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2\right]}\cos(\omega t + \theta) + \frac{\left(\omega^2 - \frac{1}{LC}\right)V\omega}{L\left[\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2\right]}\sin(\omega t + \theta)$$

Putting

$$M\cos\phi = \frac{\frac{V\omega^2 R}{L^2}}{\left(\frac{\omega R}{L}\right)^2 - \left(\omega^2 - \frac{1}{LC}\right)^2}$$

$$M\sin\phi = \frac{V\left(\omega^2 - \frac{1}{LC}\right)\omega}{L\left[\left[\frac{\omega R}{L}\right]^2 - \left(\omega^2 - \frac{1}{LC}\right)^2\right]}$$

To find M and ϕ we divide one equation by the other

$$\frac{M\sin\phi}{M\cos\phi} = \tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$\phi = \tan^{-1}\left[\left(\omega L - \frac{1}{\omega C}\right)/R\right]$$

Squaring both the Equations and adding, we get

$$M^2\cos^2\phi + M^2\sin^2\phi = \frac{V^2}{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

$$M = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

The particular current becomes

$$i_p = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left[\omega t + \theta + \tan^{-1}\frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}\right]$$

The complementary function is similar to that of DC series RLC ckt

To find out the complementary function, we have the characteristic equation

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0 \quad \text{--- (5)}$$

The roots of eq (5) are

$$D_1, D_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$K_1 = -\frac{R}{2L} \text{ and } K_2 = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$D_1 = K_1 + K_2 \text{ and } D_2 = K_1 - K_2$$

K_2 becomes positive, when $\left(R/2L\right)^2 > 1/LC$

The roots are real and unequal, which gives an overdamped response. Then eq (5) are

$$\cancel{D_1 D_2} = [D - (K_1 + K_2)][D - (K_1 - K_2)]i = 0$$

The complementary function for the above equation is

$$i_c = C_1 e^{(K_1 + K_2)t} + C_2 e^{(K_1 - K_2)t}$$

Therefore the complete solution is $i = i_c + i_p = C_1 e^{(K_1+K_2)t} + C_2 e^{(K_1-K_2)t} + \frac{V}{\ldots}$

The complete solution is $i = i_c + i_p$

$$= C_1 e^{(k_1+k_2)t} + C_2 e^{(k_1-k_2)t} + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \cos\left[\omega t + \theta + \tan^{-1}\left(\frac{1}{\omega CR} - \frac{\omega L}{R}\right)\right]$$

k_2 becomes negative, when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

Then the roots are complex conjugate, which gives an under damped response.

$$\left[D-(k_1+jk_2)\right]\left[D-(k_1-jk_2)\right] i = 0$$

The solution for the above equation is

$$i_c = e^{k_1 t}\left[c_1 \cos k_2 t + c_2 \sin k_2 t\right]$$

Therefore the complete solution is

$$i = i_c + i_p$$

$$i = e^{k_1 t}\left[c_1 \cos k_2 t + c_2 \sin k_2 t\right] + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \cos\left[\omega t + \theta + \tan^{-1}\left(\frac{1}{\omega CR} - \frac{\omega L}{R}\right)\right]$$

k_2 becomes zero, when $\left(\frac{R}{2L}\right)^2 = 1/LC$

Then the roots are equal which gives critically damped response.

Then the eq becomes $(D-k_1)(D-k_1) i = 0$

The complementary function for the above equation is

$$i_c = e^{k_1 t}\left(c_1 + c_2 t\right)$$

Therefore the complete solution is $i = i_c + i_p$

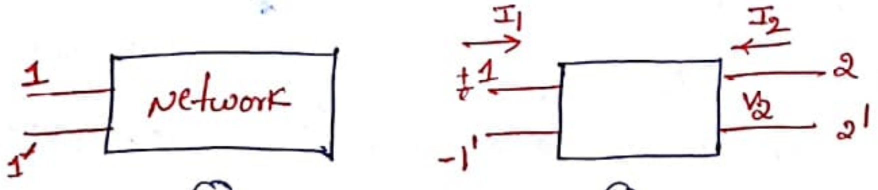
$$i = e^{k_1 t}\left[c_1 + c_2 t\right] + \frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}-\omega L\right)^2}} \cos\left[\omega t + \theta + \tan^{-1}\left(\frac{1}{\omega CR} - \frac{\omega L}{R}\right)\right]$$

UNIT-IV

TWO-PORT NETWORKS

Generally any network may be represented schematically by a rectangular box. A network may be used for representing either source (or) load (or) for a variety of purposes. A pair of terminals at which a signal may enter (or) leave a network is called a port.

A two port network is simply a network inside a black box, and the network has only two ~~points~~ ~~of~~ pairs of accessible terminals; usually one pair represents the input and the other represents the output. such a building block is very common in electronic systems, communication systems, transmission and distribution systems.



(a) (b)

Fig (b) shows a two port network, (or) two terminal pair network, in which the four terminals have been paired into ports 1-1' and 2-2'. The terminals 1-1' together constitute a port. similarly the terminals 2-2' consistute another port.

* Two ports containing no sources in their branches are called passive ports, among them are power transmission lines and transformers. Two ports containing sources in their branches are called active ports. A voltage and current ~~assigned to each of~~ ~~at~~ the two ports. The voltage and current at the input terminals are V_1 and I_1; whereas V_2 and I_2 are specified at the output port. It is also assumed that the currents I_1 and I_2 are entering into the network at the upper terminals 1 and 2, respectively. The variables of the two port network are V_1, V_2, and I_1, I_2. Two of these are dependent variables, the other two are

independent variables. The number of possible combinations generated by the four variables, taken two at a time, is six. Thus there are six possible sets of equations describing a two-port network.

Open circuit impedance parameters:

Fig(a)

A general linear two-port network is defined, in the above concept which does not contain any independent sources is shown in Fig(a)

The Z-parameters of a two port for the positive directions of voltages and currents may be defined by expressing the port voltages V_1 and V_2 in terms of the currents I_1 and I_2. Here V_1 and V_2 are dependent variables, and I_1, I_2 are independent variables. The voltage at port 1-1' is the response produced by the two currents I_1 and I_2

Thus

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{--- (1)}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{--- (2)}$$

similarly

Z_{11}, Z_{12}, Z_{21} and Z_{22} are the network functions, and are called impedance (Z) parameters, and are defined by the equations (1) and (2)

These parameters can be represented by matrices.

$$[V] = [Z][I]$$

where V is the column matrix = $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z is square matrix = $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

and we may write $[I]$ in the column matrix $= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero. suppose port 2-2' is left open circuited, then $I_2 = 0$

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

where Z_{11} is the driving point impedance at port 1-1' with port 2-2' open circuited. It is called the open circuit input impedance.

similarly $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$

where Z_{21} is the transfer impedance at port 1-1' with port 2-2' open circuited. It is also called the open circuit forward transfer impedance. suppose port 1-1' is left open circuited, then $I_1 = 0$

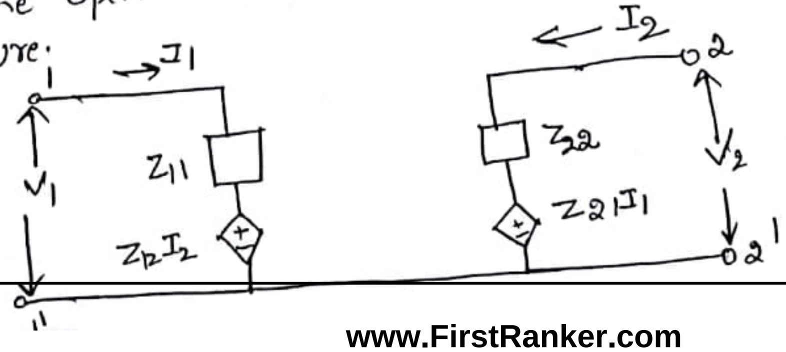
$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

where Z_{12} is the transfer impedance at port 2-2' with port 1-1' open circuited. It is also called the open circuit reverse transfer impedance.

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

where Z_{22} is the open ckt driving port impedance at port 2-2' with port 1-1' open circuited. It is also called the open circuit output impedance.

The open circuit impedance parameters is shown in below figure:

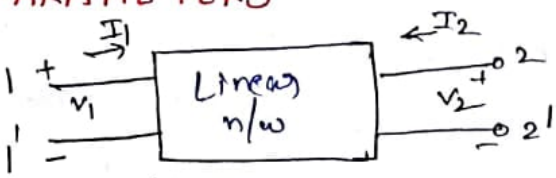


If the network under study is reciprocal (or) bilateral, then in accordance with reciprocity principle

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$Z_{21} = Z_{12}$$

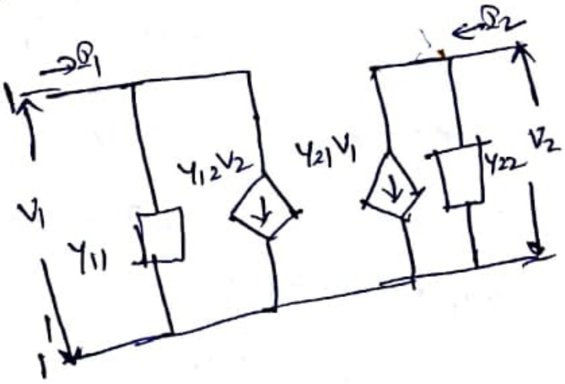
SHORT CIRCUIT ADMITTANCE (Y) PARAMETERS



The Y parameters of a two port for the positive directions of voltages and current may be defined by expressing the port currents I_1 and I_2 in terms of the voltages V_1 and V_2. Here I_1, I_2 are dependent variables and V_1 and V_2 are independent variables. I_1 may be considered to be the superposition of two components, one caused by V_1 and the other by V_2. Thus,

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{—①}$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{—②}$$

Y_{11}, Y_{12}, Y_{21} and Y_{22} are the network functions and are also called the admittance parameters.



$$[I] = [Y][V]$$

$$I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$Y_{12} = Y_{21}$

It is observed that all the parameters have the dimensions of admittance which are obtained by short circuiting either the o/p (or) i/p port from which the parameters also derive their name. i.e., the short circuit admittance parameters.

TRANSMISSION (ABCD) PARAMETERS

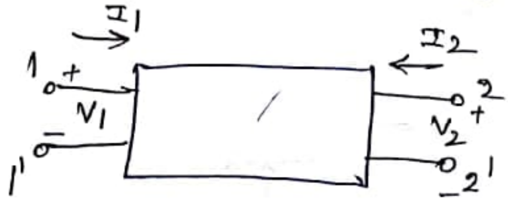
Transmission parameters are widely used in transmission line theory and cascade networks. The transmission parameters provide a direct relationship between input and output. Transmission parameters are also called general circuit parameters.

$$V_1 = AV_2 - BI_2 \quad \text{--- (1)}$$

$$I_1 = CV_2 - DI_2 \quad \text{--- (2)}$$

The negative sign is used with I_2, and not for the parameter B and D. Both the port currents I_1 and $-I_2$ are directed to the right, i.e., with a negative sign in eq (1), (2) the current at port 2-2' which leaves the port is designated as positive.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$



- with port 2-2' open i.e., $I_2 = 0$, applying a voltage V_1 at the port 1-1' using eq (1)

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad \text{and} \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$\frac{1}{A} = \frac{V_2}{V_1}\Big|_{I_2=0} = g_{21}\Big|_{I_2=0}$$

$1/A$ is called the open circuit voltage gain, a dimensionless parameter. And $\frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21}$, which is the open circuit transfer impedance. with port 2-2' short circuited i.e., with $V_2=0$, applying voltage V_1 at port 1-1' from the

$$-B = \frac{V_1}{I_2}\Big|_{V_2=0} \quad \text{and} \quad -D = \frac{I_1}{I_2}\Big|_{V_2=0}$$

$-\frac{1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0} = Y_{21}$, which is the short circuit transfer admittance.

$-\frac{1}{D} = \frac{I_2}{I_1}\Big|_{V_2=0} = \alpha_{21}\Big|_{V_2=0}$, which is the short current gain, a dimensionless parameter.

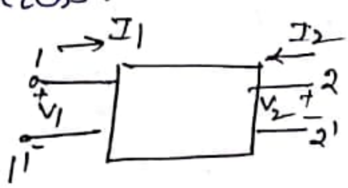
Inverse transmission parameters $(A'B'C'D')$

The voltage and current at the receiving end can also be expressed in terms of the sending end voltage and current. If the voltage and current at port 2-2' is expressed in terms of voltage and current at port 1-1'

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$

The coefficients A', B', C' and D' in the above Equations are called inverse transmission parameters.

Thus when port 1-1' is open $I_1 = 0$

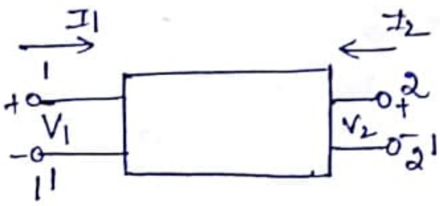


$$A' = \frac{V_2}{V_1}\Big|_{I_1=0}; \quad C' = \frac{I_2}{V_1}\Big|_{I_1=0}$$

If port 1-1' is short circuited, $V_1 = 0$

$$B' = \frac{-V_2}{I_1}\Big|_{V_1=0}; \quad D = \frac{-I_2}{I_1}\Big|_{V_1=0}$$

Hybrid parameters:



Hybrid parameters find extensive use in transistor circuits. They are well suited to transistor ckts as these parameters can be most conveniently measured. The hybrid matrices describe a two-port, when the voltage of one port and the current of other port are taken as independent variables.

If the voltage at port 1-1' and current at port 2-2' are taken as dependent variables, we can express them in terms of I_1 and V_2.

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{--- (1)}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{--- (2)}$$

The coefficients in the above equations are called hybrid parameters.

In matrix notation.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

From the equations (1) and (2) the individual h parameters may be defined by letting $I_1 = 0$ and $V_2 = 0$

when $V_2 = 0$, the port 2-2' is short circuited.

Then $h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ short circuit input impedance $\left(\frac{1}{Y_{11}}\right)$

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ short circuit forward current gain $\left(\frac{Y_{21}}{Y_{11}}\right)$

$$\frac{1}{A} = \frac{V_2}{V_1}\Big|_{I_2=0} = g_{21}\Big|_{I_2=0}$$

$1/A$ is called the open circuit voltage gain, a dimensionless parameter. And $\frac{1}{C} = \frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21}$, which is the open circuit transfer impedance. with port 2-2' short circuited ie., with $V_2=0$, applying voltage V_1 at port 1-1' from the

$$-B = \frac{V_1}{I_2}\Big|_{V_2=0} \quad \text{and} \quad -D = \frac{I_1}{I_2}\Big|_{V_2=0}$$

$$-\frac{1}{B} = \frac{I_2}{V_1}\Big|_{V_2=0} = Y_{21}$$, which is the short circuit transfer admittance.

$$-\frac{1}{D} = \frac{I_2}{I_1}\Big|_{V_2=0} = \alpha_{21}\Big|_{V_2=0}$$, which is the short current gain, a dimensionless parameter.

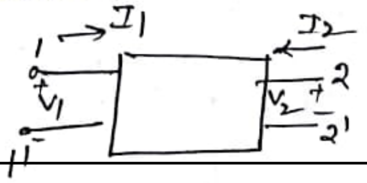
Inverse transmission parameters $(A'B'C'D')$

The voltage and current at the recieving end can also be expressed in terms of the sending end voltage and current. If the voltage and current at port 2-2' is expressed in terms of voltage and current at port 1-1'

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$

The coefficients A', B', C' and D' in the above Equations are called inverse transmission parameters.

Thus when port 1-1' is open $I_1=0$

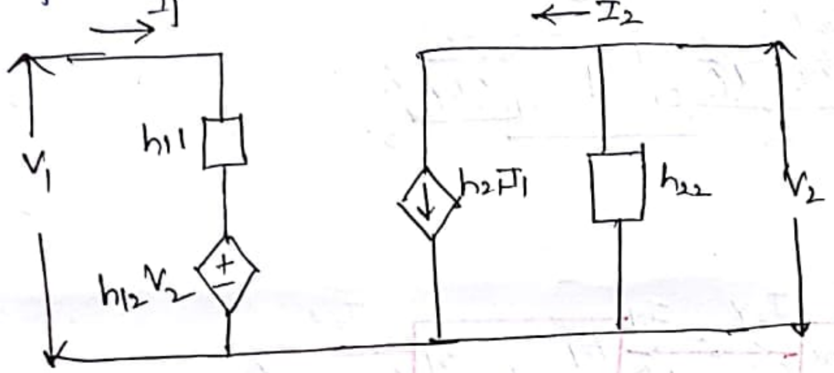


similarly, by letting port 1-1 open, $I_1=0$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$ open circuit reverse voltage gain $\left(\frac{Z_{12}}{Z_{22}}\right)$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$ open circuit output admittance $\left(\frac{1}{Z_{22}}\right)$

Since h parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called hybrid parameters. An Equivalent circuit of a two port network in terms of hybrid parameters is shown in Fig.



Inverse hybrid parameters:

The current at the input port I_1 and the voltage at the output port V_2 can be expressed in terms of I_2 and V_1.

$$I_1 = g_{11}V_1 + g_{12}I_2 \quad \text{—(1)}$$
$$V_2 = g_{21}V_1 + g_{22}I_2 \quad \text{—(2)}$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

It can be verified that $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$

The individual g parameters may be defined by letting $I_2=0$ and $V_1=0$ in eq (1) (2)

Thus when $I_2=0$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0} = \text{open ckt i/p admittance } \left(\frac{1}{z_{11}}\right)$$

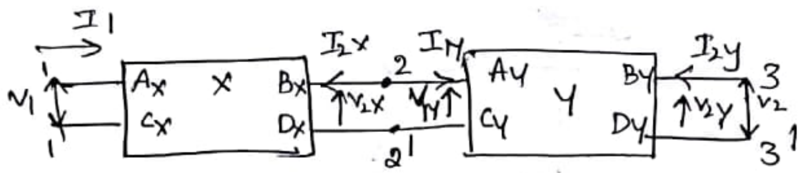
$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0} = \text{open ckt voltage gain}$$

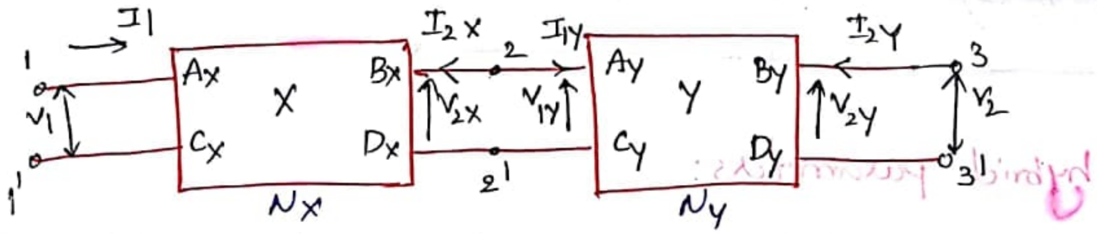
when $V_1 = 0$

$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0} = \text{short ckt reverse current gain}$$

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0} = \text{short ckt o/p impedance } \left(\frac{1}{y_{22}}\right)$$

Cascade Connection:





The main use of the transmission matrix is in dealing with a cascade connection of two-port network.

Consider two two-port networks N_x and N_y connected in cascade with port voltages and currents as indicated. The matrix representation of ABCD parameters for the network X is as under

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} V_{2x} \\ -I_{2x} \end{bmatrix}$$

And for the network Y the matrix representation is

$$\begin{bmatrix} V_{1y} \\ I_{1y} \end{bmatrix} = \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_{2y} \\ -I_{2y} \end{bmatrix}$$

It can also be observed that at for 2-2'

$$V_{2x} = V_{1y} \text{ and } I_{2x} = -I_{1y}$$

Combining the results, we have

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transmission parameters matrix for the overall network. Thus the transmission matrix of a cascade of two port networks is the product of transmission matrices of the individual two-port networks. This property is used in the design of telephone systems, microwave networks, radars.

UNIT - V

Network Synthesis

Concept of stability of a system from pole zero concept:

Let a system be represented as (denoted by ratio of polynomials)

$$F(s) = \frac{s(s-1+j2)(s-1-j2)}{(s+2)(s+j1)(s-j1)}$$

It may be observed that $F(s)$

→ $F(s)$ becomes zero for $s=0$, $(1-j2)$ and $(1+j2)$

→ $F(s)$ becomes infinity for $s=-2, -j1$ and $j1$

→ Any of the roots of the denominator of $F(s)$ define a "pole."

→ Any of the roots of the numerator of the same System defines a "zero"

→ The pole is denoted by a "small cross ×"

→ zero is denoted by a "small circle o"

Necessary conditions of stability of a Network function $F(s)$:

The network function $F(s)$ is stable when the following three conditions must be satisfied.

1. $F(s)$ cannot have poles in the right half of s-plane.
2. $F(s)$ should not have any multiple poles in the $j\omega$ axis.
3. The degree of the numerator of $F(s)$ can not exceed the ~~order~~ degree of the denominator by more than unity, ie, if

$$F(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0}{b_m s^m + b_{m-1} s^{m-1} + \dots + b_m s + b_0}$$

then the order of n cannot exceed the order of m by more than unity ie, $n-m \leq 1$. However if $n-m>1$, it would simple mean multiple poles at $s=\infty$ which impairs the stability.

Hurwitz Polynomials:

The system function represented by ratio of two polynomials is to be stable, the poles must exist at the left half of s-plane. Also the poles should be simple. The denominator polynomial P(s) of the system function is then termed as "Hurwitz polynomial".

Then a polynomial is said to be Hurwitz if

(i) P(s) is real when s is real.

(ii) the roots of P(s) have real parts which are to be zero or negative.

Properties of Hurwitz Polynomials:

A Hurwitz polynomial P(s) can have following three types of roots.

a) $s=-r$, r being real and positive

b) $s=\pm j\omega$, ω being real

c) $s=-\sigma \pm j\omega$, σ being real and positive.

∴ The polynomial can be written as

$$P(s) = (s+r)(s+\omega^2)\left[(s+\sigma)^2+\omega^2\right]$$

The Hurwitz polynomial has the following properties

$$P(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0.$$

a) The coefficients of the s terms must be Positive

b) Both the odd and even parts of the Hurwitz polynomial have roots on the imaginary axis ($j\omega$ axis) only

c) The continued fraction expansion of the ratio of even to odd parts of the Hurwitz polynomial gives all positive quotient terms.

ie, supposing $Z(s) = \frac{M(s)}{N(s)} \left[\text{or } \frac{N(s)}{M(s)}\right]$ the continued fraction can be written as

$$Z(s) = \alpha_1 s + \cfrac{1}{\alpha_2 s + \cfrac{1}{\alpha_3 s + \cfrac{1}{\alpha_4 s + \cdots + \cfrac{1}{\alpha_n s}}}}$$

where $\alpha_1, \alpha_2, \cdots \alpha_n$ are the quotients in each step of the division

$$\frac{M(s)}{N(s)} \text{ or } \frac{N(s)}{M(s)}$$

These coefficients must be Positive when the polynomial $P(s)$ is Hurwitz.

d) If the Polynomial satisfies the condition of Hurwitz, the

Polynomial then is Hurwitz to an even multiplicative factor $w(s)$;

ie., If $P_1(s) = W(s)\,P(s)$ and $P(s)$ is Hurwitz, $P_1(s)$ must be Hurwitz.

e) If the ratio of polynomial $P(s)$ and its derivative gives a continued fraction expansion with all positive coefficients then $P(s)$ is Hurwitz. In case the polynomial is either only even or only odd, the method of continued fraction is not possible and this method is then beneficial.

Check whether the polynomial $s^5 + 9s^4 + 7s^3 + s^2 + 4s$ is Hurwitz or not.

$$P(s) = s^5 + 9s^4 + 7s^3 + s^2 + 4s$$

$$= M(s) + N(s)$$

where $M(s) = 9s^4 + s^2 = s^2(9s^2 + 1)$

$$N(s) = s^5 + 7s^3 + 4s = s(s^4 + 7s^2 + 4)$$

Let $$Z(s) = \frac{N(s)}{M(s)} = \frac{\cancel{s}\,(s^4 + 7s^2 + 4)}{s^{\cancel{2}}\,(9s^2 + 1)}$$

using continued fraction method, the function

$Z(s) = \dfrac{N(s)}{M(s)}$ is tested as follows.

$$9s^3 + s)\; s^4 + 7s^2 + 4 \;(s/9$$
$$s^4 + s^2/9$$

$$\frac{62}{9}s^2 + 4)\; 9s^3 + s \;\left(\frac{81}{62}s\right.$$
$$9s^3 + \frac{4 \times 81}{62}s$$

$$\left(\frac{-13}{5}\right) \quad \frac{62}{9}s^2+4 \; ($$

Since the continued fraction contains negative quotients hence the given polynomial is not Hurwitz.

Positive Real Functions:

$$F(s) = \frac{A(s)}{B(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1}s + b_m}$$

The function $F(s)$ is called a positive real function if

a) $F(s)$ is real for s real

b) $B(s)$ is Hurwitz polynomial

c) If $F(s)$ has poles on $j\omega$ axis, the poles are simple and the residues there of are real and Positive.

d) Real $F(j\omega) \geq 0$ for all values of ω.

Properties of PR functions:

Let $f(s) = \frac{A(s)}{B(s)}$

The properties of PR functions are as below:

(i) Both $A(s)$ and $B(s)$ polynomials are Hurwitz. They have factors of the form $(s^2+\omega^2)$ ie, the poles and zeros of a PR function cannot have +ve real parts ie, they cannot be in the right half of the s-plane. only simple poles with +ve real residues can exist on the $j\omega$ axis.

(ii) The highest and lowest powers of $A(s)$ and $B(s)$ differ by unity. This condition prohibits multiple pole and zeros at $s=\infty$ and $s=0$ respectively.

(iii) If $F(s)$ is a PR function, the reciprocal of $F(s)$ is also a PR function

(iv) The sum of PR functions is also a PR function

Statement: All real parts of the driving point immitances (impedance or admittance) of RLC networks (passive networks) must be PR functions.

For a passive network, the average power dissipated has always a Positive real value.

ie, $P_{av} = \frac{1}{2} Re[Z_{in}(j\omega)]|i|^2 \geq 0$

or, $Re[Z_{in}(j\omega)] \geq 0$

$\therefore$ for any passive network, $Re[Z_{in}(j\omega)] \geq 0$.

ie, the real parts of the driving point impedance are a PR function.

The necessary and sufficient conditions of positive realness of a given rational function are as follows;

1. $F(s)$ must have simple poles on $j\omega$ axis with real and Positive residues.

2. If $F(S) = \frac{P(S)}{Q(S)}$ then $P(S) + Q(S)$ must be Hurwitz.

3. If $F(s) = \frac{P(s)}{Q(s)} = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$, M being the even parts and N being the odd parts, then for positive realness of $F(s)$, $M_1M_2 - N_1N_2 |_{s=j\omega} \geq 0$ for all ω such that $Re[F(j\omega)] \geq 0$ for all ω.

Concept of Network synthesis:

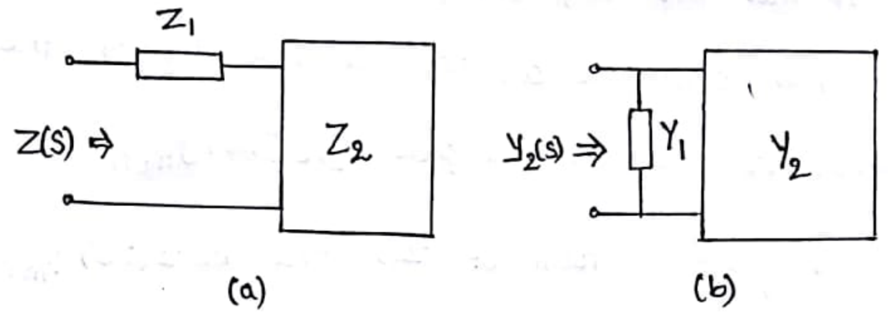


fig: Sum of Networks in impedance and admittance form

Let us ~~start~~ consider two one pole networks. Basically these two networks are equivalent provided.

$$Z(S) = Z_1(S) + Z_2(S)$$

$$Y(S) = Y_1(S) + Y_2(S)$$

Network synthesis then refers to the operational procedures in which a positive real function is given for $Z(s)$ or $Y(S)$ and it is required to select appropriate forms of Z_1 and Z_2 or Y_1 and Y_2. for this we require the positive realness of each function Z_1 and Z_2 or Y_1 and Y_2 so that their sum would be obviously a PR function. when one of the functions in the sum is recognizable from the simplicity of its form of expression, the other can be determined through the steps of.

Synthesis. The synthesis is complete when all the elements of the sum are known or identified. usually expansion technique (long division procedure) is adopted in order to perform the steps of synthesis.

Procedure of Synthesis:

Step 1: The given function being a PR function, it must be either in form of $Z(s)$ or $Y(s)$. In this first step substitute $j\omega$ for s in the expression of $Z(s)$ or $Y(s)$, rationalise and find the real part of the function. find the minimum value of the real part $[Re(j\omega)]_{min}$.

Step 2: If the functions given in form of $Z(s)$ the $[Re\, Z(j\omega)]_{min}$ is then a constant term signifying the presence of a series resistance whose magnitude is equal to the minimum real value just obtained However if given in form of $Y(s)$, the obtained min value is that of a shunt resistor.

Step 3: In case the given function does not have any real part then it becomes evident that the given function $[Z(s)$ or $Y(s)]$ does not have any resistive part and contains either inductance or capacitance.

Step 4: In this step check whether the numerator of the given function has one 's' term higher. If the numerator has one 's' term higher, pole appears at $s=\infty$, on the other hand if the denominator ~~has one s term higher, Pole appears at $s=0$.~~

This can be explained as follows:

$$Z(s) = \frac{a_{n+1} s^{n+1} + a_n s^n + \cdots + a_1 s + a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

By long division,

$$Z(s) = K_0 s + \frac{\text{Remainder}}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0}$$

It can now be observed that $Z(s)$ will have a pole at $s = \infty$

On the other hand, if

$$Z(s) = \frac{a_0 + a_1 s + \cdots + a_{n-1} s^{n-1} + a_n s^n}{b_1 s + b_2 s^2 + \cdots + b_n s^n}$$

by long division

$$Z(s) = \frac{K_0}{s} + \frac{\text{remainder}}{b_1 + b_2 s + \cdots + b_{n-1} s^{n-1}}$$

It can be observed that $Z(s)$ will have a pole at $s = 0$

Step 5: Perform the first long division to obtain the first quotient. In the first case of step 4, the quotient will be $(K_0 s)$ indicating the value of the series inductor to be K_0 Henry. If the function is originally expressed in Y form, $(K_0 s)$ indicates the presence of a shunt capacitor of value K_0 farad. On the other hand if the first quotient is K_0/s in step 4, the magnitude of the series capacitor is $(1/K_0)$ farad when the original function is expressed in Z form. If the original function is expressed in Y form, this K_0/s quotient indicates that the first element being shunt indicator, its value is $(1/K_0)$ Henry.

Step 6: Once the first part of the given function is realised, the remainder is inverter and the realisation is again commenced starting from step-1

This process continuous till the last element is realised.

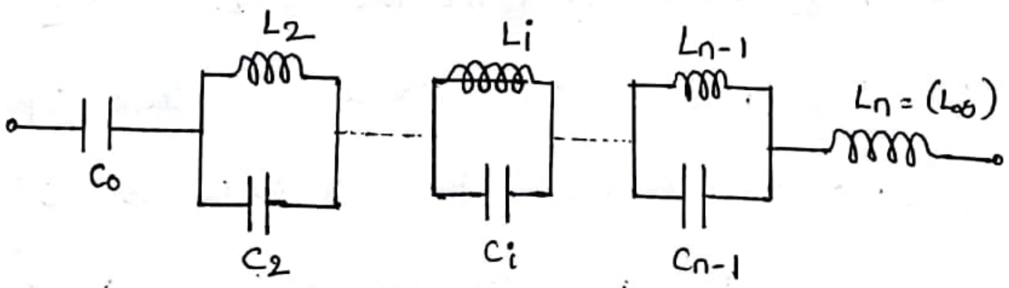
LC Network Synthesis:

Any LC network can be synthesized in two forms designated by the names of the inventors foster and cauers.

Fosters canonic form:

In foster synthesis there are two types of network for relisation of one port reactive network. The first type is a series combination of parallel LC networks while the other is the parallel combination of series LC networks.

First foster form:



first form of foster LC network (impedance form)

$Z(s)$ is a PR function having simple poles at $j\omega$ axes as well as simple poles at $s=0$, and $s=\infty$ using partial fraction $Z(s)$ can be expressed as

$$Z(s) = \frac{A_0}{s} + \sum_{i=2}^{n-1} \frac{2A_i s}{s^2+\omega_i^2} + H_1 s \quad \text{— (1)}$$

when the constituent terms in $Z(s)$ can be interpreted as

(a) $\frac{A_0}{s}$ results from a possible pole at $s=0$ ($\frac{A_0}{s}$ is the first term)

(b) $H_1 s$ results from a possible pole at $s=\infty$ [$H_1 s$ is the last n^{th} term]

(c) $\frac{2A_i s}{s^2+\omega_i^2}$ results from a pair of conjugate poles on the $j\omega$ axis and obviously,

$$\frac{2A_i s}{s^2+\omega_i^2} = \frac{A_i}{s+j\omega_i} + \frac{A_i'}{s-j\omega_i} = \frac{(A_i+A_i')s - j(A_i-A_i')\omega_i}{s^2+\omega_i^2} \quad \text{— (2)}$$

In order that they give a real term after combination it is essential that the residues A_i and A_i' must be equal.

for the parallel combination of L, C

$$Z(s) = \frac{1}{Cs+\frac{1}{Ls}} = \frac{(1/C)\, s}{s^2+\frac{1}{LC}}$$

from equation (1)

$$Z(s) = \frac{A_0}{s} + \frac{2A_2 s}{s^2+\omega_2^2} + \cdots + \frac{2A_{n-1} s}{s^2+\omega_{n-1}^2} + H_1 s \quad \text{— (3)}$$

Also $Z(s) = Z_1(s) + Z_2(s) + \cdots + Z_n(s)$ — (4)

Comparing equations (3) and (4)

$Z_1(s) = A_0/s$ $\quad Z_n(s) = H \cdot s$

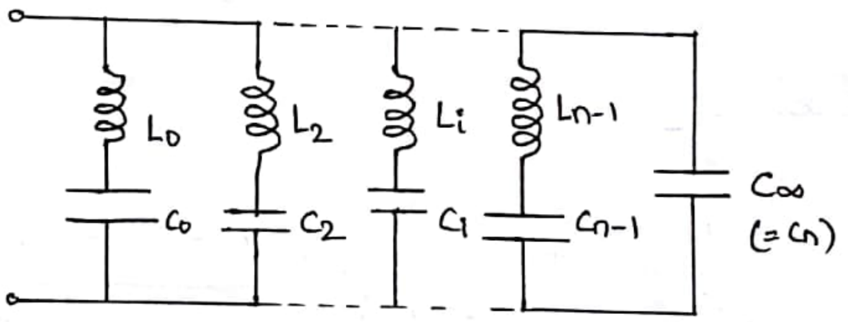
→ $Z_1(s)$ is an impedance representing a capacitor C_0 of value $\frac{1}{A_0}$

→ $Z_n(s)$ represents an inductor of inductance H henry (L_∞)

→ The remaining intermediate terms represent parallel combination of an inductor and a capacitor.

The mid terms represented as $C_i = \frac{1}{2A_i}$ and $L_i = \frac{2A_i}{\omega_i^2}$

Second foster form:



The second form of foster network is a parallel combination of series LC networks. As all the series branches are connected in parallel, it is convenient to consider the driving point admittance $Y(s)$.

$$Y_{series}(s) = \frac{1}{Ls + \frac{1}{Cs}} = \frac{(s/L)}{s^2 + 1/LC}$$

from equ (1)

$$Y(s) = \frac{B_0}{s} + \frac{2B_2 s}{s^2+\omega_2^2} + \dots + \frac{2B_i s}{s^2+\omega_i^2} + \dots + \frac{2B_{n-1} s}{s^2+\omega_{n-1}^2} + Hs$$

Total admittance for parallel combination of branch admittances

$$Y(s) = Y_1(s) + Y_2(s) + \dots + Y_n(s)$$

By comparing $Y_1(s) = B_0/s$ represent an inductor L_0 of value $1/B_0$

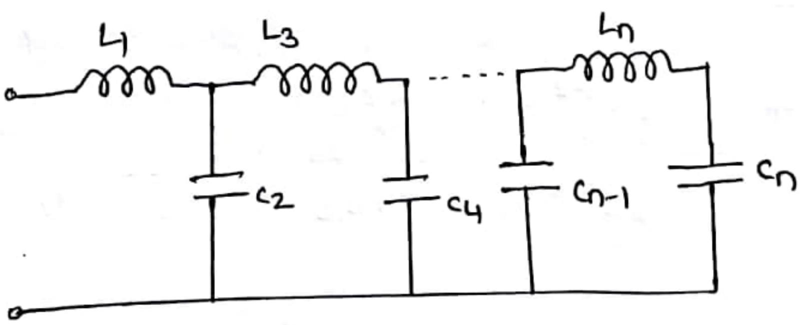
$Y_n(s) = Hs$ represents a capacitor C_∞ of value H farad

The intermediate terms represent series combination of an inductor and capacitor.

$$L_i = \frac{1}{2B_i} \qquad C_i = \frac{2B_i}{\omega_i^2}$$

Cauer form of LC network:

first form of cauer network:



$$Z(s) = \frac{a_n s^n + a_{n-2}s^{n-2} + \dots + a_0}{b_m s^m + b_{m-2}s^{m-2} + \dots + b_1 s}$$

$$\therefore \lim_{s\to\infty} \frac{a_n s^n}{b_m s^m} \begin{cases} = \infty & \text{if } n > m \\ = 0 & \text{if } m > n \end{cases}$$

(i) $\lim_{s\to\infty} Z(s) \to \infty$ if $n > m$

$\therefore$ pole is formed at $s \to \infty$ This gives L_1 having finite value

and

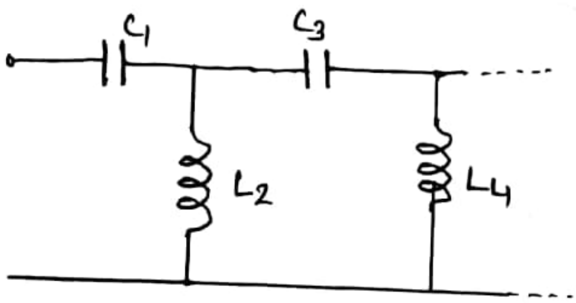
$$Z(s) = L_1 s + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \cdots}}}$$

(ii) $\lim_{s\to\infty} Z(s) \to 0$ if $n < m$

$\therefore$ zero is formed at $s \to \infty$ This is possible if $L_1 = 0$

$$Z(s) = \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s \cdots}}}$$

second form of cauer network:



$$Z(s) = \frac{a_n s^n + a_{n-2} s^{n-2} + \cdots + a_0}{b_m s^m + b_{m-2} s^{m-2} + \cdots + b_1 s}$$

(i) if $Z(s) \to \infty$ with $s \to 0$ a pole appears in $Z(s)$ and

since $Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\frac{1}{L_2 s} + \cfrac{1}{\frac{1}{C_3 s} + \cfrac{1}{L_4 s + \cdots}}}$

hence C_1 must have some definite value.

(ii) with $\lim_{s \to 0} Z(s) \to 0$

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\frac{1}{L_2 s} + \cfrac{1}{\frac{1}{C_3 s} + \cfrac{1}{L_4 s + \cdots}}}$$

hence C_1 must be zero.

UNIT - VI

Fourier analysis and Transforms

Introduction:

The analysis of networks with sinusoidal inputs is made easy by consideration of frequency domain circuits. The need for analysis of networks with non sinusoidal periodic signals arise in communication networks. Fourier has shown that any non sinusoidal periodic signal could be resolved into sinusoidal components of different frequencies. Fourier series are series of sine and cosine terms and are used to represent general periodic signals and as well as non periodic signals.

Fourier analysis deals with fourier series and fourier transforms and has several applications in mathematics, science and engineering particularly in the area of communication and signal processing

Exponential form of fourier series:

As per fourier representation

$$f(t) = a_0 + \sum_{n=1}^{\infty} M_n \cos(n\omega_0 t + \theta_n)$$

where $\omega_0 = \frac{2\pi}{T_0}$

T_0 being the time period.

Taking $n=2$, T_0 represents the two cycles within T_0 seconds while the term $M_2 \cos(2\omega_0 t + \theta_2)$ is called the 2nd harmonic

In the continued fashion thus for $n=K$, K cycles fall within T_0 seconds and $H_K \sin(K\omega_0 t + \theta_K)$ is the K^{th} harmonic term

using Euler identity,

$$f(t) = a_0 + \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$ (C_n = complex fourier coefficients)

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

fourier coefficients C_n can be evaluated as follows:

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

multiplying both sides of the above expression by $e^{-jK\omega_0 t}$ and integrating over the interval t_1 to (t_1+T_0) we get

$$\int_{t_1}^{t_1+T_0} f(t)\, e^{-jK\omega_0 t}\, dt = \int_{t_1}^{t_1+T_0} \left(\sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \right) e^{-jK\omega_0 t}\, dt$$

$$= C_K T_0$$

since $$\int_{t_1}^{t_1+T_0} e^{j(n-K)\omega_0 t}\, dt = \begin{cases} 0 & \text{for } n \neq K \\ T_0 & \text{for } n = K \end{cases}$$

$\therefore$ The fourier coefficients are defined by the expression

$$C_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

The above expression represents the exponential form of fourier series.

Trignometric form of fourier series:

$$c_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

from the above equation we can write

$$2C_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \left[\cos n\omega_0 t - j \sin n\omega_0 t\right] dt$$

using Eulers identity

$$2C_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t\, dt - j\frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t\, dt \quad \text{--- (1)}$$

However C_n being a complex coefficient,

$$2C_n = a_n - jb_n \quad \text{--- (2)}$$

comparing (1) and (2)

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin n\omega_0 t\, dt$$

Also from equation C_n C_0 being written here as a_0 and is given by

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t)\, dt$$

The average value (a_0) of the waveform then can be directly evaluated from the waveform.

Since the periodic function can be represented as a sum of sinusoidal functions we can rewrite equation as

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\alpha + \sum_{n=1}^{\infty} b_n \sin n\alpha$$

where a_0 = a constant

$a_1, a_2, a_3, \ldots a_n$ and $b_1, b_2 \cdots b_n$ are the amplitudes of different harmonics.

α = a variable and n = an integer $1, 2 \ldots$

fourier series can also be expressed in terms of either sine or cosine terms.

let us assume

$$M = a_n \cos n\alpha + b_n \sin n\alpha$$

$$= \sqrt{a_n^2 + b_n^2}\left[\frac{a_n}{\sqrt{a_n^2+b_n^2}} \cos n\alpha + \frac{b_n}{\sqrt{a_n^2+b_n^2}} \sin n\alpha\right]$$

let us now assume

$$\frac{a_n}{\sqrt{a_n^2+b_n^2}} = \sin\phi_n \qquad \frac{b_n}{\sqrt{a_n^2+b_n^2}} = \cos\phi_n$$

$$\therefore M = \sqrt{a_n^2 + b_n^2} \, (\sin\phi_n \cos n\alpha + \cos\phi_n \sin n\alpha)$$

$$= K_n \sin(n\alpha + \phi_n)$$

where $K_n = \sqrt{a_n^2 + b_n^2}$

Thus finally we can write

$$f(t) = a_0 + K_1 \sin(\alpha + \phi_1) + K_2 \sin(2\alpha + \phi_2) + \cdots$$

$$+ K_n \sin(n\alpha + \phi_n)$$

Here $\tan\phi_n = \dfrac{\sin\phi_n}{\cos\phi_n} = \dfrac{a_n}{b_n}$

on the other hand if we represent

$$\frac{a_n}{\sqrt{a_n^2+b_n^2}} = \cos\phi_n, \quad \frac{b_n}{\sqrt{a_n^2+b_n^2}} = \sin\phi_n$$

$$M = \sqrt{a_n^2 + b_n^2} \, (\cos\phi_n \sin n\alpha + \sin\phi_n \sin n\alpha)$$

$$= K_n \cos(n\alpha - \phi_n)$$

$$f(t) = a_0 + K_1 \cos(\alpha - \phi_1) + K_2 \cos(2\alpha - \phi_2) + \cdots$$

$$+ K_n \sin(n\alpha - \phi_n)$$

Here $\tan\phi_n = \dfrac{\sin\phi_n}{\cos\phi_n} = \dfrac{b_n}{a_n}$

fourier Transform:

fourier series are powerful tools in dealing various problems involving periodic functions. In several applications, many practical problems involve non periodic functions. The method of fourier series can be extended to such non periodic functions.

The exponential form of fourier series for periodic signal is given below for convenience.

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t} \quad \text{—— ①}$$

$$X_n = \frac{1}{T_0} \int_{-T/2}^{T/2} \tilde{x}(t) e^{-jn\omega_0 t} dt$$

$$x_n = \frac{1}{T_0} \int_{-T/2}^{T/2} x(t) e^{-jn2\pi f_0 t} dt \quad \text{—— ②}$$

where f_0 is the fundamental frequency of the periodic signal $x(t)$

If we define non periodic signal

$$x(t) = \tilde{x}(t) \quad \text{for } |t| < T_0/2 \quad \text{—— ③}$$

$$= 0 \quad \text{otherwise.}$$

$$x_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jn2\pi f_0 t} dt \quad \text{—— ④}$$

$$T_0 x_n = \int_{-\infty}^{\infty} x(t)\, e^{-jn2\pi f_0 t}\, dt \quad \text{--- (5)}$$

we can define the envelope $T_0 x_n$ as

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad \text{--- (6)}$$

where $\omega = 2\pi n f_0$

Therefore, the coefficient X_n becomes

$$X_n = \frac{1}{T_0} X(\omega) \quad (\text{or}) \quad X_n = \frac{1}{T_0} X(n\omega_0) \quad \text{--- (7)}$$

substituting (7) in (6) we get

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{T_0} X(n\omega_0)\right] e^{jn\omega_0 t}$$

$$(\text{or}) \quad \tilde{x}(t) = \frac{1}{T_0} \sum_{n=-\infty}^{\infty} X(n\omega_0)\, e^{jn\omega_0 t}$$

substituting $\omega_0 = \frac{2\pi}{T_0}$

$$\tilde{x}(t) = \frac{\omega_0}{2\pi} \sum_{n=-\infty}^{\infty} X(n\omega_0)\, e^{jn\omega_0 t}$$

$$\tilde{x}(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} X(n\omega_0)\, e^{jn\omega_0 t}\, \omega_0 \quad \text{--- (8)}$$

In the above complex ~~terms~~ sum let the frequency ω_0 approach to zero, the index n approach to infinity such that the product $n\omega_0$ approaches a continues frequency variable ω and the discrete sum is replaced by a continuous integral.

Therefore $\tilde{x}(t) \rightarrow x(t)$ as ω_0 approaches to zero

Then equ (6) becomes

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

and equ (8) becomes

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \quad \text{--- (10)}$$

These expressions define a fourier transform pair for the signal x(t) and are denoted by the notation

$$x(t) \longrightarrow X(\omega)$$

The integral eq (5) is referred to as fourier transform of x(t) and is denoted by $X(\omega) = F[(x(t))]$

The integral eq (10) is referred to as the inverse fourier transform and is often denoted by the symbol $x(t) = F^{-1}[X(\omega)]$

fourier transform is essentially a transformation from a function of the time variable t to the frequency variable ω. Not all functions can be transformed but satisfying the Dirichlet conditions in any finite interval and

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

are sufficient conditions for the existence of fourier transforms.

fourier transform may be represented by writing $x(\omega)$ in terms of magnitude and phase as.

$$x(\omega) = |x(\omega)| e^{-j\theta(\omega)}$$

plots of $|x(\omega)|$ and $\theta(\omega) = \angle x(\omega)$ versus frequency f are referred to as the amplitude and phase spectra of $x(t)$ respectively.

Properties of the fourier transform:

the properties of the fourier transform facilitate the transformation from the time domain to the frequency domain and viceversa.

Linearity:

The fourier transform satisfies linearity and principle of superposition. Consider two signals $x_1(t)$ and $x_2(t)$

If $x_1(t) \longleftrightarrow X_1(\omega)$

$x_2(t) \longleftrightarrow X_2(\omega)$

then $a_1 x_1(t) + a_2 x_2(t) \longleftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$

proof: $F\,(a_1 x_1(t) + a_2 x_2(t)]$

$= a_1 F[x_1(t)] + a_2 F[x_2(t)]$

$= a_1 X_1(\omega) + a_2 X_2(\omega)$

Therefore the linearity is proved.

Scaling:

consider fourier transform of $x(t)$ is $X(\omega)$

If $x(t) \longleftrightarrow X(\omega)$

then for real constant a

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

Proof: Assume $a>0$. Then fourier transform of $x(at)$ is

$$F\{x(at)\} = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

let $m = at$ then $dm = a\,dt$

$$\therefore F\{x(at)\} = \int_{-\infty}^{\infty} x(m)\, e^{-\left[j\omega \frac{m}{a}\right]} \cdot \frac{dm}{a} = \frac{1}{a} X\left[\frac{\omega}{a}\right]$$

If $a<0$, then

$$F\{x(at)\} = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

now $-m = +at$

$$F\{x(at)\} = \int_{-\infty}^{\infty} x(-m) e^{-j\left(\frac{\omega}{a}\right)(-m)} \left[\frac{-dm}{a}\right]$$

$$= \frac{-1}{a} \int_{-\infty}^{\infty} x(-m)\, e^{-j\left(\frac{-m}{a}\right)\omega}\, dm$$

$$= -\frac{1}{a} X(\omega)$$

Combining these two values, we get

$$x(at) \longleftrightarrow \frac{1}{|a|} X(\omega)$$

Symmetry:

If $x(t) \longleftrightarrow X(\omega)$

then $X(t) \longleftrightarrow 2\pi X(-\omega)$

proof: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega$

then replacing the dummy variable ω by ω'

$$2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega') e^{-j\omega' t} \, d\omega'$$

Now replace t by ω

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(\omega') e^{-j\omega' \omega} \, d\omega'$$

finally replace ω' by t, thus

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t) e^{-j\omega t} \, dt = F\{X(t)\}$$

$$X(t) \longleftrightarrow 2\pi X(-\omega)$$

Convolution:

fourier transform makes the convolution of two signals into the product of their fourier transforms. There are two types of convolution properties, one for the time domain and one for the frequency domain.

(i) Time convolution

If $x_1(t) \leftrightarrow X_1(\omega)$

$x_2(t) \leftrightarrow X_2(\omega)$

then
$$y(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau \leftrightarrow Y(\omega)$$
$$= X_1(\omega) \cdot X_2(\omega)$$

(ii) frequency convolution

If $x_1(t) \leftrightarrow X_1(\omega)$

$x_2(t) \leftrightarrow X_2(\omega)$

then $x_1(t)\, x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$

frequency Differentiation and Integration

If $x(t) \leftrightarrow X(\omega)$

then $(-jt)\, x(t) \leftrightarrow \frac{dx(\omega)}{d\omega}$

Time shifting:

If $x(t) \leftrightarrow X(\omega)$

then $x(t-t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$

from the above, we can say that shift in time domain implies shift in phase in the transform.

Frequency shifting:

If $x(t) \leftrightarrow X(\omega)$

then $x(t)\, e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0)$

frequency shifting or translation has an important significance in communication engineering. this process is known as modulated signal.

	Properties of the fourier Transform	
1.	Transformation	$x(t) \leftrightarrow X(\omega)$
2.	linearity	$a_1x_1(t) + a_2x_2(t) \leftrightarrow a_1X_1(\omega) + a_2X_1(\omega)$
3.	Scaling	$x(at) \leftrightarrow \frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
4.	Symmetry	$X(jt) \leftrightarrow 2\pi\, X(-\omega)$
5.	Time shifting	$x(t-t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$
6.	frequency shifting	$e^{j\omega_0 t} x(t) \leftrightarrow X(\omega-\omega_0)$
7.	Time convolution	$x_1(t) * x_2(t) \leftrightarrow X_1(\omega)\, X_2(\omega)$
8.	frequency convolution	$x_1(t)\, x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
9.	Reversal	$x(-t) \leftrightarrow X(-\omega)$
10.	Time differentiation	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega)$
11.	Time integration	$\int_{-\infty}^{t} x(t)\,dt \leftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$
12.	frequency differentiation	$-jt\, x(t) \leftrightarrow dX(\omega)/d\omega$
13	frequency integration	[illegible] $\leftrightarrow \int_{-\infty}^{\omega} X(\omega)\, d\omega$

Discrete & line spectra and phase spectra:

The coefficient K_n is called a line or discrete spectrum since the value of K_n exists only for some discrete values of n (n being an integer). It represents a complex spectrum and has both magnitude and phase spectra.

The fourier coefficient K_n of the exponential form is a complex quantity and can be represented by

$$K_n = Re[K_n] + j\,Im[K_n]$$

The real part of K_n is

$$Re[K_n] = \frac{1}{T}\int_0^T f(t)\cos\omega_0\, dt$$

The imaginary part of K_n is

$$Im[K_n] = \frac{1}{T}\int_0^T f(t)\sin\omega_0\, dt$$

$Re[K_n]$ is an even function of n, whereas $Im[K_n]$ is an odd function of n. Therefore the amplitude spectrum of fourier series is given by

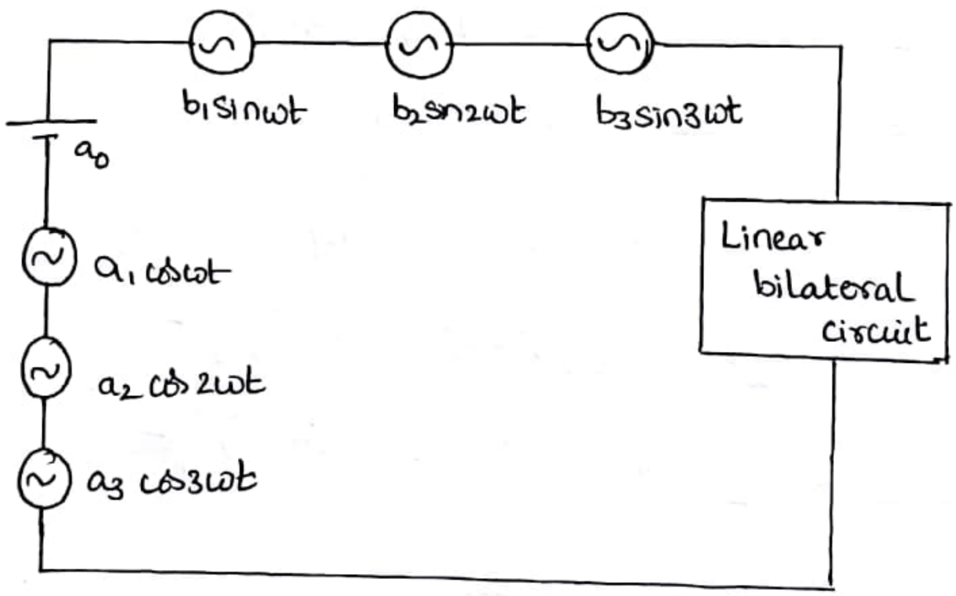
$$|K_n| = \sqrt{Re[K_n]^2 + Im[K_n]^2}$$

while the phase spectrum is given by

$$\angle K_n = \theta_n = \tan^{-1}\left\{\frac{Im[K_n]}{Re[K_n]}\right\}$$

Applications in circuit Analysis:

A voltage $v(t)$ represented by fourier series can be applied to linear circuit to obtain the corresponding harmonic terms of the current series. This result is obtained by Superposition Principle. we consider each term of the fourier series representing the voltage as a single source as shown in figure. The equivalent impedance of the network at each harmonic frequency is used to compute the current at that harmonic. The sum of these individual responses is that total response i in series form due to the applied voltage.



The non sinusoidal voltage $v(t)$ is represented by a fourier Series

$$v(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

The effective value of a voltage waveform is given by

$$V_{eff} = \sqrt{\frac{1}{T}\int_0^T |v(t)|^2 dt}$$

$$V_{eff} = \left\{ \frac{1}{T}\int_0^T \left[a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)\right]^2 dt \right\}^{1/2}$$

$$= \left\{ a_0^2 + \frac{1}{2}\left[a_1^2 + a_2^2 + a_3^2 + \cdots + b_1^2 + b_2^2 + b_3^2 + \cdots\right]\right\}^{1/2}$$

$$= \left[A_0^2 + \frac{1}{2}\left(A_1^2 + A_2^2 + A_3^2 + \cdots\right)\right]^{1/2}.$$

where $A_n^2 = a_n^2 + b_n^2$, A_0 is the average value and A_1, A_2, A_3 ... are the amplitudes of the harmonics.

If $v(t) = V_0 + \sum V_n \sin(n\omega t + \phi_n)$ and

$i(t) = I_0 + \sum I_n \sin(n\omega t + \theta_n)$

Then their effective values are given by

$$V_{rms} = \left[V_0^2 + \frac{1}{2}(V_1^2 + V_2^2 + V_3^2 + \cdots)\right]^{1/2}$$

$$I_{rms} = \left[I_0^2 + \frac{1}{2}(I_1^2 + I_2^2 + I_3^2 + \cdots)\right]^{1/2}$$

The average power is given by

$$P = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt$$

$$= \frac{1}{T}\int_0^T \left[V_0 + \sum V_n \sin(n\omega t + \phi_n)\right]\left[I_0 + \sum I_n \sin(n\omega t + \phi_n)\right] dt$$

After simplification we get

$$P = \frac{1}{T}\int_0^T V_0 I_0 \, dt + \frac{1}{T}\int_0^T V_n \sin(n\omega t + \phi_n) I_n \sin(n\omega t + \theta_n) \, dt$$

$$= V_0 I_0 + \sum \frac{1}{2} V_n I_n \cos(\phi_n - \theta_n)$$

$$= V_0 I_0 + \sum V_{rms\,n} I_{rms\,n} \cos\psi_n$$

where $\psi_n = \phi_n - \theta_n$

and $V_{rms\,n} = \frac{V_n}{\sqrt{2}}$, $I_{rms\,n} = \frac{I_n}{\sqrt{2}}$