FLUID MECHANICS & HYDRAULIC MACHINES

SYLLABUS

Objective: The students completing this course are expected to understand the properties of fluids, its kinematic and dynamic behavior through various laws of fluids like continuity, Euler's, Bernoulli's equations, energy and momentum equations. Further, the student shall be able to understand the theory of boundary layer, working and performance characteristics of various hydraulic machines like pumps and turbines.

UNIT I

Objective: After studying this unit student will know the concept of fluid and its properties, manometry, hydrostatic forces acting on different surfaces and also problem solving techniques.

Fluid statics: Dimensions and units: physical properties of fluids- specific gravity, viscosity and its significance, surface tension, capillarity, vapor pressure. Atmospheric gauge and vacuum pressure –measurement of pressure. Manometers-Piezometer, U-tube, inverted and differential manometers. Pascal's law, hydrostatic law.

Buoyancy and floatation: Meta center, stability of floating body. Submerged bodies. Calculation of metacenter height. Stability analysis and applications.

UNIT II

Objective: In this unit student will be exposed to the basic laws of fluids, flow patterns, viscous flow through ducts and their corresponding problems.

Fluid kinematics: Introduction, flow types. Equation of continuity for one dimensional flow, circulation and vorticity, Stream line, path line and streak lines and stream tube. Stream function and velocity potential function,differences and relation between them. Condition for irrotational flow, flow net, source and sink, doublet and vortex flow.

Fluid dynamics: surface and body forces –Euler's and Bernoulli's equations for flow along a stream line,momentum equation and its applications, force on pipe bend.

Closed conduit flow: Reynold's experiment- Darcy Weisbach equation- Minor losses in pipes- pipes in series and pipes in parallel- total energy line-hydraulic gradient line.

UNIT III

Objective: At the end of this unit student will be aware of the concepts related to boundary layer theory, flow
separation, basic concepts of velocity profiles, dimensionless numbers and dimensional analysis.

Boundary Layer Theory: Introduction, momentum integral equation, displacement, momentum and energy thickness, separation of boundary layer, control of flow separation, Stream lined body, Bluff body and its applications, basic concepts of velocity profiles.

Dimensional Analysis: Similitude and modelling – Dimensionless numbers

UNIT IV

Objective: In this unit student will know the hydrodynamic forces acting on vanes and their performance
evaluation.

Basics of turbo machinery: hydrodynamic force of jets on stationary and moving flat, inclined, and curved vanes, jet striking centrally and at tip, velocity diagrams, work done and efficiency, flow over radial vanes.

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UNIT V

Objective: At the end of this unit student will be aware of the importance, function and performance of hydro
machinery.

Centrifugal pumps: classification, working, work done – manometric head- losses and efficiencies- specificspeed- pumps in series and parallel-performance characteristic curves, cavitation & NPSH.

Reciprocating pumps: Working, Discharge, slip, indicator diagrams.

UNIT VI

Objective: After studying this unit student will be in a position to evaluate the performance characteristics of
hydraulic turbines. Also a little knowledge on hydraulic systems and fluidics is imparted to the student.

Hydraulic Turbines: classification of turbines, impulse and reaction turbines, Pelton wheel, Francisturbine and Kaplan turbine-working proportions, work done, efficiencies, hydraulic design –draft tube- theoryfunctions and efficiency.

Performance of hydraulic turbines: Geometric similarity, Unit and specific quantities, characteristic curves,governing of turbines, selection of type of turbine, cavitation, surge tank, water hammer. Hydraulic systemshydraulicram, hydraulic lift, hydraulic coupling. Fluidics – amplifiers, sensors and oscillators. Advantages,limitations and applications.

Text Books:

1. Hydraulics, fluid mechanics and Hydraulic machinery MODI and SETH.
2. Fluid Mechanics and Hydraulic Machines by Rajput.
3. Fluid Mechanics and Hydraulic Machines/ RK Bansal/Laxmi Publications (P) Ltd.

Reference Books:

1. Fluid Mechanics and Fluid Power Engineering by D.S. Kumar, Kotaria & Sons.
2. Fluid Mechanics and Machinery by D. Rama Durgaiah, New Age International.
3. Hydraulic Machines by Banga & Sharma, Khanna Publishers.
4. Instrumentation for Engineering Measurements by James W. Dally, William E.Riley ,John Wiley & Sons Inc. 2004 (Chapter 12 – Fluid Flow Measurements)
5. Fluid Mechanics and Hydraulic Machines by Domkundwar & Domkundwar, Dhanpatrai &Co.

Introduction:-

Hydraulics:

Hydraulics may be defined as follows:

"It is that branch of Engineering-science, which deals with water (at rest or in motion)"

(or)

" It is that branch of Engineering science which is based on experimental observation of water flow."

Fluid mechanics:

Fluid mechanics may be defined as that branch of Engineering-science which deals with the behaviour of fluid under the conditions of rest and motion.

The fluid mechanics may be divided into three parts.

- Statics
- kinematics
- Dynamics.

- Statics: The study of incompressible fluids under static conditions is called hydrostatics and that dealing with the compressible static gases is termed as aerostatics.

- kinematics: It deals with the velocities, accelerations and the patterns of flow only. Forces or energy causing velocity and acceleration are not dealt under this heading.

- Dynamics: It deals with the relations between velocities, accelerations of fluid with the forces or energy causing them.

The matter can be classified on the basis of the spacing between the molecules of the matter as follows:

- Solid State
- Fluid state

i, Liquid state and ii, Gaseous state

In solids, the molecules are very closely spaced whereas in liquids the spacing between the different molecules is relatively large and in gases the spacing between the molecules is still large.

Inter molecular cohesive forces are large in solids, smaller in liquids and extremely small in gases, and on account of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movement. So that the gases fill the container completely in which they are placed.

Physical properties of the Fluid:

* 1. Density:

i, Mass density: The density also known as (mass density or specific mass) of a liquid may be defined as the mass per unit volume $\left(\frac{m}{V}\right)$ at a standard temperature and pressure. It is usually denoted by ρ (rho)

Its unit are kg/m^3 i.e., $\boxed{\rho = \frac{m}{V}}$

ii, Weight density: The weight density also known as specific weight is defined as the weight per unit volume at the standard temperature and pressure. It is usually denoted by ω. $\boxed{\omega = \rho g}$

For the purpose of all calculations relating to Hydraulics and hydraulics machines, the specific weight of water is taken as follows:

In S.I units : $w = 9.81\ kN/m^3$ (or $9.81 \times 10^{-6}\ N/mm^3$)

In M.K.S units : $w = 1000\ kgf/m^3$

iii, Specific Volume : It is defined as volume per unit mass of fluid. It is denoted by v.

Mathematically,

$$\boxed{v = \frac{V}{m} = \frac{1}{\rho}}$$

Kinematic Viscosity : Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by ν (called nu)

Mathematically, $\nu = \frac{\text{Viscosity}}{\text{density}} = \frac{\mu}{\rho}$ $\boxed{\nu = \frac{\mu}{\rho}}$

Units:

In S.I units : m^2/sec.

In MKS units : m^2/sec

In C.G.S units the kinematic viscosity is also known as stoke

$= cm^2/sec$

One Stoke $= 10^{-4}\ m^2/sec$

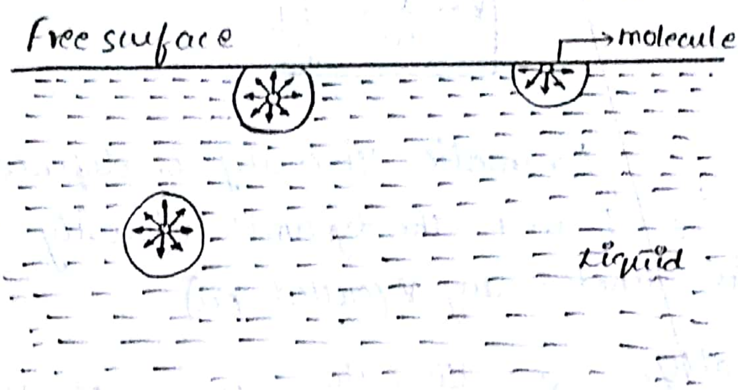
4. Surface Tension:

Cohesion : It means inter molecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one assemblage of particles. Surface tension is due to cohesion between particles at the free surface.

Adhesion: It means attraction between the molecule of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick another body.

Capillary action is due to both cohesion and adhesion.

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by another molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of molecules below it. Consequently there is a net inward force on the molecule. The force is normal to the liquid surface.



At the free surface a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface.(the layer acts as a membrane).

Some important examples of phenomenon of surface tension are as follows:

i, Rain drop A falling raindrop becomes spherical due to cohesion and surface tension.

ii, Rise of sap in a tree.

iii, Bird can drink water from ponds.

iv, Capillary rise and capillary siphoning.

v, collection of dustparticles on water surface.

vi, Break up of liquid jets.

<u>Dimensional formula for Surface tension:</u>

The dimensional formula for surface tension is given by:

$$\left[\frac{F}{L}\right] \text{ or } \left[\frac{M}{T^2}\right]$$

It is usually expressed in N/m. The value of surface tension depends upon the following factors:

i, Nature of liquid
ii, Nature of surrounding matter (E.g., solid, liquid or gas)
iii, Kinetic energy (and hence the temperature of the liquid molecules)

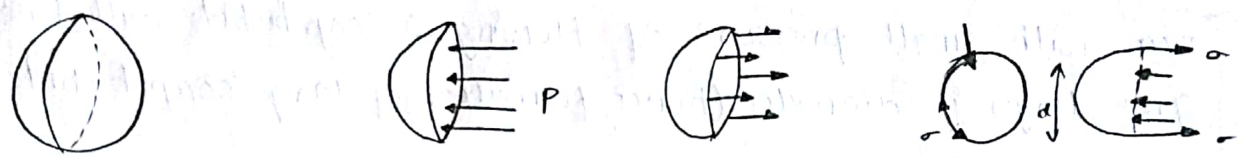
Water – air ---- 0.073 N/m at 20°C.
Water – air ---- 0.058 N/m at 100°C
Mercury – air --- 0.1 N/m length.

Pressure inside a water droplet, soap bubble and a liquid jet:

1. **Water Droplet:**

Let $p =$ pressure inside the droplet above outside pressure
(i.e., $\Delta p = p - 0 = p$ above atm. pressure)

$d =$ diameter of droplet ; $\sigma =$ surface tension of liquid.



(a) Water droplet (b) pressure forces (c) surface tension (d) free body diagram

Fig: Pressure inside a water droplet

From the free body diagram,

i, Pressure force $= p \times \frac{\pi}{4} d^2$, and
ii, Surface tension force acting around the circumference $= \sigma \times \pi d$

Under equilibrium conditions these two forces will be equal & opposite. i.e.,

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

$$\frac{pd}{4} = \sigma$$

$$\boxed{p = \frac{4\sigma}{d}}$$

With an increase in size of the droplet the pressure intensity decreases.

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Soap (or hollow) bubble:

Soap bubble have two surfaces on which surface tension σ acts.

Free body diagram

Fig: Pressure inside soap bubble

From the free body diagram, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

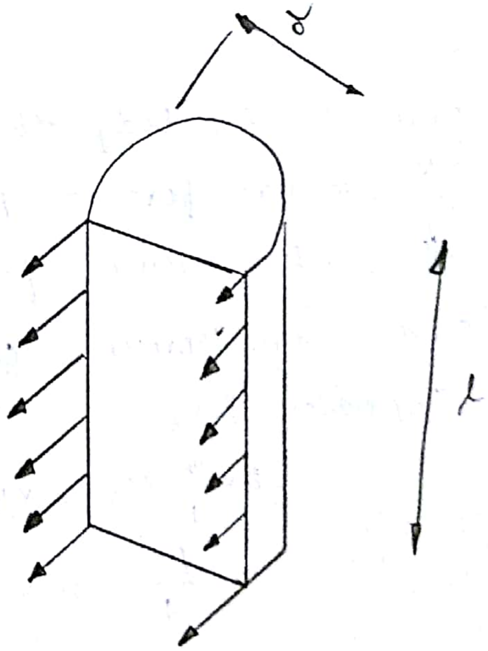
$$\frac{pd}{4} = 2\sigma$$

$$\boxed{p = \frac{8\sigma}{d}}$$

Since the soap solution has a high value of surface tension σ, even with small pressure of blowing a soap bubble will tend to grow larger in diameter (hence formation of large soap bubbles).

3. **Liquid Jet:**

Let us consider a cylindrical liquid jet of diameter d and length l.



Pressure force $= p \times l \times d$.

Surface tension force $= \sigma \times 2l$.

Equating the two forces,

we have,

$$p \times l \times d = \sigma \times 2l$$

$$pd = 2\sigma$$

$$\boxed{p = \frac{2\sigma}{d}}$$

Vapour Pressure :

All liquids have a tendency to evaporate or vaporize (i.e., to change from liquid to gaseous state). Molecules are continuously projected from free surface to the atmosphere. These ejected molecules are in a gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid (P_v). If the surface above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When the equilibrium condition is reached, the vapour pressure is called saturation vapour pressure (P_{vs}).

The following points are worth nothing :

i, If the pressure on the liquid surface is lower than or equal to the saturation vapour pressure, boiling takes place.

ii, Vapour pressure increases with the rise in temperature.

iii, Mercury has a very low vapour pressure and hence, it is an excellent fluid to be used in a barometer.

Absolute and Gauge Pressures :

* Atmospheric pressure :

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure. The atmospheric pressure is also known as Barometric pressure.

The atmospheric pressure at sea level (above absolute zero) is called standard atmospheric pressure.

* Gauge Pressure :

It is the pressure, measured with the help of pressure measuring instrument, in which the atmospheric pressure is taken

... datum the atmospheric pressure on the scale is marked as zero.

Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is below the local atmospheric pressure, then the gauge is designated as "vacuum gauge" and the recorded value indicates the amount by which the pressure of the liquid is below local atmospheric pressure. i.e., negative pressure.

Vacuum pressure is defined as the pressure below the atmospheric pressure.

Absolute pressure:

It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum.

Any pressure measured above the absolute zero of pressure is termed as an "absolute pressure".

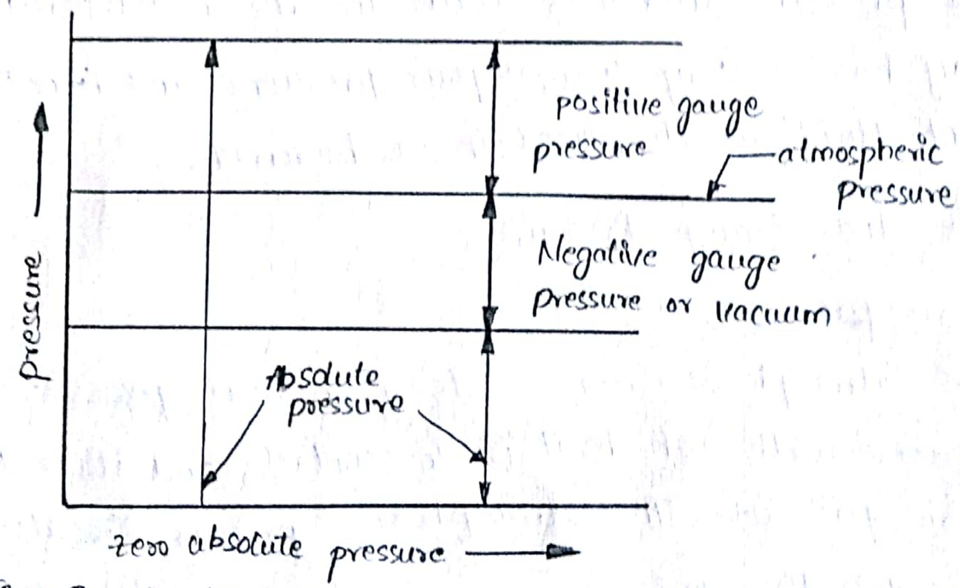


Fig: Relationship between pressures.

Mathematically,

1. Absolute pressure = Atmospheric pressure + gauge pressure

$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

2. Vacuum Pressure = Atmospheric pressure − Absolute pressure

Units for Pressure:

The fundamental S.I unit of pressure is newton per square meter (N/m^2). This is also known as pascal.

Low pressures are often expressed in terms of mm of mercury. This is an abbreviated way of saying that the pressure is such that will support a liquid column of stated height.

Standard atmospheric pressure has the following equivalent values:

101.3 kN/m^2 (or) 101.3 kPa ; 10.3m of water, 760mm of mercury 1013 mb (millibar) ; $\simeq$ 1 bar $\simeq$ 100kPa = 10^5 N/m^2.

Measurement of Pressure:

The pressure of a fluid may be measured by the manometers.

1. Manometers: Manometers are defined as the devices used for measuring the pressures at a point in a fluid by balancing the column of fluid by the same or another column of liquid. These are classified as follows:

- (a) Simple manometers:
  - i. Piezometer
  - ii. U-tube manometer
  - iii. Single column manometer.
- (b) Differential manometers:

(a) Simple manometers:

A simple manometer is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other remains open to atmosphere.

i. piezometer:

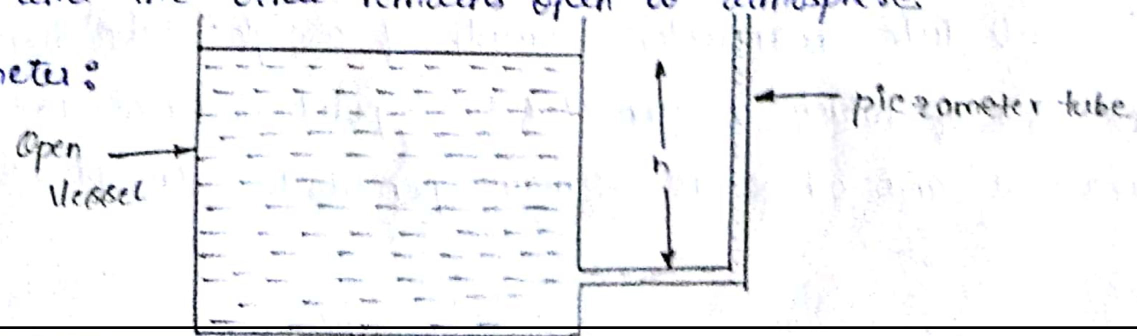
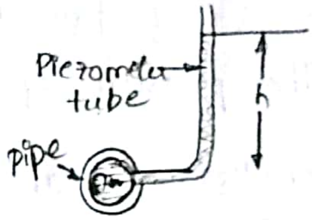


fig. piezometer tube filled to open vessel

A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids. It consists of a glass tube interested in the wall of a vessel or of a pipe, containing liquid whose pressured is to be measured. The tube extends vertically upwards to such a height that liquid can freely rise in it without over flowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it. Thus if w is the specific weight of the liquid, then the pressure at point A(p) is given by $\boxed{p = wh}$



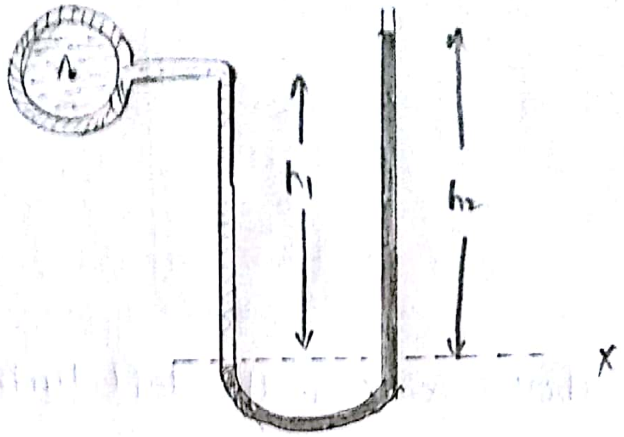
Piezometer measures gauge pressure only (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is not suitable for measuring negative pressure; as in such case the air will enter in pipe through the tube.

2. U- tube manometer:

Piezometers cannot be employed when large pressure in the lighter liquids due are to be measured, since this would require very long tubes, which cannot be handled conveniently. Further more gas pressures cannot be measured by the piezometers because a gas forms no free atmospheric surface. These limitations can be overcome by the use of U-tube manometers.

U-tube manometers consists of a glass tube bent in U-shape, One end of which is connected to a point at which pressure is to be measured and other end remain open to the atmosphere.

i. For positive pressure:



Let A be the point at which pressure is to be measured. X-X is the datum line as shown in figure.

Let h_1 = Height of the light liquid in the left limb above the datum line,

h_2 = Height of the heavy liquid in the right limb above the datum line.

h = pressure in pipe, expressed in terms of head.

S_1 = Specific gravity of the light liquid.

S_2 = Specific gravity of the heavy liquid.

The pressures in the left and right limb above the datum line X-X are equal (as the pressures at two points at the same level in a continuous homogeneous liquid are equal.)

Pressure head above X-X in the left limb = $h + h_1 S_1$

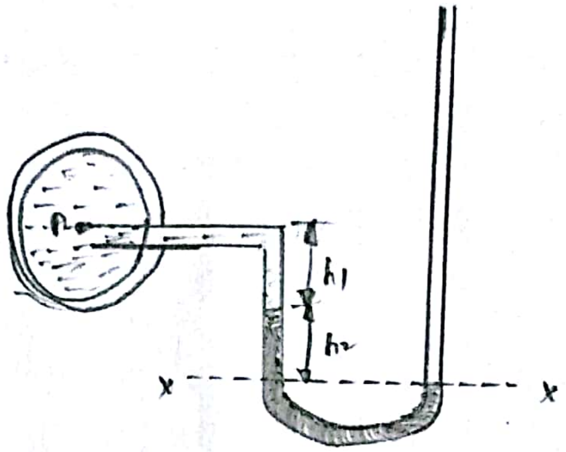
Pressure head above X-X in the right limb = $h_2 S_2$.

Equating these two pressures,

$$h + h_1 S_1 = h_2 S_2$$

(or) $\boxed{h = h_2 S_2 - h_1 S_1}$

ii, For negative Pressure:



Pressure head above x–x in the left limb

$$= h + h_1 S_1 + h_2 S_2$$

Pressure head above x–x in the right limb = 0.

Equating these two pressures,

$$h + h_1 S_1 + h_2 S_2 = 0$$

$$\boxed{h = -(h_1 S_1 + h_2 S_2)}$$

Problems:

1. Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of $6 m^3$ and weight of 44 N

Sol: Volume of the liquid = ~~44 N.~~ $6 m^3$

Weight of the liquid = 44 N.

Specific weight ω:

$$\omega = \frac{\text{weight of liquid}}{\text{Volume of liquid}}$$

$$= \frac{44}{6}$$

$$= 7.333 \; KN/m^3$$

Specific mass (or) density ρ:

$$\rho = \frac{w}{g} = \frac{7.333 \times 1000}{9.81} = 747.5 \ kg/m^3$$

Specific volume, $v = \frac{1}{\rho} = \frac{1}{747.5}$

$$= 0.00134 \ m^3/kg$$

specific Gravity s:

$$s = \frac{w_{liquid}}{w_{water}} = \frac{7.333}{9.81} = 0.747$$

2. If the surface tension at air-water interface is 0.069 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.009 mm?

Sol: Given $\sigma = 0.069$ N/m

$d = 0.009$ mm

An air bubble has only one surface, hence

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}}$$

$$= 30667 \ N/m^2$$

$$= 30.667 \ KN/m^2$$

$$= 30.667 \ KPa$$

3. If the surface tension at soap-air interface is 0.09 N/m. Calculate the internal pressure in a soap bubble of 28mm diameter.

Sol: Given $\sigma = 0.09$ N/m, $d = 28$ mm $= 28 \times 10^{-3}$ m

In soap bubble there are two interfaces,

Hence $p = \frac{8\sigma}{d} = \frac{8 \times 0.09}{28 \times 10^{-3}} = 25.71 \ N/m^2$

above atmospheric pressure)

2. Specific Gravity:

Specific gravity is the ratio of the specific weight of the liquid to the specific weight of standard fluid. It is dimensionless and has no units.

- It is represented by S.

For liquids, the standard fluid is pure water at 4°c.

$$\therefore \text{Specific Gravity} = \frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}}$$

$$\boxed{S = \frac{w_{liquid}}{w_{water}}}$$

3. Viscosity:

Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow. It is primarily due to cohesion and molecular momentum, exchange between fluid layers, and as flow occurs, these effects appear as shearing stresses between the moving layers of fluid.

An ideal fluid has no viscosity. There is no fluid which can be classified as perfectly ideal fluid. However, the fluids which with very little viscosity are sometimes considered as ideal fluids.

Viscosity of fluid is due to cohesion and interaction between particles.

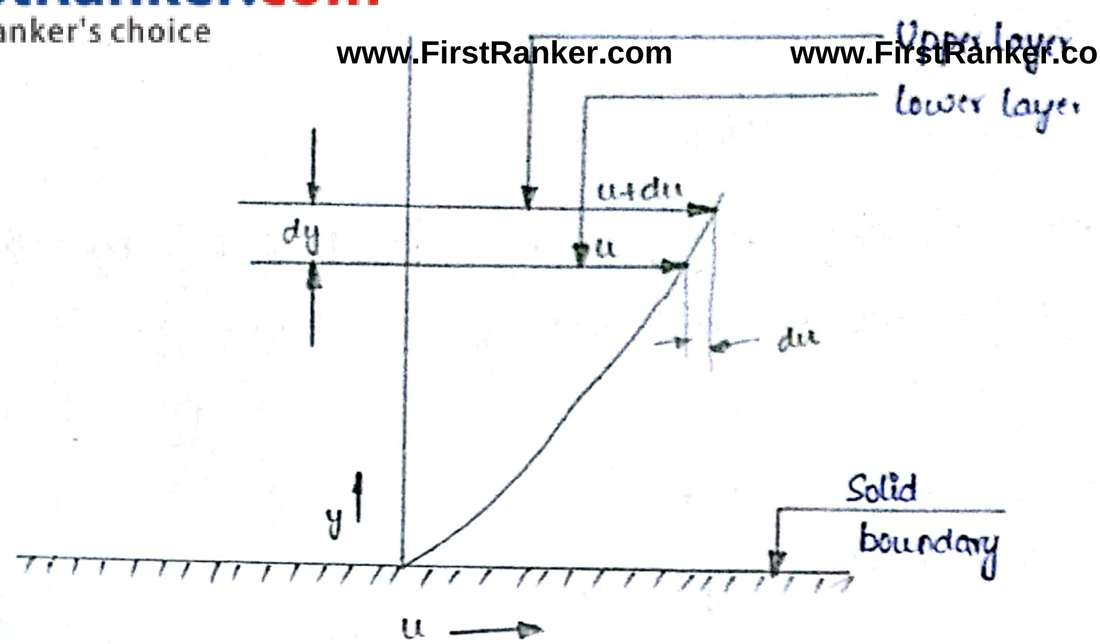


Fig: Velocity variation near a solid boundary.

When two layers of fluid, at a distance 'dy' apart, move one over the other at different velocities, say u and u+du, the viscosity together with relative velocity causes a sheer stress acting between the fluid layers. The top layers cause a sheer stress on the adjacent top layer.

This sheer stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ (called Tau)

Mathematically,

$$\tau \propto \frac{du}{dy}$$

(or) $$\boxed{\tau = \mu \cdot \frac{du}{dy}}$$

where μ = constant of proportionality and is known as co-efficient of dynamic viscosity (or) simply viscosity.

$\frac{du}{dy}$ = Rate of sheer stress or rate of sheer deformation or velocity gradient.

We have

$$\mu = \frac{\tau}{du/dy}.$$

Thus viscosity may also be defined as the shear stress required to produce unit rate of shear strain.

Units:

In S.I units : $N \cdot s/m^2$

In M.K.S units : $kgf \cdot sec/m^2$

The units of viscosity in C.G.S unit is called poise.

$$\text{poise} = \frac{\text{dyne-sec}}{cm^2}$$

$$\text{one poise} = \frac{1}{10} N.S/m^2$$

Kinematics of Fluid Flow:

Kinematics is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. The velocity at any point in a flow field at any time is studied in this branch of fluid mechanics. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

Description of Fluid motion:

The motion of fluid particles may be described by two methods.

1. Lagrangian Method.
2. Eulerian Method.

In the Lagrangian method, a single fluid particle is followed during its motion and its velocity, acceleration, density etc are described.

In case of Eulerian method, the velocity, acceleration, pressure and density etc are described at a flow field.

The Eulerian method is commonly used in Fluid Mechanics.

Description of the flow pattern (or) Types of flow:

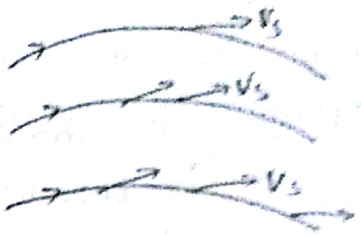
Whenever a fluid in motion its innumerable particles move along certain lines depending upon the condition of flow. Although flow lines are of several types, yet some important flow pattern may described as

1. Stream line
2. Path line
3. Streak line
4. Stream tube

1. Stream line:

A stream line is an imaginary line drawn in a flow field such that a tangent drawn at any point on this line represent the direction of the velocity vector.

From the deflection, it follows that there can be no flow across a stream line.



Considering a particle moving along a Streamline for a very short distance ds having its components dx, dy and dz along the three mutually perpendicular co-ordinate axes. Let the components of the velocity vector V_s along x, y and z directions be u, v & w respectively. The time taken by a fluid particle to move a distance ds along the stream line with velocity V_s is $t = ds/V_s$

which is same as $t = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

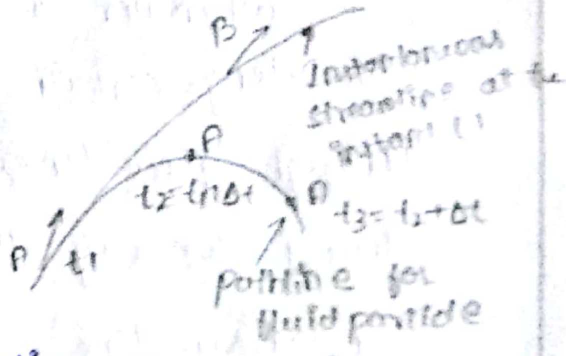
Hence differential equation of the stream line may be written as:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

→ Path Line:

A path line is the locus of a fluid particle as it moves along. In other words, a pathline is a current traced by a single fluid particle during its motion.

Fig. shows a streamline at time t_1 indicating the velocity vectors for particles A and B. At times t_2 and t_3, the particle A is shown to occupy the successive positions. The line connecting these various positions of A represents its path line.



Streak Line:

When a dye is injected in a liquid or smoke in a gas, so as to trace the subsequent motion of fluid particles passing a fixed point, the path followed by the dye or smoke is called streak line. Thus, a streakline connects all particles passing through a given point.

In steady flow, the streamlines remain fixed with respect to the co-ordinate axes. Streamlines in steady flow also represent the pathlines and streaklines. In unsteady flow, a fluid particle will not, in general, remain on the same streamline (except for unsteady uniform flow), hence streamlines and pathlines do not coincide in unsteady nonuniform flow.

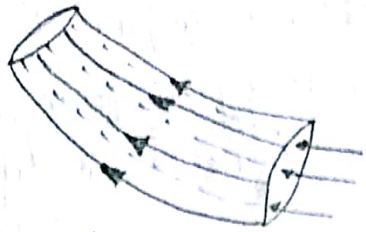
Instantaneous Stream Line:

In fluid motion which is independent of time, the position of streamline is fixed in space and a fluid particle following a streamline will continue to do so. In case of time-dependent flow, a fluid particle follows a stream line for only a short interval of time, before changing over to another streamline. The streamlines in such cases are not fixed in space, but change with time. The position of a stream line at a given instant of time is known as instantaneous stream line. For different instants of time, we shall have different instantaneous stream-lines in the same space. The stream line, path line and streak lines are one and the same, if the flow is steady.

Stream Tube:

If streamlines are drawn through a closed curve, they form a boundary surface across which fluid cannot penetrate. Such a surface bounded by streamline is a sort of tube, and known as a stream tube.

From the definition of streamline, it is evident no fluid can across the bounding surface of the stream tube. This implies that the quantity (mass) of fluid entering the stream tube at one end must be the same as the quantity leaving it at the other end. The stream tube is generally assumed to be a small cross-sectional area so that the velocity over it could be considered uniform.



The concept of stream tube can be extended, and the entire flow region may be composed of innumerable stream tubes of small cross-section. The stream tubes may be of any shape regular (or) irregular. The solid boundaries of flow represent the surface containing the stream lines.

Types of Fluid Flow:

The fluid flow is classified as:

i, Steady & unsteady flows:
ii, Uniform & non-uniform flows:
iii, Laminar & turbulent flows:
iv, Compressible & incompressible flows.
v, Rotational & irrotational flows.
vi, One, two and three dimensional flows.

i, Steady & Unsteady Flows:

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time.

Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure and density at a point changes with respect to time. Thus, mathematically, for unsteady flow.

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0.$$

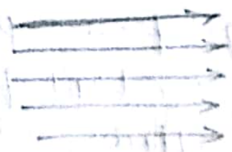
$$\left(\frac{\partial P}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \quad \text{etc.}$$

→ **Uniform and Non-uniform Flows:**

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e., length of direction of the flow).

Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$



where, ∂V = Change of velocity.

∂s = Length of flow in the direction's.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus mathematically, for non-uniform flow,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

→ **Laminar & Turbulent Flows:** Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths (or) stream line and all the stream-lines are straight and parallel. Thus the particles move laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream.line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way. Due to the movement of the fluid particles in a zig-zag way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

Where D = Diameter of pipe.
V = Mean velocity of flow in pipe.
ν = kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 & 4000, the flow may be laminar (or) turbulent.

Compressible & Incompressible Flows:

Compressible flow is that type in which density of fluid changes from point to point (or) in other words density (ρ) is not constant for the fluid. Thus, mathematically, for compressible fluid,

$$\rho \neq \text{constant}$$

Incompressible flow is that type in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible,

Mathematically, for incompressible flow. ρ = constant

Rotational & Irrotational Flows:

Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

One, Two and Three - Dimensional Flows:

One-dimensional Flow: It is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence, Mathematically, for one-dimensional flow

$$u = f(x)$$
$$v = 0$$
$$w = 0$$

where u, v and w are velocity compone in x, y & z directions respectively.

Two-dimensional Flow: It is that type of flow in which the velocity is a function of time and two rectangular space-co-ordinates say x and y. For a steady two dimensional flow the velocity is a function of two space co-ordinate only. The variation of velocity in the third direction is negligible. Thus, mathematically for 2-dimensional flow

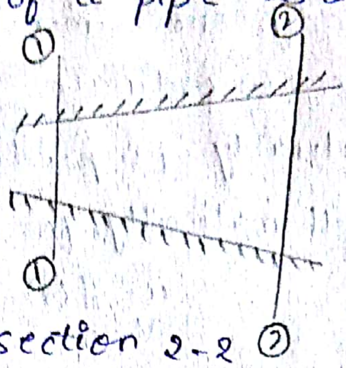
$$u = f_1(x, y)$$
$$v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional Flow: It is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, Mathematically, for three-dimensional flow.

$$u = f_1(x, y, z)$$
$$v = f_2(x, y, z)$$
$$w = f_3(x, y, z)$$

Continuity Equation:

The eqn. based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure.



Let V_1 = average velocity at cross-section 1-1

ρ_1 = Density at section-1-1

A_1 = Area of pipe at section 1-1

V_2, ρ_2, A_2 are corresponding values at section 2-2

The rate of flow at section 1-1 = $\rho_1 A_1 V_1$

Rate of flow at section 2-2 = $\rho_2 A_2 V_2$.

According to law of conservation of mass.

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

The above equation is applicable to the compressible as well as incompressible fluids and is called continuity eqn. If the fluid is incompressible, then $\rho_1 = \rho_2$ and the above continuity equation reduces to

$$\boxed{A_1 V_1 = A_2 V_2}$$

1. The direction of a pipe at the sections 1 & 2 are 10cms and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s. Determine also the velocity at section 2.

Sol: Given Data:

At section 1, $D_1 = 10cm = 0.1m$

$V_1 = 5m/s$

$A_1 = \frac{\pi}{4}(D_1)^2 = \frac{\pi}{4}(0.1)^2$

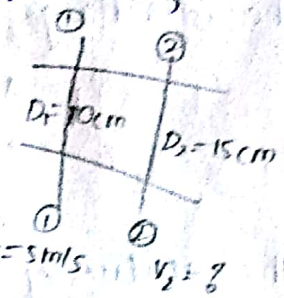
$= 0.007854\ m^2$.

At section 2, $D_2 = 15\,cm = 0.15\,m$

$$A_2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

i, Discharge through pipe is given by the equation,

$$Q = A_1 V_1$$



We know that equation (continuity)

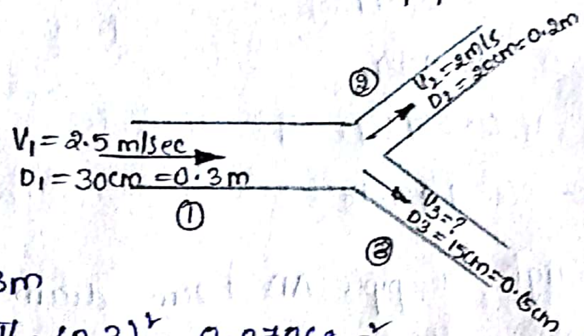
$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

$$= \frac{0.03927}{0.01767}$$

$$= 2.22\ m/s$$

2. A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15cm respt. If the average velocity in the 30cm diameter pipe is 2.5m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s.



$D_1 = 30\,cm = 0.3\,m$

$A_1 = \pi/4\ D_1^2 = \pi/4\,(0.3)^2 = 0.07068\,m^2$

$V_1 = 2.5\ m/s$

$D_2 = 20\ cm = 0.2\,m$

$A_2 = \pi/4\ D_2^2 = \pi/4\,(0.2)^2 = 0.0314\,m^2$

$V_2 = 2\ m/s$

$D_3 = 15\,cm = 0.15\,m$

$A_3 = \pi/4\ D_3^2 = \pi/4\ (0.15)^2 = 0.01767\ m^2$

Find i, Discharge in pipe 1 (or) Q_1

ii, Velocity in pipe diameter 15cm (or) V_3.

Let Q_1, Q_2 & Q_3 are discharges in pipe 1, 2 & 3 respt.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

i, The discharge in pipe 1 is given by

$$Q_1 = A_1V_1$$
$$= 0.07068 \times 2.5$$
$$= 0.1767 \text{ m}^3/\text{sec}$$

ii, The value of V_3.

$$Q_2 = A_2V_2$$
$$= 0.0314 \times 2$$
$$= 0.0628 \text{ m}^3/\text{sec}$$

substitute the values of Q_1 and Q_2 in continuity equation

$$Q_1 = Q_2 + Q_3$$

$$0.1767 = 0.0628 + (0.01767 \times V_3)$$

$$0.1767 - 0.0628 = 0.01767\, V_3$$

$$V_3 = \frac{0.1767 - 0.0628}{0.01767}$$

$$V_3 = \frac{0.1139}{0.01767}$$

$$V_3 = 6.44 \text{ m/s}$$

3. Water flows through a pipe AB 1.2cm diameter at 3m/s and then passes through a pipe BC 1.5m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC the velocity in CD and the diameter of CE.

Given that, diameter of pipe AB, $D_{AB} = 1.2\text{m}$

Velocity of flow through AB, $V_{AB} = 30 \text{ m/s}$

Diameter of pipe, BC $D_{BC} = 1.5\text{m}$

Diameter of branched pipe CD, $D_{CD} = 0.8\text{m}$

Velocity of flow in pipe CE, $V_{CE} = 2.5 \text{ m/s}$

Let the flow rate in pipe AB = Q m^3/s

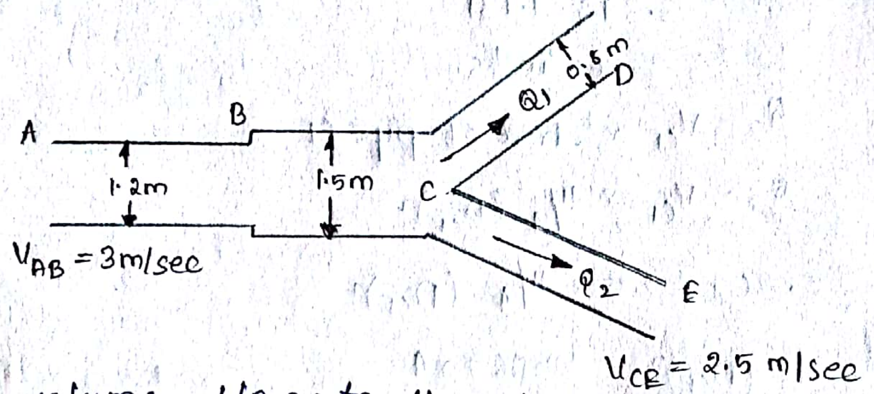
Velocity of flow in pipe BC = V_{BC} m/s.

Velocity of flow in pipe CD = V_{CD} m/s.

Diameter of pipe CE = D_{CE}

Then flow rate through CD = $Q/3$

flow rate through CE = $Q - Q/3 = \frac{2Q}{3}$.



i, Now volume flow rate through AB,

$$Q = V_{AB} \times \text{Area of } AB$$

$$= 3.0 \times \frac{\pi}{4}(D_{AB})^2$$

$$= 3.0 \times \frac{\pi}{4}(1.2)^2$$

$$= 3.393 \ m^3/s$$

ii, Applying continuity equation to pipe AB and pipe BC,

$V_{AB} \times$ Area of pipe AB = $V_{BC} \times$ Area of pipe BC

$$3.0 \times \frac{\pi}{4}(D_{AB})^2 = V_{BC} \times \frac{\pi}{4}(D_{BC})^2$$

$$3.0(1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = \frac{3.0(1.2)^2}{(1.5)^2}$$

$$= 1.92 \ m/s$$

iii, The flow rate through pipe CD

$$= Q_1 = \frac{Q}{3} = \frac{3.393}{3} = 1.131 \ m^3/s$$

$Q_1 = V_{CD} \times$ Area of pipe CD

$1.131 = V_{CD} \times \pi/4\,(0.8)^2$

$1.131 = 0.5026\; V_{CD}$

$$V_{CD} = \frac{1.131}{0.5026} = 2.25\ m/s$$

iv. Flow rate through CE,

$Q_2 = Q - Q_1$

$= 3.393 - 1.131$

$= 2.262\ m^3/s$

$Q_2 = V_{CE} \times$ area of pipe CE

$Q_2 = V_{CE} \times \pi/4\,(D_{CE})^2$

$2.262 = 2.5 \times \pi/4\,(D_{CE})^2$

$$(D_{CE})^2 = \frac{2.262 \times 4}{2.5\pi}$$

$$D_{CE} = \sqrt{\frac{2.262 \times 4}{2.5\pi}} = \sqrt{1.152} = 1.0735\ m$$

$\therefore$ Diameter of pipe CE = 1.0735 m

4. A 25 cm diameter pipe carries oil of sp. gr. 0.9 at a velocity of 3 m/s. At another section the diameter is 20 cm. Find the velocity at this section and also mass rate of flow of oil.

Sol: Given data,

at section 1, $D_1 = 25\ cm = 0.25\ m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.25)^2 = 0.049\ m^2$

$V_1 = 3\ m/sec$

at section 2, $D_2 = 20\ cm = 0.2\ m$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$

$V_2 = ?$

Mass rate of flow of oil = ?

Applying continuity eqn. at sections 1 and 2,

$A_1 V_1 = A_2 V_2$

$0.049 \times 3 = 0.0314 \times V_2$

$$V_2 = \frac{0.049 \times 3.0}{0.0314}$$

$$V_2 = 4.68 \text{ m/s}$$

Mass rate of flow of oil = Mass density x a

$= \rho A_1 V_1$

$$\text{Specific gravity of oil} = \frac{\text{Density of oil}}{\text{Density of water}}$$

Density of oil = sp.gr of oil x Density of water

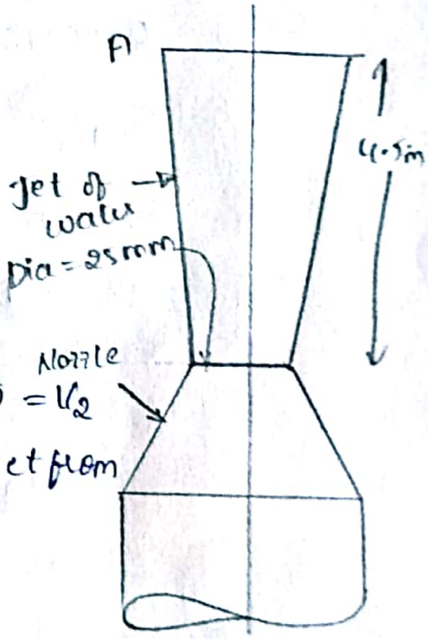
$= 0.9 \times 1000$

$= 900 \text{ kg/m}^3$

$\therefore$ Mass rate of flow $= 900 \times 0.049 \times 3.0$

$= 132.23 \text{ kg/s}$

5. A jet of water from a 25mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, that will be the diameter at a point 4.5m above the nozzle, if the velocity with which the jet leaves the nozzle is 12m/s.



Sol: Given that

Diameter of nozzle, $D_1 = 25\text{mm} = 0.025\text{m}$

Velocity of jet at nozzle, $V_1 = 12 \text{ m/s}$

Height of point A, $h = 4.5\text{m}$

Let the velocity of jet at a height 4.5 $= V_2$

consider the vertical motion of the jet from the outlet of the nozzle to the point A (neglecting any loss of energy)

Initial velocity, $u = V_1 = 12 \text{ m/s}$

Final velocity $V = V_2$

Value of $g = -9.81 \text{ m/s}^2$ and $h = 4.5\text{ m}$

using $V^2 - u^2 = 2gh$

$V_2^2 - 12^2 = 2 \times (-9.81) \times 4.5$

$V_2^2 = 12^2 - (2 \times 9.81 \times 4.5)$

$V_2^2 = 144 - 88.29$

$V_2 = 144 - 88.29$

$V_2 = \sqrt{144 - 88.29}$

$V_2 = 7.46 \text{ m/s}$

Now applying continuity eqn. to outlet of nozzle and at point A,

we get $\quad A_1V_1 = A_2V_2$

$$A_2 = \frac{A_1V_1}{V_2}$$

$$A_2 = \frac{\pi/4 \, D_1^2 \times V_1}{V_2}$$

$$A_2 = \frac{\pi \times (0.025)^2 \times 12}{4 \cdot 7.46} = 0.0007896$$

Let D_2 = diameter of a jet at point A.

Then $A_2 = \pi/4 \, D_2^2$

$0.0007896 = \pi/4 \, D_2^2$

$$D_2 = \sqrt{\frac{0.0007896 \times 4}{\pi}}$$

$D_2 = 0.0317 \text{ m}$

$D_2 = 31.7 \text{ mm}$

Velocity and Acceleration:

Let V is the resultant velocity at any point in a fluid flow. Let u, v and w are its component in x, y, z directions. The velocity components are functions of space-co-ordinates and time. Mathematically velocity components are given as

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

Resultant velocity, $V = ui + vj + wk$
$$= \sqrt{u^2 + v^2 + w^2}$$

Let a_x, a_y, a_z are the total acceleration in x, y and z directions respectively. Then by the chain rule of differentiation, we have

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial t} + \frac{\partial u}{\partial t}$$

But $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$, $\frac{dz}{dt} = w$

$$\boxed{\begin{aligned} a_x &= \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \\ a_y &= \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \\ a_z &= \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \end{aligned}}$$

For steady flow, $\frac{\partial V}{\partial t} = 0$

Where V = resultant velocity.

$$\frac{\partial u}{\partial t} = 0, \quad \frac{\partial v}{\partial t} = 0 \quad \text{and} \quad \frac{\partial w}{\partial t} = 0$$

Hence accelerations in $x, y,$ and z directions becomes

$$a_x = \frac{du}{dt} = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z}$$

Acceleration vector $A = a_x i + a_y j + a_z k$

$$= \sqrt{a_x^2 + a_y^2 + a_z^2}$$

→ **Local Acceleration and convective Acceleration:**

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow fluid. The expressions $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ is known as local acceleration.

convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particle in a fluid flow. The expressions other than $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ in the above (acceleration) equation are known as convective acceleration.

6. The velocity vector in a fluid flow is given

$$V = 4x^3 i - 10x^2 y j + 2t K.$$

Find the velocity and acceleration of a fluid particle at (2,1,3) at time t=1

Sol: Given data $V = 4x^3 i - 10x^2 y j + 2t K$

This is compared with $V = ui + vj + wk$.

velocity components are $u = 4x^3$, $v = -10x^2 y$, $w = 2t$

For the point (2,1,3) we have $x=2, y=1, z=3$ at time $t=1$.

Hence velocity components at (2,1,3) are

$$u = 4(2)^3 = 4 \times 8 = 32 \text{ units.}$$

$$v = -10(2)^2(1) = -10 \times 4 = -40 \text{ units}$$

$$w = 2(1) = 2 \text{ units.}$$

Velocity vector at (2,1,3) $= 32i - 40j + 2k$

Resultant velocity $= \sqrt{u^2 + v^2 + w^2}$

$$= \sqrt{(32)^2 + (-40)^2 + (2)^2}$$

$$= \sqrt{1024 + 1600 + 4}$$

$$= 51.26 \text{ units.}$$

We know the acceleration is

$$a_x = u\cdot\frac{\partial u}{\partial x} + v\cdot\frac{\partial u}{\partial y} + w\cdot\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\cdot\frac{\partial v}{\partial x} + v\cdot\frac{\partial v}{\partial y} + w\cdot\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\cdot\frac{\partial w}{\partial x} + v\cdot\frac{\partial w}{\partial y} + w\cdot\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

From the velocity components.

$$\frac{\partial u}{\partial x} = 12x^2, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial u}{\partial z} = 0, \quad \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial v}{\partial x} = -20xy, \quad \frac{\partial v}{\partial y} = -10x^2, \quad \frac{\partial v}{\partial z} = 0, \quad \frac{\partial v}{\partial t} = 0$$

$$\frac{\partial w}{\partial x} = 0, \quad \frac{\partial w}{\partial y} = 0, \quad \frac{\partial w}{\partial z} = 0, \quad \frac{\partial w}{\partial t} = 2$$

Substituting the values, the acceleration components at (2,1,3) at time $t=1$ are

$$a_x = 4x^3(12x^2) + (-10x^2y)(0) + 2t(0) + 0$$
$$= 48x^5$$
$$= 48(2)^5$$
$$= 48\times32 = 1536 \text{ units}$$

$$a_y = 4x^3(-20xy) + (-10x^2y)(-10x^2) + 2t(0) + 0$$
$$= -80x^4y + 100x^4y$$
$$= 20x^4y = 20(2)^4(1) = 20\times16 = 320 \text{ units}$$

$$a_z = 4x^3(0) + (-10x^2y)(0) + 2t(0) + 2$$
$$= 2 \text{ units}$$

Acceleration vector, $A = 1536i + 320j + 2k$

Resultant of acceleration $A = \sqrt{1536^2 + 320^2 + 2^2}$

$$= \sqrt{2359296 + 102400 + 4}$$

$$= 1568.9 \text{ units}$$

7. The following cases represent the two velocity components, determine the third component of velocity such that they satisfy the continuity equation.

i, $u = x^2+y^2+z^2$; $v = xy^2 - yz^2 + xy$

ii, $v = 2y^2$; $w = 2xyz$

Sol: The continuity eqn. for incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

i, $u = x^2+y^2+z^2$ $\quad \frac{\partial u}{\partial x} = 2x$

$v = xy^2 - yz^2 + xy$ $\quad \frac{\partial v}{\partial y} = 2xy - z^2 + x$

substituting the values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ in continuity equation

$$2x + 2xy - z^2 + x + \frac{\partial w}{\partial z} = 0$$

$$3x + 2xy - z^2 + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial z} = -3x - 2xy + z^2$$

$$\partial w = (-3x - 2xy + z^2)\,\partial z$$

Integrating on both sides

$$\int \partial w = \int (-3x - 2xy + z^2)\,dz$$

$$w = [-3xz - 2xyz + z^3/3] + \text{constant of integration}$$

Where constant of integration cannot be a function of z. But it can be function of x and y that is $f(x,y)$

$$w = [-3xz - 2xyz + z^3/3] + f(x,y)$$

Case (ii) : $v = 2y^2$, $w = 2xyz$

$$\frac{\partial v}{\partial y} = 4y \, , \quad \frac{\partial w}{\partial z} = 2xy$$

substituting values of $\frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}$ in continuity eqn.

$$\frac{\partial u}{\partial x} + 4y + 2xy = 0$$

$$\frac{\partial u}{\partial x} = -4y - 2xy$$

$$\partial u = (-4y - 2xy)\,\partial x$$

Integrating on both sides,

$$\int du = \int(-4y - 2xy)\,dx$$

$$u = -4xy - 2 \cdot \frac{x^2}{2} \cdot y$$

$$= -4xy - x^2 y + \text{const. of integration}$$

$$u = -4xy - x^2 y + f(y,z)$$

8. A flow field is given by

$$V = x^2 y i + y^2 z j - (2xyz + yz^2) k$$

P.T it is a case of possible steady incompressible fluid flow. Calculate the velocity and acceleration at the point (2,1,3)

Sol: Given that

$$V = x^2 y i + y^2 z j - (2xyz + yz^2) k$$

This eqn. is compared with $V = ui + vj + wk$

Then the velocity components are,

$$u = x^2 y \qquad \frac{\partial u}{\partial x} = 2xy.$$

$$v = y^2 z \qquad \frac{\partial v}{\partial y} = 2yz.$$

$$w = -(2xyz + yz^2) \qquad \frac{\partial w}{\partial z} = -(2xy + 2yz)$$

substituting the values of $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial w}{\partial z}$ in continuity eqn.

We know that the continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2xy + 2yz - (2xy + 2yz) = 0$$

$$0 = 0$$

Hence, the velocity field, $V = x^2 y i + y^2 z j - (2xyz + z^2 y) k$ is a case of possible steady incompressible fluid flow.

Velocity at (2,1,3) $x = 2, y = 1, z = 3$

$$V = (2)^2(1)\, i + (1)^2(3)\, j - [(2 \times 2 \times 1 \times 3) + (1)(3)^2] k$$

$$= 4i + 3j - (12 + 9) k$$

$$= 4i + 3j - 21k$$

Resultant of velocity $= \sqrt{\ldots + (3)^2 + (-21)^2}$

$= \sqrt{16+9+441}$

$= \sqrt{466}$

$= 21.587$ units.

Acceleration at (2,1,3):

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \cdot \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$u = x^2y \quad \frac{\partial u}{\partial x} = 2xy \quad \frac{\partial u}{\partial y} = x^2 \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = 0$

$v = y^2z \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial y} = 2yz \quad \frac{\partial v}{\partial z} = y^2 \quad \frac{\partial v}{\partial t} = 0$

$w = -(2xyz + z^2y) \quad \frac{\partial w}{\partial x} = -(2yz) \quad \frac{\partial w}{\partial y} = -(+2xz + z^2)$

$\frac{\partial w}{\partial z} = -(2xy + 2zy) \quad \frac{\partial w}{\partial t} = 0$

$a_x = x^2y(2xy) + y^2z(x^2) - (2xyz + z^2y)(0) + 0$

$= 2x^3y^2 + x^2y^2z$

$= 2(2)^3(1)^2 + (2)^2(1)^2(3)$

$= 16 + 12 = 28$ units

$a_y = x^2y(0) + y^2z(2yz) - (2xyz + z^2y)y^2 + 0$

$= -2xy^3z + 2y^3z^2 - y^3z^2$

$= -[2(2) \times (1)^3(3)] + [2(1)^3(3)^2] - [(1)^3(3)^2]$

$= \cancel{18} - 12 - 9$

$= 18 - 21$

$= -3$ units.

$a_z = x^2y(-2yz) + y^2z(-2xz - z^2) + (2xyz + z^2y)(2xy + 2zy) + 0$

$= -2x^2y^2z - 2xy^2z^2 - y^2z^3 + 4x^2y^2z + 4xy^2z^2 + 2xy^2z^2 + 2y^2z^3$

$= 2x^2y^2z + 4xy^2z^2 - y^2z^3$

$= [2(2)^2(1)^2(3)] + [4(2)(1)^2(3)^2] - [(1)^2(3)^3] = 24 + \quad = 105$

Acceleration $= 28i - 3j + 105k$

Resultant acceleration $= \sqrt{(28)^2 + (-3)^2 + (105)^2}$

$= \sqrt{784 + 9 + 11025}$

$= \sqrt{11818}$

$= 108.71$ units/m²

→ **Velocity Potential Function and Stream Function:**

Velocity Potential Function: It is defined as a scalar function of space and time such that its negative derivative with respect to any direction given the fluid velocity in that direction. It is defined by ϕ (phi). Mathematically, the velocity potential is defined as $\phi = f(x, y, z)$ for steady flow such that

$$\boxed{\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} \\ v &= -\frac{\partial \phi}{\partial y} \\ w &= -\frac{\partial \phi}{\partial z} \end{aligned}\right\}} \qquad \text{---} (1)$$

Where u, v, w are components of velocity in x, y and z directions respectively.

The continuity eqn. for an incompressible steady flow is

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \qquad \text{---} (2)$$

Substituting the values of u, v, w in above continuity eqn.

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0} \qquad \text{---} (3)$$

This eqn. is known as Laplace Equation.

For two-dimension case the laplace Equation reduces to

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0} \qquad \text{---} (4)$$

Properties of the potential function:

We know the rotational components are given by

$$\left.\begin{aligned} \omega_z &= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\ \omega_y &= \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \omega_x &= \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \end{aligned}\right\} \longrightarrow (5)$$

substituting the values of u, v, w from (1) in eqn (5)

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial\phi}{\partial x}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2\phi}{\partial x\partial y} + \frac{\partial^2\phi}{\partial y\partial x}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial}{\partial z}\left(-\frac{\partial\phi}{\partial x}\right) - \frac{\partial}{\partial x}\left(-\frac{\partial\phi}{\partial z}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2\phi}{\partial z\partial x} + \frac{\partial^2\phi}{\partial x\partial z}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial}{\partial y}\left(-\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(-\frac{\partial\phi}{\partial y}\right)\right] = \frac{1}{2}\left[-\frac{\partial^2\phi}{\partial y\partial z} + \frac{\partial^2\phi}{\partial z\partial y}\right]$$

If ϕ is a continuous function, then

$$\frac{\partial^2\phi}{\partial x\partial y} = \frac{\partial^2\phi}{\partial y\partial x}\ ; \quad \frac{\partial^2\phi}{\partial z\partial x} = \frac{\partial^2\phi}{\partial x\partial z}\ ; \quad \frac{\partial^2\phi}{\partial y\partial z} = \frac{\partial^2\phi}{\partial z\partial y}.$$

$$\therefore \omega_z = \omega_y = \omega_x = 0$$

When rotational components are zero, the flow is called irrotational. Hence the properties of the potential function are;

1. If velocity potential (ϕ) exits, the flow should be irrotational.
2. If velocity potential (ϕ) satisfies, the Laplace equation it represents the possible steady incompressible irrotational flow.

→ Stream Function:

It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (Psi) and defined only for two-dimensional

Mathematically, for steady flow it is defined as

$\psi = f(x, y)$ such that

$$\left.\begin{aligned}\frac{\partial \psi}{\partial x} &= v \\ \frac{\partial \psi}{\partial y} &= -u\end{aligned}\right\} \quad \text{------} \quad (1)$$

The continuity equation for two-dimensional flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{------} \quad (2)$$

Substituting the values of u and v from ① to ②

$$\frac{\partial}{\partial x}\left(\frac{-\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right) = 0$$

$$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x} = 0.$$

Hence existence of ψ means a possible case of fluid flow. The flow may be rotational (or) irrotational. The rotational component ω_z is given by.

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Substituting the values of u and v from ① in above eqn.

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right.$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

For irrotational flow, $\omega_z = 0$. Hence above equation becomes as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

Which is Laplace Equation for ψ.

The properties of stream function are:

1. If stream function (ψ) exists, it is a possible case of fluid flow which may be rotational (or) irrotational.
2. If stream function (ψ) satisfies the Laplace equation, it is a possible case of an irrotational flow.

→ **Equipotential Line:**

A line along which the velocity potential ϕ is constant, is called equipotential line.

For equipotential line ϕ = constant

$$\partial\phi = 0$$

But $\phi = f(x,y)$ for steady flow

$$\partial\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy$$

$$= -u \cdot dx - v \cdot dy$$

$$= -(udx + vdy)$$

For equipotential line, $\partial\phi = 0$.

$$-(udx + vdy) = 0$$

$$udx + vdy = 0$$

$$udx = -vdy$$

$$\boxed{\frac{dy}{dx} = \frac{-u}{v}} \quad \text{——— (1)}$$

But $\frac{dy}{dx}$ = slope of equipotential line.

→ **Line of constant Stream Function:**

$$\psi = \text{constant}$$

$$d\psi = 0$$

But $d\psi = \frac{\partial\psi}{\partial x}dx + \frac{\partial\psi}{\partial y}dy$

$$= vdx - udy$$

For a line of constant stream function.

$$d\psi = 0$$

$$vdx - udy = 0$$

$$vdx = udy$$

$$\frac{dy}{dx} = \frac{v}{u} \quad \text{——— (2)}$$

But $\frac{dy}{dx}$ is slope of stream line.

For eqn. (1) and (2) it is clear that the product of the slope of the equipotential line and the slope of the stream line at the point of intersection is equal to -1. Thus the equipotential lines are orthogonal to the stream lines at all points of intersection.

→ **Flow Net:**

A grid obtained by drawing a series of equipotential lines are streamlines is called a flow net. The flow net is an important tool in analysing two-dimensional irrotational flow problems.

→ **Relation between stream function and velocity potential function:**

We know that $u = -\dfrac{\partial \phi}{\partial x}$ and $v = -\dfrac{\partial \phi}{\partial y}$

and $u = -\dfrac{\partial \psi}{\partial y}$ and $v = \dfrac{\partial \psi}{\partial x}$

Thus, we have $u = -\dfrac{\partial \phi}{\partial x} = -\dfrac{\partial \psi}{\partial y}$

$$v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\therefore \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \; ; \; \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$$

9. The velocity potential function (ϕ) is given by an expression,

$$\phi = -\frac{xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$$

i, Find the velocity components in x and y direction.

ii, Show that ϕ represents a possible case of flow.

Sol: Given $\phi = -\dfrac{xy^3}{3} - x^2 + \dfrac{x^3 y}{3} + y^2$

The partial derivatives of ϕ w.r.to x and y are

$$\frac{\partial \phi}{\partial x} = -\frac{y^3}{3} - 2x + \frac{3x^2 y}{3} \quad \text{——(1)}$$

$$\frac{\partial \phi}{\partial y} = -\frac{3xy^2}{3} + \frac{x^3}{3} + 2y \quad \text{——(2)}$$

(i) The velocity components u and v are given by:

$$u = -\frac{\partial \phi}{\partial x}$$

$$u = -\left[-\frac{y^3}{3} - 2x + \frac{3x^2 y}{3}\right]$$

$$u = \frac{y^3}{3} + 2x - x^2 y$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$= -\left[-\frac{3xy^2}{3} + \frac{x^3}{3} + 2y\right]$$

$$= \frac{3xy^2}{3} - \frac{x^3}{3} - 2y$$

$$v = xy^2 - \frac{x^3}{3} - 2y$$

ii, The given value of ϕ, will represent a possible case of flow if it satisfies the Laplace equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

From eqn's (1) & (2) we have

$$\frac{\partial \phi}{\partial x} = -\frac{y^3}{3} - 2x + x^2 y$$

$$\frac{\partial^2 \phi}{\partial x^2} = -2 + 2xy$$

and $$\frac{\partial \phi}{\partial y} = -xy^2 + \frac{x^3}{3} + 2y$$

$$\frac{\partial^2 \phi}{\partial y^2} = -2xy + 2$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (-2 + 2xy) + (-2xy + 2) = 0$$

Laplace equation is satisfied and hence ϕ represent a possible case of flow.

10. The velocity potential function is given by $\phi = 5(x^2 - y^2)$. Calculate the velocity components at the point (4, 5)

Sol: $$\phi = 5(x^2 - y^2)$$

$$\frac{\partial \phi}{\partial x} = 10x$$

$$\frac{\partial \phi}{\partial y} = -10y$$

But velocity components are given by eqn

$$u = -\frac{\partial \phi}{\partial x} = -10x$$

$$v = -\frac{\partial \phi}{\partial x} = -(-10y) = 10y$$

The velocity components at the point (4,5) i.e., at x=4 & y=5.

$$u = -10 \times 4 = -40 \text{ units}$$

$$v = 10 \times 5 = 50 \text{ units.}$$

11. A stream function is given by $\psi = 5x - 6y$. Calculate the velocity components and also magnitude and direction of the resultant velocity at any point.

Sol:

$$\psi = 5x - 6y$$

$$\frac{\partial \psi}{\partial x} = 5 \quad \text{and} \quad \frac{\partial \psi}{\partial y} = -6$$

But the velocity components u and v in terms of stream function are given by equation as.

$$u = -\frac{\partial \psi}{\partial y} = -(-6) = 6 \text{ units/sec.}$$

$$v = \frac{\partial \psi}{\partial x} = 5 \text{ units/sec.}$$

$$\text{Resultant velocity} = \sqrt{u^2+v^2}$$

$$= \sqrt{6^2+5^2}$$

$$= \sqrt{61}$$

$$= 7.81 \text{ units/sec}$$

Direction is given by, $\tan\theta = \frac{u}{v} = \frac{5}{6} = 0.833$

$$\theta = \tan^{-1}(0.833)$$

$$\theta = 39^\circ 48'$$

12. If for two-dimensional potential flow, the velocity potential is given by $\phi = x(2y-1)$

determine the velocity at the point P(4,5). Determine also the value of stream function ψ at point P.

Given: $\phi = x(2y-1)$

i, The velocity components in the direction of x and y are

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial}{\partial x}[x(2y-1)] = -[2y-1] = 1-2y$$

$$v = -\frac{\partial \phi}{\partial y} = -\frac{\partial}{\partial y}[x(2y-1)] = -[2x] = -2x$$

At point p(4,5) i.e., at $x = 4$ & $y = 5$

$u = 1-2(5) = 1-10 = -9$ units/sec.

$v = -2(4) = -8$ units/sec

velocity at $P = -9i - 8j$

Resultant velocity at $P = \sqrt{9^2+8^2} = \sqrt{81+64}$

$= 12.04$ units/sec

ii, Value of stream function at p.

WKT $\frac{\partial \psi}{\partial y} = -u = -(1-2y) = 2y-1$ ——— (i)

$\frac{\partial \psi}{\partial x} = v = -2x$ ——— (ii)

Integrating equation i, with respect to y.

$$\int d\psi = \int (2y-1)\,dy$$

$\psi = 2 \cdot y^2/2 - y +$ constant of integration.

The constant of integration is not a function of y but it can be a function of x.

Let the value of constant of integration is K. Then

$\psi = y^2 - y + K$ ——— iii,

Differentiating the above eqn. w.r.to x.

$$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x}.$$

But $\frac{\partial \psi}{\partial x} = -2x$ from eqn ii,

Equating the value of $\frac{\partial \psi}{\partial x}$, we get $\frac{\partial K}{\partial x} = -2x$.

Integrating this equation,

$$K = \int -2x \cdot dx = -\frac{2x^2}{2} = -x^2$$

substituting the value ... in eq (iii),

$$\psi = y^2 - y - x^2$$

$\therefore$ stream function ψ at $P(4,5) = (5)^2 - 5 - (4)^2$

$= 25 - 5 - 16$

$= \underline{4 \text{ units}}$

13. The stream function for a two-dimensional flow is given by $\psi = 2xy$, calculate velocity at the point $P(2,3)$. Find the velocity potential function.

Sol: Given $\psi = 2xy$

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(2xy) = -2x$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(2xy) = 2y$$

At the point $P(2,3)$ we get

$u = -2 \times 2 = 4$ units/sec.

$v = 2 \times 3 = 6$ units/sec.

Resultant velocity at $P = \sqrt{4^2 + 6^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52}$

$= 7.21$ units/sec.

Velocity potential function ϕ

$$\frac{\partial \phi}{\partial x} = -u = -(-2x) = 2x \longrightarrow (i), \quad \frac{\partial \phi}{\partial y} = -v = -2y \longrightarrow (ii),$$

Integrating eqn (i) we get $\int d\phi = \int 2x\,dx$

$$\phi = \frac{2x^2}{2} + c = x^2 + c \longrightarrow (iii),$$

$c \rightarrow$ constant independent of x but can be a function of y

differentiating eqn (iii), w.r.to 'y' $\frac{\partial \phi}{\partial y} = \frac{\partial c}{\partial y}$

But from eqn (ii), $\frac{\partial \phi}{\partial y} = -2y \Rightarrow \frac{\partial c}{\partial y} = -2y$

Integrating the eqn, we get $c = \int -2y \cdot dy \Rightarrow c = \frac{-2y^2}{2} = -y^2$

substituting the value of c in eqn (iii), $\underline{\phi = x^2 - y^2}$

14. Sketch the stream lines represented by $\psi = x^2+y^2$. Also find out the velocity and its direction at point (1, 2).

<u>Sol:</u> Sketch of stream lines:

$$\psi = x^2+y^2$$

Let $\psi = 1, 2, 3$ and so on

$$1 = x^2+y^2$$
$$2 = x^2+y^2$$
$$3 = x^2+y^2$$

Each eqn. is a eqn. of circle. Thus we shall get concentric circles of differential diameters as shown in figure.

Given, $\psi = x^2+y^2$

The velocity components u and are given by

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2+y^2) = -2y$$

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2) = 2x$$

At the point (1,2) the velocity components are

$$u = -2(2) = -4 \text{ units/sec.}$$
$$v = 2(1) = 2 \text{ units/sec}$$

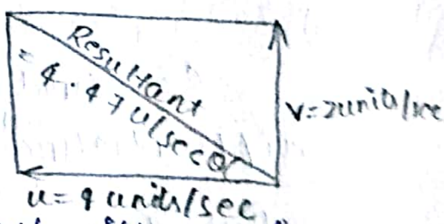
Resultant velocity $= \sqrt{u^2+v^2}$

$= \sqrt{(-4)^2+(2)^2}$

$= \sqrt{20}$

$= 4.47$ units/sec

$$\tan\theta = \frac{u}{v} = \frac{2}{+4} = \frac{+1}{2}$$



$$\theta = \tan^{-1}\left(\frac{+1}{2}\right) = 26°.34'$$

Resultant velocity makes an angle 26°34' with x-axis.

15. The velocity components in a two-dimensional flow field for an incompressible fluid are as follows:

$$u = \frac{y^3}{3} + 2x - x^2y \quad \text{and} \quad v = xy^2 - 2y - \frac{x^3}{3}.$$

Obtain an expression for the stream function ψ.

Sol: Given: $u = \frac{y^3}{3} + 2x - x^2 y$

$V = xy^2 - 2y - \frac{x^3}{3}$

The velocity components in terms of stream function are

$$\frac{\partial \psi}{\partial x} = V = xy^2 - 2y - \frac{x^3}{3} \quad \text{——— (i)}$$

$$\frac{\partial \psi}{\partial y} = -u = -\frac{y^3}{3} - 2x + x^2 y \quad \text{——— (ii)}$$

Integrating equation (i) w.r.to 'x'

$$\psi = \int \left(xy^2 - 2y - \frac{x^3}{3}\right) dx$$

$$= \frac{x^2}{2} \cdot y^2 - 2xy - \frac{x^4}{12}$$

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} + k \quad \text{——— (iii)}$$

Where k = constant of integration, which is independent of x but dependent of y, function of y

differentiating eqn (iii) w.r.to y

$$\frac{\partial \psi}{\partial y} = \frac{x^2}{2}(2y) - 2x + \frac{\partial k}{\partial y}$$

$$= x^2 y - 2x + \frac{\partial k}{\partial y}$$

but from equation (ii) $\frac{\partial \psi}{\partial y} = -\frac{y^3}{3} - 2x + x^2 y$

$$\therefore \quad -\frac{y^3}{3} - 2x + x^2 y = x^2 y - 2x + \frac{\partial k}{\partial y}$$

$$\frac{\partial k}{\partial y} = -\frac{y^3}{3} \Rightarrow k = -\frac{y^4}{12}$$

substituting the value of k in eqn (ii)

$$\psi = \frac{x^2 y^2}{2} - 2xy - \frac{x^4}{12} - \frac{y^4}{12}$$

16. In a two-dimensional incompressible flow, the fluid velocity components are given by $u = x - 4y$ & $V = -y - 4x$

S.T velocity potential exists and determine is form. Find also the stream function.

Given $u = x - 4y$ and $v = -y - 4x$

$$\frac{\partial u}{\partial x} = 1 \quad \text{and} \quad \frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

Hence flow is continued and velocity potential exists

Let ϕ = velocity potential

Velocity components in terms of velocity potential is given by.

$$\frac{\partial \phi}{\partial x} = -u = -x + 4y \quad \text{—— i,}$$

$$\frac{\partial \phi}{\partial y} = -v = y + 4x \quad \text{—— ii,}$$

Integrating eqn i, we get

$$\int d\phi = \int (-x + 4y)\, dx$$

$$\phi = -\frac{x^2}{2} + 4xy + c \quad \text{—— iii,}$$

Where c = constant of integration which is independent of x.

The constant can be function of y.

differentiating the eqn iii, w.r.to y.

$$\frac{\partial \phi}{\partial y} = 4x + \frac{\partial c}{\partial y}$$

but from eqn ii, $\frac{\partial \phi}{\partial y} = y + 4x$

$$y + 4x = 4x + \frac{\partial c}{\partial y}$$

$$\frac{\partial c}{\partial y} = y$$

integrating the above eqn, $c = \frac{y^2}{2} + c$

Where c_1 = constant of integration which is independent of x and y

$$\therefore c = y^2/2$$

substituting the value of c in eqn iii,

$$\phi = -\frac{x^2}{2} + 4xy + \frac{y^2}{2}$$

$$\Rightarrow \boxed{\phi = \frac{y^2}{2} - \frac{x^2}{2} + 4xy} \text{ velocity potential.}$$

Stream function:

Velocity components in terms of stream function are given by $\frac{\partial \psi}{\partial x} = v = -y - 4x$ ——(iv)

$\frac{\partial \psi}{\partial y} = -u = -x + 4y$ ——(v)

Integrating eqn (iv) we get

$$\int d\psi = \int (-y - 4x)\,dx$$

$$\psi = -yx - \frac{4x^2}{2} + K$$

$$\psi = -yx - 2x^2 + K \text{ ——(vi)}$$

Where K = constant of integration which is independent of x. But it is a function of y.

differentiating equation (iii) w.r.to y

$$\frac{\partial \psi}{\partial y} = -x + \frac{\partial K}{\partial y}$$

But from eqn (v) $\frac{\partial \psi}{\partial y} = -x + 4y$

$$-x + 4y = -x + \frac{\partial K}{\partial y}$$

$$\frac{\partial K}{\partial y} = 4y$$

$$\Rightarrow K = \frac{4y^2}{2} + K_1$$

Where K_1 = constant of integration which independent of x and y

$$K = 2y^2$$

Substituting the value of K in equation (vi),

$$\psi = -yx - 2x^2 + 2y^2$$

$\psi = 2y^2 - 2x^2 - yx$ stream function.

Dynamics of Fluid flow

UNIT-II

→ The study of the fluid motion the forces and energies that are involved in the flow are required to be considered. This aspect of fluid motion is known as dynamics of fluid flow.

→ The various forces acting on the fluid mass may be classified as

i, body (or) volume forces.
ii, Surface forces.
iii, Line forces.

* i, Body or volume Forces: The body (or) volume forces are the forces which are proportional to the volume of the body.
  Eg: weight, centrifugal force, magnetic force, electromotive force etc

* ii, Surface forces: The surface forces are the forces which are proportional to surface area.
  Eg: pressure force, shear (or) tangential force, force of compressibility, force due to turbulence etc.

* iii, Line forces: These are forces which are proportional to length.
  Eg: Surface tension.

→ **Equation of motion:**

Newton's second law of motion states that the resultant force on any fluid element must equal to the product of the mass and the acceleration of the element and the acceleration vector has the direction of the resultant force vector.

$$\Sigma F = Ma$$

where ΣF = the resultant external force acting on the fluid element of mass M.

a = total acceleration.

→ **Forces acting on Fluid in Motion:**

The various forces that may influence the motion of a fluid are due to gravity, pressure, viscosity, turbulence and compressibility

- For steady rotational flow of a fluid Bernoulli's equation can be derived for the points lying on the same stream line.

- Stream lines, streak lines & path lines are all identical in case of Steady flow.
- Navier-Strokes equ. is useful in the analysis of viscous flow.

Euler's equation of motion can be integrated when it is assumed that the fluid is incompressible.

- In irrotational flow of an ideal fluid a velocity potential exists.

If stream function $\psi = 2xy$ then the velocity at a point $(1,2)$ is equal to $\sqrt{20}$.

- A equipotential line has no velocity component tangent to it.
- The continuity equation fullfilled by the flow of any fluid, real or ideal, laminar or turbulent.
- A source in two-dimensional flow is a line from which fluid is imagined to flow uniformly in all directions.

- The gravity force (F_g) is due to the weight of the fluid
- The pressure force (F_p) is exerted on the fluid mass if there exists a pressure gradient between the two points in the direction of flow.
- The viscous force (F_v) is due to the viscosity of the flowing fluid and thus exists in the case of all real fluids.
- The turbulent force (F_t) is due to the turbulence of the flow. In the turbulent flow the fluid particles move from one layer to the other and therefore, there is a continuous momentum transfer between adjacent layers, which results in developing additional stresses (called Reynolds stresses) for the flowing fluid.
- The compressibility force F_c is due to the elastic property of the fluid and it is important only either for compressible fluids or in the case of flowing fluids in which the elastic properties of fluids are significant.

→ **Equations of Motion:**

According to Newton's second law of motion

$$\boxed{F_x = ma_x}$$

In above eqn, the net force,

$$\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x}$$

i, If the force due to compressibility F_c is negligible, the resulting net force,

$$\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x}$$

and eqn. of motion are called Reynold's equation of motion:

ii, For flow, where (F_t) is negligible, the resulting equations of motion are known as Navier-Stokes Equation

$$\boxed{F_x = (F_g)_x + (F_p)_x + (F_v)_x}$$

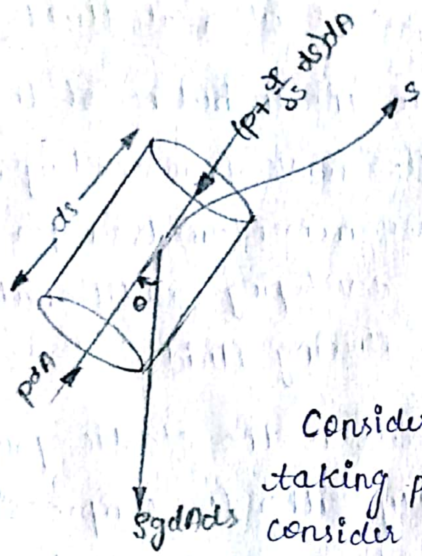
iii, If the flow is assumed to be ideal, viscous force (F_v) is zero and eqns of motion are known as Euler's Equation of motion.

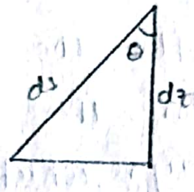
$$\boxed{F_x = (F_g)_x + (F_p)_x}$$

Euler's equation of Motion:

This is eqn. of motion in which the forces due to gravity and pressure are taken into consideration.

This is derived by considering the motion of a fluid element along a stream-line as:





Consider a stream-line in which flow is taking place in s-direction as shown in fig. Consider a cylindrical element of cross-section dA and length ds. The forces acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow.
2. Pressure force $(p+\frac{\partial p}{\partial s}\cdot ds)dA$ opposite to the direction flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction s.

$$pdA - \left(p+\frac{\partial p}{\partial s}\cdot ds\right)dA - \rho g\, dA\, ds \cos\theta = \rho\, dA ds \times a_s \quad \text{——(1)}$$

Where, a_s is the acceleration in the direction of s.

Now $a_s = \frac{dV}{dt}$, where V is function of s and t.

$$= \frac{\partial V}{\partial s}\cdot\frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$= V\cdot\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \qquad \left\{\because \frac{ds}{dt} = V\right\}$$

If the flow is steady, $\frac{\partial V}{\partial t} = 0$.

$$\therefore a_s = V\cdot\frac{dV}{ds}$$

Substituting the value [illegible] (1)

$$p\,dA - \left(p + \frac{\partial p}{\partial s}\cdot ds\right)dA - \rho g\cdot dA\,ds\cos\theta = \rho\,dA\,ds \times V\cdot\frac{\partial V}{\partial s}$$

$$p\,dA - p\,dA - \frac{\partial p}{\partial s}ds\,dA - \rho g\cdot dA\cdot ds\cos\theta = \rho\,dA\cdot ds \times V\cdot\frac{\partial V}{\partial s}$$

$$-\frac{\partial p}{\partial s}ds\cdot dA - \rho g\cdot dA\,ds\cos\theta = \rho\,dA\cdot ds \times V\cdot\frac{\partial V}{\partial s}$$

dividing on both sides by $\rho\,ds\,dA$,

$$\frac{-1}{\rho}\cdot\frac{\partial p}{\partial s} - g\cos\theta = V\cdot\frac{\partial V}{\partial s}$$

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial s} + g\cos\theta + V\cdot\frac{\partial V}{\partial s} = 0$$

But we know that $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial s} + g\cdot\frac{dz}{ds} + V\cdot\frac{\partial V}{\partial s} = 0$$

$$\Rightarrow \frac{dp}{\rho} + g\,dz + V\cdot dV = 0$$

This equation is called Euler's equation of motion.

Bernoulli's Equation From Euler's equation:

Bernoulli's eqn. is obtained by integrating the Euler's eqn. of motion as $\int\frac{dp}{\rho} + \int g\cdot dz + \int V\cdot dV = \text{constant}$.

If flow is incompressible, ρ is constant.

$$\frac{p}{\rho} + gz + \frac{V^2}{2} = \text{constant}$$

dividing with g.

$$\frac{p}{\rho g} + z + \frac{V^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{\rho g} + \frac{V^2}{2g} + z = \text{constant}}$$

This is Bernoulli's equation, in which

$\frac{p}{\rho g}$ = Pressure energy per unit weight of fluid (or) pressure Head.

$\frac{V^2}{2g}$ = Kinetic energy per unit weight of fluid (or) kinetic Head.

z = Potential energy per unit weight of fluid (or) Potential Head.

Assumptions:

The following are the assumptions made in the derivation of Bernoulli's theorem.

1. The fluid is ideal i.e., viscosity is zero.
2. The flow is steady.
3. The flow is incompressible.
4. The flow is irrotational.

1. Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above datum line.

Sol: Given, diameter of pipe = 5 cm = 0.5 m

Pressure, $P = 29.43 \ N/cm^2$
$= 29.43 \times 10^4 \ N/m^2$

Velocity, $V = 2.0$ m/sec

Datum head, $z = 5m$

Total head = Pressure head + Velocity head + Datum head

$$\text{Pressure head} = \frac{P}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30\,m.$$

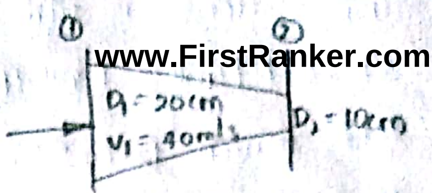
$$\text{Velocity head} = \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204\,m$$

Datum head $= z = 5m$

$$\text{Total head} = \frac{P}{\rho g} + \frac{V^2}{2g} + z.$$
$$= 30 + 0.204 + 5$$
$$= 35.204\,m$$

2. A pipe through which water is flowing is having diameters, 20cm and 10cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at section 1 and 2 and also rate of discharge

Sol: Given Data:



$D_1 = 20cm = 0.2 cm$

$\therefore$ Area $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2$

$= 0.0314 \, m^2$

$V_1 = 4.0 \, m/sec$

$D_2 = 0.1 \, m$

$A_2 = \pi/4 \, (0.1)^2 = 0.00785 \, m^2$

(i) Velocity head at section 1

$= \frac{V_1^2}{2g}$

$= \frac{4.0 \times 4.0}{2 \times 9.81}$

$= 0.815$ cm m.

(ii) Velocity head at section 2 $= \frac{V_2^2}{2g}$

To find V_2, apply continuity equation at 1 and 2.

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2}$

$= \frac{0.0314 \times 4.0}{0.00785}$

$= 16.0 \, m/sec$

$\therefore$ Velocity heat at section $2 = \frac{V_2^2}{2g}$

$= \frac{16 \times 16}{2 \times 9.81}$

$= 3.047 \, m$

(iii) Rate of discharge $= A_1 V_1$ (or) $A_2 V_2$

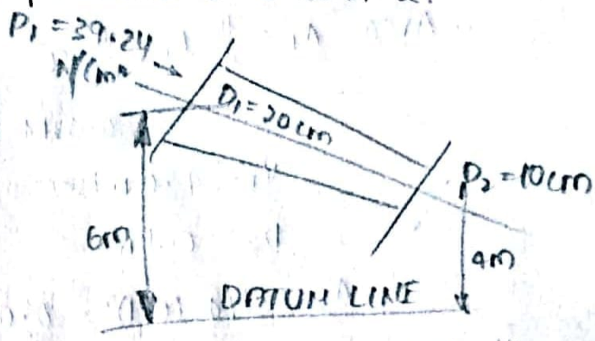
$= 0.0314 \times 4.0$

$= 0.1256 \, m^3/sec$

$= 125.6$ litres / sec.

3. The water is flowing through a pipe having diameters 20 cm and 10 cm at section 1 and 2, respectively. The rate of flow through pipe is 35 litres/sec. The section 1 is 6m above datum

and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm², Find the intensity of pressure at section 2.



Sol: Given Data:

At Section 1, $D_1 = 20cm = 0.2m$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2$$
$$= 0.0314 \, m^2$$

$$P_1 = 39.24 \, N/cm^2$$
$$= 39.24 \times 10^4 \times N/m^2$$
$$z_1 = 6m$$

At Section 2, $D_2 = 0.10cm$

$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.10)^2$$
$$= 0.00785 \, m^2$$
$$z_2 = 4m$$
$$P_2 = ?$$

Rate of flow, $q = 35$ lit/sec

$$= \frac{35}{1000} m^3/sec = 0.035 \, m^3/sec$$

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \, m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \, m/sec$$

Applying Bernoulli's eqn. at section 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4$$

$$40 + 0.063 + 6 = \frac{P_2}{9810} + 1.012 + 4$$

$$46.063 = \frac{P_2}{9810} + 5.012$$

$$\frac{P_2}{9810} = 46.062 - 5.011$$

$$\frac{P_2}{9810} = 41.051$$

$$P_2 = 41.051 \times 9810 \ N/m^2$$

$$= \frac{41.051 \times 9810}{10^4} \ N/cm^2$$

$$= 40.27 \ N/cm^2$$

Intensity of pressure at section $2 = 40.27 \ N/cm^2$

4. Water is flowing through a pipe having diameter 300mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/sec.

Sol: Given,

Section 1, $D_1 = 300mm = 0.3m$

$P_1 = 24.525 \ N/cm^2$

$= 24.525 \times 10^4 \ N/m^2$

Section 2, $D_2 = 200mm = 0.2m$

$P_2 = 9.81 \ N/cm^2$

$= 9.81 \times 10^4 \ N/m^2$

Rate of flow = 40 lit/sec.

$$Q = \frac{40}{1000} = 0.04 \ m^3/sec$$

Now, $A_1V_1 = A_2V_2 = $ rate of flow $= 0.04$

$$V_1 = \frac{0.04}{A_1} = \frac{0.04}{\pi/4 D_1^2}$$

$$= \frac{0.04}{\pi/4 (0.3)^2}$$

$$= 0.5658 \ m/sec$$

$$\simeq 0.566 \ m/sec$$

$$V_2 = \frac{0.04}{A_2} = \frac{0.04}{\pi/4 D_2^2}$$

$$= \frac{0.04}{\frac{\pi}{4}(0.2)^2}$$

$$= 1.274 \text{ m/sec}$$

Applying Bernoulli's equation at (1) and (2):

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{0.566 \times 0.566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2.$$

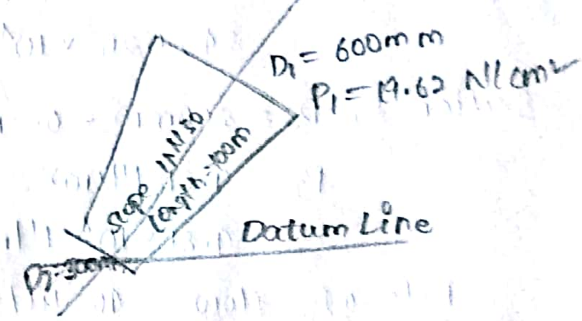
$$25 + 0.32 + z_1 = 10 + 1.623 + z_2$$

$$25.32 + z_1 = 11.623 + z_2$$

$$z_2 - z_1 = 25.32 - 11.623 = 13.697 \simeq 13.70 \text{ m}$$

Difference in datum head $= z_2 - z_1 = 13.70$ m.

5. The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm²



Sol: Given that;

length of pipe, $L = 100$m

Diameter at the upper end,

$D_1 = 600\text{mm} = 0.6\text{m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.6)^2 = 0.2827 \text{ m}^2$

$P_1 =$ Pressure at upper end $= 19.62$ N/cm².

$= 19.62 \times 10^4$ N/m²

Diameter at lower end, $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2$

$= \frac{\pi}{4}(0.3)^2 = 0.07068$ m.

$Q =$ rate of flow $= 50$ litres/s $= \frac{50}{1000} = 0.05 \text{ m}^3/\text{sec}$.

Let the datum line is passing through the centre of lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m.

Also, we know $Q = A_1V_1 = A_2V_2$

$$V_1 = \frac{Q}{A} = \frac{0.05}{0.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}.$$

$$V_2 = \frac{Q}{A} = \frac{0.5}{0.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}.$$

Applying Bernoulli's equation at section (1) and (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{(0.177)^2}{2 \times 9.81} + \frac{10}{3} = \frac{P_2}{\rho g} + \frac{(0.707)^2}{2 \times 9.81} + 0$$

$$20 + 0.001596 + 3.334 = \frac{P_2}{\rho g} + 0.0254$$

$$23.335 - 0.0254 = \frac{P_2}{1000 \times 9.81}$$

$$P_2 = 23.3 \times 9810$$

$$= 228573 \text{ N/m}^2$$

$$= 22.857 \text{ N/cm}^2$$

Bernoulli's Equation For Real Fluid:

The Bernoulli's eqn. was derived on the assumption that fluid is non-viscous and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's eqn. these losses have to be taken into consideration. Thus, the Bernoulli's eqn. for real fluids between point 1 and 2 is given as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

Where, h_L = loss of energy between point 1 and 2.

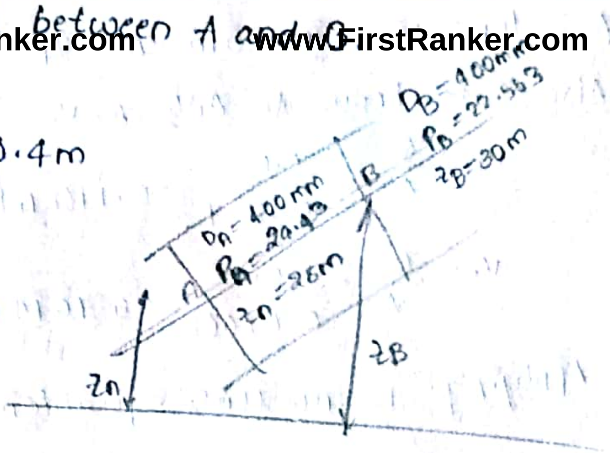
6. A pipe of diameter 400mm carries water at a velocity of 25m/s. The pressure at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively. While the datum heat at A and B are

28m and 30m. Find the loss of head between A and B.

Sol: Given

Diameter of pipe, $D = 400mm = 0.4m$

velocity, $V = 25 m/sec$



At point A: $P_A = 29.43 \; N/cm^2$

$= 29.43 \times 10^4 \; N/m^2$

$z_A = 28m$

$V_A = 25 m/sec$

Total energy at point A $= \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28$$

$$E_A = 89.85m$$

At point B: $P_B = 22.563 \; N/cm^2 = 22.563 \times 10^4 \; N/m^2$

$z_B = 30 m$

$V_B = 25 m/sec$

Total Energy at B, $E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{(25)^2}{2 \times 9.81} + 30$$

$$= 23 + 31.85 + 30$$

$$= 84.85m$$

Loss of head between A and B

$= E_A - E_B$

$= 89.85 - 84.85$

$= \underline{5m}$

7. A conical tube of length 2m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5m/s while at the lower end it is 2m/s. The pressure head at the smaller end is 2.5m of liquid. The loss of head in the tube is $\frac{0.35(V_1 - V_2)^2}{2g}$, where V_1 is the velocity at the smaller end and V_2

to the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.

Sol: Let the smaller end is represented by (1) and lower end by (2)

Given: Length of tube, $L = 2.0\,m$

$V_1 = 5\,m/s$

Pressure head $\frac{P}{\rho g} = 2.5\,m$ of liquid

$V_2 = 2\,m/s$

Loss of head $= h_L = \frac{0.35(V_1 - V_2)^2}{2g}$

$= \frac{0.35(5-2)^2}{2 \times 9.81}$

$= \frac{0.35 \times 9}{2 \times 9.81}$

$= 0.16\,m$

$\therefore$ Pressure at lower end $\frac{P_2}{\rho g} = ?$

Applying Bernoulli's theorem at sections (1) and (2),

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$2.5 + \frac{(5)^2}{2 \times 9.81} + 2.0 = \frac{P_2}{\rho g} + \frac{(2)^2}{2 \times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{P_2}{\rho g} + 0.203 + 0.16$$

$$\frac{P_2}{\rho g} = (2.5 + 1.27 + 2.0) - (0.203 + 0.16)$$

$= 5.77 - 0.363$

$= 5.407\,m$ of fluid.

8. A pipe line carrying oil of specific gravity 0.87 changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 m at a higher level. If the pressures at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively and the discharge is 200 lt/s. determine the loss of head and direction of flow.

Sol: Discharge, $Q = 200$ lit/sec

$= 0.2\,m^3/sec$

Specific gravity of oil = 0.87

$\therefore$ density of oil, $\rho = 0.87 \times 1000$

$= 870 \ kg/m^3$

At Section A, $D_A = 200mm = 0.2m$

Area, $A_A = \frac{\pi}{4}(D_A)^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \ m^2$

$P_A = 9.81 \ N/cm^2 = 9.81 \times 10^4 \ N/m^2$

If datum line is passing through A, then $z_A = 0$

$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \ m/sec$

At Section B, $D_B = 500mm = 0.5m$

Area, $A_B = \frac{\pi}{4} D_B^2 = \frac{\pi}{4}(0.5)^2 = 0.1963 \ m^2$

$P_B = 5.886 \ N/cm^2$

$= 5.886 \times 10^4 \ N/m^2$

$z_B = 4.0 \ m$

$V_B = \frac{Q}{A_B} = \frac{0.2}{0.1963} = 1.018 \ m/sec$

Total Energy at A is given by

$$E_A = \frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A$$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0$$

$$E_A = 11.49 + 2.067 = 13.557m$$

Total Energy at B is given by

$$E_B = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B$$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{1.018}{2 \times 9.81} + 4$$

$$E_B = 6.896 + 0.052 + 4.0 = 10.948m$$

i, Direction of flow: As E_A is more than E_B and hence flow is taking place from A to B.

ii, Loss of head $= h_L = E_A - E_B = 13.557 - 10.948$

$= 2.609m$

The momentum Equation:

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass "m" is given by the Newton's second law of motion.

$$\boxed{F = m \times a}$$

Where a = acceleration acting in the same direction as force F.

But $a = \frac{dV}{dt}$

Substitute the value of a in above equation

$$F = m \cdot \frac{dV}{dt}$$

$$F = \frac{d(mV)}{dt}$$

{∵ m is constant and can be taken inside the differential}

$$\boxed{F = \frac{d(mV)}{dt}}$$

This equation is known as the momentum principle.

$$\boxed{F\,dt = d(mV)}$$

Which is known as impulse-momentum equation and states that the impulse of a force F acting on a fluid mass m in a short interval of time dt is equal to the change of momentum d(mV) in the direction of the force.

Force exerted by a flowing fluid on a pipe-Bend:

The impulse momentum equation is used to determine the resultant force exerted by a flowing fluid on a pipe-bend.

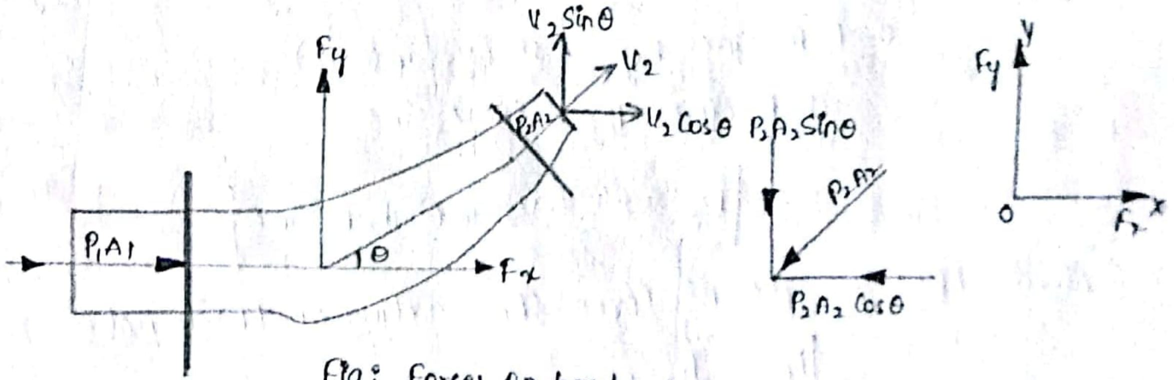


Fig: Forces on bend

Consider two sections (1) and (2) as shown in fig.

Let V_1 = Velocity of flow at section (1)

P_1 = Pressure intensity at section (1).

A_1 = Area of cross-section of pipe at section (1)

V_2, P_2, A_2 = Corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the following fluid on the bend in x and y directions respectively. Then the force exerted by the bend on the fluid in the direction of x and y will be equal to F_x and F_y but in the opposite directions.

Hence components of the force exerted by bend on the fluid in the x-direction $= -F_x$.

and in the direction of $y = -F_y$.

The other external forces acting on the fluid are P_1A_1 and P_2A_2 on the section (1) and (2) respectively. Then momentum eqn. in x-direction is given by

Net force acting on the fluid in the direction of x } = { Rate of change of momentum in x-direction.

$P_1A_1 - P_2A_2\cos\theta - F_x$ = (Mass per sec) × (change of velocity)

= ρQ (Final velocity in the direction of x − Initial velocity in the direction of x)

$$P_1A_1 - P_2A_2\cos\theta - F_x = \rho Q(V_2\cos\theta - V_1)$$

$$F_x = P_1A_1 - P_2A_2\cos\theta - \rho Q(V_2\cos\theta - V_1)$$

$$\boxed{F_x = P_1A_1 - P_2A_2\cos\theta + \rho Q(V_1 - V_2\cos\theta)}$$

Similarly, the momentum equation in y-direction gives

$$0 - P_2A_2\sin\theta - F_y = \rho Q(V_2\sin\theta - 0)$$

$$-P_2A_2\sin\theta - F_y = \rho Q V_2\sin\theta$$

$$\boxed{F_y = \rho Q(-V_2\sin\theta) - P_2A_2\sin\theta}$$

Now the resultant force (F_R) acting on the bend

$$\boxed{F_R = \sqrt{F_x^2 + F_y^2}}$$

...nd the angle made by the resultant force with horizontal direction is given by.

$$\boxed{\tan\theta = \frac{F_y}{F_x}}$$

1. A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600mm and 300mm respectively. Find the force exerted by water on the bend if the intensity of the pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/sec.

Sol: Given data :

Angle of bend, $\theta = 45^\circ$

Diameter of at inlet $D_1 = 600mm = 0.6m$

$\therefore$ Area, $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.6)^2 = 0.2827\ m^2$

Diameter at outlet, $D_2 = 300mm = 0.3m$

$\therefore$ Area, $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

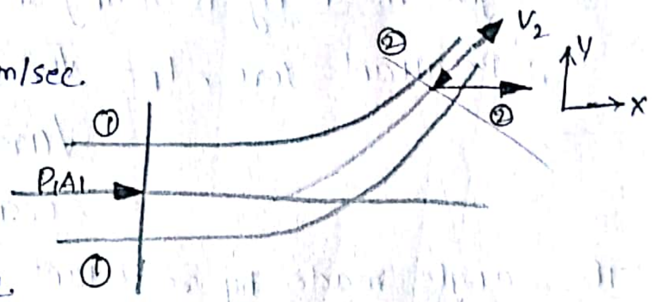
Pressure at inlet, $P_1 = 8.829\ N/cm^2 = 8.829\times10^4\ N/m^2$

$Q = 600\ lit/sec = 0.6\ m^3/sec$

$V_1 = \frac{Q}{A_1} = \frac{0.6}{0.2827} = 2.122\ m/sec.$

$V_2 = \frac{Q}{A_2} = \frac{0.6}{0.07068}$

$= 8.488\ m/sec.$



Applying Bernoulli's eqn. at sections (1) and (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{8.829\times10^4}{1000\times9.81} + \frac{2.122^2}{2\times9.81} = \frac{P_2}{1000\times9.81} + \frac{8.488^2}{2\times9.81}$$

$$9 + 0.2295 = \frac{P_2}{980} + 3.672$$

$\frac{P_2}{980} + 0.2295 = 3.672$

$\frac{P_2}{980} = 5.5575$ m of water

$P_2 = 5.5575 \times 980$

$P_2 = 5.45 \times 10^4 \; N/m^2$.

Forces on the bend in x and y directions are given by equations,

$F_x = \rho Q(V_1 - V_2 \cos\theta) + P_1 A_1 - P_2 A_2 \cos\theta$

$= \{1000 \times 0.6\,[2.122 - 8.488 \cos 45^\circ]\} + (8.829 \times 10^4 \times 0.2827)$
$\quad - (5.45 \times 10^4 \times 0.07068 \times \cos 45^\circ)$

$F_x = -2327.9 + 24959.6 - 2720.3$

$= 24959.6 - 5048.2$

$= 19911.4$ N

$F_y = \rho Q(-V_2 \sin\theta) - P_2 A_2 \sin\theta$

$= +1000 \times 0.6(-8.488 \sin 45^\circ) - (5.45 \times 10^4 \times 0.07068 \times \sin 45^\circ)$

$= -3601.1 - 2721.1$

$= -6322.2$ N

-ve sign mean F_y is acting in the downward direction.

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(19911.4)^2 + (-6322.2)^2}$

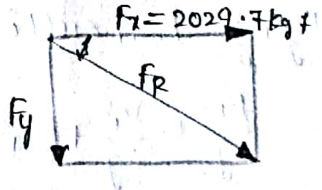
$= 20890.9$ N

The angle made by resultant force with x-axis is given by

$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$

$\theta = \tan^{-1}(0.3175)$

$\theta = 17^\circ 36'$



2. 250 lit/sec of water is flowing in a pipe having a diameter of 300mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm²

Sol: Given data:

Pressure, $P_1 = P_2 = 39.24 \; N/cm^2 = 39.24 \times 10^4 \; N/m^2$

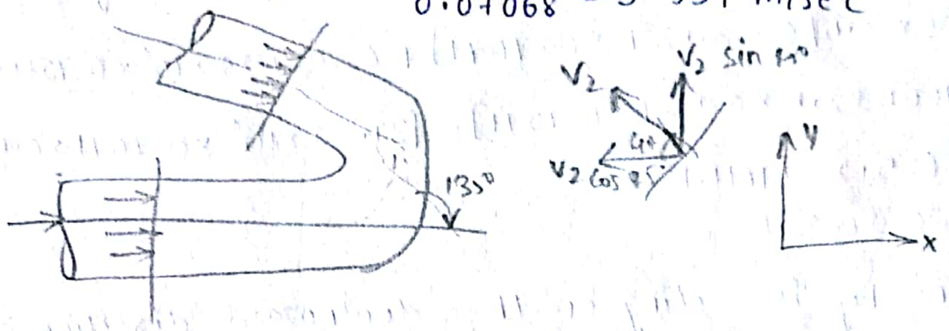
Discharge $Q = 250$ litres/s $= 0.25\ m^3/sec$

Diameter of bend at inlet and outlet, $D_1 = D_2 = 300mm = 0.3m$.

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2 = 0.07068\ m^2$

Velocity of water at (1) and (2)

$$V = V_1 = V_2 = \frac{Q}{Area} = \frac{0.25}{0.07068} = 3.537\ m/sec$$



Force along X-axis:

$$F_x = \rho Q [V_{1x} - V_{2x}] + P_{1x} A_1 + P_{2x} A_2$$

Where, V_{1x} = Initial velocity in the direction of $x = 3.537\ m/sec$

V_{2x} = Final velocity in the direction of $x = -V_2 \cos 45°$

$= -3.537 \cos 45° = -3.537 \times 0.7071$

P_{1x} = Pressure at section (1) in x-direction

$= 39.24 \times 10^4\ N/m^2$

P_{2x} = Pressure at section (2) in x-direction.

$= P_2 \cos 45°$

$= 39.24 \times 10^4 \times 0.7071$

Substituting all the values in the equation of F_x.

$\therefore F_x = 1000 \times 2.5 [3.537 + (3.537 \times 0.7071)]$

$+ (39.24 \times 10^4 \times 0.07068) + (39.24 \times 10^4 \times 0.07068 \times 0.7071)$

$F_x = 1000 \times 2.5 [3.537 + (3.537 \times 0.7071)]$

$+ 39.24 \times 10^4 \times 0.07068 [1 + 0.7071]$

$= 1509.4 + 47346$

$F = 48855.4\ N$

Force along y-axis:

$$F_y = \rho Q [V_{1y} - V_{2y}] + (P_1 A_1)_y + (P_2 A_2)_y$$

Where V_{1y} = Initial velocity in y-direction $= 0$

V_{2y} = Final Velocity in y-direction $= V_2 \sin 45^0$

$= 3.537 \times \sin 45^0 = 3.537 \times 0.7071$

$(P_1A_1)_y$ = Pressure force at section (1) in y-direction $= -P_2 \sin 45^0 A_2$

$= -39.24 \times 10^4 \times 0.7071 \times 0.07068$

Substituting all the above values in the equation of F_y.

$$F_y = 1000 \times 2.5[0 - 3.537 \times 0.7071] + 0 - (39.24 \times 10^4 \times 0.7071 \times 0.07068)$$

$$= -[1000 \times 2.5 \times 3.537 \times 0.7071] - [39.24 \times 10^4 \times 0.7071 \times 0.07068]$$

$$= -625.2 - 19611.1$$

$$= -20236.3 \text{ N}$$

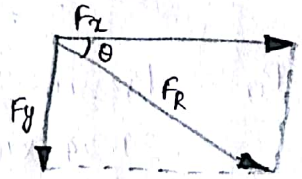
-ve sign means F_y is acting on the downward direction.

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{48855.4^2 + 20236.3^2}$$

$$= 52880.6 \text{ N}$$

The direction of the resultant force F_R, with the x-axis is given as



$$\tan\theta = \frac{F_y}{F_x} = \frac{20236.3}{48855.4} = 0.4142$$

$$\theta = \tan^{-1}(0.4142)$$

$$\theta = 22^\circ 30'$$

3. A 300mm diameter pipe carries water under a head of 20m with a velocity of 3.5 m/s. If the axis of pipe turns through 45°, find the magnitude and direction of the resultant force at the bend.

Sol: Diameter of pipe, $D = D_1 = D_2 = 300\text{mm} = 0.3\text{m}$

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 \text{ m}^2$

Velocity, $V = V_1 = V_2 = 3.5$ m/sec

$\theta = 45^0$ discharge $Q = AV = 0.07068 \times 3.5$
$= 0.2475 \text{ m}^3/\text{s}$

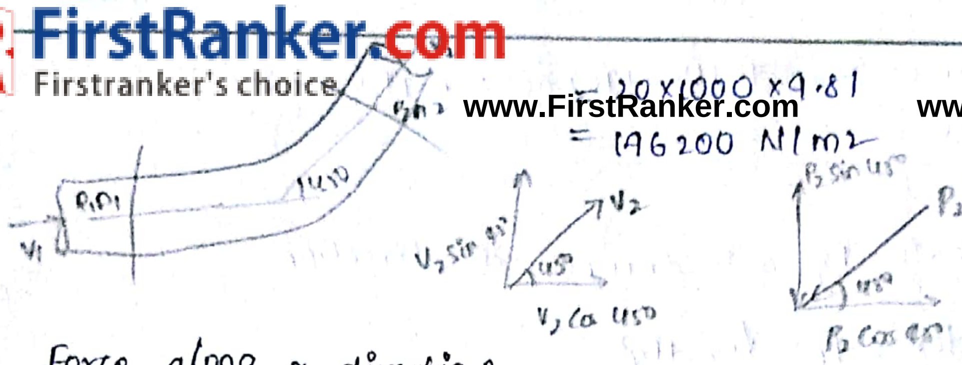
Pressure head = 20m of water

$\frac{P}{\rho g} = 20\text{m}$ of water

Pressure intensity, $P = P_1 = P_2 = 20 \times \rho g$

$= 20 \times 1000 \times 9.81$

$= 196200 \ N/m^2$



<u>Force along x-direction:</u>

$$\boxed{F_x = \rho Q [V_{1x} - V_{2x}] + (P_1 A_1)_x + (P_2 A_2)_x}$$

V_{1x} = Initial velocity in x-direction = 3.5 m/sec

V_{2x} = Final velocity in x-direction = $V_2 \cos 45^\circ = 3.5 \times 0.7071 =$

$(P_1 A_1)_x$ = pressure force at section (1) in x-direction.

$= 196200 \times 0.07068$

$(P_2 A_2)_x$ = Pressure force at section (2) in x-direction.

$= -P_2 \cos 45^\circ A_2$

$= -196200 \times 0.7071 \times 0.07068$.

Substituting all the above values in the equation of F_x

$F_x = 1000 \times 0.2475 [3.5 - (3.5 \times 0.7071)] + (196200 \times 0.07068)$
$- (196200 \times 0.7071 \times 0.07068)$

$= 253.68 + 13871.34 - 9808.04$

$= 4316.98 \ N$

<u>Force along y-direction:</u>

$F_y = \rho Q [V_{1y} - V_{2y}] + (P_1 A_1)_y + (P_2 A_2)_y$

Where, V_{1y} = Initial velocity in y-direction = 0

V_{2y} = Final velocity in y-direction = $V_2 \sin 45^\circ =$

$= 3.5 \times 0.7071$

$(P_1 A_1)_y$ = Pressure force at section (1) in y-direction = 0

$(P_2 A_2)_y$ = Pressure force at section (2) in y-direction

$= -P_2 \sin 45^\circ \times A_2$

$= -196200 \times 0.7071 \times 0.07068$

Substituting all the above values in equation of F_y.

$F_y = 1000 \times 0.2475 [0 - 3.5 \times 0.7071) + 0 - 196200 \times 0.7071 \times 0.07068$

$= -(1000 \times 0.2 \ldots \times 3.5 \times 0.7071) - (196200 \times 0.7071 \times 0.07068)$

$= -612.44 - 9808$

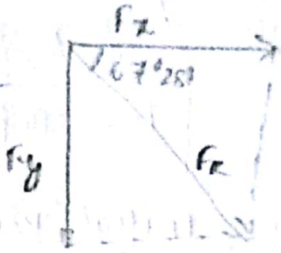
$= -10420.44\ N$

-ve sign indicates F_y acts downward direction.

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{4316.96^2 + 10420.44^2}$

$= 11279\ N$



The angle made by F_R with x-axis

$$\tan\theta = \frac{F_y}{F_x} \Rightarrow \tan\theta = \frac{10420.44}{4316.96} = 2.411$$

$$\theta = \tan^{-1}(2.411)$$
$$= 67^\circ 28'$$

Applications of Impulse - Momentum Equation:

The impulse momentum eqn. is used in the following types of Engineering problems:

1. To determine the resultant force acting on the boundary of flow passage by a stream of fluid as the stream changes its direction, magnitude (or) both.

• Problem of this types are:

i, Pipe bend
ii, Reducers
iii, Moving vanes
iv, Jet propulsion etc.

2. To determine the characteristic of flow when there is an abrupt change of flow section.

• Problems of this type are:

Sudden enlargement in pipe
Hydraulic jump in a channel etc.

CLOSED CONDUIT FLOW

Pipe:

A pipe is a closed conduit which is used for carrying fluids under pressure. Pipes are commonly circular in section. If the pipes are running completely full then we consider flow of fluids through pipes under pressure.

If pipes are partially full in case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure, then the flow of fluid in the pipe is not under pressure.

Example: Flow of water through open channels.

→ The fluid flowing in a pipe is always subjected to resistance due to shear forces between fluid particles and the boundary walls of the pipe and between the fluid particles themselves resulting from the viscosity of the fluid.

The resistance to the fluid flow is in general known as frictional resistance. since certain amount of energy possessed by the flowing fluid will be consumed in overcoming this resistance to the flow, there will be always be some loss of energy in the direction of flow, which however depends upon type of flow i, laminar ii, turbulent

→ **REYNOLDS EXPERIMENT:**

The type of flow is determined from the Reynold number i.e., $\frac{\rho V \times d}{\mu}$. This was demonstrated by O. Reynold in 1883

The apparatus consists of;

(i) A tank containing water at constant head,

(ii) A small tank containing some dye.

(iii) A glass tube having a bell-mouthed entrance at one end and a regulating valve at other end.

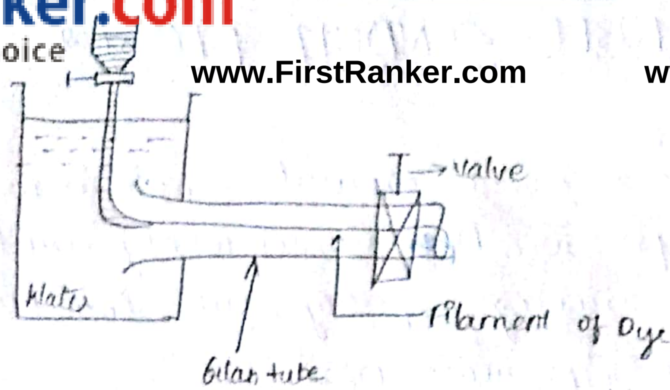
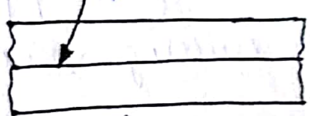


Fig: Reynold apparatus

The water from the tank was allowed to flow through the glass tube. The velocity of flow was varied by the regulating valve. A liquid dye having same specific weight as water was introduced into the glass tube

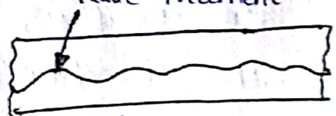
The following observations were made by Reynold:

(i) When the velocity of flow was low, the dye filament in the glass tube was in the form of a straight line. This straight line of dye filament was parallel to the glass tube which was the case of laminar flow.



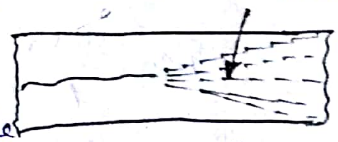
(a) Laminar flow

(ii) With the increase of velocity of flow, the dye-filament was no longer a straight-line but it became a wavy one. This shows that flow is no longer laminar.



Transition

(iii) With further increase of velocity of flow, the wavy dye-filament broke-up and finally diffused in water. This means that the fluid particles of the dye at this higher velocity are moving in random fashion, which shows the case of turbulent flow. Thus in case of turbulent flow the mixing of dye filament and water is intense and flow is irregular, random and disorderly.



Turbulent flow

Hydraulic Gradient and Total Energy Line :-

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes.

Hydraulic Gradient Line:

It is defined as the line which gives the sum of pressure head $(\frac{p}{w})$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (P/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L (Hydraulic Gradient Line)

Total Energy Line:

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L (Total Energy Line)

1) The rate of flow of water pumped into a pipe ABC, which is 200m long, is 20 lit/s. The pump is laid on an upward slope of 1 in 40. The length of the portion AB is 100m and its diameter 100mm, while the length of portion BC is also 100m but its diameter is 200mm. The change of diameter at B is sudden. The flow is taking place from A to C where the pressure at A is $19.62\ N/cm^2$ and end C is connected to a tank. Find the pressure at C and draw the hydraulic gradient and total energy line. Take $f = 0.008$

Sol: Given;

Length of pipe, $ABC = 200\ m$

Discharge, $Q = 20\ lit/sec = 0.02\ m^3/s$.

Slope of pipe $i = 1$ in $40 = \frac{1}{40}$.

Length of pipe $AB = 100\ m$, dia of pipe $AB = 100mm = 0.1m$

Length of pipe $BC = 100m$, dia of pipe $BC = 200\ mm = 0.2m$

Pressure at A, $P_A = 19.62\ N/cm^2 = 19.62 \times 10^4\ N/m^2$

Coefficient of friction, $f = 0.008$

Velocity of water in pipe AB, $V_1 = \frac{\text{discharge}}{\text{Area of AB}}$

$$= \frac{0.02}{\frac{\pi}{4}(0.1)^2} = 2.54 \text{ m/sec}$$

Velocity of water in pipe BC, $V_2 = \frac{\text{discharge}}{\text{Area of BC}}$

$$= \frac{0.02}{\frac{\pi}{4}(0.2)^2} = 0.63 \text{ m/sec}$$

Applying Bernoulli's eqn. to points A and C,

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{total loss from A to C} \longrightarrow (i)$$

Total loss from A to C = loss due to friction in pipe AB + loss of head due to enlargement at B + loss of head due to friction in BC $\longrightarrow$ (ii)

Loss of head due to friction in AB,

$$h_{f_1} = \frac{4fLV^2}{d \times 2g} = \frac{4 \times 0.008 \times 100 \times (2.54)^2}{0.1 \times 2 \times 9.81}$$

$$= 10.52 \text{m}$$

Loss of head due to friction in pipe BC,

$$h_{f_2} = \frac{4fLV^2}{d \times 2g} = \frac{4 \times 0.008 \times 100\,(0.63)^2}{0.2 \times 2 \times 9.81} = 0.323 \text{m}$$

Loss of head due to enlargement at B,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.54 - 0.63)^2}{2 \times 9.81} = 0.186 \text{ m}$$

Total loss from A to C $= h_{f_1} + h_e + h_{f_2}$

$$= 10.52 + 0.186 + 0.323$$

$$= 11.029 \simeq 11.03 \text{m}$$

Substituting this value in eqn. (i)

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + z_c + 11.03 \text{ ———— (iii)}$$

Taking datum line passing through A, we have

$$z_A = 0$$

$$z_c = \frac{1}{40} \times \text{total length of pipe}$$

$$= \frac{1}{40} \times 200 = 5\text{m}$$

$$P_A = 19.62 \times 10^4 \text{ N/m}^2$$

$$V_A = V_1 = 2.54 \text{ m/s}, \quad V_c = V_2 = 0.63 \text{ m/s}$$

... values in eqn iii.

$$\frac{19.62\times10^4}{1000\times9.81}+\frac{2.54^2}{2\times9.81}+0=\frac{P_c}{\rho g}+\frac{0.63^2}{2\times9.81}+5+11.03$$

$$20+0.328=\frac{P_c}{\rho g}+0.02+5+11.03$$

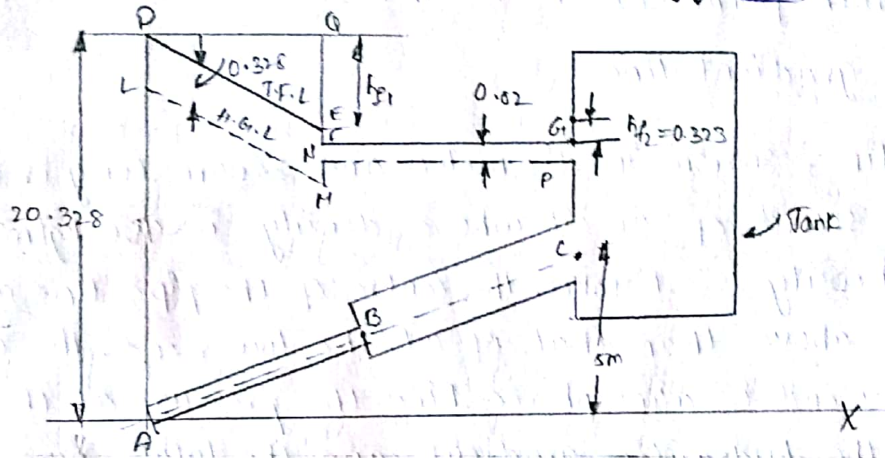
$$20.328=\frac{P_c}{\rho g}+16.05$$

$$\frac{P_c}{\rho g}=20.328-16.05=4.278\,m$$

$$P_c=4.278\times1000\times9.81\ N/m^2$$

$$=4.278\times9.81/10\ N/cm^2=4.196\ N/cm^2$$

Hydraulic gradient and total Energy line:



Pipe AB: Assuming the datum line passing through A, then total energy at A

$$=\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+Z_A$$

$$=\frac{19.62\times10^4}{1000\times9.81}+\frac{(2.54)^2}{2\times9.81}+0$$

$$=20+0.328=20.328m$$

Total Energy at B = Total Energy at A − h_{f_1}

$$=20.328-10.52$$

$$=9.808m$$

Also $\frac{V_c^2}{2g}=\frac{(0.63)^2}{2\times9.81}=0.02$

Total Energy Line: Draw a horizontal line AX as shown in fig. The centre line of the pipe is drawn in such a way that the slope of pipe is 1 in 40. Thus the point C will be at a height of $\frac{1}{40}\times200=5m$ from the line AX. Now draw a vertical line AD equal to energy at A. i.e., AD = 20.328m. From point D draw a horizontal line and point B, a vertical line, meeting at Q. From Q take vertical distance $QE = h_{f_1} = 10.52m$. Join DE. From E, take $EF = h_e = 0.166m$. From F, draw a horizontal line and from C, a vertical line meeting at R. From R take $RG = h_{f_2} = 0.323m$. Join F and G. Then DEFG represents the total energy line.

<u>Hydraulic Gradient line</u>: Draw the line LM parallel to the line DE at a distance in the downward directions equal to 0.328m. Also draw the line PN parallel to the line GF at a distance of $\frac{V_c^2}{2g} = 0.02$. Join points M and N. Then the line LMNP represents the hydraulic gradient line.

2) A pipe line, 300mm in diameter and 3200m long is used to pump up 50kg per second of an oil whose density is 950 kg/m³ and whose kinematic viscosity is 2.1 stokes. The centre of the pipe line at the upper end is 40m above than that of at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.

<u>Sol</u>: Given:

Diameter of pipe, $d = 300mm = 0.3m$

length of pipe, $L = 3200m$

Mass, $M = 50\ kg/s = \rho q$

$\therefore$ Discharge, $Q = \frac{50}{\rho} = \frac{50}{950} = 0.0526\ m^3/s$

$\because$ density, $\rho = 950\ kg/m^3$

kinematic viscosity, $\nu = 2.1$ stokes $= 2.1\ cm^2/s$

$= 2.1 \times 10^{-4}\ m^2/s$

Height of upper end = 40m

pressure at upper end = atmospheric = 0

Reynold number, $R_e = \frac{V \times d}{\nu}$

where $V = \frac{\text{discharge}}{\text{Area}} = \frac{0.0526}{\frac{\pi}{4}(0.3)^2} = 0.744\ m/s$

2.1×10^{-4} $= 1062.8$

$\therefore$ Co-efficient of friction, $f = \frac{16}{Re} = \frac{16}{1062.8} = 0.015$

Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$

$$= \frac{4 \times 0.015 \times 3200 \times (0.744)^2}{0.3 \times 2 \times 9.81} = 18.05 \text{ m of oil}$$

Applying the Bernoulli's eqn. at the lower and upper end of the pipe and taking datum line passing through the lower end, we have,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \quad \text{———(1)}$$

But $z_1 = 0$; $z_2 = 40$m, $V_1 = V_2$ as diameter is same

$P_2 = 0$ $h_f = 18.05$m

Substituting the values in eqn.(1)

$\frac{P_1}{\rho g} = 40 + 18.05 = 58.05$ m of oil

$P_1 = 58.05 \times 950 \times 9.81$

$= 540997$ N/m²

$= 540997/10^4$ N/cm²

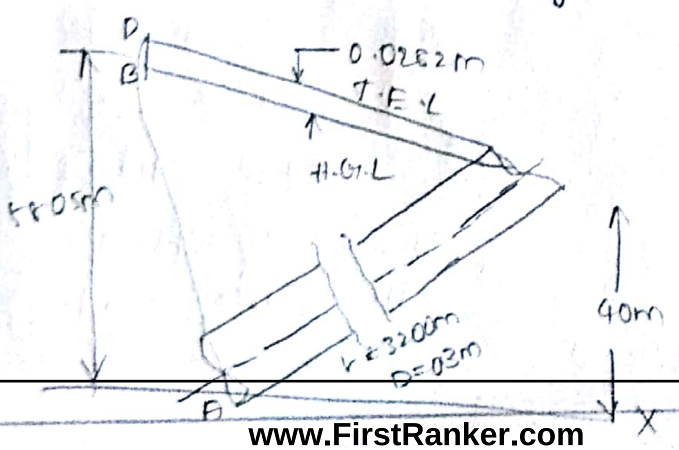
$= 54.099$ N/cm² $[\because \rho \text{ for oil} = 950]$

H.G.L and T.E.L

$$\frac{V^2}{2g} = \frac{(0.744)^2}{2 \times 9.81}$$

$$= 0.0282 \text{m}$$

$\frac{P_1}{\rho g} = 58.05$m of oil $-$ $\frac{P_2}{\rho g} = 0$



...rizontal line AX as shown in fig. From A draw the

centre line of the pipe in such a way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05 m. Join B with C. Then BC is the hydraulic gradient line.

Draw a line DE parallel to BC at a height of 0.0782m above the hydraulic gradient line. Then DE is the total Energy line.

Reynold observed that loss of head is approximately proportional to the square of velocity. More exactly the loss of head, $h_f \propto V^n$, where n varies from 1.75 to 2.0.

LOSS OF ENERGY IN PIPES:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as

Energy losses

1. Major Energy losses

This is due to friction and it is calculated by the following formulae:

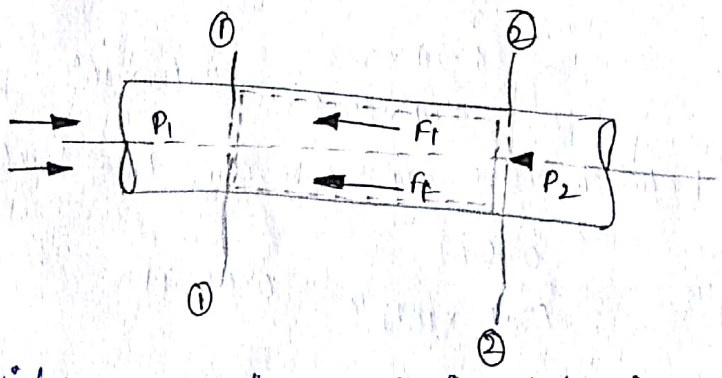
(a) Darcy – Weisbach Formula
(b) Chezy's formula

2. Minor Energy losses

This is due to
(a) Sudden expansion of pipe.
(b) Sudden contraction of pipe.
(c) Bend in pipe.
(d) Pipe fittings etc
(e) An obstruction in pipe.

LOSS OF ENERGY (OR HEAD) DUE TO FRICTION:

(a) Darcy – Weisbach Formula:



Consider a uniform horizontal pipe, having steady flow as shown in figure.

Let 1-1 and 2-2 are two sections of pipe

let P_1 = Pressure intensity at section 1-1.

V_1 = Velocity of flow at section 1-1.

L = length of the pipe between sections 1-1 & 2-2

d = diameter of the pipe.

f' = frictional resistance per unit wetted area per unit velocity.

The loss of head (or energy) in pipes due to friction is calculated from Darcy-weisbach equation.

$$\boxed{h_f = \frac{4fLV^2}{2gd}}$$

Where, h_f = loss of head due to friction.

f = co-efficient of friction which is a function of Reynold-number.

$= \frac{16}{R_e}$ for $R_e < 2000$ viscous flow

$= \frac{0.079}{Re^{1/4}}$ for Re varying from 4000 to 10^6

L = Length of the pipe.

V = mean velocity of flow.

d = diameter of the pipe.

(b) Chezy's Formula for loss of head due to friction in pipes

We know the equation,

$$\boxed{h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2}$$

Where h_f = loss of head due to friction.

P = Wetted perimeter of pipe.

A = area of cross-section of pipe

L = length of pipe.

V = mean velocity of flow.

Now the ratio of $\frac{A}{P} = \left(\frac{\text{Area of flow}}{\text{perimeter (wetted)}}\right)$ is called hydraulic mean depth (or) hydraulic radius and is denoted by m.

Hydraulic mean depth, $m = \frac{A}{P} = \frac{\pi/4\, d^2}{\pi d} = \frac{d}{4}$

substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (1)

$$h_f = \frac{f'}{\rho g} \cdot \frac{1}{m} \cdot L \times V^2$$

$$V^2 = h_f \times \frac{\rho g}{f'} \cdot m \cdot \frac{1}{L}$$

The forces acting on the fluid between sections 1-1 and 2-2 are;

1. pressure force at section 1-1 $= P_1 \times A$

where A = Area of pipe

2. Pressure force at section 2-2 $= P_2 \times A$
3. Frictional force F_1

Resolving all the forces in the horizontal direction.

$$P_1A_1 - P_2A_2 - F_1 = 0$$

$$(P_1 - P_2)A = F_1$$

$$P_1 - P_2 = \frac{F_1}{A}$$

$$P_1 - P_2 = \frac{f' P L V^2}{A}$$

But from eqn (i) $P_1 - P_2 = \rho g h_f$

$$\rho g h_f = \frac{f' P L V^2}{A} \rightarrow \boxed{h_f = \frac{f'}{\rho g}\frac{P}{A} L V^2} \quad \text{———} (3)$$

In eqn. (3) $\frac{P}{A} = \frac{\text{Wetted Perimeter}}{\text{area}}$

$$= \frac{\pi d}{\frac{\pi}{4}d^2} = 4/d$$

Substitute the value of $\frac{P}{A}$ in eqn (3)

$$\boxed{h_f = \frac{f'}{\rho g} \cdot \frac{4}{d} \cdot L V^2} \quad \text{———} (4)$$

Putting $\frac{f'}{\rho g} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (4) becomes as $h_f = \frac{f}{2g} \cdot \frac{4}{d} L V^2$

$$\boxed{h_f = \frac{4 f L V^2}{2gd}}$$

The above eqn. is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes. Sometimes the Darcy-Weisbach eqn. is written as

$$\boxed{h_f = \frac{f \cdot L \cdot V^2}{2gd}}$$

Then f is known as friction factor.

- Friction factor f is not constant. It depends on roughness condition of pipe surface and Reynolds number of the flow.

h_f = loss of head due to friction.
and P_2, V_2 = Values of pressure intensity and velocity at Section 2-2.

Applying Bernoulli's eqn. at sections 1-1 and 2-2.
Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal.
$V_1 = V_2$ as diameter of pipe is same at 1-1 and 2-2.

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$\boxed{h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g}} \quad \text{———— (1)}$$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now,
frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity².

$F_1 = f' \times \pi d L \times V^2$
$= f' P L V^2$

[∵ Wetted area $= \pi d L$
Velocity $V = V_1 = V_2$
P = perimeter $= \pi d$]

$$\boxed{\therefore F_1 = f' P L V^2} \quad \text{———— (2)}$$

4. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300mm at the rate of 500 lit/sec. Find the head lost due to friction and power required to maintain the flow for a length of 1000m. Take ν=0.29 stokes.

Sol: Given:

Specific gravity of oil, $s = 0.7$
diameter of pipe, $d = 300\text{mm} = 0.3\text{m}$
discharge, $Q = 500$ litres/sec.
length of pipe, $L = 1000\text{m}$

Area $= \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073\ m/s$

$\therefore$ Reynold number, $R_e = \frac{V \times d}{\nu}$

$= \frac{7.073 \times 0.3}{0.29 \times 10^{-4}}$

$= 7.316 \times (10)^4$

Coefficient of friction, $f = \frac{0.79}{Re^{1/4}} = \frac{0.79}{(7.316 \times 10^4)^{1/4}}$

$= 0.0048$.

$\therefore$ Head lost due to friction,

$h_f = \frac{4 f L V^2}{d \times 2g} = \frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81}$

$= 163.18\ m$

Power required $= \frac{\rho g \cdot Q \cdot h_f}{1000}$ kW

4. An oil of specific gravity 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 lit/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take $\nu = 0.29$ stokes.

Sol: Given:

Specific gravity of oil, $s = 0.7$

diameter of pipe, $d = 300mm = 0.3m$

discharge, $Q = 500$ lit/sec $= 0.5\ m^3/sec$

Length of pipe, $L = 1000m$

Velocity, $V = \frac{Q}{Area} = \frac{0.5}{\pi/4\, d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2}$

$= 7.073\ m/s$.

$\therefore$ Reynold number, $R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}}$

$= 7.316 \times (10)^4$

coefficient of friction, $f = \frac{0.79}{Re^{1/4}}$

$= \frac{0.79}{(7.316 \times 10^4)^{1/4}}$

$= 0.0048$

Head lost due to friction, $h_f = \dfrac{4fLV^2}{d \times 2g}$

$$= \frac{4 \times 0.0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81}$$

$= 163.18\ m$

power required $= \dfrac{\rho g \cdot Q \cdot h_f}{1000}$ kW

where ρ = density of oil $= 0.7 \times 1000 = 700\ kg/m^3$

$\therefore$ power required $= \dfrac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$

$= \underline{560.28\ kW}$

5. Calculate the discharge through a pipe of diameter 200mm when the difference of pressure head between the two ends of a pipe 500m apart is 4m of formula, $h_f = \dfrac{4fLV^2}{d \times 2g}$.

Sol: Given,

Diameter of pipe, $d = 200mm = 0.2m$

Length of pipe, $L = 500m$

Difference of pressure head, $h_f = 4m$ of water

$f = 0.009$

$$h_f = \frac{4fLV^2}{d \times 2g}$$

$$4 = \frac{4 \times 0.009 \times 500 \times V^2}{0.2 \times 2 \times 9.81}$$

$$V^2 = \frac{4 \times 0.2 \times 2 \times 9.81}{4 \times 0.009 \times 500}$$

$V^2 = 0.872$

$V = \sqrt{0.872} = 0.9338 \simeq 0.934\ m/s$

$V = 0.934\ m/s$

$\therefore$ Discharge, Q = Area $\times$ velocity

$= \dfrac{\pi}{4} d^2 \times V$

$= \dfrac{\pi}{4} (0.2)^2 \times 0.934$

$= 0.0293\ m^3/s$

$= 29.3$ lit/s.

6. Water is flowing through a pipe of diameter 200mm with a velocity of 3m/s. Find the head lost due to friction for a length of 5m if the co-efficient of friction is given by $f = 0.002 + \frac{0.09}{Re^{0.3}}$ where Re = Reynold number. The kinematic viscosity of water = 0.01 stoke.

Sol: Given:

Diameter of pipe, $d = 200mm = 0.2m$

Velocity, $V = 3 m/sec$

length of the pipe, $L = 5m$

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01 \times 10^{-4} m^2/sec$

$\therefore$ Reynold number, $Re = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{0.01 \times 10^{-4}}$

$= 6 \times 10^5$

Value of $f = 0.002 + \frac{0.09}{Re^{0.3}}$

$= 0.002 + \frac{0.09}{(6 \times 10^5)^{0.3}}$

$= 0.002 + \frac{0.09}{54.13}$

$= 0.002 + 0.00166$

$= 0.00366$

$\therefore$ Head lost due to friction,

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.02166 \times 5.0 \times 3^2}{2 \times 9.81 \times 0.2}$$

$= 0.1678$ m of water

7. An oil of specific gravity 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200mm at the rate of 600 lit/s. Find the head lost due to friction for a 500m length of pipe. Find the power required to maintain this flow.

Sol: Given: Specific gravity of oil, $s = 0.9$

viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10} N\text{-}s/m^2$

diameter of pipe, $d = 200mm = 0.2m$

Discharge $Q = 60$ litres/s $= 0.06 m^3/sec$.

Length of pipe, L = 500m

Density, $\rho = 0.9 \times 1000 = 900\ kg/m^3$

$\therefore$ Reynold number, $R_e = \frac{\rho V d}{\mu} = \frac{900 \times V \times 0.2}{0.06/10}$

where, $V = \frac{Q}{Area} = \frac{0.6}{\frac{\pi}{4}(0.2)^2} = 1.909\ m/s \simeq 1.91\ m/s$

$$R_e = \frac{900 \times 1.91 \times 0.2}{0.06/10}$$

$$= 57300$$

R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}}$$

$$= \frac{0.079}{(57300)^{0.25}} = 0.0051$$

Head lost due to friction, $h_f = \frac{4fLV^2}{2gd}$

$$= \frac{4 \times 0.0051 \times 500 \times 1.91^2}{2 \times 9.81 \times 0.2}$$

$$= 9.48\ m \text{ of water}$$

$\therefore$ Power required $= \frac{\rho g Q h_f}{1000}$

$$= \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000}$$

$$= 5.02\ kW$$

MINOR ENERGY (HEAD) LOSSES:

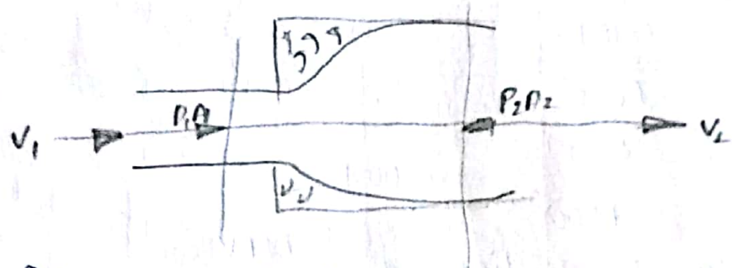
The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases:

1. Loss of head due to sudden enlargement.
2. Loss of head due to sudden contraction.
3. Loss of head at the entrance of a pipe.

4. loss of head at the exit of a pipe.
5. loss of head due to an obstruction in a pipe.
6. loss of head due to bend in the pipe.
7. loss of head in various pipe fittings

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in with the loss of head due to friction.

Loss of head due to sudden enlargement:



Consider a liquid flowing through a pipe which has sudden enlargement as shown in fig. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

Let P_1 = Pressure intensity at section (1) – (1)

V_1 = Velocity of flow at section 1-1

A_1 = area of pipe at section 1-1

P_2, V_2, A_2 = Corresponding values at section 2-2.

Due to sudden change of diameter of pipe from D_1 to D_2 the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown. The loss of head (or energy) takes place due to the formation of the eddies.

Let P' = Pressure intensity of the liquid eddies on the area $(A_2 - A_1)$.

h_e = loss of head due to sudden enlargement.

Applying Bernoulli's eqn. to section 1-1 & 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\boxed{h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)} \longrightarrow (1)$$

Consider the control volume of liquid between section 1-1 and 2-2, Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = P_1 A_1 + P'(A_2 - A_1) - P_2 A_2$$

But experimentally it is found that $P_1 = P'$

$$F_x = P_1 A_1 + P_1 (A_2 - A_1) - P_2 A_2$$

$$F_x = P_1 A_1 + P_1 A_2 - P_1 A_1 - P_2 A_2$$

$$F_x = P_1 A_2 - P_2 A_2$$

$$\boxed{F_x = (P_1 - P_2) A_2} \longrightarrow (2)$$

Momentum of liquid/sec at section 1-1 = mass × velocity

$$= \rho A_1 \times V_1 V_1$$

$$= \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 $= \rho A_2 V_2 \times V_2$

$$= \rho A_2 V_2^2$$

Change of momentum/sec $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

But from continuity equation, we have

$$A_1 V_1 = A_2 V_2 \text{ (or) } A_1 = \frac{A_2 V_2}{V_1}$$

∴ Change of momentum/sec $= \rho A_2 V_2^2 - \rho \frac{A_2 V_2}{V_1} V_1^2$

$$= \rho A_2 V_2^2 - \rho A_2 V_2 V_1$$

$$= \rho A_2 [V_2^2 - V_2 V_1] \longrightarrow (3)$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating (2) & (3) eqn.s

$$(P_1 - P_2) A_2 = \rho A_2 [V_2^2 - V_2 V_1]$$

$$P_1 - P_2 = \rho (V_2^2 - V_2 V_1)$$

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_2 V_1$$

Dividing by "g" on both sides, we have

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_2 V_1}{g}$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_2 V_1}{g}$$

Substituting the value of $\frac{P_1}{\rho g} - \frac{P_2}{\rho g}$ in equation (1)

$$h_e = \frac{V_2^2 - V_2 V_1}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

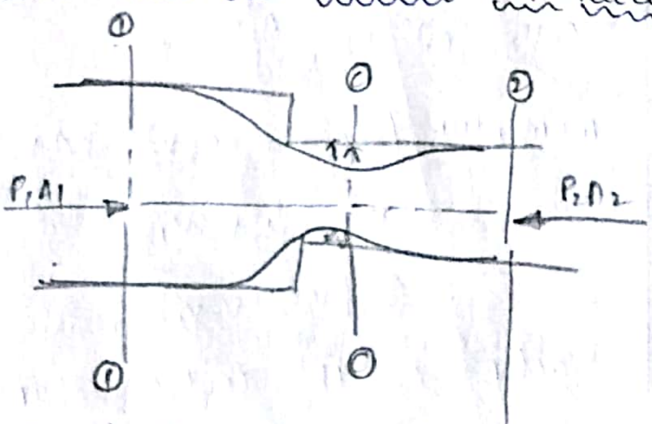
$$= \frac{V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 - 2V_1V_2 + V_1^2}{2g_2}$$

$$= \frac{(V_2 - V_1)^2}{2g}$$

$$\therefore \boxed{h_e = \frac{(V_2 - V_1)^2}{2g}}$$

Loss of head due to sudden contraction:



Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in fig. Consider two sections 1-1 and 2-2 before and after contraction. As the liquids flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes min. at section c-c as shown in fig. 3. This section c-c called vena-contracta. After section c-c a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

Let A_c = Area of flow at section c-c
V_c = Velocity of flow at section c-c
A_2 = Area of flow at section 2-2
V_2 = Velocity of flow at section 2-2
h_c = loss of head due to sudden contraction.

Now, h_c = actually loss of head due to enlargement from section c-c to section 2-2 and is given by

$$\boxed{h_c = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2} \quad \text{—— i,}$$

From continuity equation, we have

$$A_c V_c = A_2 V_2 \Rightarrow \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c} \quad \left[\because V_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in eqn. i,

$$\boxed{h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

$$= k \cdot \frac{V_2^2}{2g} \quad \text{where } k = \left[\frac{1}{C_c} - 1\right]$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Then h_c becomes as $\boxed{h_c = \frac{kV_2^2}{2g} = 0.375 \frac{V_2^2}{2g}}$

If the value of C_c is not given then the head loss due to sudden contraction is taken as $0.5 \frac{V_2^2}{2g}$

$$\boxed{h_c = 0.5 \frac{V_2^2}{2g}}$$

Loss of head at the entrance of pipe:

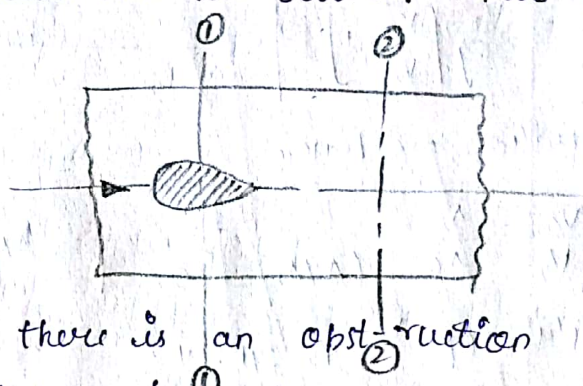
This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at entrance is taken

$= 0.5 \frac{V^2}{2g}$ where V = velocity of liquid in pipe.

This loss is denoted by h_i.

$$\boxed{h_i = 0.5 \frac{V^2}{2g}}$$

Loss of head at the end of pipe:



Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in fig.

Consider a pipe of area of cross-section A having an obstruction.

Let a = Maximum area of obstruction,

A = Area of pipe.

V = Velocity of liquid in pipe.

Then $(A-a)$ = Area of flow of liquid at section 1-1

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

Let V_c = Velocity of liquid at vena-contracta.

Then loss of head due to obstruction $\Big\}$ = $\Big\{$ loss of head due to enlargement from vena-contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \quad \text{——— (i)}$$

From continuity equation, we have

$$\boxed{a_c V_c = A \times V} \quad \text{——— (ii)}$$

Where a_c = area of cross-section at vena-contracta.

If C_c = coefficient of contraction.

$$C_c = \frac{\text{area at vena-contracta}}{(A - a)}$$

$$C_c = \frac{a_c}{(A-a)}$$

$$\boxed{a_c = C_c \times (A - a)}$$

Substituting the value of a_c in eqn. (ii)

$$C_c \times (A - a) V_c = A \times V$$

$$\boxed{V_c = \frac{A \times V}{C_c (A - a)}}$$

Substituting the value of V_c in equation (i), we get

$$\boxed{\begin{aligned} &\text{Head loss due to obstruction} \\ &= \frac{(V_c - V)^2}{2g} = \left(\frac{AV}{C_c(A-a)} - V\right)^2 \Big/ 2g \\ &= \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2 \end{aligned}}$$

→ **Loss of Head due to Bend in pipe:**

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$\boxed{h_b = \frac{KV^2}{2g}}$$

where, h_b = loss of head due to bend

V = velocity of flow.

K = coefficient of bend.

The value of K depends on

i, Angle of bend

ii, Radius of curvature of bend.

iii, Diameter of pipe.

Loss of Head in various Pipe Fittings:

The loss of head in the various pipe fittings such as valves, coupling etc is expressed as

$$= \frac{KV^2}{2g}$$

where V = velocity of flow

k = coefficient of pipe fitting.

1) Find the loss of head when a pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 litres/sec

Sol: Diameter of small pipe, D_1 = 200mm = 0.2m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141\ m^2$

Diameter of large pipe, D_2 = 400mm = 0.4m

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.4)^2 = 0.12564\ m^2$

Discharge, Q = 250 litres/sec

$= 0.25\ m^3/sec$

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96\ m/s$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12564} = 1.99\ m/s$

Loss of head due to sudden expansion is given by,

$$h_e = \frac{(V_1-V_2)^2}{2g}$$

$$= \frac{(7.96-1.99)^2}{2\times 9.81}$$

$$= 1.816 \text{ m of water}$$

2) At a sudden enlargement of water main from 240mm to 480mm diameter the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Sol: Given

Diameter of smaller pipe, $D_1 = 240\text{mm} = 0.24\text{m}$

area, $A_1 = \frac{\pi}{4}(0.24)^2 =$

Diameter of larger pipe, $D_2 = 480\text{mm} = 0.48\text{m}$

area, $A_2 = \frac{\pi}{4}(0.48)^2$

Rise of hydraulic gradient, $\left[z_2 + \frac{P_2}{\rho g}\right] - \left[z_1 + \frac{P_1}{\rho g}\right] = 10\text{mm} = \frac{1}{100}\text{m}.$

Let the rate of flow $= Q$

Applying Bernoulli's equation theorem to both sections.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \longrightarrow (1)$$

But head loss due to enlargement is given by,

$$h_e = \frac{(V_1-V_2)^2}{2g} \quad \text{———} \quad (2)$$

From continuity equation we have,

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2V_1}{A_1} = \frac{\frac{\pi}{4}D_2^2 V_2}{\frac{\pi}{4}D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$= \left(\frac{0.48}{0.24}\right)^2 V_2$$

$$V_1 = 4V_2$$

Substituting the value of V_1 in eqn. (2)

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the values of V_1 and h_e in eqn(1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

$$\frac{6V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

$$\frac{3V_2^2}{2g} = \left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right]$$

But hydraulic gradient rise, $\left[\frac{P_2}{\rho g} + z_2\right] - \left[\frac{P_1}{\rho g} + z_1\right] = \frac{1}{100}$

$$\frac{3V_2^2}{2g} = \frac{1}{100}$$

$$V_2^2 = \frac{2g}{300}$$

$$V_2 = \sqrt{\frac{2 \times 9.81}{300}} = 0.1808 \simeq 0.181 \text{ m/s}$$

Discharge, $Q = A_2 V_2$

$$= \frac{\pi}{4}(0.48)^2 (0.181)$$

$$= 0.03275 \text{ m}^3/\text{sec}$$

$$= 32.75 \text{ litres/sec}$$

3) The rate of flow of water through a horizontal pipe is $0.25 \text{ m}^3/\text{s}$. The diameter of the pipe which is 200mm is suddenly enlarged to 400mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine.

(i) loss of head due to sudden enlargement.
(ii) Pressure intensity in the large pipe.
(iii) power lost due to enlargement.

Sol: Given;

Discharge, $Q = 0.25 \text{ m}^3/\text{s}$

Diameter of smaller pipe, $D_1 = 200\text{mm} = 0.2\text{m}$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03141 m^2$

Diameter of large pipe, $D_2 = 400mm = 0.4 m$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.4)^2 = 0.12566 m^2$

pressure in smaller pipe, $P_1 = 11.772 N/cm^2$
$= 11.772 \times 10^4 N/m^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{0.03141} = 7.96 m/s.$

velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{0.12566} = 1.99 m/s.$

(i) Loss of head due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(7.96 - 1.99)^2}{2 \times 9.81}$$

$$= 1.816 m$$

(ii) Let the pressure intensity in larger pipe $= P_2$.

The applying Bernoulli's eqn. before and after sudden elargement,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$\frac{P_2}{\rho g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$\frac{P_2}{1000 \times 9.81} = 12.0 + 3.229 - 0.2018 - 1.8160$$

$$\frac{P_2}{9810} = 15.229 - 2.0178 = 13.21 m \text{ of water}$$

$$P_2 = 13.21 \times 9810 = 12.96 \times 10^4 N/m^2 \Rightarrow 12.96 N/cm^2$$

(iii) power lost due to sudden enlargement,

$$P = \frac{\rho g Q h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = 4.453 kW$$

... pipe of diameter 500mm is suddenly contracted to a diameter of 250mm. The pressure intensities in the larger and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine the rate of flow of water.

Sol: Given :

Diameter of large pipe, $D_1 = 500mm = 0.5m$

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.5)^2 = 0.1963 \, m^2$

Diameter of smaller pipe, $D_2 = 250mm = 0.25 \, m$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.25)^2 = 0.04908 \, m^2$

Pressure in large pipe, $P_1 = 13.734 \, N/cm^2$
$= 13.734 \times 10^4 \, N/m^2$

Pressure in smaller pipe, $P_2 = 11.772 \, N/cm^2$
$= 11.772 \times 10^4 \, N/m^2$.

Head lost due to contraction, $h_c = \frac{V_2^2}{2g} \left[\frac{1}{C_c} - 1\right]^2$

$$= \frac{V_2^2}{2g} \left[\frac{1}{0.62} - 1\right]^2$$

$$\boxed{h_c = \frac{V_2^2}{2g} \, 0.375}$$

From continuity eqn, we have

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2 V_2}{A_1}$$

$$V_1 = \frac{\frac{\pi}{4} D_2^2 V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$V_1 = \left(\frac{0.25}{0.5}\right)^2 V_2$$

$$\boxed{V_1 = \frac{1}{4} V_2}$$

Applying Bernoulli's eqn. before and after contraction,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{\rho g} + z_2 + h_c$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c \quad \text{---(1)}$$

substitute the values of h_c and V_1 in eqn. (1)

$$\frac{13.734 \times 10^4}{1000 \times 9.81} + \frac{(1/4 V_2)^2}{2 \times 9.81} = \frac{11.772 \times 10^4}{2 \times 9.81} + \frac{V_2^2}{2g} + 0.375 \frac{V_2^2}{2g}$$

$$14.0 + \frac{V_2^2}{16 \times 2 \times 9.81} = 12.0 + 1.375 \times \frac{V_2^2}{2 \times 9.81}$$

$$14 - 12 = \frac{V_2^2}{2 \times 9.81}\left[1.375 - \frac{1}{16}\right]$$

$$2 = 1.3125 \frac{V_2^2}{2 \times 9.81}$$

$$V_2^2 = \frac{2 \times 2 \times 9.81}{1.3125}$$

$$V_2 = \sqrt{\frac{2 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s}$$

(i) loss of due to sudden contraction.

$$h_c = 0.375 \frac{V_2^2}{2g}$$

$$= 0.375 \times \frac{(5.467)^2}{2 \times 9.81}$$

$$= 0.571 \text{ m}$$

(ii) Rate of flow of water $= A_2 V_2$

$$= \frac{\pi}{4}(0.5)^2 (0.571)$$

$$= 0.04908 \times 0.571$$

$$= 0.2683 \text{ m}^3/\text{s}$$

$$= 268.3 \text{ litres/sec}$$

6. If in the above problem, the rate of flow of water is 300 lit/sec, other data remaining the same, find the value of co-efficient of contraction.

Sol: Given:

$D_1 = 0.5$ m $P_1 = 13.734 \times 10^4$ N/m² $Q = 300$ lit/s

$D_2 = 0.25$ m $P_2 = 11.772 \times 10^4$ N/m² $= 0.3$ m³/s

$$V_1 = \frac{V_2}{4} \quad \text{where} \quad V_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi}{4}(0.5)^2} = 1.528 \text{ m/s}$$

$$V_2 = 4V_1$$

$= 4 \times 1.528 = 6.112\ m/s$

From Bernoulli's eqn. we have,

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

$$\frac{13.734 \times 10^4}{1000 \times 9.81} + \frac{1.528^2}{2 \times 9.81} = \frac{11.772}{1000 \times 9.81} + \frac{6.112^2}{2 \times 9.81} + h_c$$

$$14.0 + 0.119 = 12.0 + 1.904 + h_c$$

$$14.119 = 13.904 + h_c$$

$$h_c = 14.119 - 13.904 = 0.215$$

We know that loss of heat due to sudden contraction,

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$0.215 = \frac{(6.112)^2}{2 \times 9.81}\left[\frac{1}{C_c} - 1\right]^2$$

$$\left[\frac{1}{C_c} - 1\right]^2 = \frac{0.215 \times 2 \times 9.81}{(6.112)^2}$$

$$\left[\frac{1}{C_c} - 1\right]^2 = 0.1129$$

$$\frac{1}{C_c} - 1 = \sqrt{0.1129} = 0.336$$

$$\frac{1}{C_c} = 1 + 0.336$$

$$\frac{1}{C_c} = 1.336 \Rightarrow C_c = \frac{1}{1.336} = \underline{0.748}$$

Coefficient of contraction, $C_c = 0.748$.

7) A 150mm diameter pipe reduces in diameter abruptly to 100mm diameter. If the pipe carries water at 30 lit/sec, calculate the pressure loss across the contraction. Take the coefficient of contraction as 0.6.

Sol: Diameter of larger pipe, $D_1 = 150mm = 0.15\ m$

Diameter of smaller pipe, $D_2 = 100\ mm = 0.1\ m$

$\therefore$ area, $A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$

area, $A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ m^2$

Discharge, $Q = 30$ lit/sec

$= 0.03\ m^3/sec$

coefficient of contraction, $C_c = 0.6$

From continuity eqn. $A_1V_1 = A_2V_2 = Q$

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82 \text{ m/sec}$$

Applying Bernoulli's eqn. before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_c \text{ ——— (i)} \qquad (\because z_1 = z_2)$$

h_c = loss of head due to sudden contraction.

$$= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= \frac{(3.82)^2}{2\times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

substituting all the values in eqn (i)

$$\frac{P_1}{\rho g} + \frac{(1.697)^2}{2\times 9.81} = \frac{P_2}{\rho g} \times \frac{(3.82)^2}{2\times 9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271 \text{ m}$$

$$P_1 - P_2 = 0.9271 \times 9.81 \times 1 \text{ N/m}^2$$

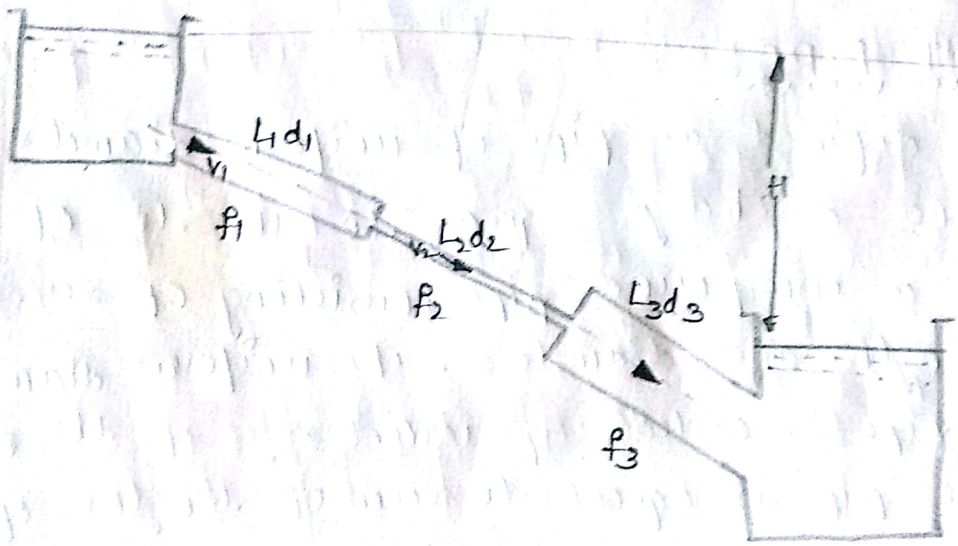
$$= 0.909 \times 10^4 \text{ N/m}^2$$

$$= 0.909 \text{ N/cm}^2$$

Pressure loss across contraction, $(P_1 - P_2) = 0.909$ N/cm^2.

FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES :

pipes in series (or) compound pipes, is defined as the pipe of different lengths and different diameters connected end to end (in series) to form a pipe line.



Let L_1, L_2, L_3 = length of pipes 1, 2, and 3 respectively.

d_1, d_2, d_3 = diameter of pipes 1, 2, and 3 respectively.

V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3

f_1, f_2, f_3 = coefficient of frictions for pipes 1, 2, 3.

H = difference of water level in the two tanks.

The discharge passing through each pipe is same.

$$Q = A_1V_1 = A_2V_2 = A_3V_3.$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\boxed{H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}}$$

If minor losses are neglected then eqn. (i) becomes as ——— (i)

$$\boxed{H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}} \quad \text{——— (ii)}$$

If the co-efficient of friction is same for all pipes $f_1 = f_2 = f_3 = f$ then eqn. (ii) becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$\boxed{H = \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]}$$

→ **EQUIVALENT PIPE:**

This is defined as pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1 d_1 = dia of pipe 1

L_2 = length of pipe 2 d_2 = dia of pipe 2

L_3 = length of pipe 3 d_3 = dia of pipe 3.

H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe.

Then $\boxed{L = L_1 + L_2 + L_3}$

Total head loss in the compound pipe, neglected minor losses.

$$\boxed{H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}}$$

Assuming $\boxed{f_1 = f_2 = f_3 = f}$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3$

$$Q = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}d_3^2$$

$$\boxed{V_1 = \frac{4Q}{\pi d_1^2}\ ;\ V_2 = \frac{4Q}{\pi d_2^2}\ ;\ V_3 = \frac{4Q}{\pi d_3^2}}$$

substituting these values in eqn. (i)

$$H = \frac{4fL_1\left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$\boxed{H = \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]}$$

Head loss in the equivalent pipe, $H = \frac{4fLV^2}{d \times 2g}$

[Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$\boxed{H = \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16Q^2 f}{\pi^2 2g}\left[\frac{L}{d^5}\right]}$$

∴ loss in compound pipe and in equivalent pipe is same hence equating eqn's i, and ii,

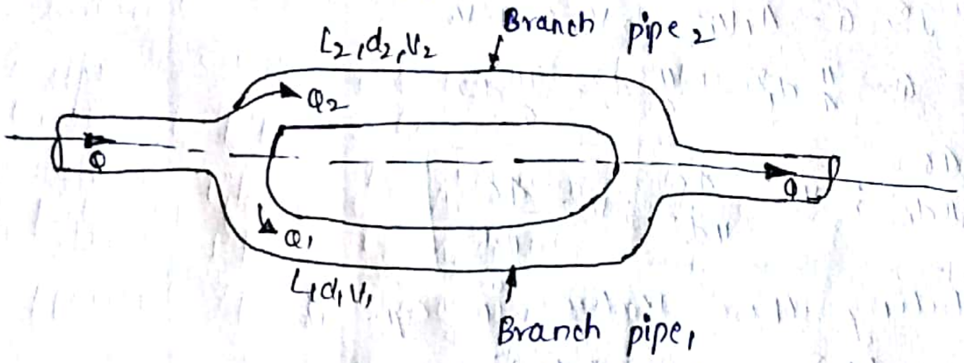
$$\frac{4 \times 16 f Q^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5}\right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left(\frac{L}{d^5}\right)$$

$$\boxed{\frac{L_1}{d_1^5}+\frac{L_2}{d_2^5}+\frac{L_3}{d_3^5} = \frac{L}{d^5}} \quad \text{——— iii,}$$

The eqn. iii, is known as Dupuit's eqn. In this eqn. $L = L_1+L_2+L_3$ In this equation $L = L_1+L_2+L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, i.e., value of d obtained.

FLOW THROUGH PARALLEL PIPES:

Consider a main pipe which divides into two (or) more branches as shown in fig. below.



and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from fig.

$$\boxed{Q = Q_1 + Q_2}$$

In this arrangement, the loss of head for each branch pipe is same.

∴ loss of head for branch pipe 1 = loss of head for branch pipe 2

$$\frac{f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{f_2 L_2 V_2^2}{d_2 \times 2g}$$

If $f_1 = f_2$ then

$$\boxed{\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}}$$

1. The difference in water surface levels in two levels which are connected by three pipes in series of lengths 300m, 170m and 210m and of diameters 300mm, 200mm and 400mm respectively, is 12m. Determine the rate of flow of water if co-efficient of friction are 0.005, 0.0052 and 0.0048 respectively, considering
   i, minor losses also.
   ii, Neglecting minor losses.

Sol: Given:

difference of water level, $H = 12m$

length of pipe 1, $L_1 = 300m$ diameter, $d_1 = 0.3m$

length of pipe 2, $L_2 = 170m$ diameter, $d_2 = 0.2m$

length of pipe 3, $L_3 = 210m$ diameter $d_3 = 0.4m$

co-efficient of friction, $f_1 = 0.005$

$f_2 = 0.0052$

$f_3 = 0.0048$

i, Considering minor losses:

Let V_1, V_2, V_3 are the velocities in the 1st, 2nd & 3rd pipe respectively.

From continuity eqn., we have

$$A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} d_1^2 V_1}{\frac{\pi}{4} d_2^2} = \left(\frac{d_1}{d_2}\right)^2 V_1 = \left(\frac{0.3}{0.2}\right)^2 V_1$$

$$= 2.25 V_1$$

$$V_3 = \frac{A_1 V_1}{A_3} = \frac{\frac{\pi}{4} d_1^2 V_1}{\frac{\pi}{4} d_3^2} = \left(\frac{d_1}{d_3}\right)^2 V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1$$

$$= 0.5625 V_1$$

We know the equation,

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2g} + \frac{V_3^2}{2g}$$

$$12.0 = \frac{0.5V_1^2}{2g} + \frac{4 \times 0.005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5(2.25V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0052 \times 170 \times (2.25V_1)^2}{0.2 \times 2g} + \frac{(2.25V_1 - 0.05625V_1)^2}{2g}$$

$$+ \frac{4 \times 0.0048 \times 210 \times (0.5625V_1)^2}{0.4 \times 2g} + \frac{(0.5625V_1)^2}{2g}$$

$$12 = \frac{V_1^2}{2g}[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$12 = \frac{V_1^2}{2g}(118.87)$$

$$V_1^2 = \frac{12 \times 2 \times 9.81}{118.87}$$

$$V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.87}} = 1.407 \text{ m/s}$$

∴ Rate of flow, Q = Area × velocity

$$= A_1V_1$$

$$= \frac{\pi}{4}d_1^2 \times V_1$$

$$= \frac{\pi}{4}(0.3)^2 \times 1.407$$

$$= 0.09945 \text{ m}^3/\text{sec}$$

$$= 99.45 \text{ lit/sec}$$

ii, Neglecting minor losses:

$$\boxed{H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}}$$

$$12 = \frac{V_1^2}{2g}\left[\frac{4 \times 0.005 \times 300}{0.3} + \frac{4 \times 0.0052 \times 170(2.25)^2}{0.2} + \frac{4 \times 0.0048 \times 210(0.5625)^2}{0.4}\right]$$

$$12 = \frac{V_1^2}{2g}[20 + 89.505 + 3.189]$$

$$12 = \frac{V_1^2}{2g}(112.694)$$

$V_1^2 = \frac{12 \times 2 \times 9.81}{112.694} \Rightarrow V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{112.694}} = 1.446$ m/s

$\therefore$ Rate of flow, $Q = A_1 V_1$

$= \frac{\pi}{4} d_1^2 V_1$

$= \frac{\pi}{4} (0.3)^2 \times 1.446$

$= 0.1021$ m³/s

$= 102.1$ lit/sec

2. A three pipes of 400mm, 200mm and 300mm diameters have lengths of 400m, 200m and 300m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference of water levels is 16m. If co-efficient of friction for these pipes is same and equal to 0.005, determine the discharge through the compound pipe neglecting first the minor losses and then including them.

Sol: Given:

Difference of water levels, $H = 16$m

Length and dia of pipe 1, $L_1 = 400$m

$d_1 = 400$mm $= 0.4$m

Length and dia of pipe 2, $L_2 = 200$m

$d_2 = 200$ mm $= 0.2$m

Length and dia of pipe 3, $L_3 = 300$m

$d_3 = 300$mm $= 0.3$m

$f_1 = f_2 = f_3 = 0.005$

i, Discharge through the compound pipe first neglecting minor losses,

Let V_1, V_2, V_3 are the velocities in the 1st, 2nd & 3rd pipes respectively

From continuity eqn. we have

$$\boxed{A_1 V_1 = A_2 V_2 = A_3 V_3}$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\pi/4\, d_1^2 V_1}{\pi/4\, d_2^2} = \left(\frac{d_1}{d_2}\right)^2 V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

$$V_3 = \frac{A_1 V_1}{A_3} = \frac{\pi/4\, d_1^2 V_1}{\pi/4\, d_3^2} = \left(\frac{d_1}{d_3}\right)^2 V_1 = \left(\frac{0.4}{0.3}\right)^2 V_1 = 1.77\, V_1$$

We know the eqn.

$$\boxed{H = \frac{4f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4f_3 L_3 V_3^2}{d_3 \times 2g}}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2g} + \frac{4 \times 0.005 \times 200 (4V_1)^2}{0.2 \times 2g} + \frac{4 \times 0.005 \times 300 \times (1.77 V_1)^2}{0.3 \times 2g}$$

$$16 = \frac{V_1^2}{2 \times 9.81}\left[\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 1.77^2}{0.3}\right]$$

$$16 = \frac{V_1^2}{2 \times 9.81}\left[20 + 320 + 63.14\right]$$

$$16 = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$$V_1^2 = \frac{16 \times 2 \times 9.81}{403.14} \Rightarrow V = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

Discharge through the compound pipe

$$Q = A_1 V_1$$
$$= \frac{\pi}{4} d_1^2 \times V_1$$
$$= \frac{\pi}{4} (0.4)^2 \times 0.882$$
$$= 0.1108 \text{ m}^3/\text{s} = 110.8 \text{ lit/s}$$

ii, Discharge through the compound pipe considering minor losses also.

Minor losses are:

(a) at inlet, $h_i = \frac{0.5 V_1^2}{2g}$

(b) Between first pipe and second pipe due to contraction,

$$h_c = \frac{0.5 V_2^2}{2g} = \frac{0.5 (4V_1)^2}{2g}$$

$$= \frac{0.5 \times 16 V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(c) Between second pipe and 3rd pipe due to sudden enlargement.

$$h_c = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_1 - 1.77V_1)^2}{2g}$$

$$= (2.23)^2 \times \frac{V_1^2}{2g}$$

$$= 4.973 \frac{V_1^2}{2g}.$$

(d) At the outlet of 3rd pipe,

$$h_o = \frac{V_3^2}{2g} = \frac{(1.77V_1)^2}{2g} = 1.77^2 \frac{V_1^2}{2g} = 3.1329 \frac{V_1^2}{2g}$$

The major losses are;

$$= \frac{4f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4f_3 L_3 V_3^2}{d_3 \times 2g}$$

$$= \frac{4 \times 0.005 \times 400 V_1^2}{0.4 \times 2g} + \frac{4 \times 0.005 \times 200 (4V_1)^2}{0.2 \times 2g} + \frac{4 \times 0.005 \times 300 \times (1.77 V_1)^2}{0.3 \times 2g}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

∴ Sum of minor losses and major losses

$$= \left[\frac{0.5V_1^2}{2g} + \frac{8V_1^2}{2g} + \frac{4.973 V_1^2}{2g} + \frac{3.1329 V_1^2}{2g}\right] + 403.14 \frac{V_1^2}{2g}$$

$$= 419.746 \frac{V_1^2}{2g}$$

But total loss must be equal to H

$$419.746 \frac{V_1^2}{2g} = H$$

$$V_1^2 = \frac{H \times 2g}{419.746} \Rightarrow V_1 = \sqrt{\frac{H \times 2g}{419.746}}$$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864 \text{ m/s}$$

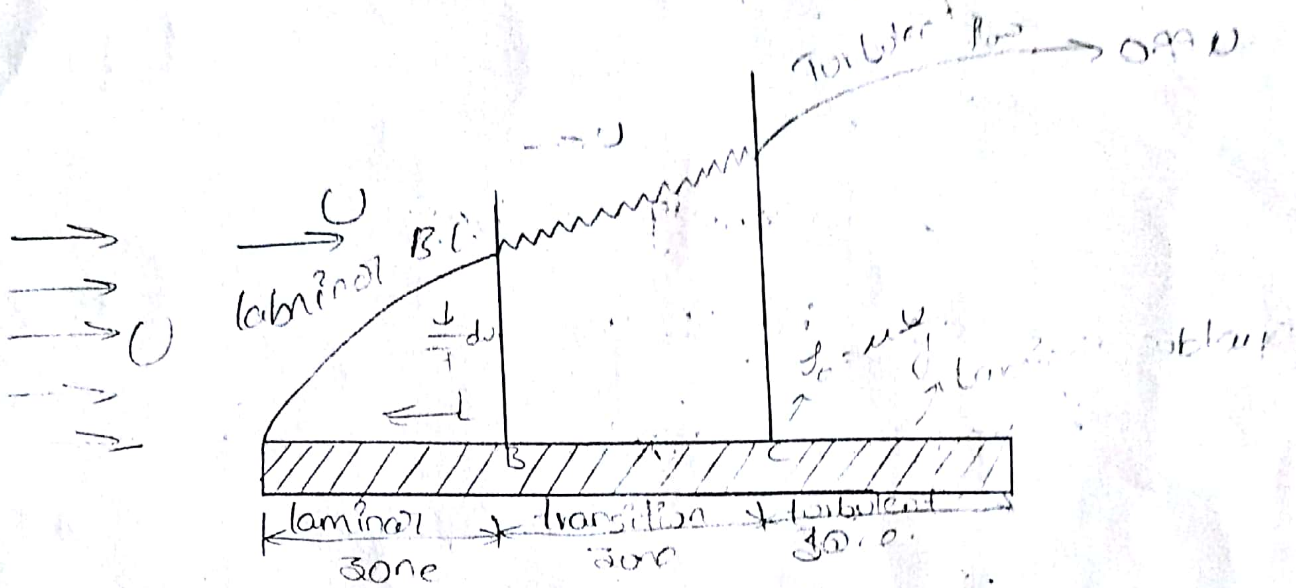
∴ Discharge, $Q = A_1 V_1$

$$= \frac{\pi}{4} (0.4)^2 \times 0.864$$

$$= 0.1085 \text{ m}^3/\text{s}$$

$$= 108.5 \text{ lit/sec}$$

UNIT-II Boundary Layer Theorem.



We have shear stress $\tau = \mu \frac{du}{dy}$.

Boundry layer thickness:-

The ⊥ler distance b/w the boundry layer to top of the flow.

δ_{lam} – laminar flow thickness

δ_{tub} – turbulent flow thickness

δ_{sb} – ~~laminar~~ sublayer thickness boundry layer thickness

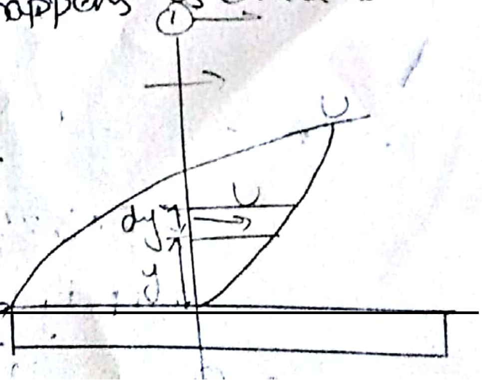
Displacement Thickness:- The distance ⊥ler to boundary due to which ~~flo~~ the free flow happens is called D.T.

It is denoted by δ^*

mass flow through the strip.

$\rho \times V \times A$.

$= \rho \times u \times b \times dy \rightarrow (1)$



without plate

$$\text{mass}/\text{sec} = \rho \times U \times b \times dy \rightarrow ②$$

loss of mass flow/sec

$$\delta^* = ② ①$$

$$= \rho U \times b \times dy - \rho u\, b \times dy$$

$$= \rho b dy (U-u).$$

$$\rho \times b \times dy \int_0^{\delta} (U-u) = \rho U \delta^* b$$

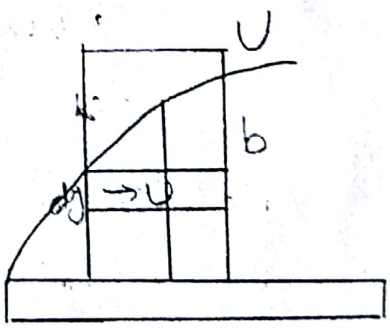
$$\delta^* = \int_0^{\delta} \frac{(U-u)}{U} dy$$

$$\boxed{\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy}$$ loss in mass volume.

Momentum thickness (θ).

$= \text{mass} \times \text{velocity}$ → loss in momentum

$$= (\rho \times U \times b \times dy) U = \rho \times U^2 b dy$$



By plate:

$$(\rho \times u \times b \times dy) u = \rho \times u^2 b dy.$$

$$\text{loss of mass} = \rho b dy (U-u) u$$

$$= (\rho \times U \times \theta \times b) U.$$

$$= \rho b dy \int_0^{\infty} (U-u) u$$

$$= \int_0^\infty \left(\frac{U-u}{U^2}\right) u\,dy.$$

$$= \int_0^\infty \left(\frac{Uu-u^2}{U^2}\right) dy.$$

$$= u\int_0^\infty \left(\frac{U-u}{U^2}\right) dy.$$

$$= \frac{u}{U}\left(\int_0^\delta 1-\frac{u}{U}\right) dy.$$

Energy thickness (δ^{**}).

$K.E = \frac{1}{2}mv^2$ → loss in K.E.

$$= \frac{1}{2}(\rho u b\,dy)u^2$$

$$= \frac{1}{2}\rho b\,dy\,(U^3-u^3).$$

$$= \frac{1}{2}(\rho U \delta^{**} b)U^2.$$

$$\frac{1}{2}(\rho \times U^3 \delta^{**} b) = \frac{1}{2}\rho b\,dy\,(U^3-u^3).$$

$$\delta^{**} = \int_0^\delta \left(\frac{U^3-u^3}{U^3}\right) dy.$$

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right) dy.$$

Rate of $F_D = \tau_0 \times b \times \Delta x.$

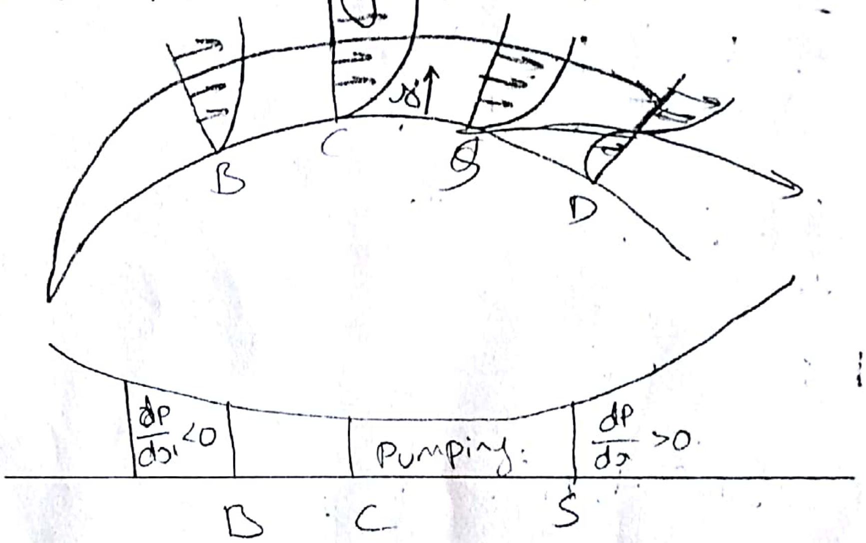
Total rate of change of momentum is equal to BL is equal to drag force. (F_D)

$M_{BC} - M_{AD} - M_{DC}$

$= M_{AD} + \frac{d}{dx}(M_{AD})\Delta x - M_{AD} - M_{DC} = FD. = \tau_0 \times b \times \Delta x.$

$$\frac{\tau_0}{\rho U^2} = \frac{\partial \theta}{\partial x}.$$

Seperation of boundry layer.



$\frac{\partial u}{\partial y} > 0$ $\frac{\partial u}{\partial y} < 0$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} < 0.$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = 0$$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} > 0.$$

[illegible] methods of B.L.

1) Motion of a solid body:-

2\. 2) Acceleration of fluid in the boundry layer

3) Suction of fluid from boundry.

4) Stream line of body shapes.

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\Rightarrow \delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right) dy$$

$$= \int_0^\delta 1\, dy - \int_0^\delta 2\frac{y}{\delta} dy + \int_0^\delta \frac{y^2}{\delta^2} dy$$

$$= y\Big]_0^\delta - \frac{2}{\delta}\frac{y^2}{2}\Big]_0^\delta + \frac{1}{\delta^2}\frac{y^3}{3}\Big]_0^\delta$$

$$= 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \left(1 - 2\frac{y}{\delta} + \frac{y^2}{\delta^2}\right) dy$$

$$= 2\int_0^\delta \frac{y}{\delta} - 2\int \frac{y^2}{\delta^2} \cdot 1\, dy -$$

$$= \cancel{2}\frac{y^2}{\cancel{2}\delta} - 2\frac{y^3}{3\delta^2} - 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^3$$

$$= \frac{\delta}{3}$$

Energy thickness:

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy.$$

$$= \int_0^\delta 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \left(1 - \left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right)^2 dy.$$

$$= 2\cdot\frac{y^2}{2\delta}\Big]_0^\delta - \frac{y^3}{3\delta}\Big]_0^\delta \left(1\,dy\Big]_0^\delta - \left(2\frac{y^2}{2\delta}\right)_0^\delta - \frac{y^3}{3\delta}\right)_0^\delta$$

$$= \frac{y^2}{\delta}\,\delta$$

$$= \cancel{\delta} - \frac{\delta^3}{3\delta}\left(\delta - \frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta}\right.$$

$$= \delta.$$

$$= \cancel{\delta^2} - \cancel{\delta^2} - \frac{\delta^3}{3} - \frac{\delta^3}{3} - \delta^3 - \frac{\delta^4}{9}$$

$$= \int_0^\delta \frac{u}{U} - \frac{u^2}{U^2}\,dy.$$

$$= \int_0^\delta \left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right) dy - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)^2 dy.$$

$$= \delta/3 - \int_0^\delta 4\frac{y^2}{\delta^2} - 4\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\Big)\,dy.$$

$$= 4\cdot\frac{y^3}{3\delta^2} - \cancel{4}\cdot\frac{y^4}{\cancel{4}\delta^3} + \frac{y^5}{5\delta^4}\Big)_0^\delta$$

$$= \frac{4}{3}\frac{\delta^3}{\delta^2} - \frac{\delta^4}{\delta^3} + \frac{\delta^5}{5\delta^4}$$

$$= \frac{4}{3}\delta - \delta + \delta = \frac{4}{3}\delta - \delta + \frac{\delta}{5} + \frac{\delta}{3}.$$

$$= \frac{4}{3}\delta + \frac{\delta}{3} = \frac{20\delta - 15\delta + 3\delta + 5\delta}{15} = \frac{3\delta}{15}.$$

$$\frac{\tau}{\rho U^2} = \frac{\partial \theta}{\partial x}.$$

$$\tau = \mu \frac{du}{dy}.$$

$$F_D = \tau \times \Delta x \times b.$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A}.$$

$$F_D = \frac{C_D \times \rho A U^2}{2}.$$

For the velocity profile for ~~the~~ laminar boundary layer given us. $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$. Find an expression for boundary layer thickness δ shear stress & τ and co-efficient of drag C_D in terms of Reynold's number.

we have,

$$\frac{\tau}{\rho U} = \frac{d}{dx}\left(\int_0^\delta \left(1-\frac{u}{U}\right)\frac{u}{U}\, dy\right).$$

$$= \frac{\partial}{\partial x}\left(\frac{3\delta}{15}\right) dy.$$

$$= \frac{3}{15}\frac{\partial}{\partial x}(\delta).$$

$$\tau = \frac{3\rho U^2}{15}\frac{\partial}{\partial x}(\delta).$$

$$\frac{u}{U} = 2 \cdot \frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2.$$

let $u = U\left(2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right)$.

$$\tau = \mu \frac{du}{dy}.$$

$$= \frac{2U}{\delta^2} \times (\delta - y).$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2U}{\delta^2}\left(\delta - 0\right)$$

$$= \frac{2U}{\delta} \quad \text{max.}$$

$$\tau = \mu \frac{2U}{\delta}.$$

$$\tau = \frac{3\rho U^2}{15} \cdot \frac{\partial}{\partial x}(\delta).$$

$$\mu \frac{\partial u}{\partial y} = \frac{3\rho U^2}{15}.$$

$$\mu \cdot \frac{2U}{\delta} = \frac{3\rho U^2}{15} \frac{\partial}{\partial x}(\delta).$$

$$\mu \cdot \frac{2U}{\delta} = \frac{3\rho U^2}{15} \cdot \frac{\partial}{\partial x}(\delta)$$

$$2\mu = \frac{3}{15} \rho U \times \frac{\partial}{\partial x}(\delta).$$

$$\frac{\partial}{\partial x}(\delta) = 2\mu \frac{15}{3\rho U}.$$

$$= \frac{10\mu}{\rho U}.$$

$$\delta \, \partial \delta = \frac{10\mu}{\rho U} \partial x.$$

$$\frac{\delta^2}{2} = \frac{10\mu}{\rho U} x + C.$$

$$\left(\because C = 0\right).$$

$$\delta^2 = \frac{20\mu}{\rho U} x$$

$$\delta^2 = \frac{20x}{Re} \qquad \left(\because Re = \frac{\rho U}{\mu}\right).$$

$$\therefore \delta = \sqrt{\frac{20x}{Re}}.$$

Pg no 617 & 622.

$$C_D = \frac{F_D}{\frac{1}{2}\rho A U^2}$$

$$F_D = \int_0^L \tau \times \Delta x \times b.$$

Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

Given distribution.

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Displacement thickness

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy.$$

$$= \int_0^\delta 1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right] dy.$$

$$= \int_0^\delta \left(1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right) dy$$

$$= \int_0^\delta 1\,dy - 2\int_0^\delta \frac{y}{\delta}\,dy + \int_0^\delta \frac{y^2}{\delta^2}\,dy.$$

$$= \left. y - \cancel{2}\frac{y^2}{\cancel{2}\delta} + \frac{y^3}{3\delta^2}\right]_0^y$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2}$$

$$= \delta - \delta + \frac{\delta}{3}$$

$$\therefore = \frac{\delta}{3}.$$

Momentum thickness:

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy.$$

$$= \int_0^\delta \frac{u}{U} - \frac{u^2}{U^2}$$

$$= \int_0^\delta 2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\left(1-\left(2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right)\right)dy.$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right)dy$$

$$\Rightarrow \int_0^\delta \left(\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right)dy$$

$$\Rightarrow \left[\frac{\cancel{2}y^2}{\cancel{2}\delta} - 5\cdot\frac{y^3}{3\delta^2} + \cancel{4}\cdot\frac{y^4}{\cancel{4}\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$\Rightarrow \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$\Rightarrow \frac{15\delta - 25\delta + 15\delta - 3\delta}{15}$$

$$\Rightarrow \frac{2\delta}{15}$$

Energy thickness:

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right)dy.$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1-\left[\frac{2y}{\delta}-\frac{y^2}{\delta^2}\right]^2\right)dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1-\left(\frac{4y^2}{\delta^2} - 4\cdot\frac{y^3}{\delta^3} + \frac{y^4}{\delta^4}\right)\right)dy.$$

$$\Rightarrow \int_0^\delta \left(\frac{2y}{\delta} - \frac{8y^3}{\delta^3} + \frac{8y^4}{\delta^4} - \frac{2y^5}{\delta^5} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5}\right)dy$$

$$= \frac{\partial}{\partial x}\int_0^{\delta}\left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right]dy$$

$$= \frac{\partial}{\partial x}\left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^{\delta}$$

$$\Rightarrow \frac{\partial}{\partial x}\left[\delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}\right]$$

$$\Rightarrow \frac{\partial}{\partial x}\left[\frac{15\delta - 25\delta + 15\delta - 3\delta}{15}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{30\delta - 28\delta}{15}\right] = \frac{\partial}{\partial x}\left(\frac{2\delta}{15}\right)$$

$$= \frac{2}{15}\frac{\partial}{\partial x}(\delta)$$

$$\tau_0 = \rho U^2 \times \frac{2}{15}\frac{\partial}{\partial x}(\delta)$$

$$= \frac{2}{15}\rho U^2 \frac{\partial[\delta]}{\partial x}$$

The shear stress at the boundary in laminar flow is also given by Newton's law of viscosity as

$$\tau_0 = \mu\left(\frac{dU}{dy}\right)_{y=0} \longrightarrow (2)$$

$$u = U\left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]$$

$$\frac{du}{dy} = U\left[\frac{2}{\delta} - \frac{2y}{\delta^2}\right]$$

$$\left(\frac{du}{dy}\right)_{y=0} = U\left[\frac{2}{\delta} - \frac{2\times(0)}{\delta^2}\right] = \frac{2U}{\delta}$$

Subs the value in eqn (2) we get:

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta} \longrightarrow (3)$$

Equating the two values of τ_0 given by eqn (3) to

$$\left(\tau_0 = \rho U^2 \times \frac{2}{15}\frac{\partial}{\partial x}(\delta)\right) = \frac{2}{15}\rho U^2\frac{\partial(\delta)}{\partial x}$$

we get

$$\tau_0 = \mu \times \frac{2U}{\delta} = \frac{2\mu U}{\delta} \rightarrow$$

$$\frac{2}{15}\rho U^2 \frac{\partial}{\partial x}(\delta) = \frac{2\mu U}{\delta}$$

$$\delta \frac{\partial}{\partial x}[\delta] = \frac{15\mu U}{\rho U^2} = \frac{15\mu}{\rho U}$$

$$\delta \partial[\delta] = \frac{15\mu}{\rho U} \partial x.$$

As the boundary layer thickness (δ) is a function of x only. Hence partial derivative can be changed to total derivative.

$$\delta d[\delta] = \frac{15\mu}{\rho U} dx. \quad \left[\because \frac{\mu}{\rho U} \text{ is constant}\right]$$

On integration, we get $\frac{\delta^2}{2} = \frac{15\mu}{\rho U}x + C.$

$x=0,\ \delta=0$ and hence $C=0$.

$$\frac{\delta^2}{2} = \frac{15\mu x}{\rho U}.$$

$$\delta = \sqrt{\frac{2\times 15\mu x}{\rho U}}$$

$$= \sqrt{\frac{30\mu x}{\rho U}} = 5.48\sqrt{\frac{\mu x}{\rho U}}.$$

$$= 5.48\sqrt{\frac{\mu x \times x}{\rho U \times x}}$$

$$= 5.48\frac{x}{\sqrt{Re_x}}.$$

Shear stress τ_0 in terms of Reynold's number.

From eqn (3), we have $\tau_0 = \frac{2\mu U}{\delta}.$

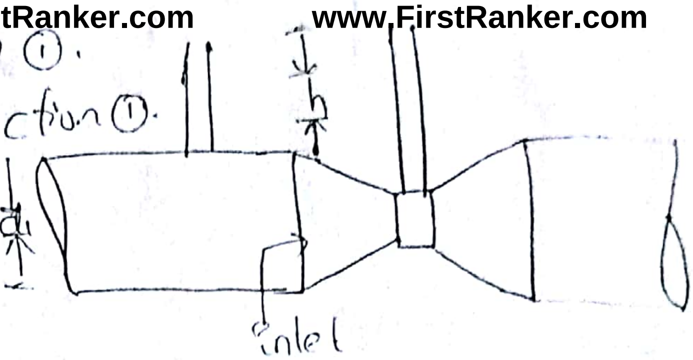
Sub. the value of δ from eqn $5.48\frac{x}{\sqrt{Re_x}}$, we get

$$\tau_0 = \frac{2\mu U}{5.48\frac{x}{\sqrt{Re_x}}} = \frac{2\mu U\sqrt{Re_x}}{5.48x} = 0.365\frac{\mu U}{x}\sqrt{Re_x}$$

let a_1 = area of section ①.

d_1 = diameter of section ①.

v_1 = velocity of fluid at section ①.



P_1 = Pressure of fluid at the inlet or section ①.

P_2, v_2, a_2, d_2 are the corresponding values at section

Apply Bernoulli's theorem

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2.$$

As pipe is horizontal.

$$z_1 = z_2.$$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}.$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}.$$

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}. \rightarrow ①.$$

By continuity eqn $a_1 v_1 = a_2 v_2$.

$$v_1 = \frac{a_2 v_2}{a_1}.$$

$$h = \frac{v_2^2}{2g} - \left(\frac{\frac{a_2 v_2}{a_1}}{2g}\right)^2.$$

$$h = \frac{v_2^2}{2g}\left(1 - \frac{a_2^2}{a_1^2}\right).$$

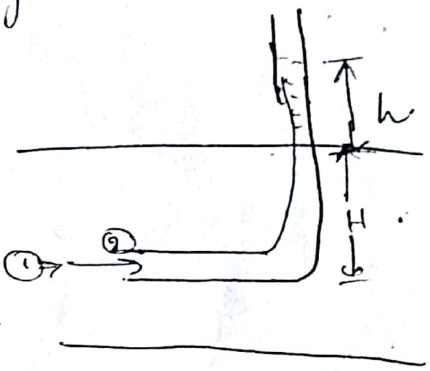
$$\Rightarrow v_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right). \Rightarrow v_2 = \sqrt{2gh} \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

$Q = a_2 v_2$.

$\Rightarrow a_2 \cdot \sqrt{2gh} \cdot \frac{a_1}{\sqrt{a_1^2 - a_2^2}}$.

$\Rightarrow \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$.

$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gh}$



$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{\rho g} + Z_2$.

$Z_1 = Z_2$.

$V_2 = 0$.

$\frac{P_1}{\rho g} = H$.

$\frac{P_2}{\rho g} = (h + H)$.

$H + \frac{V_1^2}{2g} = (H + h) + 0$.

$\frac{V_1^2}{2g} = h$.

$\Rightarrow V_1^2 = 2gh$

$V_1 = \sqrt{2gh}$.

$\therefore V_{act} = C_v \sqrt{2gh}$.

where C_v - coefficient of pitot tube.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1 - \left(2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right)\right) dy$$

$$= \left[y - 2\cdot\frac{y^2}{2\delta} + \frac{y^3}{3\delta^2} \right]_0^\delta$$

$$= \cancel{\delta} - \cancel{\delta} + \frac{\delta}{3}$$

$$= \frac{\delta}{3}$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\Rightarrow \int_0^\delta 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2 \left(1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right] dy$$

$$\Rightarrow \int_0^\delta 2\cdot\frac{y}{\delta} - 4\left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right]$$

$$\Rightarrow \int_0^\delta \left(2\cdot\frac{y}{\delta} - 5\left(\frac{y}{\delta}\right)^2 + 4\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right]$$

$$\Rightarrow \left. \cancel{2}\frac{y^2}{\cancel{2}\delta} - 5\cdot\frac{y^3}{3\delta^2} + \cancel{4}\cdot\frac{y^4}{\cancel{4}\delta^3} - \frac{y^5}{5\delta^4} \right]_0^\delta$$

$$= \frac{\delta^2}{\delta} - \frac{5}{3}\cdot\frac{\delta^3}{\delta^2} + \frac{\delta^4}{\delta^3} - \frac{\delta^5}{5\delta^4}$$

$$= \delta - \frac{5}{3}\delta + \delta - \frac{\delta}{5}$$

$$\Rightarrow \frac{15\delta - 25\delta + 15\delta - 3\delta}{15}$$

$$\Rightarrow \frac{2\delta}{15}$$

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\left(\frac{u}{U}\right)^2\right)dy$$

$$\Rightarrow \int_0^\delta 2\left(\frac{y}{\delta}\right)\left(\frac{y}{\delta}\right)^2\left(1-\left(2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2\right)^2\right)dy$$

$$\Rightarrow \int_0^\delta 2\left(\frac{y}{\delta}\right)\left(\frac{y}{\delta}\right)^2\left(1-\left(4\frac{y^2}{\delta^2}-4\left(\frac{y}{\delta}\right)^3+\left(\frac{y}{\delta}\right)^4\right)\right)dy.$$

$$\Rightarrow \int_0^\delta \Big[2\left(\frac{y}{\delta}\right)-8\left(\frac{y}{\delta}\right)^3+8\left(\frac{y}{\delta}\right)^4-2\left(\frac{y}{\delta}\right)^5-\left(\frac{y}{\delta}\right)^2+$$

$$4\left(\frac{y}{\delta}\right)^4-4\left(\frac{y}{\delta}\right)^5+\left(\frac{y}{\delta}\right)^6\Big)dy.$$

$$\Rightarrow \Big[\cancel{2}\left(\frac{y^2}{\cancel{2}\delta}\right)-8\left(\frac{y^4}{4\delta^3}\right)+8\left(\frac{y^5}{5\delta^4}\right)-2\cdot\left(\frac{y^6}{6\delta^5}\right)-\frac{y^3}{3\delta^2}+$$

$$4\left(\frac{y^5}{5\delta^4}\right)-4\left(\frac{y^6}{6\delta^5}\right)+\frac{y^7}{7\delta^6}\Big]_0^\delta.$$

$$\Rightarrow \frac{\delta^2}{\delta}-2\cdot\frac{\delta^4}{\delta^3}+\frac{8}{5}\cdot\frac{\delta^5}{\delta^4}-\frac{2}{6}\cdot\frac{\delta^6}{\delta^5}-\frac{\delta^3}{3\delta^2}+$$

$$\frac{4}{5}\frac{\delta^5}{\delta^4}-\frac{4}{6}\frac{\delta^6}{\delta^5}+\frac{\delta^7}{7\delta^6}.$$

$$\Rightarrow \delta-2\delta+\frac{8}{5}\delta-\frac{1}{3}\delta-\frac{1}{3}\delta+\frac{4}{5}\delta-\frac{2}{3}\delta+\frac{1}{7}\delta.$$

$$\frac{22\,\delta}{105}$$

Dimension Analysis:- We have 4 System of Units which (1) are universally used & recognise. They are

1. $\boxed{\text{F.P.S UNIT}}$

The System of unit based on the feets, pounds & Seconds. Length in foots, Mass in pounds & time in Seconds.

2. C.G.S Unit :- The system of units based on Centimeters, grams & Seconds. Length in centimeters, mass in grams, time in Seconds.

3. M.K.S unit:- The System of Units based on meters, kilograms, and second. Length in meters, mass in kilograms time in Seconds.

4. S.I Unit:- There are system of International units which is world most widely used modern form a M.K.S System.

System	Prefer	Mult.
kilo	K	10^3
Mega	M	10^6
Giga	G	10^9
centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
nano	n	10^{-9}

* Force - Which ever causes & tense to cause motion is a force - Newton.
* Power - Rate of doing work - Watts.
* Velocity - Rate of change of displacement - m^3
* Acceleration:- Rate of change of velocity - m/sec^2.
* Dimension:- Five important physical qualities are involved in study of fluid.

 Mass (M), Length (L), Force (F) Temperature (θ) Time (T).

* **Dimension Analysis:-**

Representing any important parameter in terms of 5 physical quantities is called dimension analysis.

Two types of dimensional analysis.

1. Independent of Mass (M).
2. Independent of force (F).

Ex:- Power $P = \frac{F \times S}{t} = \frac{mg \cdot S}{T} = \frac{MLT^{-2}L}{T} = ML^2T^{-3}$

* **Dimensions and Units:-**

Dimension is a measure by which a physical variable is express quantiting.

Unit:- Unit is the particular way of attaching a number to the quantiting dimension.

Ex:- Length, it is a dimension measured by a physical Variables such as distance, displacement width, deflection & height while cm & mts are both numerical units for expressing length.

Primary dimensions.

1. Force (F) 2. Mass (M) 3. Length (L), 4. Time (T).

P.D	M.K.S	S.I.
Force (F)	kg	kgm/sec^2
Mass (M)	$kg\,sec^2/m$	Kilogram.
Length (L)	m	m.
Time (T)	sec	sec.

* **Table for Conversions:-**

Name	Quantity.	Symbol	Value in SI Unit.
Minute	time	min	1min = 60 sec.
hour	time	h	1hr = 60 min = 3600 sec.
litre	Volume	L	$1L = 10^{-3}m^3 = 1000L$.
tonne	mass	t	1t = 1000 kg.

Derived quantity	S I	C.G.S & M.K.S.
Area	m^2	m^2
Volume	m^3	m^3.
Velocity	m/sec	m/sec.
Angular Velocity	rad/sec	rad/sec
Acceleration	m/sec^2	m/sec^2
frequency	Hz	1/sec.
Angular acceleration	rad/sec^2	rad/sec^2
Discharge	m^3/sec	m^3/sec
Mass density	kg/m^3	Ml/cubic mt.
Force.	$N = kgm/sec^2$	kg (f).
Dynamic Viscosity.	$N \cdot S/m^2$	$kg(f) \cdot Sec/m^2$
kinematic Viscosity	m^2/sec	m^2/sec.
Power	J/sec.	kg(f).m/sec.
Surface tension	N/m	kg(f)./m.
Work Energy	kgm/sec	Metric Slug. m/sec.

* Model Analysis:- The performance of the hydraulic structures Eg:- dams, Spillway etc. & hydraulic machines eg:- turbines, pumps. etc. before actuall constructing & manufacturing them, their models are made & tested to get the required information. The model is Small Scale of the actual structure & Machine. The actual structure & machine is called Prototype. The models are not always smaller than the prototype, in some cases a model may be even larger & of the Same Size as prototype depending upon the need & purpose.

* Applications:-

1. Civil Engg. structures Such as dams, weirs, canals etc
2. Flood control, investigation of Silting & Scour in rivers

3. Turbines, pumps

4. Design of harbours, ships.

5. Aeroplanes, rockets.

6. Tall building

* Similitude:- To find solutions to numerous complicated problems in hydraulic engineering & fluid mechanics.

1. Geometric Similarity:- $\frac{L_m}{L_p} = \frac{H_m}{H_p} \cdot \frac{D_m}{D_p} = L_r$

L_m, H_m, D_m are the Length, Height, Diameter models.
L_p, H_p, D_p " " " " Prototype.
L_r:- Scale ratio

A_r Area ratio $= \frac{A_m}{A_p} = L_r^2$

V_r volume ratio $= \frac{V_m}{V_p} = L_r^3$.

2. Kinematic Similarity:-

Velocity ratio $V_r = \frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p}$.

Acceleration ratio $a_r = \frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p}$.

Directions of the velocities in the model & prototype should be same.

3. Dynamic Similarity:- It is Similarity of forces.

$(F_i)_m$ = Inertia force.

$(F_v)_m$ = Viscous force.

$(F_g)_m$ = Gravity force.

Force Ratio $F_r = \frac{(F_i)_m}{(F_i)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_g)_m}{(F_g)_p}$.

The directions of the corresponding forces at the corresponding points in the model & prototype should also be same.

Forces Influencing Hydraulic Phenomena:

(1) Inertia force (F_i) :- It always exists in the fluid flow problem. It is equal to the product of mass & acceleration of the flowing fluid & acts in the direction opposite to the direction of acceleration.

(2) Viscous force (F_v) :- It is present in fluid flow problems where viscosity to play an important role.
* It is equal to the product of shear stress (τ) due to viscosity & surface area of the flow.

(3) Gravity force (F_g) :- * It is present in case of open surface flow.
* It is equal to the product of mass & acceleration due to gravity.

(4) Pressure force (F_p) :- It is present in case of pipe flow.
* It is equal to the product of pressure intensity & cross-sectional area of the flowing fluid.

(5) Surface Tension force (F_s) :- It is equal to the product of surface tension & length of surface of the flowing fluid.

(6) Elastic force (F_e) :- It is equal to the product of elastic stress & area of the flowing fluid.

* Dimensionless Numbers & their Significance :-

The dimensionless numbers also called Non-Dimensional parameters.
They are 5 types of dimensionless numbers.

1. Reynolds number.
2. Froude's number.
3. Euler's number.
4. Weber's number.
5. Mach's number.

Reynolds number:- It is defined as the ratio of inertia force to the viscous force.

$$\boxed{R_e = \frac{F_i}{F_v}}$$

inertia force (F_i) = mass $\times$ acceleration.

$= \rho \times \text{Volume} \times \frac{\text{Velocity}}{\text{time}}$

$= \rho \times AV \times V$ [$\because$ Volume per Second $= A \times V$]

$= \rho A V^2$

Viscous force (F_v) = shear stress $\times$ Area $= \tau \times A$.

$= \left[\mu \frac{du}{dy}\right] \times A$ [$\because \frac{du}{dy} = \frac{V}{L}$]

$= \mu \frac{V}{L} \times A$.

$\therefore$ Reynolds number $R_e = \frac{F_i}{F_v} = \frac{\rho A V^2}{\mu \times \frac{V}{L} \times A} = \frac{\rho V L}{\mu}$

$= \frac{VL}{\mu/\rho}$ [$\because \nu = \mu/\rho$]

$= \frac{VL}{\nu}$

For flow pipe $\boxed{R_e = \frac{Vd}{\nu}}$

② Froude's number (F_r):-

It is defined as the square root of the ratio of the inertia force & the gravity force.

$$F_r = \sqrt{\frac{F_i}{F_g}}$$

where $F_i = \rho A V^2$

F_g = Mass $\times$ acceleration due to gravity

$= \rho \times \text{Volume} \times g$ [$\because L^2 = A$ = area]

$= \rho L^3 g = \rho L^2 L g = \rho A L g$

$$F_r = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \frac{V}{\sqrt{Lg}}$$

3. Euler's Number (Eu) :-

It is defined as the square root of the ratio of the inertia force to the pressure force.

$$Eu = \sqrt{\frac{F_i}{F_p}}$$

$F_i = \rho A V^2$, F_p = intensity of pressure × Area.

$= P \times A$.

$$Eu = \sqrt{\frac{\rho A V^2}{P \times A}} = \sqrt{\frac{V^2}{P/\rho}} = \frac{V}{\sqrt{P/\rho}}$$

4. Weber's Number (We) :-

It is defined as the square root of the ratio of the inertia force to the surface tension force.

$$We = \sqrt{\frac{F_i}{F_s}}$$

$F_i = \rho A V^2$, F_s = Surface tension force.

= Surface tension × length.

$$= \sqrt{\frac{\rho A V^2}{\sigma L}} = \sqrt{\frac{\rho \times L^2 \times V^2}{\sigma L}} \qquad [\because A = L^2]$$

$$= \sqrt{\frac{\rho L \times V^2}{\sigma}} = \frac{V}{\sqrt{\sigma / \rho L}}$$

5. Mach Number (M) :-

It is defined as the square root of the ratio of the inertia force to the elastic force.

$$M = \sqrt{\frac{F_i}{F_e}} \qquad F_i = \rho A V^2$$

Fe = Elastic force = Elastic stress × Area.

$= K \times A = K \times L^2$

$$M = \sqrt{\frac{\rho A V^2}{K L^2}} = \sqrt{\frac{\rho L^2 V^2}{K L^2}} = \frac{V}{\sqrt{K/\rho}}$$

$\sqrt{K/\rho} = C$ = velocity of Sound in the fluid.

$$\boxed{M = \frac{V}{C}}$$

Impact of Free Jets

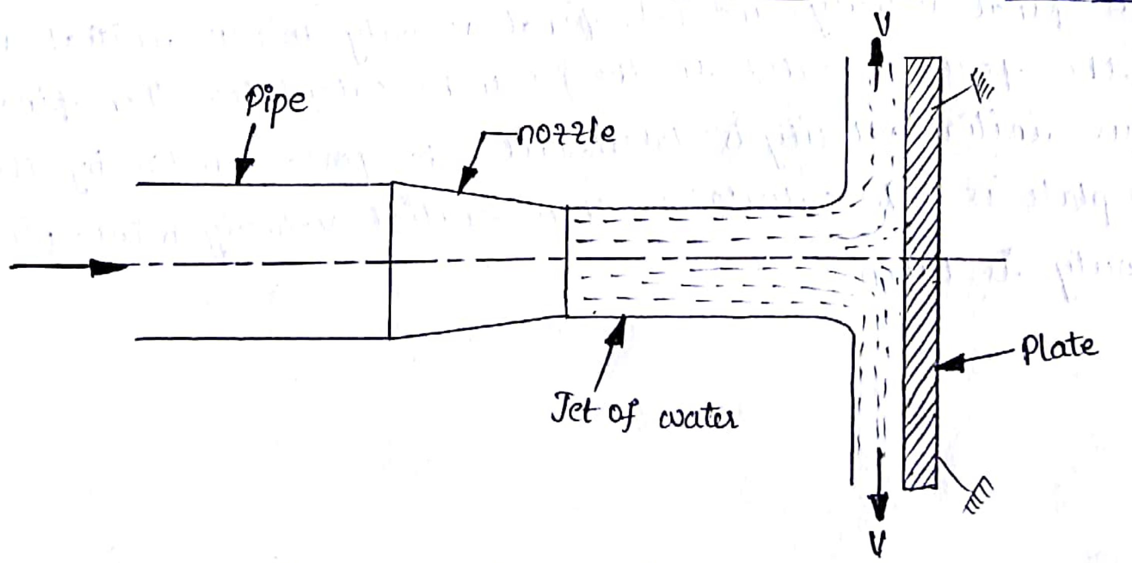
The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure. If some plate, which may be fixed or moving, is placed in the path of jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion (or) from Impulse-momentum equation. Thus impact of jet means the force exerted by the jet on a plate which may be stationary (or) moving.

The following cases of the impact of jet i.e., the force exerted by the jet on a plate, will be considered.

(a) Force exerted by the jet on a stationary plate when
  i. plate is vertical to the jet.
  ii. plate is inclined to the jet.
  iii. plate is curved.

(b) Force exerted by the jet on a moving plate when
  i. plate is vertical to the jet.
  ii. plate is inclined to the jet.
  iii. Plate is curved.

Force exerted BY THE JET ON A STATIONARY VERTICAL PLATE:



Consider a jet of water coming out from the nozzle, strikes a flat vertical plate as shown in fig.

V = velocity of the jet.

d = diameter of the jet.

a = area of cross-section of the jet = $\frac{\pi}{4}d^2$.

The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will get deflected through 90°. Hence the component of the velocity of jet, in the direction of jet, after striking will be zero.

The force exerted by the jet on the plate in the direction of jet,

F_x = Rate of change of momentum in the direction of force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$F_x = \frac{(\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} (\text{Initial velocity} - \text{Final velocity})$$

= (Mass/sec) [velocity of jet before striking − velocity of jet after striking]

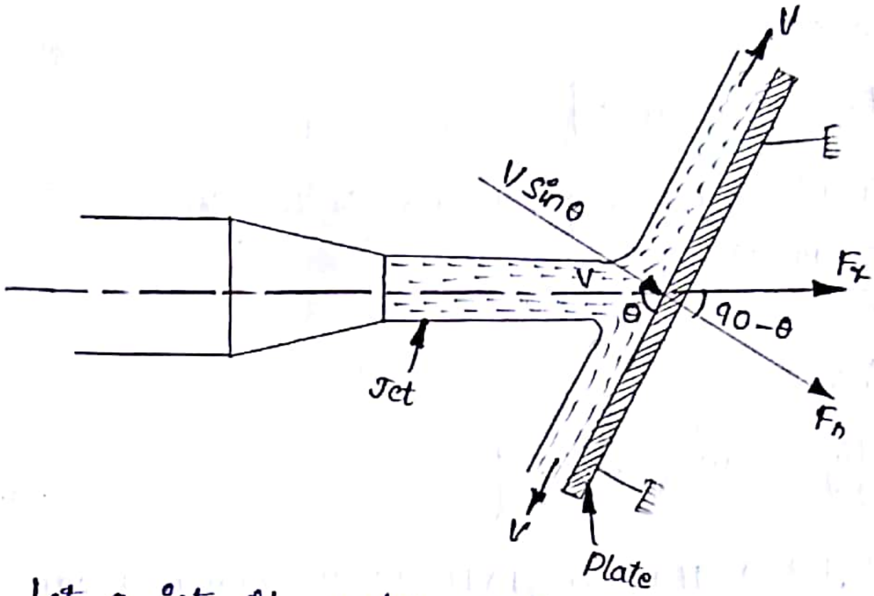
$= \rho a V [V - 0]$ $\quad\because$ mass/sec $= \rho \times a V$

$= \rho a V^2$

$$\therefore \boxed{F_x = \rho a V^2}$$

For deriving above equation, we have taken initial velocity minus final velocity and not final velocity minus initial velocity. If the force exerted on the jet to be calculated then final velocity minus initial velocity is taken. But the force exerted by the jet on the plate is to be calculated, then initial velocity minus final velocity is taken.

FORCE EXERTED BY A JET ON STATIONARY INCLINED FLAT PLATE:



Let a jet of water, coming out from the nozzle strikes an inclined flat plate as shown in fig.

Let V = Velocity of jet in the direction of x.

θ = Angle between the jet and plate.

a = Area of cross-section of the jet.

Then mass of water per second striking the plate $= \rho \times aV$

If the plate is smooth and if it is assumed that there is no loss of energy due to impact of the jet, the jet will move over the plate after striking with a velocity equal to initial velocity i.e., with a velocity V. Let us find the force exerted by the jet on the plate in the direction normal to the plate.

Let this force is represented by F_n.

Then F_n = mass of jet striking per second × [Initial velocity of jet before striking in the direction of n − Final velocity of jet after striking in direction of n]

$= \rho aV [V \sin\theta - 0]$

$$\boxed{F_n = \rho a V^2 \sin\theta}$$

This force can be resolved into two components, one in the direction of jet and other perpendicular to the direction of flow. Then we have

F_x = component of F_n in the direction of flow

$= F_n \cos(90-\theta)$

$= F_n \sin\theta$

$= \rho a V^2 \sin\theta \cdot \sin\theta$

$= \rho a V^2 \sin^2\theta$

$$\boxed{F_x = \rho a V^2 \sin^2\theta}$$

and F_y = component of F_n, perpendicular to the flow.

$= F_n \sin(90-\theta)$

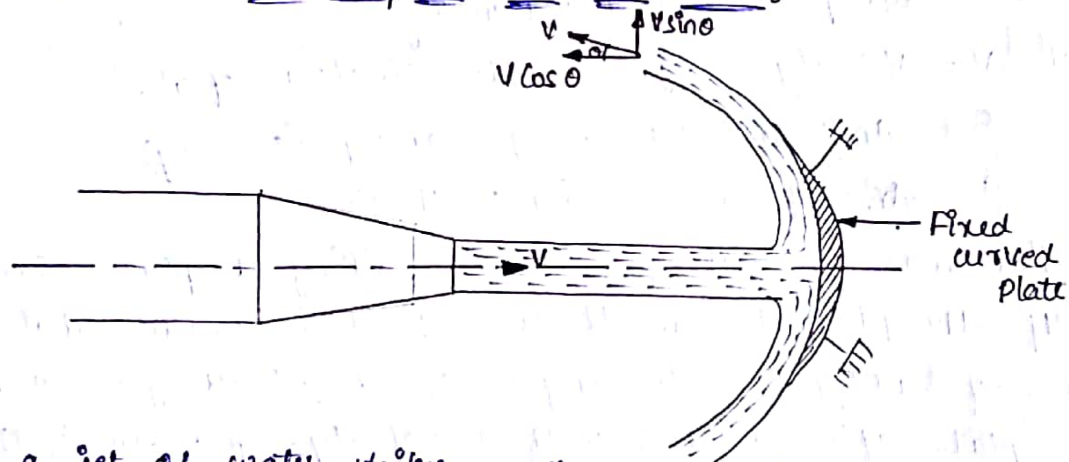
$= F_n \cos\theta$

$= \rho a V^2 \sin\theta \cdot \cos\theta$

$$\boxed{F_y = \rho a V^2 \sin\theta \cdot \cos\theta}$$

→ FORCE EXERTED BY A JET ON STATIONARY CURVED PLATE:

(a) Jet strikes the curved plate at the centre:



Let a jet of water strikes a fixed curved plate at the centre as shown in fig. The jet after striking the plate, comes out with the same velocity if the plate is smooth and there is no loss of energy due to impact of jet, in the tangential direction of the curved plate. The velocity at outlet ~~of~~ the plate can be resolved into two components, one in the direction of jet and other perpendicular to the direction of jet.

Component of velocity in the direction of jet $= -V\cos\theta$ (-ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out from nozzle)

component of velocity perpendicular to the jet $= V\sin\theta$.

Force exerted by the jet in the direction of jet,

F_x = Mass per sec $\times [V_{1x} - V_{2x}]$

where V_{1x} = Initial velocity in the direction of jet = V.

V_{2x} = Final velocity in the direction of jet = $-V\cos\theta$.

$$F_x = \rho a V[V - (-V\cos\theta)]$$
$$= \rho a V[V + V\cos\theta]$$
$$= \rho a V \cdot V(1+\cos\theta)$$
$$= \rho a V^2(1+\cos\theta)$$

$$\boxed{F_x = \rho a V^2(1+\cos\theta)}$$

Similarly, F_y = mass per sec $[V_{1y} - V_{2y}]$

where V_{1y} = Initial velocity in the direction of $y = 0$.

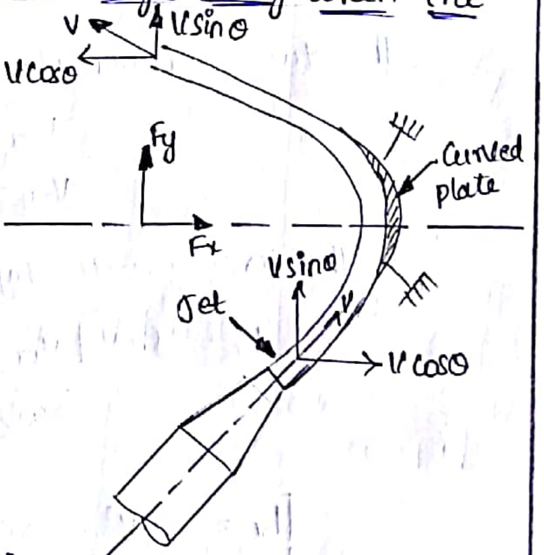
V_{2y} = Final velocity in the direction of $y = V\sin\theta$.

$$\therefore F_y = \rho a V[0 - V\sin\theta]$$
$$= \rho a V(-V\sin\theta)$$
$$= -\rho a V^2\sin\theta$$

$$\boxed{F_y = -\rho a V^2\sin\theta}$$

Negative sign means the force is acting in the downward direction. In this case the angle of deflection of the jet = $(180-\theta)$.

(b) Jet strikes the curved plate at one end tangentially when the plate is symmetrical:



Let the jet strikes the curved fixed plate at one end tangentially as shown in fig. Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let V = velocity of jet of water.

θ = Angle made by jet with x-axis at inlet tip of the curved plate.

If the plate is smooth and loss of energy due to impact is zero, then the velocity of water at the outlet tip of the curved plate will be equal to V. The forces exerted by the jet of water in the direction of x and y are F_x = mass/sec $[V_{1x} - V_{2x}]$

$= \rho a v [v \cos\theta - (-v \cos\theta)]$

$= \rho a v (2 v \cos\theta)$

$$\boxed{F_x = 2\rho a v^2 \cos\theta}$$

$F_y = $ mass/sec $[V_{1y} - V_{2y}]$

$= \rho a v [v \sin\theta - v \sin\theta]$

$= \rho a v (0)$

$= 0$

$$\boxed{F_y = 0}$$

(C) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:

When the plate is unsymmetrical about x-axis, then the angles made by the tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let θ = angle made by tangent at inlet tip with x-axis.

ϕ = angle made by tangent at outlet tip with x-axis.

The two components of velocity at inlet are,

$V_{1x} = v \cos\theta$

$V_{1y} = v \sin\theta$

The two components of velocity at outlet are

$V_{2x} = -v \cos\phi$

$V_{2y} = v \sin\phi$

∴ The force exerted by the jet of water in the directions of x and y are:

$F_x = \rho a v [V_{1x} - V_{2x}] = \rho a v [v \cos\theta - (-v \cos\phi)]$

$= \rho a v [v \cos\theta + v \cos\phi]$

$= \rho a v^2 (\cos\theta + \cos\phi)$

$$\boxed{F_x = \rho a v^2 (\cos\theta + \cos\phi)}$$

$F_y = \rho a v [V_{1y} - V_{2y}]$

$= \rho a v [v \sin\theta - v \sin\phi]$

$= \rho a v \cdot v (\sin\theta - \sin\phi)$

$= \rho a v^2 (\sin\theta - \sin\phi)$

$$\boxed{F_y = \rho a v^2 (\sin\theta - \sin\phi)}$$

Find the force exerted by a jet water of diameter 75mm on a stationary flat plate when the jet strikes the plate normally with a velocity of 20 m/sec.

Sol: Given:

diameter of jet, $d = 75mm = 0.075m$

velocity of jet, $V = 20 m/sec$

area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.075)^2 = 0.00417 m^2$

The force exerted by the jet of water on a stationary vertical plate is given by,

$$F = \rho a V^2$$
$$= 1000 \times 0.00417 \times (20)^2$$
$$= \underline{1766.8 N}$$

2) Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100mm and the head of water at the centre of nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95

Sol: diameter of the nozzle, $d = 100mm = 0.1m$

Area of nozzle, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 0.007854 m^2$

Head of water, $H = 100m$

Co-efficient of velocity $C_v = 0.95$

Theoritical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \ m/sec.$$

But $C_v = \frac{\text{Actual velocity}}{\text{Theoritical velocity}}$

Actual velocity $= C_v$ (theoritical velocity)

$$= 0.95 \times 44.294$$
$$= 42.08 \ m/sec.$$

3. Force on a fixed vertical plate is given by

$$F = \rho a V^2$$
$$= 1000 \times 0.007854 \times (42.08)^2$$
$$= 13907.2 N$$
$$= \underline{13.9 \ kN}$$

3) A jet of water of diameter 75mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by jet on the plate.

(i) in the direction normal to the plate.
(ii) in the direction of the jet.

sol: diameter of jet, $d = 75mm = 0.075m$

area, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.075)^2 = 0.004417 m^2$

velocity of jet, $V = 25$ m/sec

angle between jet and plate, $\theta = 60°$.

(i) The force exerted by the jet of water in the direction normal to the plate is given by,

$$F_n = \rho a V^2 \sin\theta$$
$$= 1000 \times 0.004417 \times (25)^2 \sin 60.$$
$$= \underline{2390.7\ N}$$

(ii) The force exerted by the jet of water in the direction of the jet.

$$F_x = \rho a V^2 \sin^2\theta$$
$$= 1000 \times 0.004417 \times (25)^2 \sin^2 60$$
$$= \underline{2070.4 N}$$

4) A jet of water of diameter 50mm strikes a fixed in such a way that the angle between the place and jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.

sol: Diameter of jet, $d = 50mm = 0.05m$

angle between plate and jet, $\theta = 30°$

Force exerted in direction of jet, $F_x = 1471.5N$

area of jet, $a = \frac{\pi}{4}(0.05)^2 = 0.001963 m^2$

Force in the direction of jet is given by

$$F_x = \rho a V^2 \sin\theta$$
$$1471.5 = 1000 \times 0.001963 \times V^2 \sin 30°$$
$$1471.5 = 0.49 V^2$$
$$V^2 = \frac{1471.5}{0.49} = 3003.06$$
$$V = 54.8 m/s$$

$\therefore$ Discharge, Q = Area x Velocity

$= 0.001963 \times 54.8$

$= 0.10757 \ m^3/sec$

$= 107.57$ lit/sec

5) A jet of water of diameter 50mm moving with a velocity of 40m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of jet, if the is deflected through an angle of 120° at the outlet of the curved plate.

sol: diameter of the jet, $d = 50mm = 0.05m$

Area, $a = \frac{\pi}{4}(0.05)^2 = 0.001963 m^2$

Velocity of jet, $V = 40 m/s$

Angle of deflection, $\theta = 120°$

The angle of deflection $= 180 - \theta$

$= 180 - 120 = 60°$

Force exerted by the jet on the curved plate in the direction of the jet is given by

$$F_x = \rho a V^2 (1 + \cos\theta)$$

$$= 1000 \times 0.001963 \ (40)^2 [1 + \cos 60°]$$

$$= 4711.2 N$$

6) A jet of water of diameter 75mm moving with a velocity of 30m/s, strikes a curved fixed plate at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by jet on the plate in the horizontal and vertical direction.

sol: diameter of the jet, $d = 75mm = 0.075m$

$\therefore$ Area, $a = \frac{\pi}{4}(0.075)^2 = 0.004417 m^2$

Velocity of jet, $V = 30 m/s$

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$.

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of x is given by

$$F_x = \rho a V^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times 0.004417 \times (30)^2 [\cos 30 + \cos 20]$$

$$= 7178.2 N$$

$F_y = \rho a V^2 [\sin\theta - \sin\phi]$

$= 1000 \times 0.004417 \times (30)^2 [\sin 30 - \sin 20]$

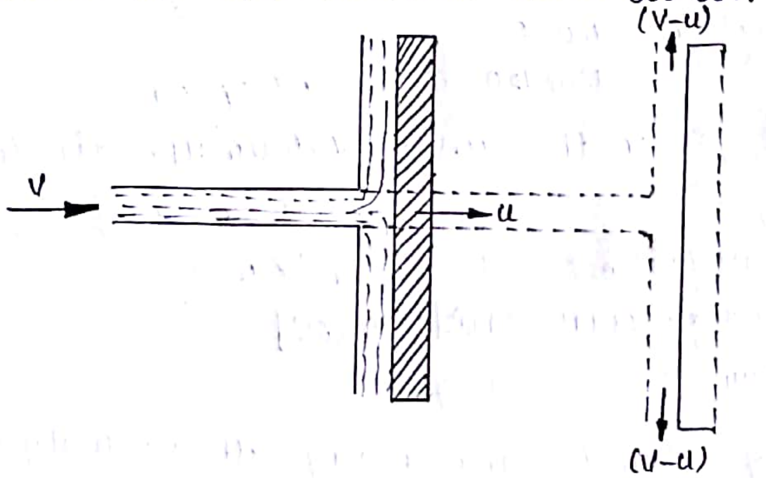
$= \underline{628.02\,N}$

→ **FORCE EXERTED BY A JET ON MOVING PLATES:**

The following cases of the moving plates will be considered.

1. Flat vertical plate moving in the direction of the jet away from the jet.
2. Inclined plate moving in the direction of the jet.
3. Curved plate moving in the direction of the jet (or) in the horizontal direction.

→ **Force on Flat vertical plate moving in the direction of Jet:**



Above fig. shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity of the absolute jet,

a = area of cross-section of the jet,

u = Velocity of the plate.

In this case, the jet doesnot strike the plate with a velocity V, but it strikes with a relative velocity, which is equal to the absolute velocity of jet of water minus the velocity of plate.

Hence relative velocity of jet with respect to plate $= (V-u)$

Mass of water striking the plate per sec.

$= \rho \times$ Area of jet $\times$ Velocity with which jet strikes the plate

$= \rho a (V-u)$

∴ Force exerted by the jet on the moving plate in the direction of the jet,

= Mass of water striking per sec × [Initial velocity with which water strikes − Final velocity]

$= \rho a (V-u)[(V-u)-0]$

$$\boxed{F_x = \rho a (V-u)^2}$$

In this case the work will be done by the jet on the plate, as plate is moving. For the stationary plate the work will be zero.

∴ Workdone per second by the jet on the plate

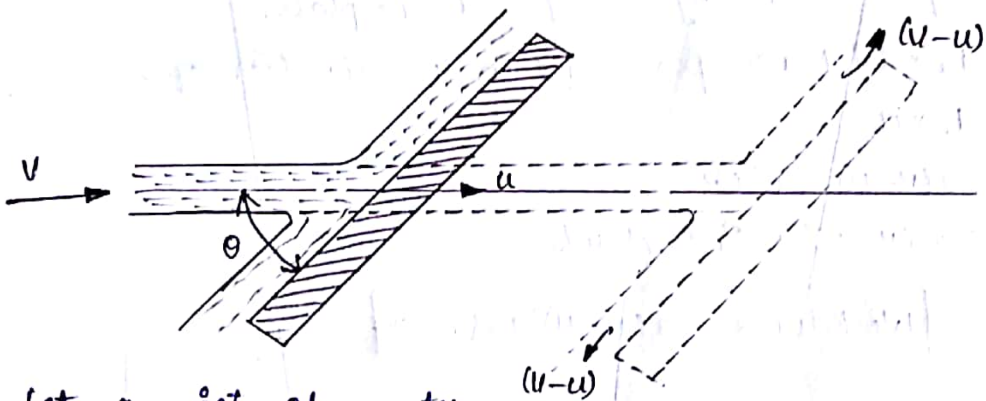
$= \text{Force} \times \dfrac{\text{distance in the direction of force}}{\text{Time}}$

$= F_x \times u$

$= \rho a (V-u)^2 u$

$$\boxed{\text{Workdone} = \rho a (V-u)^2 u}$$

→ Force on the inclined plate moving in the direction of the Jet:



Let a jet of water strikes a inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in fig.

Let V = absolute velocity of jet of water

u = velocity of the plate in the direction of jet

a = cross-sectional area of jet.

θ = Angle between jet and plate.

Relative velocity of jet of water $= (V-u)$

∴ The velocity with which jet strikes $= (V-u)$

Mass of water striking per second $= \rho \times a \times (V-u)$

If the plate is smooth and loss of energy due to impact of jet is assumed as zero, the jet of water will leave the inclined plate with a velocity equal to $(V-u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as,

F_n = Mass striking per second × [Initial velocity in the normal direction with which jet strikes − Final velocity]

$= \rho a(V-u)[(V-u)\sin\theta - 0]$

$= \rho a(V-u)^2 \sin\theta$

$$\boxed{F_n = \rho a(V-u)^2 \sin\theta}$$

The normal force F_n is resolved into two component namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$F_x = F_n \sin\theta = \rho a(V-u)^2 \sin^2\theta$

$F_y = F_n \cos\theta = \rho a(V-u)^2 \sin\theta \cdot \cos\theta$

$$\boxed{\begin{aligned} F_x &= \rho a(V-u)^2 \sin^2\theta \\ F_y &= \rho a(V-u)^2 \sin\theta \cdot \cos\theta \end{aligned}}$$

Workdone per second by the jet on the plate,

$= F_x \times$ distance per second in the direction of x.

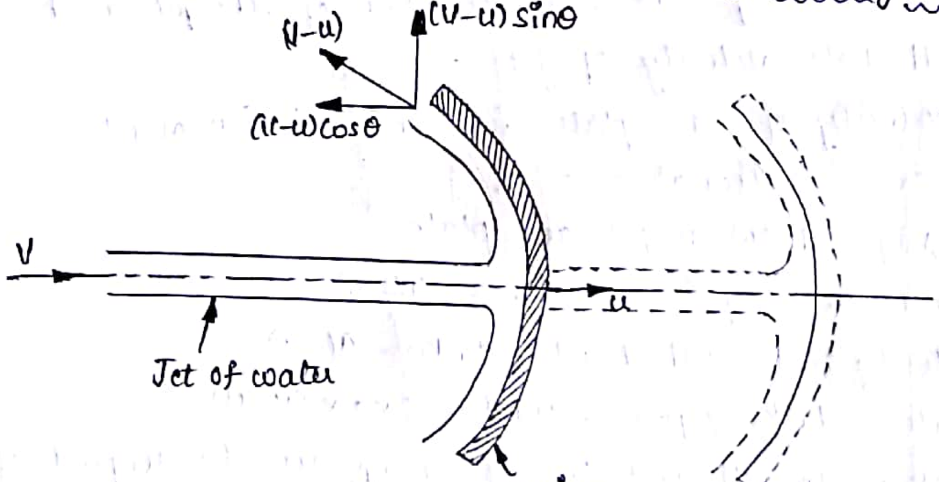
$= F_x \times u$

$= \rho a(V-u)^2 \sin^2\theta \cdot u$

$= \rho a(V-u)^2 u \sin\theta$ N-m/sec

$$\boxed{\text{Workdone} = \rho a(V-u)^2 u \sin^2\theta}$$

→ Force on the curved plate when the plate is moving in the direction of jet:



Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of jet as shown in fig.

V = Absolute velocity of jet,
a = Area of jet,
u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate $= (V-u)$.

If the plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane $= (V-u)$.

This velocity can be resolved into two components, one in the direction of jet and other perpendicular to direction of jet.

Component of velocity in the direction of jet $= -(V-u)\cos\theta$.

(-ve sign is taken as at the outlet, the component is in opposite direction of the jet)

Component of velocity in direction perpendicular to the direction of jet $= (V-u)\sin\theta$

Mass of the water striking the plate
$= \rho \times a \times$ velocity with which jet strikes the plate.
$= \rho a (V-u)$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of jet

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet − Final velocity]

$= \rho a (V-u)[(V-u) - (-(V-u)\cos\theta)]$

$= \rho a (V-u)[1+\cos\theta](V-u)$

$= \rho a (V-u)^2 (1+\cos\theta)$

$$\boxed{F_x = \rho a (V-u)^2 (1+\cos\theta)}$$

Work done by the jet on the plate per second

$= F_x \times$ Distance travelled per second in the direction of x.

$= F_x \times u$

$= \rho a (V-u)^2 (1+\cos\theta) u$

$= \rho a (V-u)^2 u (1+\cos\theta)$

$$\boxed{\text{Workdone} = \rho a (V-u)^2 u (1+\cos\theta)}$$

Force exerted by a jet of water on an un-symmetrical moving curved plate when jet strikes tangentially at one of the tips:

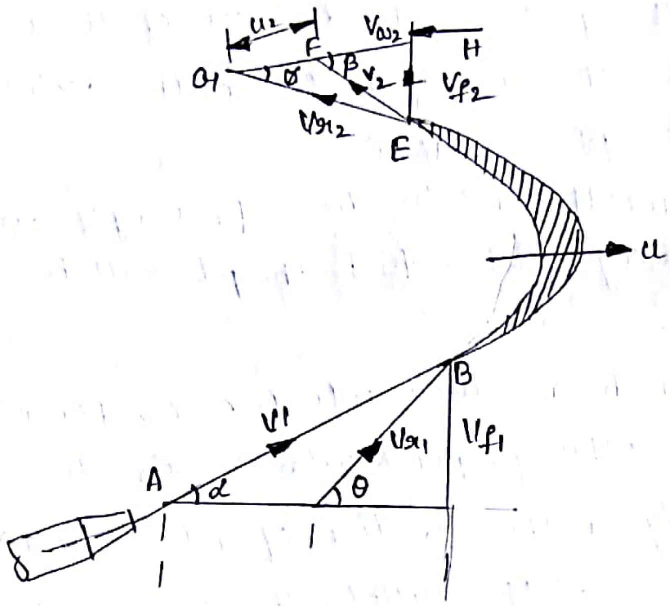


Figure shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to relative velocity of jet with respect to plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of the jet and velocity of the plate at inlet.

Let V_1 = Velocity of the jet at inlet

u_1 = velocity of plate at inlet

α = Angle between the direction of the jet and direction of motion of plate, also called guide blade angle

V_{r_1} = Relative velocity of jet and plate at inlet.

θ = Angle made by relative velocity (V_{r_1}) with the direction of motion at inlet also called vane angle at inlet.

V_{w_1} and V_{f_1} = The components of the velocity of jet V_1 in the direction of motion and perpendicular to direction of motion of vane respectively.

V_{w_1} = Also known as velocity of whirl at inlet

V_{f_1} = also known as velocity of flow at inlet

V_2 = Velocity of jet, leaving the vane or velocity of jet at outlet of the vane.

u_2 = Velocity of vane at outlet.

V_{r_2} = relative velocity of jet with respect to vane at outlet.

β = Angle made by velocity V_2 with direction of motion of the vane at outlet.

ϕ = Angle made by relative velocity V_{r_2} with the direction of motion of vane at outlet and also called vane angle at outlet.

V_{w_2} and V_{f_2} = components of the velocity V_2 in the direction of motion of the vane at outlet and also called vane angle at outlet

V_{w_2} = Also called velocity of whirl at outlet.

V_{f_2} = Velocity of flow at outlet.

The triangles ABD and EGH are called velocity triangles at inlet and outlet.

1. Velocity of triangles at Inlet: Take any point A and draw a line $AB = V_1$ in magnitude and direction which means line AB makes an angle α with the horizontal line AD. Next draw a line $AC = u_1$ in magnitude join C to B. Then CB represents the relative velocity of the jet at inlet.

If the loss of energy at inlet due to impact is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D

Then BD = Velocity of flow at inlet $= V_{f_1}$.

AD = Velocity of whirl at Inlet $= V_{w_1}$.

$\angle BCD$ = Vane angle at inlet $= \theta$.

2. Velocity triangle at outlet: If the vane surface is assumed to be very smooth the loss of energy due to friction will be zero. The water will be gliding over the surface of vane with a relative velocity equal to V_{r_1} and will come out of the vane with relative velocity V_{r_1}. This means that the relative velocity at outlet $V_{r_2} = V_{r_1}$. And also the relative velocity at outlet should be in tangential direction to the vane at outlet.

Draw EG in the tangential direction of the vane at outlet and cut $EG = V_{r_2}$. From G, draw a line GF in the direction of vane at outlet equal to u_2, the velocity of vane at outlet. Join EF. Then EF represents the absolute velocity of jet at outlet in magnitude and direction. From E draw a vertical line EH to meet the line GF produced at H.

Then EH = Velocity of flow at outlet $= V_{f_2}$.

FH = Velocity of whirl at outlet $= V_{w_2}$.

ϕ = Angle of vane at outlet.

β = Angle made by V_2 with the direction of motion of vane at outlet.

If the vane is smooth and having velocity in the direction of motion at inlet and outlet equal then

$u_1 = u_2 = u$ = velocity of vane in the direction of motion

$$V_{r_1} = V_{r_2}$$

Now mass of water striking vane per sec $= \rho a V_{r_1}$ ——— (i)

where a = Area of jet of water

V_{r_1} = Relative velocity at inlet.

$\therefore$ Force exerted by the jet in the direction of motion,

F_x = Mass of water striking per sec × [Initial velocity with which jet strikes in the direction of motion − Final velocity of jet in the direction of motion] ——— (ii)

But initial velocity with which jet strikes the vane $= V_{r_1}$

The component of this velocity in the direction of motion

$$= V_{r_1}\cos\theta = (V_{w_1} - u_1)$$

Similarly, the component of relative velocity at outlet in the direction of motion $= -V_{r_2}\cos\phi$

$$= -[u_2 + V_{w_2}]$$

−ve sign is taken as the component of V_{r_2} in the direction of motion is in the opposite direction.

Substituting the eqn. (i) and all above values of the velocities in eq (ii)

$$F_x = \rho a V_{r_1}\left[(V_{w_1} - u_1) - \{-(u_2 + V_{w_2})\}\right]$$

$$= \rho a V_{r_1}\left[V_{w_1} - u_1 + u_2 + V_{w_2}\right]$$

$[\because u_1 = u_2]$

$$F_x = \rho a V_{r_1}\left[V_{w_1} + V_{w_2}\right] \text{ ——— (iii)}$$

The eqn (iii) is true only when angle β is an acute angle. If $\beta = 90°$, the $V_{w_2} = 0$, then eqn-(iii), becomes as $F_x = \rho a V_{r_1} w_1$

If β is an obtuse angle, the expression for F_x will become,

$$F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]$$

Thus, in general, F_x is written as,

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]}$$

Workdone per second on the vane by the jet

= Force x Distance per second in direction of force

$= F_x \times u$

$$\boxed{W.D = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}$$

∴ Workdone per second per unit ~~length~~ weight of fluid striking per second

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{Wt. of fluid striking/sec} \cdot \frac{Nm/s}{N/s}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{g \times \rho a V_{r_1}}$$

$$= \boxed{\frac{1}{g} [V_{w_1} + V_{w_2}] \times u \quad N\text{-}m/N}$$

Workdone/sec per unit mass of fluid striking per second

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{mass of fluid striking/sec} \cdot \frac{N\text{-}m/s}{kg/s}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{\rho a V_{r_1}}$$

$$= \boxed{[V_{w_1} \pm V_{w_2}] \times u \quad N\text{-}m/kg}$$

) A jet of water of diameter 10cm strikes a flat plate normally with a velocity of 15 m/sec. The plate is moving with a velocity of 6m/sec, in the direction of jet and away from the jet. Find

(i) Force exerted by jet on the plate.
(ii) Workdone by jet on the plate per second.
(iii) power (iv) Efficiency

Sol: Given;

diameter of the jet, $d = 10cm = 0.1m$

$\therefore$ Area, $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.1)^2 = 0.007854\ m^2$

velocity of jet, $V = 15m/sec$

velocity of plate, $u = 6m/sec$

(i) The force exerted by the jet on a moving flat vertical plate is given by $F_x = \rho a(V-u)^2$

$= 1000 \times 0.007854(15-6)^2$

$= \underline{636.17\ N}$

(ii) work done per second by the jet

$= F_x \times u$

$= 636.17 \times 6$

$= 3817.02\ N\text{-}m/sec$

(iii) power of the jet, in kW

$$P = \frac{\text{Workdone per second}}{1000}$$

$$= \frac{3817.02}{1000} = 3.817\ kW$$

(iv) Efficiency of the jet

$$\eta = \frac{\text{Output of jet per second}}{\text{Input of jet per second}}$$

where output of jet per second = workdone by jet per second = 3817.02 N-m/s

Input per second = kinetic energy of jet /sec

$= \frac{1}{2}\left(\frac{mass}{sec}\right)V^2 = \frac{1}{2}(\rho aV)V^2 = \frac{1}{2}(\rho aV^3)$

$= \frac{1}{2} \times 1000 \times 0.007854 \times (15)^3$

$= 13253.6\ N\text{-}m/s$

$\therefore \eta$ of jet $= \frac{3817.02}{13253.6} = 0.288 = \underline{28.8\%}$

A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to axis of jet. Find the normal force on the plate : (i) When the plate is stationary.

(ii) When the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of jet when the plate is moving.

Sol: diameter of jet, $d = 7.5\ cm = 0.075\ m$

$\therefore$ area, $a = \frac{\pi}{4}(0.075)^2 = 0.004417\ m^2$

Angle between the jet and plate, $\theta = 90 - 45 = 45°$

velocity of jet, $V = 30\ m/s$

(i) When the plate is stationary, the normal force on the plate is given by,

$F_n = \rho a V^2 \sin\theta$

$= 1000 \times 0.004417 (30)^2 \sin 45°$

$= \underline{2810.96\ N}$

(ii) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by

$F_n = \rho a (V-u)^2 \sin\theta$

$= 1000 \times 0.004417 \times (30-15)^2 \sin 45$

$= 702.74\ N$

(iii) The power and efficiency of jet, when plate is moving is obtained as, Workdone per second by jet = Force in direction jet × distance moved by plate in direction of jet /sec

$= F_x \times u$

$= F_n \sin\theta \times u$

$= 702.74 \sin 45 \times 15$

$= 496.9 \times 15 = 7453.5\ N\text{-}m/s$

$\therefore$ Power in kW $= \frac{\text{Workdone per second}}{1000} = \frac{7453.5}{1000} = \underline{7.4535\ kW}$

Efficiency of the jet $= \frac{\text{output}}{\text{input}}$

$= \frac{\text{Workdone per second}}{\text{K.E of jet}} = \frac{7453.5}{\frac{1}{2}(\rho a V)V^2}$

$= \frac{7453.5}{1/2 \times 1000 \times 0.004417 \times 30^3} = 0.1249 \simeq 0.125 = 12.5\,\%$

A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of jet. The jet is deflected through an angle of 165°. Assuming the plate smooth:

(i) Force exerted on the plate in direction of jet.
(ii) Power of jet, and
(iii) Efficiency of jet.

Sol: Diameter of jet, $d = 7.5\,cm = 0.075\,m$

Area, $a = \frac{\pi}{4}(0.075)^2 = 0.004417\,m^2$

velocity of jet, $V = 20\,m/s$.

velocity of plate, $u = 8\,m/s$.

angle of deflection of jet $= 165°$

$\therefore$ angle made by the relative velocity at the outlet of plate,

$$\theta = 180 - 165 = 15°$$

(i) Force exerted by jet on plate in direction of jet is given by

$$F_x = \rho a (V-u)^2 (1+\cos\theta)$$
$$= 1000 \times 0.004417 (20-8)^2 (1+\cos 15°)$$
$$= 1250.38\,N$$

(ii) Workdone by jet on the plate per second

$$= F_x \times u$$
$$= 1250.38 \times 8 = 10003.04\ N\text{-}m/s$$

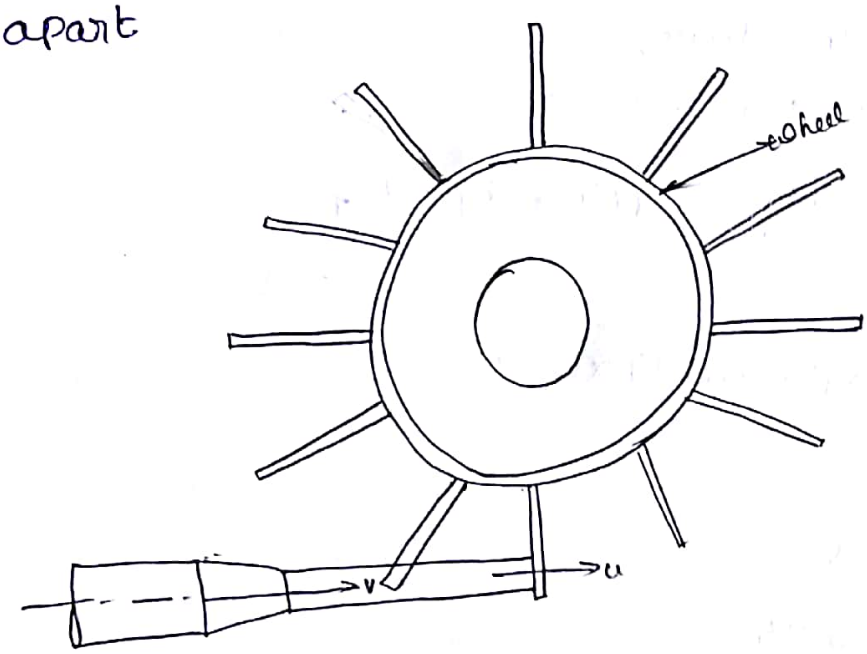
$$\therefore \text{power of jet} = \frac{10003.04}{1000}$$
$$= 10\,kW$$

(iii) Efficiency of jet $= \dfrac{\text{output}}{\text{Input}} = \dfrac{\text{Workdone by jet/sec}}{\text{K.E of jet/sec}}$

$$= \frac{1250.38}{\frac{1}{2}\rho a u^3}$$
$$= \frac{1250.38}{\frac{1}{2} \times 1000 \times 0.004417 \times 20^3}$$
$$= 0.564$$
$$= 56.4\,\%$$

Force Exerted by a Jet of water on a Series of Vanes:

The force exerted by a jet of water on a single moving plate (which may be flat or curved) is not Practically feasible. This case is only a theoritical case. In actual Practice, a large number of Plates are mounted on the circumference of a wheel at a fixed distance apart



The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving and the second plate mounted on the wheel appears before the jet, which again exerts the force on the second plate. Thus each plate appears successively before the jet and the jet exerts force on each plate.

The wheel starts moving at a constant speed,

let $V =$ velocity of jet

$d =$ diameter of jet

a = cross-sectional area of jet

$= \frac{\pi}{4} d^2$

u = velocity of vane

In this case the mass of water coming out from the nozzle per second is always in contact with the plates, when all the plates are considered. Hence mass of water per second striking the series of plates = $\rho a v$.

Also the jet strikes the plate with a velocity = $V - u$

After striking, the jet moves tangential to the plate and hence the velocity component in the direction of plate is equal to zero.

$\therefore$ The force exerted by the jet in the direction of motion of plate,

F_x = Mass per second [Initial velocity - final velocity]

$= \rho a v [(v-u) - 0]$

$= \rho a v (v-u)$

$$\boxed{\therefore F_x = \rho a v [v-u]}$$

Work done by the jet on the series of plates per second = Force x distance per second in the direction of force

$= F_x \times u$

$= \rho a v [v-u] u$

Kinetic energy of jet per second

$= \frac{1}{2} m v^2 = \frac{1}{2} \rho a v v^2 = \frac{1}{2} \rho a v^3$

$\therefore$ Efficiency $\eta = \frac{\text{work done per second}}{\text{Kinetic energy per second}}$

$$\frac{\rho a v [v-u] u}{\frac{1}{2} \rho a v^3} \quad \frac{2u[v-u]}{v^2}$$

$$\boxed{\eta = \frac{2u[v-u]}{v^2}}$$

Condition for maximum Efficiency: The above equation gives the value of the efficiency of the wheel. For a given jet velocity v, the efficiency will be maximum when

$$\frac{d\eta}{du} = 0$$

$$\frac{d}{du}\left[\frac{2u(v-u)}{v^2}\right] = 0$$

$$\frac{d}{du}\left[\frac{2uv-2u^2}{v^2}\right] = 0$$

$$\frac{1}{v^2}[2v-4u] = 0$$

$$2v-4u = 0$$

$$2v = 4u$$

$$\boxed{v = 2u} \quad \text{or} \quad u = \frac{v}{2}$$

Maximum Efficiency: Substituting the value of $v = 2u$ in efficiency equation

$$\eta_{max} = \frac{2u[2u-u]}{4u^2}$$

$$= \frac{2u^2}{4u^2} = \frac{1}{2} \text{ (or) } 50\%$$

$$\boxed{\eta_{max} = 50\%}$$

Force Exerted on a Series of Radical Curved Vanes:

For a radical curved vane, the radius of the vane at inlet and outlet is different and hence the tangential velocities of the radial vane at inlet and outlet will not be equal. Consider a series of radial curved vanes mounted on a wheel as shown in fig. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

let R_1 = Radius of wheel at inlet of the vane

R_2 = Radius of wheel at outlet of the vane

ω = Angular speed of the wheel.

Then $u_1 = \omega R_1$ and $u_2 = \omega R_2$

The velocity triangles at inlet and outlet are drawn as shown in fig.

The mass of water striking per second for a series of vanes = mass of water coming out from nozzle per sec

$= \rho a v_1$

where a = Area of jet

v_1 = velocity of jet

Momentum of water striking the vanes in the tangential direction per sec. at inlet

= Mass of water per second × component of V_1 in the tangential direction.

$= \rho a V_1 \times V_{w_1}$

[∵ component of V_1 in the tangential direction $= V_1 \cos\alpha = V_{w_1}$]

Similarly momentum of water at outlet per sec.

$= \rho a V_1 \times$ component of V_2 in the tangential direction

$= \rho a V_1 \, (-V_2 \cos\beta)$

$= -\rho a V_1 \, V_{w_2}$ [∵ $V_2 \cos\beta = V_{w_2}$]

−ve sign is taken as the velocity V_2 at outlet is an opposite direction

Now angular momentum per second at inlet

= momentum at inlet × Radius at inlet

$= \rho a V_1 V_{w_1} \times R_1$

Angular momentum per second at outlet

= momentum at outlet × Radius at outlet

$= -\rho a V_1 V_{w_2} \times R_2$

Torque exerted by the water on the wheel

T = Rate of change of angular momentum

= [Initial angular momentum per sec − Final angular momentum per sec]

$= \rho a V_1 \times V_{w_1} \times R_1 - [-\rho a V_1 V_{w_2} R_2]$

$= \rho a V_1 [V_{w_1} R_1 + V_{w_2} R_2]$

Workdone per second on the wheel,

= Torque × angular velocity

$= T \times \omega$

$= \rho a V_1 [V_{w_1} R_1 + V_{w_2} R_2] \times \omega$

$= \rho a V_1 [V_{w_1} R_1 \omega + V_{w_2} R_2 \omega]$ $\left[\because \begin{array}{l} u_1 = R_1 \omega \\ u_2 = R_2 \omega \end{array}\right]$

$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]$

If the angle β is an obtuse angle then workdone per second will be given as

$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$

$\therefore$ The general expression for the workdone per second on the wheel $= \rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]$

If the discharge is radial at outlet, then $\beta = 90°$ and workdone becomes as $= \rho a V_1 [V_{w_1} u_1]$ $\because V_{w_2} = 0$

Efficiency of Radial Curved Vane:

$$\text{Efficiency}, \eta = \frac{\text{Workdone per second}}{\text{Kinetic energy per second}}$$

$$= \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2} (\text{mass} / \text{sec}) V_1^2}$$

$$= \frac{\rho a V_1 [V_{w_1} u_1 \pm V_{w_2} u_2]}{\frac{1}{2} (\rho a V_1) V_1^2}$$

$$= \frac{2 [V_{w_1} u_1 \pm V_{w_2} u_2]}{V_1^2}$$

$$\boxed{\text{Efficiency} = \frac{2[V_{w_1} u_1 + V_{w_2} u_2]}{V_1^2}}$$

1. A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities at inlet and outlet and find:

i) The angles of vanes tips so that water enters and leaves without shock

ii) The workdone per unit weight of water entering the vanes, and

iii) The efficiency.

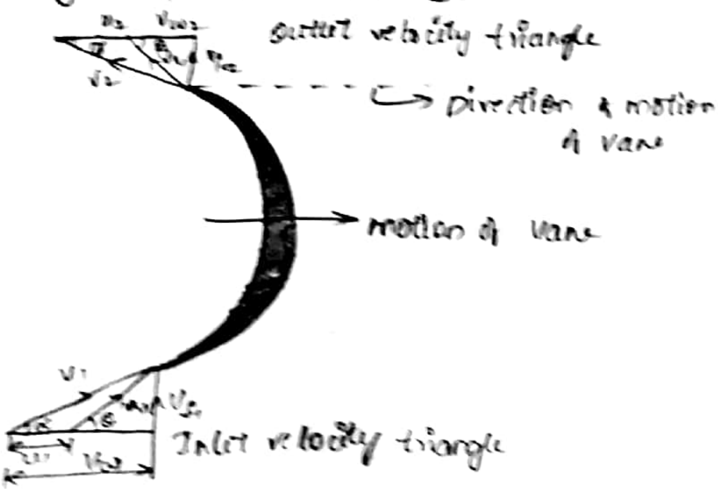
Sol: velocity of jet, $V_1 = 35$ m/s

velocity of vane, $u_1 = u_2 = 20$ m/s

Angle of jet at inlet, $\alpha = 30°$

Angle made by the jet at outlet with the direction of motion of vanes $= 120°$

$\therefore$ Angle $\beta = 180 - 120 = 60°$



a) Angles of vanes tips

From inlet velocity triangle,

$V_{w_1} = V_1 \cos\alpha = 35 \cos 30 = 30.31$ m/sec

$V_{f_1} = V_1 \sin\alpha = 35 \sin 30 = 17.50$ m/sec

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{17.50}{30.31 - 20} = 1.697$$

$$\theta = \tan^{-1}(1.697)$$

$$\underline{\underline{\theta = 60^\circ}}$$

By sine rule, $\dfrac{V_{r_1}}{\sin 90} = \dfrac{V_{f_1}}{\sin\theta}$

$$\frac{V_{r_1}}{1} = \frac{17.50}{\sin 60}$$

$$V_{r_1} = \frac{17.50}{\sin 60}$$

$$V_{r_1} = 20.25 \text{ m/s}$$

$$V_{r_2} = V_{r_1} = 20.25 \text{ m/s}$$

From outlet velocity triangle, by sine rule

$$\frac{V_{r_2}}{\sin 120} = \frac{u_2}{\sin(60-\phi)}$$

$$\frac{20.25}{\sin 120} = \frac{20}{\sin(60-\phi)}$$

$$\sin(60-\phi) = \frac{20}{20.25} \times 0.886$$

$$\sin(60-\phi) = 0.855$$

$$\sin(60-\phi) = \sin(58.75)$$

$$60-\phi = 58.75^\circ$$

$$\phi = 60 - 58.75^\circ$$

$$\phi = 1.25^\circ$$

ii) Work done per unit weight of water entering

$$= \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u_1$$

The value of V_{w_2} is obtained from outlet velocity triangle, $V_{w_2} = V_2 \cos\phi - u_2$

$$= 20.25 \cos 1.25 - 20$$

$= 0.24\ m/s$

$\therefore$ Workdone /unit weight $= \frac{1}{9.81}[30.31 + 0.24] \times 20$

$= 62.28\ N\text{-}m/N$

C. Efficiency, $\eta = \dfrac{\text{workdone per Kg}}{\text{Energy supplied per Kg}}$

$= \dfrac{62.28}{V_1^2/2g} = \dfrac{62.28 \times 2 \times 9.81}{35 \times 35} = 99.74\%$

UNIT-V

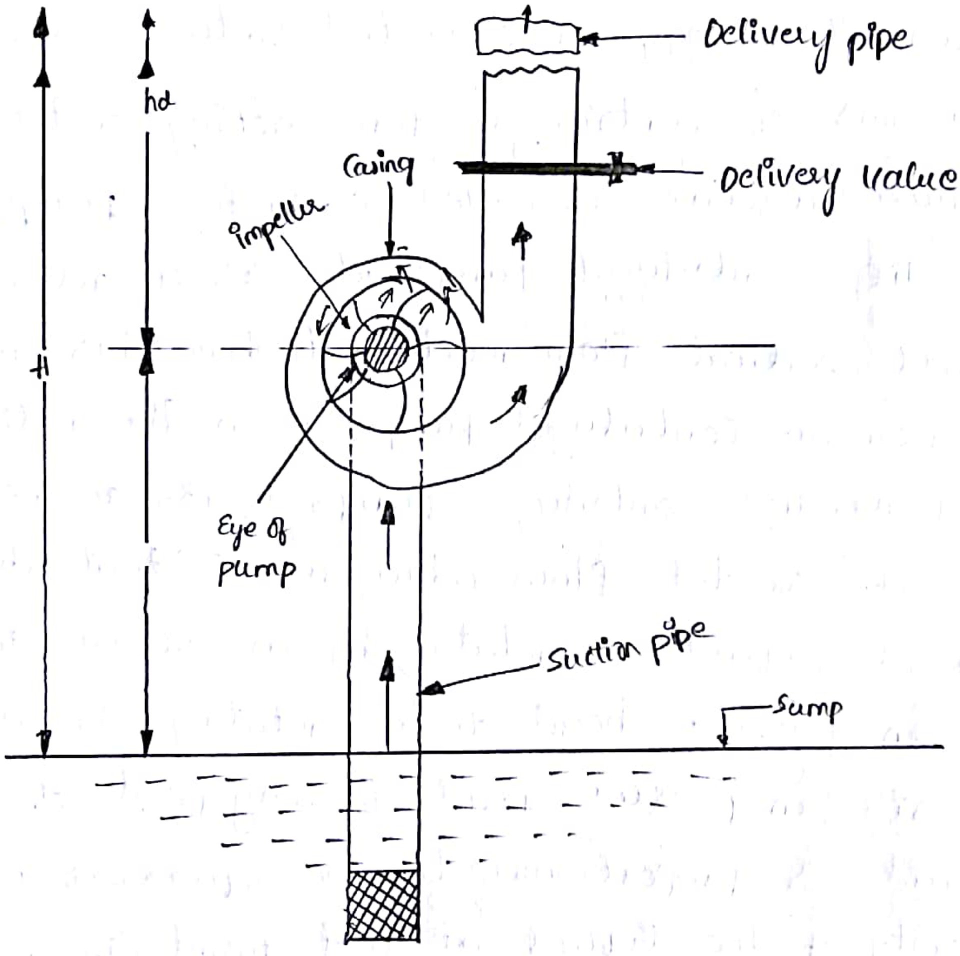
Centrifugal Pumps:

The hydraulic machines which convert the mechanical energy into hydraulic energy are called pumps. The hydraulic energy is in the form of pressure energy. If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called centrifugal pump.

The centrifugal pump acts as a reversed of an inward radical flow reaction turbine. This means that the flow in centrifugal pumps is in the radical outward directions. The centrifugal pumps works on the principle of forced vortex flow which means that when a certain mass of liquid is rotated by an external torque, the rise in pressure head of the rotating liquid takes place. The rise in pressure head of any point of the rotating liquid is proportional to the square of tangential velocity of the liquid at that point [i.e, rise in pressure head $= \frac{V^2}{2g}$ or $\frac{\omega^2 r^2}{2g}$]. Thus at the out let of the impeller where radius is more, the rise in pressure head will be more and the liquid will be discharged at the outlet with a high pressure head. Due to this high pressure head, the liquid can be lifted to a high level.

Main Parts of a Centrifugal Pump

The following are the main parts of a centrifugal Pump:
1. Impeller
2. Casing
3. Suction pipe with a foot valve and a strainer.
4. Delivery Pipe.



* 1. Impreller: The rotating Part of a centrifugal Pump is called Impreller. It consists of a series of backward curved vanes. The impeller is mounted as a shaft which is connected to the shaft of an electric motor.
* 2. Casing: It is an air-tight Passage Surrounding the impeller and is designed in such away that the

kinetic energy of the water discharged at the outlet of the impeller is converted into pressure energy before the water leaves the casing and enters the delivery pipe, The following three types of the casing are commonly adopted.

a. volute casing
b. vortex casing
c. casing with guide blades.

a. volute casing: The above figure shows the volute casing, which surrounds the impeller. It is of spiral type in which area of flow increases gradually. The increase in area of flow decreases the velocity of flow. The decrease in velocity increases the pressure of the water flowing through the casing. It has been observed that in case of volute casing, the efficiency of the pump increases slightly as a large amount of energy is lost due to the formation of eddies in this type of casing.

b. Vortex casing:

If a circular chamber is introduced between the casing and the impeller as shown in fig. the casing is known as vortex casing. By introducing the circular chamber, the loss of energy due to the formation of eddies is reduced to a considerable extent. Thus the

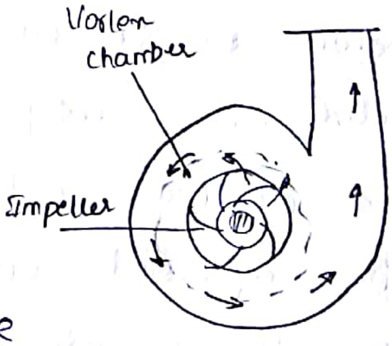
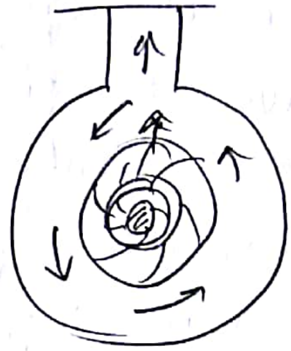


Fig: Vortex casing.

efficiency of the pump is more than the efficiency when only volute casing is provided.

c. Casing with guide blades:

This casing is shown in figure, in which the impeller is surrounded by a series of guide blades mounted on a ring which is known as diffuser. The guide vanes are designed in which a way that the water from the impeller enters the guide vanes without stock.



Also the area of the guide blades vanes increases. thus red reducing the velocity of flow through guide vanes and consequently increasing the pressure of water. The water from the guide vanes then passes through the surrounding casing which is in most of the cases concentric with the impreller.

3. Suction pipe with a foot-value and a stainer:

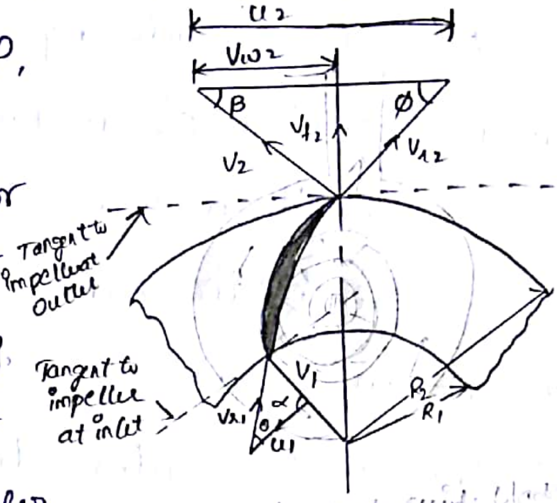
A pipe whose one end is connected to the inlet of the pump and other end dips into water in a sump is known as suction pipe. A foot valve which is a non-return valve or one-way type of valve is fitted at the lower end of the suction pipe. The foot valve opens only in the upward direction. A strainer is also fitted at the lower end of the suction pipe.

4. Delivery pipe: A pipe whose one end is connected to the outlet of the pump and other end delivers

the water at a required height is known as delivery pipe.

Workdone by the centrifugal pump (or by impeller) on water:

In case of the centrifugal pump, work is done by the impeller on the water. The expression for the workdone by the impeller on the water is obtained by drawing velocity triangles at inlet and outlet of the impeller in the same way as for a turbine,



The water enters the impeller radially at inlet for best effeciency of the pump, which means the absolute velocity of water enters the imakes an angle of 90° with the direction of motion of the impeller at inlet. Hence angle $\alpha = 90°$ and $V_{w_1} = 0$, for drawing the velocity triangles, the same notations are used as that for turbines. Figure shows the velocity triangles at the inlet and outlet tips of the vane fixed to an impeller.

Let N = Speed of the impeller in rpm.

D_1 = Diameter of impeller at inlet

u_1 = Tangential velocity of impeller at inlet,

$$u_1 = \frac{\pi D_1 N}{60}$$

D_2 = Diameter of impeller at outlet.

u_2 = Tangential velocity of impeller at outlet.

$$= \frac{\pi D_2 N}{60}$$

V_1 = Absolute velocity of water at inlet.

V_{r_1} = Relative velocity of water at outlet,

α = angle made by absolute velocity (V_1) at inlet with the direction of motion of vane.

θ = angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane, and

V_2, V_{r_2}, β and ϕ are the corresponding values at outlet.

As the water enters the impeller radially which means the absolute velocity of water at inlet is in the radial direction and hence angle $\alpha = 90°$ & $V_{w_1} = 0$

A centrifugal pump is the reverse of a radially inward flow reaction turbine, the workdone by the water on the runner per second per unit weight of the water striking per second is given by.

$$= \frac{1}{g}\left[V_{w_1}u_1 - V_{w_2}u_2\right]$$

$\therefore$ Workdone by the impeller on the water per second per unit weight of water striking per second

$$= -\left[\text{workdone in case of turbine}\right]$$

$$= -\left[\frac{1}{g}\left(V_{w_1}u_1 - V_{w_2}u_2\right)\right]$$

$$= \frac{1}{g}\left[V_{w_2}u_2 - V_{w_1}u_1\right]$$

$$= \frac{1}{g}V_{w_2}u_2 \qquad \left[\because V_{w_1} = 0 \text{ hw}\right]$$

workdone by impeller on water per second $= \frac{W}{g}V_{w_2}u_2$

where W = weight of water $= \rho \times g \times Q$

Q = volume of water

Q = Area x velocity of flow

$= \pi D_1 B_1 \times V_{f_1}$

$= \pi D_2 B_2 \times V_{f_2}$

where B_1 and B_2 are width of impeller at inlet and outlet and V_{f_1} and V_{f_2} are velocities of flow at inlet and outlet.

<u>Definitions of heads and efficiencies of a centrifugal pump</u>:

1) Suction head (h_s): It is the vertical height of centre line of centrifugal pump above the water surface in the tank or pump from which water is to be lifted. This height is also called suction lift and is denoted by 'h_s'.

2) Delivery head (h_d): The vertical distance between the centre line of the pump and the water surface in tank to which water is delivered is known as delivery head is known as delivery head. This is denoted by 'h_d'.

3) Static head (H_s): The sum of suction head and delivery head is known as static head. This is represented by H_s is written as

$$\boxed{H_s = h_s + h_d}$$

4) Manometric head (H_m): This is defined as head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by following expressions.

(a) H_m = Head imparted by impeller to the water − loss of head in pump

$$= \frac{V_{w_2} u_2}{g} - \text{loss of head in impeller and casing.}$$

$$= \frac{V_{w_2} u_2}{g} \text{ ----- if loss of pump is zero.}$$

(b) H_m = Total head at outlet of pump − Total head at inlet of pump.

$$= \left[\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + z_o\right] - \left[\frac{P_i}{\rho g} + \frac{V_i^2}{2g} + z_i\right]$$

where $\frac{P_o}{\rho g}$ = pressure head at outlet of pump = h_d

$\frac{V_o^2}{2g}$ = Velocity head at outlet of pump

= Velocity head in delivery pump = $\frac{V_d^2}{2g}$

z_o = vertical height of outlet of pump from datum line and

$\frac{P_i}{\rho g}$, $\frac{V_i^2}{2g}$, z_i = corresponding values of pressure head, velocity head and datum head at inlet of pump.

i.e., h_s, $\frac{V_s^2}{2g}$ and z_s respectively.

$$\boxed{H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}}$$

where h_s = suction head.

h_d = Delivery head.

h_{f_s} = Frictional head loss in suction pipe,

h_{f_d} = Frictional head loss in delivery pipe

V_d = velocity of water in delivery pipe

5) Efficiencies of a centrifugal pump: In case of a centrifugal pump, the power is transmitted from the shaft of electric motor to shaft of the pump and then to impeller. From impeller, the power is given to the water. Thus power is decreasing from the shaft of the pump to the impeller and then to water. The following are important efficiencies of centrifugal pump

(a) Monometric efficiency, η_{man}

(b) Mechanical efficiency, η_m and

(c) Overall efficiency, η_o

(a) Monochrometic Efficiency (η_{man}): The ratio of the manometric head to the head imparted by impeller to the water is known as manometric efficiency. Mathematically, it is written as

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$\eta_{man} = \frac{H_m}{\frac{V_{w_2} u_2}{g}} = \frac{g H_m}{V_{w_2} u_2}$$

The power at the impeller of pump is more than power given to the water at outlet of pump. The ratio of power given to water at outlet of pump to power available at impeller, is known as manometric efficiency.

The power given to water at outlet of pump $= \frac{W H_m}{1000}$ kW.

The power at the impeller $= \frac{\text{Workdone by impeller per second}}{1000}$ kW

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000} \text{ kW}$$

$$\eta_{man} = \frac{\frac{W \times H_m}{1000}}{\frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}}$$

$$\boxed{\eta_{man} = \frac{g \times H_m}{V_{w_2} u_2}}$$

(b) Mechanical Efficiency (η_m) : The power at shaft of centrifugal pump is more than the power available at impeller of pump. Ratio of power available at impeller to power at shaft of centrifugal pump.

$$\eta_m = \frac{\text{power at impeller}}{\text{power at shaft}}$$

The power at impeller in kW $= \frac{\text{Workdone by impeller per second}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

$$\boxed{\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{S.P}}$$

S.P = Shaft Power

(c) Overall Efficiency (η_o) : It is defined as ratio of power output of pump to power input to pump. The power output of pump in kW.

$$= \frac{\text{Weight of water lifted} \times H_m}{1000}$$

$\frac{WHm}{1000}$

Power input to pump = Power supplied by electric motor
= Shaft power of pump

$$\eta_o = \frac{\left(\frac{WHm}{1000}\right)}{S.P}$$

$$\eta_o = \eta_{man} \times \eta_m .$$

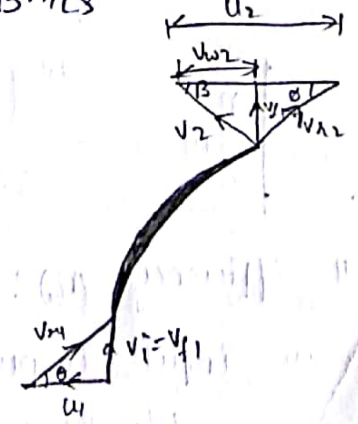
1) The internal and external diameters of impeller of centrifugal pump are 200 mm and 400mm respectively. Pump is running at 1200 rpm. The vane angles of impeller at inlet and outlet are 20° and 30° respectively. The water enters impeller radially and velocity of flow is constant. Determine the workdone by impeller per unit weight of water.

Sol: Internal diameter of impeller, $D_1 = 200mm = 0.20m$
External diameter of impeller, $D_2 = 400mm = 0.40m$
Speed, $N = 1200$ rpm
Vane angle at inlet, $\theta = 30°$
Vane angle at outlet, $\phi = 30°$.
Water enters radially means, $\alpha = 90°$, $V_{\omega_1} = 0$.
Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller and inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1200}{60} = 12.56 \text{ m/s}.$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.40 \times 1200}{60} = 25.13 \text{ m/s}$$



From inlet velocity triangle,

$$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$$

$$\tan 20 = \frac{V_{f_1}}{12.56}$$

$$V_{f_1} = (12.56) \times \tan 20 = 4.57 \text{ m/s}$$

$$V_{f_2} = V_{f_1} = 4.57 \text{ m/s}$$

From outlet triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$

$$\tan 30 = \frac{4.57}{25.13 - V_{w_2}}$$

$$25.13 - V_{w_2} = \frac{4.57}{\tan 30}$$

$$V_{w_2} = 25.13 - \frac{4.57}{\tan 30}$$

$$V_{w_2} = 25.13 - 7.915$$

$$= 17.215 \text{ m/s}$$

The workdon by impeller per kg of water per second is given by $= \frac{1}{g} V_{w_2} u_2$

$$= \frac{17.215 \times 25.13}{9.81}$$

$$= \underline{\underline{44.1}} \text{ N-m/N}$$

2. A centrifugal pump is to discharge $0.118 \text{ m}^3/\text{s}$ at a speed of 1450 rpm against a head of 25 m. The impeller diameter is 250 mm its width at outlet is 50 mm and manometric efficiency is 75%. determine the vane angle at the outer periphery of the impeller

Sol : discharge, $Q = 0.118 m^3/s$

speed, $N = 1450 rpm$

Head, $H_M = 25 m$

diameter at outlet, $D_2 = 250 mm = 0.25 m$

width at outlet, $B_2 = 50 mm = 0.50 m$

Manometric efficiency, $\eta_{man} = 75\% = 0.75$

let vane angle at outlet $= \phi$

Tangential velocity of impeller at outlet

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 m/s$$

Discharge is given by $Q = \pi D_2 B_2 \times V_{f_2}$

$$V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times 0.5}$$

$$= 3.0 m/s$$

manometric efficiency is given by $\eta_{man} = \frac{g H_M}{V_{w_2} u_2}$

$$0.75 = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$$

$$V_{w_2} = \frac{9.81 \times 25}{0.75 \times 18.98}$$

$$= 17.23 m/s$$

From outlet velocity triangle

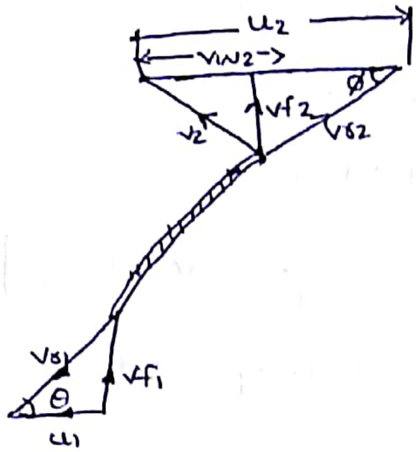
$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan\phi = \frac{30}{18.98-17.23}$$

$$\tan\phi = 1.7143$$

$$\phi = \tan^{-1}(1.7143)$$

$$= 59.74^\circ.$$



Specific speed of a centrifugal pump

The specific speed of a centrifugal pump is define as the speed of a geomotrically similar pump which would diliver and cubic metre of lequid per second against a head of one metre It is denoted by Ns.

Expression for specific speed of pump : The discharge Q, for a centrifugal pump is given by

$$Q = \text{Area} \times \text{velosity of flow}$$

$$= \pi D \times B \times V_{f_1}$$

(or) $\boxed{Q \propto D \times B \times V_{f_1}}$ — (i)

Where D = diameter of the impeller of the pump

B = Width of the impeller

We know that $B \propto D$

From equation (i), we have $\boxed{Q \propto D^2 \times V_{f_1}}$ —(ii)

We also know that tangential velocity is given by

$\boxed{u = \frac{\pi D N}{60} \propto DN}$ - (iii)

Now the tangential velocity (u) and velocity of flow of (Vf) are related to the manometric head (Hm) as

$$\boxed{u \propto V_f \sqrt{H_M}} \quad \text{--- (iv)}$$

Substituting the value of u in equation (iii)

$$\sqrt{H_M} \propto DN$$

$$(\text{or}) \quad \boxed{D \propto \frac{\sqrt{H_m}}{N}} \quad \text{--- (v)}$$

Substituting the value of D in equation (ii)

$$Q \propto \frac{H_M}{N^2} \times V_f$$

$$\propto \frac{H_m}{N^2} \times \sqrt{H_m}$$

$$\propto \frac{H_m^{3/2}}{N^2}$$

$$\boxed{Q = k \frac{H_m^{3/2}}{N^2}} \quad \text{--- (vi)}$$

Where k = constant of proportionality

If Hm = 1m and Q = 1m³/s N becomes = Ns

Substituting these values in equation (vi)

We get

$$1 = \frac{k \cdot (1)^{3/2}}{N_s^2}$$

$$k = N_s^2$$

Substituting the value of k in equation (ii)

$$Q = Ns^2 \cdot \frac{H_m^{3/2}}{N^2}$$

$$\sqrt{Q} = Ns \cdot \frac{H_m^{3/4}}{N}$$

$$\boxed{Ns = \frac{N\sqrt{Q}}{H_m^{3/4}}}$$

Model testing of centrifugal pumps :

Before manufacturing the large size pumps, their models which are complete similarity with the actual pumps also (called prototypes) are mode Tests are conducted on the models and performence of the prototypes are predicted. The complex similarity between the model and actual pump (prototype) will exist if the following conditions are satisfied.

1. Specific speed of model = Specific speed of prototype

$$(Ns)_m = (Ns)_p$$

$$\boxed{\left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_m = \left(\frac{N\sqrt{Q}}{H^{3/4}}\right)_p}$$

2. Tangential velocity (u) is given by, $u = \frac{\pi DN}{60}$

also $u \propto \sqrt{H_m}$

3. $Q \propto D^2 V_{f_1}$

$V_{f_1} \propto u \propto DN \quad Q \; D^2 \times DN$

$Q \propto D^3 N$

$$\frac{Q}{D^3 N} = \text{constant}$$

$$\boxed{\left(\frac{Q}{D^3 N}\right)_m = \left(\frac{Q}{D^3 N}\right)_p}$$

4. Power of the pump

$P \propto Q \times H_m$

$\propto D^3 N \times H_m \qquad \because Q \propto D^3 N$

$\propto D^3 N \times D^2 N^2 \qquad \sqrt{H_m} \propto DN$

$\propto D^5 N^3 \qquad \frac{P}{D^5 N^3} = \text{constant}$

$$\boxed{\left(\frac{P}{D^5 N^3}\right) = \left(\frac{P}{D^5 N^3}\right)_p}$$

A single stage centrifugal pump with impeller diameter of 30 cm rotates at 200 rpm and lifts 3 m³ of water per second to a height of 30m with an efficiency of 75%. Find the number of stages and diameter of each impeller of a similar multi stage pump to lift 5 m³ of water per second to a height of 200 metres when rotating at 1500 rpm.

Sol :

Single - stage pump

Diameter of impeller, $D_1 = 30$ cm $= 0.30$ m

speed , $N_1 = 2000$ rpm

discharge , $Q_1 = 3 m^3/s$

height $Hm_1 = 30m$

Efficiency , $\eta_{man} = 75\% = 0.75$

multystage similar pump :

Dischange , $Q_2 = 5m^3/s$

Total height $= 200m$

Let the height per stage $= H_{M_2}$

speed $N_2 = 1500$

Diameter of each impeller $= D_2$

specific speed should be same

$$\left(\frac{N\sqrt{Q}}{Hm^{3/4}}\right)_1 = \left(\frac{N\sqrt{Q}}{Hm^{3/4}}\right)_2$$

$$\frac{N_1\sqrt{Q_1}}{Hm_1^{3/4}} = \frac{N_2\sqrt{Q_2}}{Hm_2^{3/4}}$$

$$\frac{2000\sqrt{3}}{(30)^{3/4}} = \frac{1500\sqrt{5}}{Hm_2^{3/4}}$$

$$\therefore Hm_2^{3/4} = \frac{1500\sqrt{5} \times (30)^{3/4}}{2000\sqrt{3}}$$

$$H_{m2}^{3/4} = \frac{1500}{2000}\sqrt{\frac{5}{3}} \times (30)^{3/4}$$

$$= 12.818(30)^{3/4}$$

$$\simeq 12.411$$

$$\simeq 154$$

$$H_{m2} = (12.411)^{4/3} = 28.71\ m$$

$\therefore$ Number of stages $= \dfrac{\text{total head}}{\text{head per stage}}$

$$= \frac{200}{28.71} = 6.76 \simeq 7$$

We know the equation

$$\frac{\sqrt{H_{m1}}}{D_1 N_1} = \frac{\sqrt{H_{m2}}}{D_2 N_2}$$

$$\frac{\sqrt{30}}{0.30 \times 2000} \times \frac{\sqrt{28.71}}{D_2 \times 1500} \quad D_2 = \frac{28.71 \times 0.30 \times 2000}{1500}$$

$$D_2 = 0.3913\ m$$

$$D_2 = 391.3\ mm$$

Two geometrically similar pumps are running at the same speed of 1000 rpm. on pump has can impeller diameter of 0.30 metre and lifts water at the rate of 20 litres per second against a head of 15 mitres. Determine the head and impeller diameter of the other pump the delivers half the discharge

Sol: For pump No. 1:

Speed, $N_1 = 1000$ rpm

diameter $D_1 = 0.30m$

discharge, $Q_1 = 20$ litres/s $= 0.020\ m^3/s$.

Head, $H_{m_1} = 15m$.

For pump No. 2,

Speed, $N_2 = 1000$ rpm

discharge, $Q_2 = \frac{Q_1}{2} = \frac{20}{2} = 10$ litres/s.

$= 0.01\ m^3/s$.

Let D_2 = Diameter of impeller

H_{m_2} = Head developed.

$$\frac{N_1\sqrt{Q_1}}{H_{m_1}^{3/4}} = \frac{N_2\sqrt{Q_2}}{H_{m_2}^{3/4}}$$

$$\frac{1000\sqrt{0.02}}{(15)^{3/4}} = \frac{1000\sqrt{0.01}}{H_{m_2}^{3/4}}$$

$$H_{m_2}^{3/4} = \frac{1000\sqrt{0.01}}{1000\sqrt{0.02}} \times (15)^{3/4}$$

$$H_{m_2}^{3/4} = 5.389 \longrightarrow H_{m_2} = (5.389)^{4/3} = 9.44m$$

We know the equation,

$$\left(\frac{\sqrt{H_m}}{DN}\right)_1 = \left(\frac{\sqrt{H_m}}{DN}\right)_2$$

$$\frac{\sqrt{H_{m_1}}}{D_1 N_1} = \frac{\sqrt{H_{m_2}}}{D_2 N_2}$$

$$\frac{\sqrt{15}}{0.3 \times 1000} = \frac{\sqrt{9.44}}{D_2 \times 1000}$$

$$D_2 = \frac{\sqrt{9.44} \times 0.3}{\sqrt{15}} = 0.238m = 238.0mm$$

→ Priming of a centrifugal Pump:

Priming of a centrifugal pump is defined as operation in which the suction pipe, casing of pump and a portion of delivery pipe upto delivery valve is completely filled up from outside source with the liquid to be raised by pump before starting pump. Thus air from these parts of the liquid to be pumped.

...workdone by the impeller per unit weight of liquid per sec is known as the head generated by the pump as $= \frac{1}{g} V_{w_2} u_2$ m.

This equation is independent of the density of the liquid. This means that when pump is running in air, the head generated is in terms of metre of air. If the pump is primed with water, the head generated is same metre of water. But as the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible and hence the water may not be sucked from the pump. To avoid this difficulty, priming necessary.

Characteristic curves of centrifugal Pumps:

These are defined which are plotted from the results of a number of tests on the centrifugal pump. These curves are necessary to predict the behaviour and performance of the pump, when the pump is working under different flow rate, head and speed. The following are the important characteristics curves for pumps:

1. Main charactoristic curves.
2. Operating charactoristic curves.
3. Constant efficiency (or) Muschel curves.

* 1. Main characteristic curves:

The main characteristic curve of a centrifugal pump consists of variation of head (monometric head, H_m) power and discharge with respect to speed. For plotting curves of monometric head versus speed, discharge is kept constant. For plotting curves of power versus speed, the manometric head and discharge are kept constant.

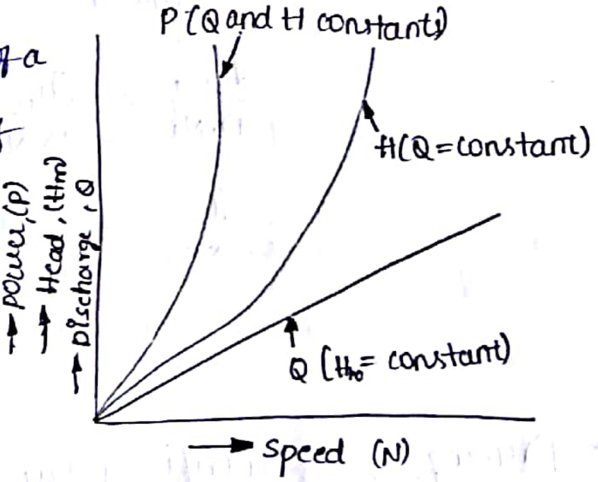


Fig: Main charactoristic curves of a pump.

* 2. Operating characteristic curves:

If the speed is kept constant, the variation of manometric head, power and efficiency with respect to discharge gives operating characteristics of pump.

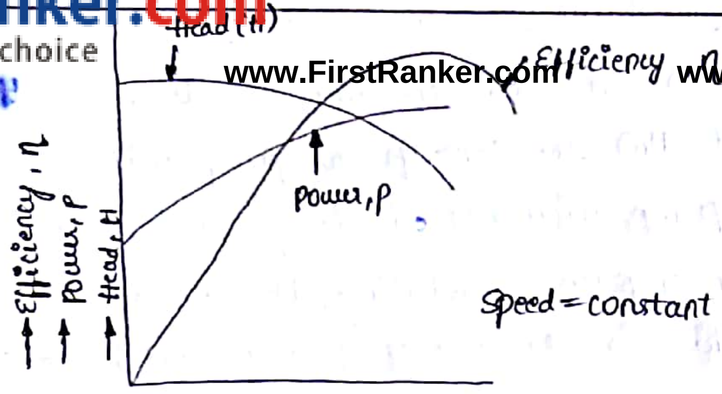


Fig: Operating characteristics of a pump.

* 3. Constant Efficiency Curves:

For obtaining constant efficiency curves for a pump the head versus discharge curves for a pump, and efficiency versus discharge curves for different speeds are used. Fig (a) shows the head versus discharge curves for different speeds. The efficiency versus discharge curves for the different speeds are shown in fig (b). By combining these curves (H~Q curves and η~Q curves), constant efficiency curves are obtained as shown in fig (a).

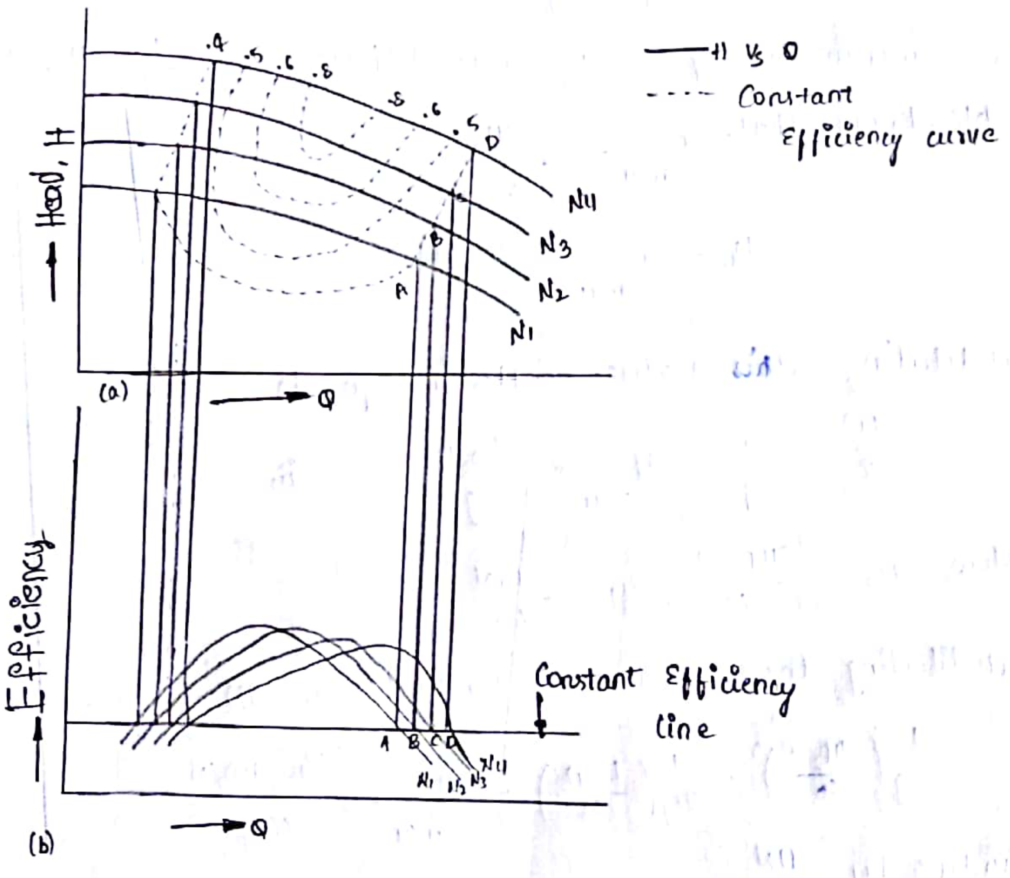


Fig: Constant Efficiency curves of a pump

Minimum Speed for starting a centrifugal pump:

If the pressure rise in the impeller is more than or equal to manometric head (H_m) the centrifugal pump will start delivering water. Otherwise, the pump will not discharge any water, though the impeller is rotating. When impeller is rotating, the water in contact with the impeller is also rotating. This is the case of forced vortex. In case of forced vortex, the centrifugal head or head due to pressure rise in the impeller

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

where, ωr_2 = Tangential velocity of impeller at outlet $= u_2$.

ωr_1 = Tangential velocity of impeller at inlet $= u_1$.

$\therefore$ Head due to pressure rise in impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

The flow of water will commence only if Head due to pressure rise in impeller $= \geq H_m$

(or) $$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m$$

For minimum speed, we must have, $\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$ —— (i)

We know that, $$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$H_m = \eta_{man} \times \frac{V_{w_2} u_2}{g}$$

substituting this value of H_m in eqn. (i)

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \eta_{man} \times \frac{V_{w_2} u_2}{g} \quad \text{—— (ii)}$$

Now, $u_2 = \frac{\pi D_2 N}{60}$, $u_1 = \frac{\pi D_1 N}{60}$

substituting these values of u_1 and u_2 in (ii)

$$\frac{1}{2g}\left(\frac{\pi D_2 N}{60}\right)^2 - \frac{1}{2g}\left(\frac{\pi D_1 N}{60}\right)^2 = \eta_{man} \times \frac{V_{w_2} \pi D_2 N}{60 g}$$

Dividing by $\frac{\pi N}{g \times 60}$,

$$\frac{\pi N D_2^2}{120} - \frac{\pi N D_1^2}{120} = \eta_{man} \times V_{w_2} \times D_2$$

$\frac{\pi N}{120}(D_2^2 - D_1^2) = \eta_{man} V_{w_2} D_2$

$$\boxed{N = \frac{120 \times \eta_{man} \times V_{w_2} D_2}{(D_2^2 - D_1^2)}}$$

The above equation gives the minimum speed of the centrifugal pump.

→ The diameter of an impeller of a centrifugal pump at inlet and outlet are 30cm and 60cm respectively. Determine the mini starting speed of the pump if it works against a head of 30m.

Sol: Diameter of Impeller at inlet, $D_1 = 30cm = 0.30m$

Diameter of Impeller at outlet, $D_2 = 60cm = 0.60m$

Head, $H_m = 30$ cm

Let the minimum starting speed $= N$

We know the eqn,

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 N}{60} = 0.03141 N$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times N}{60} = 0.0157 N$$

$$\frac{N^2}{2g}\left[(0.03141)^2 - (0.0157)^2\right] = 30$$

$$N^2 = \frac{30 \times 2 \times 9.81}{0.03141^2 - 0.0157^2} = \frac{588.6}{0.0009866 - 0.0002465}$$

$$N^2 = 795297.9$$

$$N = \sqrt{795297.9}$$

$$N = \underline{891.8 \text{ rpm}}$$

→ **Reciprocating pumps:**

The pumps are the hydraulic machines which convert the mechanical energy is converted into hydraulic energy which is mainly in the form of pressure energy. If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on liquid and creates in hydraulic energy (pressure energy), the pump is known as reciprocating pump.

Main parts of a Reciprocating pump:

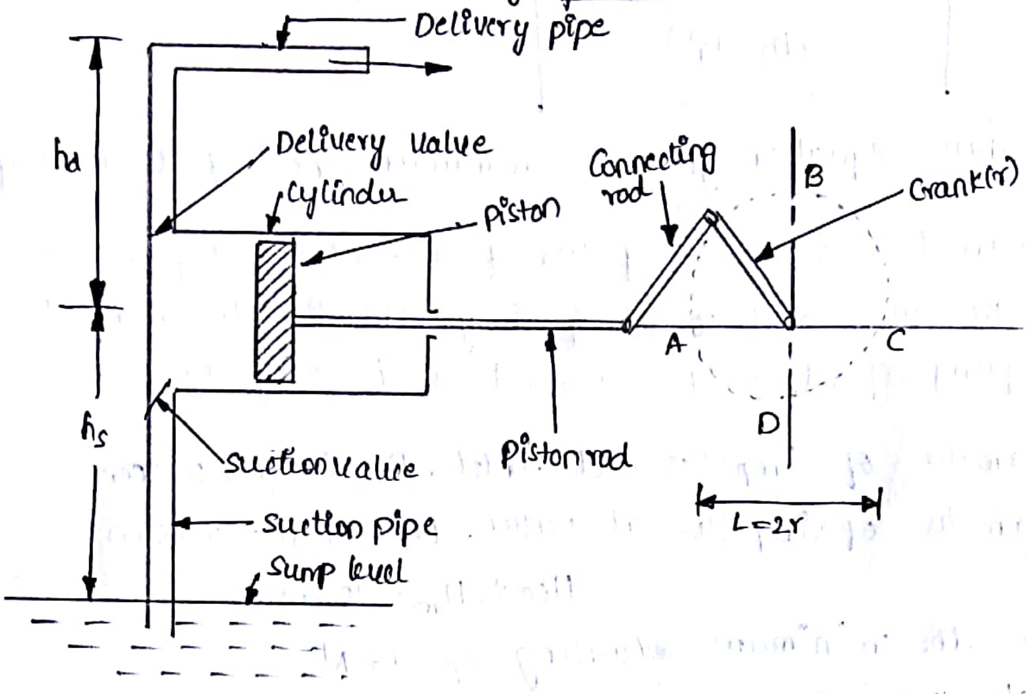


Fig: Main parts of a reciprocating pump.

1. A cylinder with a piston, piston rod, connecting rod and a crank
2. Suction pipe
3. Delivery pipe
4. Suction valve and
5. Delivery valve

Working of a reciprocating pump:

The above fig. shows a single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of piston is obtained by connecting the piston rod and crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction & delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction & delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allow water from suction pipe to cylinder which delivery valve allows water from cylinder to delivery pipe only.

When crank starts rotating, the piston moves to and fro to cylinder.

When crank is at A, the piston is at the extreme left position in the cylinder. As crank is rotating from A to C (i.e., from $\theta = 0°$ to $180°$), the piston is moving towards right in the cylinder. The movement of the piston towards right creates a partial vacuum in the cylinder. But on the surface of liquid in sump atm pressure is acting, which is more than the pressure inside the cylinder. Thus the liquid is forced in the suction pipe from sump. This liquid open the suction valve and enters the cylinder.

When crank is rotating from C to A (i.e., from $\theta = 180°$ to $360°$), the piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of liquid inside the cylinder more than the atm pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to required height.

Discharge through a Reciprocating pump:

Consider a single acting reciprocating pump.

Let D = diameter of cylinder.

A = cross-sectional area of piston or cylinder,

$= \frac{\pi}{4} D^2$

r = Radius of crank

N = rpm of the crank

L = length of the stroke $= 2 \times r$

h_s = Height of axis of cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution.

$= $ Area $\times$ length of stroke

$= A \times L$

Number of revolution per second $= \frac{N}{60}$

$\therefore$ Discharge of pump per second,

Q = Discharge in one revolution $\times$ No. of revolution per second

$$Q = A \times L \times \frac{N}{60} = \frac{ALN}{60}$$

Weight of water delivered per second

$$W = \rho \times g \times Q = \frac{\rho g ALN}{60}$$

→ **Workdone by Reciprocating pump:**

Workdone by reciprocating pump per second is given by the relation as

Workdone per second = Wt. of water lifted per second × Total height through which water is lifted.

$$= W(h_s + h_d) \longrightarrow (i)$$

where, $h_s + h_d$ = Total height through which water is lifted.

Weight W is given by,

$$W = \frac{\rho g \times ALN}{60}$$

Substituting the value of W in (i)

$$\text{Workdone per second} = \frac{\rho g \times ALN}{60}(h_s + h_d)$$

$$\boxed{W.D/s = \frac{\rho g ALN}{60}(h_s + h_d)}$$

∴ Power required to drive the pump in kW

$$P = \frac{\text{Workdone per second}}{1000}$$

$$= \frac{\rho g ALN (h_s + h_d)}{1000 \times 60}$$

$$\boxed{P = \frac{\rho g ALN (h_s + h_d)}{60{,}000}} \text{ kW}$$

→ **Slip of Reciprocating pump:**

Slip of a pump is defined as the difference between the theotrical discharge and actual discharge of pump. The discharge of a single acting pump and a double acting pump are theoritical discharge. The actual discharge of a pump is less than the thearitical discharge due to leakage. The difference of theoritical dischaege and

...actual discharge is known as slip of pump.

Hence, mathematically

$$\boxed{Slip = Q_{the} - Q_{act}}$$

But slip is mostly expressed as percentage slip which is given by percentage slip $= \frac{Q_{the} - Q_{act}}{Q_{the}} \times 100$

$$\boxed{\%\ Slip = 1 - \frac{Q_{act}}{Q_{the}} \times 100}$$

$\because C_d = \frac{Q_{act}}{Q_{the}}$

$= (1 - C_d) \times 100$

where $C_d =$ Co-efficient of discharge

* **Negative slip of reciprocating pump:** slip is equal to the diff of theoritical discharge and actual discharge. If actual disharge is more than the theoritical discharge, the slip of pump will become –ve. In that case, the slip of pump is known as ~~+e~~ –ve slip.

–ve slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

→ **Classification of Reciprocating pumps:**

The reciprocating pumps may be classified as

1. according to water being in contact with one side or both sides of the piston, and
2. according to no. of cylinders provided.

If water is in contact with one side of piston, the pump is known as single-acting. On other hand, if water is in contact with both side of piston, the pump called as double-acting. Hence classification according to contact of water:

(i) single-acting pump
(ii) Double-acting pump.

according to no. of cylinders provided.

(i) single cylinder pump.
(ii) Double cylinder pump.
(iii) Triple cylinder pump.

A single acting reciprocating pump, running at 50 rpm, delivers $0.01\ m^3/s$ of water. The diameter of piston is and stroke length 400 mm. Determine :

(i) The theoritical discharge of pump
(ii) co-efficient of discharge and
(iii) slip and percentage slip of pump.

Sol: speed of pump, $N = 50$ rpm

actual discharge, $Q_{act} = 0.01\ m^3/s$

Diameter of piston, $D = 200mm = 0.20m$

$\therefore$ Area, $A = \pi/4\ (0.20)^2 = 0.031416\ m^2$

stroke, $L = 400\ mm = 0.40\ m$

(i) Theoritical discharge for single-acting reciprocating pump is given by,

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{0.031416 \times 0.40 \times 50}{60} = 0.01047\ m^3/sec$$

(ii) Co-efficient of discharge is given by,

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{0.01047} = 0.955$$

(iii) We know the eqn.

$$Slip = Q_{th} - Q_{act} = 0.00047\ m^3/s$$

$$\text{percentage slip} = \left(\frac{Q_{th} - Q_{act}}{Q_{th}}\right) \times 100$$

$$= \left(\frac{0.01047 - 0.01}{0.01047}\right) \times 100$$

$$= 4.489\ \%$$

→ A double-acting reciprocating pump, running at 40 rpm, is discharging $1.0\ m^3$ of water per minute. The pump has a stroke of 400mm. The diameter of piston is 200mm. The delivery and suction head are 20m and 5m respectively. Find the slip of pump and power required to drive the pump.

Sol: speed of pump, $N = 40$ rpm

actual discharge, $Q_{act} = 1.0\ m^3/min = \frac{1.0}{60}\ m^3/s = 0.01666\ m^3/s$

Stroke, L = 400mm = 0.40m

diameter of piston, D = 200mm = 0.20m

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = 0.031416 m^2$

Suction head, $h_s = 5m$

Delivery head, $h_d = 20m$

Theoritical discharge for double acting pump is given by

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times 0.031416 \times 0.4 \times 40}{60} = 0.01675 \; m^3/s$$

We know that

$$slip = Q_{th} - Q_{act}$$
$$= 0.01675 - 0.01666$$
$$= 0.00009 \; m^3/s$$

Power required to drive the double acting pump is given by

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

$$= \frac{2 \times 1000 \times 9.81 \times 0.031416 \times 0.4 \times 40 \times (50+20)}{60{,}000}$$

$$= 4.109 \; kW$$

Indicator Diagram:

The indictor diagram for a reciprocating pump is defined as the graph between the pressure head in the cylinder and the distance travelled by piston from inner dead centre for the complete revolution of the crank. As the maximum distance travelled by the piston is equal to the stroke length of the piston for the complete revolution. The pressure head is taken as ordinate and stroke length as abscissa.

2. Ideal indicator diagram :-

The graph between pressure for a reciprocating pump is defined head in the cylinder and stroke length of the piston for one complete revolution of the crank. under conditions is known as ideal indicator diagram

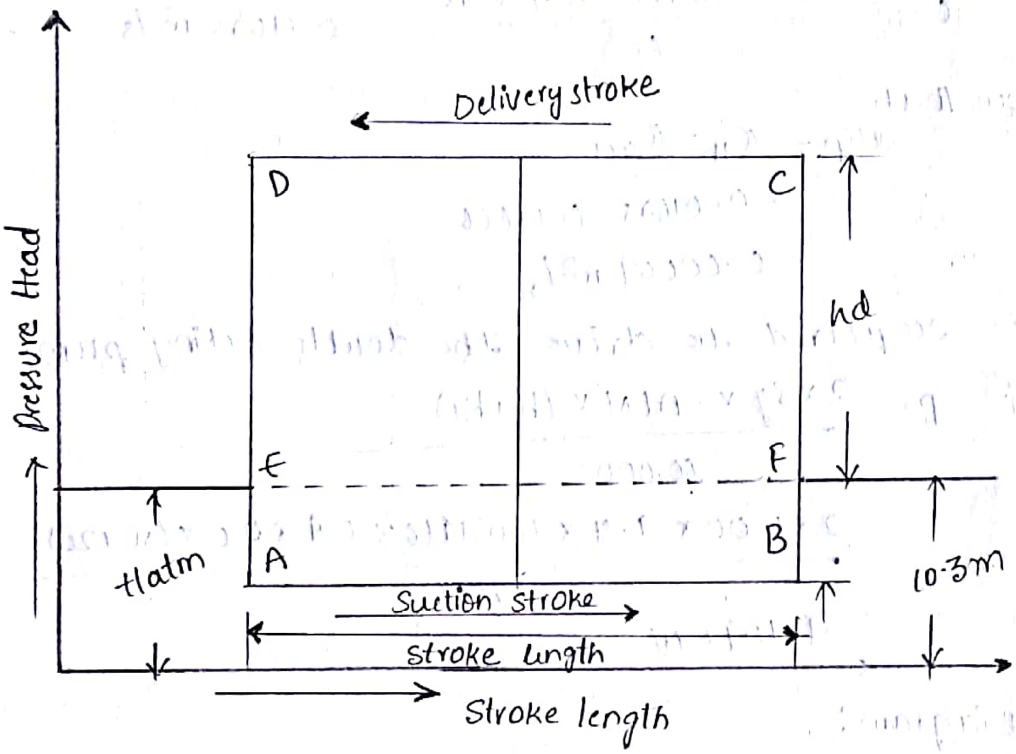


Figure shows the ideal indicator diagram in which line EF represents at atmospheric pressure head equal to 10·3m of water

Let H_{atm} = Atmospheric pressure head.
= 10·3m of water

L = length of the stroke

h_s = Suction head, and

h_d = Delivery head

During suction stroke, the pressure head in the cylinder is constant and equal the suction head (h_s). which is below.

the atmospheric pressure head (Hatm) by a height of hs. The pressure head during suction stroke is represented by a horizontal line AB which is below the line BF by a height of hs

(3) During dilvery stroke, the pressure head in the cylinder is constant and equal to delivery head (hd) which is above the atmospheric head by a height of (hd) Thus, the pressure head during delivery stroke is represented by horizontal line CD which is above the line EF by a height of hd. Thus. for one complete revolution of the crank, the pressure head in the cylinder is represented by the diagram A-B-C-D-A. This diagram is known as ideal indicator diagram.

We know that the work done by the pump per second

$$= \frac{S \times g \times ALN}{60} \times (h_s + h_d)$$

$$= KL(h_s + h_d) \quad \left[\text{Where } K = \frac{SgAN}{60} = \text{constant}\right]$$

$$\propto L \times (h_s + h_d) \quad \text{———} \quad (i)$$

By from fig area of indicator diagram

$$= AB \times BC = AB \times (BF + FC)$$

$$= L \times (h_s + h_d)$$

Substituting this value in equation (i)

Workdone by pump $\propto$ Area of indicator diagram.

Effect of Acceleration in Suction and Delivery pipes on Indicator Diagram

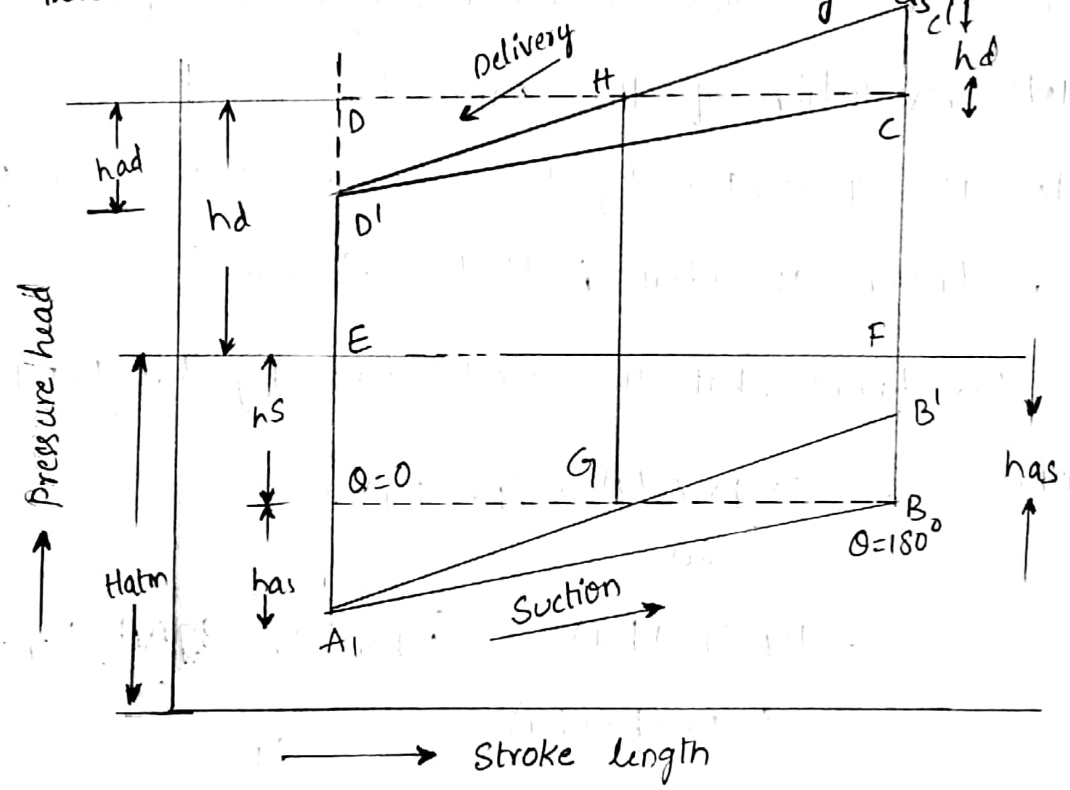
The pressure head due to acceleration in the suction pipe is given by.

$$h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \cos\theta$$

When $\theta = 0°$, $\cos\theta = 1$ and $h_{as} = \frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r$

When $\theta = 90°$; $\cos\theta = 1$ and $h_{as} = 0$

When $\theta = 180°$; $\cos\theta = -1$ and $h_{as} = -\frac{l_s}{g} \frac{A}{a_s} \omega^2 r$



Thus, the pressure head inside the cylinder during suction stroke will not be equal to h_s, as was the case for ideal indicator diagram. but it will be equal to the sum of h_s and h_{as}.

Let H_{atm} = Atmospheric pressure head
= 10.3 m of water.

(4) L = length of the stroke.

h_s = Suction head, and

h_d = Delivery head.

During suction stroke, the pressure head in the cylinder is constant and equal to suction head (h_s), which is below the atmospheric pressure head (H_{atm}) by a height of h_s. The pressure head during suction stroke is represented by a horizontal line AB which is below the line BF by a height of h_s.

During delivery stroke, the pressure head in the cylinder is constant and equal to delivery head (h_d) which is above the atmospheric head by a height of a (h_d). Thus, the pressure head during delivery stroke is represented by a horizontal line CD which is above the line BF by a height of h_d. Thus for one complete revolution of the crank, the pressure head in cylinder is represented by the diagram A-B-C-D-A. This diagram is known as ideal indicator diagram.

We know that the work done by the pump per

Second $= \frac{S \times g \times ALN}{60} \times (h_s + h_d)$

$= K_L (h_s + h_d)$ $\left[\text{where } K = \frac{S g A N}{60} = \text{constant}\right]$

$\propto L \times (h_s + h_d)$ —— (2)

But from fig area of indicator diagram.

$= AB \times BC = AB \times (BF + FC)$

$= L \times (h_s + h_d)$

Substituting this value in equation (i)

Workdone by pump $\propto$ Area of indicator diagram.

3. Expansion chamber Surge tank :

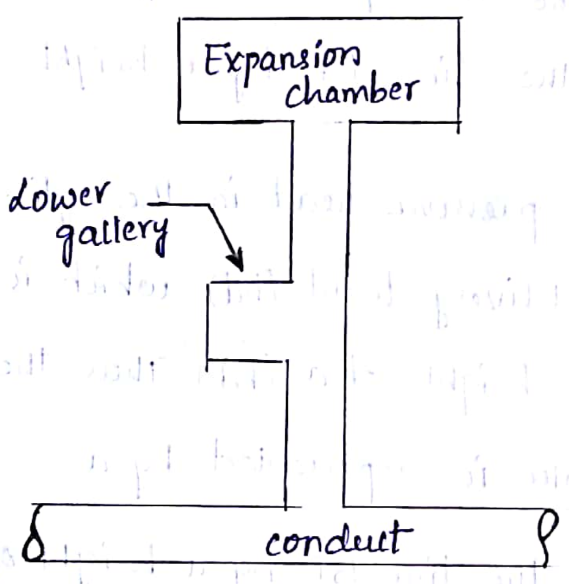


Fig. Expansion chamber Surge tank.

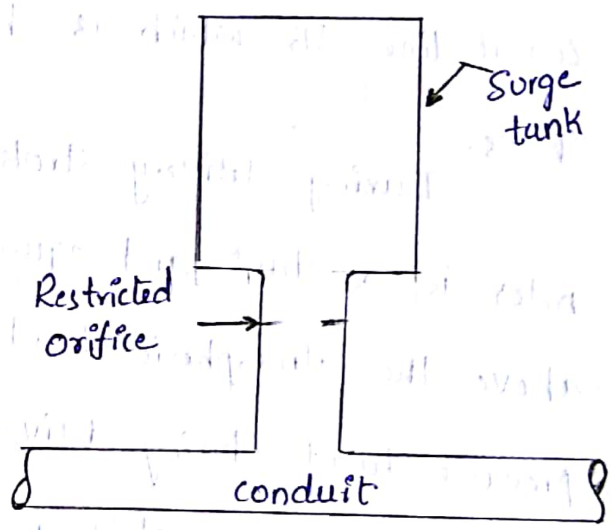


Fig :- Restricted orifice Surge tank

This type of surge tank has an expansion tank at top and expansion gallery at the bottom; these expansions limit the extreme surges. The upper expansion chamber must be above the maximum reservoit level and bottom gallery must be below the lowest steady running level in the surge tank. Besides this the intermedicte shaft should have stable minimum diameter.

* 1. Restricted orifice Surge tank: It is also called throttled surge tank. The main object of providing a throttle or resitricted orifice is a create an appreciable friction loss when the water is flowing to or from the tank. When the water is flowing to or from the tank. When the load on the turbine is reduced the surplus water passes through the throttle and retarding head equal to the loss due to throttle is built up in the conduit. The size of the throttle can be designed for any desired retarding head. The size of the throttle adopted is usually such as the intial retarding head is equal to the full load is rejected by the turbine (a case when there is closure of the gate valve).

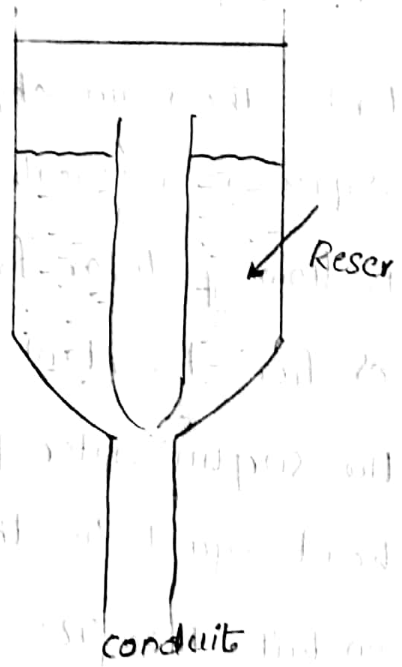
Advantage:- storage function of the tank can be separated from acceleration and retarding functions.

Disadvantages:- considered portion of water hammer pressure is transmitted directly into the low pressure conduit.

In comparisam to other types of surge tanks there less popular.

Differential Surge tank:

A differential surge tank has a riser with a small hole at its lower and through which water enters in it. The function of the surge tank depends upon the area of hole.



Hydro Electric Power Plant:

In hydro-Electric plants, energy of water is utilised to move the turbines which in turn run the electric generators. The energy of water utilised for power generation may be kinetic or potential. The kinetic energy of water is its energy in motion and is a function of mass and velocity, while the P.E is a function of the difference in level/head of water between two points. In either case continuous availability of water is a basic necessity; to ensure this, water collected in natural lakes and reservoirs of high altitudes may be utilized or water may be artificially stored by constructing dams across flowing stream. The ideal site is one in which a good system of natural lakes with substantial catchment area, exists at a high altitude. Rainfall is the primary source of water and depends upon such factors as temperature, humidity, cloudiness, wind etc. The usefulness of rainfall for power purposes further depends upon several complex factors which include its intensity, time distribution, topography of land etc, However its has been observed that only a small part of the rainfall can actually be utilized for power generation. A significant part is accounted for by direct evaporation, while another similar quantity seeps in to the soil and forms the underground storage, some water is also absorbed by vegetation. Thus only a part of water falling as rain actually flows over the ground surface as direct runoff and forms the streams which can be utilized for hydroschemes.

First hydroelectric station was probably started in America in 1882 and thereafter development took place very rapidly. In India, the first major hydro-electric development of 4.5 MW capacity named as Sivasamudram Scheme in Mysore was commi-ssioned in 1902. In 1914, a hydropower plant named khopoli project of 50 MW capacity was commissioned in Maharastra. The hydropower capacity, upto 1947, was nearly 500 MW.

Hydropower is a conventional renewable source of energy which is clean, free from pollution and generally has a good environmental effect. However the following facts are major obstacles in the utilisation of hydropower resources:

(i) Large investments
(ii) Long gestation period
(iii) Increased cost of power transmission.

Next to thermal power, hydropower is important in regard to power generation. The hydroelectric power plants provide 30% of the total power of world. The total hydropotential of the world is about 5000 GW. In some countries (like Norway) almost total power generation is hydrobased.

Application of Hydroelectric Power Plants:

Earlier hydro-electric plants have been used as exclusive source of power, but the trend is towards use of hydro-power in an inter-connected system with thermal stations. As a self-contained and independent power source a hydroplant is most effective with adequate storage capacity otherwise the maximum load capacity of the station has to be based on the year. This increases the per unit cost of installation. By interconnecting hydropower with steam, a great deal of saving in cost can be effected due to:

(i) reduction in necessary reserve capacity
(ii) diversity in construction programmes
(iii) higher utilisation factors on hydroplants, and
(iv) higher capacity factors on efficient steam plants.

In an inter-connected system the base load is supplied by hydropower when the maximum flow demand is less than the stream flow while steam supplies the peak. When stream flow is lower than the maximum demand the hydroplant supplies the peak load and steam plant the base load.

Advantages and Disadvantages of hydro-electric power plants:

Advantages and Disadvantage of Hydro-electric Power Plants:

* Advantages:

1. No fuel charges.
2. A hydroelectric plant is highly reliable
3. Maintenance and Operation charges are very low.
4. Running cost of the plant is low.
5. The plant has no standby losses.
6. The plant efficiency doesnot change with age.
7. It takes a few minutes to run and synchronise the plant.
8. less supervising staff is required.
9. No fuel transportation problem.
10. NO ash problem and atmosphere is not polluted since no smoke is produced in the plant.
11. In addition to power generation; these plants are also used for flood control and irrigation purposes.
12. such a plant has compairitively a long life (100 to 125 years as against 20-45 years of a thermal plant).
13. The number of operations required is considerably small compared with thermal power plants.
14. The machines used in hydro-electric plants are more robust and generally run at low speeds at 300 to 400 rpm. where as the machines used in thermal plants run at a speed 3000 to 4000 rpm.
15. Therefore, there are no specialised mechanical problems or special alloys required for construction.

    The cost of land is not a major problem since the hydroelectric stations are situated away from the developed areas.

• Disadvantages:

1. The initial cost of the plant is very high.
2. It takes considerably long time for the erection of such plants.

such plants are usually located in hilly areas far away from the load centre and as such they require long transmission lines and losses in them will be more.

4. Power generation by the hydro-electric plant is only dependent on the quantity of water available which in turn depends on the natural phenomenon of rain. So, if the rainfall is in time and proper and the required amount of it can be collected, the plants will function satisfactorily otherwise not.

Surge Tanks:

A surge tank is a small reservoir or tank in which the water level raises or falls to reduce the pressure swings so that they are not transmitted in full to a closed. In general a surge tank is employed to serve the following purposes.

1. To reduce the distance between the free water surface and turbine thereby reducing the water hammer effect (the water hammer is defined as the change in pressure rapidly above or below normal pressure caused by sudden changes in rate of flow through the pipe according to the demand of the prime mover) on penstock and also protect upstream tunnel from high pressure rises.
2. To serve as supply tank to the turbine when the water in the pipe is accelerating during increased load conditions and storage tank when the water is decelerating during reduced load continuous.

* Types of Surge Tanks:

The different types of surge tanks in use are:

1. Simple surge tank.
2. Inclined surge tank.
3. The expansion chamber and gallery type surge tank.
4. Restricted Orifice surge tank
5. Differential surge tank.

1. **Simple Surge Tank:** A simple surge tank is a vertical stand pipe connected to the penstock. In the surge tank if the overflow is allowed, the rise in pressure can be eliminated but overflow surge tank is seldom satisfactory and usually uneconomical.

UNIT QUANTITIES:

In order to predict the behaviour of a turbine working under varying conditions of head, speed, output and gate opening, the results are expressed in terms of quantities which may be obtained when the head on the turbine is reduced to unity. The conditions of the turbine under unit head are such that the efficiency of the turbine remains unaffected. The following are the three important quantities which must be studied under unit head:

1. Unit Speed
2. Unit Power
3. Unit discharge

1. Unit Speed: It is defined as the speed of a turbine working under a unit head (i.e., under a head of 1m) It is denoted by N_u. The expression for unit speed (N_u) is obtained as:

Let N = speed of a turbine under a head H.

H = Head under which a turbine is working.

u = Tangential velocity.

The tangential velocity, absolute velocity of water and head on the turbine are related as

$$u \propto V$$

$$\boxed{u \propto \sqrt{H}} \text{ ——— (1)} \qquad \text{where } V \propto \sqrt{H}$$

Also tangential velocity (u) is given by

$$u = \frac{\pi D N}{60}$$

where D = Diameter of the turbine,

For a given turbine, the diameter (D) is constant.

$$u \propto N$$

(or) $$N \propto u$$

$$\propto \sqrt{H} \qquad \because u = \sqrt{H}$$

where k_1 = constant of proportionality

If head on the turbine becomes unity, the speed becomes unit speed (or) when $H = 1$, $N = N_u$

Substituting these values in equation (ii)

$$N_u = k_1\sqrt{1} = k_1$$

Substituting the value of k_1 in eqn. (ii)

$$N = N_u\sqrt{H}$$

$$\boxed{N_u = \frac{N}{\sqrt{H}}}$$

2. Unit Discharge: It is defined as the discharge passing through a turbine, which is working under a unit head (i.e., 1m). It is denoted by the symbol Q_u. The expression for unit discharge is given as;

Let H = Head of water on the turbine.

Q = Discharge passing through turbine when head is H on the turbine.

a = Area of flow of water.

The discharge passing through a given turbine under a head 'H' is given by,

$$Q = \text{Area of flow} \times \text{velocity}$$

But for a turbine, area of flow is constant and velocity is proportional to $\sqrt{H}$

$$Q \propto \text{Velocity} \propto \sqrt{H}$$

$$Q = k_2\sqrt{H} \quad \text{———— (iii)}$$

where k_2 = constant of proportionality

If $H = 1$, $Q = Q_u$

Substituting these values in eqn (iii)

$$Q_u = k_2\sqrt{1.0} = k_2$$

Substituting the value of k_2 in eqn (iii)

$$Q = Q_u\sqrt{H}$$

$$\boxed{Q_u = \frac{Q}{\sqrt{H}}}$$

Unit Power: It is defined as the power developed by a turbine working under a unit head (i.e., $H = 1m$). It is denoted by the symbol 'P_u'. The expression for unit power is obtained as;

Let H = Head of water on turbine.

P = Power developed by turbine under a head of H.

Q = discharge through turbine under a head H.

The overall efficiency (η_o) is given as

$$\eta_o = \frac{\text{Power developed}}{\text{Water power}}$$

$$= \frac{P}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$P = \eta_o \times \frac{\rho \times g \times Q \times H}{1000}$$

$$\propto Q \times H$$

$$\propto \sqrt{H} \times H^2$$

$$\propto \sqrt{H^3} \qquad \because Q \propto \sqrt{H}$$

$$\propto H^{3/2}$$

$$= K_3 H^{3/2} \quad \text{——— (iv)}$$

where K_3 = constant of proportionality.

when $H = 1m$, $P = P_u$

$$P = K_3 (1)^{3/2} = K_3$$

substituting the value of K_3 in eq (iv)

$$P = P_u H^{3/2}$$

$$\boxed{P_u = \frac{P}{H^{3/2}}}$$

Use of Unit Quantities: (N_u, Q_u, P_u)

If a turbine is working under different heads the behaviour of the turbine can be easily known from the values of unit quantities, i.e., from the values of unit speed, unit discharge and unit power.

Let $H_1, H_2, ----$ are the heads under which a turbine works,

$N_1, N_2, ---$ = the corresponding speeds,

$Q_1, Q_2, ----$ = the discharge

$P_1, P_2, ----$ = are the power developed by the turbine.

We know that $N_u = \frac{N}{\sqrt{H}}$; $Q_u = \frac{Q}{\sqrt{H}}$; $P_u = \frac{P}{H^{3/2}}$

using these relations.

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$$

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$$

$$P_u = \frac{P_1}{\sqrt{H_1}} = \frac{P_2}{\sqrt{H_2}}$$

Hence, if the speed, discharge and power developed by a turbine under a head are known, then by using above relations, the speed, the discharge and power developed by the same turbine under a different head can be obtained easily.

1) A turbine develops 9000 kW when running at 100 rpm. The head on the turbine is 30m. If the head on the turbine is reduced to 18m, determine the speed and power developed by the turbine.

Sol: Power developed $P_1 = 9000$ kW.

speed $N_1 = 100$ rpm.

Head $H_1 = 30$ m

let for a head $H_2 = 18$ m

Speed $= N_2$

Power $= P_2$

We know that $\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$

$$N_2 = N_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 100 \times \frac{\sqrt{18}}{\sqrt{30}} = \frac{100 \times 4.2426}{5.4772} = 77.46 \text{ rpm.}$$

Also we know that $\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$

$$P_2 = P_1 \times \left(\frac{H_2}{H_1}\right)^{3/2} = 9000 \times \left(\frac{18}{30}\right)^{3/2} = \frac{687307.78}{164.316}$$

$$= 4182.84 \text{ kW}$$

A turbine develops 500 kW power under a head of 100 metres at 200 rpm. What would be its normal speed and output under a head of 81 metres?

Sol: power, $P_1 = 500\ kW$

speed, $N_1 = 200\ m$.

Head, $H_1 = 100\ m$.

For a head, $H_2 = 81\ m$

$N_2 =$ speed

$P_2 =$ power

We know that $\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$

$$N_2 = N_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 200 \frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10} \times 200 = 180\ rpm$$

also we know that $\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H^{3/2}}$

$$P_2 = P_1 \times \frac{H_2^{3/2}}{H_1^{3/2}} = 500 \times \frac{(81)^{3/2}}{(100)^{3/2}} = 500 \times \frac{729}{1000} = 364.5\ kW.$$

3) A turbine is to operate under a head of 25m at 200 rpm. The discharge is 9 cumec. If the efficiency is 90%. determine the performance of the turbine under a head of 20m.

Sol: Head on turbine, $H_1 = 25\ m$.

speed, $N_1 = 200$ rpm.

Discharge, $Q_1 = 9\ m^3/s$.

Overall efficiency, $\eta_o = 90\% = 0.90$

performance of the turbine under a head, $H_2 = 20m$. means to find the speed, discharge and power developed by the turbine when working under the head of 20m.

Let for the head, $H_2 = 20m$,

$N_2 =$ speed

$P_2 =$ power

$Q_2 =$ discharge

We know the relation, $\eta_o = \frac{P}{W.P} = \frac{P_1}{\frac{\rho \times g \times Q_1 \times H_1}{1000}}$

$$P_1 = \frac{\eta_0 \times S \times g \times Q_1 \times H_1}{1000}$$

$$= \frac{0.90 \times 1000 \times 9.81 \times 9 \times 25}{1000}$$

$P_1 = 1986.5$ kW

using the relation, $\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$

$$N_2 = N_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 200 \times \frac{\sqrt{20}}{\sqrt{25}} = \underline{178.88 \text{ rpm}}$$

and $\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$

$$Q_2 = Q_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 9 \times \frac{\sqrt{20}}{\sqrt{25}} = \underline{8.05 \text{ m}^3/s}$$

and $\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$

$$P_2 = P_1 \times \left(\frac{H_2}{H_1}\right)^{3/2} = 1986.5 \times \left(\frac{20}{25}\right)^{3/2} = \underline{1421.42} \text{ kW}.$$

4) A pelton wheel is revolving at a speed of 190rpm and develops 5150.25 kW when working under a head of 220m with an overall efficiency of 80%. Determine unit speed, unit discharge and unit power. The speed ratio for the turbine is given as 0.47. Find the speed, discharge and power when this turbine is working under a head of 140m.

Sol: Speed, $N_1 = 190$ rpm

Power, $P_1 = 5150.25$ kW

Head, $H_1 = 220$ m

Overall efficiency, $\eta_0 = 80\% = 0.80$

Speed ratio $= 0.47$

New head of water, $H_2 = 140$ m

Overall efficiency is given as;

$$\eta_0 = \frac{P_1}{\frac{S \times g \times Q_1 \times H_1}{1000}} = \frac{1000 P_1}{S \times g \times Q_1 \times H_1}$$

$$Q_1 = \frac{1000 \times 5150.25}{0.80 \times 1000 \times 9.81 \times 220}$$

$$Q_1 = 2.983 \ m^3/s$$

We know that the unit speed is given by,

$$N_u = \frac{N_1}{\sqrt{H_1}} = \frac{190}{\sqrt{220}} = 12.81 \text{ rpm}$$

Unit discharge is given by,

$$Q_u = \frac{Q_1}{\sqrt{H_1}} = \frac{2.983}{\sqrt{220}} = 0.201 \ m^3/sec$$

unit power is given by,

$$P_u = \frac{P_1}{(H_1)^{3/2}} = \frac{5150.25}{(220)^{3/2}}$$

$$= 1.578 \text{ KW}$$

When the turbine is working under a new head of 140 m, the speed, discharge and power are given by,

For speed, $\frac{N_1}{\sqrt{H_1}} = \frac{N_2}{\sqrt{H_2}}$

$$N_2 = N_1 \times \frac{\sqrt{H_2}}{\sqrt{H_1}} = 190\sqrt{\frac{140}{220}} = 151.56 \text{ rpm}.$$

For discharge, $\frac{Q_1}{\sqrt{H_1}} = \frac{Q_2}{\sqrt{H_2}}$

$$Q_2 = Q_1\sqrt{\frac{H_2}{H_1}} = 2.983\sqrt{\frac{140}{220}} = 2.379 \ m^3/sec$$

For power, $\frac{P_1}{H_1^{3/2}} = \frac{P_2}{H_2^{3/2}}$

$$P_2 = P_1 \times \left(\frac{H_2}{H_1}\right)^{3/2} = 5150.25\left(\frac{140}{220}\right)^{3/2} = 2614.48 \text{ KW}.$$

5) A pelton wheel is supplied with water under a head of 35m at the rate of 40.5 kilo litre/min. The bucket deflect the jet through an angle of 160° and the mean bucket speed is 13 m/s. Calculate the power and hydraulic efficiency of the turbine.

Sol: Net head, $H = 35$ m

Discharge, $Q = 40.5$ kilo litre/min

$= 40.5 \times 1000$ lit/min.

$= \frac{40.5 \times 1000}{1000}$ m^3/min

$= 40.5$ m^3/min

$= \frac{40.5}{60}$ m^3/sec

$= 0.675$ m^3/sec

Angle of deflection $= 160°$.

$\therefore$ Angle $\phi = 180° - 160° = 20°$

Mean bucket speed, $u = u_1 = u_2 = 13 m/s$.

Calculate (i) power at runner

(ii) hydraulic efficiency

Taking the value of $C_v = 1.0$

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 1 \times \sqrt{2 \times 9.81 \times 35}$

$= 26.2 m/s$

$V_{r1} = V_1 - u_1 = 26.2 - 13 = 13.2 m/s$

$V_{w1} = V_1 = 26.2$ m/s

$V_{r2} = V_{r1} = 13.2$ m/s

$V_{w2} = V_{r2} \cos\phi - u_2$

$= 13.2 \cos 20 - 13$

$= 12.554 - 13 = -0.446 m/s$

(i) Power at runner:

We get the workdone by the jet on the runner per second.

Workdone /s $= \rho \times a \times V_1 [V_{w1} + V_{w2}] \times u$

$= \rho Q [V_{w1} + V_{w2}] \times u$

$= 1000 \times 0.675 [26.2 + (-0.446)] \times 13$

$= 225991$ N-m/s

$= 225991$ W

$= 225.991$ kW

$\therefore$ Power at runner $= 225.991$ kW.

(ii) Hydraulic efficiency:

Input power $= \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times 0.675 \times 35}{1000}$

$= 231.761$ kW

Power developed = 225.991 kW

$\therefore$ Hydraulic efficiency = $\frac{\text{Power developed by runner}}{\text{Input power}}$

$$= \frac{225.991}{231.761}$$

$$= 0.975$$

$$= 97.5\%$$

Characteristic curves of Hydraulic Turbines:

Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behaviour and performance of the turbine under different working conditions, can be known. These curves are plotted from the results of the tests performed on the turbine under different working conditions.

The important parameters which are varied during a test on a turbine are;

1. Speed (N)
2. Head (H)
3. Discharge (Q)
4. power (P)
5. Overall efficiency (η_o)
6. Gate opening.

Out of the above six parameters, three parameters namely speed (N), Head (H) and discharge (Q) are independent parameters.

Out of the three independent parameters, N, H, Q are one of the parameter is kept constant say H and the variation of the other four parameters with respect to any one of the remaining two independent variables say N and Q are plotted and various curves are obtained. The curves are called characteristic curves.

1. Main characteristic curves (or) Constant Head curve.
2. Operating characteristic curves (or) Constant speed curve.
3. Muschel curves (or) constant Efficiency curve.

* 1. Main characteristic curves (or) constant Head curves:

Main characteristic curves are obtained by maintaining a constant head and a constant gate opening (G.O) on the turbine. The speed of the turbine is varied by changing load on the turbine. For

...ach value of the speed, the corresponding values of the power (P) and discharge (Q) are obtained. Then the overall efficiency (η_0) for each value of the speed is calculated. From these readings the value of unit speed (N_u) unit power (P_u) and unit discharge (Q_u) are determined. Taking N_u as abscissa, the values of Q_u, P_u, P and η_0 are plotted as shown fig. By changing the gate opening the values of Q_u, P_u and η_0 are plotted.

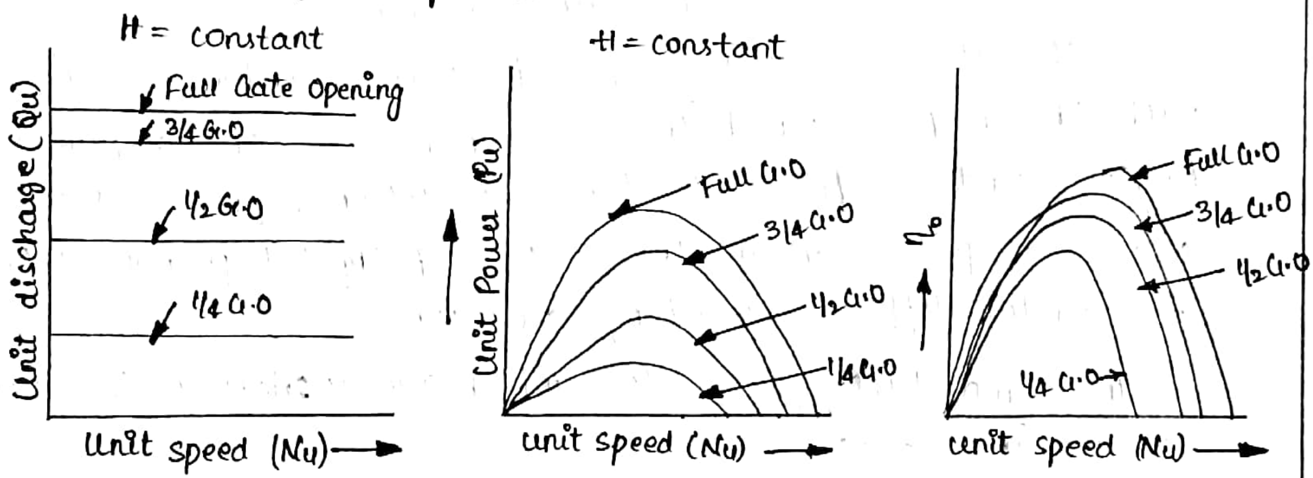
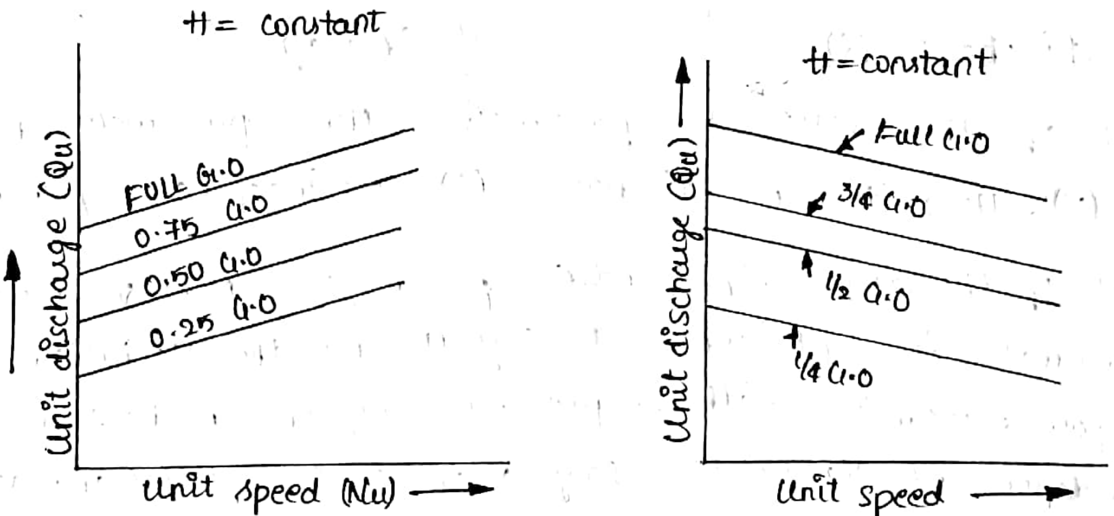


Fig: Main characteristic curves for a pelton wheel.



(a) For Kaplan Turbine (b) For Franscis Turbine

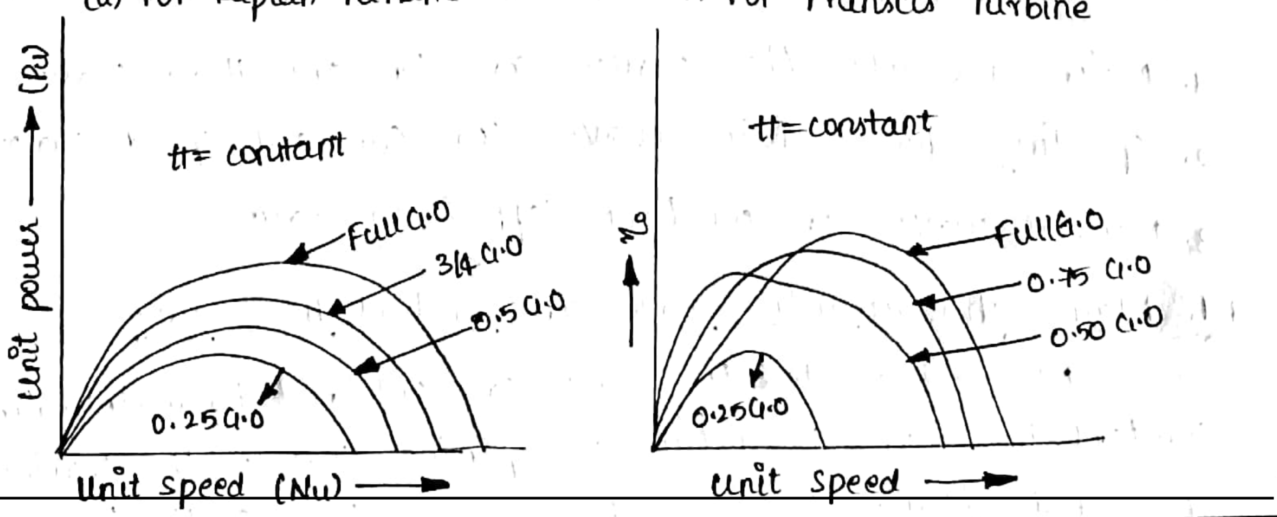


Fig: Main characteristic curves for reaction turbine

2. Operating Characteristic Curves or Constant speed curves:

Operating characteristic curves are plotted when the speed on the turbine is constant. In case of turbines, the head is generally constant. These are three independent parameters namely, N, H and Q. For operating characteristics N and H are constant and hence the variation of power and efficiency with respect to discharge are plotted. The power and efficiency curves will be slightly away from the origin on the x-axis, as to become initial friction certain amount of discharge will be required.

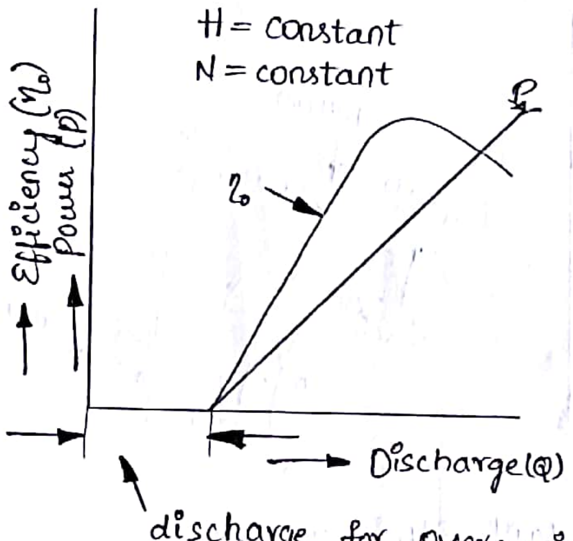


Fig: Operating characteristic curves.

3. Constant Efficiency curves or Muschel curves (or) Iso-efficiency curves:

These curves are obtained from the speed vs efficiency and speed vs. discharge curves for different gate openings. For a given efficiency from the Nu vs η_o curves, there are two speeds. From the Nu Vs Qu curves, corresponding to two values of speeds there are two values of discharge. Hence for a given efficiency there are two values of discharge for a particular gate opening. This means for a given efficiency there are two values of speeds, a two values of discharge for a given gate opening. If the efficiency is maximum there is only one value. These two values of speed and two values of discharge corresponding to a particular G.O are plotted as shown in fig. The procedure is repeated for diff. gate openings and the curves Q vs. N are plotted. The points having the same efficiencies are joined. The curves having same efficiency are called iso-efficiency curves. These curves are helpful for determining the zone of constant efficiency and for predicting the performance of the turbine at various efficiencies.

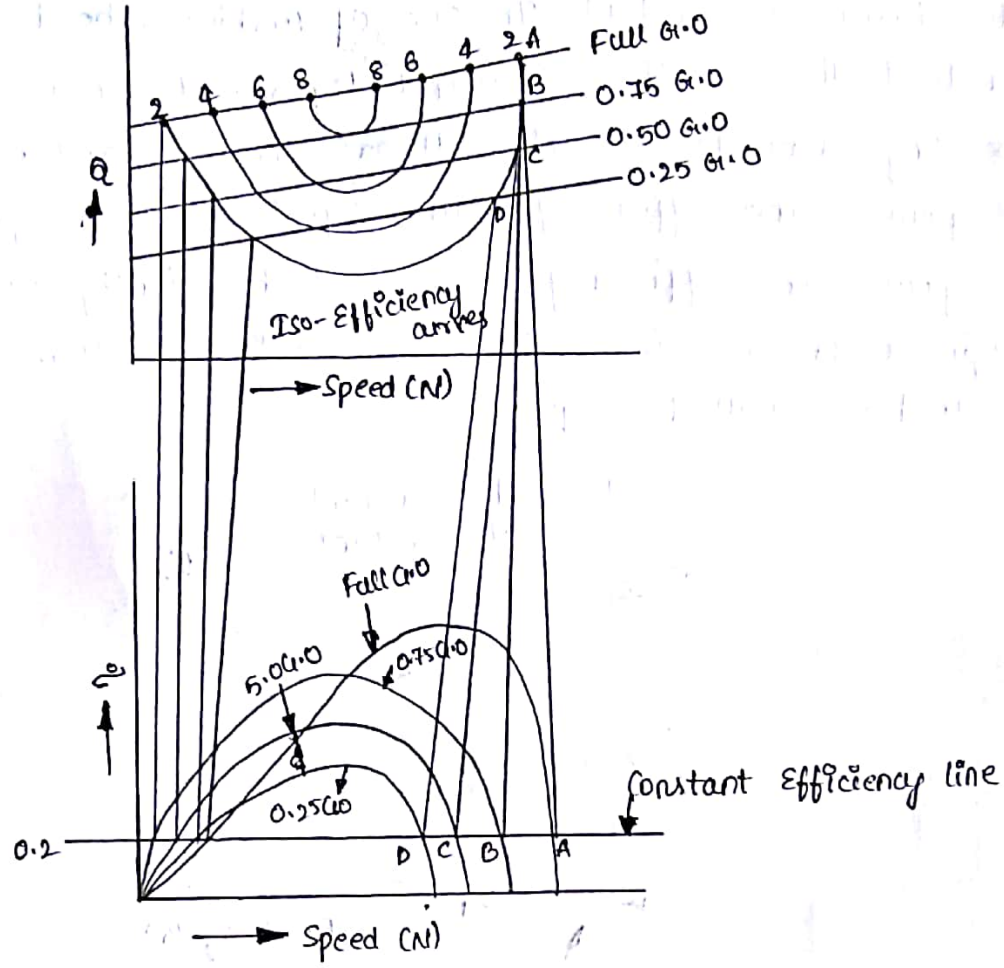


Fig: Constant Efficiency curve.

Governing of Turbines:

The governing of a turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done automatically by means of a governor, which regulates the rate of flow through the turbines according to the changing load conditions on the turbine.

Governing of a turbine is necessary as a turbine is directly coupled to an electric generator which is required to run at constant speed under all fluctuating load conditions. The frequency of power generation by a generator of constant number of pair of poles under all varying conditions should be constant. This is only possible when the speed of the generator, under all changing load condition is constant. The speed of the generator will be constant, when the speed of the turbine (which is coupled to the generator) is constant.

When the load on the generator decreases the speed of the generator increases beyond the normal speed (constant speed). Then the speed of the turbine also increases beyond the normal speed (constant speed). When the speed of the turbine also increases beyond the normal speed. If the turbine or the generator is to run at constant (normal) speed, the rate of flow of water to the turbine should be decreased it to the speed becomes normal. This process by which the speed of the turbine (and hence of generator) is kept constant under varying condition of load is called governing.

Governing of pelton Turbine (Impulse Turbine):

Governing of pelton turbine is done by means of oil pressure governor, which consists of the following as:

1. Oil sump.
2. Gear pump also called oil pump, which is driven by the power obtained from turbine shaft.
3. The servomotor also called the relay cylinder.
4. The control valve or the distribution valve or relay valve.
5. The centrifugal governor or pendulum which is driven by belt or gear from the turbine shaft.
6. Pipes connecting the oil sump with the control valve and control valve with servomotor and
7. The spear rod or needle.

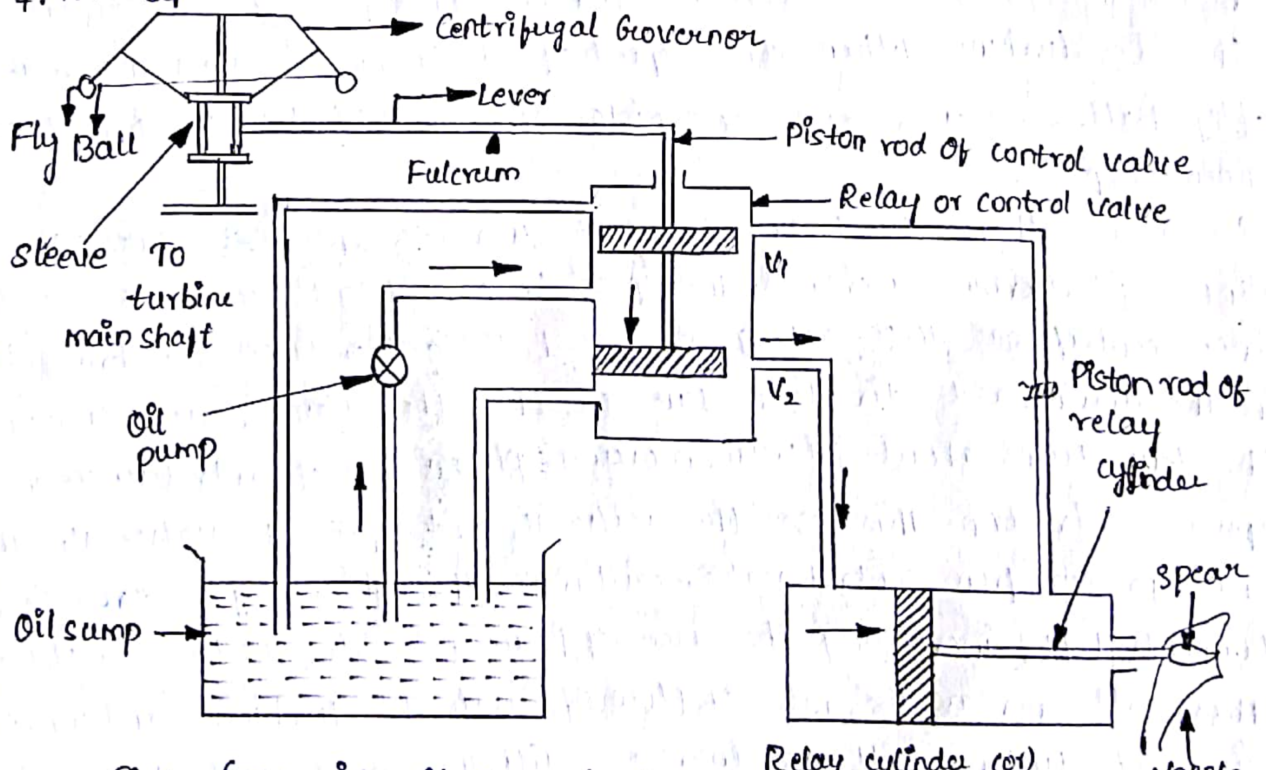


Fig: Governing of Pelton turbine

...ure shows the position of the piston in the relay cylinder, position of control or relay valve and fly-balls of the centrifugal governor, when the turbine is running at the normal speed.

When the load on the generator decreases, the speed of generator increases. This increases the speed of the turbine beyond the normal speed. The centrifugal governer, which is connected to the turbine main shaft, will be rotating at an increased speed. Due to increase in the speed of centrifugal governor, the fly-balls move upwards due to increased centrifugal force on them. Due to the upward movement of fly-balls, the sleeve will also move upward. A horizontal lever, supported over a fulcrum, connects the sleeve and the piston rod of control valve. As the sleeve moves up, the lever turns about the fulcrum and the piston rod of the control valve moves downward. This close the valve V_1 and opens the valve V_2.

The oil, pumped from the oil pump to the control valve or relay valve, under pressure will flow through the valve V_2 to the servometer (or relay cylinder) and will exert force on the face A of the piston rod and ~~spear will move~~ of relay cylinder. The piston along with piston rod and spear will move towards right. This will decrease the area of flow of water at the outlet of the nozzle. This decrease of area of flow will reduce the rate of flow of water the turbine which consequently reduces the speed of the turbine. When the speed of the turbine becomes normal, the fly-balls, sleeve, lever and piston rod of control valve come to its normal piston.

When load on generator increases, speed of generator and hence of turbine decreases. The speed of centrifugal governor also decreses and hence centrifugal force acting on fly-balls also reduces. This brings the fly-balls in the downward direction. Due to this, the sleeve moves downward and the lever turns about fulcrum, moving piston rod of control valve in the upward direction. This closes the valve V_2 and opens the valve V_1. The oil under pressure from control valve, will move through with piston rod and spear towards left, increasing the area of flow of water at the outlet of nozzle. This will increase the rate of flow of water to turbine and consequently, the speed of turbine will also increases till the speed of the turbine becomes normal.

Cavitation:

The formation, growth and collapse of vapour filled cavities or bubbles in a flowing liquid due to local fall in fluid pressure is called cavitation. When the pressure at any point in a flow field equals the vapour pressure of the liquid at that temperature vapour cavities (bubbles of vapour) begin to appear. It is presumed that a vapour cavity is formed around a dust nuclei which is in the liquid (The vapour pressure values of water at 15°c and 20°c are 1.74m and 2.38m of water column absolute). The cavities thus formed, due to motion of liquid, are carried to high pressure regions where the vapour condenses and they suddenly collapse. The adjoining liquid rushes with a very great velocity and hence with very great force to occupy the empty spaces thus created, causes series of violet, irregular, spherical shock waves. When these irregular implosions occur on the metallic surface, they produce noise and vibration.

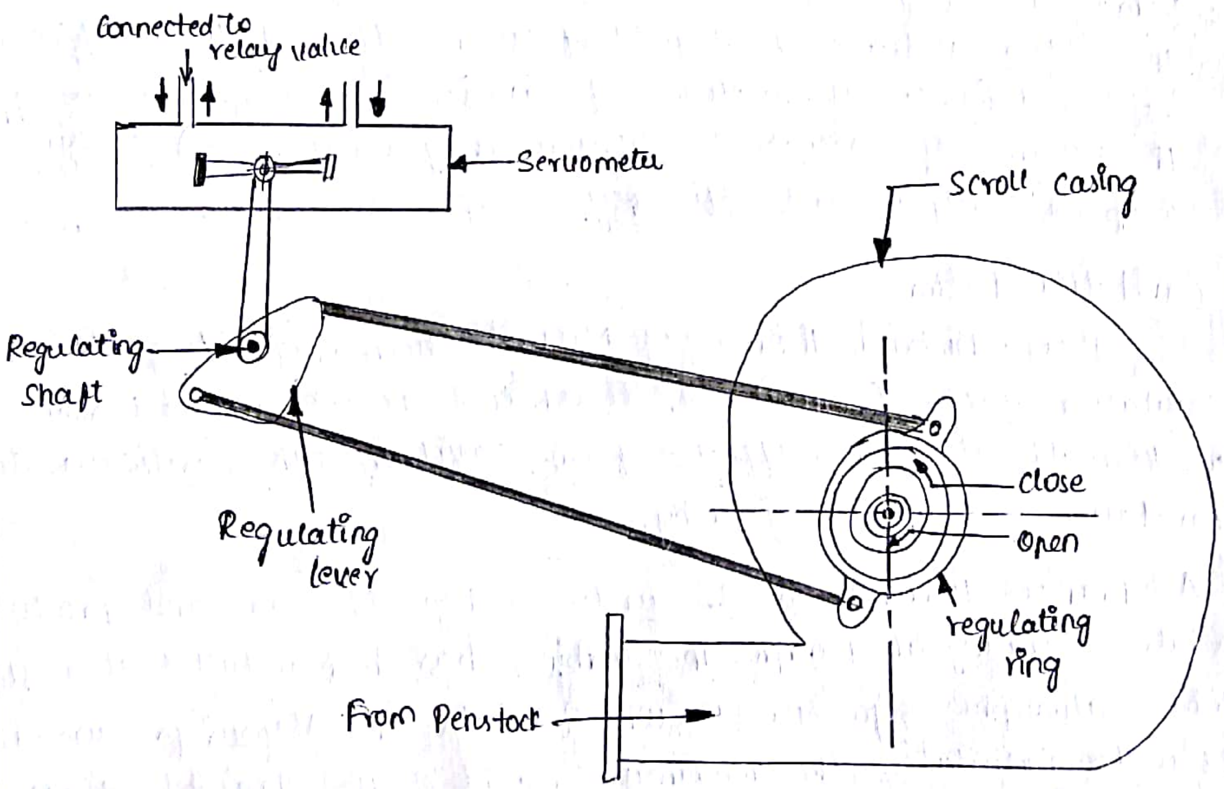


Fig: Governing Mechanism for reaction turbines.

When the cavities collapse the collapsing pressure is of the order of 100 times the atmospheric pressure on the surface of a body, due to repeated hammering action, the metal particles gives way ultimately due to fatigue and indentations are formed; this erosion of material is called pitting.

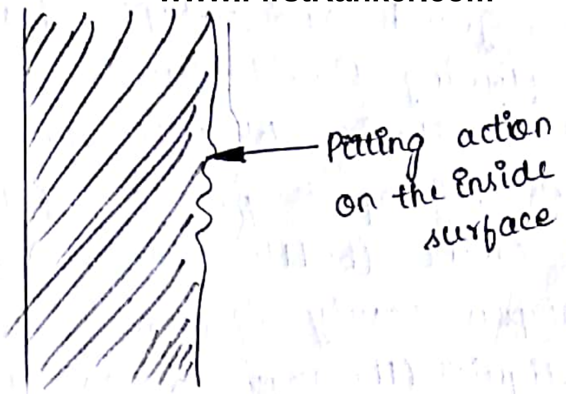


Fig: Pitting action on the inside surface (shown in large scale)

In reaction turbines the cavitation may occur at the runner exit or the draft tube inlet where the pressure is negitive. The hydraulic machinery is affected by the cavitation in the following three ways:

1. Roughening of all the surfaces takes place due to loss of material caused by pitting.
2. Vibration of parts is caused due to irregular collapse of cavities.
3. The actual volume of liquid flowing through the machine is reduced (since the volume of cavities is many times more than the volume of water from which they are formed) causing sudden drop in output and efficiency.

* Cavitation Factor:

Prof. Dietrich Thoma of Munich (Germany) suggested a cavitation factor (sigma) to determine the zone where turbine can work without being affected from cavitation. The critical value of cavitation factor (σ_c) is given by,

→ A Francis Turbine works under a head of 25m and produces 1180 kW while running at 120 rpm. The turbine has been installed at a station where atmospheric pressure is 10m of water and vapour pressure is 0.2m of water. Calculate the maximum height of the straight draft tube for the turbine.

Sol: Head under which the turbine works, $H = 25m$

power output $P = 11800 kW$

speed of the turbine, $N = 120$ rpm

atmospheric pressure, $P_a = 10m$ of water

Maximum height of the draft tube: (H_s)

$\sigma_c = 0.625 \left(\frac{N_s}{380.78}\right)^2$

$= 0.625 \left(\frac{233.2}{380.78}\right)^2$

$= 0.2344$

Also, $\sigma_c = \frac{H_a - H_v - H_s}{H}$

$0.234 = \frac{10 - 0.2 - H_s}{25}$

$0.234 \times 25 = 10 - 0.2 - H_s$

$H_s = 10 - 0.2 - (0.234 \times 25)$

$= \underline{3.94 m}$

Hence, max. permissible height of the draft tube, $H_s = \underline{3.94m}$

Selection of Hydraulic Turbines:

The following points should be considered while selecting right type of hydraulic turbine for hydro-electric power plant:

(i) 1. Specific speed:

High specific speed is essential where head is low and output is large, because otherwise the rotational speed will be low which means cost of turbo-generator and power-house will be high. On the other hand, there is practically no need of choosing a high value of specific speed for high installations, because even with low specific speed high rotational speed can be attained with medium capacity plants.

(ii) 2. Rotational Speed:

It depends on specific speed. Also rotational speed of an electric generator with which the turbine is to be directly coupled, depends on the frequency and no. of pair of poles. The value of specific speed adopted should be such that it will give the synchronous speed of the generator.

(iii) 3. Efficiency:

The turbine selected should be such that it gives the highest overall efficiency for various operating conditions.

(c) 4. Partload Operation:

In general the efficiency at partloads and overloads is less than normal. For the sake of economy the turbine should always run with maximum possible efficiency to get more revenue.

When the turbine has to run at part or overload conditions Deriaz turbine is employed. Similarly, for low heads, Kaplan turbine will be useful for such purposes in place of propeller turbine.

$$\boxed{\sigma_c = \frac{(H_a - H_v) - H_s}{H}}$$

Where H_a = Atmospheric pressure head in metres of water.

H_v = Vapour pressure head in metres of water corresponding to the water temperature.

H = Working head of turbine (difference between head race and tail level in metres), and

H_s = suction pressure head (or height of turbine outlet above tail race level in metres)

The values of critical factor depends upon the specific speed of the turbine.

The value for σ_c for different materials may be determined with the help of following empirical relations:

- For Francis turbine : $\sigma_c = 0.625\left[\frac{N_s}{380.78}\right]^2$
- For Propeller turbine : $\sigma_c = 0.28 + \left[\frac{1}{7.5}\left(\frac{N_s}{380.78}\right)^3\right]$
- For Kaplan turbine : Values of σ_c obtained by above equation (propeller turbine) should be increased by 10%.

Where N_s = Specific speed in rpm

* Suction specific speed $(N_s)_{suc}$: In addition to thomas criterion the consideration of suction specific speed provides very useful criterion for establishing similarity in respect of cavitation in the turbines. The suction speed may be defined as speed of a geometrically similar turbine such that when it is developing a power equal to 1kW the total suction head thr is equal to 1m (absolute units). It can be proved that specific speed is given by,

$$N_s = \frac{NSP}{(\sigma H)^{4/5}}$$

$$\sigma = \left[\frac{N_s}{(N_s)_{suc}}\right]^{4/5}$$

The above equations give the relation between the two parameters σ and $(N_s)_{suc}$, both of which are useful for establishing a similarity in respect of cavitation in the model and prototype turbines. The concept of suction speed, however, is more commonly used in pumps.

* Methods to avoid cavitation:

The following methods may be used to avoid cavitation:

1. Runner / turbine may be kept under water. But it is not advisable as the inspection and repair of the turbine is difficult. The other method to avoid cavitation zone without keeping the runner under water is to use the runner of low specific speed.

2. The cavitation free runner may be designed to fulfil the given conditions with extensive research.

3. It is possible to reduce the cavitation effect by selecting materials which resist better the cavitation effect. The cast steel is better than cast iron and stainless steel or alloy steel is still better than cast steel.

4. The cavitation effect can be reduced by polishing the surface, That is why the cast steel runner and blades are coated with stainless steel.

5. The cavitation may be avoided by selecting a runner of proper specific speed for given head.

(•) 5. Cavitation:

The installation of water turbines of reaction type over the tail race is affected by cavitation. The critical value of cavitation factor must be obtained to see that the turbine works in safe zone. Such a value of cavitation factor also affects the design of turbine, especially of kaplan, propeller and bulb types.

(•) 6. Disposition of turbine shaft:

Experience has shown that the vertical shaft arrangement is better of large sized reaction turbines, therefore, it is almost universally adopted. In case of large size impulse turbines, horizontal shaft arrangement is mostly employed.

(•) 7. Head:

(i) Very high Heads: (350m and above)

For heads greater than 350m, pelton turbine is generally employed and there is practically no choice except in very special cases.

(ii) High Heads: (150m and 350m)

In this range either pelton (or Francis turbine may be employed. For high specific speeds Francis turbine is more compact and economical than the pelton turbine which for same working conditions would have to be much bigger and rather cumbersome.

(iii) Medium Heads: (60m and 150m)

A Francis turbine is usually employed in this range. Whether a high or low specific speed unit would be used depends on the selection of the speed.

(iv) Low Heads: (below 60m)

Between 30 and 60m heads both Francis and Kaplan turbines may be used. The latter is more expensive but yield a higher efficiency at partloads and overloads. It is therefore preferable for variable loads. Kaplan turbine is generally employed for heads under 30m.

Propeller turbines are however, commonly used for heads upto 15m. They are adopted only when there is practically no load variations.

(v) Very low heads:

For very low heads from bulb turbines are employed these days. although Kaplan turbines can also be used for heads from 2m to 15m. but they are not economical.

Table: Criteria for Selection of Turbines

S.No	Type of Turbine	Head H (m)	Specific Speed (Ns)	Speed ratio (Ku)	Maximum Hydraulic efficiency (%)	Remarks
1.	Pelton: 1 Jet	upto 2000	12 to 30	0.43 to 0.48	89	Employed for very high head
	2 Jets	upto 1500	17 to 50			
	4 Jets	upto 500	24 to 70			
2.	Francis: High head	upto 300	80 to 50	0.6 to 0.9	93	Full load efficiency high partload efficiency lower than pelton wheel
	Medium head	50 to 150	150 to 250			
	Low head	30 to 60	250 to 400			
3.	Propeller and kaplan	4 to 60	300 to 1000	1.4 to 2	93	High part load efficiency; high discharge with low head.
4.	Bulb or tubular turbines	3 to 10	1000 to 1200	6 to 8	91	Employed for very low head-tidal power plants

Overall efficiency (η_o) of all turbines = 85 percent