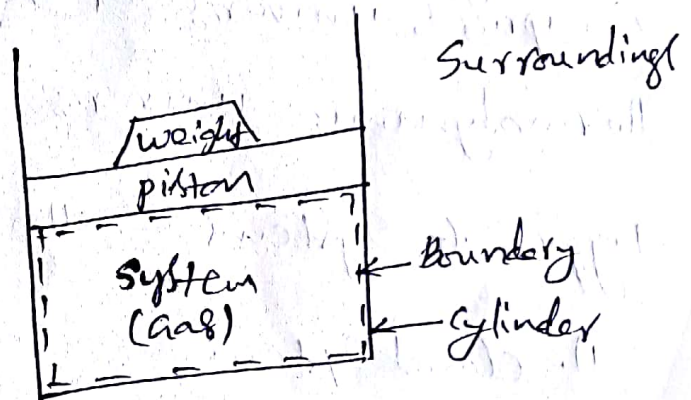
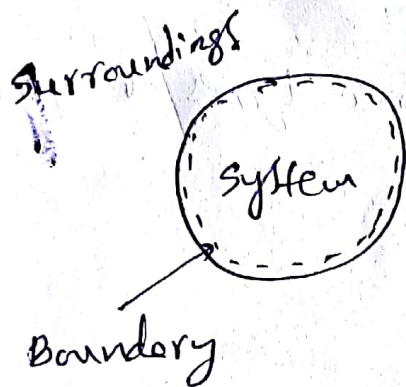


Thermodynamics:- It is the branch of science explaining the principles of conversion of heat energy into other forms of energy.

Applications:- Steam power plants, nuclear power plants, I.C. engines, air-conditioning, refrigeration, Jet propulsion, chemical process plants, compressors etc.

System:-



A definite area or a space where some thermodynamic process takes place is known as system or thermodynamic system.

Ex I.C. engine cylinder, a system may be quantity of steam or gas, etc.

Boundary:- The system is separated from the surrounding by the system boundary as shown in fig. The boundary may be either fixed or moving. A system is enclosed by a boundary.

Surrounding is the Combination of matter and

Space external to the system and outside the imaginary boundary is called surrounding.

As the system undergoes property changes, heat and work energies are transferred across the boundary to the surrounding.

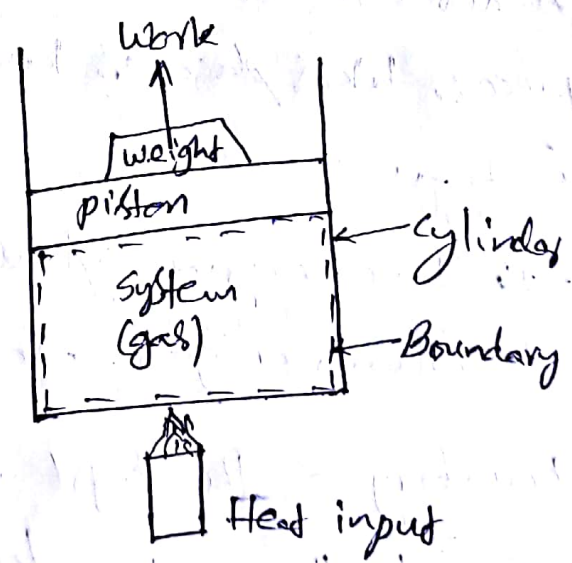
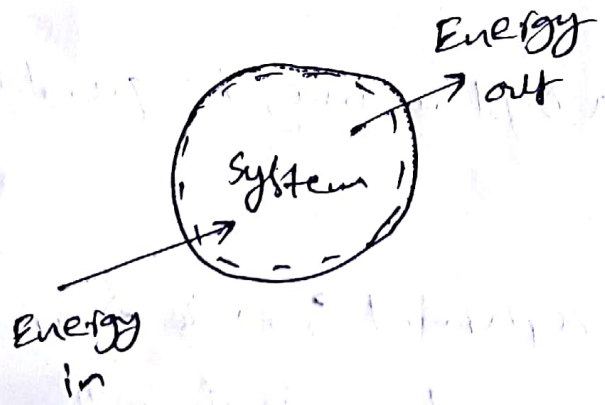
Universe:— A system and its surrounding together

comprises universe of thermodynamic universe.
Thermodynamic universe is an isolated system.

Types of Systems:— There are three types of system.

- 1). closed system
- 2). open system
- 3). Isolated system.

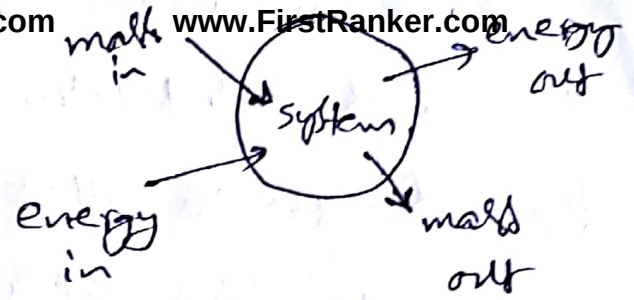
1). closed system:—



only heat and work energies are transferred across the boundary of the system, the mass remains the same.

Ex:- I.C. engine, gas in a cylinder.

Both the heat and work as well as the mass of the working substance crosses the boundary of the system. The system is known as open system.

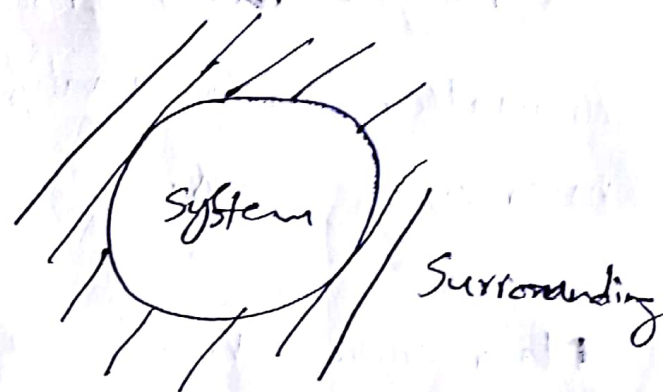


Substance crosses the boundary of the system. The system is known as open system.

Ex:- Turbine, Compressor, boiler etc.

Isolated System:-

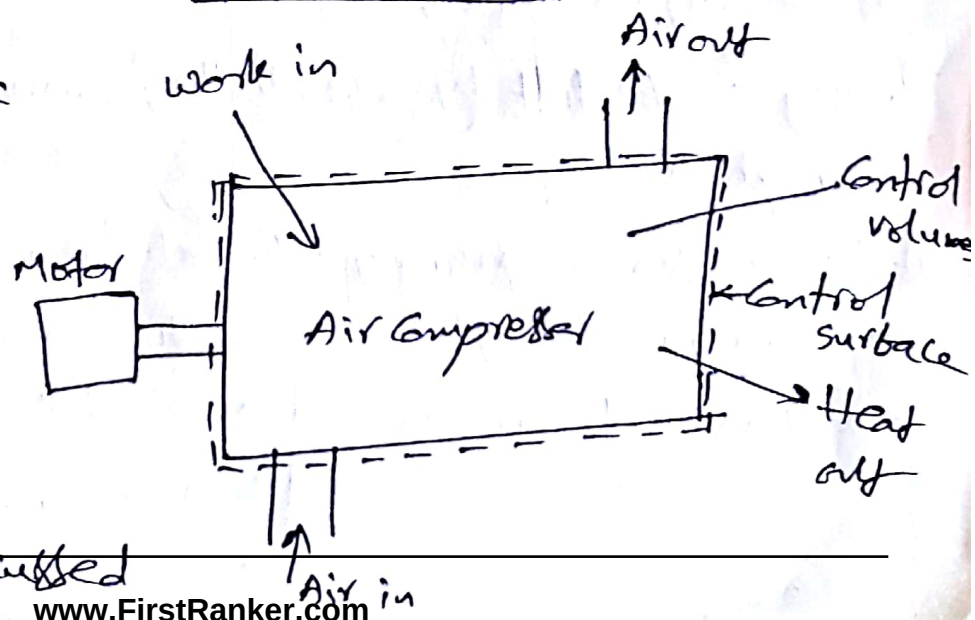
There is no interaction between the system and the surrounding. No mass or energy transfer across the system.



Ex:- Thermos flask, fluid contained in a perfectly insulated closed vessel.

Control volume and Control ~~space~~ surface:-

For thermodynamic analysis of an open system such as an air compressor



attention is focused

on a certain volume space surrounding the system
www.FirstRanker.com www.FirstRanker.com

Known as control volume. Control volume is bounded by a surface is called control surface.

Boundary will be imagined only inside the system but the control surface may be imagined outside the system.

Macroscopic and Microscopic analysis :- The behaviour of matter can be analysed by using macroscopic and microscopic views.

Macroscopic view	Microscopic view
1). No assumptions are made regarding the structure of matter 2). very few parameters are to be mentioned to define a system (pressure, temperature) 3). They can be felt by sense perception. 4). The analysis of macroscopic system required simple mathematical formula	1). Assumptions are made on molecular theory of matter. 2). large number of parameters are required to define a system. (pressure, temperature, density, etc) 3). they cannot be experienced. 4). The advanced statistical and mathematical methods are required to explain the change in system.

Concept of Continuum: — In Engineering thermodynamics we postulate that the material in our system or control volume is a continuum; that is, it is continuously distributed throughout the region of interest. Such a postulate allows us to describe a system or control volume using only a few measurable properties.

Consider the definition of density given by

$$\rho = \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V}$$

where Δm is the mass contained in the volume ΔV shown in fig; physically,

ΔV cannot be allowed to shrink to zero since, if ΔV became extremely small,

Δm would vary discontinuously, depending on the number of molecules in ΔV .

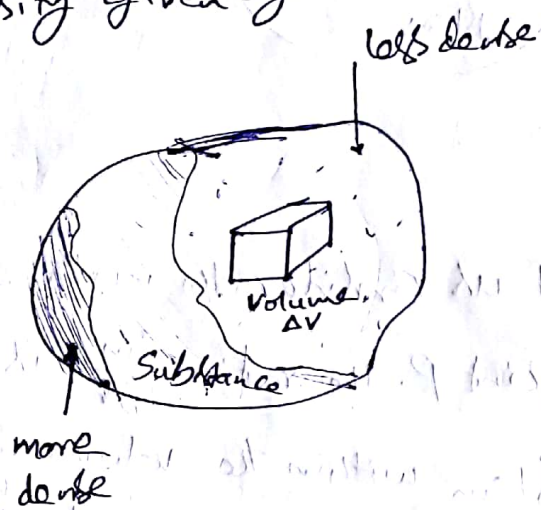
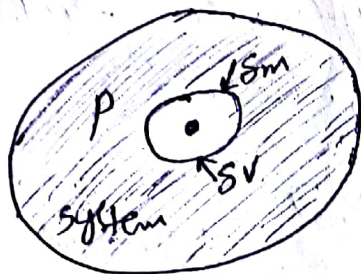


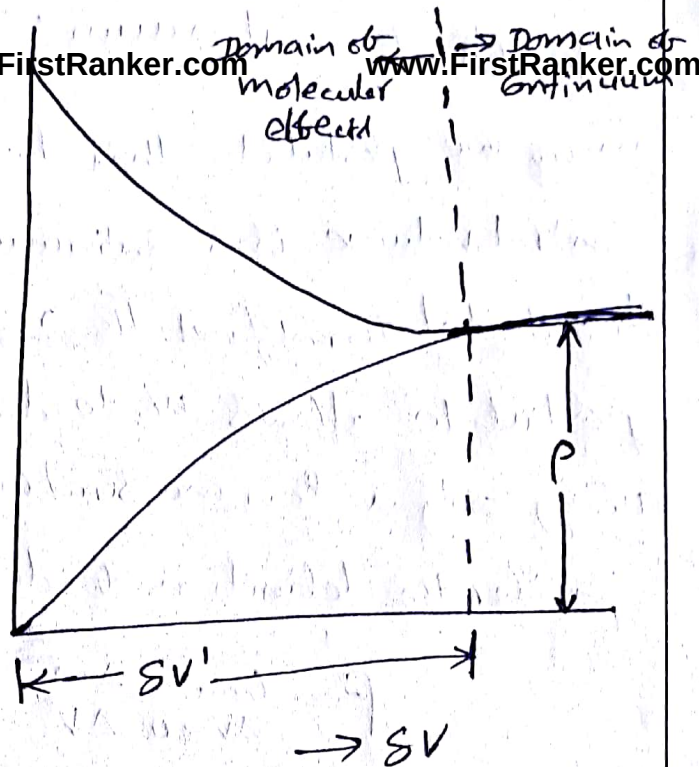
Fig: Mass as a Continuum

There are, however, situations where the continuum assumption is not valid; for example, the re-entry of satellites. At an elevation of 100 km the mean free path, the average distance a molecule travels before it collides with another molecule, is about 30 mm; the macroscopic approach with its continuum assumption is already questionable.

At 150 km the mean free path exceeds 3 m, which is comparable to the dimensions of the satellite. Under these conditions, statistical methods based on molecular activity must be used.



$\frac{S_m}{S_v}$
↑



Let us consider the mass S_m in a volume S_v surrounding the point P . The ratio $\frac{S_m}{S_v}$ is the average mass density of the system within the volume S_v . We suppose that at first S_v is rather large, and is subsequently shrunk about the point P . If we plot $\frac{S_m}{S_v}$ against S_v , the average density tends to approach an asymptote as S_v increases. However, when S_v becomes so small as to contain relatively few molecules, the average density fluctuates substantially with time as molecules pass into and out of the volume in random motion, and so it is impossible to speak of a definite value of $\frac{S_m}{S_v}$. The smallest volume which may be regarded as continuous is its S_v' . The density P of the system at a point is thus defined as

$$P = \lim_{S_v \rightarrow S_v'} \frac{S_m}{S_v}$$

Similarly, the fluid velocity at a point P is defined as the instantaneous velocity of the centre of gravity of the smallest continuous volume S_v' .

Thermodynamic analysis of an open system

Such as an air compressor attention is focused on a certain volume space surrounding the compressor known as control volume. Control volume is bounded by a surface is called control surface.

→ Boundary will be imagined only inside the ~~side~~ system but the control surface may be imagined outside the system.

* THERMODYNAMIC PROPERTIES:

Every system has a certain characteristics by which its physical condition can be described. Such characteristics are called Properties of system.

eg: Volume, Temperature, Pressure etc.

There are two types of Properties:

- (i) Extensive Properties
- (ii) Intensive Properties.

$$P = \frac{F}{A} \quad \frac{\text{kg}}{\text{m}^2}$$

$$P = \frac{\text{mass}}{\text{volume}} \quad \frac{\text{kg}}{\text{m}^3}$$

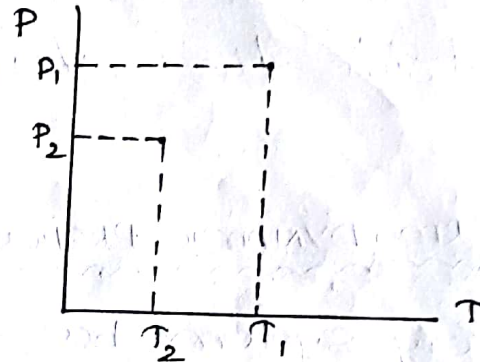
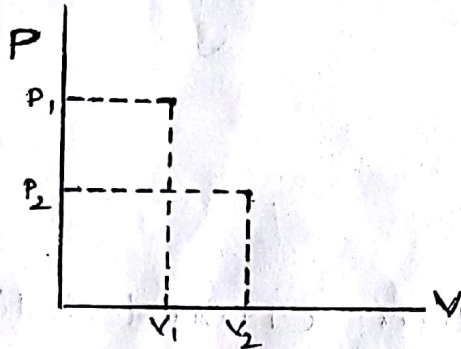
- (i) Extensive Properties: Extensive (or) Quantitative property which are dependent on the mass of the system.

eg: Weight, Volume, Energy

(ii) Intensive Property (or) Qualitative Property

These are Independent of the mass of the system ~~mass of the system~~ that is which donot dependent on the mass of the system.

eg: Pressure, temperature, Specific volume



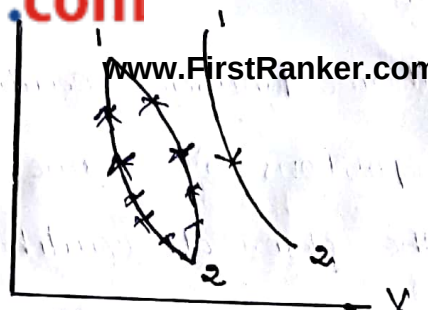
Representation of state points

* STATE OF SYSTEM:

The condition of the system which is described by means of Thermodynamic Properties is called the state of the system.

* CHANGE OF STATE: Any operation in which one (or) more properties of the system change is called change of state.

* PATH: The succession of states passed through, during a change of state is called a path.



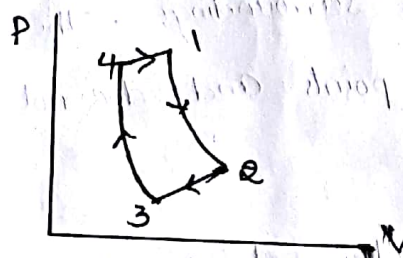
process and a cycle.

when the properties completely specified the change of state is called process.

In fig: 1-2 shows the process.

eg: IC engine contains 4 process.

* Cycle: A thermodynamic cycle is defined as a series of state change such that the final state is identical with initial state. In fig: 1-2 and 2-1 represents the cyclic process.



* HOMOGENOUS SYSTEM: Every substance can exist in any one of the three phases i.e solid, liquid, gaseous. In homogenous system no phase change will occur.

* HETEROGENOUS SYSTEM: This system is consisting more than one phase is known as ^{heterogeneous} ~~heterogeneous~~ system. In this system phase change will occur.

A system is in thermodynamic equilibrium if the temperature and pressure at all positions are same there should be no velocity gradient. the chemical equilibrium is also necessary. Thus for attaining the state of thermodynamic equilibrium the following three types of equilibrium must be achieved.

- Thermal equilibrium.
- Mechanical equilibrium.
- Chemical equilibrium.

Thermal equilibrium: The temperature of the system does not change with time and has same value ^{at} ~~that~~ all points of the system.

Mechanical equilibrium: There is no unbalanced forces within the system. (or) between the surroundings. The pressure in the system is same at all points and does not change with respect to time.

Chemical equilibrium: No chemical reactions take place in the system and the chemical composition which is same through out the system does not vary with time.

Quasi-static process:

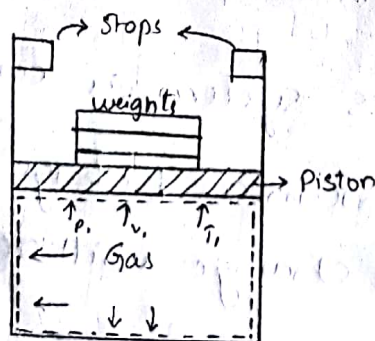


Fig: Infinitely slow transmission of a system by infinitesimal

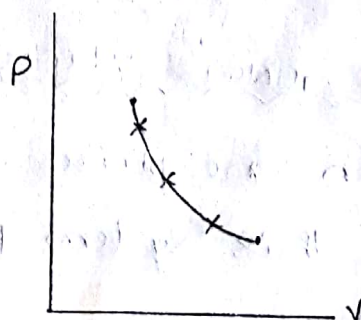


Fig: Quasi-static process

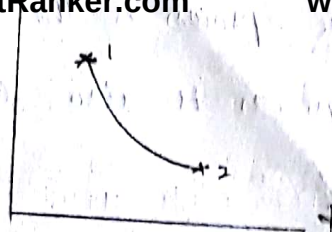
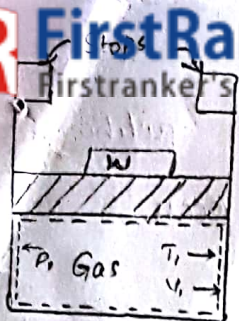


Fig: Transition between the two equilibrium states by an unbalanced force

Fig: Non-quasi-static process.

Figure ① shows the piston cylinder arrangement contains gas in it. Above the piston weight will be placed. This weight balance the forces on the system. It is state point ① shown in figure ①. Now remove the weight placed on the piston and the gas will be expanded suddenly, and piston hits the stops. At this position the system is in equilibrium condition at point ② shown in figure ②. The process ① and ② represents expansion of gases between two equilibrium conditions. But the two intermediate states passed through the system are non-equilibrium states which can not be described by thermodynamic co-ordinates.

Figure ③ and Figure ④ shows the Quasi-static process.

Figure ③ shows the piston cylinder arrangement. The no of weights are placed on the top of the piston. Now this system will be balanced at state point ①. Now remove the weights one by one very slowly from the top of the piston. At any instant of the upward travel of the piston, if the gas system is isolated. the departure of the state of the system from the thermodynamic equilibrium state will be infinitely small. So every state passed through the system will be an equilibrium state. Such a process of all equilibrium points passed through the system is known as Quasi-static process.

The Quasi-static process is the succession of all equilibrium states.

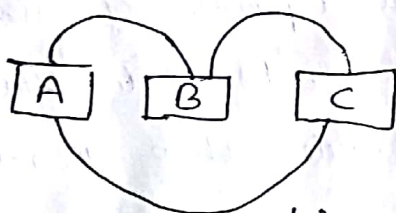
State. A Quasi-static process is also called reversible process.

Zeroth law of thermodynamics:

When body "A" is in thermal equilibrium with body "B" and also separately with a body "C" then "B" and "C" will be in thermal equilibrium with each other.

(OR)

When body "A" and "B" are in thermal equilibrium with each other and also separately body "B" and "C" is in thermal equilibrium with each other then body "A" and "C" is in thermal equilibrium.



Temperature measurement:

Thermometry: It is the measurement of temperature by using various thermometers.

Thermometric property: Every thermometer has one substance in it (mercury, gas etc). By changing the property (pressure, volume, length etc) of the substance we can estimate the change in temperature. Every common thermometer consists of a small amount of mercury in an evacuated capillary tube. In this case the extension of mercury is used as a thermometric property.

each with its own thermometric property as shown in the table.

S.No	Thermometer	Thermometric property	symbol
1.	constant volume gas thermometer	pressure.	p
2.	constant pressure gas thermometer	volume	V
3.	Electrical resistance thermometer	Resistance	R .
4.	Thermo couple	Emf	$\mathcal{E}$
5.	Mercury in glass thermometer	length	L .

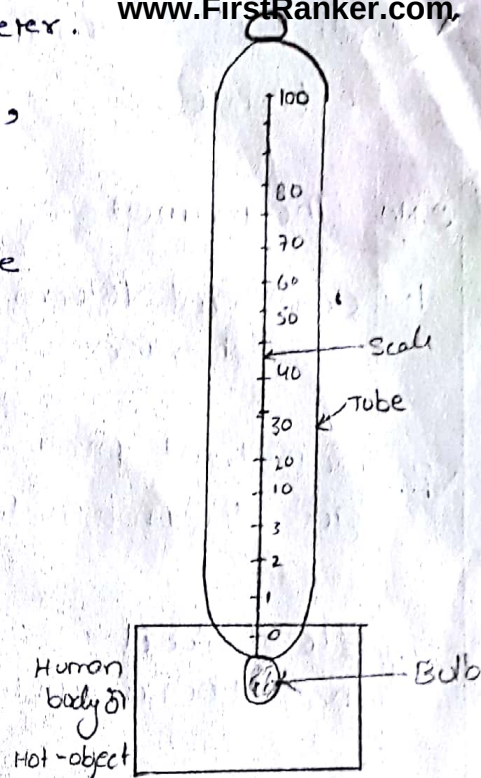
Reference points: Generally there are two fixed reference points
1) Ice point
2) Steam point.

1. Ice point: A mixture of ice and water which is in equilibrium with air saturated with vapour at 1 atm pressure is said to be at ice point.

2. Steam point (100°C): A mixture of liquid water and water vapour with no air in equilibrium at 1 atm pressure is said to be at steam point.

Figure shows the ordinary glass thermometer.

It consists the following components Bulb, scale, tube & mercury. The bulb of the thermometer is contacted whose temperature is to be measured. According to the change in temperature mercury will rise up. Here the thermometric property is change in length of mercury (L).



CONSTANT VOLUME GAS THERMOMETER

B- Bulb

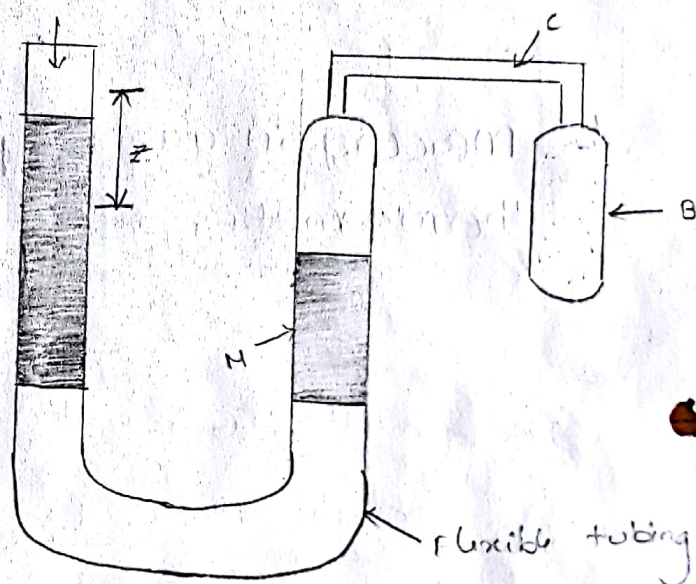
C- Capillary tube

M- Mercury manometer

Z- Distance in mercury level

$$p = \rho g z$$

$$pV = nRT$$



U-tube manometer and bulb is connected through capillary tube. U-tube manometer contains mercury, bulb (B) contains constant volume of gas in it. The bulb is connected whose temperature is to be measured. If there is any change in temperature occurs change in pressure indicates change in u-tube manometer. OR if there is any change in pressure in the object it indicates the change in temperature. The difference in mercury level indicates the change in pressure.

In this equation V is ~~change in pressure~~ ΔP and ΔT proportionately indicates the change in temperature.

ELECTRICAL RESISTANCE THERMOMETER

Wheatstone bridge contains four arms having the resistances R_1, R_2, R_3 & R_4 .

Resistance R_3 wire is connected to the object whose temperature is to be

measured. If there is no change in

temperature the bridge is ~~un~~ balanced,

if there is any change in temp. the bridge is unbalanced. It is shown in galvanometer (G). By

balancing the bridge we can estimate

the change in resistance of the wire

R_3 and accordingly we can estimate the change in temperature.

$$R = R_0(1 + At + Bt^2)$$

where R_0 is the resistance of platinum wire & A & B are constant.

TEMPERATURE SCALE - REFERENCE POINTS

Method used before 1954:

If 'X' is the thermometric property let us arbitrarily choose for the temperature common to the thermometer and to all the systems in thermal equilibrium with it. The following equation is a linear function of X.

$$\theta(X) \propto X$$

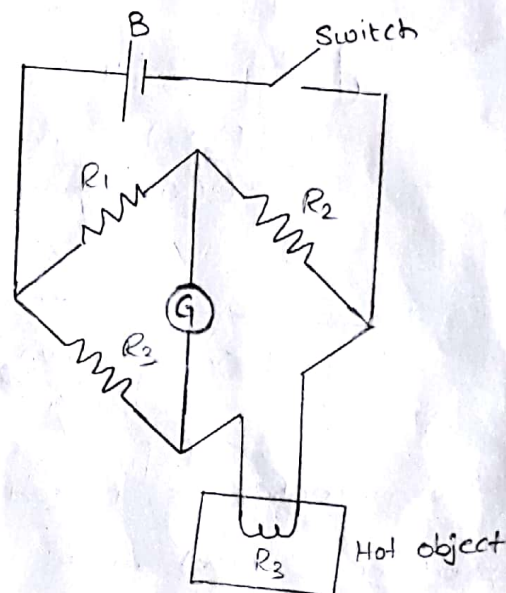
$$\theta(X) = aX \rightarrow \textcircled{1}$$

$$\theta(X_1) \propto X_1$$

$$\theta(X_1) = aX_1 \rightarrow \textcircled{2}$$

$$\theta(X_2) \propto X_2$$

$$\theta(X_2) = aX_2 \rightarrow \textcircled{3}$$



$$\frac{\Theta(x_1)}{\Theta(x)} = \frac{ax_1}{ax} = \frac{x_1}{x}$$

$$\Rightarrow \Theta(x_1) = \frac{x_1}{x} \Theta(x)$$

$$= \frac{\Theta(x)}{x} x_1 \rightarrow (4)$$

Dividing eqn (3) by (2)

$$\frac{\Theta(x_2)}{\Theta(x_1)} = \frac{ax_2}{ax_1}$$

$$= \frac{x_2}{x_1}$$

$$\Theta(x_2) = \frac{\Theta(x_1)}{x_1} x_2 \rightarrow (5)$$

The thermometer is first placed in contact with the system whose temperature is $\Theta(x)$ is to be measured and then in contact with another standard system in an easily reproducible state when the temperature is $\Theta(x_1)$. Thus from eqn (4) $\Theta(x_1) = \frac{\Theta(x)}{x} x_1$

Then the thermometer at the temperature $\Theta(x)$ is placed in contact with another standard system easily reproducible state where the temperature is $\Theta(x_2)$. Thus from equation (1) & (3)

$$\frac{\Theta(x_2)}{\Theta(x)} = \frac{ax_2}{ax}$$

$$= \frac{x_2}{x}$$

$$\Theta(x_2) = \frac{\Theta(x)}{x} x_2 \rightarrow (6)$$

Subtracting eqn (4) & (6)

$$\frac{\Theta(x_1)}{\Theta(x)} - \frac{\Theta(x_2)}{\Theta(x)} = \frac{x_1}{x} - \frac{x_2}{x}$$

$$\frac{\Theta(x_1) - \Theta(x_2)}{\Theta(x)} = \frac{x_1 - x_2}{x}$$

$$\Theta(x) = \frac{\Theta(x_1) - \Theta(x_2)}{x_1 - x_2} \cdot x$$

After 1954 only one fixed point has been used that is the triple point of water. Triple point of water value is 273.16 K

Triple Point $\rightarrow$ The state at which ice, liquid water and water vapour coexist in equilibrium that point is known as triple point.

$$\text{Triple Point} = 273.16\text{ K}$$

If θ_t is the temperature of the object and x_t is the Thermometric Property of the object, then

$$\theta_t \propto x_t$$

$$\theta_t = a x_t$$

$$\therefore a = \frac{\theta_t}{x_t} = \frac{273.16}{x_t}$$

θ is the temperature of another object, x is the thermometric Property of the object. then

$$\theta \propto x$$

$$\theta = a x$$

$$\theta = \frac{273.16}{x_t} \cdot x$$

$$\theta = 273.16 \frac{x}{x_t}$$

Comparison on thermometers:

1) constant volume gas thermometer

$$\theta(p) = 273.16 \frac{P}{P_t}$$

2) constant Pressure gas thermometer

$$\theta(V) = 273.16 \frac{V}{V_t}$$

$$\theta(R) = 273.16 \frac{R}{R_t}$$

f) Liquid in glass thermometer:-

$$\theta(L) = 273.16 \frac{L}{L_t}$$

g) thermocouple :-

$$\theta(E) = 273.16 \frac{E}{E_t}$$

Celsius Temperature Scale:-

$$t = T - 273.15^\circ$$

for 0°C

$$t = T + 273.15^\circ$$

$$= 0 + 273.15^\circ$$

$$= 273.15^\circ$$

for 100°C

$$t = T + 273.15^\circ$$

$$t = 100 + 273.15^\circ$$

$$= 373.15^\circ$$

International Practical temperature scale:-

* Normal Boiling Point of oxygen.

$$= -180.97^\circ\text{C}$$

$$= -182.97^\circ\text{C}$$

* Triple Point of water

$$= 0.01^\circ\text{C}$$

* Normal Boiling Point of water

$$= 100^\circ\text{C}$$

Temperatures at
fixed points

$$= 444.60^{\circ}\text{C}$$

* Normal Boiling Point of Antimony

$$= 630.50^{\circ}\text{C}$$

* Normal Melting Point of Silver

$$= 960.80^{\circ}\text{C}$$

* Normal Melting Point of gold

$$= 1063^{\circ}\text{C}$$

i) To measure the Temperature from 0 to 660°C

The following equation is used

$$R = R_0 (1 + At + Bt^2)$$

where Platinum Resistance thermometer is used to measure the temperature range

$R_0 \rightarrow$ Ice Point

$A \rightarrow$ Steam Point

$B \rightarrow$ Sulphur Point

the Platinum wire Diameter is 0.05 to 0.20 mm

2) Temperature range:

Temperature range from -190 to 0°C

$$R_t = R_0 [1 + At + Bt^2 + C(t-100)t^3]$$

www.FirstRanker.com

To measure this Temperature Platinum resistance Thermometer is used

$R_0 \rightarrow$ Ice Point

$A \rightarrow$ Steam Point

$B \rightarrow$ Sulphur Point

$C \rightarrow$ Oxygen Point

3) To measure Temperature Range 660 to 1063°C

$$E = a + bt + ct^2$$

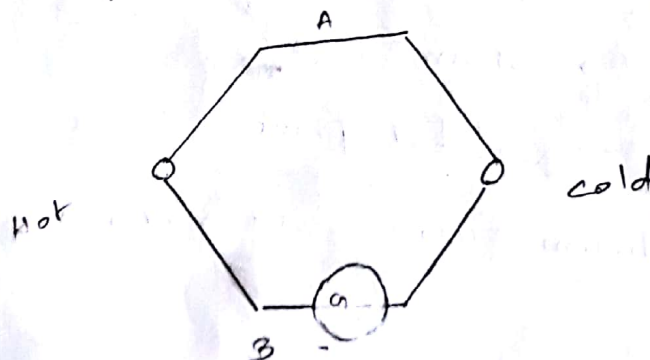
To measure this Temperature Range Thermocouple are used

$a \rightarrow$ Antimony Point

$b \rightarrow$ Silver Point

$c \rightarrow$ Gold Point

Thermocouple



If two Dissimilar materials forming two Junction maintain a Different Temperature, flow of electrons will be Take Place, i.e.) emf will

The Relation Between F and $^{\circ}C$

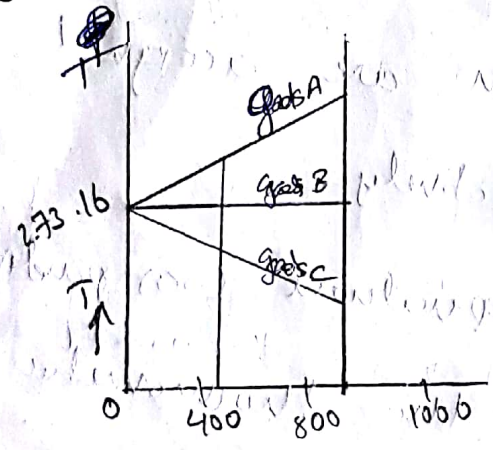
$$\frac{F - 32}{180} = \frac{C}{100} = \frac{R}{80}$$

at $-40^{\circ}C$ F and $^{\circ}C$ Temperature is

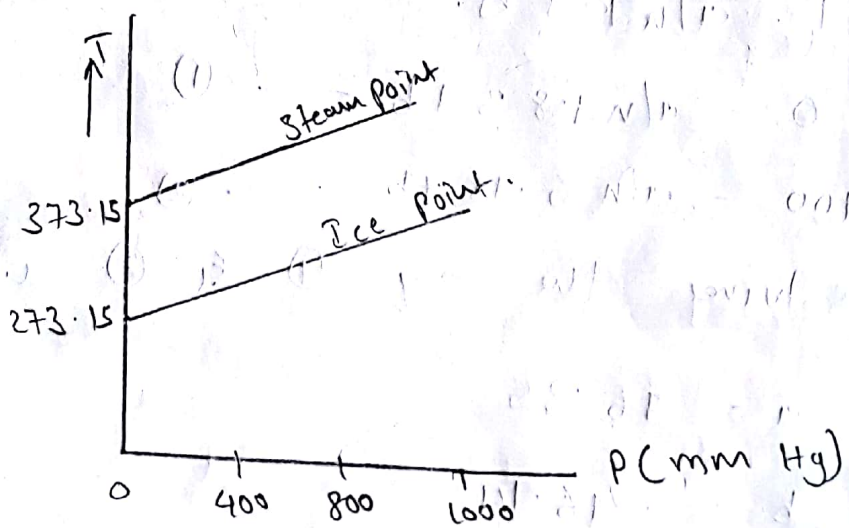
equal

Ideal gas or Perfect gas Temperature scale:-

P, V, T plots
No experimental data obtained from a constant volume gas thermometer using three different gases at different pressure



$273.16 \frac{P_1}{1000} = T_1$
 $273.16 \frac{P_2}{1000} = T_2$
 $273.16 \frac{L}{P_i}$



Temperature on a thermometric scale is defined in terms of a property 'k' by the relation

$$T = a \ln k + b$$

$$T = a \ln k + b$$

where a & b are constants

The value of k are found to be 1.83 and 6.78 at the ice point and steam point, the temperature of which are assigned the numbers 0 and 100 respectively.

Determine the temperature corresponding to a reading of k = 2.42 on the thermometric scale

Sol:- $T = a \ln k + b$ — (A)

$$0 = a \ln 1.83 + b$$
 — (1)

$$100 = a \ln 6.78 + b$$
 — (2)

By solving the eq (1) & (2) we get.

$$a = 76.35$$

$$b = -46.14$$

Sub a & b in eq (A)

$$T = 76.35 \ln k - 46.14$$

If $k = 2.42$

$$T = 76.35 \ln 2.42 - 46.14 = 21.33^\circ\text{C}$$

8, 15, 16, 19, 26,
24, 38, 39, 41, 45,
49, 52,

A platinum wire is used as a resistance thermometer. The wire resistance was found to be $10\ \Omega$ and $16\ \Omega$ at ice and steam point respectively, and $30\ \Omega$ at sulphur boiling point of 444.6°C . Find the constants a & b in equation $R = R_0(1 + aT + bT^2)$ where T is in degree $^\circ\text{C}$ also find the resistance of the wire at 500°C .

Sol: Given $R = 10\ \Omega$ at 0°C

$$R = 16\ \Omega \text{ at } 100^\circ\text{C}$$

$$R = 30\ \Omega \text{ at } 444.6^\circ\text{C}$$

Sub the values in equation above

$$R = R_0(1 + aT + bT^2)$$

$$10 = R_0(1 + a \times 0 + b \cdot 0^2)$$

$$\therefore R_0 = 10$$

Sub $R = 16\ \Omega$ at 100°C and $R_0 = 10$ in the above equation

$$R = R_0(1 + aT + bT^2)$$

$$16 = 10(1 + a \times 100 + b \times (100)^2) \quad \text{--- (i)}$$

$$16 = 10(a \times 1000 + b \times 100000) \quad \text{--- (ii)}$$

Sub $R = 30\ \Omega$ at 444.6°C and $R_0 = 10$ in the above equation.

$$R = R_0 (1 + \alpha t + \beta t^2)$$

$$30 = 10 (1 + a(444.6) + b(444.6)^2) \quad \text{--- (1)}$$

Solve eq. (1) & (2) we get.

$$a = 6.43 \times 10^{-3}$$

$$b = -4.35 \times 10^{-6}$$

Sub a & b values and also R_0 value and also

$T = 500^\circ\text{C}$ in the main equation.

$$R = 10 (1 + 6.43 \times 10^{-3} \times 500 - 4.35 \times 10^{-6} (500)^2)$$

$$R = \underline{31.26 \Omega}$$

- 3) A platinum Resistance Thermometer has a resistance of 2.8Ω at 0°C and 3.8Ω at 100°C . Calculate the temperature when the resistance indicated is 5.8Ω . Assume the resistance varies linearly with temperature as per the relation,

$$(R = R_0(1 + \alpha t)) \quad \text{--- (1)}$$

Substitute $t = 0$, $R = 2.8 \Omega$

$$2.8 = R_0(1 + \alpha \times 0)$$

$$2.8 = R_0$$

$$\therefore R_0 = 2.8 \Omega$$

Substitute $R_0 = 2.8$, $t = 100$, $R = 3.8$ in equation (1)

$$3.8 = 2.8(1 + \alpha \times 100)$$

$$\Rightarrow \alpha = 3.5 \times 10^{-3}$$

Substitute $\alpha = 3.5 \times 10^{-3}$, $R_0 = 2.8$, $R = 5.8$ in equation (1)

$$5.8 = 2.8(1 + 3.5 \times 10^{-3} \times t)$$

$$\therefore t = \underline{300.12^\circ\text{C}}$$

The emf in a thermocouple with the test junction at T_c and reference junction at ice point is given by $E = 0.20T - 5 \times 10^{-4} T^2$ mV

The mv meter is calibrated at ice and steam point. what will this thermometer read in place where gas thermometer reads 50°C .

The given $E = 0.20T - 5 \times 10^{-4} T^2$ mV.

at $T = 0^\circ\text{C}$

$$E = 0.20 \times 0 - 5 \times 10^{-4} \times 0^2$$

$$E = 0$$

at $T = 100^\circ\text{C}$

$$E = 0.20 \times 100 - 5 \times 10^{-4} \times 100^2$$

$$E = 15 \text{ mV}$$

at $T = 50^\circ\text{C}$

$$E = 0.20 \times 50 - 5 \times 10^{-4} \times 50^2$$

$$E = 8.75 \text{ mV}$$

when the gas thermometer reads 50°C the thermocouple will read

$$= \frac{100}{15} \times 8.75$$

$$= \underline{\underline{58.33^\circ\text{C}}}$$

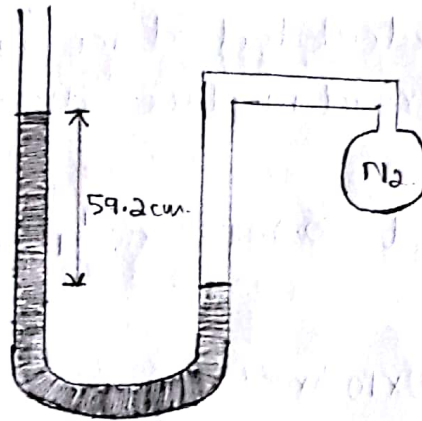
- 5- A constant volume gas thermometer containing nitrogen is brought in contact with the system of unknown temperature and then into a contact with a system maintaining at the triple point of water. The mercury column attached to the device reading of 59.2 and 2.28 cm's respectively for the two systems. If the barometric pressure 960 bar. what is the unknown temperature in kelvin. If $g = 9.806 \text{ m/sec}^2$, specific gravity of mercury may be taken

as 13.6

Soln:-

If the given, $g = 9.806 \text{ m/sec}^2$.

specific gravity of mercury = 13.6.



$$\text{specific gravity} = \frac{\text{density of fluid}}{1000}$$

$$13.6 = \frac{P}{1000}$$

$$\Rightarrow P = 13.6 \times 10^3 \text{ kg/m}^2$$

$$P_{\text{gauge}} = Pgh$$

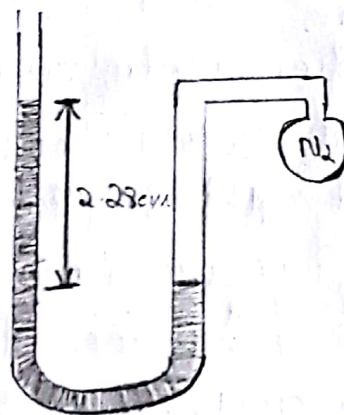
$$= 13.6 \times 10^3 \times 9.806 \times 59.2 \times 10^{-2}$$

$$= 789.5 \text{ bar}$$

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$= 960 + 789.5$$

$$P_{\text{abs}} = 1749.5 \text{ bar}$$



$$P_{\text{gauge}} = Pgh$$

$$= 13.6 \times 10^3 \times 9.806 \times 2.28 \times 10^{-2}$$

$$= 3040.6 \text{ pascal}$$

$$= 30.406 \text{ bar}$$

$$P_{abs} = P_{atm} + P_{gauge}$$

$$= 960 + 30.406$$

$$= 990.6 \text{ bar}$$

$$T = 273.16 \frac{P}{P_t}$$

$$= 273.16 \times \frac{1749.5}{990.6}$$

$$T = \underline{\underline{482.5 \text{ K}}}$$

* A U-tube manometer with one arm to atmosphere is used to measure the pressure of steam flowing through the pipe. Mercury is used as manometric fluid. The height of the column of mercury in the open end is 10 cm greater than mercury level in the arm connected to the steam pipe. Steam condensed in the manometer on the steam side. The column of condensate is 3.5 cm height. The atmospheric pressure is 76 cm of mercury. Find the absolute pressure of steam in the pipe in bar. Density of mercury is 13600 kg/m^3 .

Soln: Difference of Hg column = 10 cm

$$= 100 \text{ mm of Hg}$$

Column of Condensate

$$= 3.5 \text{ cm of water}$$

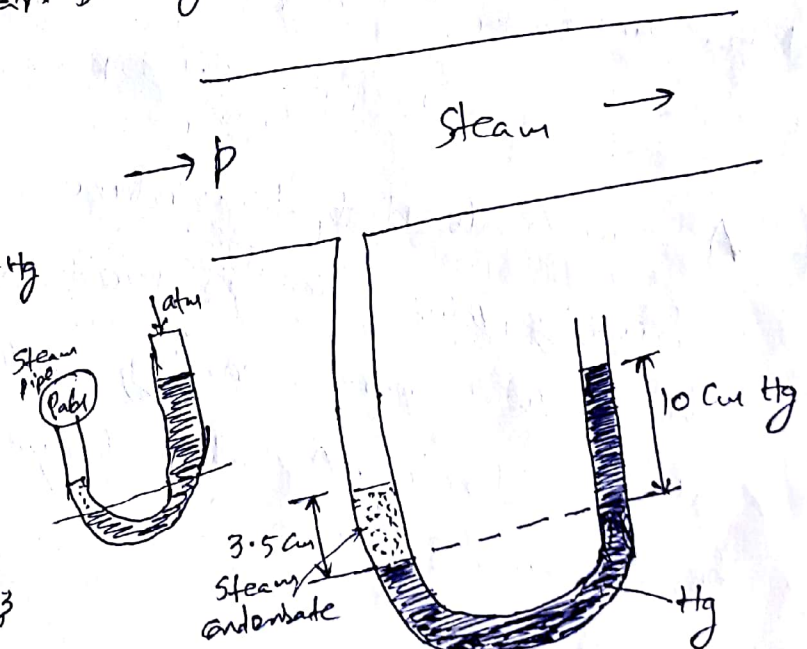
$$= 35 \text{ mm of water}$$

$$\text{Density of Hg} = 13600 \text{ kg/m}^3$$

$$\text{Atmospheric pressure} = 76 \text{ cm Hg}$$

$$= 760 \text{ mm Hg}$$

$$35 \text{ mm of water column} = \frac{35}{13.6} = 2.573 \text{ mm of Hg}$$



equivalent manometer Head = $100 - 2.573 = 97.427$
 $= 97.43 \text{ mm of Hg.}$

$$P_{abs} = P_{atm} + P_{gauge}$$

$$(\because P_{abs} = P_{atm} + \text{manometer pressure})$$

$$= 760 + 97.43$$

$$= 857.43 \text{ mm of Hg}$$

$$= 857.43 \times 0.1333$$

$$= 114.29 \text{ kPa}$$

$$= \frac{114.29 \times 1000}{100}$$

$$= 1.1429 \text{ bar}$$

$$P_{atm} = \rho g h$$

$$= 13600 \times 9.81 \times 760 \times 10^{-3}$$

$$= \text{--- Pa bar}$$

$$= \frac{\text{---}}{100} \text{ bar}$$

$$P_{gauge} = \rho g h$$

$$= 13600 \times 9.81 \times 97.43 \times 10^{-3}$$

$$= \text{--- Pa bar}$$

$$= \frac{\text{---}}{100} \text{ bar}$$

$$1 \text{ bar} = 10^5 \text{ Pa}$$

$$= 100 \text{ kPa}$$

$$= 0.1 \text{ MPa}$$

$$1 \text{ atm} = 1.01325 \text{ bar}$$

$$1 \text{ atm} = 101.325 \text{ kPa}$$

$$760 \text{ mm Hg} = 101.325 \text{ kPa}$$

$$1 \text{ mm Hg} = \frac{101.325}{760}$$

$$= 0.1333 \text{ kPa}$$

$$P_{abs} + P_{water} = P_{Hg} + P_{atm}$$

$$P_{abs} + 2.573 = 100 + 760$$

$$P_{abs} = 857.43 \text{ mm of Hg}$$

$$= 857.43 \times 10^{-3} \times 13600 \times 9.81 \times 10^3$$

A new scale N of temperature is devised in such a way that the freezing point of ice is 100°N and boiling point is 400°N .
 The freezing point of ice is 100°N and boiling point is 400°N .
 what is the temperature reading on this new scale when the temperature is 150°C . At what temperature scale readings would be the same?

Soln:- $\frac{N-100}{400-100} = \frac{C}{100-0}$ — (1)

for $C = 150^\circ \text{C}$, $\frac{N-100}{300} = \frac{150}{100}$

$$N-100 = 150 \times 3 = 450$$

$$\therefore N = 450 + 100 = 550^\circ \text{N}$$

Its reading is same on two scales

$N = C$, substitute in Eqn (1)

$$\frac{N-100}{300} = \frac{N}{100}$$

$$N-100 = 3N$$

$$N-3N = 100$$

$$-2N = 100$$

$$N = \frac{-100}{2} = -50^{\circ}\text{C}$$

* A thermometer is calibrated using ice point (0°C) and steam point (100°C) as fixed reference points. An algebraic function is chosen is $T = A \ln x + B$ is chosen to establish the temperature scale. Show that the unknown temperature $T = 100 \left[\ln \left(\frac{x}{x_i} \right) / \ln \left(\frac{x_L}{x_i} \right) \right]$ where x_i and x_L are the values of properties when $T=0$ and $T=100$ deg C respectively.

Soln:-

x_i $T=0$
 x_L $T=100$

$$T = A \ln x + B \quad \text{--- (1)}$$

$$100 = A \ln x_L + B \quad \text{--- (2)}$$

$$0 = A \ln x_i + B \quad \text{--- (3)}$$

$$\begin{aligned} (2)-(3) &= \frac{100 = A \ln(x_L - \ln x_i)}{= A \ln \left(\frac{x_L}{x_i} \right)} \end{aligned}$$

$$\Rightarrow A = \frac{100}{\ln \left(\frac{x_L}{x_i} \right)}$$

$$\text{from (3), } B = -A \ln x_i = \frac{-100 \ln x_i}{\ln \frac{x_L}{x_i}}$$

substitute A and B value in eqn ①

www.FirstRanker.com

www.FirstRanker.com

$$T = \frac{100}{\ln \frac{x_L}{x_i}} \ln x - \frac{100 \ln x_i}{\ln \frac{x_L}{x_i}}$$

$$= \frac{100 (\ln x - \ln x_i)}{\ln \left(\frac{x_L}{x_i} \right)} = \frac{100 \ln \frac{x}{x_i}}{\ln \frac{x_L}{x_i}}$$

$$= 100 \left[\ln \left(\frac{x}{x_i} \right) / \ln \left(\frac{x_L}{x_i} \right) \right]$$

* The pressure of the gas in a constant volume gas thermometer are 80 cm, 90 cm and 100 cm of Hg at the ice point, steam point and in a heated wax bath respectively. Find the temperature of the heated wax bath.

Soln:- $\frac{T}{T_1} = \frac{P}{P_1 \rightarrow \text{critical}}$

$$PV = nRT$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$T = 273.16 \frac{P}{P_r}$$

for 0°C $\frac{273}{T_1} = \frac{80}{P_1} \Rightarrow 273P_1 - 80T_1 = 0 \quad \text{--- (1)}$

for 100°C $\frac{373}{T_1} = \frac{90}{P_1 \rightarrow \text{critical}} \Rightarrow 373P_1 - 90T_1 = 0 \quad \text{--- (2)}$

Solve these two equations, and find

$$P_1 = 1.25 \text{ cm}$$

$$T_1 = 3.84^\circ\text{K}$$

$$273P_1 = 80T_1$$

$$P_1 = \frac{80}{273} T_1$$

$$\frac{T}{T_1} = \frac{P}{P_1}$$

$$\frac{T}{3.84} = \frac{100}{1.25} \Rightarrow T = 341^\circ\text{K}$$

$$= 68^\circ\text{C}$$

$$373 \left(\frac{80}{273} T_1 \right) - 90T_1 = 0$$

$$T_1 \left(\frac{373 \times 80}{273} - 90 \right) = 0$$

$$T_1 = 0$$

Discuss whether or not the following quantities can be used as properties.

①. $\int p dv$

②. $\int v dp$

③. $\int p dv + \int v dp$

Soln:- ③. $\int p dv + \int v dp = \int d(pv) = pv$

$\therefore$ It is exact differential

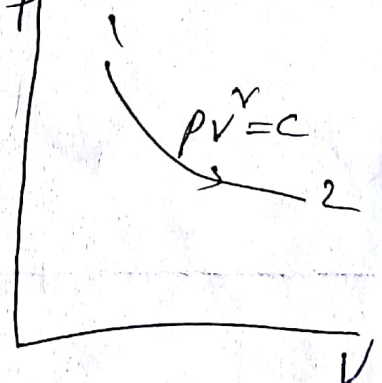
$\therefore$ It is a property of the system.

① & ② are inexact differential
 $\therefore$ those are not properties.

Adiabatic process:- A process, in which, the working substance neither receives nor gives out heat to its surroundings, during its expansion or compression, is called adiabatic process.

①. P-V-T-relation in.

$$PV^\gamma = C$$



$$P_1 V_1^\gamma = P_2 V_2^\gamma \quad \left| \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \right.$$

$$\left[\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma \right] \quad \left| \quad \frac{P_1}{P_2} = \frac{T_1}{T_2} \times \frac{V_2}{V_1} \right.$$

$$\frac{T_1}{T_2} \times \frac{V_2}{V_1} = \left(\frac{V_2}{V_1} \right)^\gamma$$

$$\left[\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1} \right]$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \times \frac{P_2}{P_1}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} \times \frac{P_1}{P_2}$$

$$= \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma}} \times \frac{P_1}{P_2}$$

$$= \left(\frac{P_1}{P_2} \right)^{\frac{1}{\gamma} + 1} = \left(\frac{P_1}{P_2} \right)^{\frac{1+\gamma}{\gamma}}$$

Side note: $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma$
 $\left(\frac{V_2}{V_1} \right)^\gamma = \left(\frac{V_1}{V_2} \right)^{-\gamma}$
 $\frac{V_1}{V_2} = \left(\frac{P_1}{P_2} \right)^{-\frac{1}{\gamma}}$
 $\frac{V_1}{V_2} = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$

$$\left[\frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

work done during adiabatic process.

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \text{for expansion}$$

$$W = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \quad \text{for compression.}$$

(3). change in internal energy

$$dU = U_2 - U_1 = m c_v (T_2 - T_1)$$

(4). Heat Supplied or heat transferred

$$Q_{1-2} = 0 \quad (\text{adiabatic process})$$

(5). change in enthalpy

$$dH = H_2 - H_1 = m c_p (T_2 - T_1)$$

Polytropic process:- is also known as the general law for the expansion and compression of gases, and it is given by the relation

$$P V^n = \text{Constant}$$

$n \rightarrow$ polytropic Index.

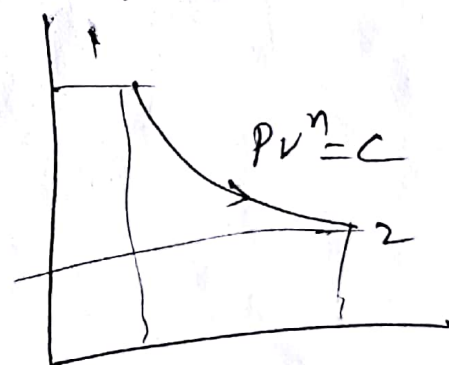
(1). $P V T$ relationship

$$P_1 V_1^n = P_2 V_2^n = C$$

$$(a). \frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$(b). \frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{n-1}{n}}$$

$$(c). \frac{T_1}{T_2} = \left(\frac{P_2}{P_1} \right)^{1/n}$$



$$w_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{R(T_2 - T_1)}{\gamma - 1}$$

③. change in internal energy

$$\Delta U = U_2 - U_1 = m c_v (T_2 - T_1)$$

④. Heat supplied or heat transfer

$$Q_{1-2} = w_{1-2} + \Delta U$$

$$= \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} + m c_v (T_2 - T_1)$$

$$= \frac{m R (T_1 - T_2)}{\gamma - 1} + m \cancel{c_p} \times \frac{R}{\gamma - 1} (T_2 - T_1) \quad (c_v = \frac{R}{\gamma - 1})$$

$$= m R (T_1 - T_2) \left[\frac{1}{\gamma - 1} - \frac{1}{\gamma - 1} \right]$$

$$= m R (T_1 - T_2) \left[\frac{(\gamma - 1) - (\gamma - 1)}{(\gamma - 1)(\gamma - 1)} \right]$$

$$= m R (T_1 - T_2) \left[\frac{\gamma - \gamma}{(\gamma - 1)(\gamma - 1)} \right]$$

$$= \frac{\gamma - \gamma}{\gamma - 1} \times \frac{m R (T_1 - T_2)}{\gamma - 1}$$

⑤. change in Enthalpy

$$\Delta H = H_2 - H_1 = m c_p (T_2 - T_1)$$

work and heat transfer :-

work → Work is defined as the result of moving force through a distance

$$\text{Work} = \text{Force} \times \text{distance}$$

$$= F \times x$$

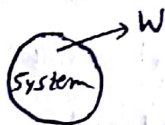
units → Newton / m (or) Joule

Power: It is defined as the rate of doing work.

$$\text{Power} = \frac{\text{Work}}{\text{time}}$$

units: Joule / sec (or) Watts

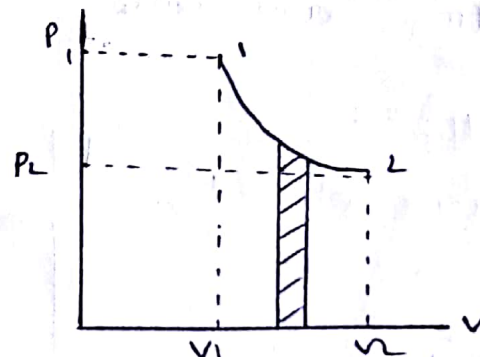
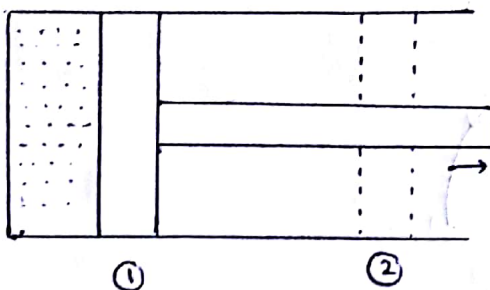
* Sign Convention for work transfer:



(a) Work is +ve

(b) Work is -ve

pdv Work (or) displacement work:-



$$dw = F \times dx$$

$$= p \cdot A \times dx$$

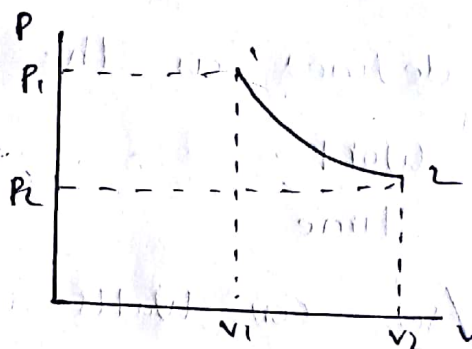
$$\Rightarrow F = p \cdot A$$

$$dw = p \cdot dv$$

$$W_{1-2} = \int_{v_1}^{v_2} p \cdot dv \rightarrow (\text{For entire process ① to ②})$$

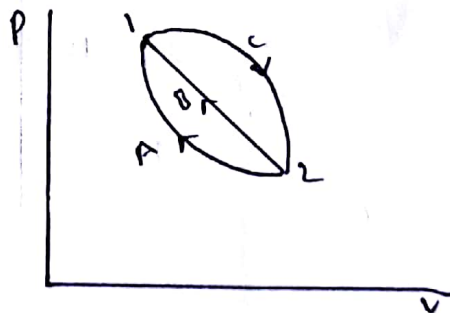
* Point Function: When two properties locate a point on the graph then these properties are called point function

Eg: Pressure
Volume
Temperature etc



* Path Function There are certain quantities which cannot be located on a graph by a point. But are given by the area on the graph. In that case the area of the graph related to the particular process is a function of the path of a process. Such quantities are called path function

Eg: Heat
work etc



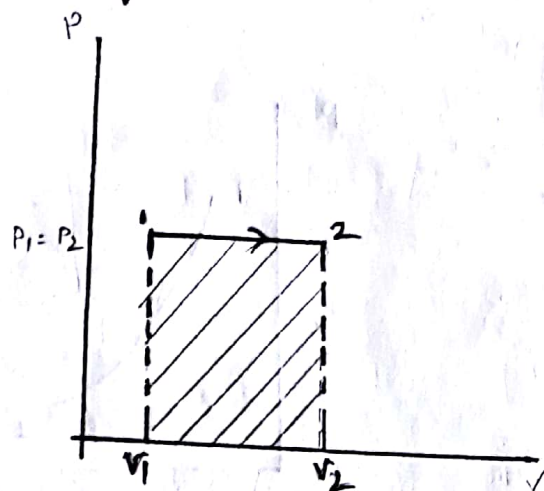
$$\int_1^2 dw \neq w_2 - w_1$$

$$w_2 \text{ or } w_{1-2}$$

Similarly $\int_1^2 dq \neq q_2 - q_1$

$$q_2 \text{ or } q_{1-2}$$

constant pressure process: (isobaric)



work done during process 1 → 2 = area bounded

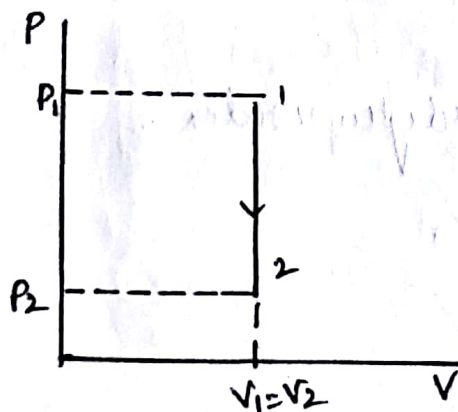
$$W_{1-2} = \int_{V_1}^{V_2} P dV$$

on integrating

$$= P \int_{V_1}^{V_2} dV$$

$$W_{1-2} = P(V_2 - V_1)$$

constant volume process: (isochoric)



$$W_{1-2} = \int_{V_1}^{V_2} P dV$$

$$= P \int_{V_1}^{V_2} dV$$

$$= P(V)$$

$$= P(V_2 - V_1)$$

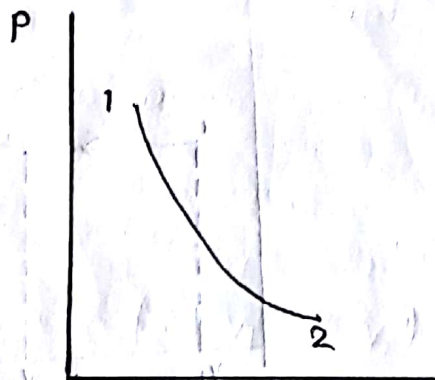
$$W_{1-2} = 0$$

$$w_{1-2} = \int_{V_1}^{V_2} P dV$$

$$w_{1-2} = \int_{V_1}^{V_2} \frac{P_1 V_1}{V} dV$$

$$(\because PV = P_1 V_1 = C \Rightarrow P = \frac{P_1 V_1}{V})$$

$$P = \frac{P_1 V_1}{V}$$



$$w_{1-2} = P_1 V_1 \int_{V_1}^{V_2} \frac{1}{V} dV = P_1 V_1 [\log V]_{V_1}^{V_2}$$

$$= P_1 V_1 (\log V_2 - \log V_1)$$

$$w_{1-2} = P_1 V_1 \log \left(\frac{V_2}{V_1} \right)$$

$$w_{1-2} = P_1 V_1 \log \frac{P_1}{P_2}$$

$$P_1 V_1 = P_2 V_2$$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

polytropic process:

$$PV^n = C$$

$n = \text{polytropic index}$

* $n=0 \Rightarrow P=C$ (Isobaric process)

* $n=1 \Rightarrow PV=C$ (Isothermal)

* $n=\infty \Rightarrow PV=C$ (Isochoric)

$$w_{1-2} = \int_{V_1}^{V_2} P dV$$

$$PV^n = P_1 V_1^n$$

$$P = \frac{P_1 V_1^n}{V^n}$$

$$w_{1-2} = \int_{V_1}^{V_2} \frac{P_1 V_1^n}{V^n} dV$$

$$= P_1 V_1^\gamma \int_{V_1}^{V_2} \frac{dy}{V^\gamma}$$

$$= P_1 V_1^\gamma \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$= \frac{P_1 V_1^\gamma}{1-\gamma} [V_2^{1-\gamma} - V_1^{1-\gamma}]$$

$$= \frac{P_2 V_2^\gamma V_2^{1-\gamma} - P_1 V_1^\gamma V_1^{1-\gamma}}{1-\gamma}$$

$$[\because P_1 V_1^\gamma = P_2 V_2^\gamma]$$

$$\boxed{w_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}}$$

$$\left(\frac{V_1}{V_2} \right)^\gamma = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{V_2}{V_1} = \left(\frac{P_1}{P_2} \right)^{1/\gamma}$$

$$= \frac{1}{\gamma-1} [P_1 V_1 - P_2 V_2]$$

$$= \frac{1}{\gamma-1} \left[P_1 V_1 \left(1 - \frac{P_2 V_2}{P_1 V_1} \right) \right]$$

$$= \frac{P_1 V_1}{\gamma-1} \left[\left(1 - \left(\frac{P_2}{P_1} \right) \left(\frac{P_1}{P_2} \right)^{1/\gamma} \right) \right]$$

$$= \frac{P_1 V_1}{\gamma-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{1-1/\gamma} \right]$$

$$w_{1-2} = \frac{P_1 V_1}{\gamma-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{1-1/\gamma} \right]$$

Adiabatic process

$$P V^\gamma = C$$

γ = adiabatic index

$$w_{1-2} = \frac{P_1 V_1}{\gamma-1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

- * A certain quantity of air has a volume of 0.08 m^3 at 1.25 bar at 25°C it is compressed to a volume of 0.0042 m^3 ~~At~~ ^{follows to} law $pV^{1.3} = \text{constant}$. Find the final temperature and work done during compression. Determine reduction in pressure at constant volume required to bring the air back to its original temperature

Given

$$V_1 = 0.08 \text{ m}^3 \times 10^{-9}$$

$$= 0.08 \times 10^{-9} \text{ m}^3$$

$$P_1 = 1.25 \text{ bar}$$

$$= 1.25 \times 10^5 \text{ N/m}^2$$

$$T_1 = 25^\circ\text{C} = 298 \text{ K}$$

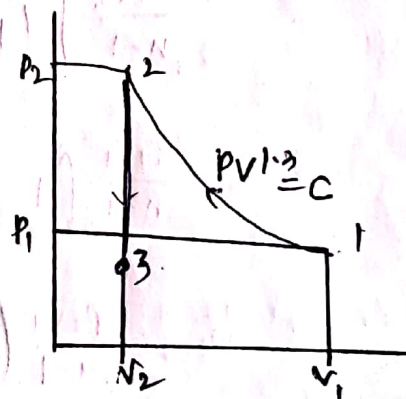
$$V_2 = 0.0042 \text{ m}^3$$

$$= 0.0042 \times 10^{-9} \text{ m}^3$$

$$T_2 = ?$$

$$W = ?$$

$$P_3 = ?$$



$$pV^{1.3} = \text{const}$$

$$\boxed{\eta = 1.3}$$

$$P_1 V_1^\eta = P_2 V_2^\eta$$

$$P_2 = \frac{P_1 V_1^\eta}{V_2^\eta} = \frac{1.25 \times 10^5 \times (0.08 \times 10^{-9})^{1.3}}{(0.0042 \times 10^{-9})^{1.3}}$$

$$= 57.63 \times 10^5 \text{ N/m}^2$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{P_1}{P_2} = \frac{V_2}{T_2} \times \frac{T_1}{V_1}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^n$$

$$\frac{V_2}{T_2} \times \frac{T_1}{V_1} = \left(\frac{V_2}{V_1} \right)^n$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^n \left(\frac{V_1}{V_2} \right)$$

$$= \left(\frac{V_2}{V_1} \right)^{n-1}$$

$$\frac{298}{T_2} = \left(\frac{0.0042 \times 10^{-9}}{0.08 \times 10^{-9}} \right)^{1.3-1}$$

$$\Rightarrow T_2 = 721.54^\circ \text{K}$$

$$= 448.54^\circ \text{C} \quad \text{Ans.}$$

$$W. D = \frac{P_2 V_2 - P_1 V_1}{n-1}$$

$$= \frac{57.63 \times 10^5 \times 0.0042 \times 10^{-9} - 1.25 \times 10^5 \times 0.08 \times 10^{-9}}{1.3-1}$$

$$= 0.4734 \times 10^{-4} \text{ Watt}$$

$$= 4.734 \times 10^{-5} \text{ Watt}$$

2-3 is constant volume process

$$P \propto T$$

$$\frac{P_2}{T_2} = \frac{P_3}{T_3}$$

$$\frac{P_2}{T_2} = \frac{P_3}{T_1}$$

$$\frac{57.63 \times 10^5}{721.54} = \frac{P_3}{298}$$

$$\Rightarrow P_3 = 23.8 \times 10^5 \text{ N/m}^2$$

$$= \underline{\underline{23.8 \text{ bar}}}$$

15/3/12

11. A piston cylinder device operates 1 kg of fluid at 20 atmospheric pressure. The initial volume is 0.04 m^3 . The fluid is allowed to expand ~~irreversibly~~ ^{reversibly} following ~~the~~ ^a process $PV^{1.45} = \text{constant}$, so that the volume becomes double. The fluid is then cooled at constant pressure until the piston comes back to its original position. Keeping the piston unaltered, heat is added reversibly to restore ~~the~~ ^{it to the} initial pressure. Calculate the work done in the cycle.

Given data $P_1 = 20 \text{ atm}$

$$= 20 \times 1.01325$$

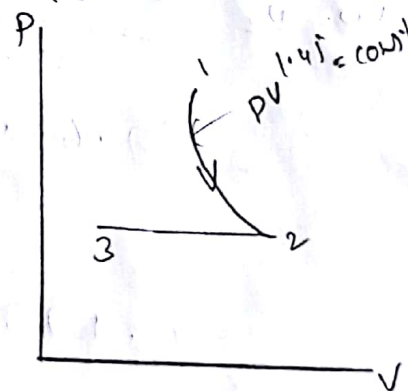
$$= 20.265 \text{ bar}$$

$$V_1 = 0.04 \text{ m}^3$$

$$V_2 = 2V_1$$

$$= 2 \times 0.04$$

$$= 0.08 \text{ m}^3$$



$$P_1 V_1^n = P_2 V_2^n$$

$$P_2 = \frac{P_1 V_1^n}{V_2^n}$$

$$= \frac{20.265 \times 10^5 \times (0.04)^{1.45}}{(0.08)^{1.45}}$$

$$= 7.41 \times 10^5 \text{ N/m}^2$$

$$= 7.41 \text{ bar}$$

Work done during the process 1 to 2

$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{n-1}$$

$$= \frac{20.265 \times 10^5 \times 0.04 - 7.41 \times 10^5 \times 0.08}{1.45 - 1}$$

$$= 0.484$$

$$= 48.4 \text{ kJ}$$

Work done during the process 2 to 3

$$W_{2-3} = -P_2(V_2 - V_1) = -7.41 \times 10^5 (0.08 - 0.04)$$

$$= -29.6 \text{ kJ}$$

Total work done

$$W = W_{1-2} + W_{2-3}$$

$$= 18.76 \text{ kJ}$$

Ques 2009

1. Air is compressed in a cylinder from 0.6 m^3 to 0.1 m^3 by a piston. The relation between pressure and volume is given by $P = 5 - 3V$ where P is bar and V is in m^3 .

Calculate the magnitude of work done on the system.

$$V_1 = 0.6 \text{ m}^3$$

$$V_2 = 0.1 \text{ m}^3$$

$$P = (5 - 3V) \text{ bar}$$

$$= (5 - 3V) \times 10^5 \text{ N/m}^2$$

$$= (5 - 3V) \times 10^2 \text{ kN/m}^2$$

$$\text{Work done} = \int_{V_1}^{V_2} P dV$$

$$W_1 - W_2 = \int_{V_1}^{V_2} (5 - 3V) \times 10^2 dV$$

$$= 10^2 \left[5V - \frac{3V^2}{2} \right]_{V_1}^{V_2}$$

$$= 10^2 \left[5V - \frac{3V^2}{2} \right]_{0.6}^{0.1}$$

$$= 10^2 \left[5(V_2 - V_1) - \frac{3(V_2^2 - V_1^2)}{2} \right]$$

$$= 10^2 \left[5(0.1 - 0.6) - \frac{3(0.1^2 - 0.6^2)}{2} \right]$$

$$= -197.5 \text{ kJ}$$

$$10^2 \left\{ \left[5(0.1) - \frac{3(0.1)^2}{2} \right] - \left[5(0.6) - \frac{3(0.6)^2}{2} \right] \right\} = -197.5 \text{ kJ}$$

~~April 2018~~

3. Determine the work transfer and its direction for system in which air is compressed from 101.3 kPa, 20°C to a pressure of 600 kPa. Following the law $PV^{1.3} = \text{constant}$.

A Given data

$$P_1 = 101.3 \text{ kPa}$$

$$= 101.3 \times 10^3 \text{ N/m}^2$$

$$T_1 = 20^\circ\text{C}$$

$$= 20 + 273 = 293 \text{ K}$$

FirstRanker.com ^{macroscopic & microscopic}
 FirstRanker's choice www.FirstRanker.com www.FirstRanker.com

idea of a petrol engine cylinder with mixture of air and petrol vapour in it. The mixture occupies a certain volume for each position of the piston and it can be easily measured. The pressure temp can also be measured for each position of piston in the cylinder. The state or condition of the system (mixture in the cylinder) can be determined by means of above-mentioned characteristics or properties. Such properties are known as macroscopic properties and these describe the system condition in macroscopic point of view. This is the method of analysis usually used by the engineers in analysing the behaviour of the thermal engines.

However, if the analysis is made on the basis of the behaviour of individual atoms and molecules of a substance under investigation, then the analysis is said to be microscopic behaviour of systems and helps to reinforce the conclusions of macroscopic studies.

Cycle :- When a process or processes are performed on a system such that starting from a certain state (by point 1), the system reaches back to its original state (i.e., 1), the system is said to have undergone a cycle.

Cyclic process :- is one in which the end states are identical i.e. the system after undergoing a series of processes returns to its original condition. (or)

- A thermodynamic cycle is defined as a series of state changes such that the final state is identical with the initial state.

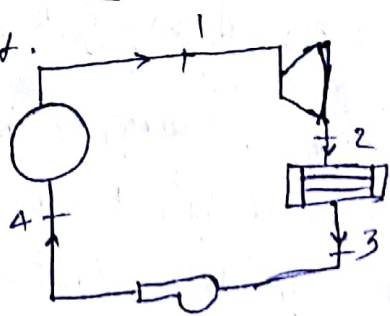
ex:- steam that circulates through steam power plant.

1-2 → Constant heat addition process

2-3 → Reversible adiabatic expansion process in the turbine.

3-4 → Constant pressure heat removal from the condenser

4-1 → A reversible adiabatic pumping process.



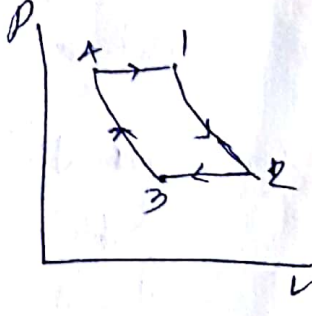


Fig: Rankine Cycle

is supplied in the boiler where isothermal evaporation of liquid takes place at constant pressure process line 4-1. The dry saturated steam undergoes frictionless adiabatic expansion in the turbine between the states 1 and 2. Heat is then removed reversibly and the vapour undergoes isothermal condensation at const. pre. process 2-3. The condensation is stopped at point 3 which lies on the isentropic line drawn from state point 4. The wet vapour is then compressed isentropically from point 3 until it is restored to original state & the liquid represented by point 4.

Energy: — Energy is the capacity to do work.
(or)

Energy is defined as the capacity to produce a change from existing conditions i.e., capacity to exert force through a distance and do work. When energy is added to or subtracted from a system, there occurs a change in one or more properties of the system.

Energy may be of two types: 1). Energy in state (i.e. stored energy)
2). Energy in transition.

Energy in state: — It is the energy contained within the system boundaries
ex:- P.E, K.E, Internal energy

Energy in transition: — The energy which crosses the system boundaries.
ex:- heat, work, electrical energy.

potential energy: — is the energy possessed by a body due to virtue of its position.
 $P.E = m \cdot g \cdot h$ or $m \cdot \frac{g}{g_c} \cdot z$
 $g \rightarrow \text{acc. due to grav.}$
 $g_c \rightarrow \text{relative gravitational acceleration.}$

(or). potential energy or energy of position is the energy possessed by a system because of its height above the earth surface. This is sometimes called gravitational potential energy. The gravitational attraction of earth exerts a force on the mass of the system.

ex:- gravitational potential energy is associated with gravitational force field

$$K.E = \frac{1}{2} m v^2$$

(Q1). The K.E. refers to the energy of a system in motion.

Internal energy: — ~~The part~~ it is sum total of activity and the configuration of molecules in a substance

$$I.E = \text{Translational K.E} + \text{Rotational K.E} + \text{Vibrational K.E} + \dots$$

(Q1). The part of the total energy which is stored in the molecular and atomic structure of the system is known as the molecular internal energy or simply, internal energy (U). Matter is composed of molecules. Each molecule can possess energy in many forms, some times called microscopic energy modes. Molecules which are in random thermal motion (for gases), can have translatory K.E, rotational K.E, vibrational energy, electronic energy, nuclear binding energy, etc.

→ As the motion and configuration of molecules depend on temp, internal energy will be a function of temp. If it is expressed in Joules. change in internal energy from one state to another state is expressed as

$$\Delta U = U_2 - U_1$$

point function: — When two properties locate a point on a graph

then those properties are called point functions.

ex:- pressure, temp, volume etc

(intensive properties)



path functions: — The quantities which can not be located on a graph by a point

but can be shown by area are called as path functions

ex:- Heat, work etc

when two properties locate a point on a graph, two properties are called point functions. In path functions, two properties do not locate a point because it is a path that has two end points.



- Exact differentials are point functions

ex: dv

$$\int_1^2 dv = v_2 - v_1$$

Inexact differentials are path functions

ex: Q, W

$$\int_1^2 \delta Q \neq Q_2 - Q_1, \text{ and it is shown as } Q_{1-2} \text{ (or } Q_2)$$

it is used to denote exact differentials

$$\int_1^2 \delta W \neq W_2 - W_1, \text{ and } W_{1-2} \text{ or } W_2$$

Const. pr. process

- A system is said to be in thermal equilibrium when the temp. at all points in the system is same. ~~Temp~~ Thermal equilibrium denotes only uniformity of temp. (a). The temp. of the system does not change with time and has same value at all points of the system.

- A system is said to be in mechanical equilibrium when the pressure at all points in the system is same. Mechanical equilibrium denotes only uniformity of pressure.

(a). There are no unbalanced forces within the system or between the surroundings. The pres. in the system is same at all points and does not change w.r.t. time.

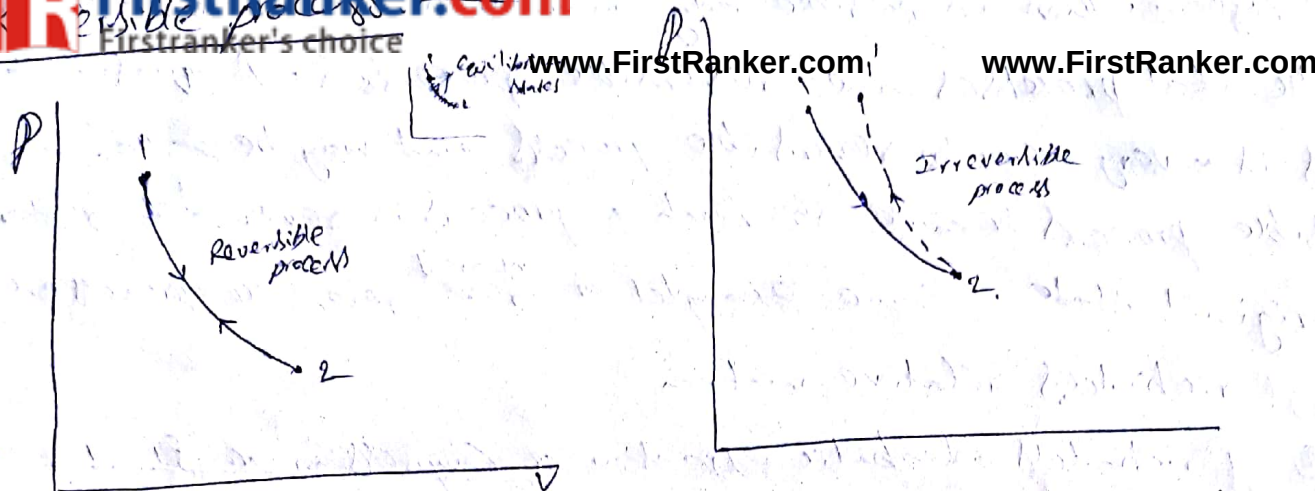
- No chemical reaction takes place in the system and chemical composition which is same throughout the system does not vary with time. Then the system is said to be in chemical equilibrium.

(a) The properties (P, T) do not change w.r.t. time if the system is isolated and it is in equilibrium condition.

- But thermodynamic equilibrium includes all types of equilibrium. Thermodynamics mainly deals with system ~~with~~ which are in thermodynamic equilibrium at the properties of state of a system which is not in equilibrium cannot be predicted.

A system is in thermodynamic equilibrium if the temp & pres. at all points are same, there should be no velocity gradient, the chemical equilibrium is also necessary.

Thus for attaining a state of thermodynamic equilibrium the following three types of equilibrium states must be achieved.



"A process is reversible if the system passes through a continuous series of equilibrium states". Hence, these states can be located on a pressure and volume diagram and a line representing the path of the process can be drawn through all the points, as shown in fig. In other words, a process is called reversible if the initial state together with all energies transferred or transformed during the process can be completely restored in both system and environment.

Definition → A reversible process (also sometimes known as quasi-static process) is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial state.

This process has the following characteristics:

- 1). No heat transfer across finite temp difference
- 2). process should be carried with slowness
- 3). At the end of process, both system and surroundings ~~can~~ reach to initial state.

Consider a piston moving in a cylinder in which gas expands from high to low pressure, some energy is expended in overcoming the friction between the piston and the cylinder; this friction produces heat. If the path is restored i.e., if the gas is compressed back to its original volume, more heat is produced, and the energy thus expended in overcoming friction is not recovered. Hence the expansion process of the fluid is not reversible. The process will be reversible ^{only} when there is no friction.

only the ideal processes are reversible processes. Therefore, this process is very near to reversible process and may be called a reversible process because if such a process is reversed it restores the original state. Some examples of ~~reversible~~ ^{nearby} reversible processes are:

Ex: Frictionless relative motion

- ①. Frictionless adiabatic expansion or compression of fluid.
- ②. extension and compression of a spring.
- ③. electrolysis
- ④. polytropic expansion or compression of a fluid
- ⑤. Isothermal expansion or compression.

Irreversible process:— If a system passes through a sequence of non-equilibrium states, then the process is said to be irreversible process.

ex: Throttling process

Ideal processes are reversible whereas natural processes are irreversible.

Causes of Irreversibility—

- 1). Lack of thermodynamic equilibrium during process
- 2). Involvement of dissipative effects during process.
- 3). mechanical & fluid friction
- 4). unrestricted expansion
- 5). Heat transfer with a finite temp difference.

Irreversibility due to lack of Equilibrium - Between system and its surroundings, if there is lack of Equilibrium i.e. mechanical, thermal, chemical & electrical, a process is caused due to spontaneous change which is irreversible.

Ex:- Heat transfer through finite temp difference

In order to transfer heat through infinitesimal temp difference an infinite area or time is needed. But actual h.t. processes are through finite temp difference in finite area and therefore irreversible due to lack of thermal equilibrium. The irreversibility increases with temp. difference.

2. Irreversibility due to Dissipative effects :- Dissipative effect with a process include the effects due to friction, viscosity,

in elasticity, electrical resistance and magnetic hysteresis. Among all these friction is more predominant.

Waste away
dissipate $\rightarrow$ gradually become
drive away (as seen in),
dissipate

The transformation of work into other form of energy, in the presence of dissipative effects, work is said to be dissipated.

Examples:-

1. Paddle wheel work transfer: Consider paddle wheel work transfer to a system in insulated container. The work transferred is dissipated adiabatically into an increase in molecular internal energy of system.
2. Flow of electric current through a resistor: Electric work is dissipated into heat and this transformation is irreversible.
3. Braking of rotating flywheel: Transformation of K.E. into molecular internal energy is irreversible.

Reversible and Irreversible processes :- Let us consider a process

(van der Waals). in which system moves from state point 1 to state point 2, causing a change in internal energy (du), transferring work ($-sw$) and rejecting heat ($-sq$) such that $-sq = du - sw$. If now we desire to move back from state point 2 to state point 1, it may or may not be possible to supply heat sq back to the system and deliver work sw to the surroundings. A process in which this reverse movement of system's path, restoring the system as well as its surroundings is

is called reversible process. That when a process is reversible, it does not leave any trace of itself in the system and the surroundings. However if heat and work are not restored back by same amounts the process will be termed as irreversible.

For reversible process "expansion and compression has to be restricted and controlled type and qualitative in character." "process involving h.t. due to finite difference of temp between the system and surroundings is irreversible".

Adiabatic & isothermal processes $\rightarrow$ reversible

There is no friction between fluid layer and moving parts $\rightarrow$ reversible

Frictionless adiabatic process is reversible in character and it called isentropic.

- In an irreversible process, there is a loss of heat

- The main cause for the irreversibility is
 - ①. mechanical and fluid friction
 - ②. unrestricted expansion
 - ③. H.T. with finite temp difference

- The conditions for the reversibility of a cycle is

- ①. The pre & temp of the working substance must not differ, appreciably, from those of the surroundings at any stage in the process.
- ②. all the processes, taking place in the cycle of operation, must be extremely slow.
- ③. The working parts of the engine must be friction free
- ④. There should be no loss of energy during the cycle of operation.

1, 3, 11, 15, 18, up to 47
17/7/03. 1st hour

1c). Reversibility :- A reversible process is one which can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states. It is an infinitely slow process.

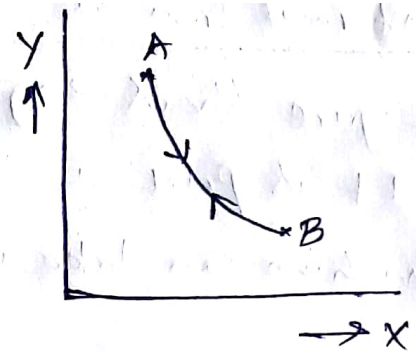
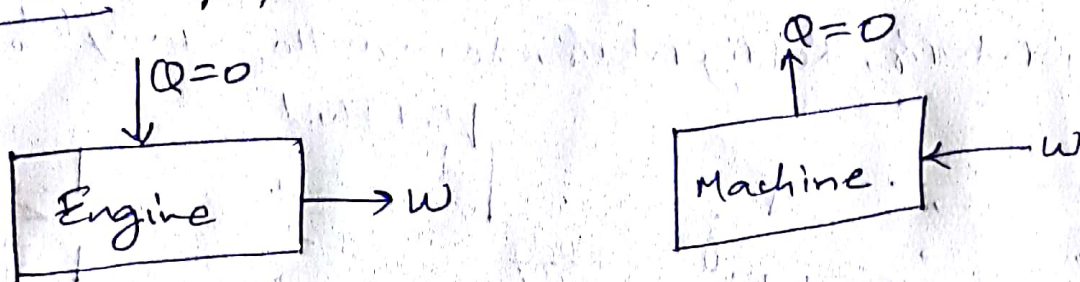


Fig: Reversible process.

Conditions for reversibility :-

- 1). No heat transfer across finite temperature difference
- 2). process should be carried with slowness
- 3). No dissipative effects
- 4). At the end of process, both the system and surroundings reach to initial states.

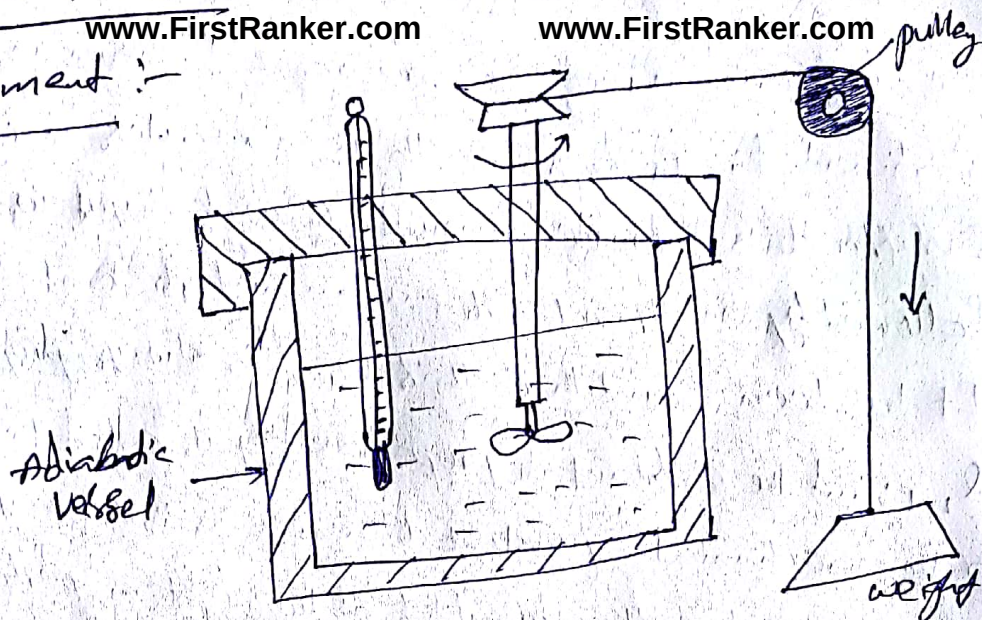
PMM - I :- perpetual motion machine of the first kind



According to law of Conservation of energy (first law of thermodynamics) energy can neither be created nor destroyed but it can be transformed from one form to another form.

* Figure shows engine can develop work without heat input, It is PMM - I

* machine can take work but no heat generated, it is PMM - I.

Joule's experiment :-

Let us consider a closed system which contains a known mass of water contained in an adiabatic vessel having a thermometer and a paddle wheel as shown in fig. Let a certain amount of work W_{1-2} be done upon the system by the paddle wheel. The quantity of work can be measured by the fall of weight which drives the paddle wheel through a pulley.

The system was initially at temperature t_1 , the same as that of atmosphere, and after work transfer let the temperature rise to t_2 . The pressure is always 1 atm. The process 1-2 undergone by the system shown in fig (2). in generalized thermodynamic coordinates X, Y .

Let the insulation now be removed.

The system and the surroundings

interacted by heat transfer till the system returns to the original temperature t_1 , attaining the condition of thermal equilibrium

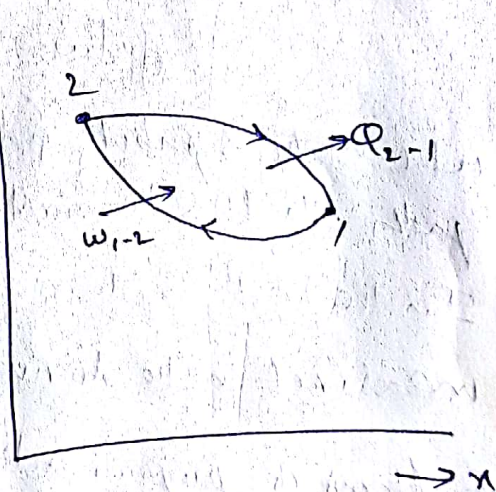


Fig. Cycle completed by a system with two energy interactions, adiabatic work transfer W_{1-2} followed by heat transfer Q_{2-1} .

The amount of heat transfer Q_{2-1} from the system during this process, $2-1$ shown in fig 2, can be estimated. The system thus executes a cycle, which exhibits a definite amount of work input w_{1-2} to the system followed by the transfer of an amount of heat Q_{2-1} from the system. It has been found that this w_{1-2} is always proportional to the heat Q_{2-1} and the constant of proportionality is called the Joule's equivalent or the mechanical equivalent of heat. In this example, there are only two energy transfer quantities as the system performs a thermodynamic cycle. If the cycle involves many more heat and work quantities, the same result will be found. Expressed algebraically

$$(\sum w)_{\text{cycle}} = J (\sum Q)_{\text{cycle}}$$

$$\begin{aligned} w &\propto Q \\ w &= JQ \end{aligned}$$

where J is the Joule's equivalent. The above equation is also expressed in the form

$$\oint dw = -J \oint dQ$$

where the symbol $\oint$ denotes the cyclic integral for the closed path. This is the first law for a closed system undergoing a cycle.

In the S.I. system of units, both heat and work are

measured in the derived unit of energy, the Joule. The

constant of proportionality, J , is therefore unity ($J = 1 \text{ Nm/J}$).

1) Work and energy are mutually convertible.

$$\oint dw = \int \delta Q$$

for the same units of heat and work

$$\oint dw = \int \delta Q$$

$$\oint dw = \int \delta Q$$

2) Energy can neither be created nor destroyed but it can be transformed from one form to another. It is the law of Conservation of Energy.

It is Q is the amount of heat

transferred to the system and

w is the amount of work transferred

from the system during the process.

The net energy transfer ($Q - w$)

will be stored in the system. Energy in storage is neither

heat nor work, and is given the name internal energy or

simply, the energy of the system.

$$Q - w = \Delta E$$

where ΔE is the increase in the energy of the system.

$$Q = \Delta E + w$$

Here $Q, w, \Delta E$ are all expressed in the same units (in Joules)

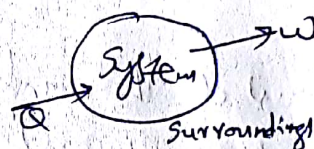
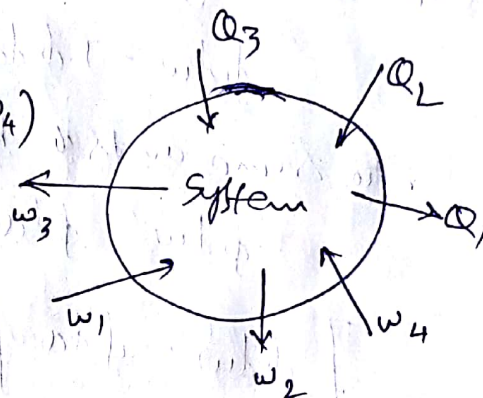


Fig: Heat and work interaction of a system with its surroundings in a process.

1st law applies to a process! — It there are more energy transfer quantities involved in the process as shown in fig.

the first law gives

$$Q_2 + Q_3 - Q_1 = \Delta E + (w_2 + w_3 - w_1 - w_4)$$



heat out from system is -ve
(out)

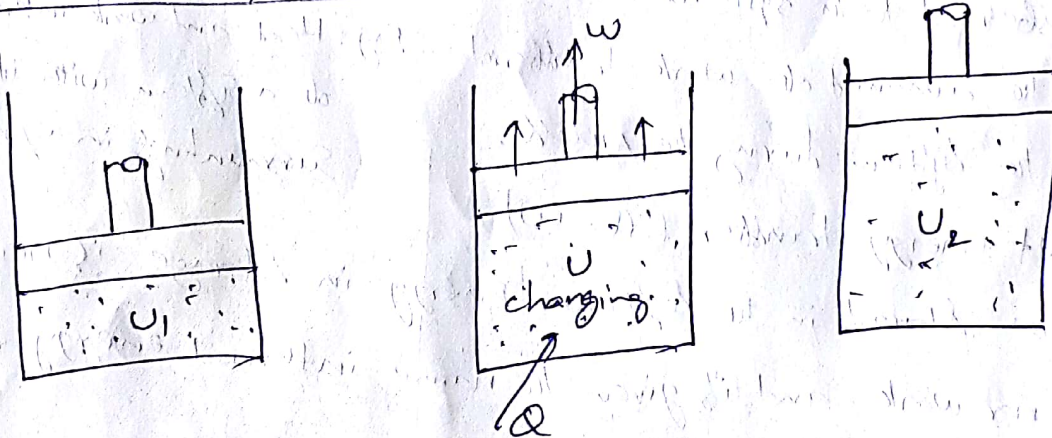
heat add to system is +ve
(in)

work done on system is -ve
(in)

work done by the system is +ve
(out)

Fig: System-surroundings interaction in a process involving many energy fluxes

First law of Thermodynamics applied to closed system! —
(non-flow process)



If the system follows a process which is not cyclic, the final state differs from the initial state, and the difference between energy input (heat transfer) and output (work transfer) results in a change in the energy content (internal energy) of the system.

$$\text{Energy input} - \text{Energy output} = \text{change in internal energy}$$

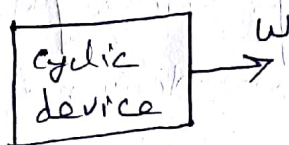
$$Q - w = U_2 - U_1$$

$$\Rightarrow Q = (U_2 - U_1) + w$$

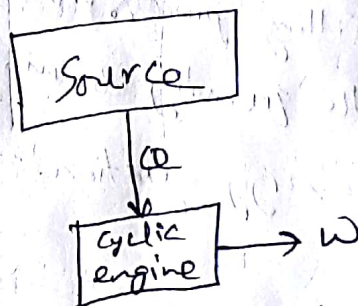
The above formula is called the non-flow energy equation. This comprises the first Corollary of the first law of Thermodynamics.

Corollaries of First law:-

- 1). For isolated system, Q and W are zero, and therefore $(U_2 - U_1) = 0$. Thus the internal energy of an isolated system is constant.
- 2). The difference between heat supplied and work done during a non-flow process results in change in the property of the closed system, and is referred as internal energy.
- 3). A perpetual motion machine of first kind is impossible. A device that will deliver work continuously without input (i.e., without receiving energy) is called perpetual motion machine of the first kind (PMM I).



a). PMM-1



b). possible engine.

First law applied to a flow system:- In an open system,

the energy as well as matter cross the boundary. Any process undergone by an open system is called a flow process.

These processes may be sub-divided into two types.

- 1). Steady flow processes, and
- 2). Unsteady flow processes.

Steady flow process: Steady flow means that the rates of flow of mass and energy across the control surface are constant, i.e., the mass of fluid entering the system is equal to the mass of fluid leaving the system in a given time, and the transfer of energy must take place at a uniform rate.

Ex:- flow of steam through a nozzle and a steam boiler operating under a constant load. In order to maintain water level in boiler, the feed pump supplies water at exactly the same rate at which the steam is drawn off from the boiler.

unsteady flow process: - unsteady flow process is one in

which the quantity of fluid (matter) in the system is not constant i.e., the mass of fluid entering the system is not the same as the mass of fluid (matter) leaving the system in a given time. Two simple cases of unsteady flow process are tank filling process and tank emptying process. In both the cases the quantity of matter in the tank is not constant.

Steady flow energy equation: -

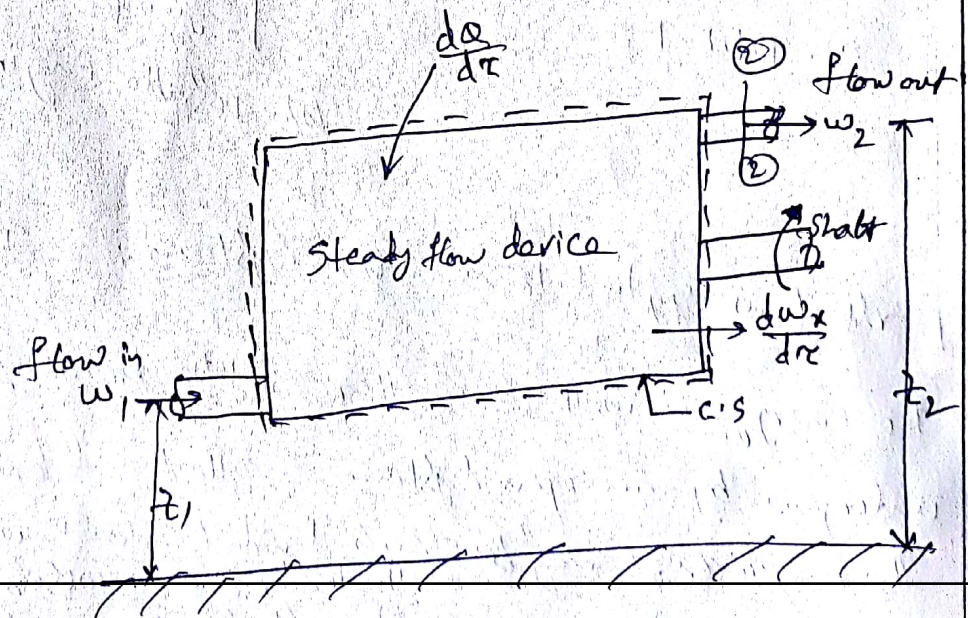


Fig shows a steady flow system, in which 7

or fluid enters and an another stream leaves the control volume. There is no accumulation of mass or energy within the control volume, and the properties at any location within the control volume are steady with time. Sections ① and ② indicate respectively the entrance and exit of the fluid across the control surface. The following quantities are

A_1, A_2 — cross-section of stream, m^2

w_1, w_2 — mass flow rate, kg/s

p_1, p_2 — pressure, absolute, N/m^2

v_1, v_2 — specific volume, m^3/kg

u_1, u_2 — specific internal energy, J/kg

V_1, V_2 — velocity, m/s

z_1, z_2 — elevation above an arbitrary datum, m

$\frac{dq}{dt}$ — net rate of heat transfer through the control surface, J/s

$\frac{dw}{dt}$ — net rate of work transfer through the control surface, J/s

t — time, s

Subscripts 1 and 2 refer to the inlet and exit sections.

Mass balance — By the conservation of mass, if there is no accumulation of mass within the control volume, the mass flow rate entering must equal the mass flow rate leaving

$$w_1 = w_2$$

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \quad \text{--- ①}$$

This equation is known as the Equation of Continuity.

Energy Balance - In a flow process, the work transfer may be of two types: the external work and the flow work.

External work is the shaft work (w_x).
The flow work is the displacement work done by the fluid of mass dm , at the inlet section 1 and that at the exit section, which are $(-p_1 \omega_1 dm_1)$ and $(+p_2 \omega_2 dm_2)$ respectively.

∴ Total work transfer

$$W = W_x - p_1 \omega_1 dm_1 + p_2 \omega_2 dm_2 \quad \text{--- (2)}$$

in the rate form

$$\begin{aligned} \frac{dW}{dt} &= \frac{dW_x}{dt} - p_1 \omega_1 \frac{dm_1}{dt} + p_2 \omega_2 \frac{dm_2}{dt} \\ &= \frac{dW_x}{dt} - p_1 \omega_1 \cdot \dot{m}_1 + p_2 \omega_2 \cdot \dot{m}_2 \quad \text{--- (3)} \end{aligned}$$

Since there is no accumulation of energy, by the conservation of energy, the total rate of flow of all energy streams entering the control volume must be equal to the total rate of flow of all energy streams leaving the control volume. This may be expressed in the following equation

$$\dot{m}_1 e_1 + \frac{dQ}{dt} = \dot{m}_2 e_2 + \frac{dW}{dt}$$

Substitute equation (3)

$$\dot{m}_1 e_1 + \frac{dQ}{dt} = \dot{m}_2 e_2 + \frac{dW_x}{dt} - \dot{m}_1 p_1 \omega_1 + \dot{m}_2 p_2 \omega_2$$

$$\dot{m}_1 e_1 + \frac{dQ}{dt} + \dot{m}_1 p_1 \omega_1 = \dot{m}_2 e_2 + \frac{dW_x}{dt} + \dot{m}_2 p_2 \omega_2 \quad \text{--- (4)}$$

where e_1, e_2 refer to the energy carried into or out of the control volume with unit mass of fluid. (9)

The specific energy e is given by

$$e = e_k + e_p + u$$

$$= \frac{v^2}{2} + z g + u$$

$$e_1 = \frac{v_1^2}{2} + z_1 g + u_1$$

$$e_2 = \frac{v_2^2}{2} + z_2 g + u_2$$

Substitute e_1 and e_2 in equation (4)

$$w_1 \left(\frac{v_1^2}{2} + z_1 g + u_1 \right) + w_1 p_1 \omega_1 + \frac{dQ}{dt} = w_2 \left(\frac{v_2^2}{2} + z_2 g + u_2 \right) + w_2 p_2 \omega_2 + \frac{dw_2}{dt}$$

$$w_1 \left[u_1 + p_1 \omega_1 + \frac{v_1^2}{2} + z_1 g \right] + \frac{dQ}{dt}$$

$$= w_2 \left[u_2 + p_2 \omega_2 + \frac{v_2^2}{2} + z_2 g \right] + \frac{dw_2}{dt}$$

$$h = u + p \omega$$

$$h_1 = u_1 + p_1 \omega_1$$

$$h_2 = u_2 + p_2 \omega_2$$

$$w_1 \left[h_1 + \frac{v_1^2}{2} + z_1 g \right] + \frac{dQ}{dt} = w_2 \left[h_2 + \frac{v_2^2}{2} + z_2 g \right] + \frac{dw_2}{dt}$$

$$\text{Let } w_1 = w_2$$

$$w = w_1 = w_2 = \frac{dm}{dt}$$

$$h_1 + \frac{v_1^2}{2} + z_1 g + \frac{dQ}{dm} = h_2 + \frac{v_2^2}{2} + z_2 g + \frac{dw_2}{dm}$$

$$\frac{dQ}{dt} \times \frac{1}{w_1}$$

$$\frac{dQ}{dt} \times \frac{1}{\frac{dm}{dt}}$$

$$\frac{dQ}{dt} \times \frac{dt}{dm} = \frac{dQ}{dm}$$

All the terms in equation (6) represents energy flow per unit mass of fluid (J/kg), whereas the terms in equation (5) represent energy flow per unit time (J/s).

Equation (6) can be written in the following form,

$$Q - W_x = (h_2 - h_1) + \frac{\bar{v}_2^2 - \bar{v}_1^2}{2} + g(z_2 - z_1) \quad (7)$$

where Q and W_x refer to energy transfer per unit mass, in the differential form, the S.F.E.E. becomes

$$dQ - dW_x = dh + \bar{v}d\bar{v} + g dz \quad (8)$$

when more than one stream of fluid enters and leaves the control volume (fig below), the mass balance and energy balance for steady flow are given below.

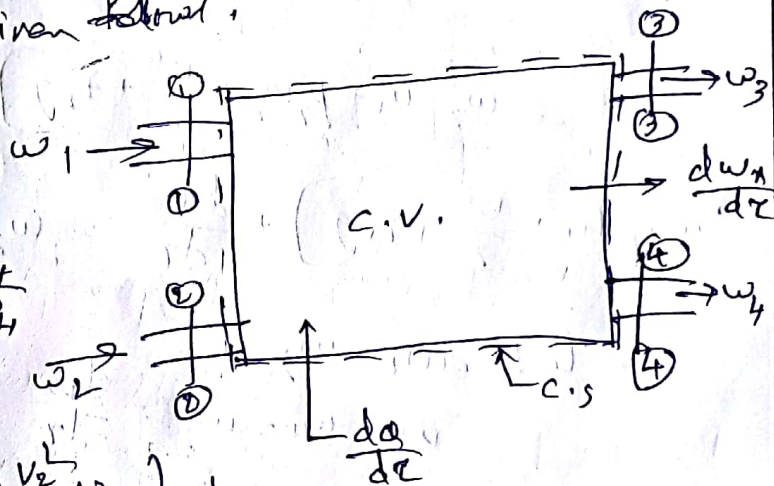
mass balance

$$W_1 + W_2 = W_3 + W_4$$

$$\frac{A_1 V_1}{C_1} + \frac{A_2 V_2}{C_2} = \frac{A_3 V_3}{C_3} + \frac{A_4 V_4}{C_4}$$

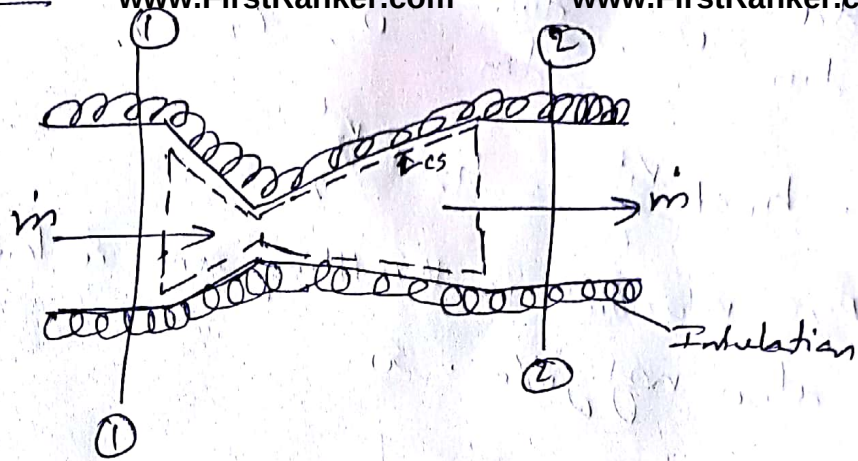
Energy Balance

$$W_1 \left(h_1 + \frac{v_1^2}{2} + z_1 g \right) + W_2 \left(h_2 + \frac{v_2^2}{2} + z_2 g \right) + \frac{dQ}{dx} = W_3 \left(h_3 + \frac{v_3^2}{2} + z_3 g \right) + W_4 \left(h_4 + \frac{v_4^2}{2} + z_4 g \right) + \frac{dW_x}{dx}$$



Nozzle:-

A nozzle is a device which increases the velocity or K.E.



of a fluid at the expense of its pressure drop. Fig. shows a nozzle which is insulated. The steady flow energy equation for the control surface gives

$$h_1 + \frac{v_1^2}{2} + z_1 g + \frac{dq}{dm} = h_2 + \frac{v_2^2}{2} + z_2 g + \frac{dw_x}{dm}$$

$$\frac{dq}{dm} = 0$$

$$\frac{dw_x}{dm} = 0$$

$$z_1 = z_2$$

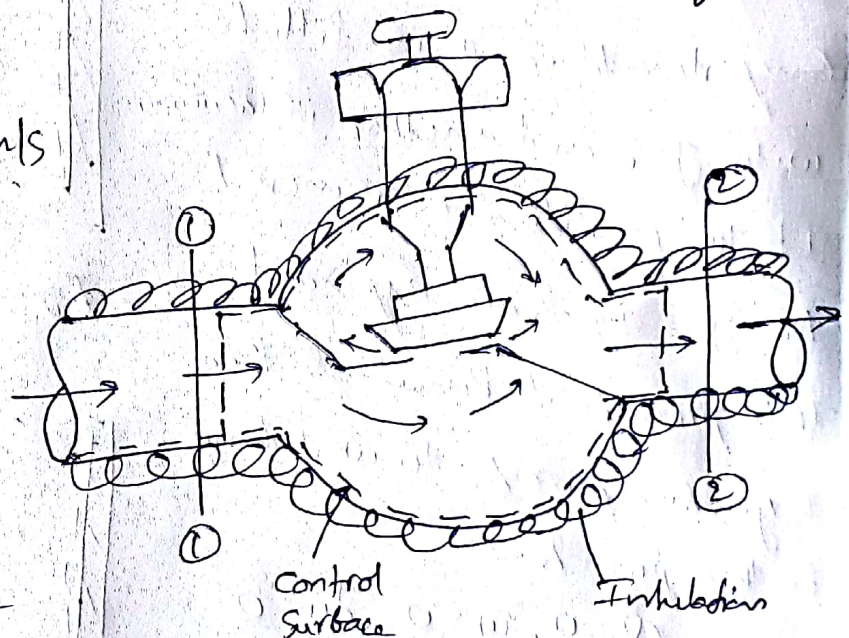
$$v_1 = \text{neglected}$$

$$h_1 = h_2 + \frac{v_2^2}{2}$$

$$\Rightarrow \boxed{v_2 = \sqrt{2(h_1 - h_2)}} \text{ m/s}$$

Throttling device:-

When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure, and the flow is said to be throttled. Fig. shows the process of throttling by a partially opened valve on a fluid flowing in an insulated pipe. In the steady flow energy equation



$$\frac{dw}{dm} = 0, \quad \frac{dw}{dm} = 0$$

$$\therefore h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

The pipe velocities in throttling are so low that the k.E. term are also negligible. So

$$h_1 = h_2$$

The enthalpy of the fluid before throttling is equal to the enthalpy of the fluid after throttling.

Free expansion process :-

A Tank is divided into two compartments. one is filled with gas and another is vacuum. If the partition is removed the gas is free expanded. This process is known as free expansion process. No work done in this process.

$$Q=0, w=0, du=0$$

$$dv=0, T_1=T_2$$

no change in temperature and hence $dh=0, (H_2=H_1)$

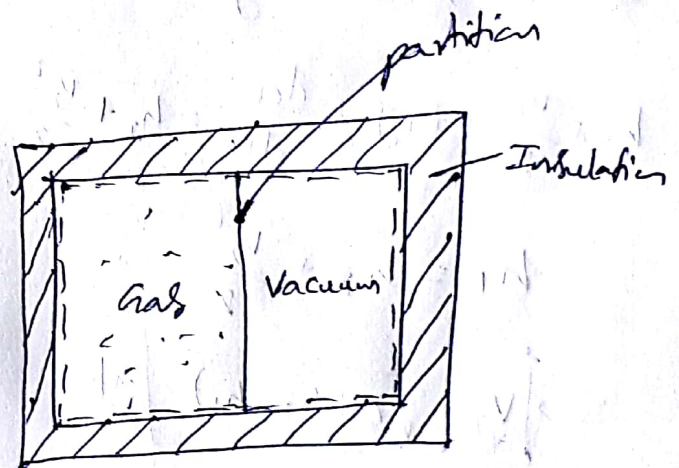


Fig: Before expansion

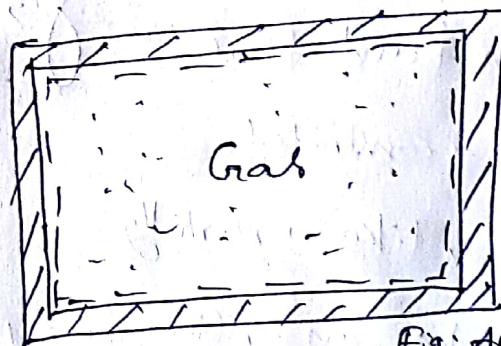


Fig: After expansion

Turbine :- For a turbine

which is well insulated,
the flow velocities are often
small, and the K.E. terms
can be neglected. The SFEE becomes

$$\frac{dQ}{dm} = 0, \quad z_1 = z_2, \quad \text{K.E. neglected}$$

$$h_1 = h_2 + \frac{dw_1}{dm}$$

$$\frac{w_1}{m} = (h_1 - h_2)$$

It is seen that work is done by the fluid at the expense of its
enthalpy.

Compressor :- For an adiabatic pump or compressor, work is done
upon the fluid and w is negative. So the SFEE becomes

$$h_1 = h_2 - \frac{w_1}{m}$$

$$\frac{w_1}{m} = h_2 - h_1$$

The enthalpy of the fluid increases by the amount of work input.

Heat Exchanger :-

K.E, P.E terms neglected,
no external work done, w_c
no external heat
interaction or heat loss

$\therefore$ SFEE becomes
 $w_s (h_2 - h_4) = w_c (h_1 - h_3)$

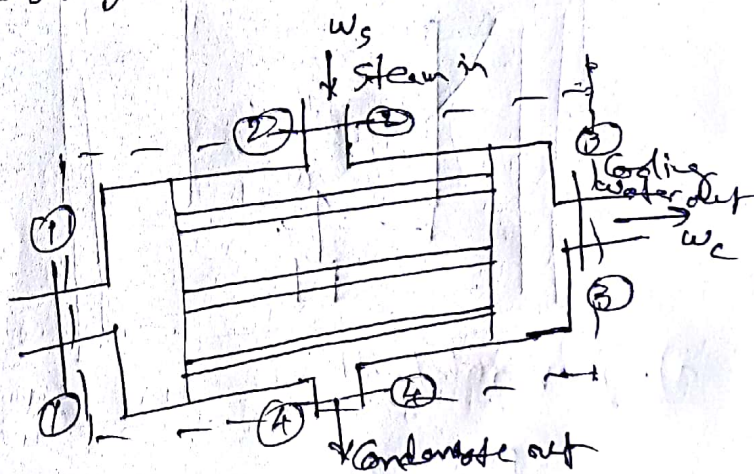


Fig: Steam Condenser

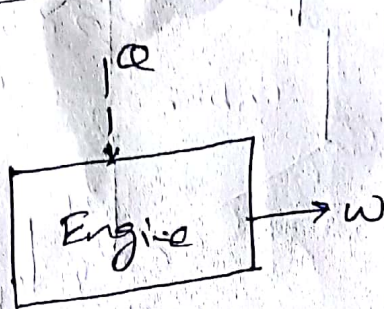


Fig 1 PMM-1

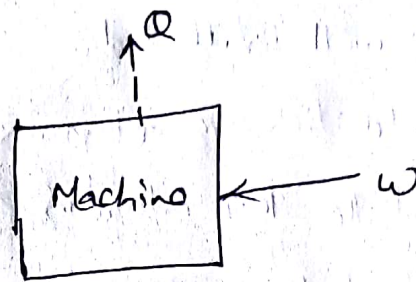


Fig 2 The conversion of PMM-1

no machine supply work without energy input. The first law states the principle of conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another.

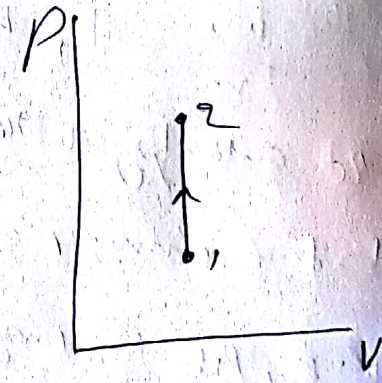
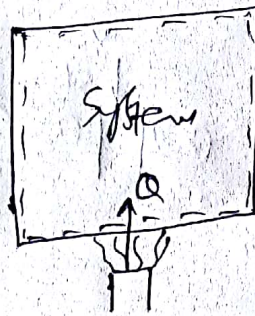
Fig 1 shows, without input, engine develop work, it's not possible.

Fig 2: machine take work but no production of heat, it's not possible.

Ref. two are PMM-I.

First law applied to closed system:-

1). Reversible Constant volume process (V constant):-



a). p-v relationship

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ (General equation)}$$

(for constant volume process)

$$W = 0$$

First law of thermodynamics

$$Q = \Delta E + W$$

$$\therefore Q = \Delta E = \Delta U$$

①. change of internal energy

$$U_2 - U_1 = m C_v (T_2 - T_1)$$

②. Heat supplied $Q = \Delta U = m C_v (T_2 - T_1)$

③. change of enthalpy

$$H_2 - H_1 = (U_2 - U_1) + (P_2 V_2 - P_1 V_1)$$

$$= m C_v (T_2 - T_1) + m R (T_2 - T_1)$$

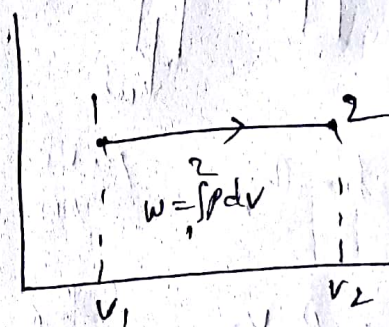
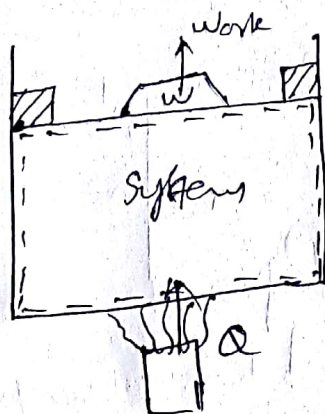
$$= m (T_2 - T_1) (C_v + R)$$

$$= m C_p (T_2 - T_1)$$

$$C_p - C_v = R$$

$$C_p = R + C_v$$

2). Reversible constant pressure process ($p = \text{constant}$) :-



a). p, V, T relation ship $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$ (general equation)

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{for } p = \text{constant})$$

$$W = p(V_2 - V_1) \\ = mR(T_2 - T_1)$$

$$pV = RT$$

$$p_1 V_1 = RT_1$$

$$p_2 V_2 = RT_2$$

$$p_2 V_2 - p_1 V_1$$

$$= R(T_2 - T_1)$$

③. change in internal energy

$$Q = \Delta E + W$$

$$\Delta U = U_2 - U_1 = mC_V(T_2 - T_1)$$

④. Heat Supplied or transferred

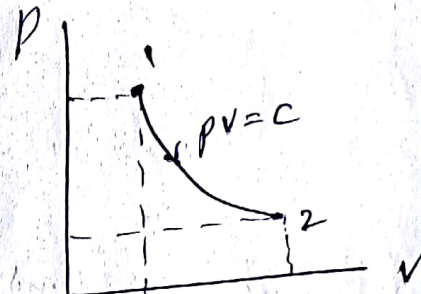
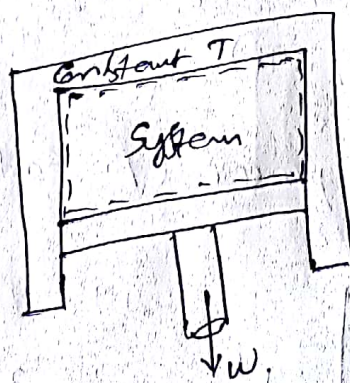
$$Q = \Delta U + W$$

$$= mC_p(T_2 - T_1)$$

⑤. change in enthalpy

$$\Delta H = H_2 - H_1 = mC_p(T_2 - T_1)$$

3). Reversible Constant Temperature process:-



a). p, V, T relationship $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

$$p_1 V_1 = p_2 V_2 \quad (\text{for } T = \text{Constant})$$

b). workdone $W = p_1 V_1 \ln \frac{V_2}{V_1}$

c). change internal energy

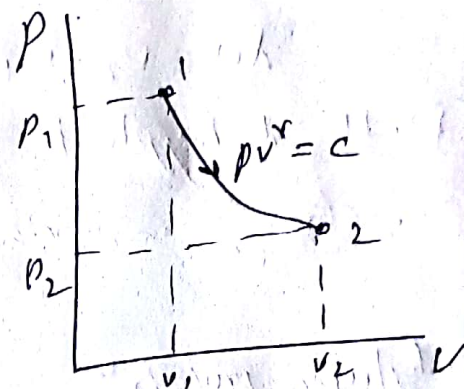
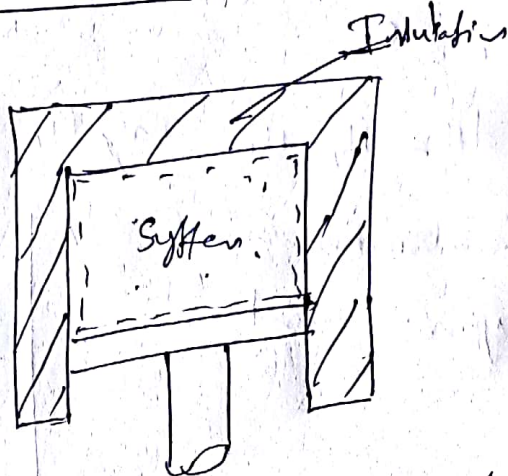
$$\Delta U = U_2 - U_1 = 0$$

d). Heat supplied $Q_{1-2} = \Delta U + W = W = p_1 V_1 \ln \frac{V_2}{V_1}$

i.e., $H_2 = H_1$, because temperature is constant.

4. Adiabatic process:-

It is a process in which neither receives nor gives heat to its surroundings



during its expansion or compression it is called reversible adiabatic process.

②. P, V, T relationship

$$pV^\gamma = c$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \times \frac{V_2}{V_1}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1} \right)^\gamma$$

$$\frac{T_1}{T_2} \times \frac{V_2}{V_1} = \left(\frac{V_2}{V_1} \right)^\gamma \Rightarrow \frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \times \frac{P_2}{P_1} = \frac{T_1}{T_2} \times \frac{P_2}{P_1}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} \times \frac{P_1}{P_2} = \left(\frac{P_1}{P_2} \right)^{1/\gamma} \times \frac{P_1}{P_2}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}}$$

③. change in internal energy

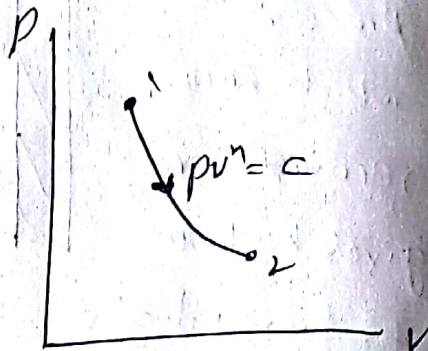
$$du = u_2 - u_1 = m c_v (T_2 - T_1)$$

④. Heat supplied $Q_{1-2} = 0$

⑤. change in enthalpy, $dh = h_2 - h_1 = m c_p (T_2 - T_1)$

polytropic process! — It is also known as the general law for the expansion and compression of gases.

$$p v^n = \text{constant}$$



⑥. p, v, T relationship

$$p_1 v_1^n = p_2 v_2^n$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{n-1}$$

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{n-1}{n}}$$

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

⑦. work done $w_{1-2} = \frac{p_1 v_1 - p_2 v_2}{n-1}$

⑧. Heat supplied or transferred

$$Q_{1-2} = w_{1-2} + du$$

$$= \frac{p_1 v_1 - p_2 v_2}{n-1} + m c_v (T_2 - T_1)$$

$$= \frac{m R (T_2 - T_1)}{n-1} + \frac{m R}{\gamma-1} (T_2 - T_1) \quad \left(\because c_v = \frac{R}{\gamma-1} \right)$$

$$= mR (T_1 - T_2) \left[\frac{1}{n-1} - \frac{1}{r-1} \right]$$
$$= mR (T_2 - T_1) \left[\frac{(r-1) - (n-1)}{(n-1)(r-1)} \right]$$

$$= mR (T_1 - T_2) \left[\frac{r-n}{(n-1)(r-1)} \right]$$

$$= \frac{r-n}{r-1} \times \frac{mR (T_1 - T_2)}{n-1}$$

①. change in internal energy

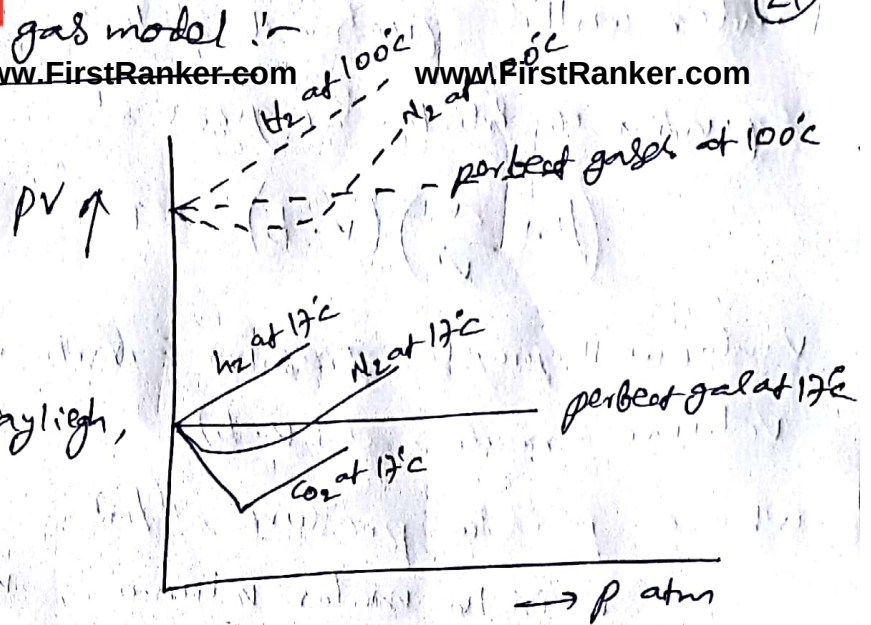
$$dU = U_2 - U_1 = mC_v (T_2 - T_1)$$

②. change in enthalpy

$$dH = H_2 - H_1 = mC_p (T_2 - T_1)$$

process	Index	Heat supplied	work done	P, V, T, relationship	change in internal energy	change in enthalpy	Specific heat
1). Constant pressure process	$n = 0$	$C_p (T_2 - T_1)$	$P(V_2 - V_1)$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$m C_v (T_2 - T_1)$	$m C_p (T_2 - T_1)$	C_p
2). Constant Volume process	$n = \infty$	$C_v (T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{P_1}{P_2}$	$m C_v (T_2 - T_1)$	$m C_p (T_2 - T_1)$	C_v
3). Constant temperature process	$n = 1$	$P_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$	$P_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$	$P_1 V_1 = P_2 V_2$	$m C_v (T_2 - T_1)$	$m C_p (T_2 - T_1)$	∞
4). Adiabatic process	$n = \gamma$	0	$\frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{R}{\gamma - 1} (T_2 - T_1)$	$P_1 V_1^\gamma = P_2 V_2^\gamma$	$m C_v (T_2 - T_1)$	$m C_p (T_2 - T_1)$	0
5). Polytropic process	$n = n$	$\frac{(P_1 - P_2) m C_v (T_2 - T_1)}{(n - 1)(C_p - C_v)}$	$\frac{P_1 V_1 - P_2 V_2}{n - 1} \cdot \frac{R}{n - 1} (T_2 - T_1)$	$P_1 V_1^n = P_2 V_2^n$	$m C_v (T_2 - T_1)$	$m C_p (T_2 - T_1)$	$C_n = C_v \left(\frac{\gamma - n}{1 - n} \right)$

Figure shows the deviations of real gases from perfect gases by the experimental results Lord Rayleigh, those curves are called isotherms.



According to Boyle's law

$PV = \text{Constant}$ at all pressures and variation of pressure with PV is a straight line. But this is only for perfect gas and real gases are not showing straight lines.

At low pressures for H₂ PV increases as P increases.

For N₂ PV decreases as P increases.

For perfect gas PV remains constant as P increases, for CO₂ and N₂, PV first decreases and then increases as P increases, for perfect gas PV remains constant as P increases.

All real gases have higher values of PV than that of a perfect gas.

Equation of State for gas is

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

Vander wall propels pressure correction as $\left(p + \frac{a}{v^2}\right)$ and
Volume correction as $(v-b)$.

Where p is the pressure, N/m^2

v is the molar volume, $m^3/kg\text{ mole}$

T is the temperature, K

R is the gas constant $= 8.314 \text{ kJ/kg mole}$

a is the constant, $N \cdot m^4/(kg\text{ mole})^2$

b is the constant, $m^3/kg\text{ mole}$

Compressibility chart:-

The Compressibility factor (z)
of any gas is a function
of only two properties,
usually temperature
and pressure, so that

$z = f(T_r, P_r)$ except near

the critical point. The

value of z for any

real gas may be less or

more than unity, depending

on pressure and temperature conditions of the gas.

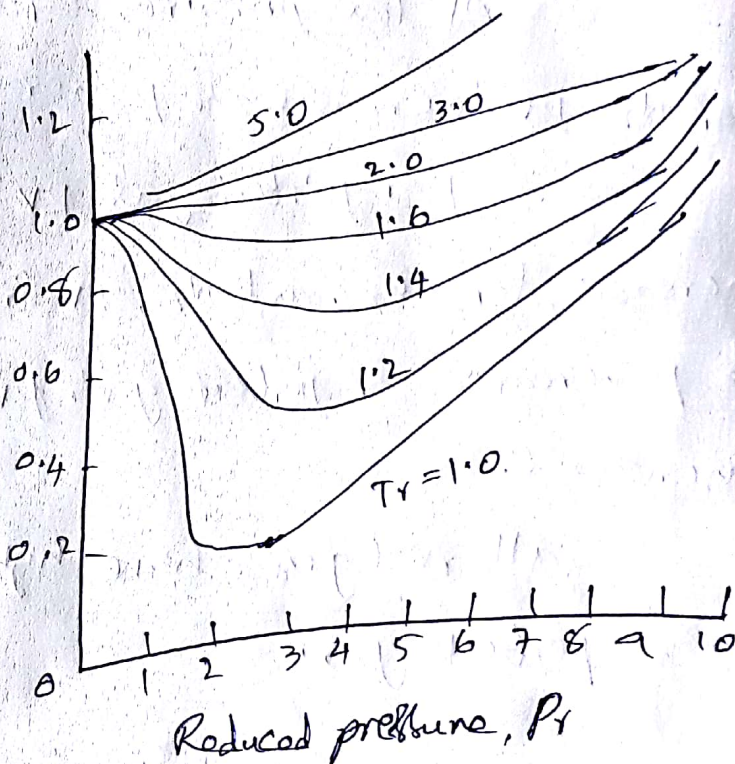
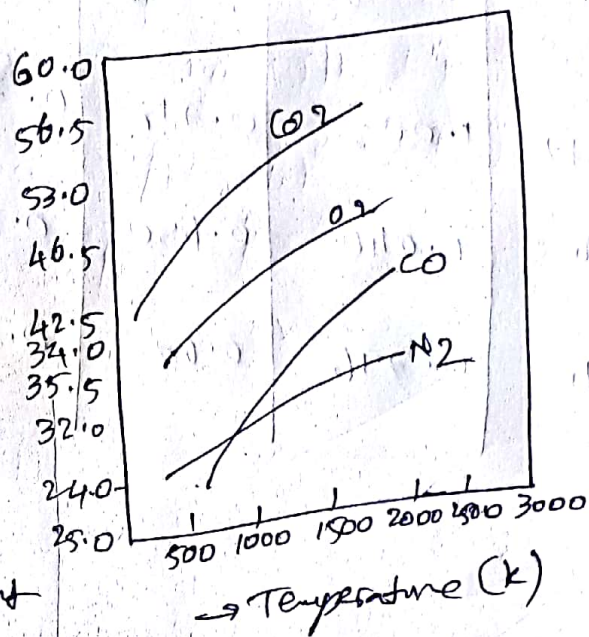


Fig: Generalized Compressibility chart

The general compressibility chart is plotted with Z versus P_r for various values of T_r . This is constructed by plotting the known data of one or more gases and can be used for any gas. Such chart is shown in fig. This chart gives best results for the regions well removed from the critical state for all gases.

Variation of specific heat with temperature:

Specific heat of any gas increases with increasing temperature because increase of vibration of molecules at high temperature. The vibration are caused because of collisions among molecules which are significant at high temperatures.



The temperature range of 300 K to 1500 K the specific heat is a linear function of temperature and may be expressed as $C_p = a + kT$, $C_v = b + kT$ where a, b, k are constants.

characteristic gas constant $R = C_p - C_v$

$$= (a + kT) - (b + kT)$$

$$= a - b$$

$$C_p = a + k_1 T$$

$$C_v = b + k_1 T + k_2 T^2$$

Gas Tables:— By using gas tables we read the properties of gases.

Gas	molecular weight	C_p (kJ/kgK)	C_v (kJ/kgK)	$R = C_p - C_v$	$\gamma = \frac{C_p}{C_v}$	$z_c = \frac{P_c V_c}{R_o T_c}$
Air	28.97	1.005	0.718	0.287	1.4	0.204
CO ₂	44	0.845	0.650	0.190	1.3	0.274
SO ₂	64	0.796	0.67	0.126	1.19	0.268

A stationary mass of gas is compressed without friction from an initial state of 0.3 m^3 and 0.105 MPa to a final state of 0.15 m^3 and 0.105 MPa , the pressure remaining constant during the process. There is a transfer of 37.6 kJ of heat from the gas during the process. How much does the internal energy of the gas change?

Soln: First law for a stationary system in a process gives

$$Q = \Delta U + W$$

$$Q_{1-2} = U_2 - U_1 + W_{1-2} \quad \text{--- (1)}$$

$$\begin{aligned} \text{Here } W_{1-2} &= \int_{V_1}^{V_2} p dV = p(V_2 - V_1) \\ &= 0.105(0.15 - 0.30) \text{ MJ} \\ &= -15.75 \text{ kJ} \end{aligned}$$

$$Q_{1-2} = -37.6 \text{ kJ}$$

∴ from Equation (1)

$$\begin{aligned} -37.6 &= U_2 - U_1 + (-15.75) \\ &= (U_2 - U_1) - 15.75 \end{aligned}$$

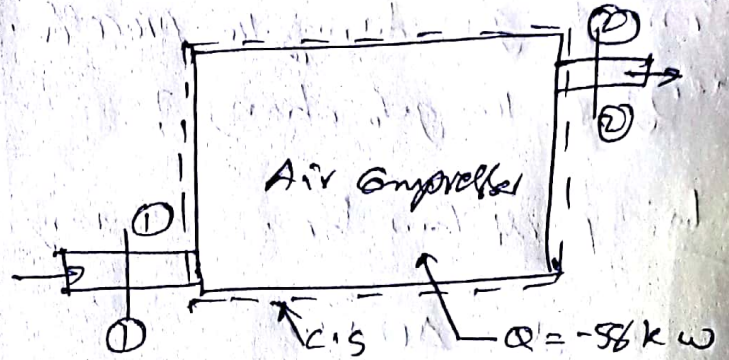
$$\therefore (U_2 - U_1) = -21.85 \text{ kJ}$$

The internal energy of the gas decreases by 21.85 kJ in the process.

Q. Air flows steadily at the rate of 0.5 kg/s through an air compressor, entering at 7 m/s velocity, 100 kPa pressure, and $0.95 \text{ m}^3/\text{kg}$ volume, and leaving at 5 m/s , 700 kPa and $0.19 \text{ m}^3/\text{kg}$. The internal energy of the air leaving is 90 kJ/kg .

in the compressor jacket & absorbs heat from the air at the rate of 58 kW. (a). Compute the rate of shaft work input to the air in kW. (b). Find the ratio of the inlet pipe diameter to outlet pipe diameter.

Soln $V_1 = 7 \text{ m/s}$
 $P_1 = 100 \text{ kPa}$
 $\rho_1 = 0.95 \text{ m}^3/\text{kg}$



$V_2 = 5 \text{ m/s}$
 $P_2 = 700 \text{ kPa}$
 $\rho_2 = 0.19 \text{ m}^3/\text{kg}$
 $u_2 = (u + 90) \text{ kJ/kg}$
 $Q = -58 \text{ kW}$

(a). writing the steady flow energy equation

$$w \left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + z_1 g \right) + \frac{dq}{dx} = w \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + z_2 g \right) + \frac{dw_x}{dx}$$

$$= \frac{dw_x}{dx} = -w \left[(u_2 - u_1) + (p_2 v_2 - p_1 v_1) + \frac{V_2^2 - V_1^2}{2} + (z_2 - z_1)g \right] + \frac{dq}{dx}$$

$$= -0.5 \left[90 + (7 \times 0.19 - 1 \times 0.95) 100 + \frac{(5^2 - 7^2) \times 10^{-3}}{2} + 0 \right] - 58$$

$$= -0.5 [90 - 38 - 0.012] - 58$$

$$= -122 \text{ kW}$$

Rate of work input is 122 kW

(b). From mass balance, we have

$$w = \frac{A_1 V_1}{\rho_1} = \frac{A_2 V_2}{\rho_2}$$

$$\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} \cdot \frac{V_2}{V_1} = \frac{0.19}{0.95} \times \frac{5}{7} = 3.57$$

$$\therefore \frac{d_1}{d_2} = \sqrt{3.57} = 1.89$$

Limitations of the first law:-

- 1). The first law does not show whether or not a system will undergo a change. Simply it states that heat and work are mutually convertible during a process.
- 2). The first law does not predict what portion of heat transferred can be converted into useful work. According to this law, probably all the heat transferred could be converted into work. However, in practice much heat is wasted in the process. Thus the process of converting heat completely into work is impossible, but it is possible to convert work completely into heat.
- 3). The first law does not specify the direction of heat and work.

Thermal Energy Reservoir (TER) :- TER is defined as a large body of infinite heat capacity, which is capable of absorbing or rejecting an unlimited quantity of heat without suffering appreciable changes in its thermodynamic coordinates. The changes that do take place in the large body as heat enters or leaves are so very slow and so very minute that all processes within it are quasi-static.

Source — the thermal energy reservoir TER_H from which heat Q_1 is

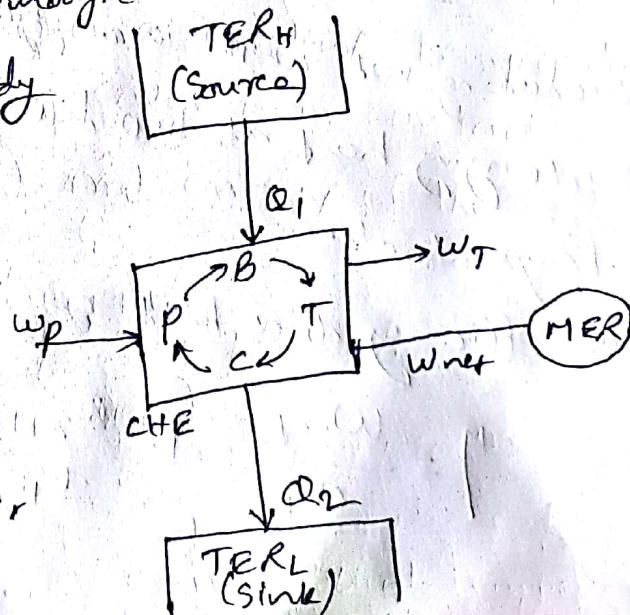


Fig: Cyclic heat engine (CHE) with Source and Sink.
www.FirstRanker.com

transferred to the system operating in a heat engine cycle is called the source.

Ex:- A typical source is a constant temperature furnace where fuel is continuously burnt.

Sink - The thermal energy reservoir TERL to which heat Q_2 is rejected from the system during a cycle is the sink.

Ex:- A typical sink is a river or sea or the atmosphere itself.

Mechanical Energy Reservoir (MER) is a large body enclosed by an adiabatic impermeable wall capable of storing work as potential energy (such as a raised weight or wound spring) or kinetic energy (such as a rotating flywheel). All processes of interest within an MER are essentially quasi-static. An MER receives and delivers mechanical energy quasi-statically.

Heat engine: Figure shows heat engine producing net work in a cycle by exchanging heat at two different temperatures.

The efficiency of a heat engine

$$\eta = \frac{W_{net}}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

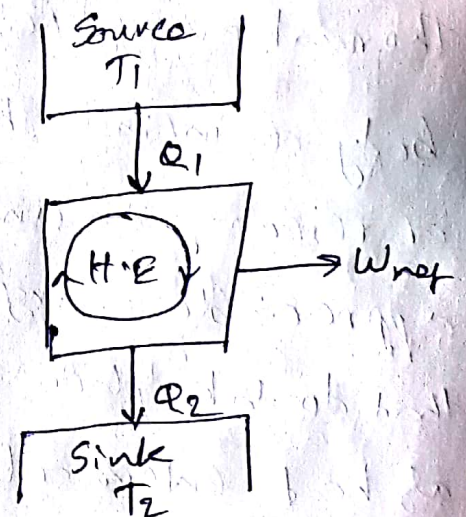


Fig: Heat engine

device which, operating in a cycle, maintains a body, say B, at a temperature higher than the temperature of the surroundings.

The heat is extracted from

the low temperature reservoir (atmosphere), and discharged into the high temperature body B, with the expenditure of work W in a cyclic device called a heat pump.

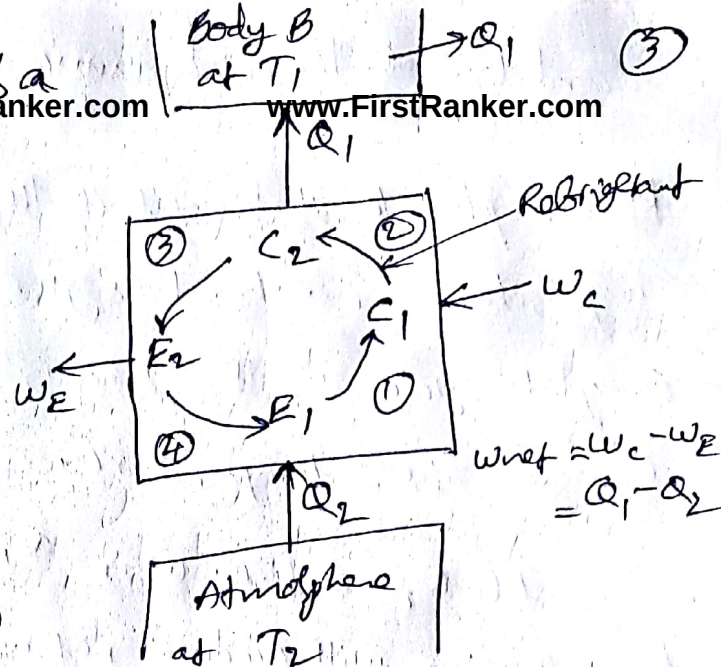


Fig: A cyclic heat pump

The working fluid operates

in a cycle flowing through the evaporator E_1 , compressor C_1 , condenser C_2 and expander E_2 , similar to a refrigerator, but

the attention is here focussed on the high temperature body B. Here Q_1 and W are of primary interest, and the COP is defined

as Coefficient of performance, $COP = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$

$$\therefore (COP)_{HP} = \frac{Q_1}{Q_1 - Q_2}$$

Kelvin-Planck Statement of the Second Law States: It is impossible for a heat engine to produce net work in a complete cycle if it exchanges heat only with bodies at a single fixed temperature.

If $Q_2 = 0$ (i.e. $w_{net} = Q_1$ or $\eta = 1.00$), the heat engine will produce net work in a complete cycle by exchanging heat with only one reservoir, thus violating the Kelvin-Planck Statement (Fig). Such a heat engine is called a perpetual motion machine of the second kind (PMM2).

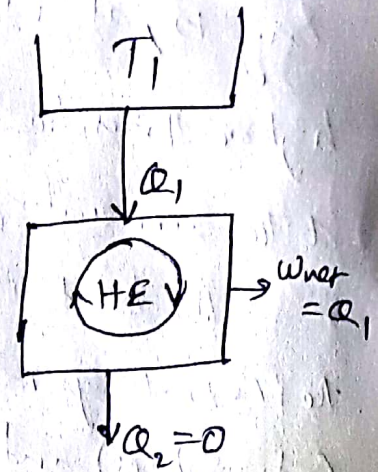


Fig: A PMM2

A PMM2 is impossible. Clausius Statement of the Second Law:—"It is impossible to

construct a device, which, operating in a cycle, will produce no effect other than the transfer of heat from a cooler to a hotter body".

Heat cannot flow of itself from a body at a lower temperature to a body at a higher temperature. Some work must be expended to achieve this.

Heat always flows from a body at a higher temperature to a body at a lower temperature. The reverse process never occurs spontaneously.

At first sight, kelvin-planck's and clausius's statements may appear to be unconnected, but it can easily be shown, that they are virtually two parallel statements of the second law, and are equivalent in all respects.

The Equivalence of the two statements will be proved if it can be shown that the violation of one statement implies the violation of the second, and vice versa.

a) Let us first consider a cyclic heat pump P which transfers heat from a low temperature reservoir (T_2) to a high temperature reservoir (T_1) with no other effect i.e. with no expenditure of work, violating clausius statement.

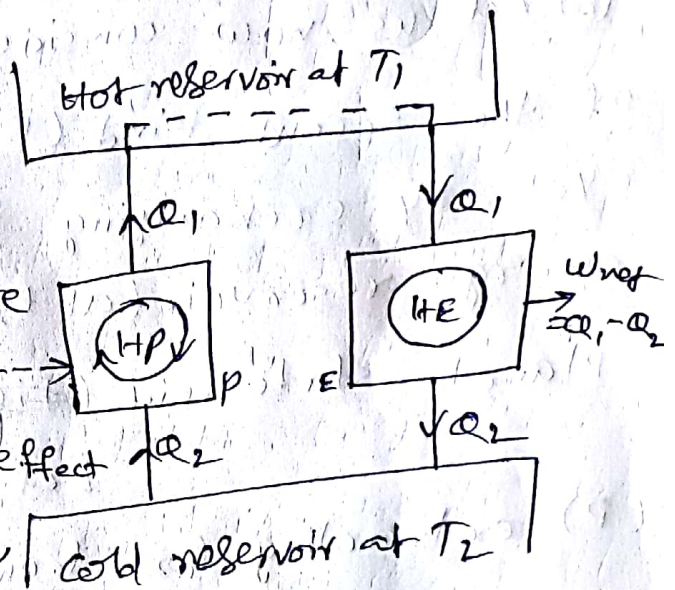


Fig: Violation of the clausius statement

Let us assume a cyclic heat engine E operating between the same thermal energy reservoirs, producing W_{net} in one cycle. The rate of working of the heat engine is such that it draws an amount of heat Q_1 from the hot reservoir equal to that discharged by the heat pump.

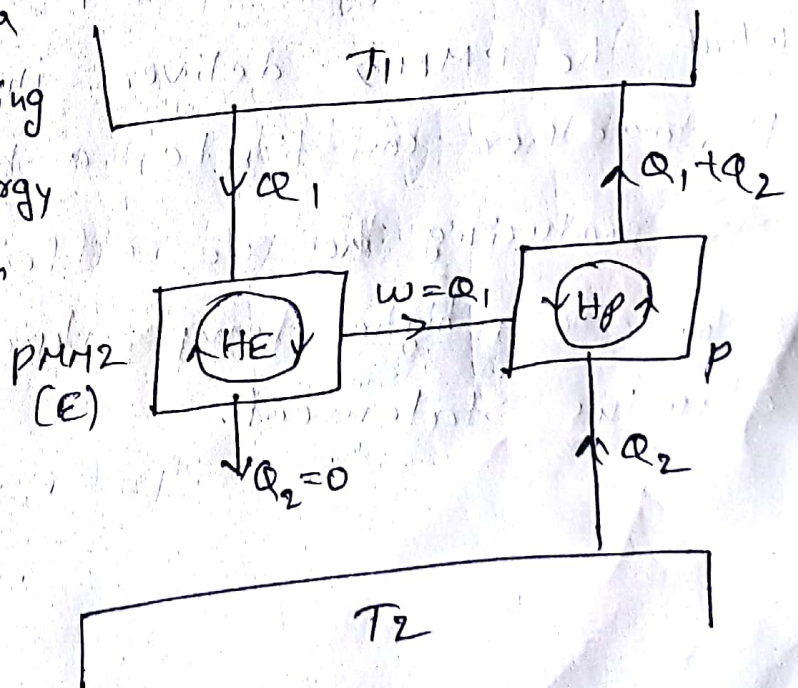


Fig: Violation of the kelvin-planck statement

the hot reservoir may be eliminated and the heat Q_1 discharged by the heat pump is fed to the heat engine. So we see that the heat pump P and the heat engine E acting together constitute a heat engine operating in cycles and producing net work while exchanging heat only with one body at a single fixed temperature. This violates the Kelvin-Planck statement.

(b) Let us now consider a perpetual motion machine of the second kind (E) which produces net work in a cycle by exchanging heat with only one thermal energy reservoir (at t_2) and thus violates the Kelvin-Planck statement.

Let us assume a cyclic heat pump (P) extracting Q_2 from a low temperature reservoir at t_2 and discharging heat to the high temperature reservoir at t_1 with the expenditure of work W equal to what the PMM-2 delivers in a complete cycle. So E and P together constitute a heat pump working in cycle and producing the sole effect of transferring heat from a lower to a higher temperature body, thus violating the Clausius statement.

It states that of all heat engine operating between a given constant temperature source and a given constant temperature sink, none has a higher efficiency than a reversible engine.

Let two heat engines E_A and E_B operate between the given source at temperature t_1 and the given sink at temperature t_2 as shown in Fig.

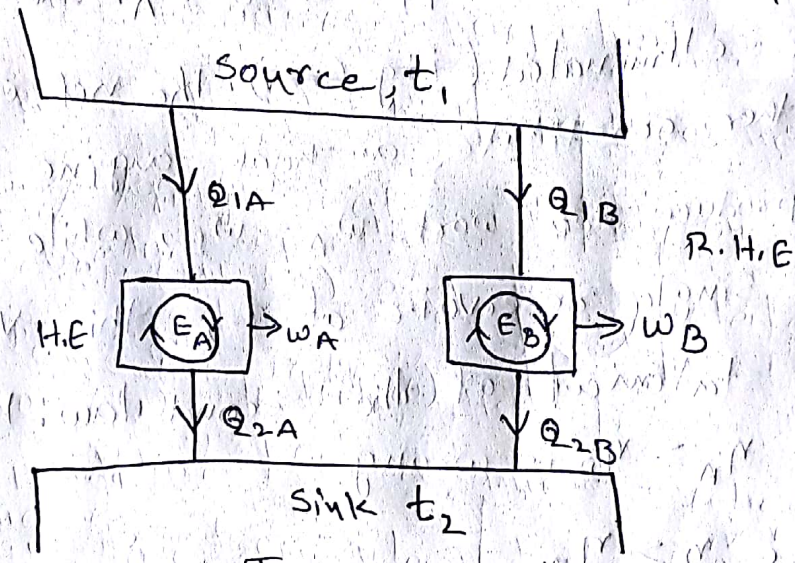


Fig. Two cyclic Heat Engines E_A and E_B operating between the same source and sink, of which E_B is Reversible

Let E_A be any heat engine and E_2 be any reversible heat engine, we have to prove that the efficiency of E_B is more than that of E_A . Let us assume that this is not true and $\eta_A > \eta_B$. Let the rates of working of the engines be such that

$$Q_{1A} = Q_{1B} = Q_1$$

$$\eta_A > \eta_B$$

$$W_A > W_B$$

$$W_A > W_B$$

Let E_B be reversed. Since E_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same, but their directions will be reversed, as shown in Fig 6.26. Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive that reversed heat engine E_B . Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by E_B may be supplied to E_A . The source may, therefore, be eliminated (~~Fig. 6.26~~). The net result is that E_A and E_B together constitute a heat engine which, operating in a cycle, produces net work $W_A - W_B$, while exchanging heat with a single reservoir at t_2 . This violates that Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.

Therefore $\eta_B \geq \eta_A$

Second Law of Thermodynamics

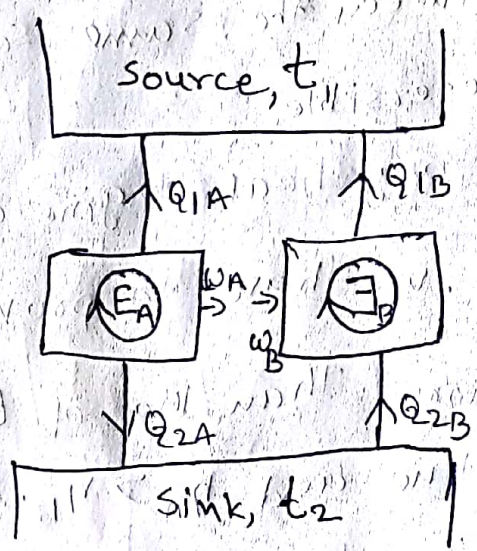


Fig. ~~6.26~~ E_B is Reversed

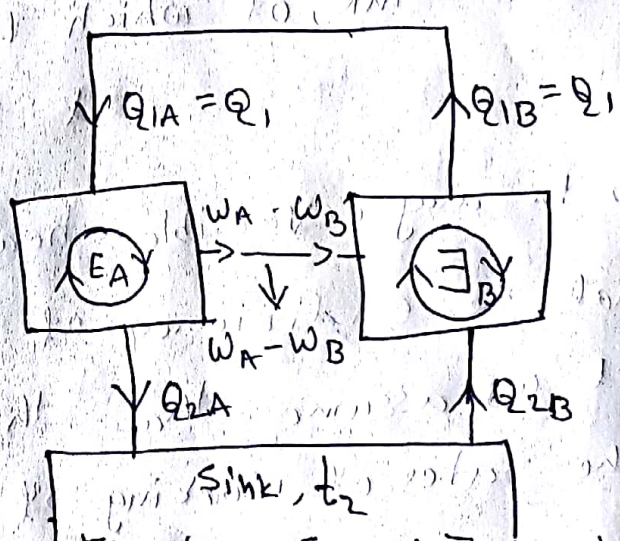


Fig. ~~6.27~~ E_A and E_B together violate the K-P statement.

FirstRanker.com OF CARNOT'S THEOREM (a)

FirstRanker's choice

www.FirstRanker.com

www.FirstRanker.com

(1) The efficiency of all reversible heat engines operating between the same temperature levels is the same.

(2) Let both the heat engines E_A and E_B (Fig 6.25) be reversible. Let us assume $\eta_A > \eta_B$. Similar to the procedure outlined in the preceding article, if E_B is reversed to run, say, as a heat pump using some part of the work output (W_A) of engine E_A , we see that the combined system of heat pump E_B and engine E_A , becomes a PMM2. So η_A cannot be greater than η_B . Similarly, if we assume $\eta_B > \eta_A$ and reverse the engine E_A , we observe that η_B cannot be greater than η_A .

Therefore $\eta_A = \eta_B$

(3) Since the efficiencies of all reversible heat engines operating between the same heat reservoirs are the same, the efficiency of a reversible engine is independent of the nature of amount of the working substance undergoing the cycle.

ABSOLUTE THERMODYNAMIC TEMPERATURE SCALE

The efficiency of any heat engine receiving heat Q_1 and rejecting heat Q_2 is given by

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad (6.18)$$

By the second law it is necessary to have a temperature difference ($t_1 - t_2$) to obtain work for any

www.FirstRanker.com

we know that the efficiency to all heat engines operating between the same temperature levels is the same. and it is independent of the working substance. Therefore, for a reversible cycle (Carnot cycle) the efficiency will depend solely upon the temperatures t_1 and t_2 , at which heat is transferred, or

$$\eta_{rev} = f(t_1, t_2) \quad (\text{Eq. 6.18})$$

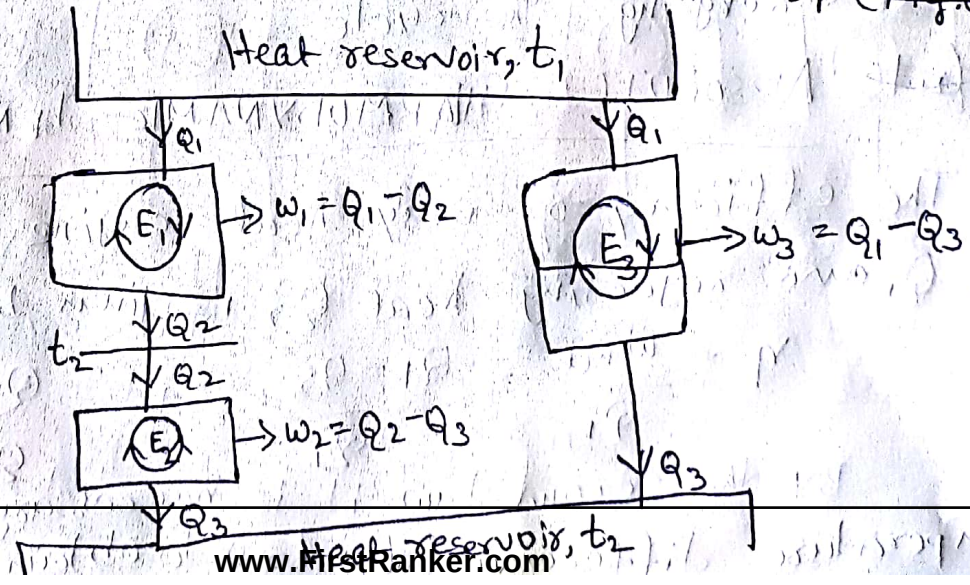
where f signifies some function of the temperature. From equation (6.18) and

$$1 - \frac{Q_2}{Q_1} = f(t_1, t_2)$$

In terms of new function $\frac{Q_1}{Q_2} = F(t_1, t_2) \quad (\text{Eq. 6.19})$

If some functional relationship is assigned between t_1, t_2 and Q_1/Q_2 , the equation becomes the definition of a temperature scale.

Let us consider two reversible heat engines E_1 receiving heat from the source at t_1 , and rejecting heat at t_2 to E_2 which, in turn, rejects heat to the sink at t_2 . (Fig. 6.22)



Now $\frac{Q_1}{Q_2} = F(t_1, t_2)$, $\frac{Q_2}{Q_3} = F(t_2, t_3)$

E_1 and E_2 together constitute another heat engine E_3 , operating between t_1 and t_3

$\therefore \frac{Q_1}{Q_3} = F(t_1, t_3)$

Now $\frac{Q_1}{Q_3} = \frac{Q_1/Q_2}{Q_2/Q_3}$

or $\frac{Q_1}{Q_2} = F(t_1, t_2) = \frac{F(t_1, t_3)}{F(t_2, t_3)}$ ~~(Equation)~~

The temperatures t_1 , t_2 and t_3 are arbitrary. t_3 is chosen. The ratio Q_1/Q_2 depends only on t_1 and t_2 and is independent of t_3 , so t_3 will drop out from the ratio on the right in equation ~~(Equation)~~. After it has been cancelled, the numerator can be written as $\phi(t_1)$, and the denominator as $\phi(t_2)$ where ϕ is another, unknown, function. Thus

$$\frac{Q_1}{Q_2} = F(t_1, t_2) = \frac{\phi(t_1)}{\phi(t_2)}$$

Since $\phi(t)$ is an arbitrary function, the simplest possible way to define the absolute thermodynamic temperature T is to let $\phi(t) = T$, as proposed by Kelvin. Then by definition

$$\frac{Q_1}{Q_3} = \frac{T_1}{T_2}$$

The absolute thermodynamic temperature scale is also known as the Kelvin scale. Two temperatures

relationship to each other as do the heats absorbed and rejected respectively by a Carnot engine operating between two reservoirs at these temperatures. The Kelvin temperature scale is, therefore, independent of the peculiar characteristics of any particular substance.

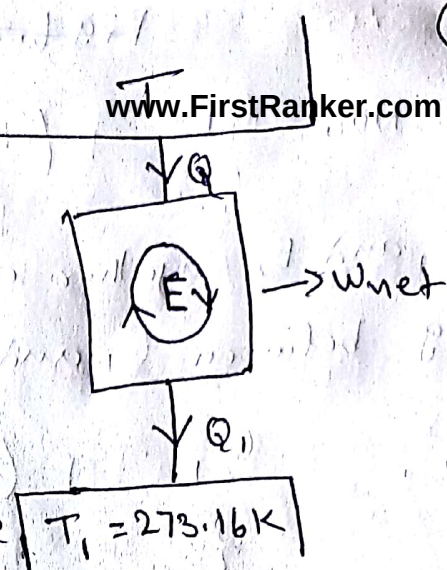


Fig. 6.29 Carnot Heat engine with Sink at Triple point of water.

The heat absorbed Q_1 and heat rejected Q_2 during the two reversible isothermal processes bounded by two reversible adiabatics in a Carnot engine can be measured. In defining the Kelvin temperature scale also, the triple point of water is taken as the standard reference point. For a Carnot engine operating between reservoirs at temperatures T and T_t , T_t being the triple point of water (Fig. 6.29), arbitrarily assigned the value 273.16 K,

$$\frac{Q_1}{Q_2} = \frac{T}{T_t}$$

$$T = 273.16 \frac{Q_1}{Q_2} \quad (\text{Eq. 6.29})$$

If this equation is compared with the equation given in ~~the text~~, it is seen that in the Kelvin scale, Q plays the role of thermometric property. The amount of heat supplied Q changes with change in temperature just like the thermodynamic temperature.

dynamic temperature scale has a definite zero point can be shown by imagining a series of

reversible engines, extending from a source at T_1 to lower, temperature (Fig. 6)

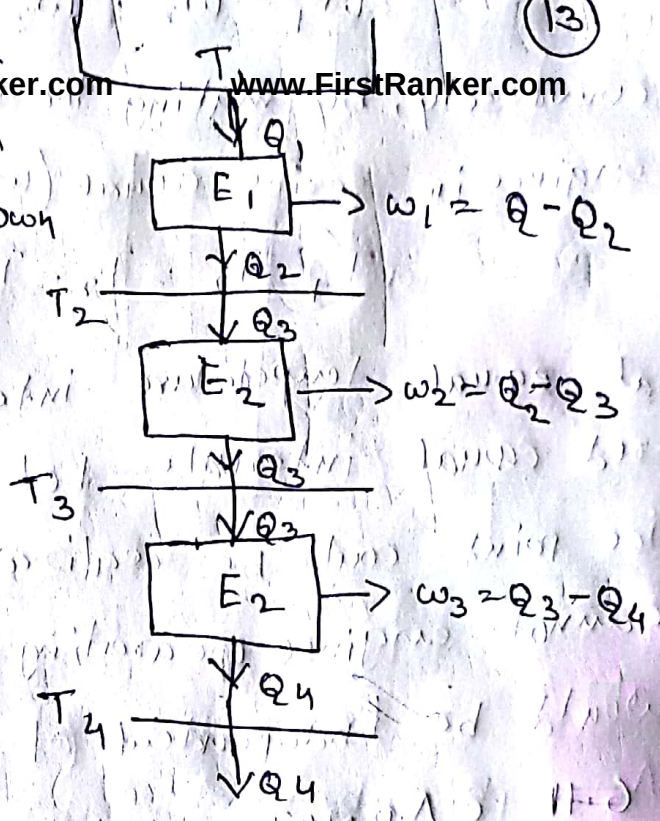


Fig. 6 Heat Engines operating in Series.

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \quad (1)$$

$$\frac{T_1 - T_2}{T_2} = \frac{Q_1 - Q_2}{Q_2}$$

or $T_1 - T_2 = (Q_1 - Q_2) \frac{T_2}{Q_2}$

similarly $T_2 - T_3 = (Q_2 - Q_3) \frac{T_3}{Q_3}$

$$= (Q_2 - Q_3) \frac{T_2}{Q_2}$$

$$T_3 - T_4 = (Q_3 - Q_4) \frac{T_4}{Q_4}$$

and so on

if $T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = \dots$, assuming equal temperature intervals or $Q_1 - Q_2 = Q_2 - Q_3 = Q_3 - Q_4 = \dots$

conversely, by making the work quantities performed by the engines in series equal ($w_1 = w_2 = w_3 = \dots$), we will get

$$T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = \dots$$

at equal temperature intervals. A scale having one hundred equal intervals between the steam point and the ice point could be realized by a series of one hundred Carnot engines operating as in Fig 6.30. Such a scale would be independent of the working substance.

CARNOT CYCLE

A reversible cycle is an ideal hypothetical cycle in which all the processes constituting the cycle are reversible. Carnot is a reversible cycle. For a stationary system, as in a piston and cylinder machine, the cycle consists of the following four successive processes (Fig-6.30);

(a) A reversible isothermal process in which heat Q_1 enters the system at t_1 reversibly from a constant temperature source at t_1 , when the cylinder cover is in contact with the diathermic cover A. The internal energy of the system increases.

From the First Law $Q_1 = U_2 - U_1 + w_{1-2}$

(for an ideal gas only, $U_1 = U_2$)

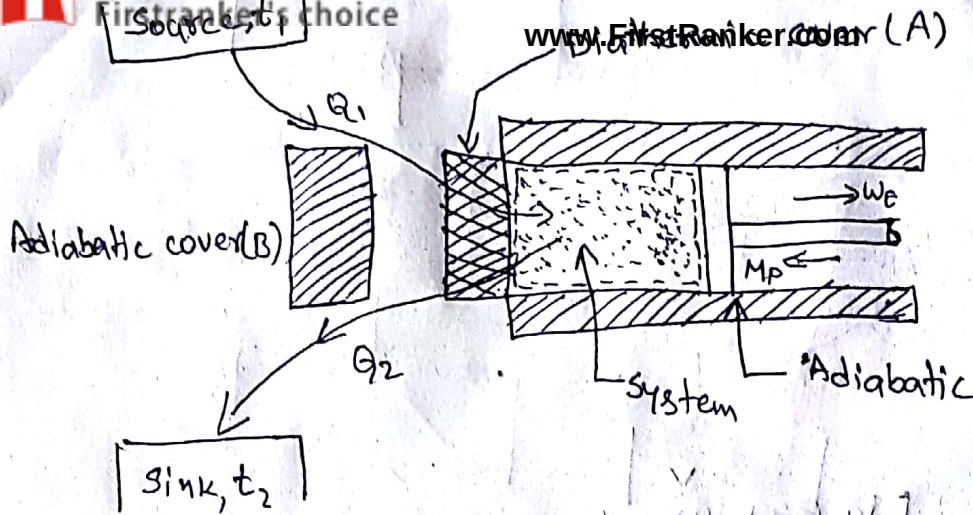


Fig. ~~1.2~~ Carnot Heat Engine - stationary system.

(b) A reversible adiabatic process in which the diathermic cover A is replaced by the Adiabatic cover B, and work W_E is done by the system adiabatically and reversibly at the expense of its internal energy, and the temperature of the system decreases from t_1 to t_2 using the first Law

$$0 = U_3 - U_2 + W_{2-3} \quad (1)$$

(c) A reversible isothermal process in which B is replaced by A, and heat Q_2 leaves the system at t_2 to a constant temperature sink at t_2 reversibly, and the internal energy of the system further decreases.

From the first Law, $-Q_2 = U_4 - U_3 - W_{3-4}$

$$U_3 = U_4$$

only for an ideal gas,

(d) A reversible adiabatic process in which B again replaces A, and work W_P is done upon the system reversibly, and adiabatically, and the internal energy of the system increases and the temperature rises from t_2 to t_1 .

Applying the first law

$$0 = U_1 - U_4 - W_{4-1} \quad (2)$$

Two reversible isotherms and two reversible adiabatics constitute a Carnot cycle which is represented in P-V

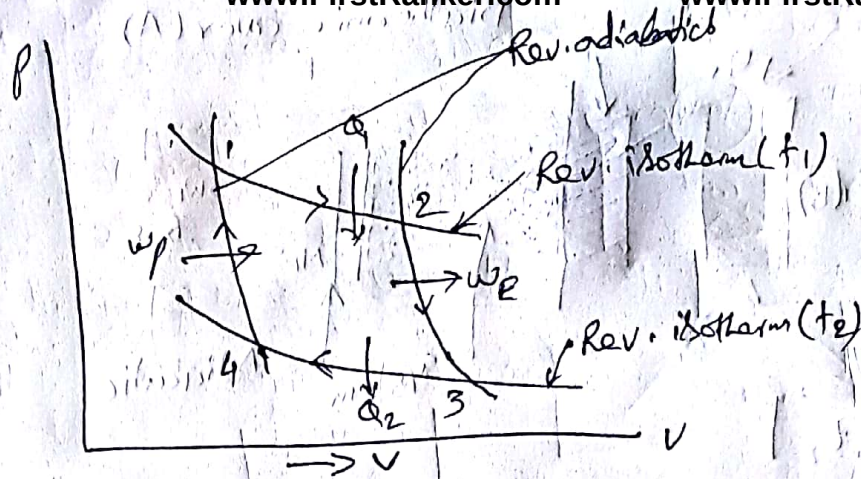


Fig. ~~6.14~~ Carnot cycle

Summing up Eqs. (6.14) to (6.17),

$$Q_1 - Q_2 = (w_{1-2} + w_{2-3}) - (w_{3-4} + w_{4-1})$$

or

$$\sum_{\text{cycle}} Q_{\text{net}} = \sum_{\text{cycle}} w_{\text{net}}$$

A cyclic heat engine operating on the Carnot cycle, is called a Carnot heat engine.

For a steady flow system, the Carnot cycle is represented as shown in Fig. ~~6.15~~. Here, heat Q_1 is transferred to the system reversibly and isothermally at t_1 in the heat exchanger A, work w_T is done by the system reversibly and adiabatically in the turbine (B), then heat Q_2 is transferred from the system reversibly and isothermally at t_2 in the heat exchanger (C) and then work w_P is done upon the system reversibly and adiabatically by the pump (D). To satisfy the conditions for the Carnot cycle, there must not be any friction or heat transfer in the pipelines through which the working fluid flows.

Example 6.1 A cyclic heat engine operates between a source temperature of 800°C and a sink temperature of 30°C . What is the least rate of heat rejection per kW net output of the engine?

Solution For a reversible engine, the rate of heat rejection will be minimum (Fig. 6.1)

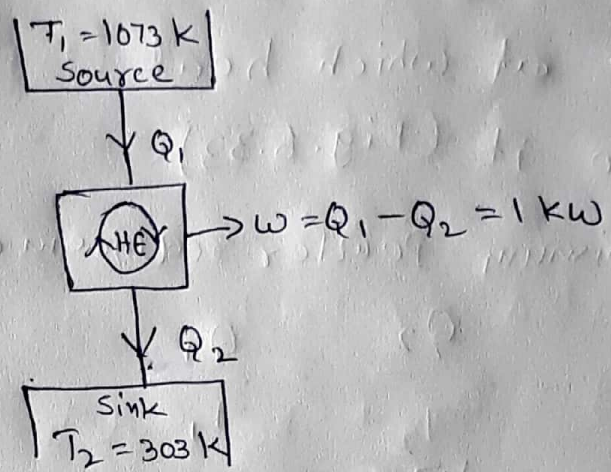


Fig. 6.1

$$\eta_{\max} = \eta_{\text{rev}} = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{30 + 273}{800 + 273}$$

$$= 1 - 0.282 = 0.718$$

Now

$$\frac{W_{\text{net}}}{Q_1} = \eta_{\max} = 0.718$$

$$Q_1 = \frac{1}{0.718} = 1.392 \text{ kW}$$

Now

$$Q_2 = Q_1 - W_{\text{net}} = 1.392 - 1 = 0.392 \text{ kW}$$

This is the least rate of heat rejection.

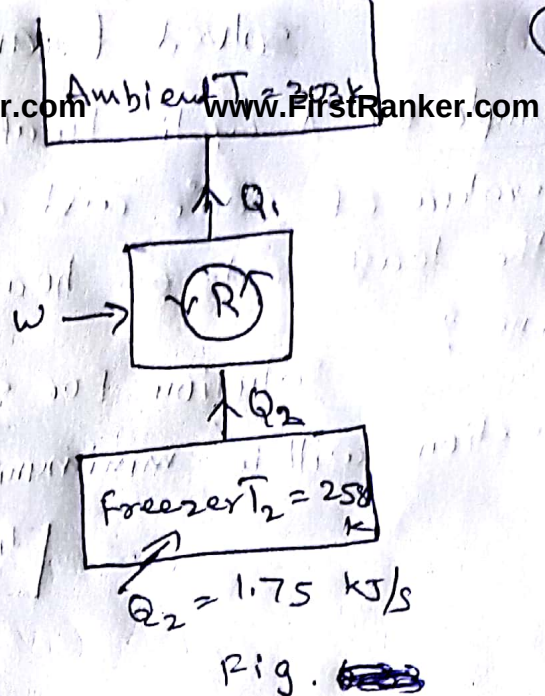
Example 6.2 A domestic food freezer maintaining a temperature of -15°C the ambient air temperature is 30°C . If heat leaks into the freezer at the continuous rate of 1.75 kJ/s what is the least power necessary to pump this heat out continuously?

$$T_2 = -15 + 273 = 258 \text{ K}$$

Ambient air temperature

$$T_1 = 30 + 273 = 303 \text{ K}$$

The refrigerator cycle removes heat from the freezer at the same rate at which heat leaks into it (Fig. 6.33)



For minimum power requirement

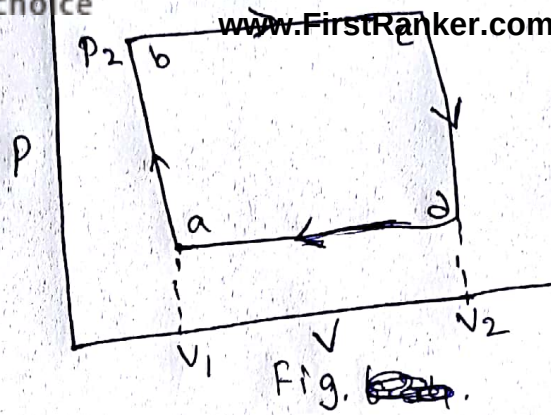
$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

$$\therefore Q_1 = \frac{1.75}{258} \times 303 = 2.06 \text{ kJ/s}$$

$$\therefore W = Q_1 - Q_2 = 2.06 - 1.75 = 0.31 \text{ kJ/s} = 0.31 \text{ kW}$$

Example 6.3 An ideal gas cycle is represented by a rectangle on a $p-v$ diagram. If p_1 and p_2 are the lower and higher pressures; and v_1 and v_2 the smaller and larger volumes, respectively then (a) calculate the work done per cycle, (b) indicate which parts of the cycle involve heat flow into the gas, (c) show that

$$\eta = \frac{\gamma - 1}{\gamma \frac{p_2}{p_1} + \frac{v_1}{v_2 - v_1}} \quad \text{if heat capacities are constant}$$



(a) $w = \text{area of the cycle (Fig. 6.34)}$
 $= (P_2 - P_1)(V_2 - V_1) \text{ Aug.}$

(b) processes ab and bc

Heat absorbed by 1 mole of gas in one cycle

$$Q = Q_{ab} + Q_{bc} = C_v(T_b - T_a) + C_p(T_c - T_b)$$

Now, $T_a = T_b \frac{P_1}{P_2}$ and $V_1 = RT_b$

$$T_c = T_b \frac{V_2}{V_1}, T_b = \frac{P_2 V_1}{R}$$

$$Q = C_v T_b \left[1 - \frac{P_1}{P_2} \right] + C_p T_b \left[\frac{V_2}{V_1} - 1 \right]$$

$$= \frac{P_2 V_1}{R} \left[C_p \frac{P_2 - P_1}{P_2} + C_p \frac{V_2 - V_1}{V_1} \right] \text{ Aug.}$$

Fundamentals of partial differentiation :- If three variables are represented by x, y, z . Their functional relationship may be expressed in the following form

$$f(x, y, z) = 0$$

$$x = x(y, z)$$

$$y = y(x, z)$$

$$z = z(x, y)$$

Let x is a function of two independent variables y and z

$$x = x(y, z)$$

Now the differential of the dependent variable x is given by

$$dx = \left(\frac{\partial x}{\partial y} \right)_z dy + \left(\frac{\partial x}{\partial z} \right)_y dz \quad \text{--- (1)}$$

$$dx = M dy + N dz \quad \text{--- (2)}$$

Partial differentiation of M and N

w.r.t. z and y , respectively, gives

$$M = \left(\frac{\partial x}{\partial y} \right)_z$$

$$N = \left(\frac{\partial x}{\partial z} \right)_y$$

$$\frac{\partial M}{\partial z} = \frac{\partial^2 x}{\partial y \partial z}$$

$$\frac{\partial N}{\partial y} = \frac{\partial^2 x}{\partial y \partial z}$$

$$\therefore \frac{\partial M}{\partial z} = \frac{\partial N}{\partial y} \quad \text{--- (4)}$$

dx is a perfect differential when Eqn (4) is satisfied for any function of x .

$$dy = \left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial y}{\partial z} \right)_x dz$$

$$z = z(x, y)$$

$$dz = \left(\frac{\partial z}{\partial x} \right)_y dx + \left(\frac{\partial z}{\partial y} \right)_x dy$$

$$dy = \left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial y}{\partial z} \right)_x \left[\left(\frac{\partial z}{\partial x} \right)_y dx + \left(\frac{\partial z}{\partial y} \right)_x dy \right]$$

$$= \left[\left(\frac{\partial y}{\partial x} \right)_z + \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y \right] dx + \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial y} \right)_x dy$$

$$= \left[\left(\frac{\partial y}{\partial x} \right)_z + \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y \right] dx + dy$$

$$\left(\frac{\partial y}{\partial x} \right)_z + \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = 0$$

$$\left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = - \left(\frac{\partial y}{\partial x} \right)_z$$

$$\therefore \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial z}{\partial x} \right)_y \left(\frac{\partial y}{\partial z} \right)_x = -1$$

In terms of p, v, T , the following relation holds good

$$\left[\left(\frac{\partial p}{\partial v} \right)_T \left(\frac{\partial T}{\partial p} \right)_v \left(\frac{\partial v}{\partial T} \right)_p = -1 \right]$$

The first law applied to a closed system undergoing a reversible process states that

$$dq = du + pdv$$

$$Tds = du + pdv$$

$$ds = \left(\frac{dq}{T} \right)_{rev}$$

$$du = Tds - pdv \quad \text{--- (1)}$$

Enthalpy is given by

$$h = u + pv$$

$$dh = du + d(pv)$$

$$= du + pdv + vdp$$

$$= Tds - pdv + pdv + vdp \quad (\text{from eqn (1)})$$

$$dh = Tds + vdp \quad \text{--- (2)}$$

Gibbs function is given by

$$g = h - TS$$

$$dg = dh - d(TS)$$

$$= dh - Tds - SdT$$

$$= Tds + vdp - Tds - SdT \quad (\text{from eqn (2)})$$

$$= vdp - SdT \quad \text{--- (3)}$$

Helmholtz function is given by

$$f = u - TS$$

$$df = du - d(TS)$$

$$= du - Tds - SdT$$

$$= Tds - pdv - Tds - SdT \quad (\text{from (1)})$$

$$df = -pdv - SdT \quad \text{--- (4)}$$

function and properties then du , dh , dg , df are exact differentials.

$\therefore$ Apply the test for exactness

If $dx = Mdy + Ndz$

$x = x(y, z)$

$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$

$dx = Mdy + Ndz$

If $dz = Mdx + Ndy$ is a property, then

$z = z(x, y)$

$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$

$dz = Mdx + Ndy$

$M = \left(\frac{\partial z}{\partial x}\right)_y$

$N = \left(\frac{\partial z}{\partial y}\right)_x$

$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial^2 z}{\partial x \partial y}\right), \left(\frac{\partial N}{\partial x}\right)_y = \left(\frac{\partial^2 z}{\partial x \partial y}\right)$

$\therefore \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

Consider Eqn (1), $du = Tds - pdv$
 $dz = Mdx + Ndy$

$M = T$
 $x = S$

$N = -p$

$y = V$

$\therefore \left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V$ — (5)

Consider Eqn (2), $dh = Tds + v \cdot dp$
 $dz = Mdx + Ndy$

$M = T$

$x = S$

$N = v$

$y = p$

$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$

$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial v}{\partial S}\right)_p$ — (6)

Compare with $dz = Mdx + Ndy$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T \quad \text{--- (7)}$$

$$M = V$$

$$N = P$$

$$M = -S$$

$$y = T$$

Consider Eqn (4), $df = -pdv - SdT$
 $dz = Mdx + Ndy$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

$$-\left(\frac{\partial P}{\partial T}\right)_V = -\left(\frac{\partial S}{\partial V}\right)_T$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \quad \text{--- (8)}$$

$$M = -P$$

$$N = V$$

$$M = -S$$

$$y = T$$

Rewrite the eqns (5), (6), (7) & (8) are

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

These four equations are known as Maxwell's relations and are used to determine relationship between the properties P, V, T , and S for any equilibrium state.

Clausius inequality, Entropy, entropy increase, Energy equation, availability and irreversibility - Thermodynamic potentials, Gibbs and Helmholtz functions, Maxwell relations, Elementary treatment of the third law of thermodynamics.

Available energy

It is not possible to convert all the heat absorbed by a system into work.

At T_1 heat Q_1 is absorbed.

By the engine (say $Q_1 = 70 \text{ kJ}$).

only some heat is utilized to convert ~~to~~ work and the some heat is rejected to atmosphere.

$$Q_1 = AE + UE$$

$$W_{\max} = AE = Q_1 - AE$$

If $Q_1 = 70 \text{ kJ}$, $W = 50 \text{ kJ}$ then $Q_2 = 20 \text{ kJ}$ rejects to atmosphere.

Therefore 50 kJ is the available energy and 20 kJ is the unavailable energy.

The area 1-2-3-4 represents the available energy.

The shaded area A-3-B-A represents the energy, which is discarded to the ambient atmosphere, and this quantity of energy cannot be converted into work and is called unavailable energy.

Irreversibility: - The actual work which a system does is always less than the idealized reversible work, and the difference between the two is called the irreversibility of the process.

Thus, Irreversibility, $I = W_{\max} - W$

This is also sometimes referred to as 'degradation' or 'dissipation'.

Available energy → is the maximum portion of energy which could be converted into useful work is known as available energy.

Entropy is a function of a quantity of heat which shows the possibility of conversion of that heat into work. The increase in entropy is small when heat is added at a high temperature and is greater when heat addition is made at lower temperature. Thus for maximum entropy, there is a minimum availability for conversion into work and for minimum entropy there is maximum availability for conversion into work.

$$ds = \left(\frac{\delta Q}{T} \right)_R$$

Entropy change, $ds = \frac{\text{Heat change } (Q)}{\text{Absolute temperature } (T)}$

or $S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_R$
(change of entropy)

Unit $\rightarrow S = \frac{J}{K} = J/K$

Third Law of Thermodynamics - States "when a system is at zero absolute temperature, the entropy of system is zero".

(or)

"The entropy of all perfect crystalline solids is zero at absolute zero temperature".

Entropy $\rightarrow$ May also be defined as the thermal property of a substance which remains constant when substance is expanded or compressed adiabatically in a cylinder.

Properties of pure Substances
pure substance $\rightarrow$ is a substance of constant chemical composition throughout its mass. It may exist in one or more phases.

It is a homogeneous and invariable chemical composition irrespective of phase.

Ex: ①. A mixture of ice, water, steam is an example of pure substance. Chemical composition of three phases is same.

②. Air is a pure substance.

Air is a mixture of N_2 and O_2 . In gaseous phase air may be heated expanded or compressed without altering its chemical composition. Hence air can be considered as a pure substance in the gaseous phase since it has invariable chemical composition.

③. mixture of CO_2 and O_2 can be considered as pure substance so long as there is no chemical reaction.

Phase Transformation:- A pure substance may exist in different aspects known as phases.

phase transformation means, change of a state of substance from one state to another state.

Ice $\xrightarrow{\text{solid}}$ water $\xrightarrow{\text{liquid}}$ vapour $\xrightarrow{\text{gas}}$

liquid $\rightarrow$ solid (freezing or solidification)

Solid $\xrightarrow{\text{to}}$ liquid $\rightarrow$ melting (fusion of ice)

liquid $\xrightarrow{\text{to}}$ vapour $\rightarrow$ vaporization

vapour $\xrightarrow{\text{to}}$ liquid $\rightarrow$ condensation.

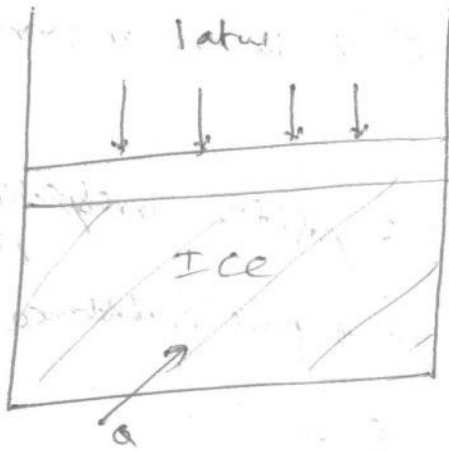


Fig: Heating of Ice at const. pre. of 1 atm

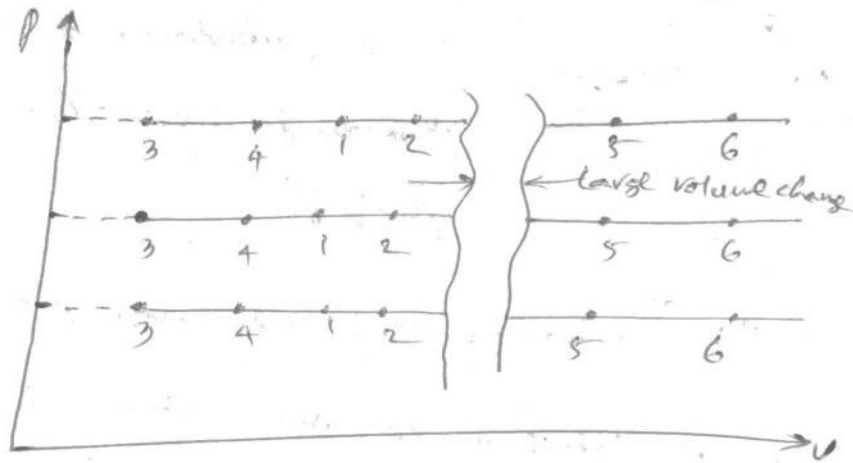


Fig: change in the volume of water during heating at constant pressure.

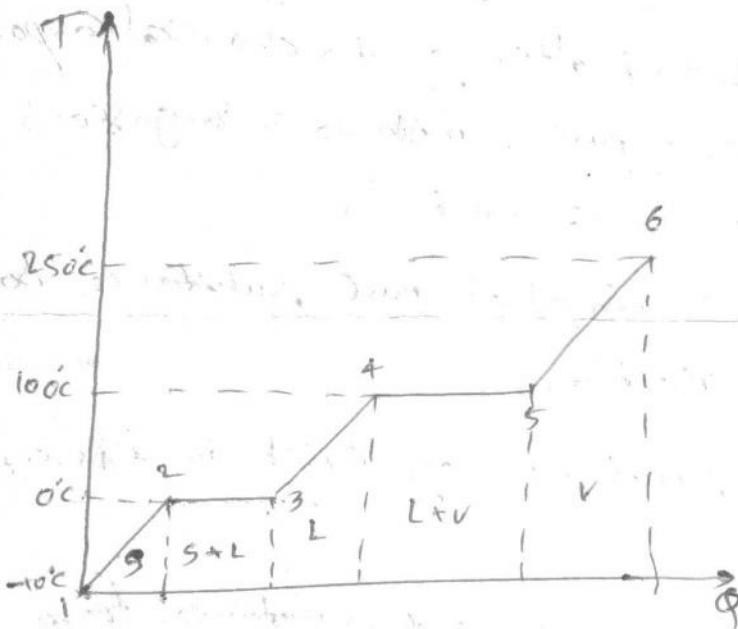


Fig: T-Q diagram for phase change of Ice into steam

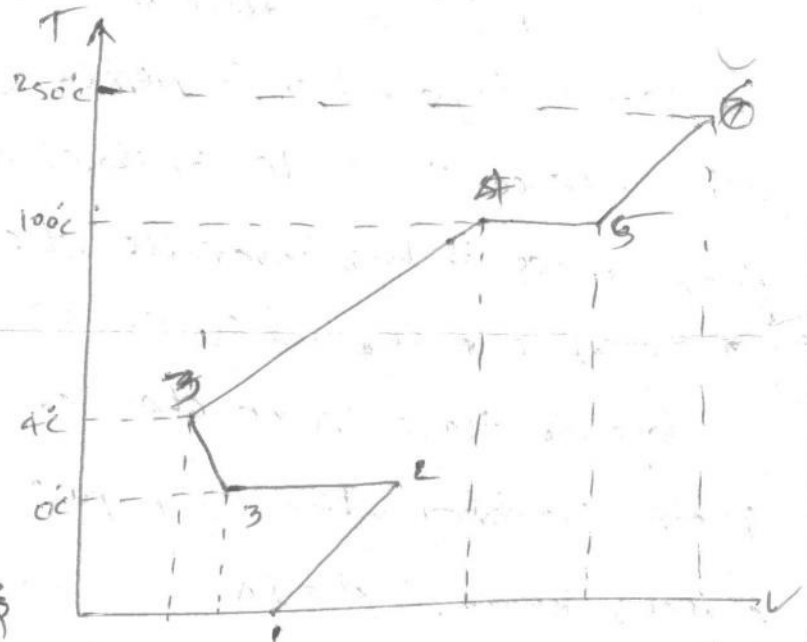


Fig: T-V diagram for phase change of ice into steam.

point 2: Saturated solid state (because a solid ~~state~~ ^{can} change into liquid at const. pre & temp. from state 2).

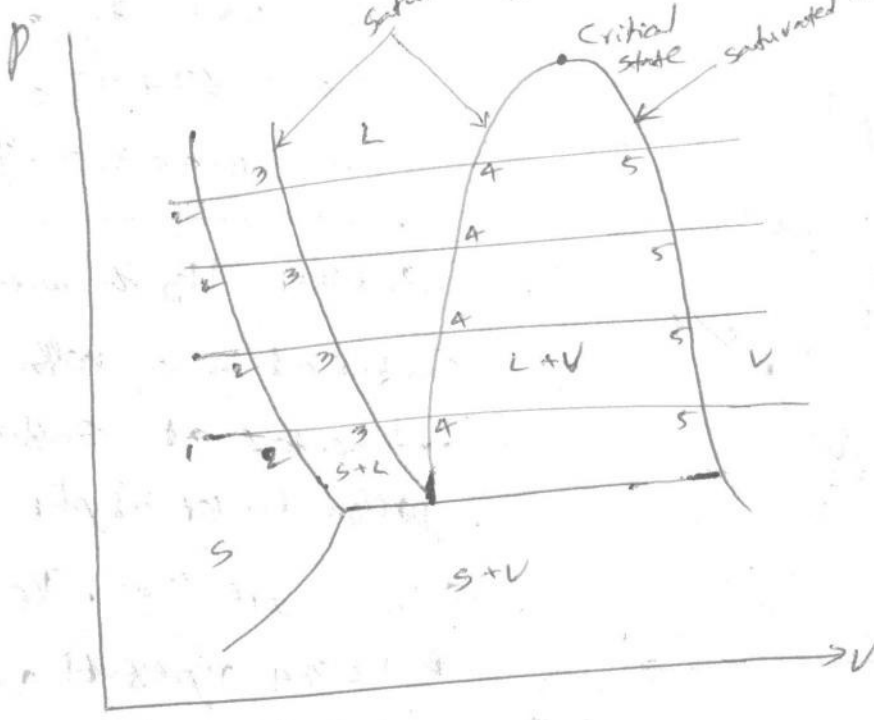
State 3: Saturated liquid state

In state 3, the liquid is saturated with respect to solidification.

In state 4, the liquid is saturated w.r.t. vaporization.

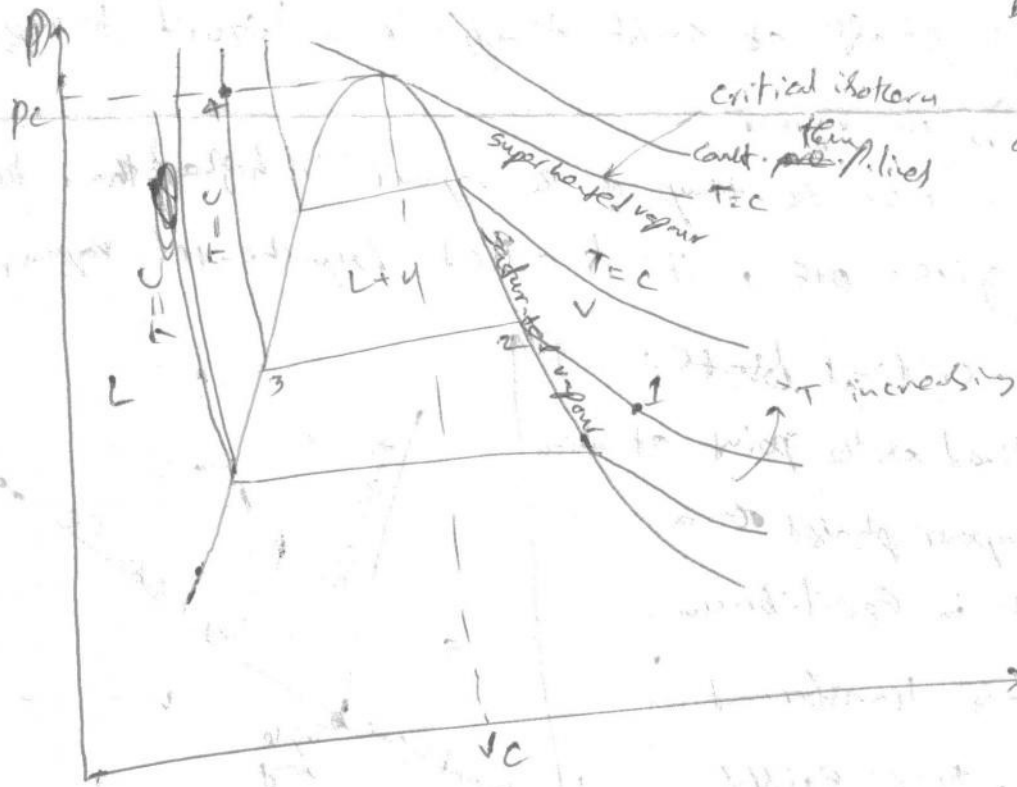
State 5: Saturated vapour state (because from state 5 the vapour can condense into liquid without a change of pre & temp).

A saturation state is a state from which phase change may occur without a change of pre & temp.



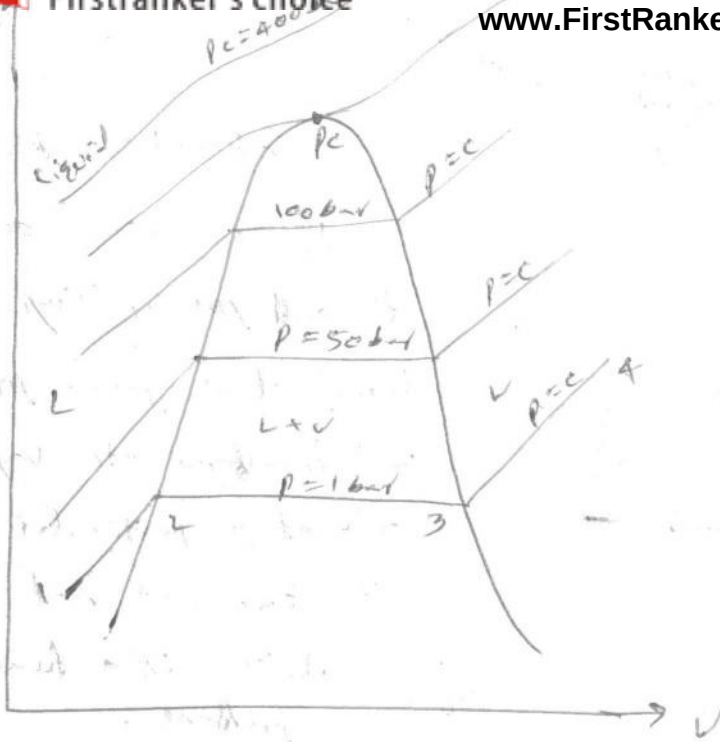
Saturation temp:-
phase change occurs at const. pres & temp. A pure liquid at a given pressure will transform into vapour only at a particular temp, known as Sat. temp, which is a function of pressure.

Fig: P-v diagram of pure substance other than water, whose volume increases on melting.



Sat. pres:- If the temp is fixed, the liquid will boil (or condense) only at a particular pressure, called the Sat. pres, which is a function of temp.

P-v diagram for water



$$P_c = 221.1 \text{ bar}$$

$$T_c = 374.15^\circ\text{C}$$

$$V_c = 0.00317 \text{ m}^3/\text{kg}$$

Consider 1 kg of water
Contained in a vessel
and heated at constant
pressure. let us plot the
constant pressure line. The line
1-2-3-4 represents a constant
pressure line at 1 atm.

Fig. 1.1 P-v diagram for water.

1-2 → Heating of liquid to sat. temp.

2-3 → change of phase at const temp. from liquid to vapour.

3-4 → Superheating of vapour.

Superheated vapour — when the temp. of the vapour is higher than the saturation temp. at a given pressure, it is called superheated vapour.

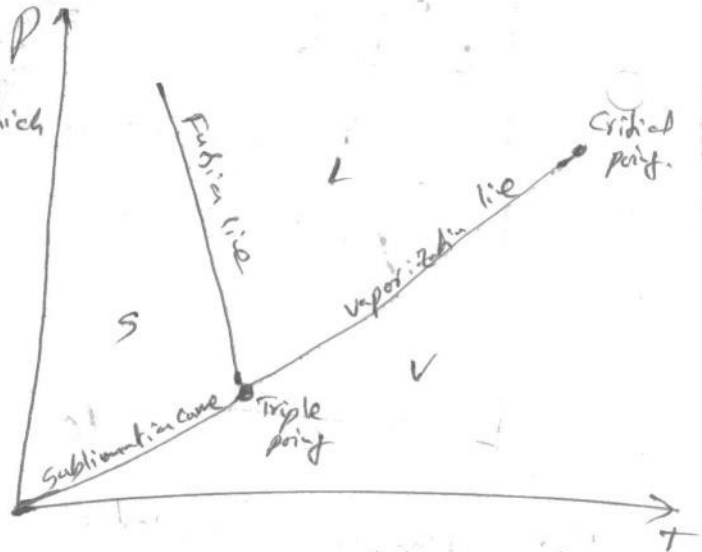
Triple point at critical state — P

Triple point is defined as the point at which
solid, liquid and vapour phases of a
substance coexist in equilibrium.

Ex — Consider phase transformation
in ice to steam, there exists

Fusion line, vaporization line,

sublimation line. If we plot the
variation of pressure with temp. the curve
shown in fig. will be obtained.



Triple point of water is at 4.88 mm Hg
and 273.16 K

(Cont. pre)
1 2 3 4 5 6 represent an isobar

4 $\rightarrow$ sat. liquid point
5 $\rightarrow$ sat. vapour point.

For different pressures, we will get again another set of sat. liquid point & sat. vapour point.
Curve joining all the saturated states is called saturation curve.

Most often liquid vapour transformation is of interest and Fig shows the liquid, vapour, & transition zones only.
With reference to T-S diagram the state of a substance can be divided into five distinguished zones as indicated below.

- Subcooled liquid
- Saturated liquid
- wet vapour
- Saturated vapour
- Superheated vapour

Subcooled liquid:-

It is represented by point A

on T-S diagram. Here the

Temp of liquid $<$ saturation temp corresponding to that pressure.

This state is known as Subcooled state.

Degree of subcooling = sat. temp - Subcooled temp.

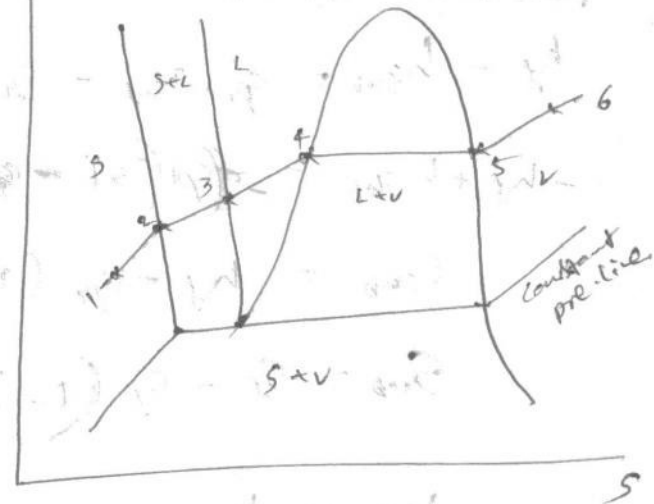
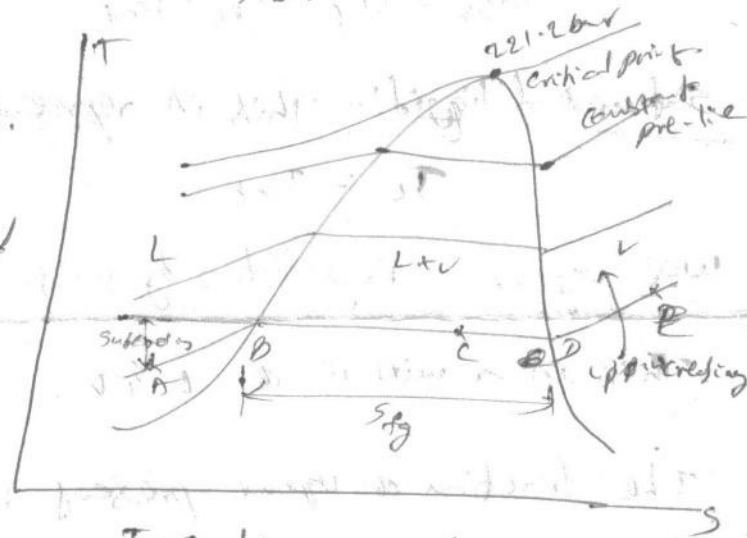


Fig: Equilibrium diagram for a pure substance



T-S diagram for water

At particular pressure,

$S_f \rightarrow$ specific entropy of saturated water

$S_g \rightarrow$ " " " " Saturated vapour

$S_{fg} \rightarrow$ Entropy change during the phase change (From L $\rightarrow$ V)

$$S_{fg} = S_g - S_f$$

The value of S_{fg} decreases as the pressure increases, and becomes zero at the critical point.

In the subcooled region $h = c_{pl} T$ if less than the enthalpy of a

saturated liquid.

$$h = c_{pl} T$$

h_f = enthalpy of sat. liquid

h_{sub} → enthalpy of subcooled liquid

$$h_f - h_{sub} = c_{pl} T_{sat} - c_{pl} T_{sub}$$

$$-h_f + h_{sub} = -c_{pl}(T_{sat} - T_{sub})$$

$$h_{sub} = h_f - c_{pl}(T_{sat} - T_{sub})$$

$$S_{sub} = S_f - c_p \left(\ln \frac{T_{sat}}{T_{sub}} \right)$$

$$v_{sub} = v_{sat}$$

$$u_{sub} = u_f - c_{vl}(T_{sat} - T_{sub})$$

Saturated liquid — This is represented by point B on the T-S diagram.

$$T_L = T_{sat}$$

Wet vapour — Represented by point C. At this point $T_{emp} = T_{sat}$ and $p_{emp} = p_{sat}$.

This is a mixture of L & V.

The fraction of vapour present in kg of mixture of liquid and vapour is known as dryness fraction (x) or quality of steam.

$$x = \frac{\text{mass of vapour}}{\text{mass of vapour} + \text{mass of liquid}} = \frac{m_v}{m_v + m_L}$$

at point B, we have 100% liquid and $x = 0$

at point D, we have 100% vapour, so $x = 1$

at point C, the dryness fraction is in between 0 and 1,

— Let V be the total volume of a liquid vapour mixture

V_f → volume of sat. liquid

V_g → vol. of sat. vapour

$$\Rightarrow v_f = m - m_g$$

$v_f \rightarrow$ sp. vol. of sat. liquid

$v_g \rightarrow$ sp. vol. of sat. vapour

$v \rightarrow$ sp. vol. of mixture

$$m v = m_f v_f + m_g v_g$$

$$= (m - m_g) v_f + m_g v_g$$

$$v = \left(1 - \frac{m_g}{m}\right) v_f + \frac{m_g}{m} v_g$$

$$v = (1 - x) v_f + x v_g$$

$$= v_f + x (v_g - v_f)$$

$$\therefore v = v_f + x v_{fg}$$

$$h = h_f + x h_{fg}$$

$$s = s_f + x s_{fg}$$

$$u = u_f + x u_{fg}$$

Similarly

$v_f, v_{fg}, u_f, u_{fg}, h_f, h_{fg}, s_f, s_{fg}$

are the saturation properties at the given pressure and temp.

$\rightarrow$ Ref values find from Steam tables.

Saturated vapour $\rightarrow$ This is represented by point D where we have completely saturated vapour. It is to be noted that during this phase the saturation temp remains same from B to D.

Superheated Vapour (E) $\rightarrow$ Temp. of substance $>$ Sat. temp.

$$\text{degree of superheat} = \text{superheat temp} - \text{sat. temp.}$$

The enthalpy of a superheated vapour is nothing but energy of the vapour when its temp is at T_{sup} . so the change in enthalpy in constant pressure process is given by

$$h_{sup} - h_g = c_p (T_{sup} - T_{sat})$$

$$h_{sup} = h_g + c_p (T_{sup} - T_{sat})$$

Similarly change in entropy $s_{sup} - s_g = c_p \ln \frac{T_{sup}}{T_{sat}}$

$$s_{sup} = s_g + c_p \ln \frac{T_{sup}}{T_{sat}}$$

relation was obtained.

$$TdS = dh - vdp$$

$$dq = dh + dw$$

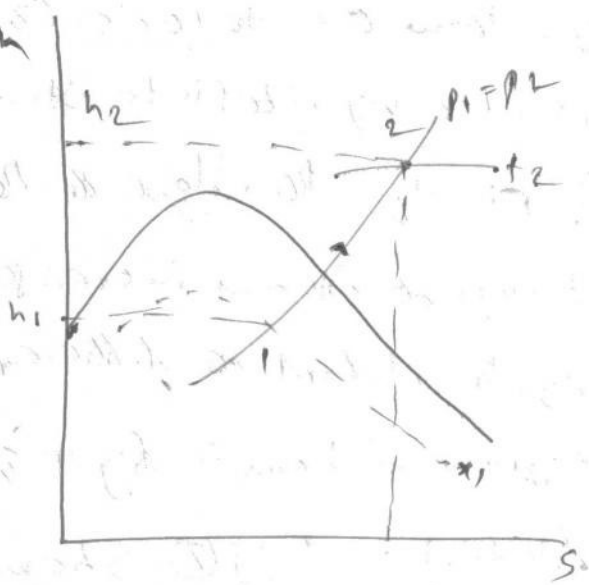
$$TdS = dh - vdp$$

$$\left(\frac{\partial h}{\partial s}\right)_p = T$$

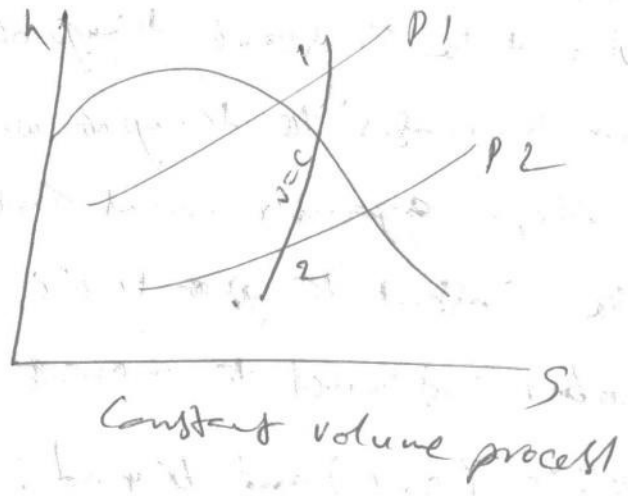
This Eqn. found the basis of the $h-s$ diagram of pure substance, also called the Mollier diagram. The slope at an isobar on the $h-s$ coordinates equal to the absolute temp. $(t_{sat} + 273)$ at that pressure. If the temp. remains constant the slope will remain constant. If the temp. increases, the slope of the isobar will increase.

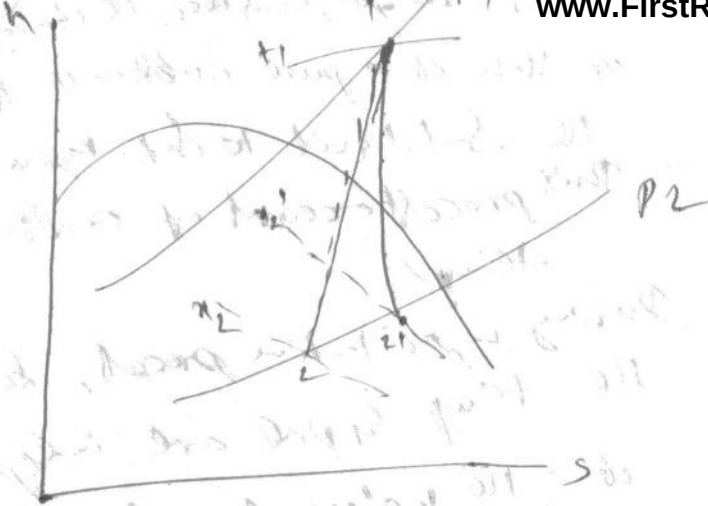
Consider the heating of a system of ice at -5°C to steam at 250°C , the pressure being maintained constant at 1 atm. The slope of the isobar of 1 atm on the $h-s$ coordinates (Fig 1) first increases as the temp of the ice increases from -5°C to 0°C (1-2). It then remains constant as ice melts into water at the constant temp of 0°C (2-3). The slope of the isobar again increases as the temp of water rises from 0°C to 100°C (3-4). The slope again remains constant as water vaporizes into steam at the constant temp of 100°C (4-5). Finally, the slope of the isobar continues to increase as the temp of steam increases to 250°C (5-6) and beyond. Similarly, the isobars at different pressures can be drawn on the $h-s$ diagram as shown in Fig 1 & 2. States 2, 3, 4, 5 are saturation states. Fig 2 shows the phase equilibrium diagram of a pure substance on the $h-s$ coordinates.

Fig (9) is the $h-s$ or the Mollier diagram indicating the liquid and vapour phases. and so the slope of the isobar also increases. Hence, the constant pressure lines diverge from one another, and the critical isobar is a tangent at the critical point, as shown. In the vapour region, the states of equal slope at various pressures are joined by lines, as shown, which are the constant temp. lines. Although the slope of an isobar remains continuous beyond the saturated vapour line, the isotherm bends towards the right and its slope decreases asymptotically to zero, because in the ideal gas region it becomes horizontal and the constant enthalpy implies constant temperature. At a particular pressure, h_f is the specific enthalpy of saturated water, h_g is that of saturated vapour, and $h_{fg} = (h_g - h_f)$ is the latent heat of vaporization at that pressure. As the pressure increases, h_{fg} decreases, and at the critical pressure, h_{fg} becomes zero.

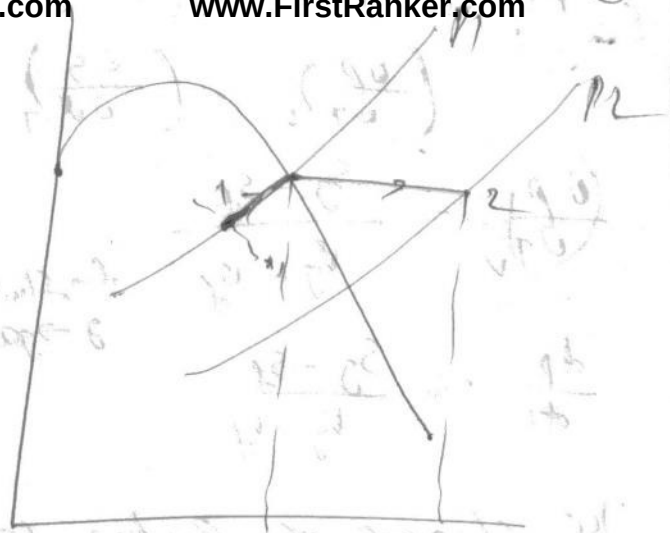


Const. pres. process





Adiabatic process



Const Temp (Isothermal)

Method of Determining the dryness fraction:-

There are four methods of determining the dryness fraction of steam experimentally. They are follows.

- 1). Bucket or Barrel Calorimeter
- 2). Separating Calorimeter
- 3). Throttling Calorimeter
- 4). Combined separating and throttling Calorimeter.

Clausius - Clapeyron Equation:- The Clapeyron equation gives us valuable data relating to the change of phase of a pure substance. This equation presents a relationship between the saturation pressure, temperature and enthalpy of evaporation and the specific volume of phases of a substance.

$$\int \frac{dp}{dT} = \frac{h_{fg}}{T v_{fg}}$$

Integrated $\frac{dp}{dT} = \frac{\Delta h}{T \Delta v}$

for Maxwell relation

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{s_g - s_f}{v_g - v_f} \quad \begin{matrix} \text{fluid} \\ \text{state} \end{matrix}$$

$$\frac{dp}{dT} = \frac{s_g - s_f}{v_g - v_f}$$

The change of entropy ($s_g - s_f$) is equal to the enthalpy of vaporization divided by absolute temp

$$(s_g - s_f) = \frac{h_g - h_f}{T}$$

$$\frac{dp}{dT} = \frac{h_g - h_f}{T(v_g - v_f)} = \frac{h_g}{T v_{fg}}$$

from solid to liquid

$$\frac{dp}{dT} = \frac{h_l - h_s}{T(v_l - v_s)}$$

Solid to vapor

$$\frac{dp}{dT} = \frac{h_g - h_s}{T(v_g - v_s)}$$

For example, solid to liquid or state of a pure substance from the solid to liquid to vapor. This process occurs at constant temp. During vaporization process, both the temp & pressure are independent of the volume, during the change of phase extensive property are linearly inter-related.

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{dp}{dT}$$

For saturated states the pressure is a function of temp alone and it is independent of volume.

1) wet steam (two-phase mixture)

When steam contains moisture it is said to be wet steam.

or
when the substance lies between the saturated liquid and saturated vapour state, it is termed as wet steam.

or
wet steam is defined as a mixture of dry steam and suspended water particles.

By applying heat to the wet steam, it becomes dry and saturated if all the suspended water particles evaporate.

The composition of the mixture is given by a term called quality of steam and the state will be within the vapor dome.

Dryness fraction or quality of steam tells the relative amount of dry steam or moisture present in wet steam.

The dryness fraction $x = \frac{\text{mass of dry steam}}{\text{total mass of mixture}}$

$m_g = m_{fg} = m_{wf} \rightarrow \text{mass of dry steam / kg of mixture}$
 $m_L = m_f = m_{wf} \rightarrow \text{mass of suspended water or fluid particles / kg of mixture.}$

$$= \frac{m_g}{m_g + m_f} \quad \text{--- (1)}$$

The above eqn shows that dryness fraction of sat. liquid is zero because vapour phase is absent whereas the dryness fraction of dry and saturated steam is unity because water particles are absent.

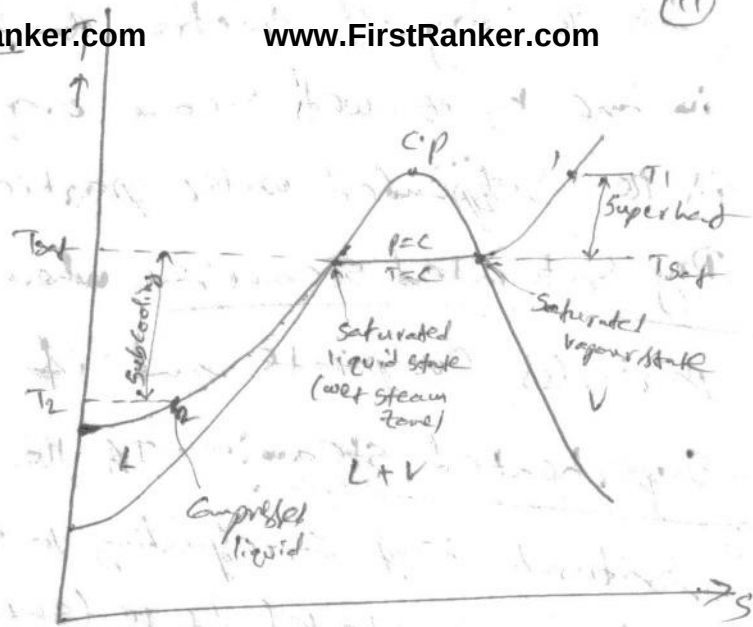


Fig: Saturated wet, superheated, and subcooling.

Saturated state $\rightarrow$ it's a state from which a phase change occurs without a change in pressure or temperature.

is the dryness fraction. It is 0.9 in wet steam and 0.1 kg is the suspended water particles.

Dry Saturated Steam:— when wet steam is heated, water particles evaporate and it is known as dry sat. steam.

Superheated steam:— If the temp of the vapour is greater than the saturation temp corresponding to the given pressure, the vapour is said to be in superheated state (state in fig). The difference between the temp of superheated vapour (t_1) and the saturation temp (t_{sat}) at that pressure is called the degree of superheat. That
degree of superheat = $t_1 - t_{sat}$.

Compressed or Subcooled liquid:— If the temperature of the liquid is less than the saturation temp at the given pressure the liquid is said to be compressed liquid (state 2 in fig). Since the properties of liquids vary little with pressure hence properties of compressed liquids are taken from the saturation tables at the temp of compressed liquid.

If the liquid is cooled below its saturation temp at a certain pressure it is called subcooled. The difference between the sat. temp and actual liquid temp (i.e. $t_3 - t_2$) is termed as the degree of subcooling.

[Note:— It is to be observed that steam as a vapour will not obey the law of perfect gases unless it is in a highly dried condition (i.e. highly superheated steam).]

5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25

Dalton's law of partial pressures :-

①



Total pressure of mixture is equal to the individual partial pressures

$$P = P_A + P_B$$

①

Similarly $P = P_A + P_B + P_C + \dots + P_i = \sum P_i$

②

where P_i is the partial pressure of a constituent

Volumetric analysis of a gas mixture :- Let us consider a

Volume V of a gaseous mixture at a temperature T , consisting of three constituents A, B and C.

the volume V contains n moles of the mixture at P & T

$$\begin{aligned} m &= m_A + m_B + m_C = \sum m_i \\ P &= P_A + P_B + P_C = \sum P_i \\ n &= n_A + n_B + n_C = \sum n_i \end{aligned}$$

(a)

V_A	V_B	V_C
P	P	P
m_A	m_B	m_C
n_A	n_B	n_C

(b)

the gas A occupies a volume V_A at P & T , and this volume contains n_A moles. Similarly there are n_B moles of gas B in volume V_B and n_C moles of gas C in volume V_C .

$$PV = nRT$$

$$\Rightarrow m = \frac{PV}{RT}$$

$$m_A = \frac{P_A V}{R_A T}$$

$$\text{and } m_A = \frac{P \cdot V_A}{R_A T} \quad \text{--- Is (b)}$$

Equating the two values for m_A , we have

$$\frac{P_A V}{R_A T} = \frac{P \cdot V_A}{R_A T}$$

$$P_A V = P \cdot V_A$$

$$V_A = \frac{P_A}{P} \cdot V$$

$V \rightarrow$ Total volume

$V_A \rightarrow$ partial volume of A

Volumetric analysis is the analysis by volume.

gravimetric analysis is the analysis by weight or mass

In general

$$V_i = \frac{P_i}{P} \cdot V$$

$$\sum V_i = \sum \frac{P_i}{P} \cdot V = \frac{V}{P} \sum P_i = \frac{V}{P} \cdot P = V$$

$$\sum P_i = P$$

$$\therefore \sum V_i = V \quad (3)$$

Total volume of mixture is equal to the sum of the volumes of the individual constituents when each exists at the same pressure and temperature as the mixture.

The mole $n = \frac{m}{M}$ (4)

$n \rightarrow$ number of moles

$m \rightarrow$ mass of gas

$M \rightarrow$ molecular weight

Avogadro's law :- The number of moles of any gas is proportional to the volume of the gas at a given pressure and temperature.

$$n \propto V$$

From Equation (3), $\sum V_i = V$

$$V_A + V_B + V_C = V$$

$\therefore$ the total number of moles in the vessel must equal the sum of the moles of the individual constituents.

$$n = n_A + n_B + n_C = \sum n_i$$

Mass fraction

$$PV = mRT$$

$$P_i V = m_i R_i T$$

$$\sum P_i V = \sum m_i R_i T$$

$$V \sum P_i = T \sum m_i R_i$$

$$PV = T \sum m_i R_i$$

$$mRT = T \sum m_i R_i$$

$$mR = \sum m_i R_i$$

$$R = \sum \frac{m_i}{m} R_i$$

Dalton's law
($\because \sum P_i = P$)

where $\frac{m_i}{m}$ = mass fraction of a constituent (x)

Mole fraction:-

$$\frac{n_i}{n} = \text{mole fraction}$$

Specific heat of a gas mixture:-

As per Gibbs - Dalton law, the internal energy of a mixture of gases is given by $mu = \sum m_i u_i$ — (1)

$$\text{also } u = c_v T \text{ — (2)}$$

from (1) & (2)

$$m c_v T = \sum m_i c_{v,i} T$$

$$m c_v = \sum m_i c_{v,i}$$

$$c_v = \sum \frac{m_i}{m} c_{v,i} \text{ — (3)}$$

$$\begin{aligned} m h &= \sum m_i h_i \\ m c_p T &= \sum m_i c_{p,i} T \quad (\because h = c_p T) \\ m c_p &= \sum m_i c_{p,i} \\ c_p &= \sum \frac{m_i}{m} c_{p,i} \text{ — (4)} \end{aligned}$$

$$H = n c_p T$$

By the Gibbs - Dalton law

$$U = \sum U_i$$

$$n c_v T = \sum n_i c_{v,i} T$$

$$c_v = \sum \frac{n_i}{n} c_{v,i}$$

$$H = \sum H_i$$

$$n c_p T = \sum n_i c_{p,i} T$$

$$c_p = \sum \frac{n_i}{n} c_{p,i}$$

According to Gibbs - Dalton law

- ①. The internal energy, enthalpy and entropy of a ~~gas~~ gaseous mixture are respectively equal to the sum of the internal energy, enthalpy and entropy of the ~~constituent~~ constituents.
- ②. Each constituent has that internal energy, enthalpy and entropy, which it would have if it occupied alone that volume occupied by the mixture at the temperature of the mixture.

→ Characteristic equation of mixture

$$pV = n R_0 T$$

$n \rightarrow$ no. of moles of mixture

$R_0 \rightarrow$ universal gas constant.

Characteristic equation of gas
gas constant $pV = RT$

$$R = \frac{p \cdot V}{n T} = \frac{287 \text{ J/kg} \cdot \text{K}}{0.287 \text{ kg/mole} \cdot \text{K}}$$

Universal gas constant or molar constant of gas

$$R_0 = M \cdot R$$

$M \rightarrow$ Molecular mass of gas in kg - mole

$$\downarrow$$

$$8314 \text{ J/kg} \cdot \text{mole} \cdot \text{K}$$

$$8.314 \text{ kJ/kg} \cdot \text{mole} \cdot \text{K}$$

$R \rightarrow$ gas constant.

Molar specific heat of a gas

$$C_m = M \cdot C$$

$C_m \rightarrow$ molar specific heat
 $M \rightarrow$ Molecular mass of the gas

$$C_{vm} = M C_v$$

$$C_{pm} = M C_p$$

molar sp. heat at constant volume

molar sp. heat at constant pressure

The molar or volumetric specific heat of a gas may be defined as the amount of heat required to raise the temperature of unit mole of gas through one degree.

Internal energy of Gas mixture = Internal energy of mixture of gases is equal to the sum of the internal energies of the individual components, each taken at the temperature and volume of the mixture (i.e. sum of the partial internal energies). Therefore, on a mass basis

$$m u_m = m_1 u_1 + m_2 u_2 + \dots + m_c u_c$$

$$u_m = \frac{m_1 u_1 + m_2 u_2 + \dots + m_c u_c}{m_1 + m_2 + \dots + m_c}$$

which is the average specific internal energy of the mixture.

Enthalpy of gas mixture - The total enthalpy of a gas mixture is the sum of the partial enthalpies.

$$m h_m = m_1 h_1 + m_2 h_2 + \dots + m_c h_c$$

$$\therefore h_m = \frac{m_1 h_1 + m_2 h_2 + \dots + m_c h_c}{m_1 + m_2 + \dots + m_c}$$

$$C_{vm} = \frac{m_1 C_{v1} + m_2 C_{v2} + \dots + m_c C_{vc}}{m_1 + m_2 + \dots + m_c}$$

$$C_{pm} = \frac{m_1 C_{p1} + m_2 C_{p2} + \dots + m_c C_{pc}}{m_1 + m_2 + \dots + m_c}$$

Entropy of gas mixtures :- Gibbs theorem states that the total entropy of a mixture of gases is the sum of the partial entropies.

The total entropy (initial)

$$S_i = m_1 S_1 + m_2 S_2 + \dots + m_c S_c$$

$$= \sum m_k S_k$$

Small's

Entropy change due to diffusion

$$S_f - S_i = - \sum m_k R_k \ln \frac{p_k}{p}$$

$$= - \left(m_1 R_1 \ln \frac{p_1}{p} + m_2 R_2 \ln \frac{p_2}{p} + \dots + m_c R_c \ln \frac{p_c}{p} \right)$$

1. A vessel of 0.35 m capacity contains 0.4 kg of carbon monoxide (molecular weight = 28) and 1 kg of air at 20°C. Calculate:

①. The partial pressure of each constituent

②. The total pressure in the vessel.

The gravimetric analysis of air may be taken as 23.3% oxygen (molecular weight = 32) and 76.7% nitrogen (molecular weight = 28).

Soln:- Capacity of vessel - $V = 0.35 \text{ m}^3$

mass of Carbon monoxide = 0.4 kg

mass of air = 1 kg

Temperature, $T = 20 + 273 = 293 \text{ K}$

Mass of oxygen present in 1 kg of air = $\frac{23.3}{100} \times 1 = 0.233 \text{ kg}$

Mass of nitrogen present in 1 kg of air = $\frac{76.7}{100} \times 1 = 0.767 \text{ kg}$

But characteristic gas constant $R = \frac{R_0}{M}$

$R_0 \rightarrow$ universal gas constant
 8.314 kJ/kgK
 $M \rightarrow$ Molecular weight

Also, the characteristic gas equation is

$$PV = mRT$$

$$PV = m \cdot \frac{R_0}{M} \cdot T$$

$$P = \frac{m R_0 T}{M \cdot V}$$

Hence, for a constituent

$$P_i = \frac{m_i R_0 T}{M_i \cdot V}$$

Substituting the values, we get the partial pressures

$$\text{For } O_2, P_{O_2} = \frac{0.233 \times 8.314 \times 10^3 \times 293}{32 \times 0.35 \times 10^5} = 0.5068 \text{ bar}$$

$$\text{For } N_2, P_{N_2} = \frac{0.767 \times 8.314 \times 10^3 \times 293}{28 \times 0.35 \times 10^5} = 1.9065 \text{ bar}$$

$$\text{For } CO_2, P_{CO_2} = \frac{0.40 \times 8.314 \times 10^3 \times 293}{28 \times 0.35 \times 10^5} = 0.9943 \text{ bar}$$

$$\therefore \text{Total pressure in the vessel, } P = \sum P_i = P_{O_2} + P_{N_2} + P_{CO_2} \\ = 0.5068 + 1.9065 + 0.9943 \\ = 3.4076 \text{ bar}$$

2). Two vessels A and B, both containing nitrogen, are connected by a Valve which is opened to allow the contents to mix and achieve an equilibrium temperature of 27°C . Before mixing the following information is known about the gases in the two vessels.

vessel A

$$P = 1.5 \text{ MPa}$$

$$t = 50^\circ\text{C}$$

$$\text{Contents} = 0.5 \text{ kg mol}$$

vessel B

$$P = 0.6 \text{ MPa}$$

$$t = 20^\circ\text{C}$$

$$\text{Contents} = 2.5 \text{ kg}$$

Calculate the final equilibrium pressure, and the amount of heat transferred to the surroundings. If the vessel had been perfectly insulated, calculate the final temperature and pressure which would have been reached. Take $\gamma = 1.4$.

Soln:-

$$P \cdot V = n R T$$

$$P_A V_A = n_A R T_A$$

$$1.5 \times 10^3 \times V_A = 0.5 \times 8.3143 \times 323$$

∴ Volume of vessel A

$$V_A = 0.895 \text{ m}^3$$

Mass of gas in vessel A

$$m_A = n_A \cdot M_A$$

$$= 0.5 \text{ kg/mol} \times 28 \text{ kg/kg mol} = 14 \text{ kg}$$

Characteristic gas constant R of nitrogen

$$R = \frac{R_0}{M} = \frac{8.3143}{28} = 0.297 \text{ kJ/kg K}$$

For vessel B, $P_B V_B = m_B R T_B$

$$0.6 \times 10^3 \times V_B = 2.5 \times 0.297 \times 293$$

$$\Rightarrow V_B = 0.363 \text{ m}^3$$

Total volume of A and B

$$V = V_A + V_B = 0.895 + 0.363 = 1.258 \text{ m}^3$$

Total mass of gas, $m = m_A + m_B = 14 + 2.5 = 16.5 \text{ kg}$

For the final condition after mixing

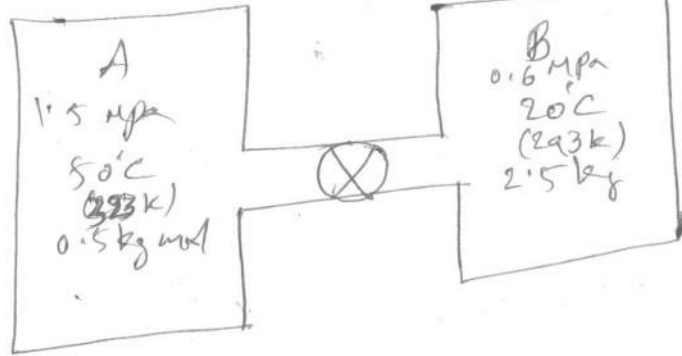
$$P V = m R T$$

$$P \times 1.258 = 16.5 \times 0.297 \times (27 + 273)$$

final equilibrium pressure

$$\Rightarrow P = 1168.16 \text{ kPa} = 11.68 \text{ MPa}$$

Temperature after mixing = 27°C



$$C_v = \frac{R}{\gamma - 1} = \frac{0.297}{1.4 - 1} = 0.743 \text{ kJ/kgK}$$

Initial internal energy (before mixing)

$$U_i = m_A C_v T_A + m_B C_v T_B$$

$$= 14 \times 0.743 \times 323 + 2.5 \times 0.743 \times 293 = 3904.1 \text{ kJ}$$

Final internal energy U_2 (after mixing)

$$= m C_v T$$

$$= 16.5 \times 0.743 \times 300$$

$$= 3677.9 \text{ kJ}$$

Amount of heat transfer (since there is no work transfer)

$$Q = U_2 - U_1 = 3677.9 - 3904.1 = -226.2 \text{ kJ}$$

If the vessel were insulated

$$U_1 = U_2$$

$$m_A C_v T_A + m_B C_v T_B = m C_v T$$

where T would ~~be~~ have been final temperature

$$T = \frac{m_A T_A + m_B T_B}{m} = \frac{14 \times 323 + 2.5 \times 293}{16.5} = 318.5 \text{ K}$$

$$= 45.5^\circ \text{C}$$

The final pressure $p = \frac{m R T}{V} = \frac{16.5 \times 0.297 \times 318.5}{1.258}$

$$= 1240.7 \text{ kPa}$$

$$= 1.24 \text{ MPa}$$

Reversible adiabatic process :- The internal energy change of an ideal gas for a reversible adiabatic process is given by

$$TdS = du + pdv = 0$$

property relation

$$\begin{aligned} du &= -pdv \\ \int_1^2 du &= - \int_1^2 pdv = - \int_1^2 \frac{C}{v^\gamma} dv \end{aligned}$$

$$\begin{aligned} \because pv^\gamma &= C \\ \therefore p &= \frac{C}{v^\gamma} \end{aligned}$$

$$u_2 - u_1 = -C \int_1^2 \frac{1}{v^\gamma} dv$$

$$= -C \int_1^2 v^{-\gamma} dv$$

$$= -C \left[\frac{v^{-\gamma+1}}{-\gamma+1} \right]_1^2 = \frac{-C}{-\gamma+1} \left[v_2^{1-\gamma} - v_1^{1-\gamma} \right]$$

$$= \frac{C}{\gamma-1} \left[v_2^{1-\gamma} - v_1^{1-\gamma} \right]$$

$$pv^\gamma = p_1 v_1^\gamma = p_2 v_2^\gamma = C$$

$$= \frac{p_2 v_2^\gamma \cdot v_2^{1-\gamma} - p_1 v_1^\gamma \cdot v_1^{1-\gamma}}{\gamma-1}$$

$$v_2^\gamma \cdot \frac{v_2}{v_2^\gamma}$$

$$= \frac{p_2 v_2 - p_1 v_1}{\gamma-1}$$

$$= \frac{R(T_2 - T_1)}{\gamma-1} = \frac{R T_1}{\gamma-1} \left[\frac{T_2}{T_1} - 1 \right]$$

$$= \frac{R T_1}{\gamma-1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$1 - \frac{1}{\gamma} = \frac{\gamma-1}{\gamma}$$

$$\begin{aligned} p_1 v_1^\gamma &= p_2 v_2^\gamma \\ \frac{p_2}{p_1} &= \left(\frac{v_1}{v_2} \right)^\gamma \Rightarrow \frac{p_2}{p_1} = \left(\frac{v_2}{v_1} \right)^{-\gamma} \\ \frac{p_2}{p_1} &= \left(\frac{p_2 v_2}{p_1 v_1} \right)^{\frac{\gamma}{\gamma-1}} \\ \frac{T_2}{T_1} &= \frac{p_2 v_2}{p_1 v_1} \\ &= \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \end{aligned}$$

The enthalpy change of an ideal gas for a reversible adiabatic process may be derived as

$$TdS = dh - vdp = 0$$

Thermodynamic
property relation

$$dh = vdp$$

$$\int_1^2 dh = \int_1^2 v dp = \int_1^2 \frac{C^{1/\gamma}}{P^{1/\gamma}} dP$$

$$PV^\gamma = C$$

$$V^\gamma = \frac{C}{P}$$

$$V = \left(\frac{C}{P}\right)^{1/\gamma}$$

$$= \frac{C^{1/\gamma}}{P^{1/\gamma}}$$

$$h_2 - h_1 = C^{1/\gamma} \int_1^2 \frac{1}{P^{1/\gamma}} dP = C^{1/\gamma} \int_1^2 P^{-1/\gamma} dP$$

$$= C^{1/\gamma} \left[\frac{P^{1-1/\gamma}}{1-1/\gamma} \right]_1^2 = C^{1/\gamma} \left[\frac{P^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} \right]_1^2$$

$$= \frac{\gamma}{\gamma-1} C^{1/\gamma} \left[P^{\frac{\gamma-1}{\gamma}} \right]_1^2$$

$$PV^\gamma = P_1 V_1^\gamma = P_2 V_2^\gamma = C$$

$$= \frac{\gamma}{\gamma-1} C^{1/\gamma} \left[P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}} \right]$$

$$= \frac{\gamma}{\gamma-1} \left[(P_2 V_2^\gamma)^{1/\gamma} \cdot P_2^{\frac{\gamma-1}{\gamma}} - (P_1 V_1^\gamma)^{1/\gamma} \cdot P_1^{\frac{\gamma-1}{\gamma}} \right]$$

$$= \frac{\gamma}{\gamma-1} \left[V_2 \cdot P_2 \cdot P_2^{\frac{\gamma-1}{\gamma}} - V_1 \cdot P_1 \cdot P_1^{\frac{\gamma-1}{\gamma}} \right]$$

$$= \frac{\gamma}{\gamma-1} C^{1/\gamma} P_1^{\frac{\gamma-1}{\gamma}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{\gamma}{\gamma-1} (P_1 V_1^\gamma)^{1/\gamma} \cdot P_1^{\frac{\gamma-1}{\gamma}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\frac{1 + \frac{\gamma-1}{\gamma}}{\frac{\gamma + \gamma - 1}{\gamma}} = \frac{2\gamma-1}{2\gamma-1}$$

$$= \frac{\gamma}{\gamma-1} R U_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$= \frac{\gamma}{\gamma-1} R T_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

3) a). The specific heats of a gas are given by $c_p = a + kT$ and $c_v = b + kT$, where a, b and k are constants and T is in $^{\circ}K$.
Show that for an isentropic expansion of this gas

$$T^b V^{a-b} e^{kT} = \text{Constant}.$$

b). 1.5 kg of this gas occupying a volume of 0.06 m^3 at 5.6 MPa expands isentropically until the temperature is $240^{\circ}C$. If $a = 0.946$, $b = 0.662$, and $k = 10^{-4}$, calculate the work done in the expansion.

Soln: a). $c_p - c_v = a + kT - b - kT$
 $= a - b$

$$c_p = a + kT$$

$$c_v = b + kT$$

$$\text{Now } ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$= (a + kT) \frac{dT}{T} + (a - b) \frac{dv}{v}$$

$$= b \cdot \frac{dT}{T} + k dT + (a - b) \frac{dv}{v}$$

For an isentropic process

$$b \ln T + kT + (a - b) \ln v = \text{Constant}$$

$$T^b V^{a-b} e^{kT} = \text{Constant}$$

(Q.E.D.)

b) $R = a - b$

$$= 0.946 - 0.662 = 0.284 \text{ kJ/kgK}$$

$$T_2 = 240 + 273 = 513 \text{ K}$$

$$T_1 = \frac{P_1 V_1}{m R} = \frac{5.6 \times 10^3 \times 0.06}{1.5 \times 0.284} = 788.73 \text{ K} = 789 \text{ K}$$

$$Tds = du + dw = 0$$

$$w_{1-2} = - \int_{T_1}^{T_2} m c_v dT$$

$$= 1.5 \int_{T_2}^{T_1} c_v dT = 1.5 \int_{513}^{789} (b + kT) dT$$

$$= 1.5 \int_{513}^{789} (0.662 + 0.0001 T) dT$$

$$= 1.5 \left[0.662 (T_2 - T_1) + 0.0001 \times \frac{1}{2} [(789)^2 - (513)^2] \right]$$

$$= 1.5 (182.71 + 19.97)$$

$$= 304 \text{ kJ}$$

$$\begin{matrix} T_2 \\ 0.662(T_2 - T_1) + 0.0001 \times \frac{1}{2} [(T_2)^2 - (T_1)^2] \end{matrix}$$

4). A certain gas has $C_p = 1.968$ and $C_v = 1.507$ kJ/kg K.

Find its molecular weight and the gas constant.

A constant volume chamber of 0.3 m^3 capacity contains 2 kg of this gas at 5°C . Heat is transferred to the gas until the temperature is 100°C . Find the workdone, the heat transferred, and the changes in internal energy, enthalpy and entropy.

Soln Gas constant $R = C_p - C_v = 1.968 - 1.507$
 $= 0.461 \text{ kJ/kg K}$

Molecular weight

$$R = \frac{R_0}{M}$$

$$M = \frac{R_0}{R} = \frac{8.3143}{0.461} = 18.04 \text{ kg/kg mol}$$

At constant volume

$$\begin{aligned} Q_{1-2} &= m C_v (T_2 - T_1) \\ &= 2 \times 1.507 [100 - 5] \\ &= 286.33 \text{ kJ} \end{aligned}$$

$$W_{1-2} = \int_1^2 p \cdot dv = 0$$

Change in internal energy $U_2 - U_1 = Q_{1-2} = 286.33 \text{ kJ}$

Change in enthalpy, $H_2 - H_1 = m C_p (T_2 - T_1) = 2 \times 1.968 [100 - 5]$
 $= 373.92 \text{ kJ}$

Change in entropy, $S_2 - S_1 = m C_v \ln \frac{T_2}{T_1} = 2 \times 1.507 \ln \frac{373}{268}$
 $= 0.721 \text{ kJ/K}$

Universal gas constant :- The general form of characteristic

gas Equation can be written as

$$PV = mRT$$

$$m \rightarrow \text{mass of gas in kg} \\ = nM$$

$n = \text{no. of kilomoles}$
 $M \rightarrow \text{Molecular weight}$
 kg/kilomole

∴ The characteristic gas Equation is

$$PV = nMRT$$

The product of molecular weight and characteristic gas constant is called universal gas constant. It will have the same value

(R_u)

for all gases.

$$\therefore PV = nR_u T$$

$$R_u = \frac{PV}{nT} \quad \text{J/kg mole K or kJ/kg mole K}$$

Its value is 8.314 ~~kJ/kg mole K~~ kJ/kg mole K
8314 J/kg mole K

$\rightarrow \text{characteristic gas constant}$
 $R \rightarrow \frac{PV}{mT}$
 $\frac{n/M \cdot T \cdot M^3}{\text{kg} \cdot \text{K}}$
 $\frac{\text{N} \cdot \text{m}^3/\text{kg} \cdot \text{K}}{\text{J/kg K}}$

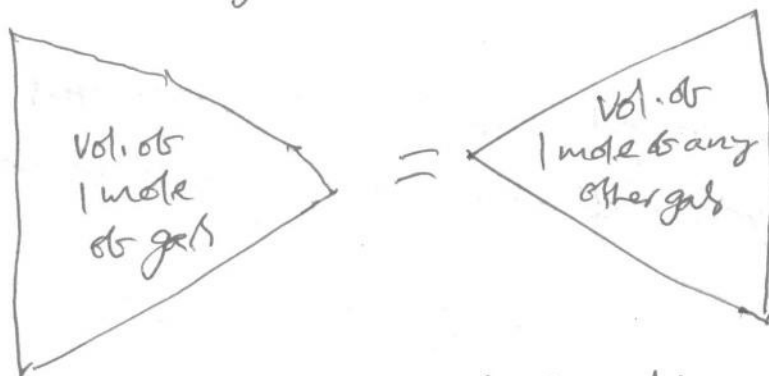
Avogadro's law :- Avogadro's law states that equal volumes of all perfect gases, under the same conditions of temperature and pressure, contain equal number of molecules.

Consider m kg of gas whose molecular weight is M in kg/kilomole. The number of kilomoles, n is obtained by the equation

$$n = \frac{m}{M}$$

If $n=1$, then $m=M$.

Kilogram mole: one kilogram mole of a substance is the amount of that substance in kilograms numerically equal to its molecular weight. It follows from Avogadro's law that the volume occupied by kg mole is the same for all gases at the same temperature and pressure.



Consider characteristic gas equation in terms specific volume,

$$p v = R T$$

multiply both sides by M

$$p(Mv) = M R T$$

The product of molecular weight and specific volume is called molar volume,

$$\therefore p V_{mol} = R_u T$$

but $R_u = \text{Constant}$ for all gases.

$\therefore$ for given conditions of p and T , V_{mol} is constant for all gases.

$$\text{At NTP conditions } V_{mol} = \frac{8.314 \times 273}{101.3} = 22.4 \text{ m}^3/\text{kg mole}$$

Hence at 0°C and 760 mm Hg (101.3 kN/m^2), the volume of 1 kg mole is 22.4 m^3 .

Thus from Avogadro's law, 1 kg. mole of any perfect gas occupies volume of 22.4 m^3 and contain 6×10^{23} molecules at NTP. The number

6×10^{23} is called Avogadro's number.

11. Air ($n_2 = 77$, $n_{O_2} = 23$ by weight) at 25°C and 12 bar is contained in a vessel of capacity 0.6 m^3 . Some quantity of CO_2 is forced into the vessel so that the temperature remains at 25°C but the pressure rises to 18 bar. Find the masses of O_2 , N_2 and CO_2 in the cylinder. Set 10, May 2013

Soln: Volume of vessel $V = 0.6 \text{ m}^3$
 $T = 25^\circ\text{C} = 25 + 273 = 298 \text{ K}$
 $P_1 = 12 \text{ bar}$
 $P_2 = 18 \text{ bar}$

Now $PV = nR_0T$

$$n = \frac{PV}{R_0T} = \frac{12 \times 10^5 \times 0.6}{8.314 \times 10^3 \times 298} = 0.29$$

mass $\rightarrow$ no. of moles
 $(m = n \times M)$ — molecular wt.

$$\text{Also } R_{\text{air}} = \left(\frac{m_{\text{O}_2}}{M_{\text{O}_2}} + \frac{m_{\text{N}_2}}{M_{\text{N}_2}} \right) \frac{R_0}{m}$$

Considering 100 kg of air

$$m_{\text{O}_2} = 23 \text{ kg}$$

$$m_{\text{N}_2} = 77 \text{ kg}$$

number 8
 $\text{O}_2 = 16 + 16 = 32$
 $\text{N}_2 = 14 + 14 = 28$
 number 7

$$\therefore R_{\text{air}} = \left(\frac{23}{32} + \frac{77}{28} \right) \frac{8.314}{100} = 0.288 \text{ kJ/kgK}$$

molecular weight of air
 $M_{\text{air}} = \frac{R_0}{R_{\text{air}}} = \frac{8.314}{0.288} = 28.87$

Now $PV = mRT$

$$m = \frac{PV}{RT} = \frac{12 \times 10^5 \times 0.6}{0.288 \times 10^3 \times 298} = 8.39 \text{ kg of air}$$

$$\text{Mass of } \text{O}_2, m_{\text{O}_2} = 0.23 \times 8.39 = 1.93 \text{ kg}$$

$$\text{Mass of } \text{N}_2, m_{\text{N}_2} = 0.77 \times 8.39 = 6.46 \text{ kg}$$

$P_{N_2} + P_{O_2} = 12 \text{ bar}$, before adding CO_2

$P_{CO_2} + (P_{N_2} + P_{O_2}) = 18 \text{ bar}$, after adding CO_2

$P_{CO_2} + 12 = 18$

$\therefore P_{CO_2} = 18 - 12 = 6 \text{ bar}$

Now $P_{CO_2} = \frac{m_{CO_2} \cdot R_{CO_2} \cdot T_{CO_2}}{V}$

$m_{CO_2} = \frac{P_{CO_2} \times V}{R_{CO_2} \cdot T_{CO_2}} = \frac{6 \times 10^5 \times 0.6}{\left(\frac{8.314}{44}\right) \times 10^3 \times 298}$
 $= 6.39 \text{ kg}$

12) Mole fraction $n = \frac{m}{M}$

Set 4
Set 2014
6(b)

$n_{O_2} = \frac{3}{32} = -$

$n_{N_2} = \frac{5}{28} = -$

$n_{CH_4} = \frac{12}{12+4=16} = -$

$x_{O_2} = \frac{n_{O_2}}{n_{O_2} + n_{N_2} + n_{CH_4}}$

$x_{N_2} = \frac{n_{N_2}}{\text{Total}}$

$x_{CH_4} = \frac{n_{CH_4}}{\text{Total}}$

Average molecular weight, $M = \frac{n_{O_2} \times \overset{32}{M_{O_2}} + n_{N_2} \times \overset{28}{M_{N_2}} + n_{CH_4} \times \overset{16}{M_{CH_4}}}{n_{O_2} + n_{N_2} + n_{CH_4}} = -$

Specific gas constant $R = \frac{R_0}{M} = \frac{8.314}{M} = - \text{ kJ/kgK}$

Mass fraction $\rightarrow$ Total mass = $3 + 5 + 12 = 20$

mass fraction of O_2 , $\frac{3}{20} = - \left(\frac{m_a}{m} \right)$

mass fraction N_2 , $\frac{5}{20} = -$

mass fraction $CH_4 = \frac{12}{20} = -$

Entirely a gas mixture that consists of 3 kg of O_2 , 5 kg of N_2 and 12 kg of CH_4 .
Determine (1) the mass fractions of each component
(2) the mole fractions of each component
and (3) the average molar mass and gas constant of the mixture.

Partial pressure

Let us consider a closed vessel of volume V (9)

at temperature T , which contains a mixture of perfect gases at a known pressure. If some of the mixture were removed, then the pressure would be less than the initial value. If the gas removed were the full amount of one of the constituents, then the reduction in pressure would be equal to the contribution of that constituent to the initial total pressure. Each constituent contributes to the total pressure by an amount which is known as the partial pressure of the constituent.

The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.

1). The gravimetric analysis of air and other data are as follows:

<u>Constituent</u>	<u>Percentage</u>	<u>Molecular weight</u>
Oxygen	23.14	32
Nitrogen	75.53	28
Argon	1.28	40
Carbon dioxide	0.05	44

Calculate: (1) Gas content for air (2) Average molecular weight.

Soln $R = \frac{R_o}{M}$

$$R_{O_2} = \frac{8.314}{32} = 0.2598 \text{ kJ/kgK}$$

$$R_{N_2} = \frac{8.314}{28} = 0.2969 \text{ "}$$

$$R_{Ar} = \frac{8.314}{40} = 0.2078 \text{ "}$$

$$R_{CO_2} = \frac{8.314}{44} = 0.1889 \text{ "}$$



Gas constant for air

$$R = \sum \left(\frac{m_i}{m} R_i \right)$$

 $\frac{m_i}{m} \rightarrow$ given (percent)

$$R = 0.2314 \times 0.2598 + 0.7553 \times 0.2969 + 0.0128 \times 0.2078 \\ + 0.0005 \times 0.1889$$

$$= 0.2871 \text{ kJ/kg K}$$

Apparent molecular weight

$$R = \frac{R_0}{M}$$

$$\therefore M = \frac{R_0}{R} = \frac{8.314}{0.2871} = 28.96$$

Psychrometry :- The properties of mixtures of air and water vapour are studied under the chapter of psychrometry. Psychrometry is the foundation for air-conditioning.

Psychrometric properties :- The definitions of psychrometric properties are :

Dry air :- It is the mixture of nitrogen (79%) and oxygen (21%) neglecting the small percentages of other gases. And the molecular weight of dry air is taken as 29 approximately.

Moist air :- It is a mixture of dry air and water vapour. The quantity of water vapour present in the air depends upon the temperature of the air.

Water vapour :- The water vapour present in air is known as moisture and its quantity in air is an important factor in all air-conditioning systems.

Dry bulb temperature (DBT) :- The temperature of air measured by ordinary thermometer is known as dry bulb temperature.



Wet bulb temperature (WBT) :- The temperature measured by the thermometer when its bulb is covered with wet cloth and is exposed to a current of moving air is known as wet bulb temperature.

Wet bulb depression :- The difference between the DBT and WBT is known as wet bulb depression.

reduced by continuous cooling than the water vapour in the air will start condensing at a particular temperature. The temperature at which the condensing starts is known as dew-point temperature. Dew point temperature is equal to the steam table saturation temperature corresponding to the actual partial pressure of the water vapour in the air.

Dew-point depression:— The difference between DBT and DPT is known as dew-point depression.

Specific humidity (humidity ratio):— It is the mass of water vapour present per kg of dry air. It is given in grams per kg of dry air.

$$w = \frac{\text{mass of water vapour in mixture}}{\text{mass of dry air in mixture}} = \frac{m_v}{m_a}$$

$$m_a = \frac{P_a \cdot V}{R_a \cdot T}$$

where $P_a \rightarrow$ partial pressure of dry air
 $V \rightarrow$ volume of mixture

$R_a \rightarrow$ gas constant for dry air

$P_v \rightarrow$ partial pressure of water vapour

$R_v \rightarrow$ gas constant for water vapour

$$m_v = \frac{P_v \cdot V}{R_v \cdot T}$$

$$w = \frac{m_v}{m_a}$$

$$= \frac{P_v \cdot V}{R_v \cdot T} \times \frac{R_a \cdot T}{P_a \cdot V} = \frac{R_a}{R_v} \cdot \frac{P_v}{P_a}$$

$$= \frac{R}{M_a} \cdot \frac{M_v}{R} \cdot \frac{P_v}{P_a}$$

$$= \frac{M_v}{M_a} \cdot \frac{P_v}{P_a} = \frac{18}{29} \cdot \frac{P_v}{P_a}$$

$$w = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v}$$

$$R_a = \frac{R}{M_a}, R_v = \frac{R}{M_v}$$

$R \rightarrow$ universal gas constant

M_a & $M_v \rightarrow$ Molecular weights of air and water vapour.

At per Dalton's law of partial pressures $P_t = P_a + P_v$

$P_t \rightarrow$ Total pressure of moist air

$P_a \rightarrow$ partial pressure of dry air

$$m_a = \frac{V}{v_a}$$

$$m_v = \frac{V}{v_v}$$

$v_v \rightarrow$ specific volume of water vapour

$V \rightarrow$ Volume

$m_a \rightarrow$ mass of air

$m_v \rightarrow$ mass of water vapour

$$\therefore w = \frac{m_v}{m_a}$$

$$= \frac{V}{v_v} \times \frac{v_a}{V}$$

$$\boxed{w = \frac{v_a}{v_v}} \quad \text{--- (2)}$$

Absolute humidity :- The weight of water vapour present in unit volume of air is known as absolute humidity.

Degree of Saturation :- It is defined as the ratio of mass of water vapour associated with unit mass of dry air to mass of water vapour associated with unit mass of dry air saturated at the same temperature.

$$M = \frac{\text{mass of water vapour associated with unit mass of dry air}}{\text{mass of water vapour associated with saturated unit mass of dry air}} = \frac{w}{w_s}$$

where $w_s \rightarrow$ specific humidity of air when air is fully saturated.

$$M = \frac{0.622 \left(\frac{P_v}{P_t - P_v} \right)}{0.622 \left(\frac{P_{vs}}{P_t - P_{vs}} \right)} = \frac{P_v}{P_{vs}} \left(\frac{P_t - P_{vs}}{P_t - P_v} \right) = \frac{P_v}{P_{vs}} \left[\frac{1 - \frac{P_{vs}}{P_t}}{1 - \frac{P_v}{P_t}} \right] \quad \text{--- (3)}$$

where $P_{vs} \rightarrow$ partial pressure of water vapour when the air is fully saturated at the same temperature of air. This can be calculated from steam tables corresponding to the DBT of the air.

of water vapour in a given volume to the mass of water vapour in the air if it is saturated at the same temperature.

$$\phi = \frac{\text{mass of water vapour in a given volume}}{\text{mass of water vapour in same volume if saturated at the same temperature.}}$$

$$= \frac{m_v}{m_{vs}} = \frac{\frac{P_v \cdot V}{R_v \cdot T}}{\frac{P_{vs} \cdot V}{R_v \cdot T}} = \frac{P_v}{P_{vs}}$$

$$\therefore \boxed{\phi = \frac{P_v}{P_{vs}}} \quad \text{--- (A)}$$

Therefore the relative humidity (ϕ) can be defined as the ratio of the partial pressure of water vapour in a given volume of mixture to the partial pressure of water vapour when the same volume of mixture is saturated at the same temperature.

Substituting the value of (A) in Equation (3)

$$u = \frac{P_v}{P_{vs}} \left[\frac{1 - \frac{P_{vs}}{P_t}}{1 - \frac{P_v}{P_t}} \right] = \phi \left[\frac{1 - \frac{P_{vs}}{P_t}}{1 - \frac{P_v}{P_t}} \right]$$

$$= \phi \left[\frac{1 - \frac{P_{vs}}{P_t}}{1 - \frac{\phi P_{vs}}{P_t}} \right]$$

$$(\because P_v = \phi \cdot P_{vs})$$

$$= \phi \left[\frac{P_t - P_{vs}}{P_t - \phi P_{vs}} \right]$$

$$\phi (P_t - P_{vs}) = M (P_t - \phi P_{vs})$$

$$= M \cdot P_t - M \phi P_{vs}$$

$$\phi [P_t - P_{vs} + M P_{vs}] = M \cdot P_t$$

$$\phi = \frac{M P_t}{P_t - P_{vs} + M P_{vs}} = \frac{M}{1 - (1-M) \frac{P_{vs}}{P_t}}$$

$\because [P_t - P_{vs} + M P_{vs}] = [P_t - P_{vs} (1-M)]$

$$\phi = \frac{M}{1 - (1-M) \frac{P_{vs}}{P_t}}$$

It is the relation between degree of saturation and relative humidity.

Relative humidity plays more important part in comfort ^{air} conditioning and industrial air-conditioning than specific humidity. Relative humidity signifies the absorption capacity of the air. More moisture will be absorbed by the air if the initial relative humidity of the air is less.

Enthalpy of moist air:- is the sum of the enthalpy of dry air and the enthalpy of water vapour associated with dry air.

$$h = h_a + w h_v$$

$h_a \rightarrow$ enthalpy of dry air
 $h_v \rightarrow$ enthalpy of water vapour associated with one kg of dry air.

$$\therefore h = c_{pa} (T_{db} - 0) + w [c_{pw} T_{dp} + (h_{fg})_{dp} + c_{pv} (T_{db} - T_{dp})]$$

where c_{pa} = specific heat of dry air = 1 kJ/kgK

c_{pw} = specific heat of liquid water = 4.18 kJ/kgK

c_{pv} = specific heat of superheated vapour = 1.88 kJ/kgK

$T_{dp} \rightarrow$ dew point temperature of air.

$(h_{fg})_{dp} \Rightarrow$ latent heat of vaporization at DPT

$$\therefore h = 1(T_{db} - 0) + w[4.18 T_{dp} + (h_{fg})_{dp} + 1.88 (T_{db} - T_{dp})] \quad \text{--- (5)}$$

$$= T_{db} + w[(h_{fg})_{dp} + 1.88 T_{db} + 2.3 T_{dp}]$$

$$\begin{array}{r} 4.18 \\ - 1.88 \\ \hline 2.30 \end{array}$$

$$= T_{db} + w[(h_{fg})_{dp} + 1.88 T_{db}]$$

$$= T_{db} + w[2500 + 1.88 T_{db}] \text{ kJ/kg of dry air} \quad \text{--- (6)}$$

or

$$h = (T_{db} \times 1 + w \times 1.88 T_b) + w[(h_{fg})_{dp} + 2.3 T_{dp}]$$

$$= (1 + 1.88 w) T_{db} + w[(h_{fg})_{dp} + 2.3 T_{dp}]$$

$$= c_{pma} T_{db} + w[(h_{fg})_{dp} + 2.3 T_{dp}] \quad \text{--- (7)}$$

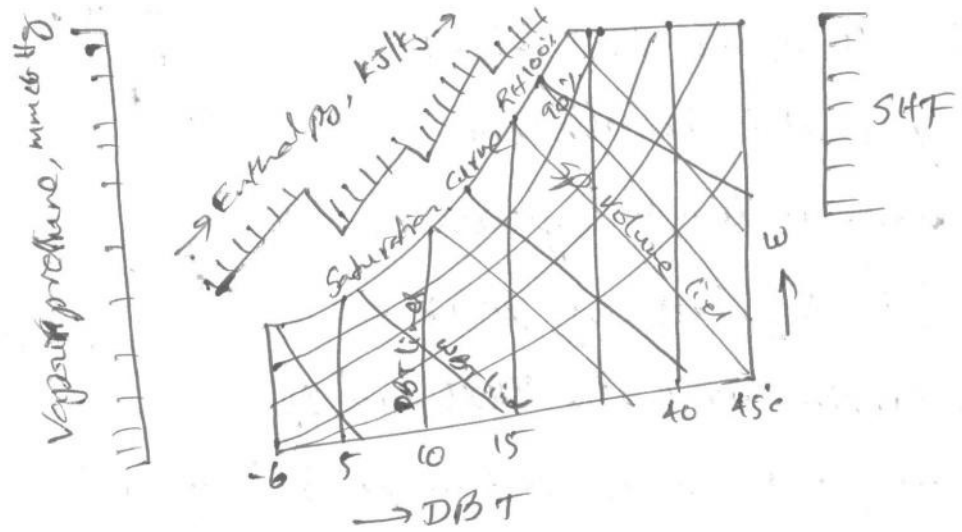
where $c_{pma} \rightarrow$ mean humid specific heat of air = 1.0216 kJ/kg of dry air.

As $1.88 w \ll 1$, therefore

$$h = 2.0216 T_{db} + w[(h_{fg})_{dp} + 2.3 T_{dp}] \quad \text{--- (8)}$$

$$\begin{array}{r} 1 + 1.0216 \\ \hline = 2.0216 \end{array}$$

to the various thermodynamic properties of moist air.

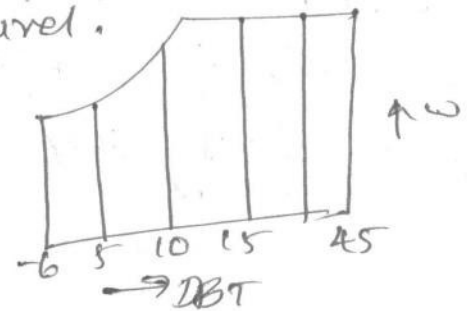


The psychrometric chart is very useful for finding out the properties of air and eliminate lot of calculations. The chart is normally drawn for standard atmospheric pressure of 760 mm of Hg (or 1.01325 bar). The saturation curve represents 100% relative humidity at various dry bulb temperatures. It also represents the wet bulb and dew point temperatures.

1. Dry bulb temperature lines:-

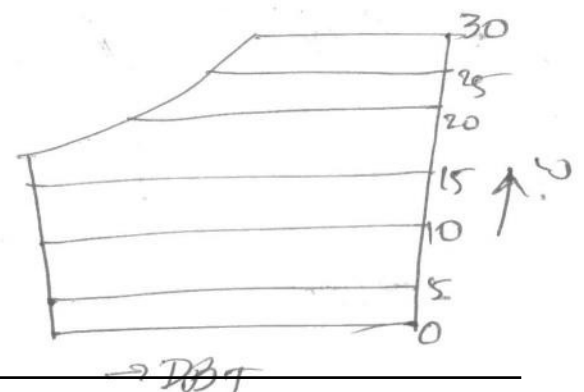
These lines are vertical i.e. parallel to the ordinate and uniformly spaced.

Generally the temperature range of these lines on psychrometric chart is from -6°C to 45°C .



2. Specific humidity lines:-

These lines are horizontal i.e. parallel to the abscissa and are also uniformly spaced as shown in figure.



These lines on chart is

from 0 to 30 g/kg or dry air (from 0 to 0.030 kg/kg or dry air)

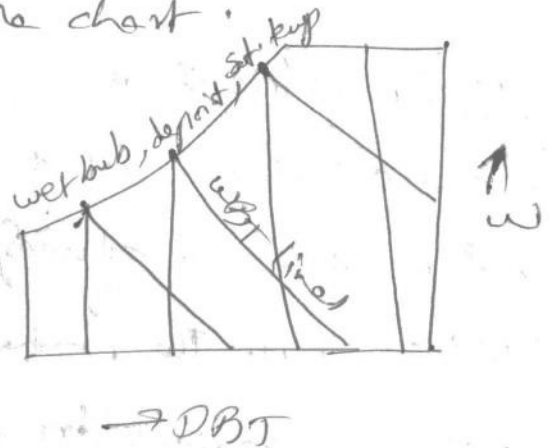
3). Dew point temperature lines :-

These lines are horizontal i.e., parallel to the abscissa and non-uniformly spaced as shown in fig. At any point on the saturation curve, the DBT and DPT are equal. The values of dew point temperature are generally given along the saturation curve of the chart.

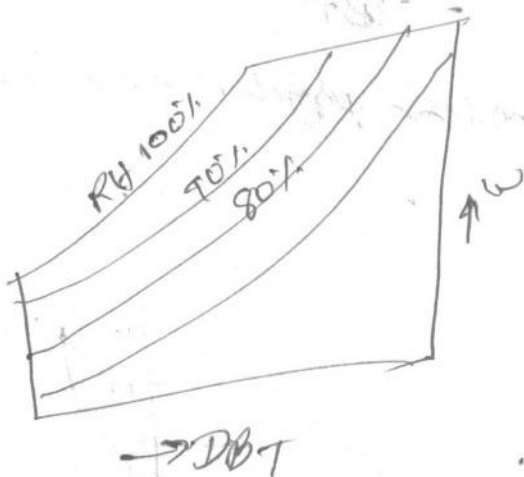


4). wet bulb temperature lines :-

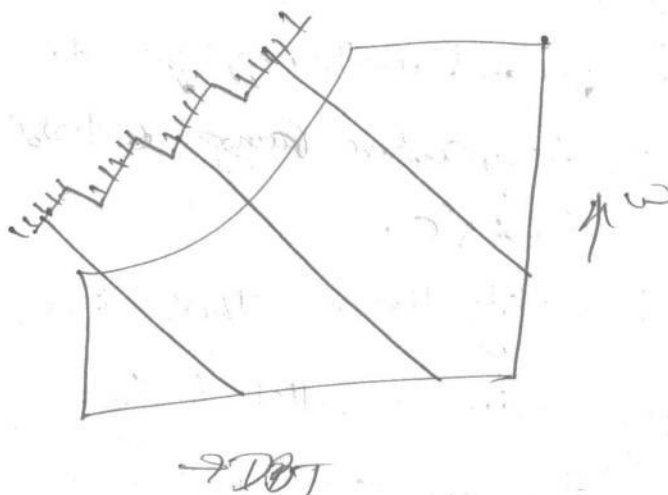
These lines are inclined straight lines and non-uniformly spaced as shown in fig. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.



Relative humidity lines



Enthalpy lines



measured by measuring the DBT of air is known as Sensible heat.

Total heat :- The total heat of the humid air is the sum of the Sensible heat of the dry air and Sensible and latent heat of water vapour associated with dry air.

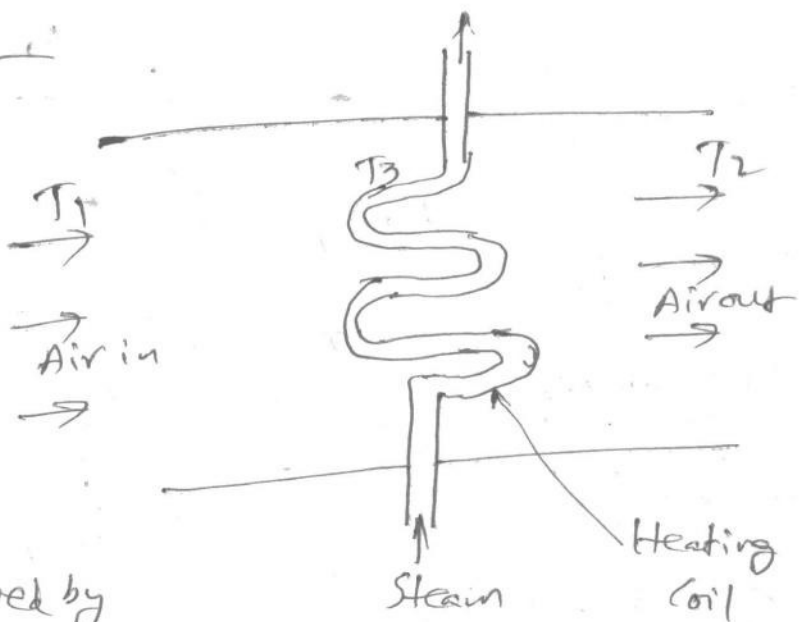
Humid specific volume :- The volume of the mixture per kg of dry air in the mixture, expressed in cubic meters, is known as humid specific volume.

Psychrometric processes :-

Sensible heating :-

Heating of air without addition of moisture is termed as sensible heating.

The heating can be achieved by passing the air over heating coil like electric resistance heating coils or steam as shown in fig.



The heating process are represented on psychrometric chart.



The heat added raises the DBT of the dry air and DBT water vapour also.

$$\therefore Q_s (\text{heat added}) = g_{pa} (T_2 - T_1) + w g_{pv} (T_2 - T_1)$$

$$= (g_{pa} + w g_{pv}) (T_2 - T_1)$$

Given: The mean specific heat = $1.0216 \text{ kJ/kg}^\circ\text{C}$

By pass factor, $B = \frac{\text{Distance 2-3}}{\text{Distance 2-1}} = \frac{T_2 - T_3}{T_2 - T_1}$

If T_1 is the inlet temperature of air and temperature of the coil is T_2 , then the temperature of air coming out of the coil will be less than the temperature T_2 , say T_3 , ($T_3 < T_2$).

Sensible cooling

DBT decreases,

Specific humidity

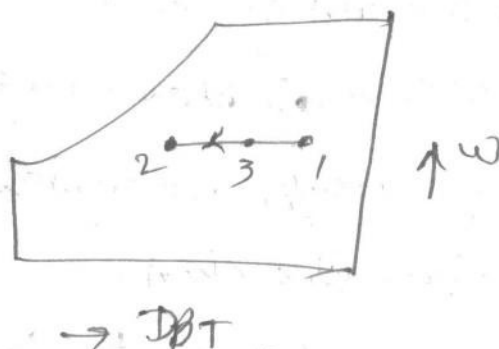
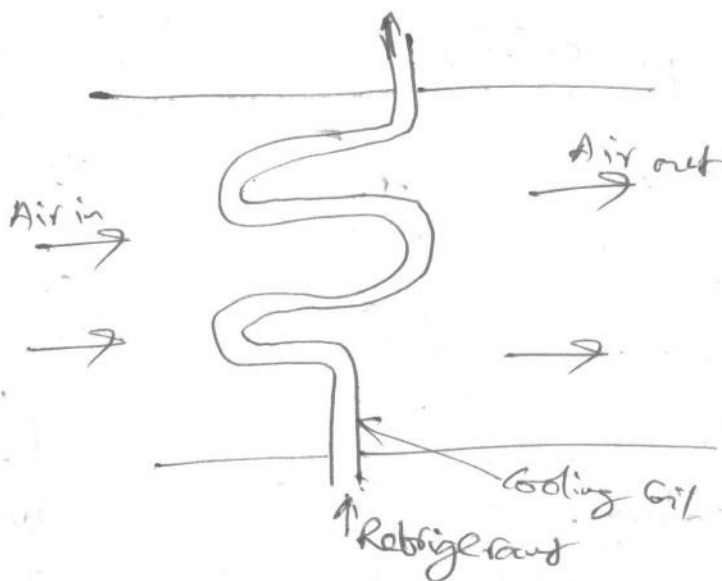
Constant. Cool

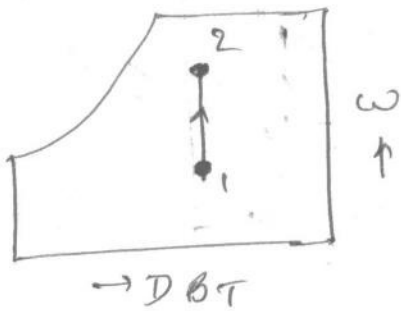
refrigerant flowing

in cooling coil

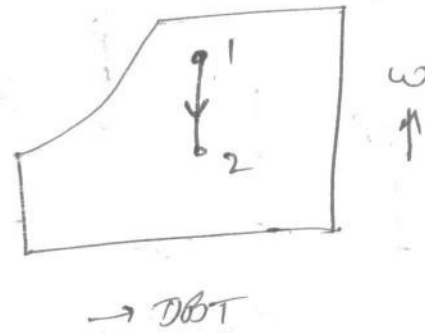
1 → Initial condition of air

2 → Final condition of air





Addition of moisture to air without changing its DBT is known as humidification.



Removing of moisture from air without ~~the~~ changing its DBT is known as dehumidification.

Cooling with dehumidification:-

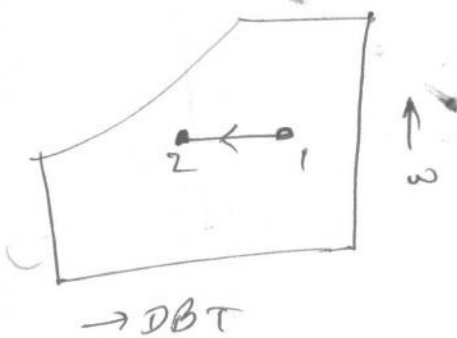


Fig: Cooling process

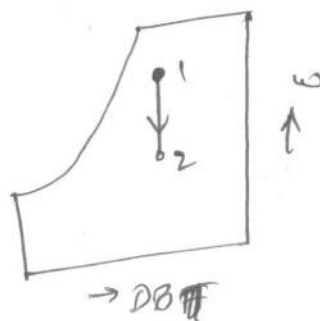


Fig: Dehumidification process

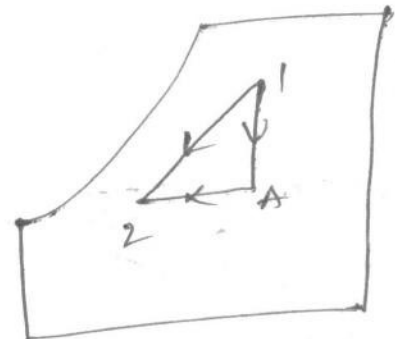


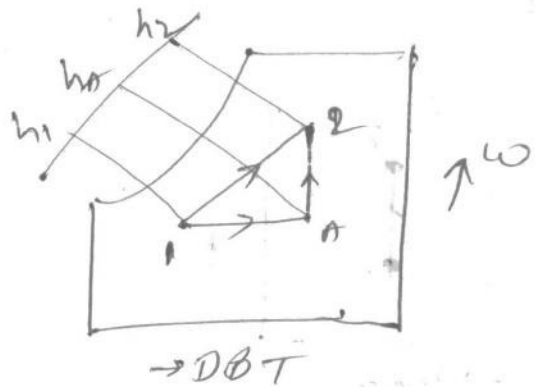
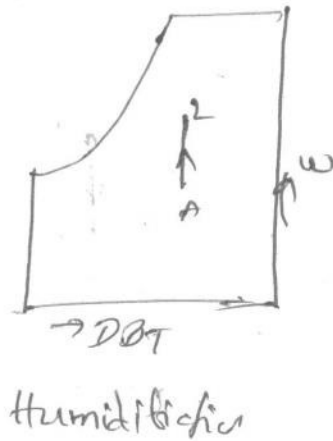
Fig: Cooling with dehumidification

1 - A → dehumidification process

A - 2 → cooling process

1 - 2 → cooling and dehumidification

Heating and humidification

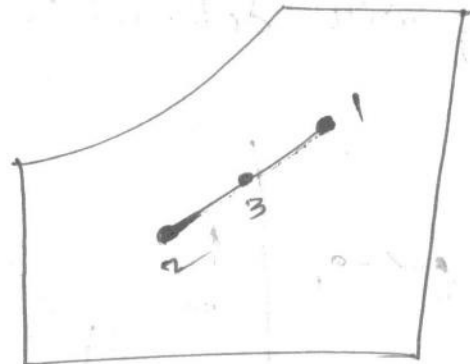
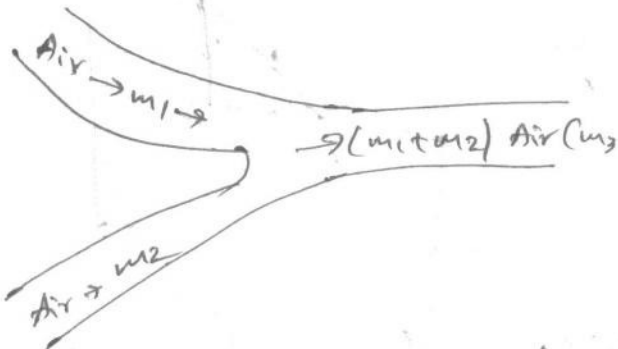


1-A → Heating

A-2 → humidification

1-2 → heating and humidification.

Mixing of air-streams:-



$$m_1 h_1 + m_2 h_2 = (m_1 + m_2) h_3 \quad \text{--- (1)}$$

$$m_1 w_1 + m_2 w_2 = (m_1 + m_2) w_3 \quad \text{--- (2)}$$

$$\therefore m_1 (h_1 - h_3) = m_2 (h_3 - h_2) \quad \text{--- (3)}$$

$$m_1 (w_1 - w_3) = m_2 (w_3 - w_2) \quad \text{--- (4)}$$

from equation (3) & (4)

$$\frac{m_1}{m_2} = \frac{h_3 - h_2}{h_1 - h_3} = \frac{w_3 - w_2}{w_1 - w_3} \quad \text{--- (5)}$$

- 1). explain 1). Saturated air 2). Vapour pressure 3). partial pressure.
2). define i). specific humidity ii). Relative humidity

Set 1, May 2013

Set 2, May 2013

Adiabatic mixing :— one kg of air at 35°C DBT and 60% RH is mixed with 2 kg of air at 20°C DBT and 13°C dew point temperature. Calculate the specific humidity of the mixture.

TE by
R.K. Raiput / R. K. Raiput
Page 456 / Arora
Ex 9.5 / Domkundwar
Page 16.34
Ex 16.12

$$\begin{array}{ll} m_1 = 1 \text{ kg} & + \quad m_2 = 2 \text{ kg} \\ \text{DBT} = 35^\circ\text{C} & \text{DBT} = 20^\circ\text{C} \\ \text{RH} = 60\% & \text{DPT} = 13^\circ\text{C} \end{array}$$

Calculate specific humidity of mixture.

For the air at 35°C DBT & 60% RH.

from Steam table at 35°C (Saturated water and steam (Temperature table))

partial pressure of water vapour when air is fully saturated (P_{vs})

$$P_{vs} = 0.0563 \text{ bar} \quad (\text{absolute pressure (bar)})$$

partial pressure of water vapour (P_v)

$$\therefore P_v = \phi \times P_{vs} = 0.6 \times 0.0563 = 0.0338 \text{ bar}$$

Specific humidity $w = 0.622 \frac{P_v}{P - P_v}$

$$= 0.622 \frac{0.0338}{1.0132 - 0.0338} = 0.0214 \text{ kg/kg of dry air.}$$

P or $P_t \rightarrow$ Total pressure

$$P = P_v + P_a$$

Standard value of atmospheric pressure

$$P = 1.0132 \text{ bar}$$

From Steam table, at $P_v = 0.0338 \text{ bar}$

at 26°C from steam table Saturated

$$T_{dp} = 26 + (27 - 26) \frac{(0.0338 - 0.0336)}{(0.0356 - 0.0336)} = 26.1^\circ\text{C}$$

at 27°C

From pressure table at $P_v = 0.0338$
roughly $T_{dp} = 26.1^\circ\text{C}$
 $0.030 - 24.1^\circ\text{C}$
 $0.035 - 26.1^\circ\text{C}$

Enthalpy $h = c_p T_{db} + w h_g$
from saturated water and steam (temperature table)

$h_g = 2565.3 \text{ kJ/kg}$

$$h = 1.005 T_{db} + w [h_g + 1.88 (T_{db} - T_{dp})]$$

$$= 1.005 \times 35 + 0.0214 [2565.3 + 1.88 (35 - 26.1)]$$

$= 1.005 \times 35 + 0.0214 [2565.3 + 1.88 (35 - 26.1)]$
 $= 90.43 \text{ kJ/kg of dry air.}$

for air at 20°C DBT and 13°C DPT :-

from steam table at 13°C

$p_v = 0.0150 \text{ bar}$

specific humidity $w = 0.622 \frac{p_v}{p - p_v}$

$= 0.622 \frac{0.0150}{1.0132 - 0.0150}$

$= 0.00935 \text{ kg/kg of dry air}$

Enthalpy $h = 1.005 T_{db} + w [h_g + 1.88 (T_{db} - T_{dp})]$

$= 1.005 \times 20 + 0.00935 [2538.1 + 1.88 (20 - 13)]$

at DBT, 20°C

$h_g = 2538.1 \text{ kJ/kg}$
 $= 43.95 \text{ kJ/kg of dry air.}$

Now enthalpy per kg of moist air

$= \frac{1}{3} \left[\frac{90.43}{1.0214} + \frac{43.95 \times 2}{1.00935} \right]$

$= 58.54 \text{ kJ/kg of moist air}$

Mass of vapour / kg of moist air

$= \frac{1}{3} \left[\frac{0.0214}{1.0214} + \frac{0.00935 \times 2}{1.00935} \right]$

$= 0.01316 \text{ kg/kg of moist air}$

$\frac{m_1}{m_1 + m_2} \left(\frac{h_1}{1 + w_1} + \frac{h_2 \times 2}{1 + w_2} \right)$

$\frac{m_1}{m_1 + m_2} \left(\frac{w_1}{1 + w_1} + \frac{w_2 \times 2}{1 + w_2} \right)$

∴ specific humidity of mixture

$$= \frac{0.01316}{1 - 0.01316}$$

$$= 0.1333 \text{ kg/kg of dry air.}$$

$$\frac{m_v}{1 - m_v}$$

- 4) Consider a mixture of several gases of identical masses. Show whether all mass fractions and mole fractions will be identical or not?
Set 10 Sep. 2014
- 5) write short notes on psychrometric chart
- 6) State Dalton's law of partial pressures. How is the partial pressure of a component in a gaseous mixture related to the mole fraction of that component.
- 7) Explain what you understand by adiabatic saturation.
- 8) Using the definitions of mass fraction and mole fraction, derive relation between them.
- 9) Explain the difference between dry bulb, wet bulb and dew point temperature.
- 10) The DBT and WBT of atmospheric air are 35°C and 23°C respectively when the barometer reads 75 cm of Hg. Determine ① Relative humidity ② humidity ratio ③ dew point temperature ④ density ⑤ enthalpy of atmospheric air.

Soln - From steam table:
at W.B.T, 23°C

$$P_{vs} = 21.05 \text{ mm of Hg}$$

$$\therefore (P_{vs})_{WBT} = 21.05 \text{ mm of Hg (from steam table).}$$

Carrier Equation

$$P_v = (P_{vs})_{WBT} - \frac{[P_t - (P_{vs})_{WBT}] (T_{DB} - T_{WB}) \times 1.8}{[2800 - 1.3 (1.8 T_{DB} + 32)]}$$

$$= 21.05 - \frac{[750 - 21.05] (35 - 23) \times 1.8}{[2800 - 1.3 (1.8 \times 35 + 32)]}$$

$$= 21.05 - 5.85 = 15.2 \text{ mm of Hg.}$$

① Relative humidity, $\phi = \frac{P_v}{P_s} = \frac{15.2}{42.5} = 0.36 = 36\%$

② Humidity ratio, $w = 0.622 \frac{P_v}{P - P_v} = 0.622 \times \frac{15.2}{750 - 15.2}$
 $= 0.01285 \text{ kg/kg of dry air}$
 $= 12.85 \text{ gms per kg of dry air}$

③ DPT is the saturation temperature at pressure (P_v) or 15.2 mm of Hg
 $\therefore \text{DPT} = 17.7^\circ\text{C}$ (from steam table)

④ Density of dry air $P_a = \frac{P_g}{R_a T} = \frac{750 - 15.2}{287 (35 + 273)}$ $P = P_a + P_v$
 $P_a = P - P_v$
 $= 1.1085 \text{ kg/m}^3$

Density of water vapor $P_v = \frac{P_v}{R_v T} = \frac{15.2 \times 133.5}{461 \times (35 + 273)}$
 $= 0.0145 \text{ kg/m}^3$

$\therefore$ Density of mixture $= P_a + P_v = 1.1085 + 0.0145$
 $= 1.123 \text{ kg/m}^3$

⑤ Enthalpy $= c_p T_{DB} + w (2500 + 1.88 T_{DB})$
 $= 1 \times 35 + 0.01285 (2500 + 1.88 \times 35)$
 $= 68.15 \text{ kJ/kg of dry air}$

In a dew point apparatus a metal beaker is cooled by gradually adding ice water to the water initially at room temperature. The moisture from the room air begins to condense on the beaker when its temp. is 12.8°C . If the room temp. is 21°C and the barometric pre. is 1.01325 bar , find the partial pre. of water vapour in the room air and parts by mass of water vapour in the room air.

Soln: partial pre. of water vapour at DPT of 12.8°C (from steam table)

$$P_v = 1.479 \text{ kN/m}^2 = 1479 \text{ N/m}^2$$

$$1 \text{ bar} = 10^5 \text{ N/m}^2$$

partial pre. of dry air

$$(\because P_a = P - P_v)$$

$$P_a = 101325 - 1479$$

$$= 99846 \text{ N/m}^2$$

specific humidity $w = \frac{m_v}{m_a} = 0.622 \frac{P_v}{P_a}$

$$= 0.622 \frac{1479}{99846}$$

$$= 0.009214 \text{ kg w.v. / kg d.a.}$$

parts by mass of water vapour:

$$\frac{m_v}{m} = \frac{w}{1+w} = \frac{0.009214}{1+0.009214}$$

$$= 0.00913 \text{ kg w.v. / kg mixture}$$

$$\frac{\frac{m_v}{m_a}}{1 + \frac{m_v}{m_a}}$$

$$\frac{m_v}{m_a + m_v}$$

$$\frac{m_v}{m}$$

A mixture of dry air and water vapour is at a temp. of 21°C under a total pre. of 736 mm Hg .

The dew point temp. is 15°C . Find:

- 1). Partial pre. of water vapour.
- 2). Relative humidity
- 3). Specific humidity
- 4). Specific enthalpy of water vapour ~~by the~~ ^{by the three methods} of fig.
- 5). Enthalpy of air per kg of dry air.
- 6). Specific volume of air per kg of dry air.

Soln:-

1). From steam tables, the partial pre. of water vapour at 15°C DPT is

$$\begin{aligned} P_v &= 12.79 \text{ mm Hg} \\ &= 12.79 (133.5) \quad \begin{matrix} 1 \text{ mm Hg} \\ = 133.5 \text{ N/m}^2 \end{matrix} \\ &= 1707.5 \text{ N/m}^2 \end{aligned}$$

2). Saturation pre. of water vapour at 21°C DBT

$$\begin{aligned} P_{vs} \text{ or } P_s &= 18.65 \text{ mm Hg. (from steam table)} \\ &= 18.65 (133.5) \\ &= 2489.8 \text{ N/m}^2 \end{aligned}$$

$$\therefore RH, \phi = \frac{P_v}{P_s} \times 100$$

$$= \frac{12.79}{18.65} \times 100 = 68.58\%$$

3) specific humidity $w = 0.622 \frac{P_v}{P - P_v}$

$$= 0.622 \frac{12.79}{736 - 12.79}$$

$$= 0.011 \text{ kg w.v. / kg d.a.}$$

4) latent heat of vaporization of water at dry bulb and dew-point temp. of air (from steam tables).

$$(h_{fg})_{21^\circ\text{C}} = 2452 \text{ kJ/kg}$$

$$(h_{fg})_{15^\circ\text{C}} = 2466.2 \text{ kJ/kg}$$

specific enthalpy of water

vapor from fig. by three methods

$$h_c = (4.1868)(21) + 2452 = 2540 \text{ kJ/kg w.v.}$$

$c_{pw} \cdot T + h_{fg}$

$$h_A = (4.1868)(15) + 2466.2 + 1.88(21 - 15)$$

$$h_B = (4.1868 t_d + (h_{fg})_d + 1.88(T - t_d)) = 2540.3 \text{ kJ/kg w.v.}$$

$$h_B = 2500 + 1.88(21)$$

$$= 2540.5 \text{ kJ/kg w.v.}$$

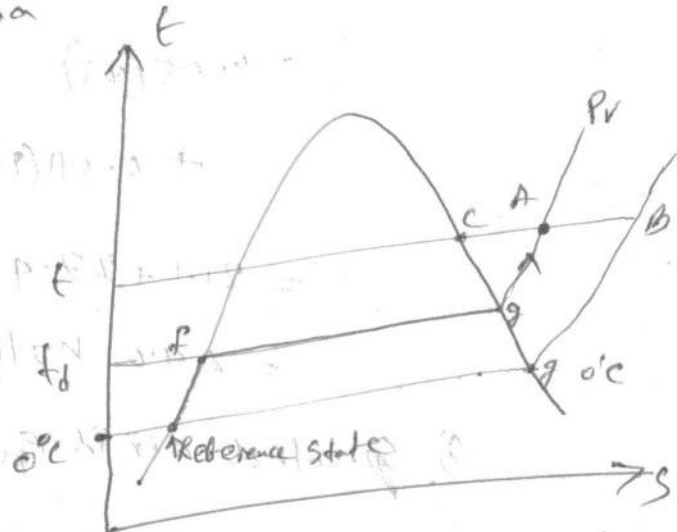


Fig. Evaluation of Enthalpy

of water vapor part.

Sp. heat of liquid water

$$c_{pw} = 4.1868 \text{ kJ/kg}^\circ\text{C}$$

$$t \rightarrow \text{DBT} = 21^\circ\text{C}$$

$$(h_{fg})_{21^\circ\text{C}} = 2456$$

$$t_d \rightarrow \text{DPT}$$

The three values are extremely close to each other.

Enthalpy of water vapour from the empirical relation

$$h = h_a + w h_v$$

$$= 1.005(21)$$

$$+ 0.011(2540.5)$$

$$= 21.1 + 27.9$$

$$= 49.0 \text{ kJ/kg d.a.}$$

$$h_a = \frac{c_p a t}{1000}$$

$$c_{pa} = 1.005$$

Sp. heat of dry air
 $t \rightarrow \text{DBT}$

9). Sp. vol. of air is equal to the volume of 1 kg of dry air or 0.011 kg of water vapour. Based on the dry air part

$$V_a = V_v = \frac{R_a T}{P_a}$$

$$P_a V = R T$$

$$= \frac{287.3 (273 + 21)}{723.21 \times 133.5}$$

$$= 0.875 \text{ m}^3/\text{kg d.a.}$$

3). Calculate ① RH ② humidity ratio ③ DPT ④ density ⑤ enthalpy of atm. air when the DBT is 35°C , WBT is 23°C and the barometer reads 750 mm Hg.

Soln:- $P_v' \rightarrow$ the sat. pre. at WBT (t')
 $t' = \text{WBT} = 23^\circ\text{C}$

$$\therefore P_v' = \text{sat. pre. at } 23^\circ\text{C} \text{ (Sat. properties of water)}$$

$$P_v = P_v' - \frac{(P - P_v')(t - t')(1.8)}{2800 - 1.3(1.8t + 32)}$$

$$t \rightarrow \text{DBT} = 35$$

$$= 21.06 - \frac{(750 - 21.06)(35 - 23)(1.8)}{2800 - 1.3(1.8 \times 35 + 32)}$$

$$= 21.06 - 5.88 = 15.18 \text{ mm Hg}$$

at DBT of 35°C

$$P_{vs} \text{ or } P_s = 42.4 \text{ mm Hg} \quad (\text{sat. pressure of water})$$

$$\therefore \text{RH} = \phi = \frac{P_v}{P_s} \times 100 = \frac{15.18}{42.4} \times 100 = 36\%$$

$$2). \text{ humidity ratio, } w = 0.622 \frac{P_v}{P - P_v}$$

$$= 0.622 \times \frac{15.18}{750 - 15.18} = 0.01285 \text{ kg w/v kg da}$$

$$3). \text{ D.P.T. at } \text{pre. } 15.18 \text{ mm Hg.} \\ (P_v)$$

$$t_d = 17.7^\circ\text{C} \quad (\text{sat. temp at } 15.18 \text{ mm Hg pressure})$$

4). partial densities

$$P_a = \frac{P_a}{R_a T} = \frac{(750 - 15.18)(133.5)}{(287.1)(35 + 273)} \quad R = P - P_v$$

$$= 1.1086 \text{ kg/m}^3$$

$$P_v = \frac{P_v}{R_v T} = \frac{13.12 \times 133.5}{(461) (273+35)}$$

$$= 0.0143 \text{ kg/m}^3$$

∴ density of moist air

$$p = p_a + p_v$$

$$= 1.086 + 0.0143 = 1.1229 \text{ kg/m}^3$$

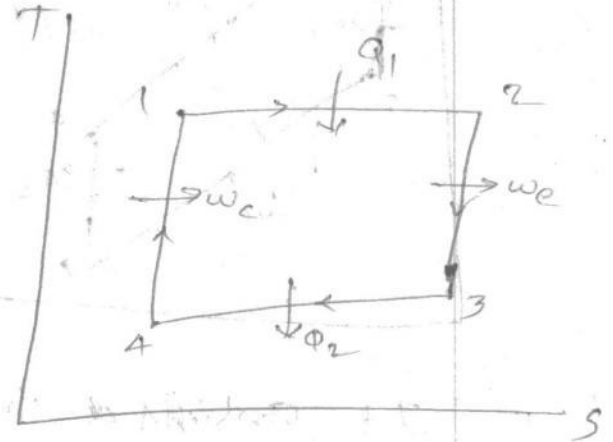
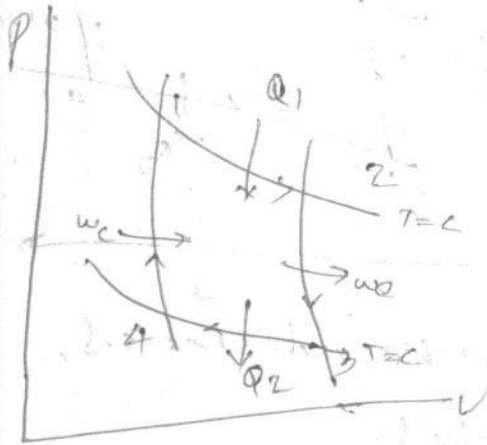
5). Enthalpy

$$h = 1.005 t + w(2500 + 1.88 t)$$

$$= 1.005 \times 35 + 0.01285 (2500 + 1.88 \times 35)$$

$$= 68.15 \text{ kJ/kg d.a.}$$

Carnot cycle (1824) :- The Carnot cycle consists of two reversible isotherms and two reversible adiabatics.



If an ideal gas is assumed as the working fluid, then for 1 kg of gas,

$$Q_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = 0$$

$$Q_{3-4} = RT_2 \ln \frac{V_4}{V_3}$$

$$Q_{4-1} = 0$$

$$W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$W_{2-3} = -C_v (T_3 - T_2)$$

$$W_{3-4} = RT_2 \ln \frac{V_4}{V_3}$$

$$W_{4-1} = C_v (T_1 - T_4)$$

$$\sum_{\text{cycle}} dQ = \sum_{\text{cycle}} dW$$

Now $\left(\frac{V_3}{V_2} \right)^{\gamma} = \left(\frac{T_2}{T_1} \right)^{1/\gamma-1}$ and $\frac{V_1}{V_4} = \left(\frac{T_2}{T_1} \right)^{1/\gamma-1}$

$$\therefore \frac{V_2}{V_3} = \frac{V_1}{V_4} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

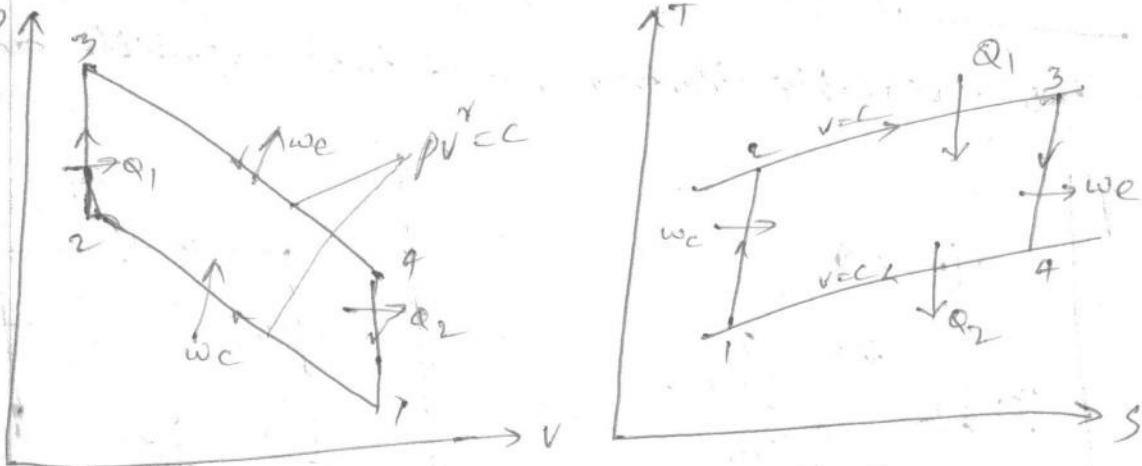
Therefore $Q_1 = \text{Heat added} = RT_1 \ln \frac{V_2}{V_1}$

$$W_{\text{net}} = Q_1 - Q_2 = R \ln \frac{V_2}{V_1} (T_1 - T_2)$$

$$\eta = \frac{W_{\text{net}}}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Otto Cycle (1876)

air Standard cycle of the S.I. Engine.



It consists of two reversible adiabatics, two constant volume processes. Air is compressed in process 1-2 reversibly and adiabatically. Heat is then added to air reversibly at constant volume process 2-3. Work is done by the air in expanding reversibly and adiabatically in process 3-4. Heat is then rejected by air reversibly at constant volume in process 4-1, and the system (air) comes back to its initial state.

Let m be the fixed mass of air undergoing the cycle or operations.

$$\text{Heat supplied } Q_1 = Q_{2-3} = m c_v (T_3 - T_2)$$

$$\text{Heat rejected } Q_2 = Q_{4-1} = m c_v (T_4 - T_1)$$

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{m c_v (T_4 - T_1)}{m c_v (T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{--- (1)}$$

process 1-2, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

process 3-4, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \frac{T_3}{T_2} = \frac{T_4}{T_1}$

$\frac{T_3}{T_2} - 1 = \frac{T_4}{T_1} - 1$

$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

From equation (1), $\eta = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}$

or $\eta_{\text{otto}} = 1 - \frac{1}{r_k^{\gamma-1}}$

where r_k is called the Compression ratio.

Diesel cycle (1832)!

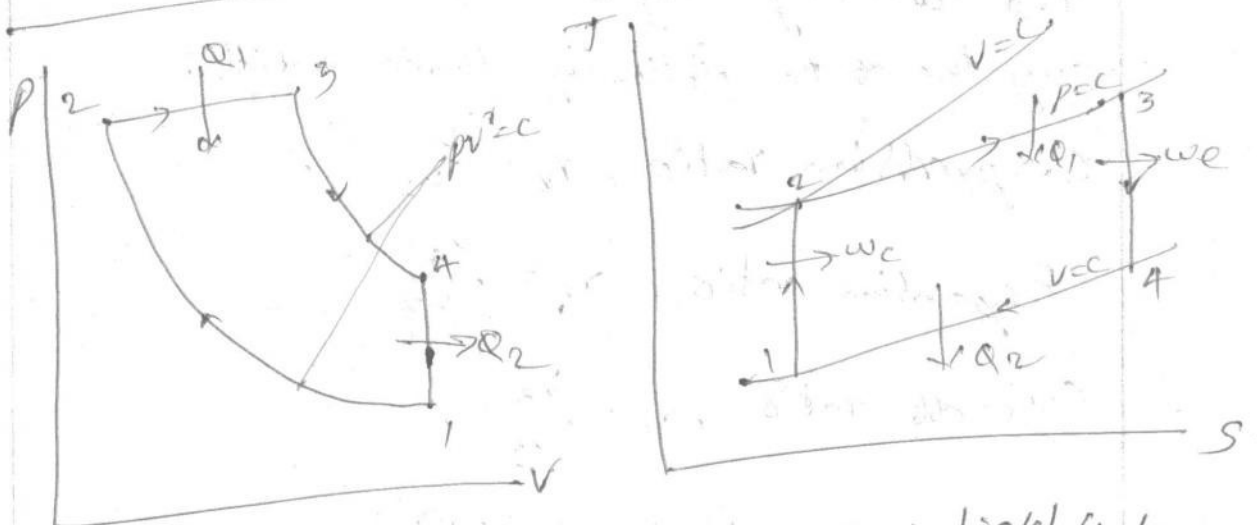


Fig. Shows the air standard cycle, called diesel cycle, corresponding to the C.I. engine. The cycle is composed of two reversible adiabatics, one reversible isobar, one reversible isochore. The air is compressed reversibly and

adiabatically in process 1-2, Heat is then added to it from an external source reversibly at constant pressure in process 2-3. Air then expands reversibly and adiabatically in process 3-4. Heat is rejected reversibly at constant volume process 4-1, and the cycle repeats itself. For m kg of air in the cylinder, the efficiency analysis of the cycle can be made as given below.

$$\text{Heat Supplied, } Q_1 = Q_{2-3} = m c_p (T_3 - T_2)$$

$$\text{Heat rejected, } Q_2 = Q_{4-1} = m c_v (T_4 - T_1)$$

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{m c_v (T_4 - T_1)}{m c_p (T_3 - T_2)}$$

$$\therefore \eta = 1 - \frac{T_4 - T_1}{\gamma (T_3 - T_2)} \quad \text{--- (i)}$$

The efficiency may be expressed in terms of any two of the following three ratios.

$$\text{Compression ratio, } r_k = \frac{V_1}{V_2} \quad \begin{matrix} \text{high value} \\ \text{low value} \end{matrix}$$

$$\text{Expansion ratio, } r_e = \frac{V_4}{V_3}$$

$$\text{Cut-off ratio, } r_c = \frac{V_3}{V_2}$$

$$\text{It is seen that } r_k = r_e \cdot r_c$$

$$\text{Process 3-4 } \frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} = \frac{1}{r_e^{\gamma-1}}$$

$$\Rightarrow T_4 = T_3 \frac{1}{r_e^{\gamma-1}}$$

$p v^\gamma = \text{const}$
 $p_1 v_1^\gamma = p_2 v_2^\gamma$
 $p_1 v_1 = p_2 v_2$
 $p_1 = p_2 \left(\frac{v_2}{v_1} \right)^\gamma$
 $p_1 = p_2 \left(\frac{v_2}{v_1} \right)^{\frac{\gamma}{\gamma-1}}$

$p v^\gamma = \text{const}$
 $p_1 v_1^\gamma = p_2 v_2^\gamma$
 $p_1 = p_2 \left(\frac{v_2}{v_1} \right)^\gamma$
 $p_1 = p_2 \left(\frac{v_2}{v_1} \right)^{\frac{\gamma}{\gamma-1}}$

$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\frac{\gamma}{\gamma-1}}$
 $\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\frac{\gamma}{\gamma-1}}$

$$\Rightarrow T_2 = T_3 \cdot \frac{1}{r_c}$$

process 1-2: $\frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma-1} = \frac{1}{r_c^{\gamma-1}}$

$$\Rightarrow T_1 = T_2 \cdot \frac{1}{r_c^{\gamma-1}} = \frac{T_3}{r_c} \cdot \frac{1}{r_c^{\gamma-1}}$$

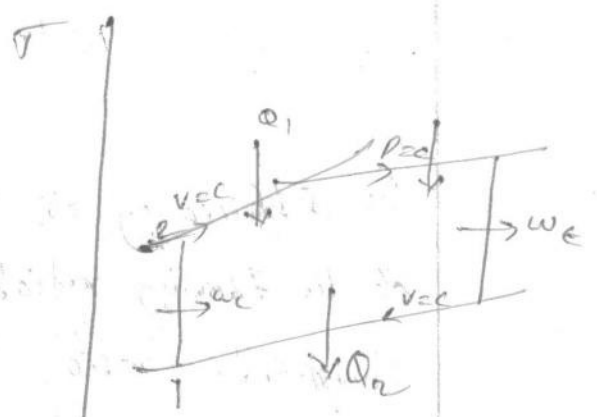
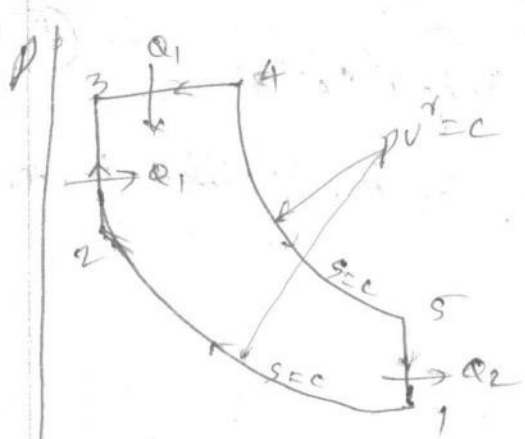
Substituting the values of T_1, T_2, T_4 in eqn (1)

$$\eta = 1 - \frac{T_3 \cdot \frac{r_c^{\gamma-1}}{r_c} - \frac{T_3}{r_c} \cdot \frac{1}{r_c^{\gamma-1}}}{r(T_3 - T_3 \cdot \frac{1}{r_c})}$$

$$\eta = 1 - \frac{1}{r} \cdot \frac{1}{r_c^{\gamma-1}} \cdot \frac{r_c^{\gamma} - 1}{r_c - 1}$$

As $r_c > 1$, $\frac{1}{r} \left(\frac{r_c^{\gamma} - 1}{r_c - 1} \right)$ is also greater than unity.
Therefore, the efficiency of diesel cycle is less than that of the otto cycle for the same compression ratio.

Dual cycle:-



Exactly the pressure-volume variation in an actual compression ignition engine, where the fuel injection is started before the end of compression stroke. A closer approximation is the limited pressure cycle in which some part of heat is added to air at constant volume and the remainder at constant pressure.

Fig. shows the $p-v$ and $T-s$ diagrams of the dual cycle. Heat is added reversibly, partly at constant volume (1-3) and partly at constant pressure (3-4).

$$\text{Heat supplied } Q_1 = m C_v (T_3 - T_2) + m C_p (T_4 - T_3)$$

$$\text{Heat rejected } Q_2 = m C_v (T_5 - T_1)$$

$$\text{Efficiency } \eta = 1 - \frac{Q_2}{Q_1}$$

$$= 1 - \frac{m C_v (T_5 - T_1)}{m C_v (T_3 - T_2) + m C_p (T_4 - T_3)}$$

$$= 1 - \frac{T_5 - T_1}{(T_3 - T_2) + \gamma (T_4 - T_3)} \quad \text{--- (1)}$$

The efficiency of the cycle can be expressed in terms of the following ratios

$$\text{Compression ratio } r_k = \frac{V_1}{V_2}$$

$$\text{Expansion ratio } r_e = \frac{V_5}{V_4}$$

$$\text{Cut-off ratio } r_c = \frac{V_4}{V_3}$$

It is seen, as before that

$$\gamma_k = \gamma_c \cdot \gamma_e$$

$$\Rightarrow \gamma_e = \frac{\gamma_k}{\gamma_c}$$

process 3-4, $\gamma_c = \frac{V_4}{V_3} = \frac{T_4 P_3}{P_4 T_3} = \frac{T_4}{T_3}$

$$\Rightarrow T_3 = \frac{T_4}{\gamma_c}$$

process 2-3, $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$

$$\Rightarrow T_2 = T_3 \frac{P_2}{P_3} = \frac{T_4}{\gamma_p \cdot \gamma_c}$$

process 1-2

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1} = \frac{1}{\gamma_k^{\gamma-1}}$$

$$\Rightarrow T_1 = \frac{T_4}{\gamma_p \gamma_c \gamma_k^{\gamma-1}}$$

process 4-5

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5} \right)^{\gamma-1} = \frac{1}{\gamma_e^{\gamma-1}}$$

$$\Rightarrow T_5 = T_4 \cdot \frac{\gamma_c^{\gamma-1}}{\gamma_k^{\gamma-1}}$$

Substituting the values of T_1, T_2, T_3 and T_5 in the

Equation (1)

$$T_4 \cdot \frac{\gamma_c^{\gamma-1}}{\gamma_k^{\gamma-1}} = \frac{T_4}{\gamma_p \gamma_c \gamma_k^{\gamma-1}}$$

$$\gamma = 1 - \frac{\left(\frac{T_4}{T_4} - \frac{T_4}{T_4} \right) + \gamma \left(T_4 - \frac{T_4}{\gamma_c} \right)}$$

$$\eta = 1 - \frac{1}{r_k^{r-1}} \cdot \frac{r p_c (r_c - 1)}{r p_c - 1 + r r_p (r_c - 1)}$$

Comparison of otto, diesel, and dual cycle! —

The three cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.

Fig. shows the comparison of otto, diesel and dual cycle for the same compression ratio

and heat rejection.

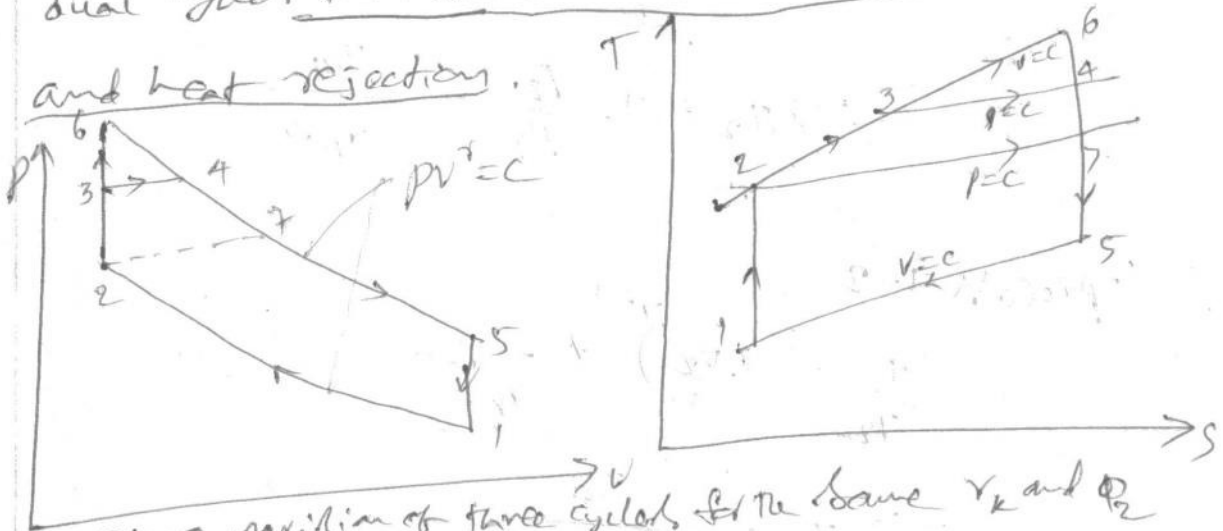


Fig: Comparison of three cycles for the same r_k and Q_2

1-2-6-5 — Otto cycle

1-2-7-5 — Diesel cycle

1-2-3-4-5 — dual cycle

For the same Q_2 , the higher the Q_1 , the higher the cycle efficiency. In the T-s diagram, the area under 2-6 represents Q_1 for Otto cycle, the area under 2-7 represents Q_1 for Diesel cycle, and the area under 2-3-4 represents the Q_1 for the dual cycle. Therefore, for the same r_k and Q_2

$$\eta_{\text{otto}} > \eta_{\text{dual}} > \eta_{\text{diesel}}$$

Same maximum pressure and temperature (state 4),
the heat rejection being also the same.

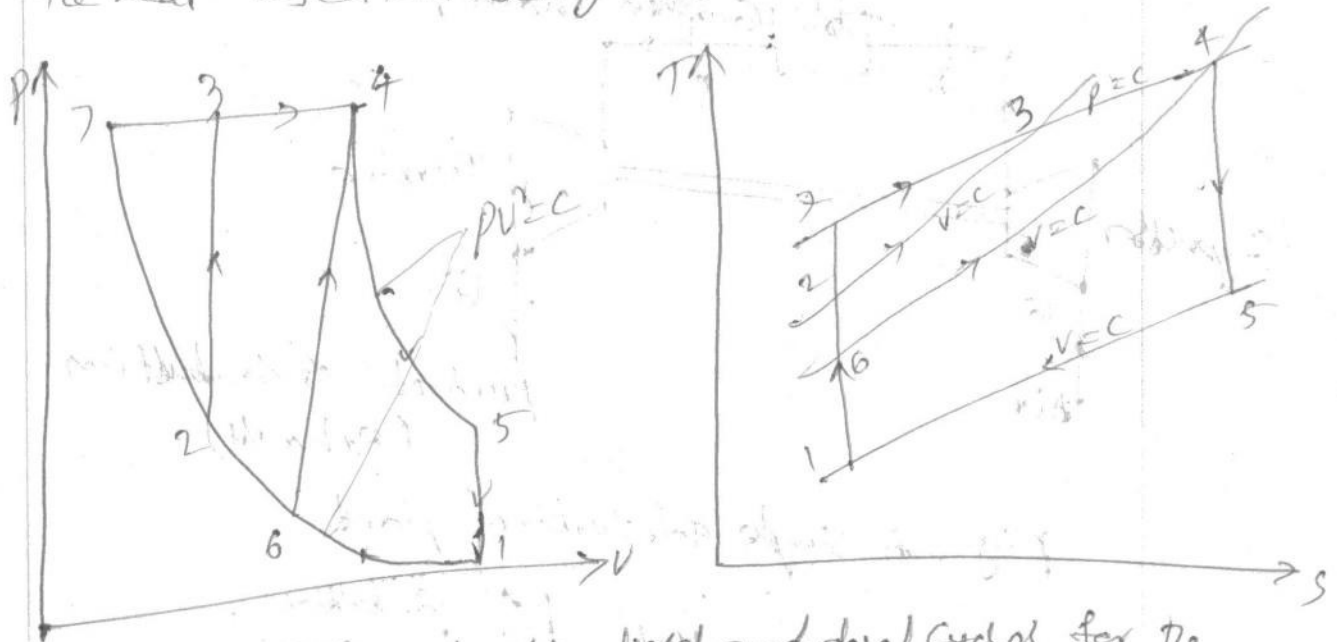


Fig. Comparison of Otto, Diesel and dual cycles for the same maximum pressure and temperature.

1-6-4-5 - Otto cycle

1-7-4-5 - Diesel cycle

1-2-3-4-5 - dual cycle

Q_1 is represented by the area under 6-4 for Otto cycle, by the area under 7-4 for the Diesel cycle and by the area under 2-3-4 for the dual cycle in the T-S plot, Q_2 being the same.

$$\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$$

This comparison is of greater significance, since the diesel cycle would definitely have a higher compression ratio than the Otto cycle.

Brayton Cycle

plant is shown in fig

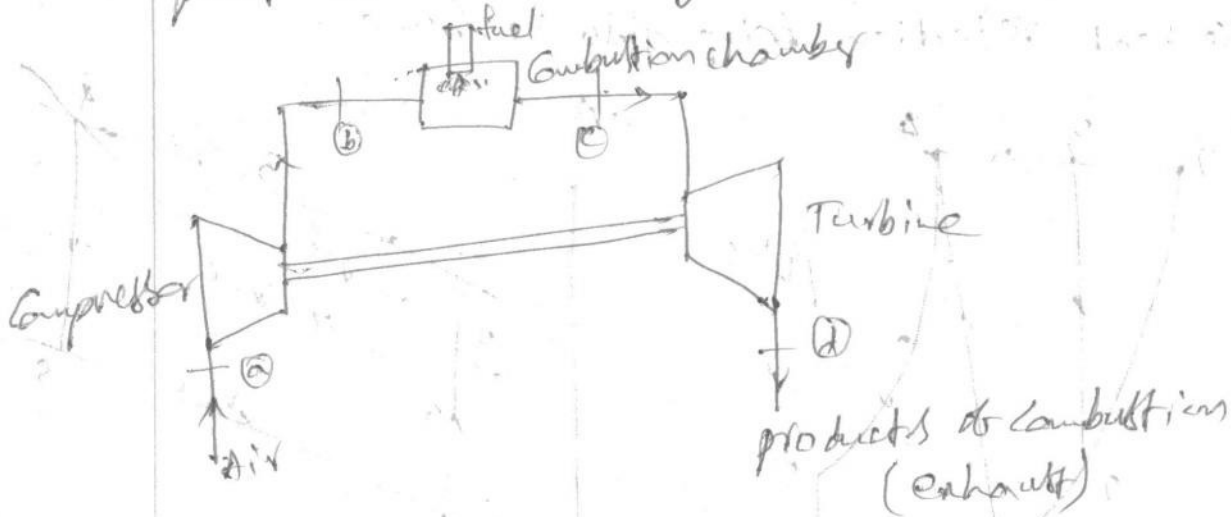


Fig: A single gas turbine plant

Air is first compressed adiabatically in process a-b, it then enters the combustion chamber where fuel is injected and burned essentially at constant pressure process b-c, and then the products of combustion expand in the turbine to the ambient pressure in process c-d and are thrown out to the surroundings. The cycle is open. The state diagram on the P-V coordinates is shown in fig. open cycle are used in air craft, automotive (buses and trucks) and industrial gas turbine installations.

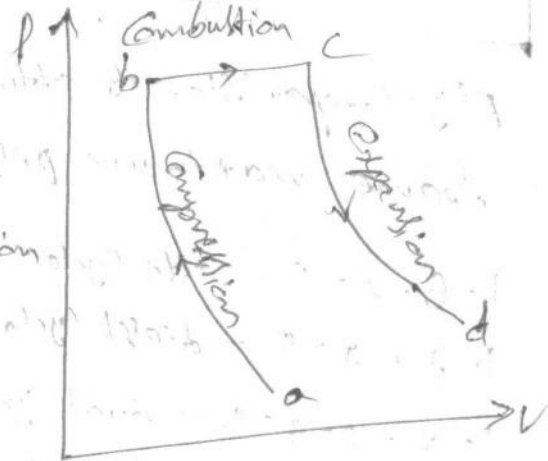
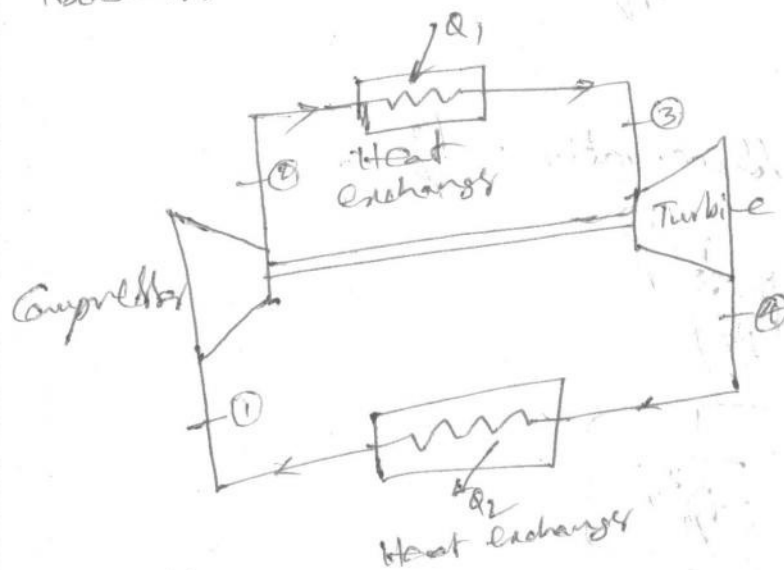
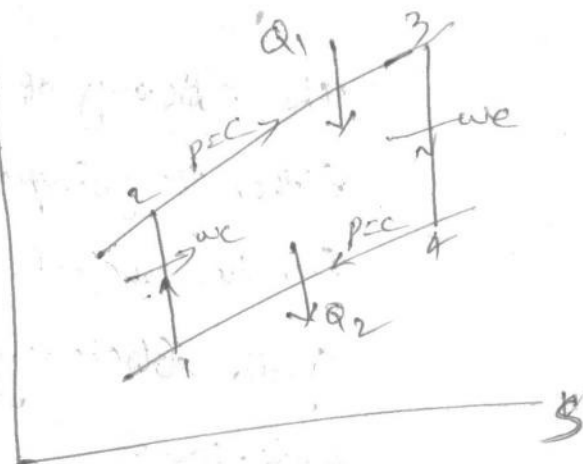


Fig: State diagram of a gas turbine plant on P-V plot

The Brayton cycle is the air standard cycle for the gas turbine power plant. Here air is first compressed reversibly and adiabatically, heat is added to it reversibly at constant pressure, air expands in the turbine reversibly and adiabatically, and heat is then rejected from the air reversibly at constant pressure to bring it to the initial state. The Brayton cycle consists of two reversible isobars and two reversible adiabatics.



For m kg of air
 $Q_1 = \text{heat supplied}$
 $= mc_p(T_3 - T_2)$
 $Q_2 = \text{heat rejected}$
 $= mc_p(T_4 - T_1)$



∴ Cycle efficiency $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$ — (1)

Now $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$

(∵ $P_2 = P_3$ and $P_4 = P_1$)

$$\frac{T_4}{T_1} - 1 = \frac{T_2}{T_2} - 1$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$

∴ $r_k = \text{Compression ratio} = \frac{V_1}{V_2}$

$$\eta = 1 - \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$

$$\boxed{\eta = 1 - \frac{1}{r_k^{\gamma-1}}}$$

∴ $r_p = \text{pressure ratio} = \frac{P_2}{P_1}$

$$\eta = 1 - \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\eta = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}$$

The efficiency of the Brayton cycle depends upon either the compression ratio or the pressure ratio. For the same compression ratio, the Brayton cycle efficiency is equal to the Otto cycle efficiency.

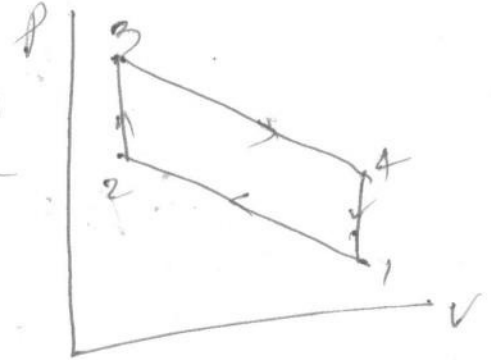
An engine working on the Otto cycle is supplied with air at 0.1 MPa, 35°C. The compression ratio is 8. Heat supplied is 2100 kJ/kg. Calculate the maximum pressure and temperature of the cycle, the cycle efficiency, and the mean effective pressure. (for air, $c_p = 1.005$, $c_v = 0.718$, and $R = 0.287$ kJ/kgK).

Soln:- $T_1 = 273 + 35 = 308$ K

$P_1 = 0.1$ MPa = 100 kN/m²

$Q_1 = 2100$ kJ/kg

$r_k = 8$, $\gamma = 1.4$



$\eta = 1 - \frac{1}{r_k^{\gamma-1}} = 1 - \frac{1}{8^{0.4}} = 1 - \frac{1}{2.3} = 0.565$ or 56.5%

$\frac{V_1}{V_2} = 8$, $V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 308}{100} = 0.884$ m³/kg

$V_2 = \frac{0.884}{8} = 0.11$ m³/kg

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (8)^{0.4} = 2.3$

$T_2 = 2.3 \times 308 = 708.4$ K

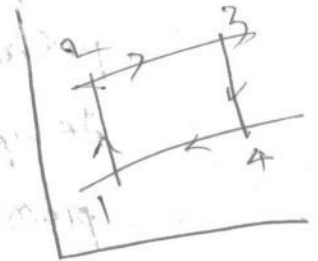
$Q_1 = c_v (T_3 - T_2) = 2100$ kJ/kg

$T_3 = 708.4 + \frac{2100}{0.718} = 3633$ K

$T_3 = T_{max} = 3633$ K

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (8)^{1.4} = 18.37$

$\therefore P_2 = 1.873$ MPa



Again $\frac{p_3 v_3}{T_3} = \frac{p_2 v_2}{T_2}$

$$p_3 = p_{max} = 1.837 \times \frac{363}{708} = 9.426 \text{ mpa}$$

$$W_{net} = Q_1 \times \eta = 2100 \times 0.565$$

$$= 1186.5 \text{ kJ/kg}$$

$$= p_m (v_1 - v_2) = p_m (0.884 - 0.11)$$

$$\therefore p_m = \text{m.e.p.} = \frac{1186.5}{0.774} = 1533 \text{ kpa}$$

$$= 1.533 \text{ mpa}$$

2) In an air standard diesel cycle, the compression ratio is 16, and at the beginning of isentropic compression, the temperature is 15°C and the pressure is 0.1 mpa . Heat is added until the temperature at the end of the constant pressure process is 1480°C . Calculate (a) the cut-off ratio (b) the heat supplied per kg of air (c) the cycle efficiency, (d) the m.e.p.

Soln: $r_k = \frac{v_1}{v_2} = 16$

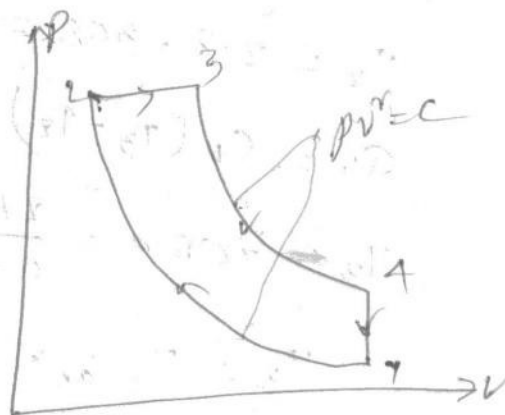
$$T_1 = 273 + 15 = 288 \text{ K}$$

$$p_1 = 0.1 \text{ mpa} = 100 \text{ kN/m}^2$$

$$T_3 = 1480 + 273 = 1753 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = (16)^{0.4} = 3.03$$

$$\Rightarrow T_2 = 288 \times 3.03 = 873 \text{ K}$$



$$a). \gamma_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1753}{873} = 2.01$$

$$b). \text{Heat supplied } Q_1 = C_p (T_3 - T_2) \\ = 1.005 (1753 - 873) \\ = 884.4 \text{ kJ/kg}$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3} \right)^{\gamma-1} = \left(\frac{V_1}{V_2} \times \frac{V_2}{V_3} \right)^{\gamma-1} = \left(\frac{16}{2.01} \right)^{0.4} = 2.29$$

$$\therefore T_4 = \frac{1753}{2.29} = 766 \text{ K}$$

$$\text{Heat rejected } Q_2 = C_v (T_4 - T_1) \\ = 0.718 (766 - 288) = 343.2 \text{ kJ/kg}$$

$$c). \text{Cycle efficiency} = 1 - \frac{Q_2}{Q_1} \\ = 1 - \frac{343.2}{884.4} = 0.612 \text{ or } 61.2\%$$

$$\eta_{\text{cycle}} = 1 - \frac{1}{\gamma} \cdot \frac{1}{\gamma_c^{\gamma-1}} \cdot \frac{\gamma_c^{\gamma} - 1}{\gamma_c - 1} \\ = 1 - \frac{1}{1.4} \cdot \frac{1}{16^{0.4}} \cdot \frac{(2.01)^{1.4} - 1}{2.01 - 1} = 0.612 \text{ or } 61.2\%$$

$$W_{\text{net}} = Q_1 \times \eta = 884.4 \times 0.612 = 541.3 \text{ kJ/kg}$$

$$V_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 288}{100} = 0.827 \text{ m}^3/\text{kg}$$

$$V_2 = \frac{0.827}{16} = 0.052 \text{ m}^3/\text{kg}$$

$$V_1 - V_2 = 0.827 - 0.052 = 0.775 \text{ m}^3/\text{kg}$$

$$d). \text{m.e.p.} = \frac{W_{\text{net}}}{V_1 - V_2} = \frac{541.3}{0.775} = 698.45 \text{ kPa}$$

An air standard Otto cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C . The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Estimate

- (a) the pressures and temperatures at the cardinal points of the cycle. (b) the cycle efficiency, and (c) the m.e.p. of the cycle, $C_v = 0.718 \text{ kJ/kgK}$, $C_p = 1.005 \text{ kJ/kgK}$.

Soln: $T_1 = 273 + 50$
 $= 323 \text{ K}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (16)^{0.4}$$

$$\Rightarrow T_2 = 979 \text{ K}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 1.0 \times (16)^{1.4} = 48.5 \text{ bar}$$

$$T_3 = T_2 \cdot \frac{P_3}{P_2} = 979 + \frac{70}{48.5} = 1413 \text{ K}$$

$$Q_{2-3} = C_v(T_3 - T_2) = 0.718(1413 - 979)$$

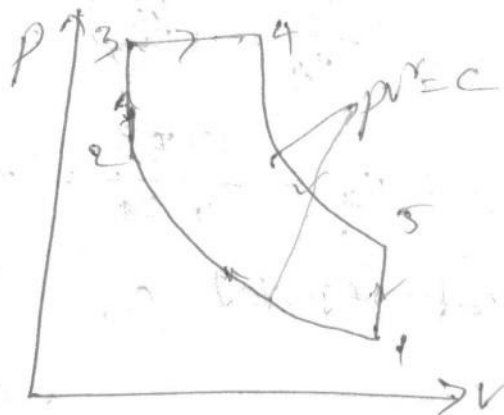
$$= 312 \text{ kJ/kg}$$

$$Q_{2-3} = Q_{3-4} = C_p(T_4 - T_3)$$

$$T_4 = \frac{312}{1.005} + 1413 = 1723 \text{ K}$$

$$\frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{1723}{1413} = 1.22$$

$$\therefore \frac{V_5}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{16}{1.22} = 13.1$$



$$T_5 = T_4 \left(\frac{V_4}{V_5} \right)^{\gamma-1} = 1723 \times \frac{1}{(13.1)^{0.4}} = 615 \text{ K}$$

$$P_5 = P_1 \left(\frac{T_5}{T_1} \right) = 1.0 \times \frac{615}{323} = 1.9 \text{ bar}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{0.718(615 - 323)}{312 + 312} = 0.665 \text{ or } 66.5 \%$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \text{ kJ/kg} \cdot \text{K} \times 323 \text{ K}}{10^2 \text{ kN/m}^2} = 0.927 \text{ m}^3/\text{kg}$$

$$v_1 - v_2 = v_1 - \frac{v_1}{16} = \frac{15}{16} v_1$$

$$w_{\text{net}} = Q_1 \times \eta = 0.665 \times 624 \text{ kJ/kg}$$

$$\therefore \text{m.e.p} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{414.96}{\frac{15}{16} \times 0.927 \text{ m}^3/\text{kg}} = 476 \text{ kN/m}^2 = \underline{\underline{4.76 \text{ bar}}}$$

Stirling cycle (1827):

It consists of

Two reversible ~~the~~ ^{isothermal} (constant temperature) _{process}

Two reversible isochores (constant volume) _{process}.

For 1 kg of ideal gas

$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = -C_V (T_2 - T_1), \quad W_{2-3} = 0$$

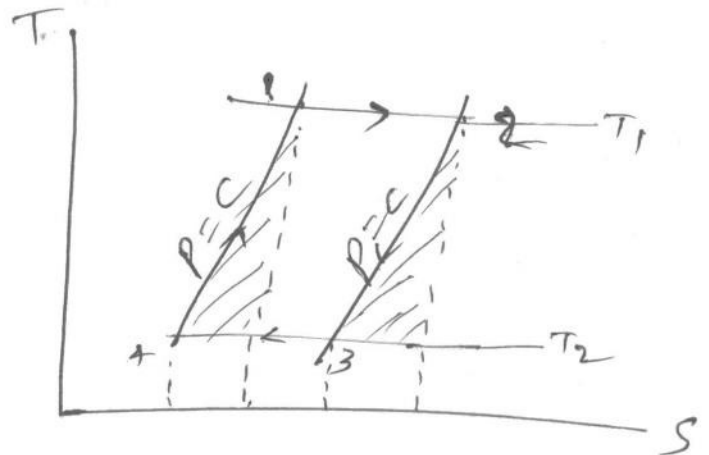
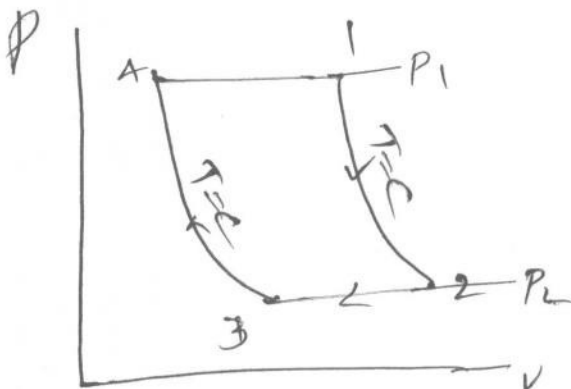
$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{V_3}{V_4}$$

$$Q_{4-1} = C_V (T_1 - T_2), \quad W_{4-1} = 0$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4}}{RT_1 \ln \frac{V_2}{V_1}}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}}$$

Erickson cycle (1850):



If regenerator is used, area under 2-3 is equal to area under 4-1, then the cycle efficiency is same as the Carnot cycle.

It consists of Two reversible isotherms (Constant temperature processes)

Two reversible isobars (Constant pressure processes)

For 1 kg of ideal gas

$$Q_{1-2} = W_{1-2} = R T_1 \ln \frac{P_1}{P_2}$$

$$Q_{2-3} = C_p (T_2 - T_1), \quad W_{2-3} = P_2 (V_3 - V_2) = R (T_2 - T_1)$$

$$Q_{3-4} = W_{3-4} = -R T_2 \ln \frac{P_1}{P_2}$$

$$Q_{4-1} = C_p (T_1 - T_2), \quad W_{4-1} = P_1 (V_1 - V_4) = R (T_1 - T_2)$$

Since part of the heat is transferred at constant pressure and part at constant temperature, the efficiency of the Ericsson cycle is less than that of the Carnot cycle. But with ideal regeneration, $Q_{2-3} = Q_{4-1}$ so that all the heat added from the external source at T_1 and all the heat is rejected to an external sink at T_2 , the efficiency of the cycle becomes equal to the Carnot cycle efficiency, since

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{R T_2 \ln \frac{P_1}{P_2}}{R T_1 \ln \frac{P_1}{P_2}} = 1 - \frac{T_2}{T_1}$$

Lenoir Cycle: — The Lenoir cycle

www.FirstRanker.com

www.FirstRanker.com

(3)

Consists of the following processes:

process 1-2 : Constant volume heat addition

process 2-3 : Reversible adiabatic expansion

process 3-4 : Constant pressure heat rejection

No compression process.

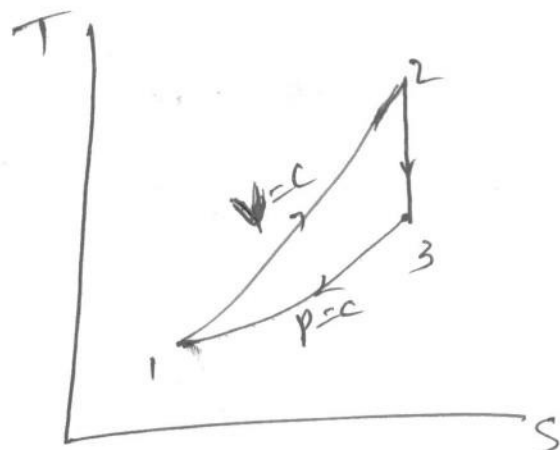
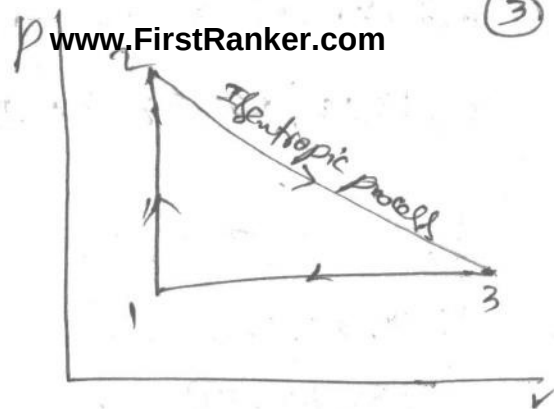
$$\eta = \frac{\text{Heat added} - \text{Heat rejected}}{\text{Heat added}}$$

$$= \frac{C_v (T_2 - T_1) - C_p (T_3 - T_1)}{C_v (T_2 - T_1)}$$

$$= 1 - \gamma \left(\frac{T_3 - T_1}{T_2 - T_1} \right)$$

$$= 1 - \gamma \left(\frac{T_1 (\gamma_p)^{\frac{1}{\gamma}} - T_1}{\gamma_p T_1 - T_1} \right)$$

$$\boxed{\eta = 1 - \gamma \left(\frac{(\gamma_p)^{\frac{1}{\gamma}} - 1}{\gamma_p - 1} \right)}$$



let $\frac{P_2}{P_1} = \gamma_p = \text{pressure ratio}$

$$\frac{T_2}{T_1} = \gamma_p$$

$$T_2 = \gamma_p \cdot T_1$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}} = (\gamma_p)^{\frac{1-\gamma}{\gamma}}$$

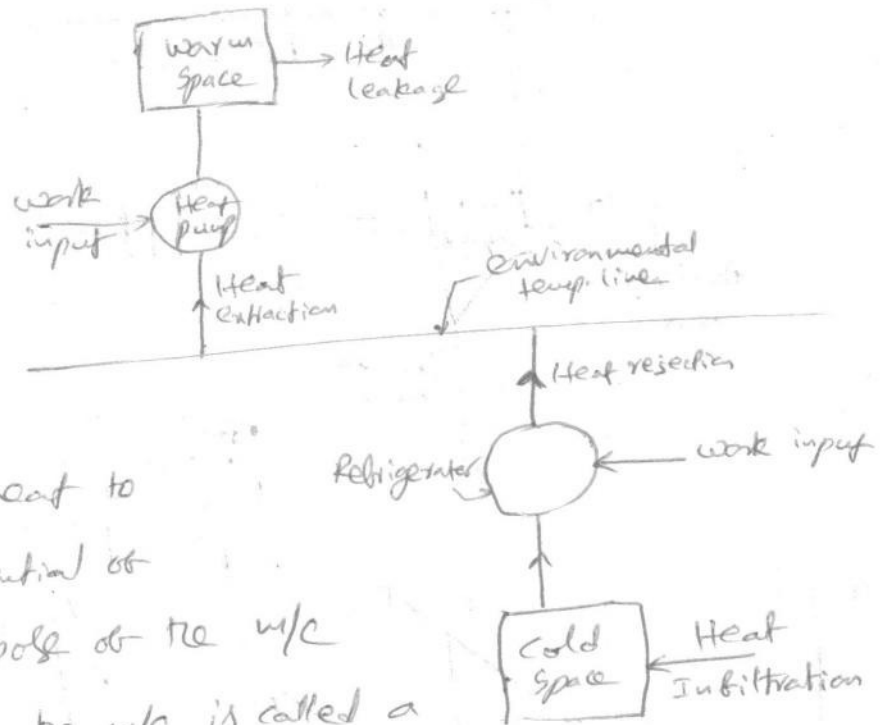
$$= (\gamma_p)^{\frac{1}{\gamma} - 1}$$

$$T_3 = T_2 (\gamma_p)^{\frac{1}{\gamma} - 1} = \gamma_p T_1 (\gamma_p)^{\frac{1}{\gamma} - 1}$$

$$= T_1 (\gamma_p)^{\frac{1}{\gamma}}$$

Refrigeration is the cooling of a system below the temp of its surrounding.

Refrigerator and heat pumps are the machines which are used to remove heat from a body at low temp level and then transfer that heat to another body at a high potential of temp. When the main purpose of the m/c is to cool some object, the m/c is called a



refrigerator. The term heat pump is applied to a machine whose main objective is to heat a medium which may already be warmer than its surrounding.

$$C.O.P = \frac{\text{Desired effect}}{\text{work input}}$$

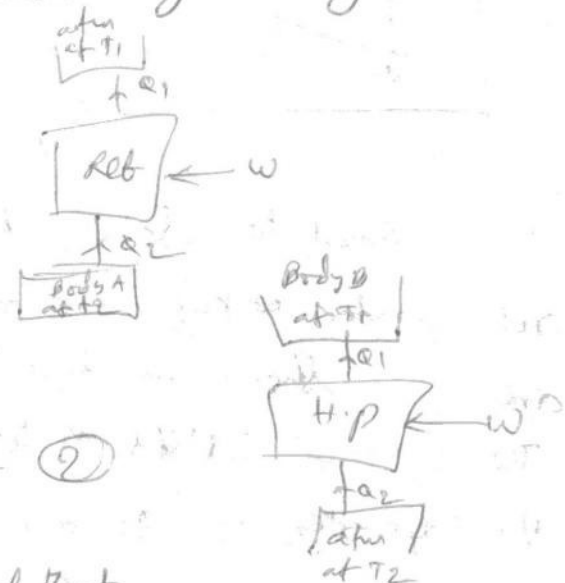
$$(COP)_{ref} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad \text{--- (1)}$$

$$(COP)_{H.P} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} \quad \text{--- (2)}$$

from (1) & (2), it is found that

$$(COP)_{HP} = (COP)_{ref} + 1$$

$$\begin{aligned} (COP)_{ref} + 1 &= \frac{Q_2}{Q_1 - Q_2} + 1 \\ &= \frac{Q_2 + Q_1 - Q_2}{Q_1 - Q_2} \\ &= \frac{Q_1}{Q_1 - Q_2} \\ &= (COP)_{HP} \end{aligned}$$



Bell-Coleman cycle.

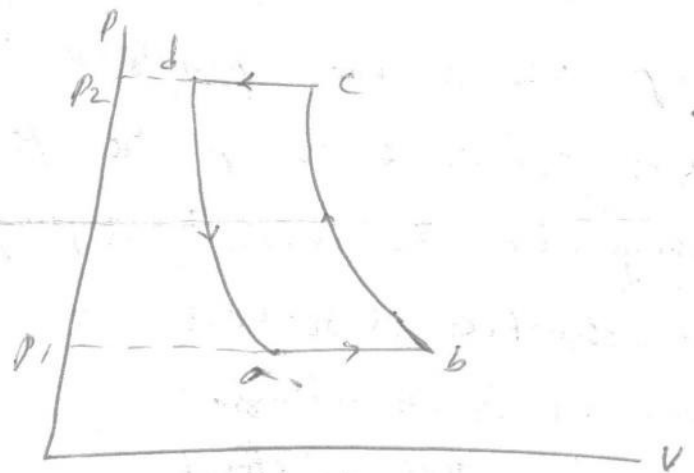
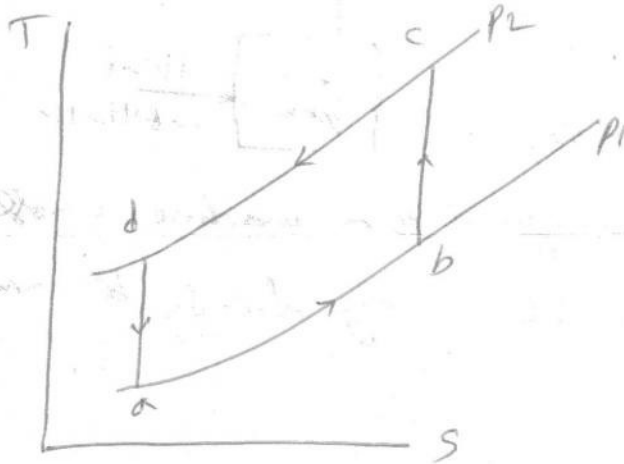
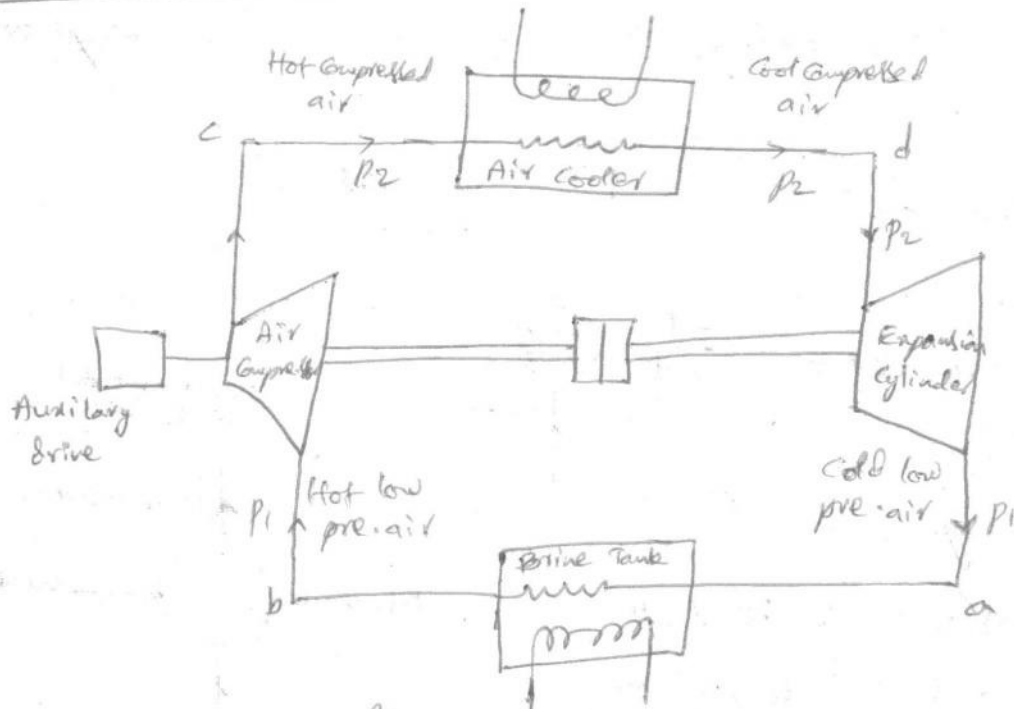


Fig. shows the Schematic diagram of the Bell-Coleman air refrigerator working on the reversed Joule cycle. $p-v$ and $T-s$ plots are also shown in fig.

The system consists of the following parts:

1. Expander:- The air at high pre (P_2) and moderate temp (T_2) is expanded in the Expanding cylinder to lower pre (P_1) and lowest temp (T_1) existing in the cycle.

2. Refrigerating chamber or cold room:- Air leaving the expansion cylinder at temp (T_1) and pre (P_1) is made to flow through the cylinder which is required to be cooled. The cold low pre.

from the air (Storage of air conditioning) at const. pre. p_1 , resulting in an increase in its temp to T_b .

3). Compressor :— Air is compressed in the compression cylinder to a relatively higher pre p_2 . The temp also increases to T_c . The work done during expansion process assists in driving the compressor unit. The remaining power comes from the auxiliary power device.

4). Air Cooler :— The cooler contains a number of pipes immersed in circulating water. The hot compressed air leaving the compressor passes through the pipes. The heat is rejected at const. pre. p_2 and the air is cooled to temp T_d .

The air then enters the expansion cylinder and the cycle is repeated.

C.O.P. for Bell-Coleman cycle :—

$$\text{Refrigerating effect, i.e. heat absorbed from cold chamber} = c_p (T_b - T_a)$$

$$\text{Heat rejected in air cooler} = c_p (T_c - T_d)$$

$$\text{work input} = \text{heat rejected} - \text{heat abstracted}$$

$$= c_p [(T_c - T_d) - (T_b - T_a)]$$

$$\text{C.O.P.} = \frac{\text{refrigerating effect}}{\text{work input}}$$

$$= \frac{T_b - T_a}{(T_c - T_d) - (T_b - T_a)}$$

are both isentropic.

Since both Expansion and Compression are with in the same pressure limits.

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} \text{ as both are equal to } \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \text{C.O.P} = \frac{\frac{T_c \cdot T_a}{T_d} - T_a}{\left(\frac{T_d T_b}{T_a} - T_d\right) - \left(\frac{T_a T_c}{T_d} - T_a\right)}$$

$$\frac{T_a \left(\frac{T_c}{T_d} - 1\right)}{T_d \left(\frac{T_b}{T_a} + 1\right) - T_a \left(\frac{T_c}{T_d} - 1\right)}$$

The terms inside the brackets in the above expression are equal

$$\therefore \text{C.O.P} = \frac{T_a}{T_d - T_a}$$

Bell-Coleman refrigerator may work on the open system or closed system. In open system the air comes in direct contact with the refrigerating chamber or the bodies being cooled. In closed system the air is passed through coils placed in the chamber. Quite often the coils carrying cold air are kept in the brine tank. The air extracts heat from the brine and the resulting chilled brine is then circulated around the cold chamber.

Brine $\rightarrow$ aqueous solution of NaCl (sodium chloride) or CaCl_2 in water.
The brine does not freeze at low temp.

Reversed Carnot cycle:-

A reversed heat engine cycle operating in the reverse way, i.e. receiving heat from a low temperature region, discharging heat to a high temperature region, and receiving the net inflow of work.

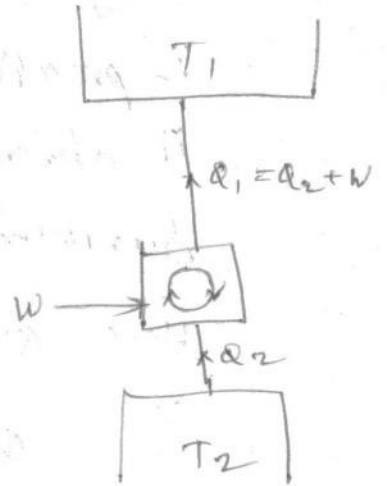


Fig: Reversed heat engine cycle

$$(COP)_{HP} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\text{and } (COP)_{Ref} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

The working fluid in a refrigeration cycle is called a refrigerant.

In the reversed Carnot cycle the refrigerant is first compressed reversibly and adiabatically in process 1-2 where the work input per kg of refrigerant is w_c , then it is expanded reversibly in process 2-3 where the heat rejection is Q_1 , the

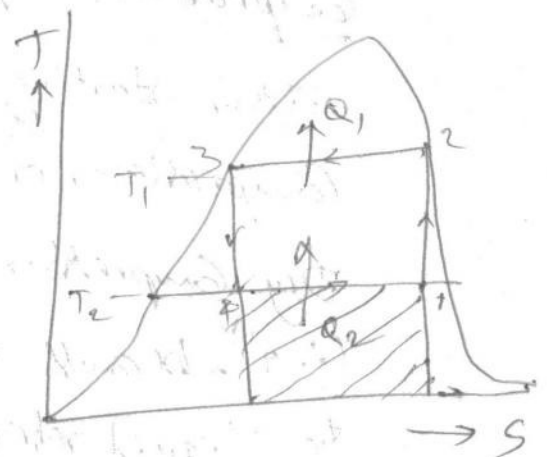
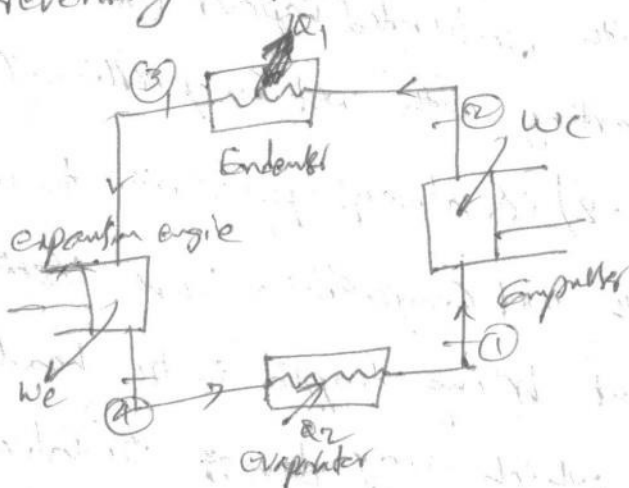


Fig: Reversed Carnot cycle

refrigerant then compressed reversibly and adiabatically in process 3-4 where the work output is w_e and finally it absorbs heat Q_2 reversibly by evaporation from the surroundings in process 4-1.

$$\text{Here } Q_1 = T_1 (S_2 - S_3)$$

$$Q_2 = T_2 (S_1 - S_4)$$

$$W_{\text{net}} = W_c - W_e = Q_1 - Q_2 = (T_1 - T_2) (S_1 - S_4)$$

where T_1 is the temperature of heat rejection
 T_2 is the temperature of heat absorption.

$$(\text{COP})_{\text{ref. rev}} = \frac{Q_2}{W_{\text{net}}} = \frac{T_2}{T_1 - T_2}$$

$$\text{and } (\text{COP})_{\text{hp rev}} = \frac{Q_1}{W_{\text{net}}} = \frac{T_1}{T_1 - T_2}$$

Vapour Compression refrigeration cycle:- The basic

operations involved in vapour compression cycle is Compression, Condensation, expansion and evaporation.

Compression:- A reversible adiabatic process 1-2 or 1'-2'

either starting with saturated vapour (state 1), called dry compression, or starting with wet compression (state 1').

Dry compression (1-2) is always preferred to wet compression (1'-2'), because with wet compression there is a danger of the liquid refrigerant being trapped in the head of the cylinder by the rising piston which may damage the valves or the cylinder head, and the droplets of liquid refrigerant may wash away

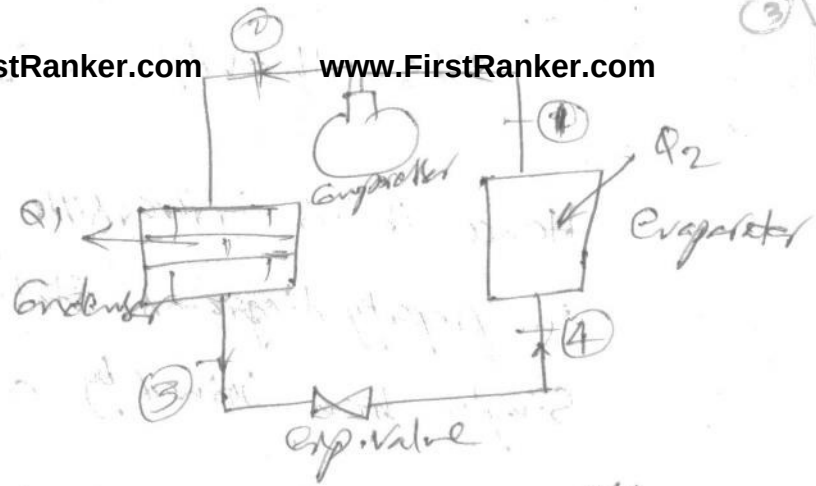
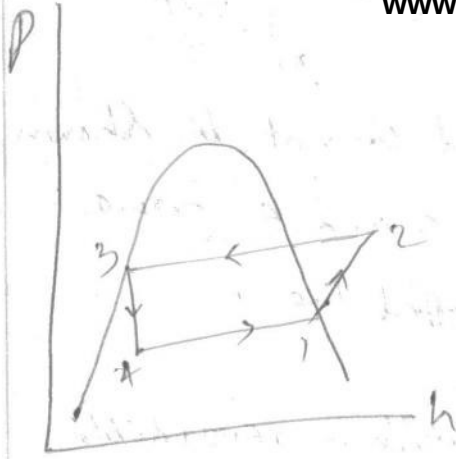


Fig: Vapour Compression refrigeration flow diagram.

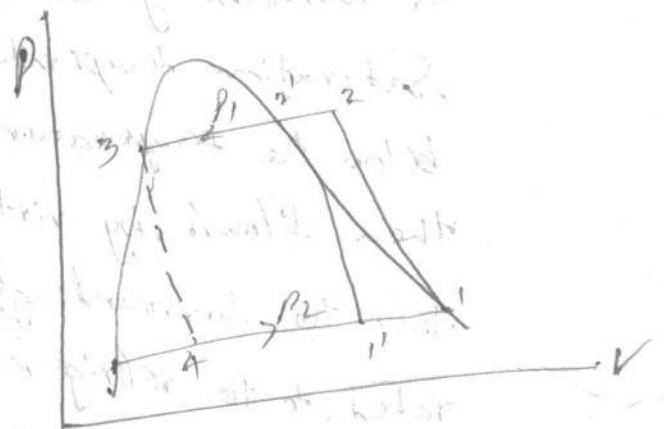
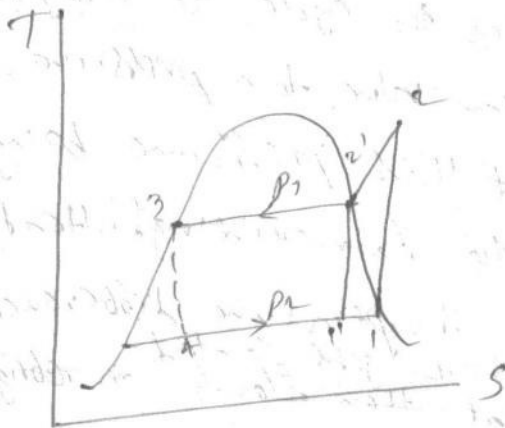


Fig: Vapour Compression refrigeration cycle - property diagrams.

The lubricating oil from the walls of the cylinder, thus accelerating wear.

Cooling and Condensing: A reversible constant pressure process, 2-3, first de-superheated and then condensed, ending with Saturated liquid. Heat Q_2 is transferred out.

Expansion: An adiabatic throttling process 3-4, first which enthalpy remains unchanged. State 3 and 4 are Equilibrium point. process 3-4 is adiabatic only but not isentropic.

$$Tds = dh - v \frac{dp}{T} = - \int_{P_1}^{P_2} \frac{v}{T} dp$$

Hence it is irreversible and cannot be shown in property diagrams. States 3 and 4 have simply been joined by a dotted line.

Evaporation:- A constant pressure reversible process, 4-1, which completes the cycle. The refrigerant is throttled by the expansion valve to a pressure, the saturation temperature at this pressure being below the temperature of the surroundings. Heat then flows, by virtue of temperature difference, from the surroundings. ~~Heat then flows~~ ^{which gets cooled} or refrigerant, which then evaporates, absorbing the heat of evaporation. The evaporator thus produces the cooling or the refrigerating effect, absorbing Q_2 from the surroundings by evaporation.

$$\text{Then for the cycle } \text{COP} = \frac{Q_2}{W_c} = \frac{h_1 - h_4}{h_2 - h_1}$$

Capacity of the refrigerating plant

$$= \frac{w (h_1 - h_4) 3600}{14,000} \text{ tonnes}$$

where w is the mass flow of refrigerant in kg/sec .

An air standard Otto cycle has a compression ratio of 16, and compression begins at 1 bar, 50°C . The maximum pressure is 70 bar. The heat transferred to air at constant pressure is equal to that at constant volume. Estimate

- (a) the pressures and temperatures at the cardinal points of the cycle. (b) the cycle efficiency, and (c) the m.e.p. of the cycle, $C_v = 0.718 \text{ kJ/kgK}$, $C_p = 1.005 \text{ kJ/kgK}$.

Soln: $T_1 = 273 + 50$
 $= 323 \text{ K}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (16)^{0.4}$$

$$\Rightarrow T_2 = 979 \text{ K}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 1.0 \times (16)^{1.4} = 48.5 \text{ bar}$$

$$T_3 = T_2 \cdot \frac{P_3}{P_2} = 979 + \frac{70}{48.5} = 1413 \text{ K}$$

$$Q_{2-3} = C_v(T_3 - T_2) = 0.718(1413 - 979)$$

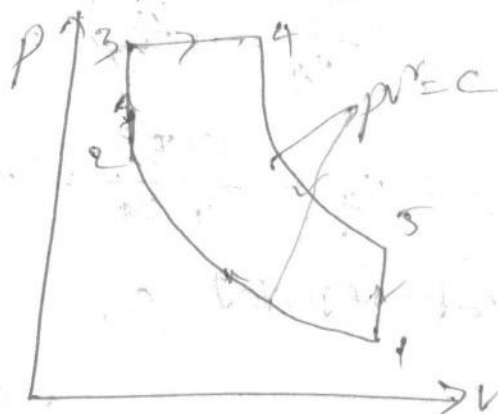
$$= 312 \text{ kJ/kg}$$

$$Q_{2-3} = Q_{3-4} = C_p(T_4 - T_3)$$

$$T_4 = \frac{312}{1.005} + 1413 = 1723 \text{ K}$$

$$\frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{1723}{1413} = 1.22$$

$$\therefore \frac{V_5}{V_4} = \frac{V_1}{V_2} \times \frac{V_3}{V_4} = \frac{16}{1.22} = 13.1$$



$$T_5 = T_4 \left(\frac{V_4}{V_5} \right)^{\gamma-1} = 1723 \times \frac{1}{(13.1)^{0.4}} = 615 \text{ K}$$

$$P_5 = P_1 \left(\frac{T_5}{T_1} \right) = 1.0 \times \frac{615}{323} = 1.9 \text{ bar}$$

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{0.718(615 - 323)}{312 + 312} = 0.665 \text{ or } 66.5 \%$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \text{ kJ/kg} \cdot \text{K} \times 323 \text{ K}}{10^2 \text{ kN/m}^2} = 0.927 \text{ m}^3/\text{kg}$$

$$v_1 - v_2 = v_1 - \frac{v_1}{16} = \frac{15}{16} v_1$$

$$w_{\text{net}} = Q_1 \times \eta = 0.665 \times 624 \text{ kJ/kg}$$

$$\therefore \text{m.e.p} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{414.96}{\frac{15}{16} \times 0.927 \text{ m}^3/\text{kg}}$$

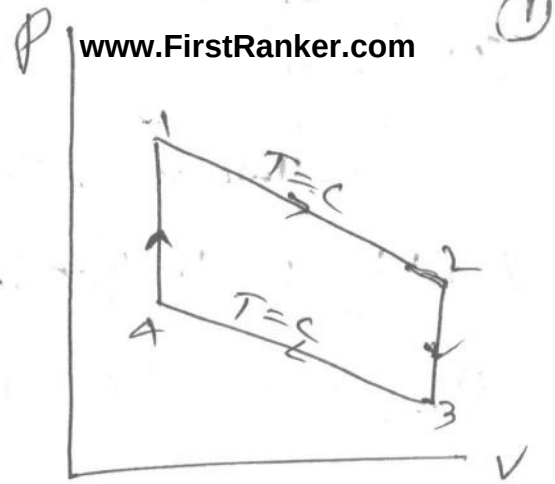
$$= 476 \text{ kN/m}^2 = \underline{\underline{4.76 \text{ bar}}}$$

Stirling cycle (1827):

It consists of

Two reversible ~~the~~ ^{isothermal} (constant temperature) _{process}

Two reversible isochores (constant volume) _{process}.



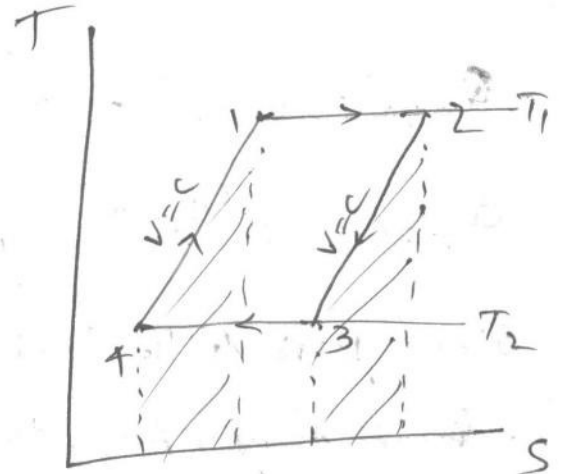
For 1 kg of ideal gas

$$Q_{1-2} = W_{1-2} = RT_1 \ln \frac{V_2}{V_1}$$

$$Q_{2-3} = -C_V (T_2 - T_1), \quad W_{2-3} = 0$$

$$Q_{3-4} = W_{3-4} = -RT_2 \ln \frac{V_3}{V_4}$$

$$Q_{4-1} = C_V (T_1 - T_2), \quad W_{4-1} = 0$$

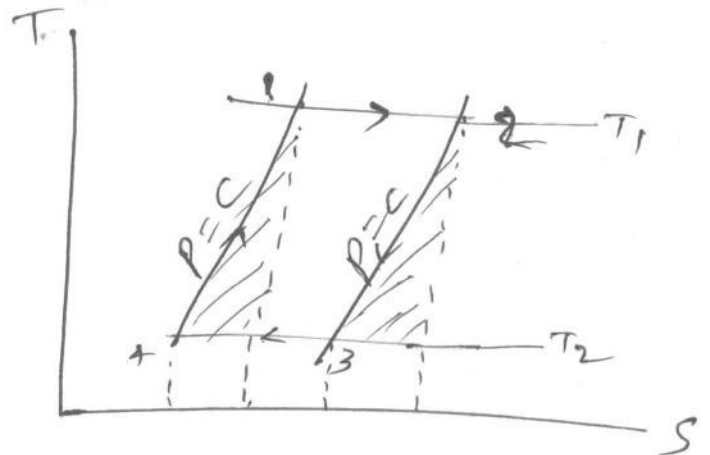
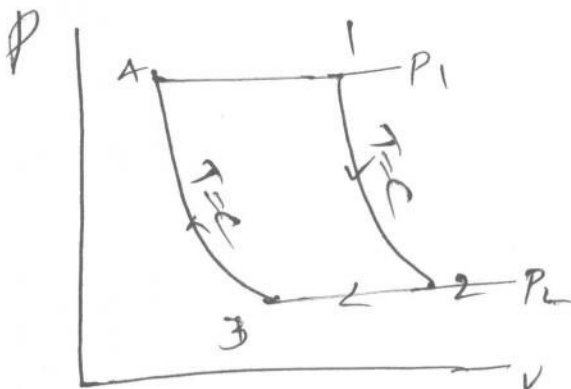


$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{RT_1 \ln \frac{V_2}{V_1} - RT_2 \ln \frac{V_3}{V_4}}{RT_1 \ln \frac{V_2}{V_1}}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}}$$

If regenerator is used, area under 2-3 is equal to area under 4-1, then the cycle efficiency is same as the Carnot cycle.

Erickson cycle (1850):



It consists of Two reversible isotherms (Constant temperature processes)

Two reversible isobars (Constant pressure processes)

For 1 kg of ideal gas

$$Q_{1-2} = W_{1-2} = R T_1 \ln \frac{P_1}{P_2}$$

$$Q_{2-3} = C_p (T_2 - T_1), \quad W_{2-3} = P_2 (V_3 - V_2) = R (T_2 - T_1)$$

$$Q_{3-4} = W_{3-4} = -R T_2 \ln \frac{P_1}{P_2}$$

$$Q_{4-1} = C_p (T_1 - T_2), \quad W_{4-1} = P_1 (V_1 - V_4) = R (T_1 - T_2)$$

Since part of the heat is transferred at constant pressure and part at constant temperature, the efficiency of the Ericsson cycle is less than that of the Carnot cycle. But with ideal regeneration, $Q_{2-3} = Q_{4-1}$ so that all the heat added from the external source at T_1 and all the heat is rejected to an external sink at T_2 , the efficiency of the cycle becomes equal to the Carnot cycle efficiency, since

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{R T_2 \ln \frac{P_1}{P_2}}{R T_1 \ln \frac{P_1}{P_2}} = 1 - \frac{T_2}{T_1}$$

Lenoir Cycle:

The Lenoir cycle

www.FirstRanker.com

www.FirstRanker.com

(3)

Consists of the following processes:

process 1-2 : Constant volume heat addition

process 2-3 : Reversible adiabatic expansion

process 3-4 : Constant pressure heat rejection

No compression process.

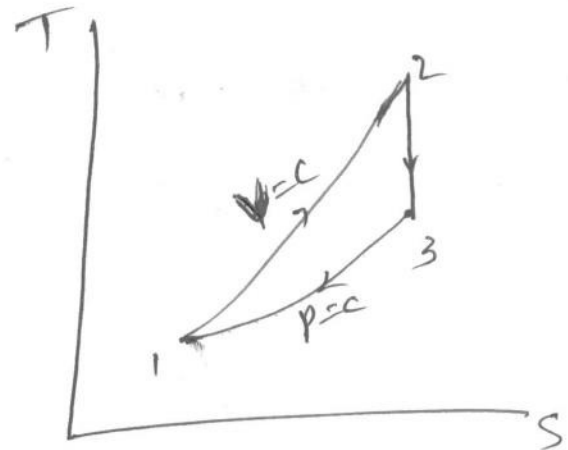
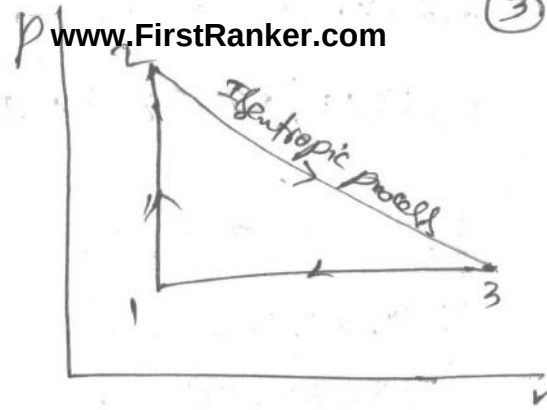
$$\eta = \frac{\text{Heat added} - \text{Heat rejected}}{\text{Heat added}}$$

$$= \frac{C_v (T_2 - T_1) - C_p (T_3 - T_1)}{C_v (T_2 - T_1)}$$

$$= 1 - \gamma \left(\frac{T_3 - T_1}{T_2 - T_1} \right)$$

$$= 1 - \gamma \left(\frac{T_1 (\gamma_p)^{\frac{1}{\gamma}} - T_1}{\gamma_p T_1 - T_1} \right)$$

$$\boxed{\eta = 1 - \gamma \left(\frac{(\gamma_p)^{\frac{1}{\gamma}} - 1}{\gamma_p - 1} \right)}$$



let $\frac{P_2}{P_1} = \gamma_p = \text{pressure ratio}$

$$\frac{T_2}{T_1} = \gamma_p$$

$$T_2 = \gamma_p \cdot T_1$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}}$$

$$= \left(\frac{1}{\gamma_p} \right)^{\frac{\gamma-1}{\gamma}} = (\gamma_p)^{\frac{1-\gamma}{\gamma}}$$

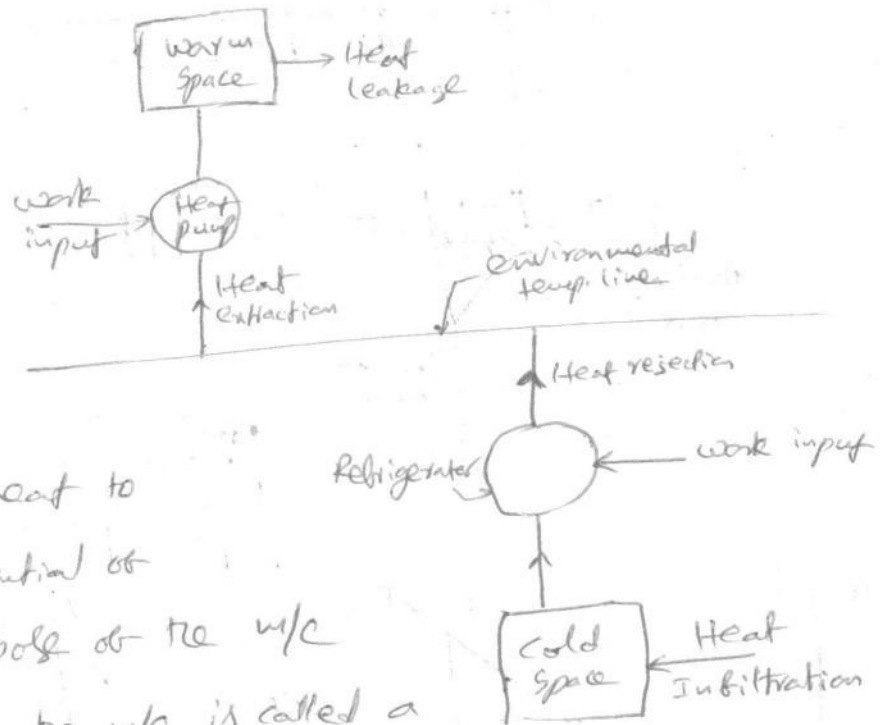
$$= (\gamma_p)^{\frac{1}{\gamma} - 1}$$

$$T_3 = T_2 (\gamma_p)^{\frac{1}{\gamma} - 1} = \gamma_p T_1 (\gamma_p)^{\frac{1}{\gamma} - 1}$$

$$= T_1 (\gamma_p)^{\frac{1}{\gamma}}$$

Refrigeration is the cooling of a system below the temp of its surrounding.

Refrigerators and heat pumps are the machines which are used to remove heat from a body at low temp level and then transfer that heat to another body at a high potential of temp. When the main purpose of the m/c is to cool some object, the m/c is called a



refrigerator. The term heat pump is applied to a machine whose main objective is to heat a medium which may already be warmer than its surrounding.

$$C.O.P = \frac{\text{Desired effect}}{\text{work input}}$$

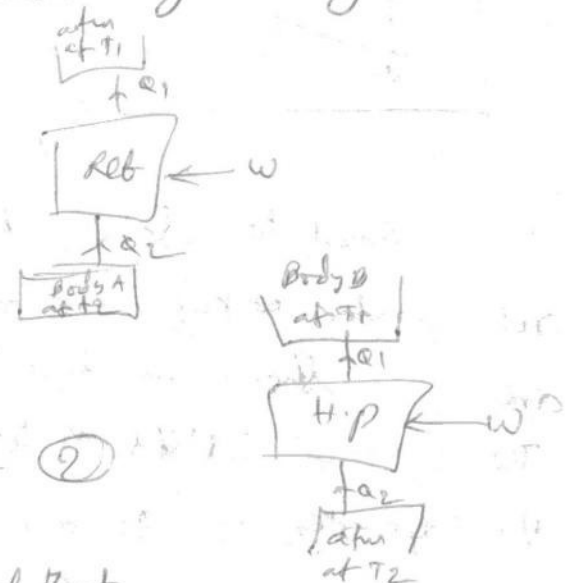
$$(COP)_{ref} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad \text{--- ①}$$

$$(COP)_{H.P} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} \quad \text{--- ②}$$

from ① & ②, it is found that

$$(COP)_{HP} = (COP)_{ref} + 1$$

$$\begin{aligned} (COP)_{ref} + 1 &= \frac{Q_2}{Q_1 - Q_2} + 1 \\ &= \frac{Q_2 + Q_1 - Q_2}{Q_1 - Q_2} \\ &= \frac{Q_1}{Q_1 - Q_2} \\ &= (COP)_{HP} \end{aligned}$$



Bell-Coleman cycle.

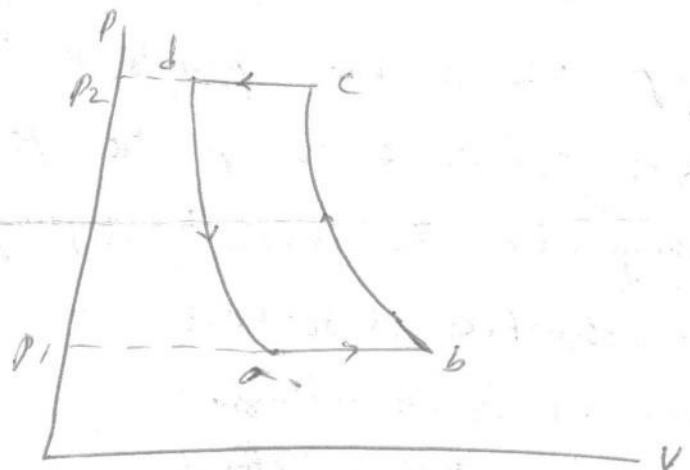
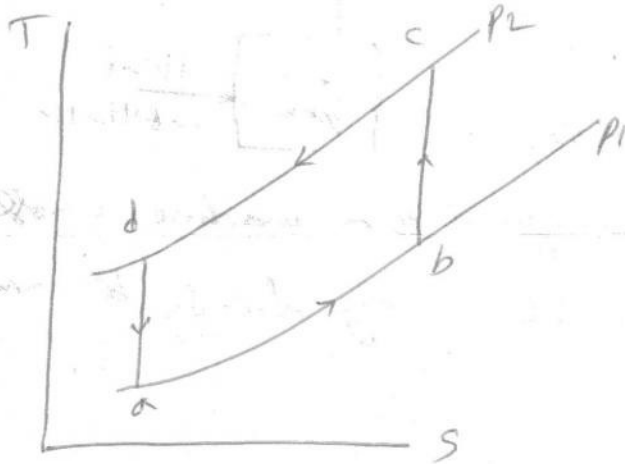
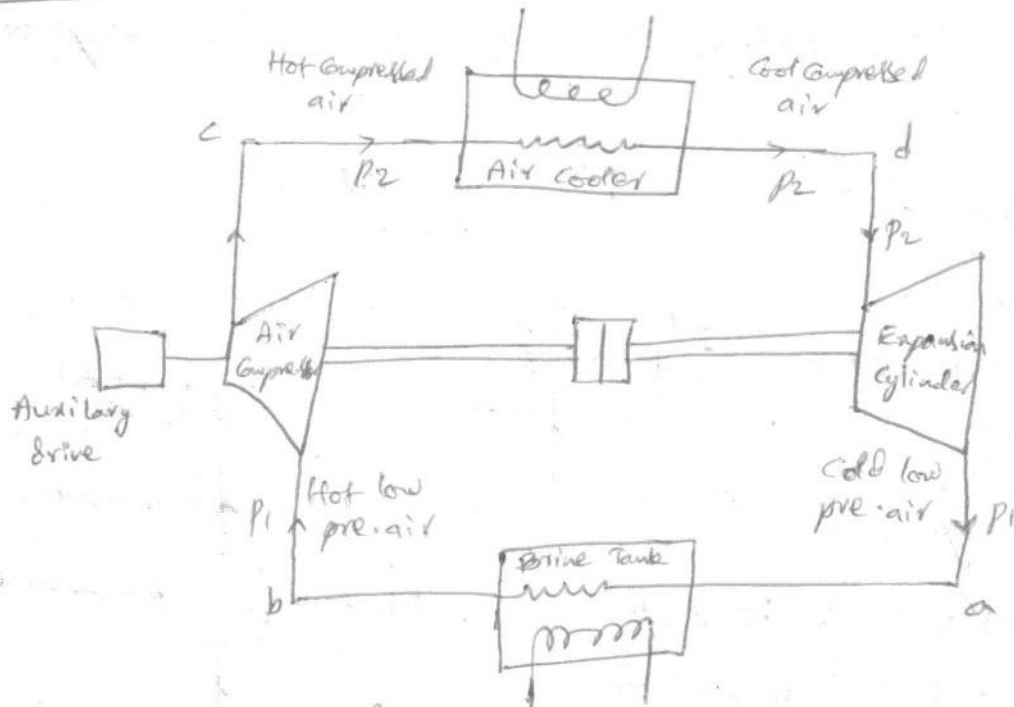


Fig. shows the Schematic diagram of the Bell-Coleman air refrigerator working on the reversed Joule cycle. $p-v$ and $T-s$ plots are also shown in fig.

The system consists of the following parts:

1. Expander:- The air at high pre (P_2) and moderate temp (T_2) is expanded in the Expanding cylinder to lower pre (P_1) and lowest temp (T_1) existing in the cycle.

2. Refrigerating chamber or cold room:- Air leaving the expansion cylinder at temp (T_1) and pre (P_1) is made to flow through the chamber which is required to be cooled. The cold low pre.

from the air (Storage of air conditioning) at const. pre. p_1 , resulting in an increase in its temp to T_b .

3). Compressor :— Air is compressed in the compression cylinder to a relatively higher pre p_2 . The temp also increases to T_c . The work done during expansion process assists in driving the compressor unit. The remaining power comes from the auxiliary power device.

4). Air Cooler :— The cooler contains a number of pipes immersed in circulating water. The hot compressed air leaving the compressor passes through the pipes. The heat is rejected at const. pre. p_2 and the air is cooled to temp T_d .

The air then enters the expansion cylinder and the cycle is repeated.

C.O.P. for Bell-Coleman cycle :—

$$\text{Refrigerating effect, i.e. heat absorbed from cold chamber} = c_p (T_b - T_a)$$

$$\text{Heat rejected in air cooler} = c_p (T_c - T_d)$$

$$\text{work input} = \text{heat rejected} - \text{heat abstracted}$$

$$= c_p [(T_c - T_d) - (T_b - T_a)]$$

$$\text{C.O.P.} = \frac{\text{refrigerating effect}}{\text{work input}}$$

$$= \frac{T_b - T_a}{(T_c - T_d) - (T_b - T_a)}$$

are both isentropic.

Since both Expansion and Compression are with in the same pressure limits.

$$\frac{T_c}{T_b} = \frac{T_d}{T_a} \text{ as both are equal to } \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \text{C.O.P} = \frac{\frac{T_c \cdot T_a}{T_d} - T_a}{\left(\frac{T_d T_b}{T_a} - T_d\right) - \left(\frac{T_a T_c}{T_d} - T_a\right)}$$

$$\frac{T_a \left(\frac{T_c}{T_d} - 1\right)}{T_d \left(\frac{T_b}{T_a} + 1\right) - T_a \left(\frac{T_c}{T_d} - 1\right)}$$

The terms inside the brackets in the above expression are equal

$$\therefore \text{C.O.P} = \frac{T_a}{T_d - T_a}$$

Bell-Coleman refrigerator may work on the open system or closed system. In open system the air comes in direct contact with the refrigerating chamber or the bodies being cooled. In closed system the air is passed through coils placed in the chamber. Quite often the coils carrying cold air are kept in the brine tank. The air extracts heat from the brine and the resulting chilled brine is then circulated around the cold chamber.

Brine $\rightarrow$ aqueous solution of NaCl (sodium chloride) or CaCl₂ in water.
The brine does not freeze at low temp.

Reversed Carnot cycle:-

A reversed heat engine cycle operating in the reverse way, i.e. receiving heat from a low temperature region, discharging heat to a high temperature region, and receiving the net inflow of work.

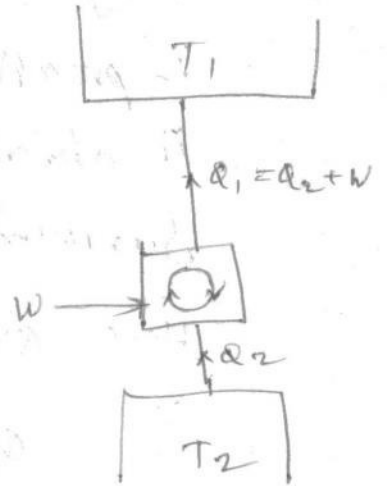


Fig: Reversed heat engine cycle

$$(COP)_{HP} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\text{and } (COP)_{Ref} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$

The working fluid in a refrigeration cycle is called a refrigerant.

In the reversed Carnot cycle the refrigerant is first compressed reversibly and adiabatically in process 1-2 where the work input per kg of refrigerant is w_c , then it is expanded reversibly in process 2-3 where the heat rejection is Q_1 , the

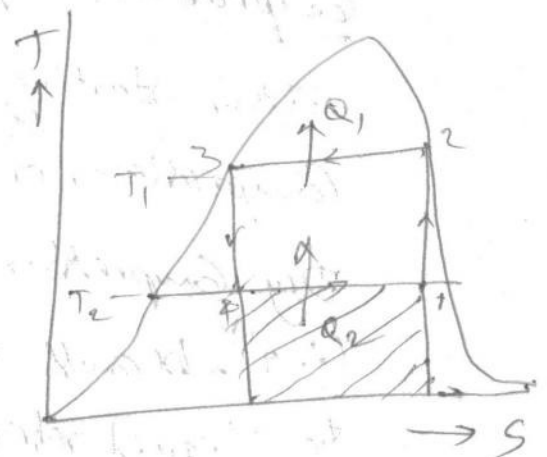
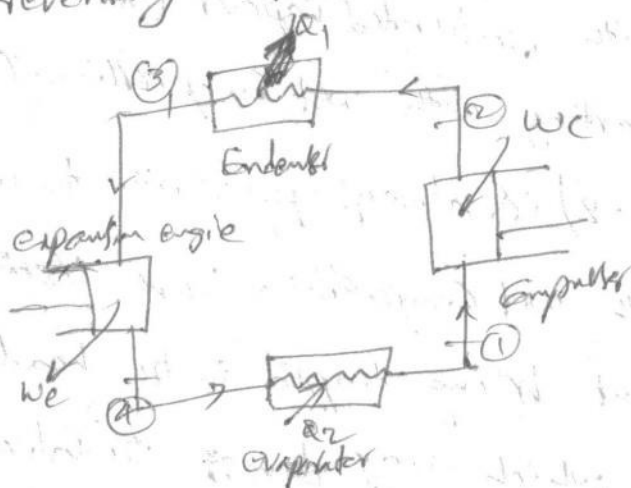


Fig: Reversed Carnot cycle

refrigerant then compressed reversibly and adiabatically in process 3-4 where the work output is w_e and finally it absorbs heat Q_2 reversibly by evaporation from the surroundings in process 4-1.

$$\text{Here } Q_1 = T_1 (S_2 - S_3)$$

$$Q_2 = T_2 (S_1 - S_4)$$

$$W_{\text{net}} = W_c - W_e = Q_1 - Q_2 = (T_1 - T_2) (S_1 - S_4)$$

where T_1 is the temperature of heat rejection
 T_2 is the temperature of heat absorption.

$$(\text{COP})_{\text{ref. rev}} = \frac{Q_2}{W_{\text{net}}} = \frac{T_2}{T_1 - T_2}$$

$$\text{and } (\text{COP})_{\text{hp rev}} = \frac{Q_1}{W_{\text{net}}} = \frac{T_1}{T_1 - T_2}$$

Vapour Compression refrigeration cycle:- The basic

operations involved in vapour compression cycle is Compression, Condensation, expansion and evaporation.

Compression:- A reversible adiabatic process 1-2 or 1'-2'

either starting with saturated vapour (state 1), called dry compression, or starting with wet compression (state 1').

Dry compression (1-2) is always preferred to wet compression (1'-2'), because with wet compression there is a danger of the liquid refrigerant being trapped in the head of the cylinder by the rising piston which may damage the valves or the cylinder head, and the droplets of liquid refrigerant may wash away

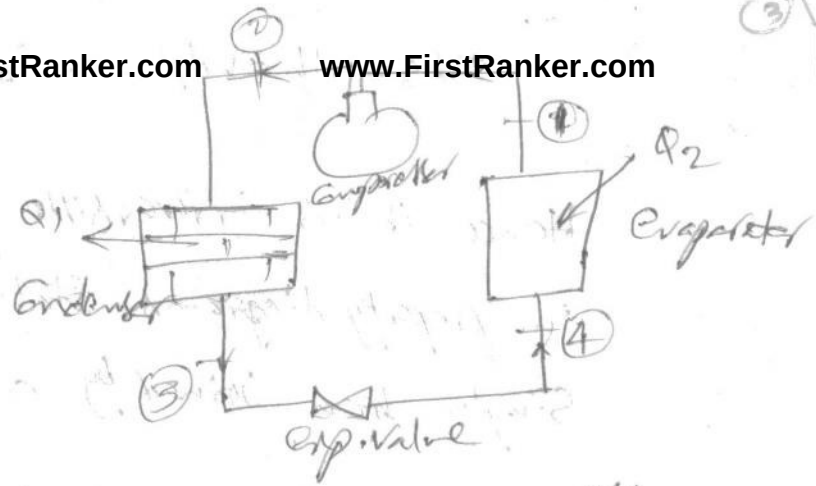
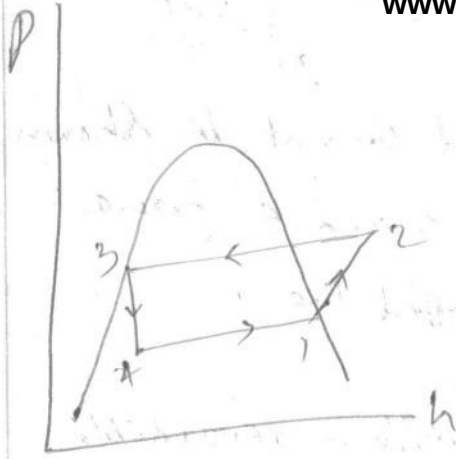


Fig: Vapour Compression refrigeration flow diagram.

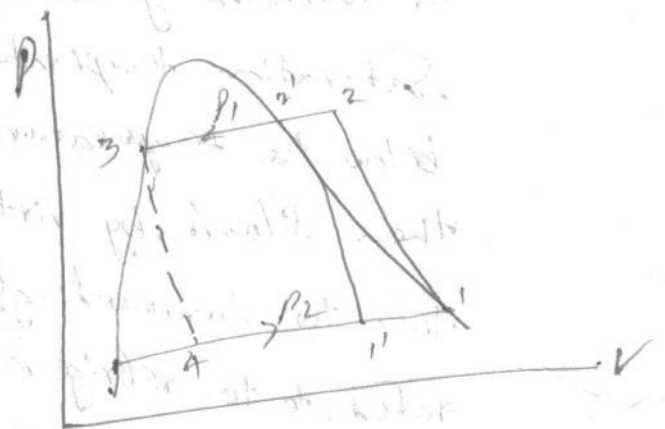
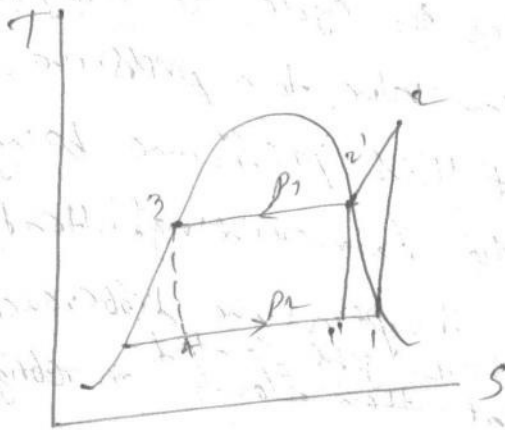


Fig: Vapour Compression refrigeration cycle - property diagrams.

The lubricating oil from the walls of the cylinder, thus accelerating wear.

Cooling and Condensing: A reversible constant pressure process, 2-3, first de-superheated and then condensed, ending with Saturated liquid. Heat Q_2 is transferred out.

Expansion: An adiabatic throttling process 3-4, first which enthalpy remains unchanged. State 3 and 4 are Equilibrium point. process 3-4 is adiabatic only but not isentropic.

$$T ds = dh - v dp \quad \int_{P_1}^{P_2} \frac{P_2}{T} = -$$

Hence it is irreversible and cannot be shown in property diagrams. States 3 and 4 have simply been joined by a dotted line.

Evaporation:- A constant pressure reversible process, 4-1, which completes the cycle. The refrigerant is throttled by the expansion valve to a pressure, the saturation temperature at this pressure being below the temperature of the surroundings. Heat then flows, by virtue of temperature difference, from the surroundings. ~~Heat then flows~~ ^{which gets cooled} or refrigerant, which then evaporates, absorbing the heat of evaporation. The evaporator thus produces the cooling or the refrigerating effect, absorbing Q_2 from the surroundings by evaporation.

$$\text{Then for the cycle } \text{COP} = \frac{Q_2}{W_c} = \frac{h_1 - h_4}{h_2 - h_1}$$

Capacity of the refrigerating plant

$$= \frac{w (h_1 - h_4) 3600}{14,000} \text{ tonnes}$$

where w is the mass flow of refrigerant in kg/sec .