Department of Basic Sciences Humanities

SUBJECT: MATHEMATICAL METHODS **YEAR /SEM : I/II** **BRANCHS: EEE/CIVIL**

UNIT-I

1. a) Find the Real root of the equation $x^2 - x - 4 = 0$ using iteration method (5M)

b) Find the Real root of the equation $x^4 - x - 10 = 0$ using Newton Raphson method (5M)

2 (a) Using Newton-Raphson method find the root of the equation **$x + \log_{10} x = 3.375$** correct to four decimal places. (5M)

b) Find the Real root of the equation $3x = e^x$ using Bisection method. (5M)

3 (a) Find the Real root of the equation $x^3 = 2x + 5$ using false position method (5M) (b)Solve $\mathbf{x^3 - 4x + 1 = 0}$ the equation by Bissection method (5M)

4 (a) Using Newton – Raphson method, find a root of the equation **2x- 3sinx =5** near x=*3* correct to three decimal places. (5M)
(b) Develop an Iterative formula to find $\frac{1}{N}$ using Newton Raphson method (5M)

5(a) Using Regular-Falsi method,find the root of $\boldsymbol{x^3 - x - 2 = 0}$, over (1, 2) (5M)
(b) By using Newton-Raphson method, find the root of $\boldsymbol{x^4 - x - 10 = 0}$, correct to three decimal places. (5M)

UNIT-II

1(a) Using Lagrange's interpolation formulae find the value of y (12) from the data (5M)

x	0	2	3	6
y	648	704	729	792

(b) Determine the value of f(x) at x = 25 for the following data (5M)

x	20	24	28	32
f(x)	24	32	35	40

2 (a) Find $f(142)$ using Newton's forward formula for the following table

x	140	150	160	170	180
Y=f(x)	3.685	4.854	6.302	8.076	10.225

(b) Calculate f(3) from the following table. (5M)

X	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

3(a) Given that f(6500) = 80.6223, f(6510) = 80.6846, f(6520) = 80.7456, f(6530) = 80.8084,
Find f(6526) using Gauss backward interpolation formula. (5M)

(b) Using gauss forward difference formula, find y(12)from the given table (5M)

x	1	6	11	16	21	26
y	5	10	14	18	24	32

4 (a) Using Gauss Backward difference polynomial, find y(5) given that (5M)

X	2	4	6	8	10
Y	5	11	13	15	17

(b) Use Gauss backward interpolation formula to find f(32) given that
$f(25) = 0.2707, f(30) = 0.3027, f(35) = 0.3386, f(40) = 0.3794.$ (5M)

5(a) Prove that $(1 + \Delta)(1 - \nabla) = 1.$ (3M)

(b)Compute f(27) using Lagranges formula from the following table

X	14	17	31	35
F(x)	68.7	64	44	39.1

6(a)Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$ (6M)

(ii) $\delta = E^{\frac{1}{2}} - E^{\frac{-1}{2}}.$ (6M)

(b) If the interval of differencing is unity, find $\Delta^2 sin(ax + b)$ (4M)

UNIT-III

1(a) Using modified Euler method solve numerically the equation $\frac{dy}{dx} = 2 + \sqrt{xy}$, with y(1)=1 to find y(1.2). (5M)

(b)Using Euler's method, solve for y (0.6) from $\frac{dy}{dx} = -2xy$, $y(0) = 1$ using step size 0.2. (5M)

2(a) Given $\frac{dy}{dx} = x + \sin y$, $y(0) = 1$, compute y(0.2) and y(0.4) using Euler's modified method. (5M) (b) A cu.rve is observed to pass through the points given in the following table 5M

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	2	2.4	2.7	2.8	3	2.6	2.1

By using simpson's rule find the area bounded by the curve and x axis between x=1 and x=4

3(a) $y' = x^2 y + 1, y(0) = 1$ using Taylors method up to 3rd degree term and compute y(0.1) (5M)

(b) Solve , $y' = y - x^2$ $y(0) = 1$ using Picard's method up to 4th approx. y(0.1).y(0.2) (5M)

4(a) Evaluate $\int_0^1 \sqrt{1 + x^4}\,dx$ by taking h=0.125 using Simpson's 1/3rd 3/8 thrule , . (5M)

(b) Evaluate $\int_0^{\pi/2} \frac{\cos x}{1+x^2} dx$ by (i) Trapezoidal rule (ii) Simpson's 3/8th Rule (5M)

5(a) Find y (0.1) using 4th order Runge-Kutta method given that , $y' = x + x^2 y$, $y(0) = 1$. (5M)

(b) Use Runge-Kutta 4th order to compute y(1.2) for the equation $y' = \frac{x^2 + y}{x}$, $y(1) = 2$ (5M)

6.. (a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 3/8th rule 5M

(b) Evaluate $\int_0^1 \frac{dx}{1+x}$ by using Trapizoidal rule. 5M

UNIT IV

1. **(a)** Obtain the Fourier series for $f(x) = e^{x}$ in the interval $0 < x < 2\pi$. 5M

   (b) Find a Fourier series to represent the function f(x) = $x - x^2$ from $x = -\pi$ to $x = \pi$ 5M

2. (a) Expand $f(x) = x \sin x$ as a Fourier series in the interval $-\pi < x < \pi$. 5M

   And show that

   $$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} \dots = \frac{\pi - 2}{4}$$

   (b) Find the Fourier series of $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \frac{\pi}{4}, & 0 < x < \pi \end{cases}$ 5M

3. (a) Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$, $0 < x < 2\pi$. 5M

   (b) Find the Fourier series of the periodic function defined as $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \end{cases}$

   Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ 5M

4. (a) Find half range cosine series of the function $f(x) = e^{x}$ $in[0,1]$ 5M

   (b) Find the half range sine series of $f(x) = \begin{cases} \frac{\pi}{2}, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$ 5M

5. (a) Find the Fourier series of the function $f(x) = \begin{cases} 0, & 0 < x < 1 \\ x^2, & 1 < x < 2 \end{cases}$ 5M

   (b) Find Fourier cosine series for $f(x) = x(x-2)$, in $0 \le x \le 2$ and hence find the sum of the series

   $$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$$ 5M

6. (a) Find the Fourier series of periodicity 2 for $f(x) = x + x^2$, in $0 < x < 2$ 5M

   (b) Find the half range cosine series of $f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x < \pi \end{cases}$ 5M

UNIT -V

1. (a) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$. 5M

(b) Using the method of separation of variables, solve
$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given that $u(0, y) = 3e^{-y} - e^{-5y}$ 5M

2. A tightly stretched string with fixed end points x=(0) and x= l is initially in a position given by

$y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from this position, find the displacement y(x, t). 10M

3. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x, 0) = 3\sin n\pi x$, $u(0, t) = 0$ and $u(l, t) = 0$, where $0 < x < l$, $t > 0$. 10M

4. Solve the laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin n\pi x/l$. 10M

6. Find the solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$, if the intial deflection is

$$f(x) = \begin{cases} \frac{2k}{l}x, & \text{if } 0 < x < \frac{l}{2} \\ \frac{2k}{l}(l - x), & \text{if } \frac{l}{2} < x < l \end{cases}$$

and intial velocity equal to 0. 10M

UNIT VI

1. (a) Using Fourier integral , Show that $\int_0^\infty \frac{\cos \lambda x}{\lambda^2+a^2} d\lambda = \frac{\pi}{2a} e^{-ax}$, $a > 0$, $x \geq 0$. 5M

   (b) Find the Fourier transform of $f(x) = \begin{cases} x , & if \ |x| \leq 1 \\ 0 , & if \ |x| > 1 \end{cases}$ 5M

2. (a) Find the Fourier transform of $\frac{1}{|x|}$ 5M

   (b) Find the Fourier sine transform of $e^{-|x|}$. 5M

3. (a)If $F(p)$ or $F(s)$ is the complex fourier transform of f(x) then the complex fourier transform of

   Then find the complex fourrier transform of f(x)= $\cos ax$ 5M

   (b) Find the Fourier sine and cosine transforms of $2e^{-5x} + 5e^{-2x}$ 5M

4. (a) Find the Fourier sine transform of $f(x) = e^{-ax}$, $a > 0$ and deduce the inversion formula. 5M
   (b) Find the inverse Fourier sine transform of $f(x)$ of $F_s(p) = \frac{p}{1+p^2}$. 5M

5. (a) Find the Fourier Cosine transform of $\frac{e^{-ax}}{x}$ 5M

   (b) Find the inverse Fourier sine transform $f(x)$ of $F_s(p) = \frac{e^{-ap}}{p}$; and show that $F_s^{-1}(1/p) = 1$. 5M

6. (a) Prove that $F\{x^n f(x)\} = (-i)^n \frac{d^n}{dp^n} F(p)$. 5M

   (b) Prove that $F\left\{\frac{d^n}{dx^n} f(x)\right\} = (-ip)^n F(p)$. where $F\{f(x)\} = F(p)$. 5M