I B.TECH IISem Question Bank

Subject : Mathematics-III Branch: ECE, EEE,Civil. (2018-2019)

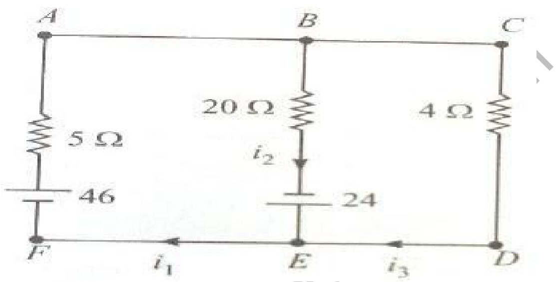
UNIT -I

1(a) Solve the system of equations $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ by Gauss Jacobi method 5M

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A = \begin{matrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 1 & 2 \end{matrix}$$ 5M

2(a) Find the currents in the following circuits 5M



(b) solve the system of equations $10x + y + z = 12$, $2x + 10y + z = 13$ and $2x + 2y + 10z = 14$ using Gauss-seidel method. 5M

3(a) Find the non singular matrices P and Q such that the normal form of A is PAQ where

$$A = \begin{matrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{matrix}$$. Hence find its rank. 5M

(b) Find the rank of 2 3 -1 -1

1 -1 -2 -4 after reducing it to Echelon form

5M 3 1 3

-2

6 3 0 -7

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4(a) Find the values of 'a' and 'b' for which equation $x + y + z = 3$; $x + 2y + 2z = 6$; $x + ay + 3z = b$ have unique solutions. 5M

(b) using Gauss-jordan method solve the system of equations $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$. 5M

5(a) Reduce the matrix A to normal form and hence find the rank of the matrix. 5M

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \end{bmatrix}$$

(b) prove that the following set of equations are consistent and solve them.

$2x - y - z = 2$; $x + 2y + z = 2$; $4x - 7y - 5z = 2$;.

UNIT – II:

1(a) Find Eigen values and Eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 5M

(b) Reduce the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$ into canonical form and find the nature, rank, index and signature. 5M

2(a)Reduce the Quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into sum of squares form by an orthogonal transformation and give the matrix transformation. 5M

(b)Find $A-1$ using Cayley –Hamilton theorem, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ 5M

3(a) what is the nature of the quadratic form $XTAX$, if $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ 5M

(b)Prove that if τ is an Eigen value of a matrix A then τ^{-1} is an Eigen value of matrix A^{-1} if it exists. 5M

4(a) If τ is an Eigen value of a non singular matrix A then show that $\frac{|A|}{\tau}$ is an Eigen value of matrix adjoint A(adjA) 5M

(b) Find A^{-1} using Cayley –Hamilton theorem, where $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ 5M

5(a) state Cayley-Hamilton theorem and find A^8 if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ 5M

(b) Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ 5M

6(a) Show that if λ is an eigen value of A, then prove that the eigen value of

$B = a_0A^2+a_1A + a_2I$ is $a_0\lambda^2 + a_1\lambda + a_2$. **5M**

(b) Is the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 2 & 5 & 7 \end{bmatrix}$ diagonalizable ? 5M

UNIT –III :

1(a) Evaluate $\int_{y=0}^{2}\int_{x=0}^{3} xy \, dxdy$ 5M

(b) Evaluate $\int_0^a \int_{x^2/a}^{2a-x} xy^2 \, dydx$ by changing the order of integration. 5M

2(a) Evaluate $\int_{x=0}^{a}\int_{y=0}^{b}(x^2 + y^2)dydx$ 5M

(b) By changing the order of integration , evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2 dxdy$ 5M

3(a) Find the moment of inertia about the initial line of the cardioid $r = a(1 - cos\theta)$. 5M

(b) Evaluate $dx\, dy\, dz$ V is the finite region of space formed by the planes

x = y = z = 0 and $2x + 3y + 4z = 12$ 5M

4(a) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{a^2-r^2} r\, dz\, dr\, d\theta$. 5M

(b) Evaluate $\int_0^4 \int_{\frac{y^2}{4}}^{y} \frac{y}{x^2+y^2}\, dx\, dy$ 5M

5(a) Evaluate $\int_0^a \int_x^a (x^2 + y^2)\, dy\, dx$ by changing the order of integration. 5M

(b) Evaluate $\iint (x^2 + y^2)\, dx\, dy$ in the positive quadrant for which $x + y \leq 1$. 5M

UNIT – IV:

1(a) Show that $\int_0^\infty x e^{-x^3} dx = \frac{\sqrt{\pi}}{3}$ 5M

(b) Show that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}} dx = \frac{\beta(m,n)}{a^n b^m}$ 5M

2(a) Prove that $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$ 5M

(b) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}} = \pi$ 5M

3(a) Evaluate $\int_0^1 \frac{x^4(1+x^5)}{(1+x)^{15}} dx$ 5M

(b) Evaluate $\int_5^7 (x-5)^6(7-x)^3\, dx$ using β and Γ functions. 5M

4(a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 5M

(b) Show that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m > 0,n > 0. 5M

5(a) Evaluate $\int_0^{\pi/2} \sin^5\theta \cos^7 2\theta\, d\theta$. 5M

(b) Evaluate $\int_0^1 x^4 \left(\log \frac{1}{x}\right)^3 dx$ 5M

6(a) Evaluate $\int_0^1 \frac{x\,dx}{\sqrt{1-x^5}}$. 5M

(b) Evaluate $\int_0^\infty x^2 e^{-x^2}\, dx$. 5M

UNIT- V :

1(a) Find unit normal vector to the surfaces $x^2 y + 2xz^2 = 8$ at the point (1,0,2) 5M (b) Prove that $div.\ (grad r^m) = m(m+1) r^{m-2}$ 5M

2(a)Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1 , 2). 5M

(b) If $\bar{A}$ is irrotational , evaluate $div(\bar{A} \times \bar{r})$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

5M 3(a) Find $div\bar{F}$, where $\bar{F} = r^n \bar{r}$. Find n if it is solenoidal.

5M

(b) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)\bar{i} + (3xz + 2xy)\bar{j} + (3xy - 2xzy + 2z)\bar{k}$ is both irrotational and Solenoidal . 5M

4(a) Find the directional derivative of $\emptyset = x^2 yz + 4xz^2$ at (1,-2,-1) in the direction of $2\bar{i} - \bar{j} - 2\bar{k}$ 5M

(b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. 5M

5(a) Show that $\nabla^2 [f(r)] = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ or $f^{II}(r) + \frac{2}{r} f^{I}(r)$ where $r = |\bar{r}|$. 5M

(b) Prove that $div(\bar{a} \times \bar{b}) = \bar{b} . curl\bar{a} - \bar{a} . curl\bar{b}$ 5M

UNIT – VI

1(a) Use Greens theorem to evaluate $\int_c (2xy - x^2)dx + (x^2 + y^2)dy$, where c is theclosed curve of the region bounded by $y = x^2$ and $y^2 = x$. 5M

(b) State Gauss divergence theorem and verify $\bar{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ over the cube $x = 0$ $x = 1, y = 0$ $y = 1, z = 0$ $z = 1.$ 5M

2(a) Evaluate $(ex\ dx + 2ydy - dz)$ where c is the curve is the curve is the curve
$x^2 + y^2 = 9$, $z = 2$, by using Stoke's theorem. 5M

(b) Compute $ax2 + by2 + cz2\ \ ds$ over the surface of the sphere $x2 + y2 + z2 = 1$. 5M

3(a) If $\bar{F} = (3x^2 + 6y)\,i - 14yzj + 20xzk$ then evaluate $\int \bar{F} \cdot d\bar{r}$ from (0,0,0) to (1,1,1) along $x = t$, $y = t^2$, $z = t^3$.

5M

(b) Apply stoke's theorem to evaluate $\int ydx + zdy + xdz$ where c is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

5M

4(a) State stoke's theorem,and verify for $\bar{F} = (x + y)\,i + (y + z)j - xk$ and S is the Surface of the plan $2x + y + z = 2$ which is in the first octant .

5M

(b) Using divergence theorem to evaluate $\int \bar{F} \cdot d\bar{s}$ where $\bar{F} = x^3 i + y^3 j + z^3 k$ and S is surface of the sphere $x^2 + y^2 + z^2 = r^2$. 5M

5(a)Verify Green's theorem in the plan for $\int (x^2 - xy^3)\,dx + (y^2 - 2xy)dy$ where C is the square with vertices (0,0), (2,0), (2,2), (0,2) 5M

(b) Evaluate by Green's theorem $\int (y - sinx)\,dx + cosx\,dy$ where C is the triangle enclosed by the lines y = 0, x = $\frac{\pi}{2}$, πy = 2x. 5M