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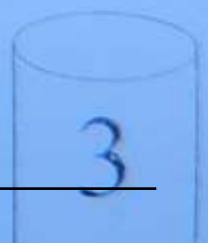


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-: HAND WRITTEN NOTES:-
OF
CIVIL ENGINEERING



-: SUBJECT:-
RCC & PRESTRESSED
CONCRETE



②

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WORKING STRESS METHOD

Introduction

RCC has two main comp. Concrete and Steel reinforcement
properties of concrete

① young's modulus of elasticity $E_c = 5000 \sqrt{f_{ck}}$ (S)

f_{ck} = Characteristic strength of concrete

→ This is the value of strength below which not more than 5% of test results are expected to fall.

g: If we take 100 no. of cubes to be tested then

17, 18, 18.5, 19, 19.5, 20, 21, 22, ...	
5% are fail	characteristic comp strength 95%

M-20 Grade Concrete $f_{ck} = 20 \text{ N/mm}^2$

f_m - Mean strength value (having max frequency)

It is the average strength

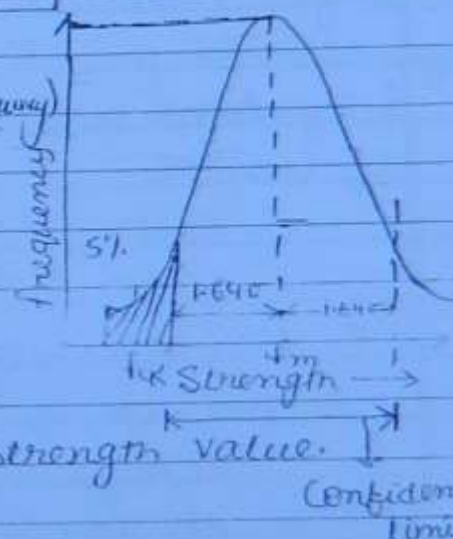
$$f_m = f_{ck} + 1.64\sigma$$

σ = Standard deviation

for M20 & M25 = 4 N/mm²

M30 & above = 5 N/mm²

→ Design mix is done for target mean strength value.



Confidence limit

Probability of a single test result is more of falling between the limit $(f_m - 1.64\sigma)$ to $(f_m + 1.64\sigma)$. This limit is called Confidence limit.

→ Probability % of lying a test result between Confidence limit is called Confidence level.

→ If % of test result falling below a value is 50% the strength value = f_m

Young's modulus of elasticity (E_c)

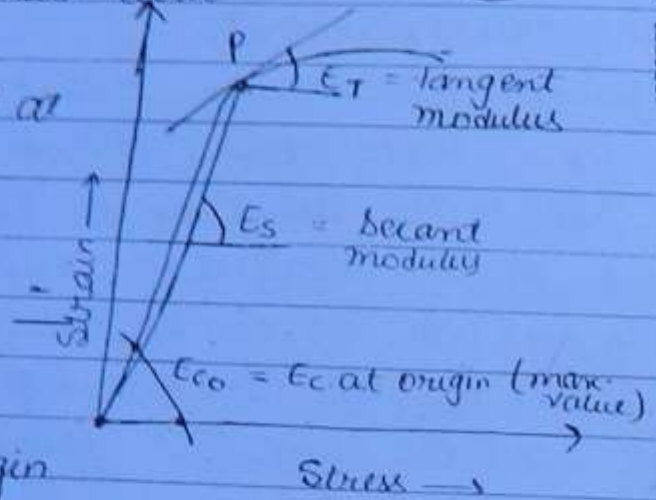
E_c is the Slope of Stress-Strain Curve

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es:- ① Young's modulus of elasticity at origin: $E_c = 5000 \sqrt{f_{ck}}$
(Considered by IS:456)

At point P on the curve

② Secant modulus - Slope of line joining any point on Stress-Strain curve with origin



③ Tangent modulus - Slope of tangent at any pt. on the curve

Long term modulus of elasticity (considering the effect of creep)

$$E_{ct} = \frac{E_c}{1+\theta} = \frac{5000 \sqrt{f_{ck}}}{1+\theta}$$

θ Creep Coefficient

As per age of testing

7 days	2.2
28 days	1.6
1 day year	1.1

θ Creep Coefficient

$$\text{Creep Coefficient} = \frac{\text{Creep Strain}}{\text{Elastic Strain}}$$

→ The value of Young's modulus of elasticity reduces 2 to 3 times due to the effect of creep.

Strength of concrete
(a) Compressive strength

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Permissible stress in L.S.M

Grades	Direct Compression σ_{cc}	Bending Compression σ_{cbc}
M15	4.0	5.0
M20	5.0	7.0
M25	6.0	8.5
M30	8.0	10.0
M35	9.0	11.5
M40	10.0	13.0

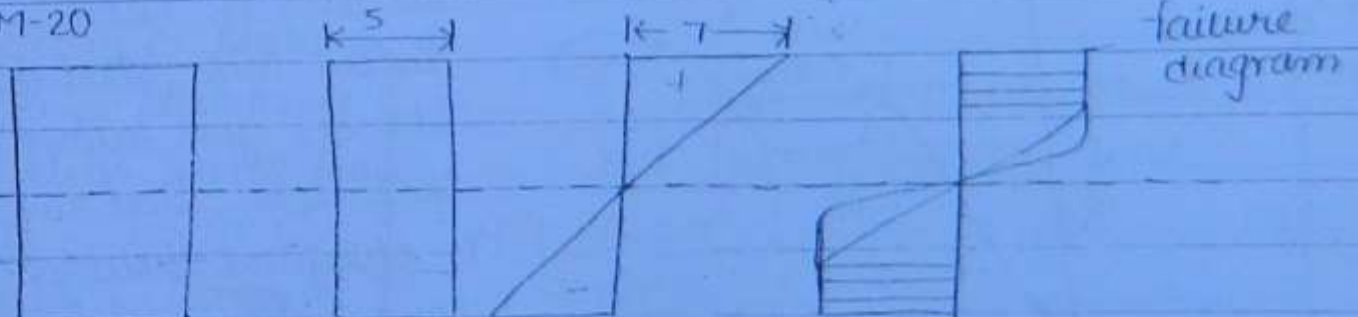
(b) Tensile stresses

flexural tensile strength (bending) of concrete

$$f_{cr} = 0.7 \sqrt{f_{ck}}$$

Grades	Direct tensile strength	Bending tensile strength
M15	2.0	2.71
M20	2.8	3.13
M25	3.2	3.5
M30	3.6	3.83
M35	4.0	
M40	4.4	

for M-20



Direct stress

Bending stress

(All fibres are stressed to same value because of failure is more)

(Only top & bottom fibre are at max. stress so permissible stress is more than that of direct stress value)

Grade Of Concrete (As per IS: 456) (Table-2/Pg-No-16)	
Ordinary Concrete	M10 to M20
Standard Concrete	M25 to M55
High Strength Concrete	M60 & above

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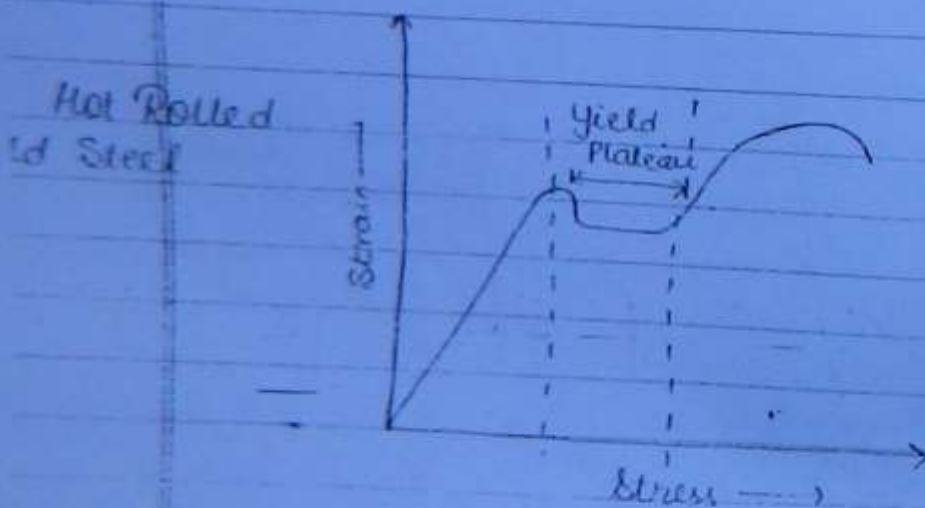
mix (M20) $\rightarrow$ f_{ck} value at 28 days strength

$\rightarrow$ Min Recommended grade of concrete for is M20 (pg no-1)

Steel :- Types :-

- ① Mild Steel $\rightarrow$ Ordinary
Flat Rolled
- ② Medium tensile Steel $\rightarrow$ Ordinary
Hot Rolled
- ③ CTD bars (Cold twisted deformed bars) or
HYSD bars (High yield strength deformed bars)
- ④ TMT bars

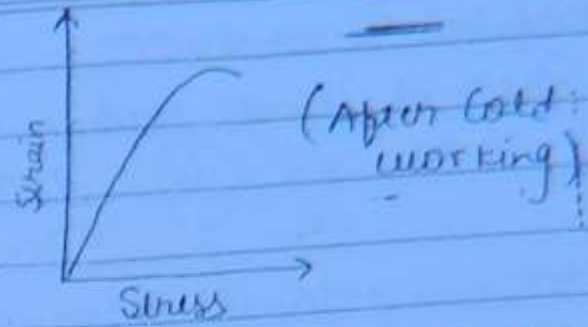
Properties of Hot Rolled mild steel bars :-



(In case of hot Rolled mild steel if stresses goes beyond yield limit, steel reinforcement is elongated resulting large deformation and cracking (i.e. undesirable).)

Remedy - (Cold working) $\rightarrow$ The steel reinforcement is stressed beyond yield limit either by stretching or twisting & then by unloading it.

Cold twisted bars are made using cold twisting process



TMT bars -

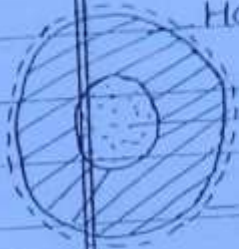
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Hard (Strength)

Soft (ductility)

Anti rusting coating

Also called as thermo mechanically treated bars



Permissible stresses in steel (Table no- 22 / P. no- 82)

W.S.M

	Mild Steel Fe 250	Medium tensile Steel	CTD bars or HYSD bars Fe 415
① Tension (σ_{st} or σ_{sc})			
upto 20 mm ϕ	140	190	230
> 20 mm ϕ	130	=	
② Compression (σ_{sc})			
All steel	130	36	130

③ Compression in bar in case of beam or slab

Case I
When Comp resistance of concrete is taking into account = $1.5 m \cdot c'$
 c' = strength in concrete surrounding steel

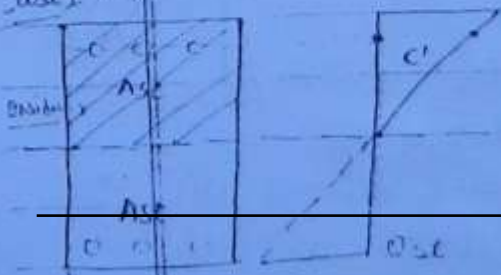
Case II
When Comp resistance is not taking into account

upto 20 mm	140	190	190
> 20 mm	130		

Case II (a)

Not considered

140	190	190
130		



SINGLY REINFORCED BEAM (L.S.M)

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1. Modular Ratio (m)

Ratio of young's modulus of elasticity of steel to concrete

$$m = \frac{E_s}{E_c} = \frac{280}{30000} \rightarrow \text{As per IS 456}$$

	M15	M20	M25	M30	M35	M40
σ_{cbc}	5	7	8.5	10	11.5	13.0
m	19	13	11	9	8	7

Consider a Column having total Area

$$A = A_c + A_s$$

If total load P

$$P = P_c + P_s$$

$$[P = A_s \cdot P_s + A_c \cdot P_c] \text{ --- (i)}$$

Deformation of Steel shall be same in Concrete & Steel

$$\delta l = \frac{P_s \cdot l}{A_s \cdot E_s} = \frac{P_c \cdot l}{A_c \cdot E_c}$$

$$\frac{P_s}{A_s \cdot E_s} = \frac{P_c}{A_c \cdot E_c} \Rightarrow \frac{P_s}{E_s} = \frac{P_c}{E_c}$$

$$\Rightarrow \frac{P_s}{P_c} = \frac{E_s}{E_c} = m$$

If strain value is same

$$p_s = m \cdot p_c$$

$$t = m \cdot c$$

$$\sigma_{st} = m \sigma_{cuc}$$

$$[c = t/m]$$

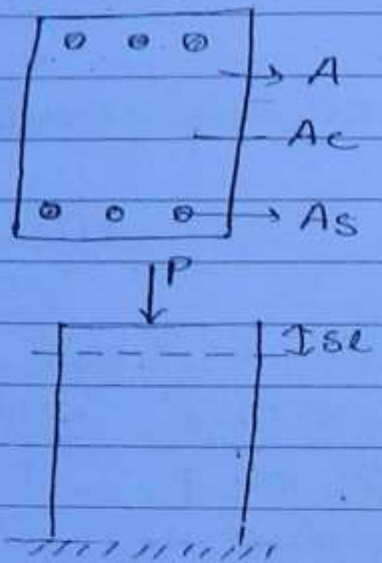
Value of p_s in (i) we get

$$P = A_s \cdot m \cdot p_c + A_c \cdot P_c$$

$$P = P_c (m A_s + A_c)$$

Total Equivalent area in terms of Concrete = $A_c + m A_s$

For A_s area of steel, equivalent area in terms of Concrete = $m A_s$



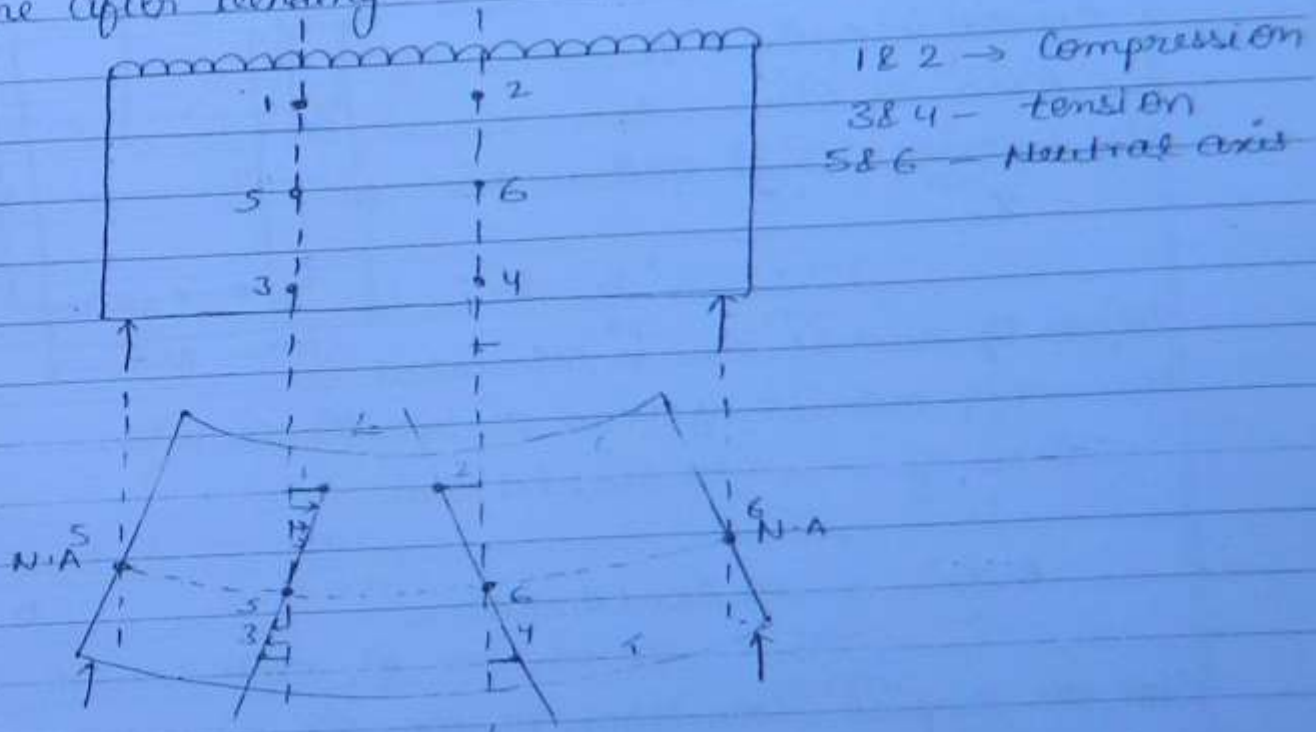
Steel	Equivalent to concrete
A_{st}	$m \cdot A_{st}$
σ_{st}	σ_{st} / m

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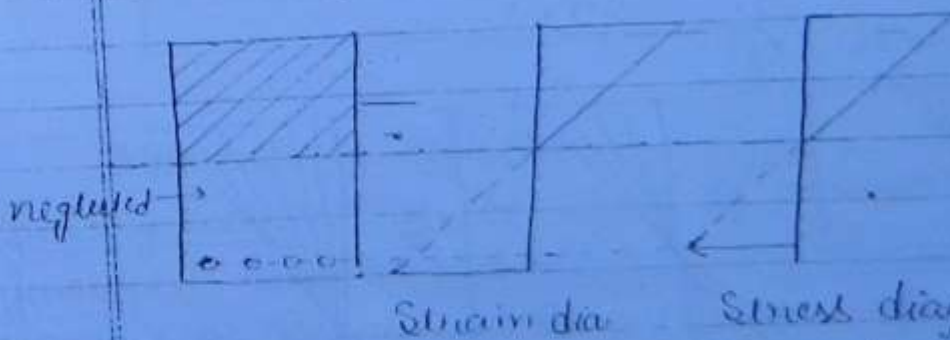
→ modular ratio value is used in working stress method & L.S.M method is also called modular ratio method.

Assumptions:- (ANNEX - B) (B-1.3 p. no. 80)

1. At any cross-section plane section before bending remains plane after bending



Result:- Strain diagram is linear



2. All tensile stresses are taken by steel only. Tensile strength of concrete is to be ignored. Concrete area in tension zone is completely neglected

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3. E_c value is constant (12)
4. Stress-strain relationship under working load is a straight line. (Stress diagram is linear)
5. Modular ratio $m = \frac{280}{3000}$

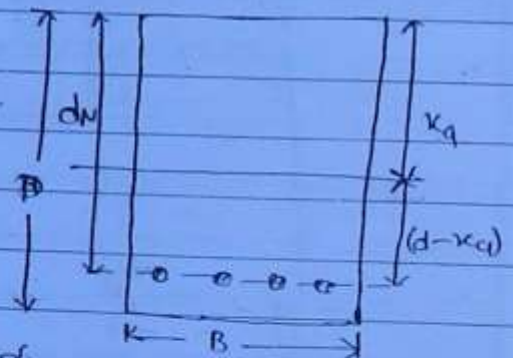
The exp given for m partially takes into account the effect of creep.

Actual depth of neutral axis
width B

Effective depth $= d$

Area of steel $= A_{st}$

If depth of actual neutral axis
 $= x_a$

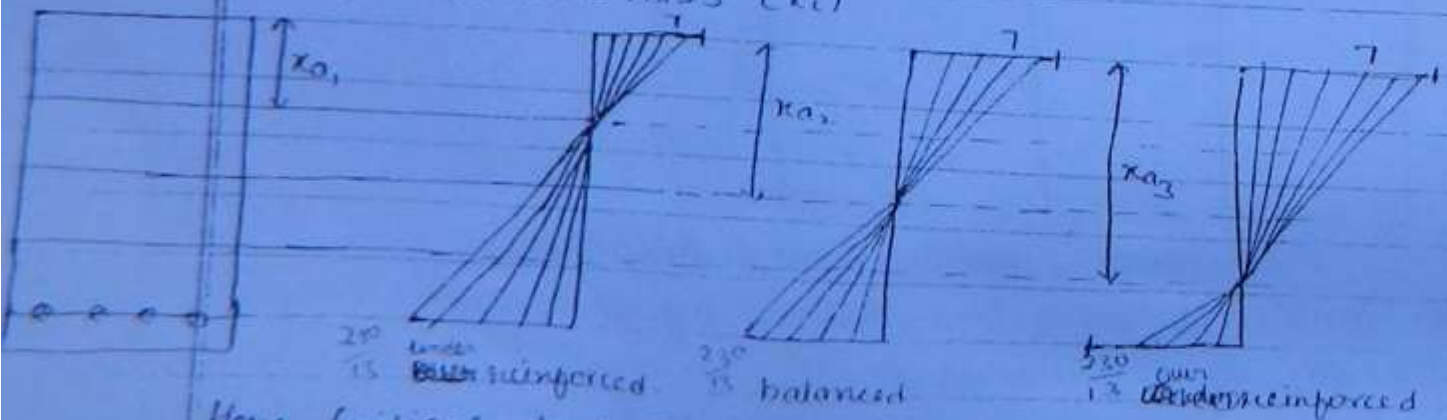


Depth of neutral axis is calculated
equating moment of area on both the side of N.A.
 $\frac{B \cdot x_a \cdot x_a}{2} = (m \cdot A_{st}) \cdot (d - x_a)$

$$\frac{B \cdot x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

For all purpose, calculation of MR, calculation of actual stress developed etc. x_a value has to be used

CRITICAL DEPTH OF NEUTRAL AXIS (x_c)



Here critical depth of N.A. $= x_{a2} = x_c$

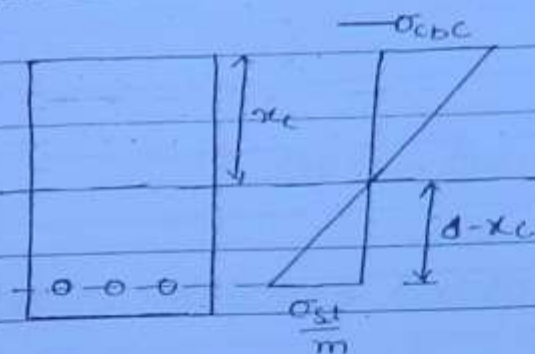
Critical depth of N.A is such that depth of N.A, so that stresses in concrete & steel are attained to its max. permissible values at the same time at $x_u = x_c$ (Balanced section)

Stress in concrete = σ_{cbc}

Stress in steel = σ_{st}

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For balanced section (for x_c)



Using similar Δ 's

$$\frac{\sigma_{cbc}}{x_c} = \frac{\sigma_{st}/m}{d-x_c}$$

$$\frac{c}{x_c} = \frac{t/m}{d-x_c}$$

$$\frac{d-x_c}{x_c} = \frac{t}{mc}$$

$$\Rightarrow \frac{d}{x_c} = \frac{t}{mc} + 1 \Rightarrow \frac{d}{x_c} = \frac{t+mc}{mc}$$

$$= \frac{x_c}{d} = \frac{mc}{t+mc}$$

$$\frac{x_c}{d} = k = \frac{mc}{t+mc}$$

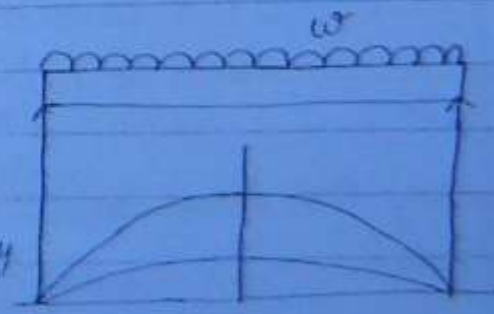
k = Critical depth of N.A factor = $\frac{mc}{t+mc}$

$$x_c = kd = \frac{mc}{t+mc} \times d$$

Moment of Resistance.

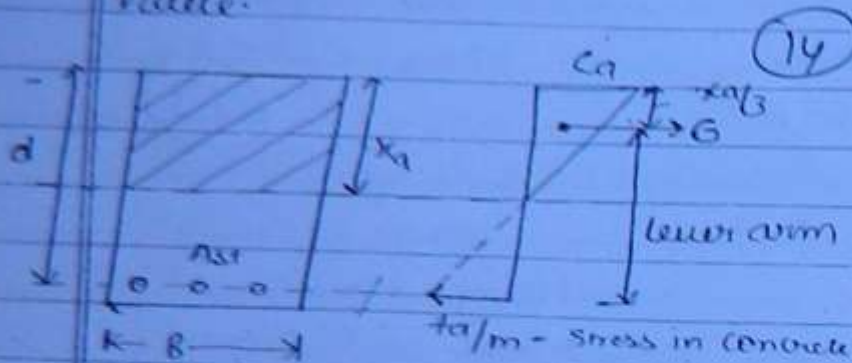
Bending moment is the value of moment developed due to load.

Moment of Resistance - It is the max. capacity of beam c/s to sustain moment



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For moment of Resistance any one material (Concrete or Steel) should be at max. permissible value.



Total Compressive force

$$C = \text{Area} \times \text{Avg. Stress} \\ B \cdot x_n \cdot \frac{c_a}{2}$$

Total tensile force

$$T = \text{Area} \times \text{Avg. Stress}$$

$$T = A_{st} \times t_a = \left(m \cdot A_{st} \times \frac{t_a}{m} \right)$$

$$\text{Lever arm} = d - \frac{x_n}{3}$$

Moment of Resistance

$$M_R = C \times \text{lever arm}$$

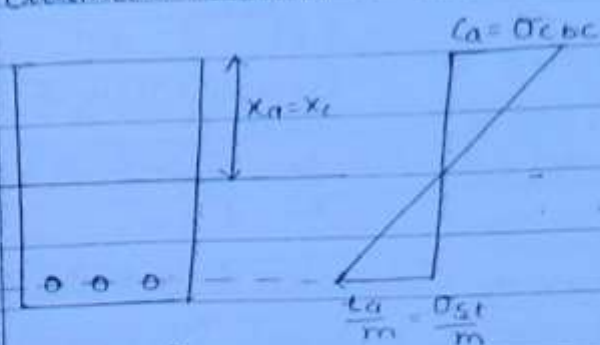
$$B \cdot x_n \cdot \frac{c_a}{2} \left(d - \frac{x_n}{3} \right) \quad \text{--- (i) (from compression)}$$

$$M_R = T \times \text{lever arm}$$

$$A_{st} \times t_a \left(d - \frac{x_n}{3} \right) \quad \text{--- (ii) (from tension)}$$

Any one stress value should be at max. permissible stress value.

CASE I Balanced Section :- In this case :- $x_a = x_c$



$$\frac{t_a}{m} = \frac{\sigma_{st}}{m}$$

$$C_a = \sigma_{cbc}$$

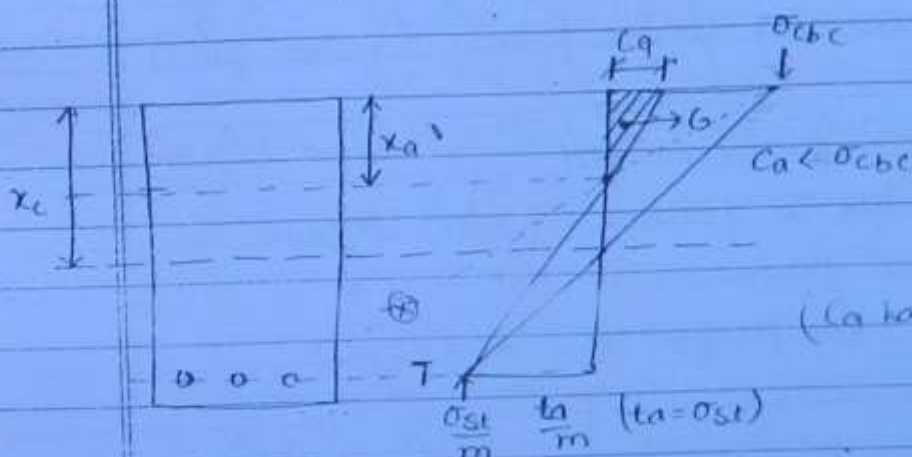
$$\text{Now } M_R = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right)$$

$$\therefore M_R = B \cdot x_c \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_c}{3} \right) \quad \text{--- (I)}$$

$$\text{again } M_R = A_{st} \cdot t_a \left(d - \frac{x_a}{3} \right)$$

$$\Rightarrow M_R = A_{st} \cdot \sigma_{st} \left(d - \frac{x_c}{3} \right) \quad \text{--- (II)}$$

CASE II Under reinforced Section :- $x_a < x_c$ (Area of steel is less than that req. for balanced sec.)



$$C_a < \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

Moment of Resistance

$$M_R = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) \quad \text{--- (1)}$$

(C_a has to be calculated by similar triangles)

$$M_R = t_a \cdot A_{st} \left(d - \frac{x_a}{3} \right)$$

$$M_R = \sigma_{st} \cdot A_{st} \left(d - \frac{x_a}{3} \right) \quad \text{--- (2)}$$

direct formula

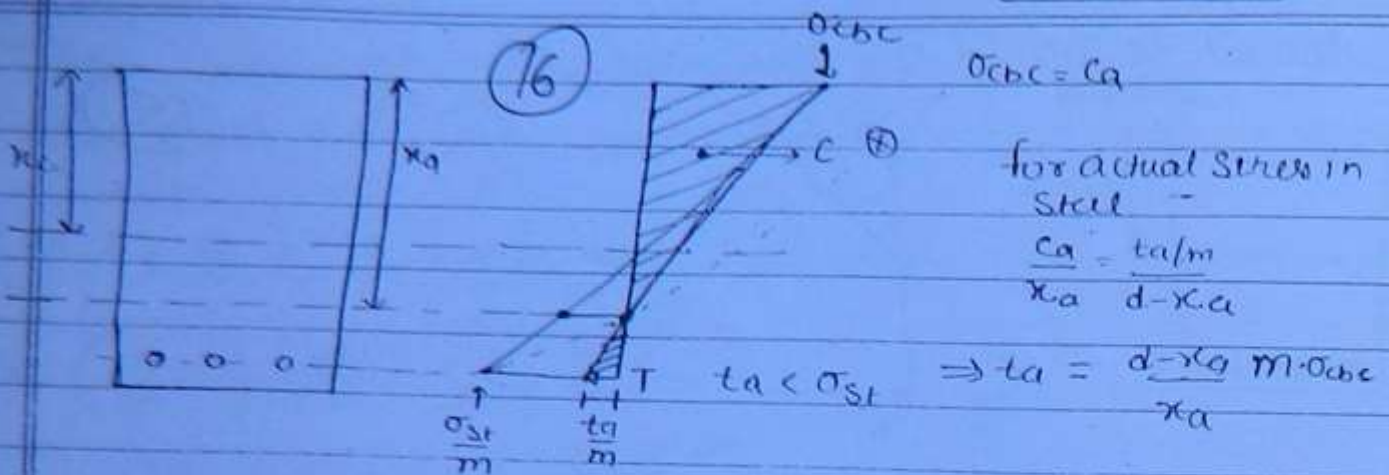
For (1) Actual stress in concrete C_a is to be calculated

$$\frac{C_a}{x_a} = \frac{\sigma_{st}/m}{d - x_a} \Rightarrow C_a = \left(\frac{x_a}{d - x_a} \right) \frac{\sigma_{st}}{m}$$

CASE III Over reinforced Section :- $x_a > x_c$; $C_a = \sigma_{cbc}$; $t_a < \sigma_{st}$

$$M_R = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_a}{3} \right) \quad \text{--- (1)}$$

$$M_R = t_a \cdot A_{st} \left(d - \frac{x_a}{3} \right) \quad \text{--- (2)}$$



In case of Over reinforced Section

- ① Concrete is at its max. permissible stress value so concrete fails first.
- ② Concrete failure is a brittle failure.
- ③ It does not give warning before failure (Sudden failure). It is to be avoided.

In case of Under reinforced Section

- ① Failure is ductile due to failure of steel because in this steel is at max. permissible stress.
- ② This section should be preferred.

Design formula

For a balanced section :- From compression side

$$M_R = B \cdot x_u \cdot \frac{C_u}{2} \left(d - \frac{x_u}{3} \right)$$

$$x_u = k d$$

$$M_R = B \cdot x_u \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_u}{3} \right)$$

$$k = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}}$$

put $x_u = k d$; $\sigma_{cbc} = C$

$$M_R = B \cdot k d \cdot \frac{C}{2} \left(d - \frac{k d}{3} \right)$$

$$= B \cdot d^2 \cdot K \cdot \frac{C}{2} \left(1 - \frac{k}{3} \right)$$

Let $\frac{1-k}{3} \rightarrow j$ Effective depth factor

$$\overline{M_R} = \frac{1}{2} c \cdot j \cdot k B d^2$$

$$M_R = Q B d^2$$

$$[Q = \frac{1}{2} c \cdot j \cdot k = \text{Moment Coefficient}]$$

$$\text{Where } k = \frac{m c}{t + m c}$$

$$j = 1 - k/3$$

For design

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STEP 1 Calculate load = w

STEP 2 Calculate bending moment

STEP 3 $B \cdot M = \text{Moment of Resistance } M_R$

$$B \cdot M = Q B d^2$$

$$\text{depth } d = \sqrt{\frac{B \cdot M}{Q \cdot B}} = \sqrt{\frac{M_R}{Q \cdot B}}$$

Area of Steel

$$B \cdot M = t_a \cdot A_{st} \left(d - \frac{x_a}{3} \right)$$

$$\sigma_{st} \cdot A_{st} \left(d - \frac{x_c}{3} \right)$$

$$\Rightarrow A_{st} = \frac{(B \cdot M = M_R)}{\sigma_{st} \left(d - \frac{x_c}{3} \right)} \Rightarrow A_{st} = \frac{B \cdot M}{\sigma_{st} \left(d - \frac{k d}{3} \right)}$$

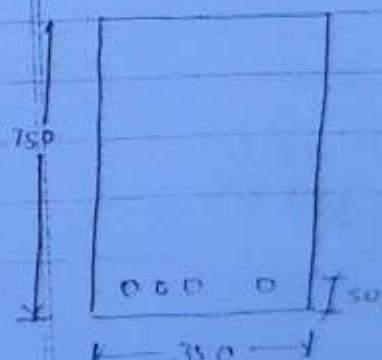
$$= \frac{B \cdot M}{\sigma_{st} \left(1 - \frac{k}{3} \right) d}$$

$$A_{st} = \frac{B \cdot M}{\sigma_{st} \cdot j \cdot d}$$

Ques:- Calculate Moment of Resistance of a RCC Rectangular beam of size 350 X 750. $f_c = 415$ & $M-20$ is used

(1) $A_{st} = 4 \text{ nos } 16 \text{ mm } \phi \text{ bars}$

(2) $A_{st} = 6 \text{ nos } 25 \text{ mm } \phi \text{ bars}$



Calculate M_R of balanced section & area of steel required for balanced section.

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Sol: - ① Critical depth of N.A x_c

$$x_c = \frac{mc}{1+mc} \times d$$

⑧

$$= \frac{13 \times 7}{230 + 13 \times 7} \times 700 = 198.44 \text{ mm}$$

① Where area of steel = 4 nos - 16mm ϕ bar

$$A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.25 \text{ mm}^2$$

Actual depth of N.A

Equating moment of area on both side of N.A

$$\frac{B \cdot x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$350 \cdot \frac{x_a^2}{2} = 13 \times 804.25 (700 - x_a)$$

$$x_a = 176.80 \text{ mm}$$

$x_a < x_c \rightarrow$ Under reinforced Section

$C_a < \sigma_{cbc}$ & $t_a = \sigma_{st}$

tension side

$$M_R = \sigma_{st} A_{st} \left(d - \frac{x_a}{3} \right) = 230 \times 804.25 \left(700 - \frac{176.80}{3} \right) \times \frac{1}{10^6}$$

$$M_R = 118.58 \text{ KN-m}$$

compression side

$$C_a = \frac{x_a}{d - x_a} \times \frac{\sigma_{st}}{m} = \frac{176.80}{700 - 176.80} \times \frac{230}{13}$$

$$C_a = 5.978 \text{ N/mm}^2$$

Now

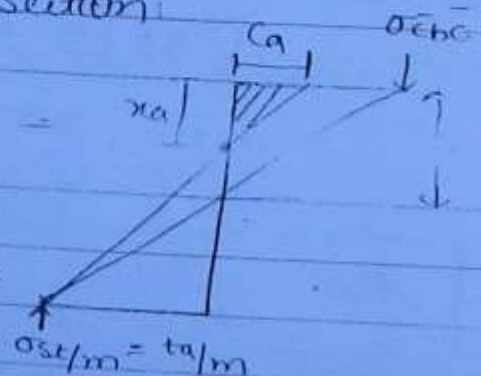
$$M_R = \frac{B \cdot x_a \cdot C_a}{2} \left(d - \frac{x_a}{3} \right)$$

$$= \frac{350 \times 176.80 \times 5.978}{2} \left(700 - \frac{176.80}{3} \right) \times \frac{1}{10^6}$$

$$= 118.57 \text{ KN-m}$$

② When area of steel = 6 nos - 25mm ϕ bar

$$6 \times \frac{\pi}{4} \times 25^2 = 2945.24 \text{ mm}^2$$



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Actual depth of N.A → By equating moment on each side of N.A

$$B \cdot x_g^2 = m \cdot A_{st} (d - x_a)$$

$$350 \cdot x_g^2 = 13 \times 2945 \cdot 24 (700 - x_a) \quad (19)$$

$$x_a = 296.95$$

$x_a > x_c \rightarrow$ Over reinforced section

$$t_a < \sigma_{st} \quad \& \quad C_a = \sigma_{cbc}$$

from direct formula of compression side

$$M_R = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2} \left(d - \frac{x_a}{3} \right)$$

$$= 350 \times 296.95 \times \frac{7}{2} \left(700 - \frac{296.95}{3} \right) \times \frac{1}{10^6} \frac{m}{m}$$

$$= 218.6 \text{ KN-m}$$

tension side $M_p = A_{st} \cdot t_a \left(d - \frac{x_a}{3} \right)$

$$\left\{ \begin{aligned} t_a &= \frac{d - x_g \cdot \sigma_{st} \cdot m}{x_a} \\ &= \frac{700 - 296.95 \times 7 \times 13}{296.95} \end{aligned} \right.$$

$$M_R = 2945 \cdot 24 \times 23.51 \left(700 - \frac{296.95}{3} \right) \times \frac{1}{10^6} \quad t_a = 123.51$$

$$M_R = 218.63 \text{ KN-m}$$

Balanced section $x_a = x_c = 198.44 \text{ mm}$

$$C_a = \sigma_{cbc} = 7 \quad ; \quad t_a = \sigma_{st} = 230$$

$$M_R = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) \Rightarrow 350 \times 198.44 \times \frac{7}{2} \left(700 - \frac{198.44}{3} \right) \times \frac{1}{10^6}$$

$$M_R = 154.08 \text{ KN-m}$$

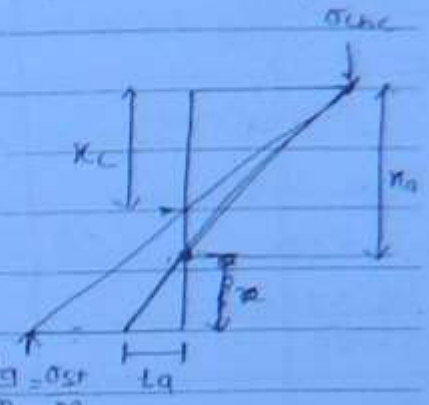
tension side $M_p = \sigma_{st} \cdot A_{st} \left(d - \frac{x_a}{3} \right)$

$$154.08 = 230 \cdot A_{st} \left(700 - \frac{198.44}{3} \right)$$

$$A_{st} = \frac{154.08 \times 10^6}{230 \left(700 - \frac{198.44}{3} \right)}$$

$$A_{st} = 1056.9 \text{ mm}^2$$

provide



Comparison

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Under Reinforced

$B = 350$

$d = 700$

$A_{st} = 4-16$

804.25 mm^2

$x = x_{bal} = 5.978$

$l_a = 230$

$= 118.57 \text{ kN-m}$

$= 176.08$

Balanced

Same

$A_{st} = 1056.9 \text{ mm}^2$

$C_a = \sigma_{cbc} = 7$

$l_a = \sigma_{st} = 230$

$M_R = 154.08 \text{ kN-m}$

$x_a = 198.44$

Overreinforced

Same

$A_{st} = 2945.24 \text{ mm}^2$

$C_a = \sigma_{cbc} = 7$

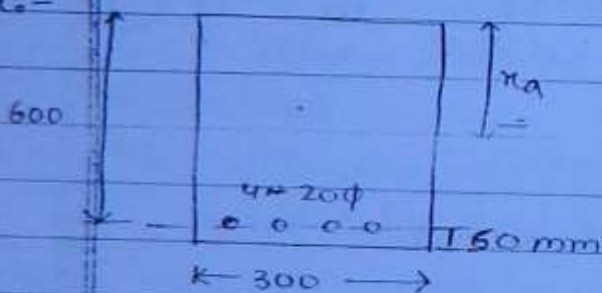
$l_a = 123.5$

$M_R = 218.63 \text{ kN-m}$

$x_a = 296.95$

Ques-2 A rectangular beam of size $300 \times 650 \text{ mm}$ is subjected to a U.D.L of 15 kN/m^2 over a S.S. span of 5.6 m (Load including self wt.). Calculate stress developed in Concrete & Steel. $A_{st} = 4 \text{ nos } 20 \text{ mm } \phi \text{ bars}$. $m = 15 \text{ mm}$ $\text{cover} = 50 \text{ mm}$

Sol:-



① Actual depth of N.A

$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$300 \cdot \frac{x_a^2}{2} = 15 \times 4 \times \frac{\pi}{4} \times 20^2 (600 - x_a)$$

$$x_a = 218.85 \text{ mm}$$

② Load = 75 kN/m

$$\text{Self wt} = 0.3 \times 0.65 \times 1 \times 25 = 4.875 \text{ kN/m}$$

$$\text{Total load } w = 19.875 \text{ kN/m}$$

$$B.M = \frac{wL^2}{8} = \frac{19.875 \times 5.6^2}{8} = 77.91 \text{ kN-m}$$

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Now $B.M = \sigma_{st} \cdot A_{st} \left(d - \frac{x_a}{3} \right)$
 $77.91 \times 10^6 = \sigma_{st} \cdot 4 \times \frac{\pi}{4} \times 20^2 \left(600 - \frac{218.85}{3} \right)$

(2) $\sigma_{st} = 117.60 \text{ N/mm}^2$ [Also calculated from similar Δ^u ~~fig~~]

$B.M = B \cdot x_a \cdot \frac{c_g}{2} \left(d - \frac{x_a}{3} \right)$

$77.91 \times 10^6 = 300 \times 218.85 \times \frac{\sigma_{cbc}}{2} \left(600 - \frac{218.85}{3} \right)$

$\sigma_{cbc} = 4.503 \text{ N/mm}^2$

A reinforced concrete beam of size $350 \times 700 \text{ mm}$ is to be designed for a B.M of (i) 100 kN-m (ii) 200 kN-m . What will be steel reinforcement required for above B.M. M20, Fe415. What will be steel required for balanced section.

Effective cover = 50 mm

Solⁿ MR of balanced section

$MR = Q \cdot B \cdot d^2$

$Q = \frac{1}{2} \times 7 \times 0.905 \times 0.2835$
 $= 0.898$

$MR = 0.898 \times 350 \times (650)^2 / 100$
 $= 132.79 \text{ kN-m}$

$Q = \frac{1}{2} \cdot \rho \cdot j \cdot k$

$k = \frac{m \cdot c}{1 + m \cdot c} = \frac{13 \times 7}{1 + 13 \times 7}$

$k = 0.2835$

$j = \left(1 - \frac{k}{3} \right) = 0.905$

(i) $B.M = 100 \text{ kN-m}$

$B.M < MR$ Singly reinforced balanced section so we need under reinforced section.

$B.M = B \cdot x_a \cdot \frac{c_g}{2} \left[d - \frac{x_a}{3} \right] \quad \text{--- (i)}$

$100 \times 10^6 = B \cdot x_a \cdot \frac{c_g}{2} \left[d - \frac{x_a}{3} \right]$

$B.M = \sigma_{st} \cdot A_{st} \left[d - \frac{x_a}{3} \right]$

DATE:

for an under reinforced section —

$$\sigma_a < \sigma_{cbc}$$

$$\epsilon_s = \epsilon_y$$

(22)

So from eqⁿ (2)

$$100 \times 10^6 = 230 \cdot A_{st} \left(\frac{650 - x_a}{3} \right) \quad \text{--- (3)}$$

Equating moment of area

$$\frac{B \cdot x_a^2}{2} = m A_{st} (d - x_a)$$

$$\frac{350 \times x_a^2}{2} = 13 \times A_{st} (650 - x_a)$$

$$A_{st} = \frac{350 x_a^2}{2 \times 13 (650 - x_a)}$$

$$A_{st} = \frac{13.46 x_a^2}{650 - x_a}$$

Putting the values of A_{st} in eq (3)

$$100 \times 10^6 = 230 \times \frac{13.46 x_a^2}{(650 - x_a)} \left(\frac{650 - x_a}{3} \right)$$

$$100 \times 10^6 = \frac{3095.8 x_a^2 (1950 - x_a)}{3(650 - x_a)}$$

$$100 \times 10^6 = \frac{1031.93 (1950 - x_a) x_a^2}{(650 - x_a)}$$

$$\frac{(1950 - x_a) x_a^2}{(650 - x_a)} = 96905.46$$

$$1950 x_a^3 - x_a^3 = 62988549 - 96905.46 x_a$$

$$x_a^3 - 1950 x_a^2 - 96905.46 x_a + 62988549 = 0$$

$$x_a = 162.5 \text{ mm}$$

$$A_{st} = \frac{13.46 x_a^2}{650 - x_a}$$

$$A_{st} = 729.08 \text{ mm}^2$$

Shortcut :- Only under-reinforced in beam

Considering a balanced section

$$x_a = x_c = 0.2835d = 0.2835 \times 650 = 184.275$$

(23)

$$A_{st} = \frac{BM}{\sigma_{st} \cdot j \cdot d} = \frac{100 \times 10^6}{230 \times 0.905 \times 650} = 739.9 \text{ mm}^2$$

Above formula can be used for slabs but not for beam.

(ii) $BM = 200 \text{ kN-m}$

$BM > MR$ (over reinforced singly or doubly)

$$200 \times 10^6 = B \cdot x_a \cdot \frac{C_g}{2} \left[d - \frac{x_a}{3} \right]$$

$$200 \times 10^6 = 350 \times x_a \cdot \frac{C_g}{2} \left[650 - \frac{x_a}{3} \right]$$

$$163265.31 = x_a \left[650 - \frac{x_a}{3} \right]$$

$$489795.9 = 1950 x_a - x_a^2$$

$$x_a = 296.14 \text{ mm}$$

from similar Δ^{ve}

$$\frac{C_g}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow t_a = \frac{(d - x_a)m \cdot C_g}{x_a}$$

$$t_a = \frac{(650 - 296.14) \times 138}{296.14}$$

$$t_a = 108.74 \text{ N/mm}^2$$

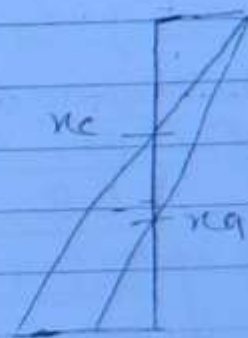
$$BM = t_a \cdot A_{st} \left[d - \frac{x_a}{3} \right]$$

$$200 \times 10^6 = 108.74 \cdot A_{st} \left[650 - \frac{296.14}{3} \right]$$

$$A_{st} = 3336.3 \text{ mm}^2$$

Area of steel required for bal. sec.

$$A_{st} = \frac{MR}{C_g \cdot j \cdot d} = \frac{132.79 \times 10^6}{230 \times 0.904 \times 650} = 982.5 \text{ mm}^2$$



Ques. Design a rectangular R.C.C beam for a live load of 45 kN over a span of 7.50 m. Use M-20 & Fe 415. load is excluding self wt. of beam. Consider width = 400 mm

Soln $B = 400 \text{ mm}$ Span = 7.50 m (effective)

1. Load / BM $\rightarrow$ I.L = 45 kN/m

$$\text{Self wt} = 0.4 \times 7.5 \times 1 \times 25 = 7.50 \text{ kN/m}$$

Assume depth = 750 mm = Span/10

$$\text{total } w = 45 + 7.50 = 52.50 \text{ kN/m}$$

$$\text{B.M} = \frac{wl^2}{8} = \frac{52.50 \times (7.5)^2}{8} = 369.14 \text{ kN-m}$$

24

Let us design for balanced section

$$x_u = x_c ; C_u = 0.85 f_{ck} ; f_u = f_{yk}$$

(i) depth required -

$$d = \sqrt{\frac{\text{BM}}{Q \cdot B}}$$

$$d = \sqrt{\frac{369.14}{1.11 \times 400}}$$

$$d = 911.8$$

eff cover = 50

$$d = 961.80 > 750 \text{ mm}$$

Let us consider $d = 1000 \text{ mm}$

$$\text{Self wt} = 0.4 \times 1.0 \times 1 \times 25 = 10 \text{ kN/m}$$

$$\text{total } w = 45 + 10 = 55 \text{ kN/m}$$

$$\text{B.M} = 386.76 \text{ kN-m}$$

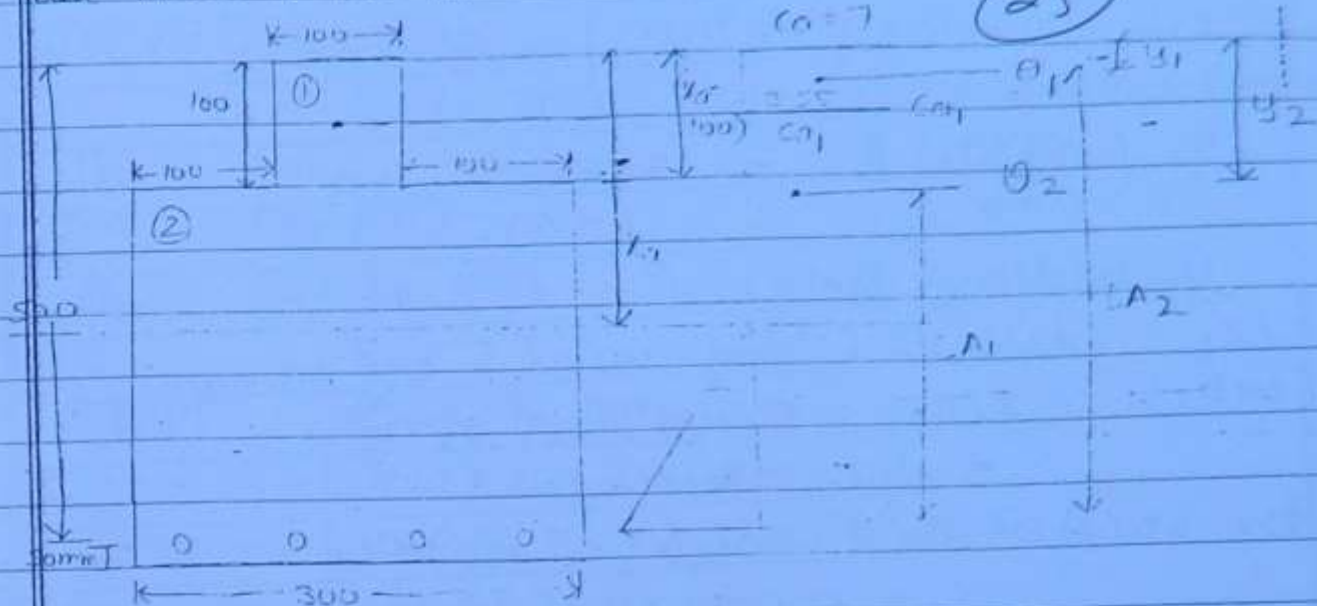
again $d = \sqrt{\frac{386.76}{1.11 \times 400}} = 933 \text{ mm}$

eff cover 50 $\Rightarrow d = 983 \text{ mm} \approx 1000 \text{ mm}$ OK

$$A_{st} = \frac{\text{B.M}}{\sigma_{st} j \cdot d} = \frac{386.76 \times 10^6}{230 \times 904 \times 950}$$

$$A_{st} = 1958 \text{ mm}^2$$

Calculate Moment of Resistance of section shown in fig.
Use M-20 & Fe 415



Critical depth of N.A $x_c = k \cdot d$

$$x_c = \frac{m_c}{1 + m_c} x_d = \frac{13 \times 7}{230 + 13 \times 7} \times 500 \Rightarrow 141.75 \text{ mm}$$

Actual depth of N.A — Equating moment area on both side
Assume $x_d > 100 \text{ mm}$

$$100 \times 100 \left(\frac{x_d - 50}{100} \right) + 300 \left(\frac{x_d - 100}{2} \right) = 66.67 \left(\frac{500 - x_d}{2} \right)$$

$$66.67 \times (x_d - 50) + (x_d - 100)^2 = 66.67 [500 - x_d]$$

$$x_d^2 - 200x_d + 10^4 + 66.67x_d - 3333.5 = 0$$

$$x_d = 202.68 > 100 \text{ mm (assumption is correct)}$$

$$x_d > x_c ; C_a = \sigma_{cbc} = 7 \text{ N/mm}^2$$

$$t_a < \sigma_{st}$$

Stress at junction

$$\frac{C_{a1}}{x_d - 100} = \frac{C_a}{x_d}$$

$$C_{a1} = \frac{(x_d - 100) \times 7}{x_d}$$

$$C_{a1} = 3.55 \text{ N/mm}^2$$

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Moment of Resistance

(26)

for bar (1)

$$C_1 = \text{area} \times \text{avg. Stress}$$

$$C_1 = 100 \times 100 \times \left[\frac{7 + 3.55}{2} \right] = 52750 \text{ N}$$

$$\bar{y}_1 = \left(\frac{a + 2b}{a + b} \right) \frac{h}{3} = \frac{7 + 2 \times 3.55}{7 + 3.55} \times \frac{100}{3}$$

$$y_1 = 44.55 \text{ mm}$$

$$LA_1 = (d - y_1) = 500 - 44.55 = 455.45 \text{ mm}$$

$$MR_1 = C_1 \times LA_1 = 52750 \times 455.45 \times 10^{-6} = 24.025 \text{ kN-m}$$

for bar (2)

$$C_2 = 300 \times (x_a - 100) \times \frac{C_1}{2} = 54677.1 \text{ N}$$

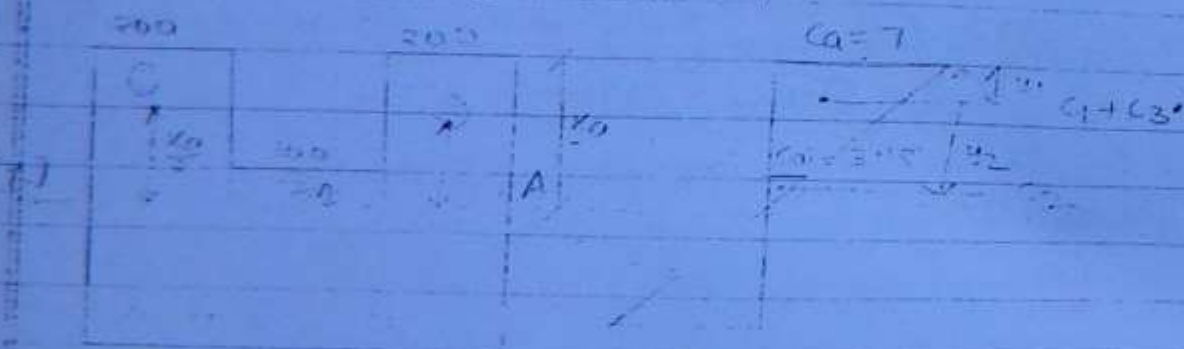
$$y_2 = 100 + \left(\frac{x_a - 100}{3} \right) = 133.89 \text{ mm}$$

$$LA_2 = d - y_2 = 366.11 \text{ mm}$$

$$(MR)_2 = C_2 \times LA_2 = 20.02 \text{ kN-m}$$

$$\text{Total M.R} = 24.025 + 20.02 = 44.04 \text{ kN-m}$$

A Reinforced Concrete beam has shown in fig. There are 4 - 20 ϕ bar. Calculate N.A & Stress developed due to a B.M. 85000 N-m



depth of N.A. Equating moment of area

$$2 \times 200 \times x_a \times \frac{x_a}{2} + 100 \times \left(-100 + x_a \right) \left(\frac{x_a - 100}{2} \right) = m \cdot A_s \cdot (d - x_a)$$

$$200 x_a^2 + 50 (x_a - 100)^2 = 15 \times \frac{\pi}{4} \times (20)^2 \times (450 - x_a)$$

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$$200 x_a^2 + 50 (x_a^2 + 100^2 - 2 \times 100 x_a) = 4712.38 (550 - x_a)$$

$$250 x_a^2 + 500000 - 10000 x_a = 2591809 - 4712.38 x_a$$

$$250 x_a^2 - 14712.38 x_a + 2091809 = 0$$

$$x_a = 181.75 \text{ mm} > 100 \text{ (Assumption is correct)}$$

$$\frac{C_a}{x_a} = \frac{C_{a1}}{(x_a - 100)} \quad (27)$$

$$C_{a1} = \frac{C_a (x_a - 100)}{x_a} = \frac{C_a (181.75)}{181.75} = 0.45 C_a$$

$$B.M = 85 \times 10^6 \text{ N-mm}$$

$$C_1 + C_3 = 2 (\text{Area} \times \text{avg. Stress})$$

$$= 2 (200 \times x_a \times \frac{C_a}{2}) = 36350 C_a$$

$$y_1 = \frac{x_a}{3} = 60.58 \text{ mm}$$

$$LA_1 = 550 - y_1 = 489.416$$

$$(MR_1 + MR_3) = (C_1 + C_3) \times LA_1$$

$$= 36350 C_a \times 489.416 = 17.79 C_a \text{ KN-m}$$

$$C_2 = 100 \times (x_a - 100) \times \frac{C_a}{2}$$

$$= 1839.37 C_a$$

$$y_2 = 100 + (x_a - 100)$$

$$y_2 = 127.25$$

$$LA_2 = 550 - 127.25 = 422.75 \text{ mm}$$

$$MR_2 = 0.77 \text{ KN-m}$$

$$\text{Total} - MR_1 + MR_2 + MR_3 = 18.56 C_a$$

$$18.56 C_a = 85 \Rightarrow C_a = 4.58 \text{ N/mm}^2$$

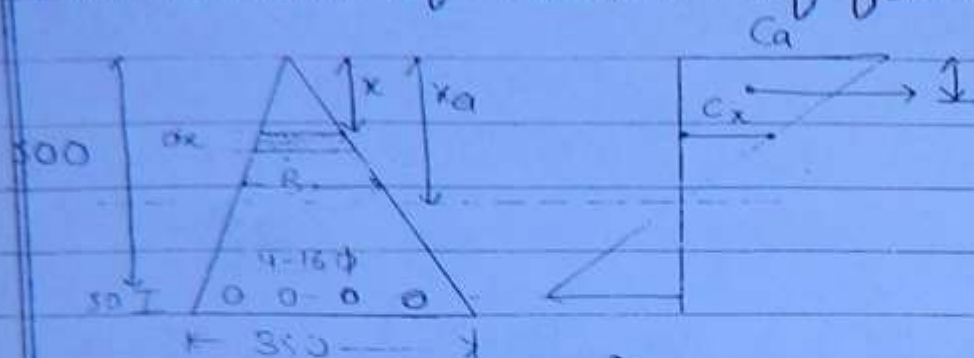
$$C_{a1} = 0.45 \times 4.58 = 2.06 \text{ N/mm}^2$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow t_a = \frac{(d - x_a)(m \cdot C_a)}{x_a}$$

$$= \frac{(550 - 181.75)(15 \times 4.58)}{181.75}$$

$$t_a = 139.19 \text{ N/mm}^2$$

Ques. Find Moment of resistance of following section.



(28)

Critical depth of N.A. $x_c = \frac{m_c}{t + m_c} \times d = \frac{13 \times 7}{230 + 13 \times 7} \times 500$
 $x_c = 141.75 \text{ mm}$

Actual depth

$$\frac{B_1}{x_a} = \frac{350}{550}$$

$$B_1 = \frac{7}{11} x_a$$

$$\frac{1}{2} \times B_1 \times x_a \cdot x_a = m \cdot A_{st} (d - x_a)$$

$$\frac{1}{2} \times \frac{7}{11} \times \frac{1}{3} \cdot x_a^3 = 13 \times 4 \times \frac{\pi}{4} \times 16^2 (500 - x_a)$$

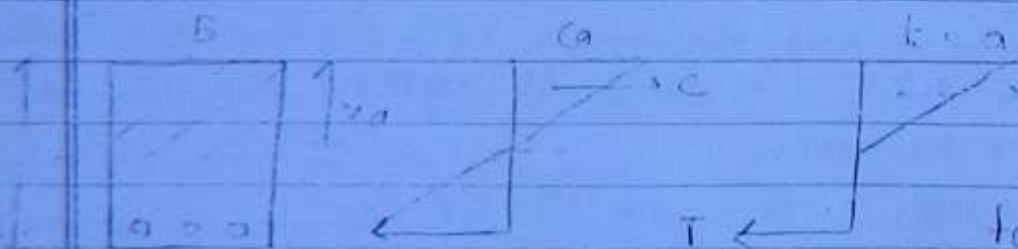
$$0.106 x_a^3 = 5227610.176 - 10455.22 x_a$$

$$0.106 x_a^3 + 10455.22 x_a - 5227610.176 = 0$$

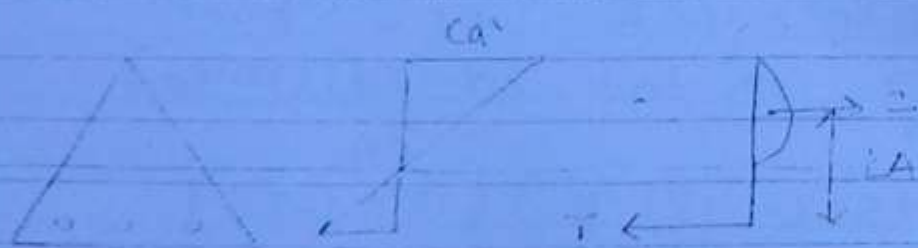
by trial & error $x_a = 279.2 \text{ mm} > x_c$
 (over reinforced)

$$C_a = 0.85 f_{ck} = 7$$

$$t_a < 0.87 f_y$$



Force in steel



Consider an elemental strip of thickness at distance x from top. $B_x = \frac{7}{11}x$ - (double)

If width of beam at NA = B_1
 $\frac{B_1}{x_a} = \frac{350}{550} = \frac{7}{11}$

$B_1 = \frac{7}{11} x_a$

(29)

$\frac{C_x}{x_a} = \frac{C_x}{(x_a - x)} \Rightarrow C_x = \frac{x_a - x}{x_a} C_a$

$\Rightarrow C_x = \frac{279.2 - x}{279.2} \times 7$

$C_x = \frac{279.2 - x}{39.88}$

M.R of elemental strip $C_x \times L \times A$
 $= \text{area} \times \text{avg. stress} \times L \times A$

$MR = (B_x \cdot dx) (C_x) (d - x)$
 $\frac{7}{11} x \cdot dx \left(\frac{279.2 - x}{39.88} \right) (500 - x)$

total M.R. $\rightarrow$

$\int_0^{x_a} MR = \int_0^{x_a} \frac{7}{11} x \left(\frac{279.2 - x}{39.88} \right) (500 - x) dx$

$\frac{7}{11} \int_0^{x_a} \left(\frac{279.2 - x - x^2}{39.88} \right) \cdot (500 - x) dx$

$= \frac{7}{11} \int_0^{x_a} \frac{279.2 \cdot x}{39.88} \cdot 500 - \frac{279.2 \cdot x^2}{39.88} - \frac{500 \cdot x^2}{39.88} + \frac{x^3}{39.88} dx$

$\frac{7}{11} \int_0^{x_a} 3500 \cdot 50x - 7x^2 - 12.53x^2 + 0.025x^3 dx$

$\frac{7}{11} \int_0^{x_a} 3500 \cdot 50x - 19.55x^2 + 0.025x^3 dx$

$\frac{7}{11} \left[\frac{0.025x^4}{4} - \frac{19.55x^3}{3} + \frac{3500 \cdot 50x^2}{2} \right]_0^{x_a}$

$MR = \frac{7}{11} \left[\frac{0.025x_a^4}{4} - \frac{19.55x_a^3}{3} + 1750 \cdot 25x_a^2 \right]$

$MR = \frac{7}{11} (0.00625x_a^4 - 6.52x_a^3 + 1750 \cdot 25x_a^2)$

30

DOUBLY REINFORCED SECTION

- # Doubly reinforced section is needed if
1. Size of beam (width & depth) both are fixed
 2. Beam has to sustain a higher B.M. $> M_R$ of singly reinforced balanced section

if size of beam = $B \times d$

M_R of singly reinforced balanced section

$$M_R = Q \cdot B d^2$$

$$Q = \frac{1}{2} c_j k$$

if $B.M. > M_R$

→ There are two options

- (i) Over-reinforced section - avoided
- (ii) Doubly-reinforced section - preferred

Permissible stress in compression steel

$$= 1.5 m \cdot c'$$

c' = Stress in concrete at the level of compression steel

Equivalent area for A_{sc} area of steel in terms of concrete

$$= 1.5 m \cdot A_{sc}$$

1.5 m value is used for compression steel, is due to Continued Compressive Strain for a long time (Creep). Due to Creep effect actual stress in steel is much higher than stress developed due to elastic strain.

Analysis

1. Critical depth of N.A

$$x_c = \frac{m c}{1 + m c} \times d$$

2. Actual depth of N.A

Equating moment of area on both side of N.A

$$B \cdot x_a \cdot \frac{x_a}{2} + 1.5 m \cdot A_{sc} (x_a - d_c) = A_{sc} (x_a - d_c) = m A_{st} (d - x_a)$$

concrete area not available

$$B \cdot \frac{x_a^2}{2} + (1.5 m - 1) A_{sc} (x_a - d_c) = m A_{st} (d - x_a)$$

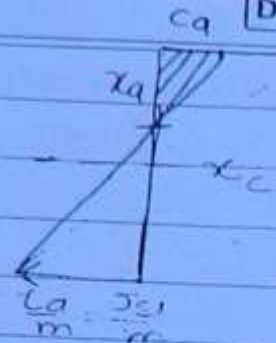
Compare —

if $x_a < x_c \rightarrow$ Under reinforced

$$C_a < \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

(32)



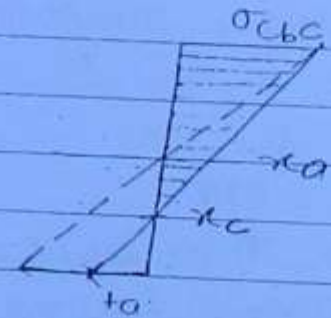
if $x_a = x_c \rightarrow$ balanced section

$$C_a = \sigma_{cbc}$$

$$t_a = \sigma_{st}$$

if $x_a > x_c \rightarrow$ Over reinforced

$$C_a = \sigma_{cbc} \quad t_a < \sigma_{st}$$



Moment of Resistance, fig (i) (Compression side)
for Concrete $C_1 = B \cdot x_a \cdot \frac{C_a}{2}$

$$L_{A1} = \left(d - \frac{x_a}{3} \right)$$

$$MR_1 = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right)$$

Compression Steel $C_2 = 1.5 m \cdot A_{sc} \cdot C' - A_{sc} \cdot C'$
 $(1.5m - 1) A_{sc} \cdot C'$

$$\text{lever arm } L_{A2} = d - d_c$$

$$MR_2 = (1.5m - 1) A_{sc} \cdot C' (d - d_c)$$

$$\text{Total M.R} = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) + (1.5m - 1) A_{sc} \cdot C' (d - d_c)$$

Moment of Resistance from tension side.

$$C_R = C_1 + C_2$$

$$y_1 = \frac{x_a}{3}; \quad y_2 = d_c \quad \text{and } \bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2}$$

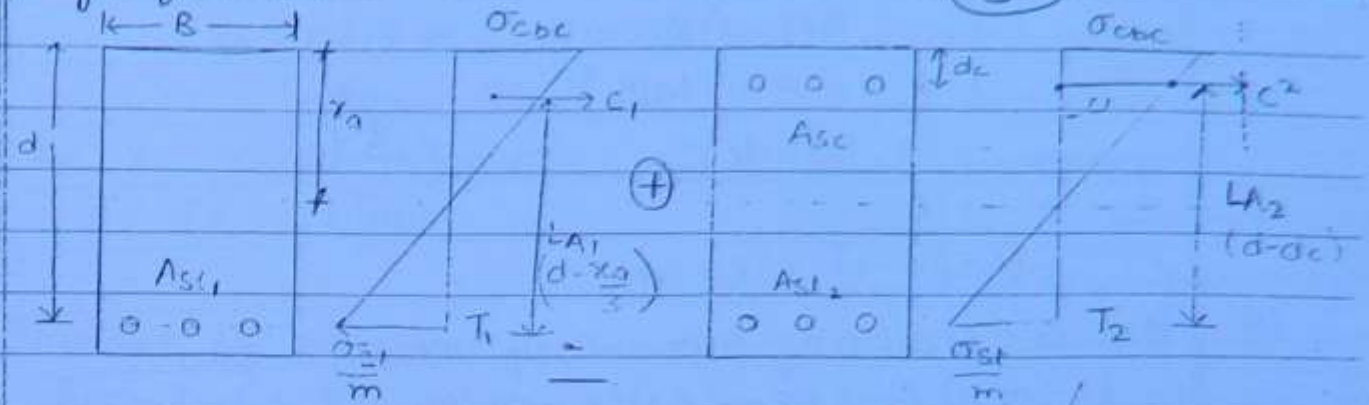
$$\text{Net lever arm} = (d - \bar{y})$$

$$M.R = C_R (d - \bar{y})$$

$$= T (d - \bar{y})$$

$$M.R = A_{st} \cdot t_a (d - \bar{y})$$

Design formula



$$C_1 = B \cdot x_a \cdot \frac{\sigma_{cbc}}{2}$$

$$LA_1 = (d - \frac{x_a}{3})$$

$$T_1 = \sigma_{st} \cdot A_{st1}$$

$$C_2 = (1.5m - 1) A_{sc} \cdot c'$$

$$LA_2 = (d - d_c)$$

$$T_2 = \sigma_{st} \cdot A_{st2}$$

$$M \cdot R = C_1 \cdot LA_1 + C_2 \cdot LA_2 \quad (\text{Compression})$$

$$= B \cdot x_a \cdot \frac{\sigma_{cbc}}{2} (d - \frac{x_a}{3}) + (1.5m - 1) A_{sc} \cdot c' (d - d_c)$$

for balanced section $M \cdot R = (M \cdot R)_\text{single reinforced section}$

$$= B \cdot x_c \cdot \frac{\sigma_{cbc}}{2} (d - \frac{x_c}{3}) + (1.5m - 1) A_{sc} \cdot c' (d - d_c)$$

$$Q b d^2 + (1.5m - 1) A_{sc} \cdot c' (d - d_c)$$

$$M \cdot R = T_1 \cdot LA_1 + T_2 \cdot LA_2 \quad (\text{Tension})$$

$$= \sigma_{st} \cdot A_{st1} (d - \frac{x_a}{3}) + \sigma_{st} \cdot A_{st2} (d - d_c)$$

Design Steps :-

1. If beam has to be designed for a B.M = B.M
 $B.M > (M \cdot R)_1 = Q b d^2$

2. Check $M \cdot R_1 = Q b d^2$

if $B.M > M \cdot R_1$, Then doubly reinforced section is required.

3. Calculate A_{st1} Equating $M \cdot R$ on both tension & compression side.

$$A_{st1} = \frac{M \cdot R_1 - Q b d^2}{\sigma_{st} (d - \frac{x_a}{3})} = \frac{M \cdot R_1}{\sigma_{st} (d - \frac{x_a}{3})}$$

4. ~~Remaining~~ Remaining B.M

$$MR_2 = BM - MR_1$$

$$Ast_2 = \frac{MR_2 = BM - MR_1}{\sigma_a \cdot (d - d_c)}$$

(34)

5 $Asc = \frac{MR_2}{(1.5m-1) \cdot C' \cdot (d - d_c)}$

Or for Asc equating moment of area for part 2
 $(1.5m-1) Asc \cdot (x_a - d_c) = m \cdot Ast_2 (d - x_a)$

$$Asc = \frac{m \cdot Ast_2 (d - x_a)}{(1.5m-1) (x_a - d_c)} \quad \checkmark \text{ (commonly used)}$$

Generally M.R is calculated from compression side.

Ques:- A rectangular beam of $230 \times 600 \text{ mm}$ c. 50 mm efb. lower is reinforced 5-25 ϕ - on tension & 3-16 ϕ - compression. It is carrying an imposed load of 50 kN/m over an effective span of 7m. Check adequacy of section (M-20) to 415.

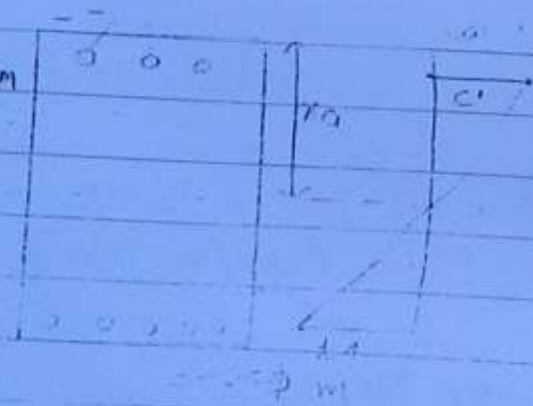
Let:- Imposed load = 50 kN/m

Self wt = $23 \times 6 \times 1 \times 25 = 345 \text{ kN/m}$

total = 53.45 kN/m

$$BM = \frac{wl^2}{8} = \frac{53.45 \times 7^2}{8}$$

$$B.M = 321.38 \text{ kN-m}$$



Moment of Resistance

$$\text{Critical depth } x_c = \frac{m c}{1 + m c} \times d = \frac{13 \times 7}{230 + 13 \times 7} \times 550 = 155.92 \text{ mm}$$

Actual depth

$$B \cdot \frac{x_a^2}{2} + (1.5m-1) Asc (x_a - d_c) = m \cdot Ast (d - x_a)$$

$$230 \cdot \frac{x_a^2}{2} + (1.5 \times 3 - 1) \cdot 603.20 (x_a - 50) = 13 \times 2454.4 (550 - x_a)$$

$$x_a^2 + 97.04x_a - 4851.83 = 152600 - 277.45x_a$$

$$x_a^2 + 374.49 - 157451.83 = 0$$

$$x_a = 251.3 \text{ mm}$$

$x_a > x_c \rightarrow$ Over reinforced $C_a = \sigma_{cbc}$ $t_a < \sigma_{st}$

$$\text{So } C_a = 7 \text{ N/mm}^2$$

(35)

Moment of Resistance

$$B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$$

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = \frac{7(251.3 - 50)}{251.3} = 5.607$$

$$M_R = 230 \times 251.3 \times \frac{7}{2} \left(550 - \frac{251.3}{3} \right) + (1.5 \times 13 - 1) 603.2 \times 5.607 (550 - 50)$$

$$M_R = 125.60 \text{ kN-m}$$

$\therefore B.M > M.R$ $\therefore$ it is not safe.

Method II - Calculate the stresses developed in the beam

$\rightarrow$ due to B.M 327.38 kN-m

1. Calculate x_a $x_a = 251.31 \text{ mm}$

If the stresses develop are $C_a =$ in concrete & $t_a =$ in steel

$$\frac{C'}{x_a - d_c} = \frac{C_a}{x_a} \Rightarrow C' = 0.8 C_a$$

Equating B.M & moment of resistance we get

$$327.38 = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) + (1.5m-1) A_{sc} \cdot C' (d - d_c)$$

$$327.38 \times 10^6 = 230 \times 251.31 \times \frac{C_a}{2} \left(550 - \frac{251.3}{3} \right) + (1.5 \times 13 - 1) 603.2 \times 0.8 C_a (550 - 50)$$

$$C_a = 18.25 \text{ N/mm}^2$$

$$C_a > 7 \text{ N/mm}^2 \quad \text{failed.}$$

$$C' = 0.8 \times 18.25 = 14.6 \text{ N/mm}^2$$

DATE:

Stress in Compression Steel

$$= 1.5 m \times c' = 284.7 \text{ N/mm}^2$$

Stress in Tension Steel

Using similar Δ^u $\frac{c_a}{x_a} = \frac{t_a}{d - x_a}$

(36)

$$\frac{t_a}{x_a} = \frac{(d - x_a)m \cdot c_a}{251.31} = \frac{(550 - 251.31)13 \times 18.25}{251.31}$$

$$t_a = 281.9 \text{ N/mm}^2 > 230 \text{ N/mm}^2$$

(Failed)

Ques:- Design a Rectangular beam of size 300x750mm overall depth for a live load of 45 kN/m over a span of 7.5m. Use M-25 & Fe 415.

Sol. (1) Live load = 45 kN/m

$$\text{Self wt} = 3 \times 7.5 \times 1 \times 25 = 5.625 \text{ kN/m}$$

$$B.M = \frac{wL^2}{8} = \frac{50.625 \times (7.5)^2}{8} = 355.95 \text{ kN-m}$$

(2) Calculate M.R of singly reinforced balanced section

$$M R_1 = Q B d^2$$

$$Q = \frac{1}{2} C \cdot j \cdot k = 1.11$$

$$k = \frac{m c}{1 + m c} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = 0.289$$

$$j = \frac{1 - k}{3} = 0.904$$

$$Q = 1.11$$

$$M R_1 = 1.11 \times 230 \times 700^2$$

$$M R_1 = 163.17 \text{ kN-m}$$

$$B.M > M R_1$$

(Doubly reinforced section)

$$A_{st} = \frac{M R_1}{\sigma_{st} (d - \frac{x_c}{3})} = \frac{M R_1}{\sigma_{st} j \cdot d}$$

$$= \frac{163.17 \times 10^6}{230 \times 0.904 \times 700} = 1121.1 \text{ mm}^2$$

DATE: _____

$$\textcircled{4} \quad MR_2 = \overline{B.M} - MR_1 \\ = 355.96 - 163.17 = 192.79 \text{ kN-m}$$

$$\textcircled{5} \quad A_{st2} = \frac{192.79}{230(700-50)} = 1289.56 \text{ mm}^2 \quad \textcircled{37}$$

$$\textcircled{6} \quad A_{sc} = \frac{m \cdot A_{st2}(d-x_a)}{(1.5m-1)(x_a-d_c)} = \frac{11 \times 1289.56(700-0.289 \times 700)}{(1.5 \times 11-1)(0.289 \times 700-50)} \\ = A_{sc} = 2990.6 \text{ mm}^2$$

$$\text{OR } A_{sc} = \frac{MR_2}{(1.5m-1)c'(d-d_c)} = \frac{192.79 \times 10^6}{(1.5 \times 11-1) \times 6.4(700-50)}$$

$$C_a = \sigma_{cbc} = 8.5 \text{ N/mm}^2$$

$$c' = \frac{x_a-d_c}{x_a} \times C_a = \frac{0.289 \times 700 - 50}{0.289 \times 700} \times 8.5$$

$$c' = 6.4 \text{ N/mm}^2$$

$$\therefore A_{sc} = 2989.9 \text{ mm}^2$$

$$\text{Total } A_{st} = A_{st1} + A_{st2} = 1121.1 + 1289.56 \\ = 2410.66 \text{ mm}^2$$

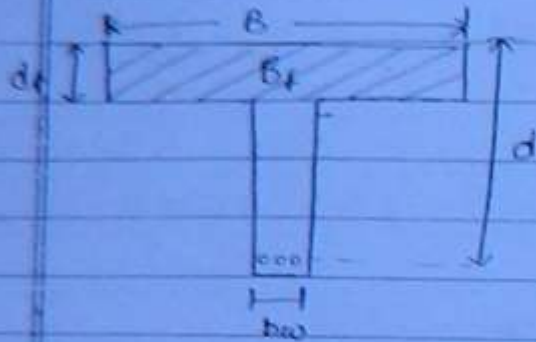
T-BEAM

PAGE NO.

DATE:

→ By working stress Method.
Isolated beam

(38)



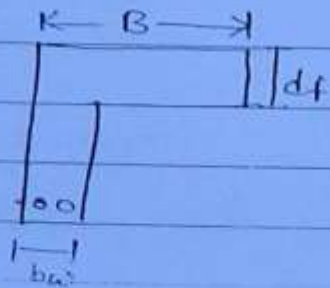
When width of flange is defined = B

Effective width of flange

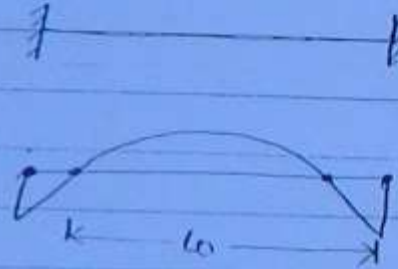
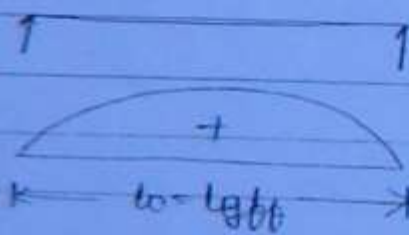
$$B_f = \frac{l_o}{\left(\frac{l_o}{B}\right) + 4} + bw \quad \text{for T-beam}$$

for L-beam

$$B_f = \frac{0.5 l_o}{\left(\frac{l_o}{B}\right) + 4} + bw$$



l_o = distⁿ betⁿ point of zero moment
- of the beam



As per IS - 456

for fixed or continuous beam

$$l_o = 0.7 l_{gtt}$$

B_o = width of outer flange

B_{eff} = width of web

for T-beam type (continuous type)

Effective width of flange

$$\text{for T-beam } B_f = \frac{l_o}{8} + bw + 6dt$$

for L-beams

$$B_f = \frac{l_o}{12} + b_w + 3d_f$$

(39)

OR for T-beam $B_f = b_w + \frac{l_1}{2} + \frac{l_2}{2}$

for L-beam $B_f = b_w + \frac{l_1}{2}$



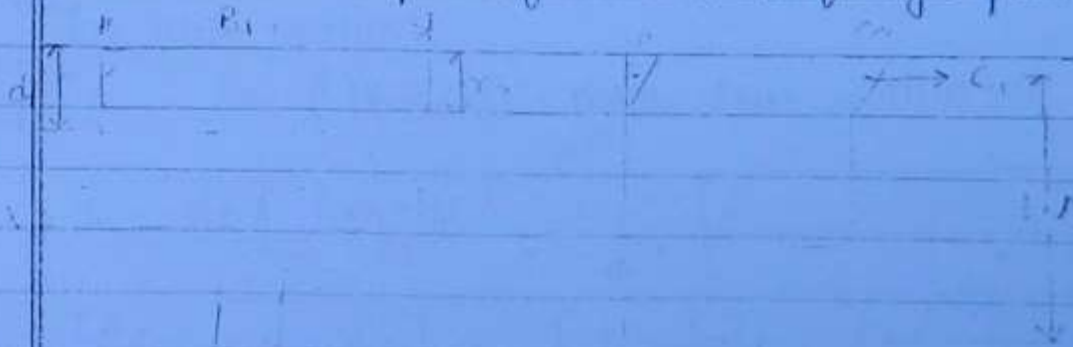
Effective width of flange is min of above two

Analysis of T-beam

(i) Critical depth of N.A

$$x_c = \frac{m c}{1 + m c} \times d$$

Case I When Actual depth of N.A is in flange portion ($x_a < d_f$)



Equating moment of area on both side of N.A

$$B_f \cdot x_a \cdot \frac{x_a}{2} = m \cdot A_{st} (d - x_a)$$

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$x_a = ?$$

Here $x_a < d_f$

(40)

(a) $x_a < x_c$ — under Reinforced Section

$$C_a < C_{cbc}$$

$$t_a = \sigma_{st}$$

(b) if $x_a = x_c$ —> Balanced Section

$$C_a = C_{cbc}$$

$$t_a = \sigma_{st}$$

(c) if $x_a > x_c$ —> over reinforced Section

$$C_a = C_{cbc}$$

$$t_a < \sigma_{st}$$

Moment of Resistance

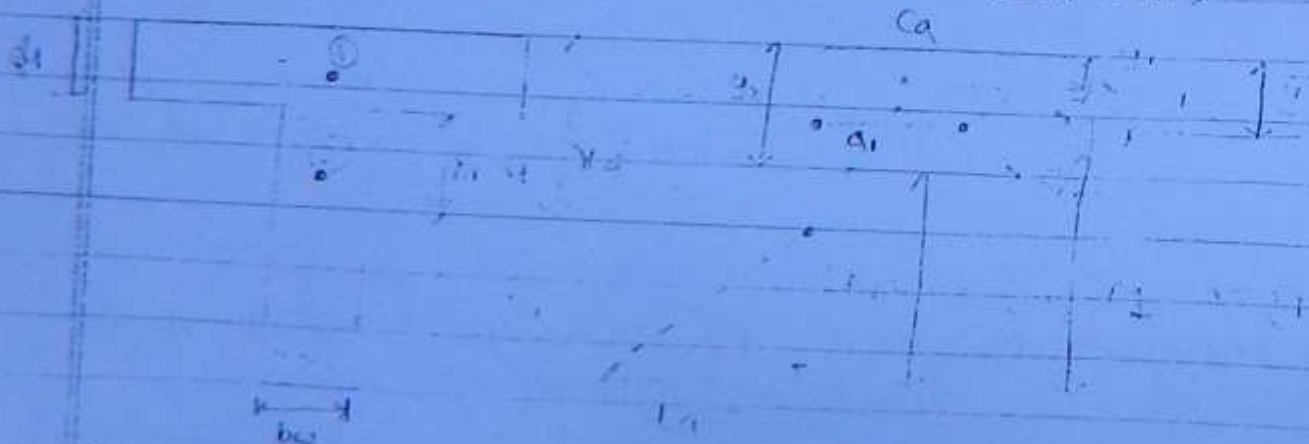
$$M_R = C_a \cdot L_A$$

$$= B_f \cdot x_a \cdot \frac{x_a}{2} \left(d - \frac{x_a}{3} \right) \quad \text{--- (1)}$$

$$M_R = t_a x A_{st} \left(d - \frac{x_a}{3} \right) \quad \text{--- (2)}$$

NOTE - Case I is same as that of Rectangular beam of size $(B_f \times d)$

CASE II When N.A is within web area ($x_a > d_f$)



(i) Actual depth of N.A

Equating moment of area on both side of N.A

$$B_f \cdot d_f \cdot (x_a - d_f/2) + b_w (x_a - d_f) \left(\frac{x_a - d_f}{2} \right) = m A_{st} (d - x_a)$$

$$B_f d_f \left(x_a - \frac{d_f}{2} \right) + b_w \left(\frac{x_a - d_f}{2} \right)^2 = m A_{st} (d - x_a)$$

(41)

2. Moment of Resistance

For flange portion - If stress at top fibre = C_g

$$\frac{C_1}{x_a - d_f} = \frac{C_g}{x_a}$$

Stress at junction $C_1 = \frac{x_a - d_f}{x_a} \times C_g$

Flange portion $C_1 = B_f \cdot D_f \left(\frac{C_g + C_1}{2} \right)$

$$y_1 = \left(\frac{a + 2b}{a + b} \right) \frac{h}{3} \Rightarrow \left(\frac{C_g + 2C_1}{C_g + C_1} \right) \frac{d_f}{3}$$

$$LA_1 = d - y_1$$

$$LA_1 = d - \left(\frac{C_g + 2C_1}{C_g + C_1} \right) \frac{d_f}{3}$$

M.R.₁ (for flange) = $C_1 \times LA_1$

$$= B_f \times d_f \cdot \left(\frac{C_g + C_1}{2} \right) \times \left[d - \left(\frac{C_g + 2C_1}{C_g + C_1} \right) \frac{d_f}{3} \right]$$

For web portion

$$C_2 = b_w (x_a - d_f) \frac{C_1}{2}$$

$$y_2 = d_f + \frac{(x_a - d_f)}{3}$$

$$LA_2 = d - y_2 = d - \left[d_f + \frac{(x_a - d_f)}{3} \right]$$

$$MR_2 = C_2 \cdot LA_2$$

$$= b_w (x_a - d_f) \frac{C_1}{2} \left[d - d_f - \frac{(x_a - d_f)}{3} \right]$$

Total Moment of Resistance, $MR = MR_1 + MR_2$

$$= B_f d_f \left(\frac{C_g + C_1}{2} \right) \left[d - \left(\frac{C_g + 2C_1}{C_g + C_1} \right) \frac{d_f}{3} \right] + b_w (x_a - d_f) \frac{C_1}{2} \left[d - d_f - \frac{(x_a - d_f)}{3} \right]$$

$$\left[d = d_f - \left(\frac{x_a - d_f}{3} \right) \right]$$

(42)

Moment of Resistance from tension side

Total Resultant Comp. force $C_R = C_1 + C_2$

$$\bar{y} = \frac{C_1 Y_1 + C_2 Y_2}{C_1 + C_2}$$

Net lever arm = $(d - \bar{y})$

$$MR = C_R (d - \bar{y}) = T (d - \bar{y}) = t_a A_{st} (d - \bar{y})$$

Design formula

It is based on following assumption

① web area is neglected

② lever arm = $0.9d$

$$MR = B_f \cdot d_f \cdot \left(\frac{C_a + C_1}{2} \right) \times 0.9d$$

$$= B_f \cdot d_f \left[C_a + \left(\frac{x_a - d_f}{x_a} \right) C_a \right] \times 0.9d$$

$$= B_f \cdot d_f \cdot C_a \left[\frac{2x_a - d_f}{x_a} \right] \times 0.45d$$

$$MR = B_f \cdot d_f \cdot \sigma_{cbc} \left[\frac{2kd - d_f}{kd} \right] \times 0.45d$$

$$MR = 0.45 \cdot B_f \cdot d_f \cdot \sigma_{cbc} \left[\frac{2kd - d_f}{k} \right]$$

Equating $BM = MR$ we get value of d required

$$BM = t_a \cdot A_{st} \cdot 0.9d$$

$$A_{st} = \frac{BM}{t_a \times 0.9d}$$

Calculate HR of an Isolated T beam.

Supported over a simply supported span = $l_m = l_o$ ① Eff width of flange (B_f)

$$B_f = \frac{l_o}{\left(\frac{l_o}{b} \right) + 4} + b_w$$

$$B_f = \frac{8000}{\frac{8000}{1500} + 4} + 300 = 1157.14$$

Assuming N.A in flange area

$$B_f \cdot \frac{x_a^2}{2} = m \cdot A_{st} (d - x_a)$$

$$1157.14 \cdot \frac{x_a^2}{2} = 13 \times 4 \times \frac{\pi}{4} (25)^2 (800 - x_a) \pm 50 \text{ mm}$$

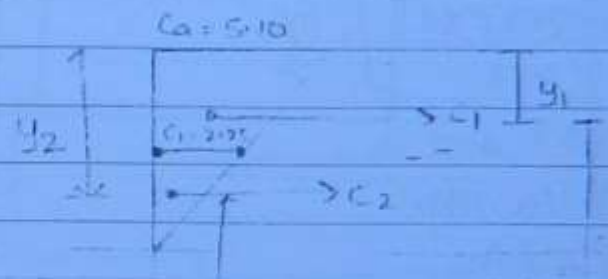
$$x_a^2 = 14768290.47 (800 - x_a)$$

$$x_a^2 + 14768290.47 x_a = 1.1814 \times 10^{10}$$

$$x_a = 167.09 \text{ mm} > 100 \text{ mm}$$

∴ Assumption is not correct

→ N.A is in web area



$$B_f \cdot df \left(x_a - \frac{df}{2} \right) + b_w \left(\frac{x_a - df}{2} \right)^2 = m \cdot A_{st} \cdot (d - x_a)$$

$$1157.14 \times 100 \times \left(x_a - \frac{100}{2} \right) + 300 \left(\frac{x_a - 100}{2} \right)^2 = 13 \times 4 \times \frac{\pi}{4} (25)^2 (800 - x_a)$$

$$771.43 x_a - 38571.67 + x_a^2 - 200 x_a + 10^4 = 136135.68 - 170.17 x_a$$

$$x_a^2 + 741.6 x_a - 164707.35 = 0$$

$$x_a = 178.93 \text{ mm}$$

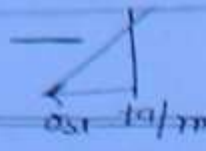
(2) Critical depth of N.A - $x_c = \frac{m_c}{1 + m_c} \times d = \frac{B \times T}{2.5 \alpha + 13.87} \times 800$

$$= 226.8 \text{ mm}$$

$x_a < x_c$ ∴ It is under reinforced case

$$C_a < C_{bc} \quad \& \quad f_{sc} = \sigma_{st} = 230 \text{ N/mm}^2$$

$$C_1 = \frac{178.93}{800 - 178.93} \times \frac{230}{13} = 5.10 \text{ N/mm}^2$$



at junction $\frac{C_1}{x_a - d_f} = \frac{C_2}{x_a} \Rightarrow C_1 = \frac{x_a - d_f}{x_a} \times C_2$

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$$C_1 = \frac{178.93 - 100}{178.93} \times 5.10 = 2.25 \text{ N/mm}^2$$

③ Moment of Resistance

$$M_R = C_1 \cdot b_f \cdot d_f \cdot \left(\frac{C_1 + C_2}{2} \right)$$

$$= 11.57 \times 100 \left(\frac{5.10 + 2.25}{2} \right)$$

$$= 425248.95 \text{ N}$$

$$\left(\frac{C_1 + 2C_2}{C_1 + C_2} \right) \frac{d_f}{3}$$

$$\left[\frac{5.10 + (2 \times 2.25 \times 2)}{5.10 + 2.25} \right] \times \frac{100}{3}$$

$$= 48.537$$

$$= 756.46 \text{ mm}$$

$$= C_1 \times LA_1$$

$$= 756.46 \times 425248.95$$

$$= 321.68 \text{ kN-m}$$

→ Web - $C_2 = \frac{b_w (x_a - d_f)}{3} \times C_1$

$$= 300 \times \frac{(178.93 - 100)^2}{3} \times \frac{2.25}{3}$$

$$C_2 = 26638.875 \text{ N}$$

$$y_2 = d_f + \frac{x_a - d_f}{3} = 100 + \frac{(178.93 - 100)}{3}$$

$$= 126.31$$

$$LA_1 = (d - y_2) = 800 - 126.31 = 673.69 \text{ mm}$$

$$MR_2 = C_2 \times LA_2 = 26638.875 \times \frac{673.69}{10^6}$$

$$\Rightarrow MR_2 = 17.94 \text{ kN-m}$$

$$\text{Total } MR = MR_1 + MR_2$$

$$= 321.685 + 17.94$$

$$MR = 339.625 \text{ kN-m}$$

R Design a T-beam for a Commercial Complex with reference to the data as stated below

Clear span of T-beam = 10 m

Spacing of T-beam = 2.5 m

Live load = 4 kN/m^2

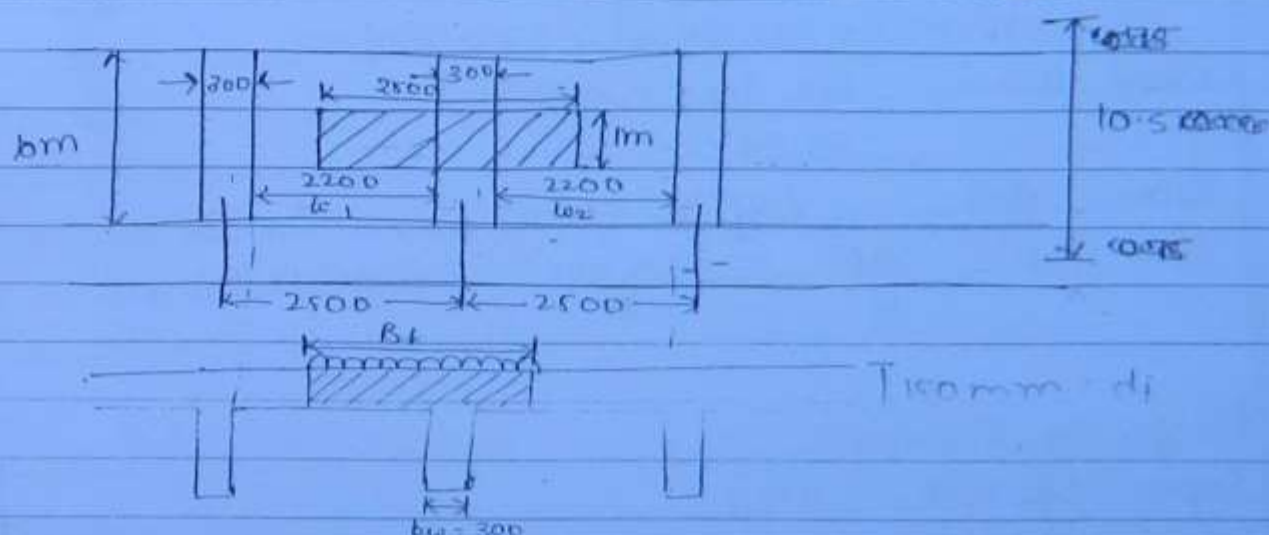
Thickness of slab = $150 \text{ mm} = 0.15 \text{ m}$

M20 concrete

Effective span of beam = 10.5 m

$b_w = 300 \text{ mm}$

Sol:-



① Effective width of flange for Cont Slab + beam type

$$B_f = \frac{l_o + b_w + 6d_f}{6}$$

$$= \frac{10500 + 300 + 6 \times 150}{6} = 2950 \text{ mm}$$

② Max. B_f = c/c distance between adjacent slab

$$= b_w + \frac{l_{o1}}{2} + \frac{l_{o2}}{2} = 300 + \frac{2200}{2} + \frac{2200}{2}$$

$$B_f = 2500 \text{ mm}$$

③ Load Calculation

one T-beam per in length of beam

④ dead

$$\text{Live load} = 1 \times 2.5 \times 4 = 10 \text{ kN/m}$$

Dead load of slab

$$= 0.15 \times 1 \times 2.5 \times 2.5 = 9.37$$

(46)

Self wt. of web portion of slab

Assume depth of T-beam

$$= \frac{\text{Span}}{20} = \frac{10000}{20} = 500 \text{ mm}$$

$$\text{Slab thickness} = 150 \text{ mm}$$

$$\text{Web depth} = 350 \text{ mm}$$

Dead load of web

$$= 0.30 \times 0.35 \times 1.0 \times 2.5 = 2.625 \text{ kN/m}$$

$$\text{Total load} = 10 + 9.37 + 2.625 = 21.995$$

~~Max. BH~~

say 22 kN/m

$$\text{Max B.M} = \frac{wL^2}{8} = \frac{22 \times 10.5^2}{8} = 303.2 \text{ kN/m}$$

Design of T-beam

$$\text{Depth of N.A} = 0.9d$$

Neglecting web portion

$$B.M = M_R = 0.45 \cdot B_f \cdot d_f \cdot \sigma_{cbc} \left(\frac{2k_d - d_f}{k} \right)$$

$$303.2 \times 10^6 = 0.45 \times 2500 \times 7 \times 150 \left(\frac{2 \times 2835 \cdot d - 150}{0.2835} \right)$$

$$d = 393.89$$

$$\text{say } d = 400 \text{ mm}$$

$$D = 400 + 150 = 450 \text{ mm}$$

Area of steel $A_{st} = \frac{B.M}{\sigma_{st} \cdot j \cdot d}$

$$= \frac{303.2 \times 10^6}{230 \times 0.90 \times 400} = 3661.8 \text{ mm}^2$$

$$\text{No of } 25 \text{ mm } \phi \text{ bars} = \frac{3661.8}{\frac{\pi}{4} (25)^2} = 745 \text{ (say 6 nos.)}$$

DATE: _____

2nd Of Option

If deflection criteria is considered, Min effective depth required

$$= \frac{\text{Span}}{20} = \frac{10500}{20} = 525 \text{ mm}$$

$$\text{Say } 550 \text{ mm} \\ + 50 \text{ mm}$$

$$\text{Total} = 600 \text{ mm}$$

$$A_{st} = \frac{303.2 \times 10^6}{230 \times 0.9 \times 550} = 2663 \text{ mm}^2$$

$$\text{No. Of } 25 \text{ mm } \phi \text{ bars} = \frac{2663}{\frac{\pi}{4} \times 25^2} = 5.42 \text{ (Say 6 Nos)}$$

(47)

LIMIT STATE METHOD

PAGE NO. 5

DATE:

① Material Strength

(a) Characteristic Strength - The value of strength below which not more than 5% of test result are expected to fall
for concrete $\rightarrow$ M25 $f_{ck} = 25 \text{ N/mm}^2$
for steel - Fe 415 $f_y = 415 \text{ N/mm}^2$

(b) Design Strength

= $\frac{\text{Characteristic Strength}}{\text{Partial factor of safety}}$

① For concrete $\gamma_{mc} = 1.50$

Permissible stress in concrete = $\frac{0.67 f_{ck}}{1.5}$ (As per IS-456)

$$= 0.45 f_{ck}$$

② for steel $\gamma_{ms} = 1.15$

$$\text{Permissible stress} = \frac{f_y}{1.15} = 0.87 f_y$$

③ Load

Characteristic load - The load value that has 95% probability of not being exceeded during its life time of structure. This value is called Characteristic load & used for design purpose.

Design load (Tn IS-800)

$$F_d = \gamma_f \times F$$

F = Characteristic load

Value of γ_f

Partial safety factor for loads (Table - 18) P-6B

Limit State of Collapse Limit State of Serviceability

	D.L	L.L	W.L	D.L	L.L	W.L
D.L + L.L	1.5	1.5	-	1.0	1.0	-
D.L + (L.L or W.L)	1.5	-	1.5	1.0	-	1.0
D.L + L.L + (W.L or C.L)	1.2	1.2	1.2	1.0	0.8	0.8

Example - If DLM = 50 kN-m

LLM = 40 kN-m & C.G.M = 20 kN-m

① $1.5 \times 50 + 1.5 \times 40 = 135 \text{ kN-m}$ - Not Critical

② $1.5 \times 50 + 1.5 \times 20 = 105 \text{ kN-m}$

③ $1.2 \times 50 + 1.2 \times 10 + 1.2 \times 20 = 132 \text{ kN-m}$

(49)

NOTE ① $\gamma = 1.90$ value is Consider when Stability against overturning is Critical

② WL & C.G.L are not Considered simultaneously.

Limit State Of Collapse	Limit State Of Serviceability
① Flexure	Deflection [IS 456]
② Shear	Cracking
③ Compression	Vibration
④ Torsion - -	Corrosion

→ Comparison

Working Stress Method

Limit State Method

1. Concrete

1. Concrete

Factor of Safety for stress

Stress factor of Safety

① $M15 = \frac{\text{Charac. strength}}{\text{Permissible stress}} = \frac{15}{5} = 3$

$= \frac{\text{Charac. strength}}{\text{Permissible stress}} = \frac{f_{ck}}{\sigma_{stk}} = 2.22$

$M20 = \frac{20}{7} = 2.86$

Load factor of Safety = 1.50

$M25 = \frac{25}{8.5} = 2.94$

Net FOS = $2.22 \times 1.50 = 3.33$

Average = 3.0 times

which is higher than that

Factor of Safety for load = 1.0

of WSM

Net factor of Safety = $3.0 \times 1.0 = 3.0$

W.S.M

L.S.M

2. Steel

Stress F.O.S

Mild Steel = f_y

Permissible

$$250 = 250/140 = 1.78$$

$$415 = 415/250 = 1.6$$

Stress F.O.S = 1.6

Load F.O.S = 1.00

$$\text{Net F.O.S} = 1.6 \times 1 = 1.60$$

Steel

$$\text{Stress F.O.S} = \frac{f_y}{1.87 f_y} = 1.15$$

$$\text{Load F.O.S} = 1.50$$

$$\text{Net F.O.S} = 1.15 \times 1.50 = 1.725$$

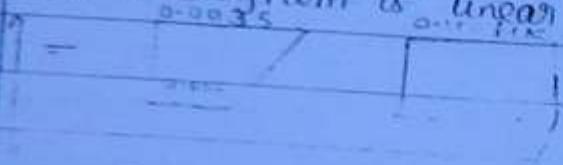
less than that of W.S.M

Limit
of Method

SINGLY REINFORCED BEAM

Assumptions (38.1 / Pg-69)

- ① Plane section before bending remains plane after bending
→ Strain diagram is linear



- ② The max. strain in concrete at outermost compression fibre is 0.0035

- ③ An acceptable stress-strain curve is shown in fig 21 for concrete

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→ For design purpose the Comp. strength of concrete shall be assumed to be 0.67 times characteristic strength. The partial F.O.S shall be applied in addition to it.

(57)

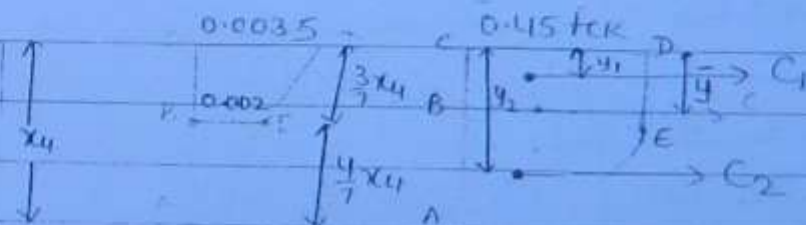
④ Tensile strength of concrete is ignored.

⑤ The stress in reinforcement are derived from stress-strain curve for the type of steel used. Curves are given in fig 23. For design purpose partial safety factor of 1.15 shall be applied.

$$\text{Permissible stress in steel} = \frac{f_y}{1.15} = 0.87 f_y$$

⑥ The max. strain in tension reinforcement in the section at the time of failure shall not be less than $\frac{f_y}{1.15 E_s} + 0.002$

Analysis of stress block parameter



K - B - K

$$T = 0.87 f_y A_{st}$$

If x_u = Depth of neutral axis (Actual depth of N.A)

$$\frac{AB}{AC} = \frac{0.002}{0.0035} = \frac{4}{7}$$

$$AB = \frac{4}{7} AC = \frac{4}{7} x_u$$

BC = $\frac{3}{7} x_u$ (Depth of rectangular portion of stress diagram)

DATE:

Compressive force

$$C_1 = (\text{width of beam} \times \text{Area of stress diagram})$$

$$= B \times 0.45 \text{ t/c} \times \frac{3}{7} x_u$$

$$= 0.1928 \text{ t/c} \cdot x_u \cdot B$$

(52)

$$y_1 = \frac{1}{2} \times \frac{3}{7} x_u = \frac{3}{14} x_u$$

$$C_2 = B \times \frac{2}{3} \times 0.45 \text{ t/c} \times \frac{4}{7} x_u$$

$$= 0.1714 \text{ t/c} \cdot x_u \cdot B$$

$$y_2 = \frac{3}{7} x_u + \frac{3}{8} \times \frac{4}{7} x_u = 0.6429 x_u$$

$$\textcircled{a} C = C_1 + C_2$$

$$= 0.1928 \text{ t/c} \times x_u \cdot B + 0.1714 \text{ t/c} \cdot x_u \cdot B$$

$$= 0.3642 \text{ t/c} \cdot x_u \cdot B$$

$$\text{As per IS} = 0.36 \text{ t/c} \cdot x_u \cdot B$$

$$\bar{y} = \frac{C_1 y_1 + C_2 y_2}{C_1 + C_2} = \frac{0.1928 \times \frac{3}{14} + 0.1714 \times 0.6429}{0.3642} \times x_u$$

$$= 0.416 x_u$$

$$\boxed{\bar{y} = 0.42 x_u} \quad \text{As per IS 45.6}$$

$$\text{Total tensile force} = T = 0.87 f_y A_{st}$$

Lever arm

$$= \text{distance between } C \text{ \& } T$$

$$(d - 0.42 x_u)$$

$$\textcircled{b} \text{ Actual depth of Neutral Axis } (x_u)$$

$$\text{Total Compressive force} = \text{Total tensile force}$$

$$C = T$$

$$0.36 \text{ t/c} \cdot x_u \cdot B = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 \text{ t/c} \cdot B} \quad \text{emp}$$

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② Moment of Resistance.

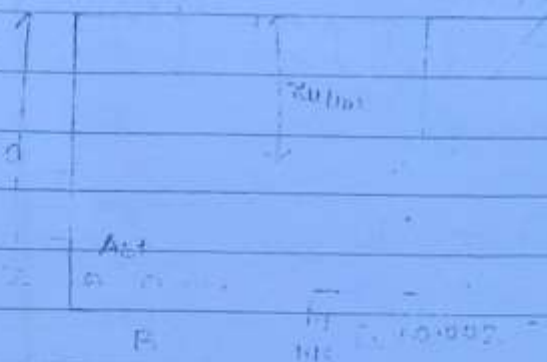
$$M \cdot R = \text{Total force of Comp.} \times \text{lever arm} \\ = 36 \text{ kN} \times 0.42 \times 1100 \text{ mm} \text{ --- imp}$$

$$M \cdot R = \text{Total force of Tension} \times \text{lever arm} \\ = 87 \text{ kN} \times A_{st} (d - 0.42 \times 1100) \text{ --- imp.}$$

(53)

③ Limiting depth of Neutral Axis

(based on strain diagram)



At $x_{u,lim}$ - value of strain should be at max limiting value

from similar triangles

$$\frac{x_{u,lim}}{d - x_{u,lim}} = \frac{0.0035}{\frac{f_y}{E_s} + 0.002}$$

$$\Rightarrow \frac{d - x_{u,lim}}{x_{u,lim}} = \frac{\frac{f_y}{E_s} + 0.002}{0.0035}$$

$$\frac{d}{x_{u,lim}} = \frac{0.87 f_y + 0.002 E_s}{0.0035 E_s} + 1$$

$$\frac{d}{x_{u,lim}} = \frac{0.87 f_y + 0.0055 E_s}{0.0035 E_s}$$

$$\frac{d}{x_{u,lim}} = \frac{0.87 f_y + 0.0055 \times 2 \times 10^5}{0.0035 \times 2 \times 10^5}$$

$$\frac{d}{x_{u,lim}} = \frac{0.87 f_y \times 1100}{700}$$

$$\frac{x_{u,lim}}{d} = \frac{700}{0.87 f_y + 1100} \quad K$$

$x_{u,lim}/d$

0.53

0.48

0.46

f_y

10 250

10 415

10 500

$x_{u,lim}$

0.53d

0.48d

0.46d

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$$x_u \leq x_{u\lim}$$

$$x_u \neq x_{u\lim}$$

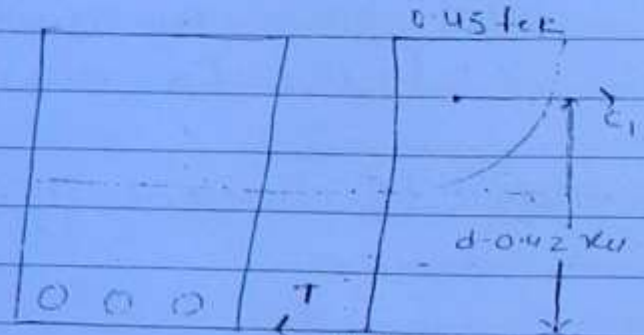
(54)

Moment of Resistance

1. Under reinforced Section

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B}$$

$$x_{u\lim} = \left. \begin{array}{l} 0.53d \\ 0.48d \\ 0.46d \end{array} \right\}$$



If $x_u < x_{u\lim}$ - It is under reinforced secⁿ $0.87 f_y$

Use x_u for M.R

$$M.R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) \quad \text{--- (1)}$$

$$= 0.87 f_y A_{st} (d - 0.42 x_u) \quad \text{--- (2)}$$

Limiting Section

If $x_u = x_{u\lim}$

Use $x_{u\lim}$ for M.R

$$M.R = 0.36 f_{ck} B \cdot x_{u\lim} (d - 0.42 x_{u\lim}) \quad \text{--- (1)}$$

$$= 0.87 f_y A_{st} (d - 0.42 x_{u\lim}) \quad \text{--- (2)}$$

Over reinforced Section

If $\left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} B} > x_{u\lim} \right)$ It is an over reinforced secⁿ

In this case x_u is limited to $x_{u\lim}$

$$M.R = 0.36 f_{ck} B \cdot x_{u\lim} (d - 0.42 x_{u\lim}) \quad \text{--- (1)}$$

Design formula

for a limiting secⁿ

$$B.M = M.R = 0.36 f_{ck} B \cdot x_{u\lim} (d - 0.42 x_{u\lim})$$

$$x_{u\lim} = K \cdot d$$

$$B.M = 0.36 f_{ck} B \cdot K \cdot d (d - 0.42 K \cdot d)$$

$$B.M = [0.36 f_{ck} \cdot B \cdot K \cdot (1 - 0.42 K)] d^2$$

$$B.M = Q B d^2$$

$$d = \sqrt{\frac{B.M}{Q \cdot B}}$$

(3)

$$\text{for Fe 415 } Q = 0.36 \times f_{ck} (1 - 0.48 \times 0.42) \cdot 48$$

$$= 0.138 f_{ck}$$

$$M20 \quad Q = 2.75$$

$$M25 \quad Q = 0.138 \times 25 = 3.45$$

Ques- Calculate H.R of a beam size 400X600mm (overall depth)

Eff. Cover = 50mm, M20, Fe 415 is used. Area of steel is

(i) 4 Nos 18 mm ϕ bars (ii) 8 Nos 25 mm ϕ bars

Calculate H.R of limiting section also & area of steel req for this section

$$\text{Sol}^n \quad (i) x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B} = \frac{0.87 \times 415 \times 4 \times \pi \times 18^2}{0.36 \times 20 \times 400}$$

$$x_u = 127.61 \text{ mm}$$

$$x_{u \text{ lim}} = 0.48 \times d = 0.48 \times 550$$

$$= 264 \text{ mm}$$

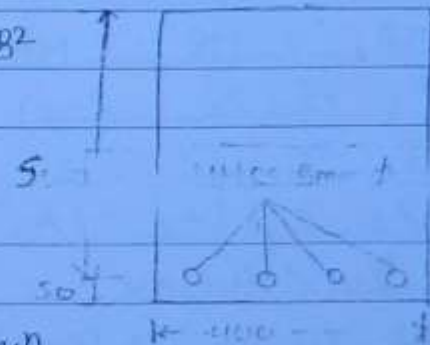
$x_u < x_{u \text{ lim}}$ - Under reinforced secⁿ

$\therefore x_u$ is used for the H.R

$$M.R = 0.36 f_{ck} B x_u (d - 0.42 x_u)$$

$$= 0.36 \times 20 \times 400 \times 127.61 (550 - 0.42 \times 127.61)$$

$$= 18244 \text{ kN-m}$$



(ii) Area of steel = 8 Nos 25 mm ϕ bars

$$A_{st} = 8 \times \frac{\pi}{4} \times 25^2 = 3926.99$$

$$x_u = \frac{0.87 \times 415 \times 3926.99}{0.36 \times 20 \times 400} = 492.30$$

$$x_{u \text{ lim}} = 0.48 \times d = 264 \text{ mm}$$

$x_u > x_{u \text{ lim}}$ - It is over reinforced secⁿ

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$X_u = X_{u\lim}$

$M_R = 0.36 f_{ck} B \cdot X_{u\lim} (d - 0.42 X_{u\lim})$ 264

$= 0.36 \times 20 \times 400 \times 1000 (550 - 0.42 \times 1000) \times \frac{1}{10^6}$

$= 333.87 \text{ kN-m}$

M.R of box limiting secⁿ

$= 333.87 \text{ kN-m}$

$333.87 \times 10^6 = 0.87 \times 415 \times A_{st} (550 - 0.42 \times 264)$

$A_{st} = \frac{333.87 \times 10^6}{158544.276}$

$A_{st} = 2105.85 \text{ mm}^2$

W-B-H $X_{a1} < X_{a2} < X_{a3}$

$C_1 < C_2 < C_3$

$LA_1 > LA_2 > LA_3$

$MR_1 < MR_2 < MR_3$

L-S-H $C_1 < (C_2 = C_3)$

$T_1 < (T_2 = T_3)$

$LA_1 > (LA_2 = LA_3)$

$X_u > X_{umax}$

but $X_{u\lim}$ is considered

$MR_1 < (MR_2 = MR_3)$

Actual $(T_3 > T_2)$ but not considered.

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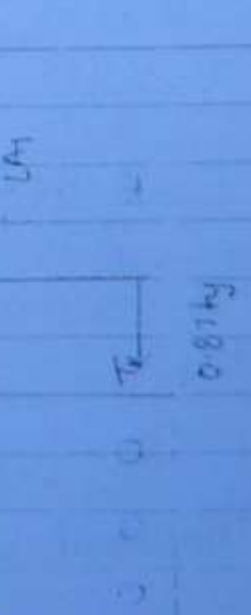
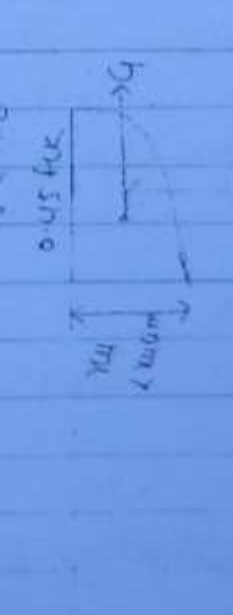
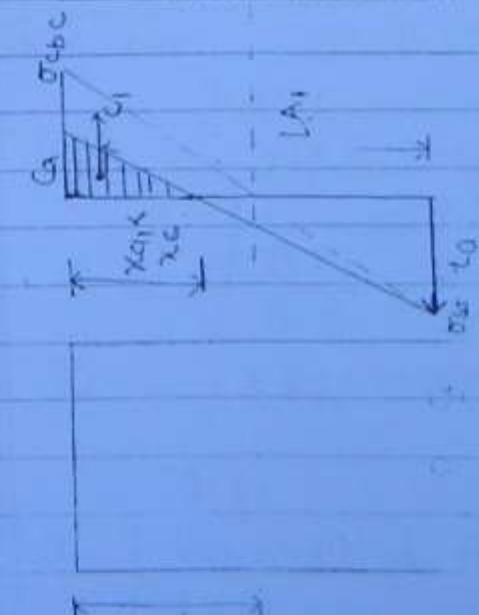
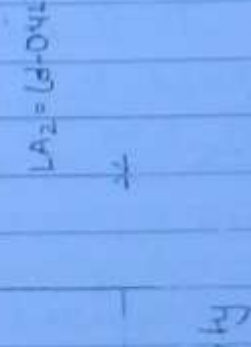
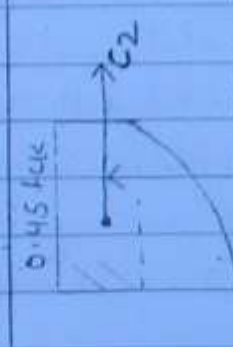
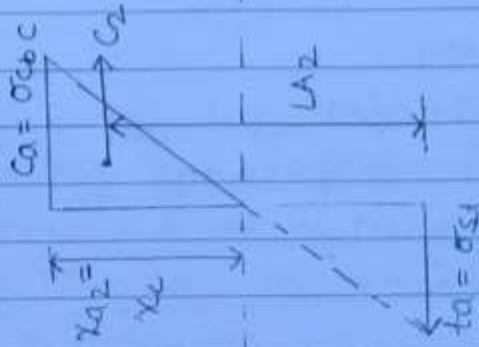
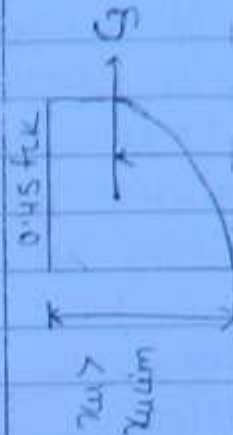
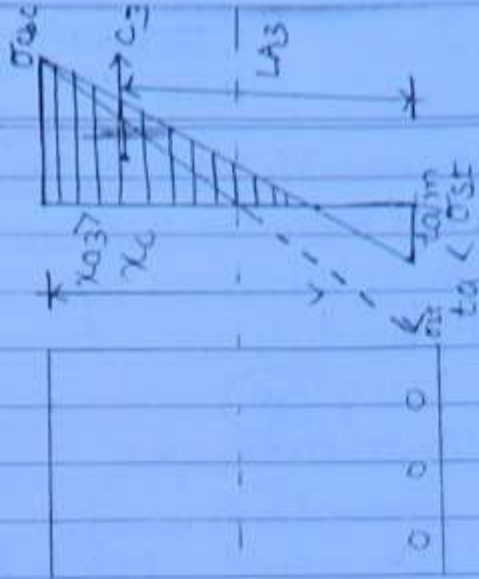
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OVER REINFORCED SECTION

SECTION

BALANCED

UNDER REINFORCED SECTION



Working Stress Method

Limit State Method

Ques - Use ISM to design a reinforced concrete rectangular beam having an eff. S.S. span of 6m. The beam is required to have live (service) load = 14 kN/m. & Superimposed load = 9.5 kN/m. Use Fe-415 & M-20.
Adopt $d/b = 2$

$P_{self} = 0.955 \gamma_c$, unit wt. of concrete $\frac{25 \text{ kN/m}^3}{1 \text{ m}}$

Solⁿ 1. Load

live load = 14 kN/m

S.T. D.L = 9.5 kN/m

Self wt of beam = $0.6 \times 3 \times 1 \times 25 = 4.5 \text{ kN/m}$

Total load (G) = 28 kN/m

factored load = $1.5 \times 28 = 42 \text{ kN/m}$

Max. B.M = $\frac{wl^2}{8} = \frac{42 \times 6^2}{8} = 189 \text{ kN-m}$

② Depth required

$$d = \sqrt{\frac{BM}{R \cdot B}}$$

$$R = 0.36 f_{ck} (1 - 0.42 k) = 0.138 \times f_{ck} = 2.75$$

$$d = \sqrt{\frac{B \cdot M \times 2}{R \cdot d}}$$

$$d^2 = \frac{2BM}{R \cdot d}$$

$$d^3 = \frac{2 \times 189 \times 10^6}{2.75} = \boxed{516.08 \text{ mm} = d}$$

Say $d = 520 \text{ mm}$

$$b = \frac{d}{2} = \frac{520}{2} = 260$$

$$D = 520 + 50 = 570$$

Load considered is slightly higher but Ok.

Area of Steel Required

$$A_{st} = \frac{B.M}{\sigma_{st} \times (d - 0.42 \times \text{width})}$$

$$= \frac{189 \times 10^6}{87 \times 415 \quad (520 - 0.42 \times 0.48 \times 520)}$$

$$= 1261 \text{ mm}^2$$

(59)

Check Limiting $A_{st} = \frac{0.955}{100} \times 260 \times 520$

$$= 1291 \text{ mm}^2$$

Ques A simply supported Rec. rectangular beam is supported over a span of 8.0 m. line load over beam = 30 kN/m. M25 / Fe 415 is used. Design the beam by WSM & LTM & compare the results. Consider width of beam 350 mm in both case.

Sol: Working Stress Method

line load = 30 kN/m

Span = 8 m ; Depth = $\frac{8000}{10} = 800 \text{ mm}$
width = 350 mm

Self wt = $0.35 \times 8 \times 1 \times 25 = 7 \text{ kN/m}$

Total = 37 kN/m

$$B.M = \frac{37 \times 8^2}{8} = 296 \text{ kN-m}$$

$$d = \sqrt{\frac{B.M}{Q \cdot B}}$$

$$Q = \frac{1}{2} \times 8.5 \times 289 \times 0.904$$

$$= 1.11$$

$$d = \sqrt{\frac{296 \times 10^6}{1.11 \times 350}} = 872.87 > \phi$$

$$922 \text{ mm}$$

$$Q = \frac{1}{2} C_j \cdot K$$

$$K = \frac{m_c}{m_{ert}}$$

$$C = 230 - m - 11$$

$$C = 8 \cdot C$$

$$K = \frac{11 \times 8.5}{11 \times 8.5 + 230}$$

$$= 0.289$$

$$= 0.904$$

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Consider $D = 950 \text{ mm}$
 $d = 900 \text{ mm}$

(60)

Self wt - $35 \times 85 \times 1 \times 25 = 9.83125 \text{ kN/m}$

total $w = 38.3125 \text{ kN/m}$

$B.M = \frac{wl^2}{8} = 306.5 \text{ kN-m}$

$d = \sqrt{\frac{306.5 \times 10^6}{1.11 \times 350}} = 888 \text{ mm} < 900 \text{ mm}$
OK.

Area of steel = $\frac{B.M}{0.87 f_y d}$
 $= \frac{306.5 \times 10^6}{230 \times 0.87 \times 900}$
 $= 1638 \text{ mm}^2$

Limit State method

live load 30 kN/m

Self wt - $35 \times 85 \times 1 \times 25 = 9.83125 \text{ kN/m}$
 $w = 37 \text{ kN/m}$

factored $= 1.5 \times 37 = 55.5 \text{ kN/m}$

$B.M = \frac{wl^2}{8} = \frac{55.5 \times 8^2}{8} = 444 \text{ kN-m}$

$d = \sqrt{\frac{B.M}{Q \cdot B}}$

$Q = 0.138 f_{ck} = 0.138 \times 25 = 3.45$

$d = \sqrt{\frac{444 \times 10^6}{3.45 \times 350}} = 606.3 \text{ mm} < 750 \text{ mm}$

So consider $d = 600$
 $D = 650 \text{ mm}$

Self wt - $35 \times 65 \times 1 \times 25 = 5.6875$

$w = 1.5 \times 35.6875 = 53.53 \text{ kN/m}$

$$d = \frac{428.25}{8} = 53.53125$$

$$d = \sqrt{\frac{428.25 \times 10^6}{3.45 \times 350}} = 596 < 600 \text{ mm OK}$$

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$$A_{st} = \frac{B.M}{\sigma_{st} \times f_y \times (d - 0.42 \times u \times d)}$$

$$= \frac{428.25 \times 10^6}{\sigma_{st} \times 415 \times (600 - 0.42 \times 0.48 \times 600)}$$

$$= 2476 \text{ mm}^2$$

(61)

Results :-

	W.S.M	L.S.M (Preferred)
① Depth D	950 mm	650 mm ^{less}
② Eff depth d	900 mm	600 mm ^{less}
③ Area of Steel	1638 mm ²	2476 mm ²

Area of Steel

Reinforced section / (limiting section or under reinforced section)

$$A_{st} = 0.5 f_{ck} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}} \right] B d$$

eg. Size of beam = 400 x 700 mm & designed for 20 kN/m over a span of 7.5 m eff. using 25 / fe 415. Calculate Area of Steel

$$L.L = 20 \text{ kN/m}$$

$$\text{Self wt} = 0.4 \times 7 \times 1 \times 25 = 7 \text{ kN/m}$$

$$W = 1.5 \times 27 = 40.5 \text{ kN/m}$$

$$B.M = \frac{40.5 \times 7.5^2}{8} = 284.77 \text{ kN-m}$$

Moment of Resistance of singly Reinforced beam as per

Maxim $0.87 f_y A_{st} d$

$$= 0.87 \times 415 \times 2476 \times 650 / 10^6$$

$$= 1383.05 \text{ kN-m}$$

B.M < Maxim $0.87 f_y A_{st} d$ Under reinforced section

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(62)

$$B \cdot M = M \cdot R$$

$$284.76 \times 10^6 = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u)$$

$$= 0.36 \times 25 \times 400 x_u (650 - 0.42 x_u)$$

$$0.42 x_u^2 - 650 x_u + 79100 = 0$$

$$x_u = 133.147 \text{ mm}$$

$$A_{st} = \frac{B \cdot M}{\sigma_{st} f_y (d - 0.42 x_u)}$$

$$= \frac{284.76 \times 10^6}{87 \times 415 (650 - 0.42 \times 133.147)}$$

$$= 1327.60 \text{ mm}^2$$

2nd Method Using direct formula we get

$$A_{st} = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}} \right] B d$$

$$= \frac{0.5 \times 25}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 284.76 \times 10^6}{25 \times 400 \times 650^2}} \right] 400 \times 650$$

$$= 1326.30 \text{ mm}^2$$

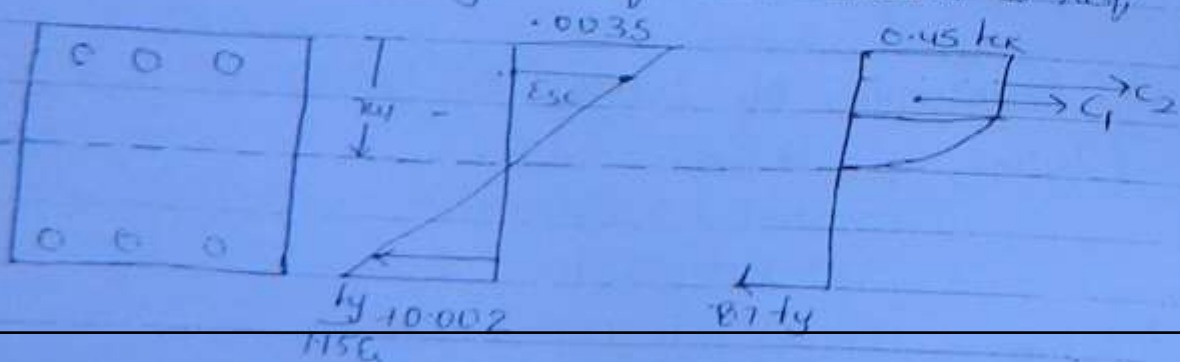
DOUBLY REINFORCED SECTION

If width & depth both are restricted and beam has to sustain B.M higher than moment of resistance of singly reinforced section. Doubly reinforced beam is required.

B and D/r are defined

$M_R = 0.87 f_y A_{st} d$ — for singly reinforced section

$B \cdot M > M_R$ Doubly reinforced section is req.



Max. Permissible Stress

for concrete = $0.45 f_{ck}$

for tensile steel = $0.87 f_y$

(63)

for Comp. Steel

As per assumption is 456 - 2000

Stress in reinforcement is derived from Stress-Strain Curve

If strain at Comp. Steel = ϵ_{sc}

$$\frac{\epsilon_{sc}}{x_u - d_c} = \frac{0.0035}{x_u}$$

$$\epsilon_{sc} = \left(\frac{x_u - d_c}{x_u} \right) \times 0.0035$$

As per value of ϵ_{sc} stress in Comp. Steel is read from Stress-Strain Curve as per table.

Stress-Strain table

$f_c 415$

$f_c 500$

Strain (ϵ_{sc})	Stress (f_{sc})	Strain	Stress
0.00144	288.7 ($5 \times 87 f_y$)	0.00174	347.8
0.00163	306.7	0.00195	369.6
0.00192	324.8	0.00226	391.3
0.00241	342.8	0.00277	413.0
0.00276	351.8	0.00312	423.9
0.00380	360.9	0.00417	434.8

Analysis of doubly reinforced section

(1) Limiting depth of N.A

$$x_{u,lim} = 0.53 d \quad \text{--- } f_c 250$$

$$= 0.48 d \quad \text{--- } f_c 415$$

$$0.46 d \quad \text{--- } f_c 500$$

(2) Actual depth of N.A

Equating total force of comp. to total force of tension

$$C = T$$

$$f_c A_c = f_y A_s$$

0.36 f_{ck} B · x_u + A_{sc} × f_{sc} - A_{sc} × 0.45 f_{ck} = 0.87 f_y A_{st}

(64)

Stress in
conc

0.36 f_{ck} B · x_u + A_{sc} (f_{sc} - 0.45 f_{ck}) = 0.87 f_y A_{st}

x_u < x_{u,lim} It is a under reinforced section

Calculate moment of resistance using x_u
Moment of resistance

= C₁ × L_{A1} + C₂ × L_{A2}

M_u = 0.36 f_{ck} B · x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)

(ii) If x_u = x_{u,lim} ← limiting case

use x_{u,lim} in place of x_u

M_u = 0.36 f_{ck} B · x_{u,lim} (d - 0.42 x_{u,lim}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)

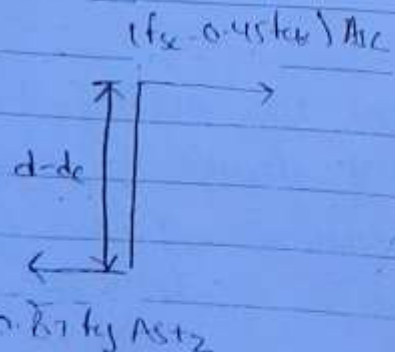
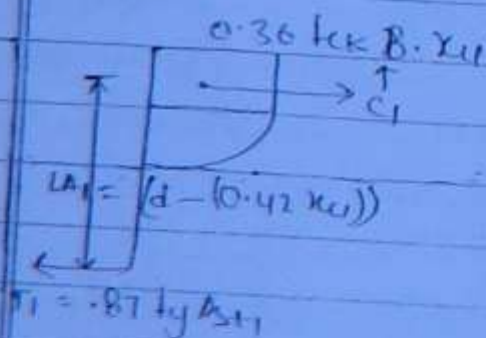
= 0.87 f_y A_{st} (d - d_c)

= M_{R1} + M_{R2}

(iii) If x_u > x_{u,lim} use x_u = x_{u,lim} and moment of resistance shall be limited to M_{u1}

Part I

Part II



C₁ = 0.36 f_{ck} B · x_u

L_{A1} = 0.87 f_y A_{st1}

L_{A1} = (d - 0.42 x_u)

C₂ = (f_{sc} - 0.45 f_{ck}) · A_{sc}

L_{A2} = 0.87 f_y A_{st2}

L_{A2} = (d - d_c)

$$M_R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c) \\ = 0.87 f_y A_{st1} (d - 0.42 x_u) + 0.7 f_y A_{st2} (d - d_c)$$

$$M_R = (Q B d^2) + (M_R = B M - M_{R1})$$

(65)

Design formula

Step ① The beam has to be designed for B.M. Calculate M.R of singly reinforced limiting section

$$M_{R1} = Q B d^2 = 0.36 f_{ck} B \cdot x_{u \text{ lim}} (d - 0.42 x_{u \text{ lim}})$$

If $B.M > M_{R1}$ — Then a doubly reinf. Secⁿ is required

Step ② A_{st1} for M_{R1}

$$A_{st1} = \frac{M_{R1}}{0.87 f_y (d - 0.42 x_{u \text{ lim}})}$$

Step ③ Remaining B.M

$$M_{R2} = B.M - M_{R1}$$

Step ④ Calculate A_{st2}

$$A_{st2} = \frac{(M_{R2} = B.M - M_{R1})}{0.87 f_y \cdot (d - d_c)}$$

Step ⑤ Find E_{sc}

$$E_{sc} = \left(\frac{x_{u \text{ lim}} - d_c}{x_{u \text{ lim}}} \right) \times 0.0035$$

Step ⑥ A_{sc} for M_{R1}

$$A_{sc} = \frac{M_{R1}}{(f_{sc} - 0.45 f_{ck}) (d - d_c)}$$

Ques Determine ultimate moment capacity of a doubly reinf. Secⁿ

$$B = 300 \text{ mm} ; D = 600 \text{ mm}$$

$$A_{st} = 2060 \text{ mm}^2$$

$$E_{ff} \text{ Cover} = 50 \text{ mm}$$

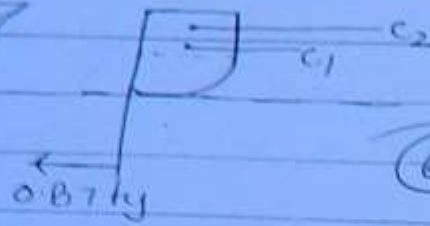
$$A_{sc} = 804 \text{ mm}^2$$

$$M20 / Fe - 415$$

Sol.

$$M_u = M_{R1} + M_{R2}$$

DATE: _____



(26)

Limiting depth of N.A

$$x_{u,lim} = 0.48 \times d = 0.48 \times 550 = 264 \text{ mm}$$

Actual depth of N.A

$$C_1 + C_2 = T$$

$$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + (f_{sc} - 0.45 \times 20) 804 = 0.87 \times 41.5 \times 2060$$

$$2160 x_u + 804 f_{sc} = 750999$$

$$2.68 x_u + f_{sc} = 934.08 \quad \text{--- (A)}$$

trial ①

$$\text{Use } f_{sc} = 350 \text{ N/mm}^2$$

$$x_u = 217.94$$

$$E_{sc} = \left(\frac{x_u - d_c}{x_u} \right) \times 0.0035$$

$$= \left(\frac{217.94 - 50}{217.94} \right) \times 0.0035 = 0.00269$$

$$f_{sc} = 342 + \frac{351 - 342}{(0.00276 - 0.00241)} \times (0.00269 - 0.00241)$$

$$= 349.46$$

$$x_u = 217.94$$

$$x_u < x_{u,lim}$$

So It is under reinforced beam

$$M_R = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

$$= 0.36 \times 20 \times 300 \times 217.94 (550 - 0.42 \times 217.94) + (349.46 - 0.45 \times 20) \times 804 \times (550 - 50)$$

$$M_R = 191.71 \times 10^6 + 136.86 \times 10^6 = 328.77 \times 10^6$$

$$M_R = 328.77 \text{ kN-m} \quad \text{Ans}$$

The size of a R.C beam is restricted to 250 x 500 mm. It carries a S.I load of 25 kN/m over a span of 6m. Determine the reinf. by L.S.D method M20/fe-415. Eff. cover = 40mm. 67

Stress	0.8 f _{yd}	0.85 f _{yd}	0.90 f _{yd}	0.95 f _{yd}	0.975 f _{yd}	1.0 f _{yd}
Strain	0.00144	0.00163	0.00192	0.00241	0.00276	0.00386
(f _{sc})	288.84	306.89	324.95	343	352.02	361.05

$$f_{yd} = 0.87 f_y$$

$$= 0.87 \times 415 = 361.05$$

Load and B.M

Super imposed load = 25 kN/m

$$\text{Self wt} = 0.25 \times 0.50 \times 1 \times 25 = 3.152 \text{ kN/m}$$

$$w = 28.152 \text{ kN/m}$$

$$\text{factored load} = 1.5 \times 28.152 = 42.2 \text{ kN/m}$$

$$\text{Max. B.M} = \frac{wL^2}{8} = \frac{42.2 \times 6^2}{8}$$

M.R. of singly reinforced section

$$MR_1 = 0.87 f_{ck} B d^2$$

$$= 0.36 f_{ck} B \times x_{ulim} (d - 0.42 x_{ulim})$$

$$= 0.138 f_{ck} B d^2$$

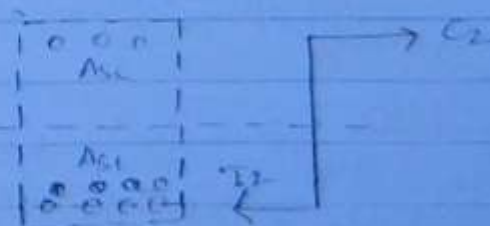
$$= 0.138 \times 20 \times 250 \times 450^2 \times \frac{1}{10^6}$$

$$MR_1 = 145.96 \text{ kN-m}$$

Req. MR > MR₁ So we have to use doubly reinf. section.



$$MR_1 = 145.96$$



$$MR_2 = 43.94$$

$$BM > MR_1$$

$$MR_2 = B.M - MR_1 = 189.9 - 145.96 = 43.94 \text{ KN-M}$$

Area of Steel A_{st1} for part 1

$$A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{ulim})}$$

$$= \frac{145.96 \times 10^6}{0.87 \times 415 (460 - 0.42 \times 48 \times 460)}$$

$$= 1100.75 \text{ mm}^2$$

$$A_{st1} = 1100.75 \text{ mm}^2$$

Area of Steel for part 2 A_{st2}

$$A_{st2} = \frac{MR_2}{0.87 \times 415 \times (460 - 40)}$$

$$= \frac{43.94 \times 10^6}{0.87 \times 415 \times (460 - 40)}$$

$$= 289.76 \text{ mm}^2$$

$$A_{st2} = 289.76 \text{ mm}^2$$

Total area of Steel $A_{st} = A_{st1} + A_{st2}$

$$= 1100.75 + 289.76$$

$$= 1390.51 \text{ mm}^2$$

Value of ϵ_{sc}

$$\epsilon_{sc} = \frac{x_{ulim} - d_c}{x_{ulim}} \times 0.0035$$

$$= \frac{0.48 \times 460 - 40}{48 \times 460} \times 0.0035$$

$$\epsilon_{sc} = 0.00286$$

$$f_{sc} = 352 + \frac{361 - 352}{(0.00380 - 0.00276)} \times (0.00286 - 0.00276)$$

$$f_{sc} = 352.8$$

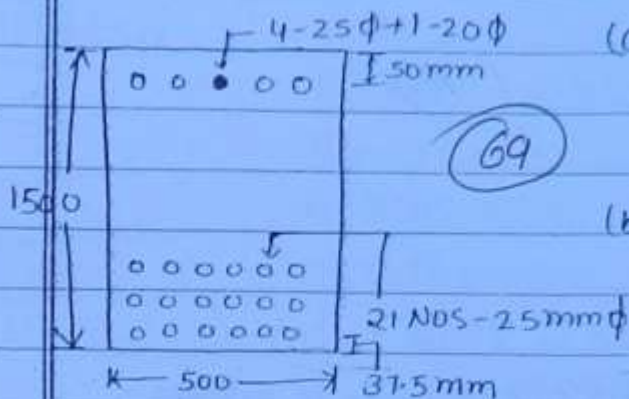
$$A_{sc} = \frac{MR_2}{(f_{sc} - 0.45 f_{ck}) (d - d_c)}$$

$$= \frac{43.94 \times 10^6}{(352.8 - 0.45 \times 20) \times (460 - 40)}$$

$$A_{sc} = 304.94 \text{ mm}^2$$

$$A_{sc} = 304.94 \text{ mm}^2$$

Ques A simply supported 18m effective span R.C.C. rectangular beam 500×1500 mm. M25 / Fe-415

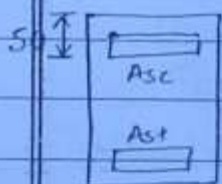


(a) Max. B.M At limit-state of Collapse.

Stress in comp. rein - $0.9566 f_{yd}$

(b) what can be the Super imposed load that can be applied over beam at working conditions.

Soln $M_u = MR_1 + MR_2$



$$A_{sc} = \frac{4 \times \pi \times 25^2}{4} + \frac{\pi \times 1 \times 20^2}{4} = 2277 \text{ mm}^2$$

$$A_{st} = \frac{21 \times \pi \times 25^2}{4} = 10308.35 \text{ mm}^2$$

Eff. Cover on tension side

$$= \frac{37.5 + 25 + 25 + 25}{2} = 100 \text{ mm}$$

$$B = 500 \text{ mm}$$

$$d = 1400 \text{ mm}$$

$$A_{sc} = 2277 \text{ mm}^2$$

$$A_{st} = 10308 \text{ mm}^2$$

① Limiting depth of $x_u = 0.48 \times 1400 = 672 \text{ mm}$

② Actual depth of N.A $C_1 + C_2 = 1$

$$0.36 f_{ck} B \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 500 \times x_u + (345.20 - 0.45 \times 25) \times 2277 = 0.87 \times 415 \times 10308$$

$$x_u = 658.06 \text{ mm}$$

Beam is under reinforced ($x_u < x_{u,lim}$)

$$MR = 0.36 f_{ck} B \cdot x_u (d - 0.42 x_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 500 \times 658.16 (1400 - 0.42 \times 658.16) + (345.20 - 0.45 \times 25) \times 2277 \times (1400 - 50)$$

$$M_u = 4354.25 \text{ KN-m}$$

$$(4) \quad B.M = \frac{w_u l^2}{8}$$

$$= \frac{8 \times 4354.25}{16^2} = w_u$$

$$w_u = 107.50 \text{ KN/m}$$

$$\text{Service load / unfactored load} = 71.66 \text{ KN/m}$$

$$\text{deduct self wt} = 0.50 \times 1.5 \times 1 \times 25 = 18.75 \text{ KN/m}$$

day

12011

2006

$$\text{Service load that can be applied over beam} = 52.91 \text{ KN/m}$$

Ques - A SSB of 5m eff. span is subjected to 24 KN/m live load $f_{ck} = 20 \text{ N/mm}^2$, $f_y = 415 \text{ N/mm}^2$. The overall depth of beam is 400 & width is 250 mm. Design the reinfⁿ of the beam $K = 0.138$; $j = 0.80$.

Solⁿ By limit state method we can solve this question

$$j = \left(1 - 0.42 \frac{x_{u, \max}}{d}\right) = \left(1 - 0.42 \times 0.48\right) = 0.7984 = \text{say } 0.80$$

$$Q = K \cdot f_{ck}$$

$$Q = 0.138 \times 20 = 2.76$$

1. Load & B.M

$$\text{Line load} = 24 \text{ KN/m}$$

$$\text{Self wt} = 0.4 \times 25 \times 1 \times 25 = 2.5 \text{ KN/m}$$

$$\therefore \text{Total load } w = 26.5 \text{ KN/m}$$

$$B.M = \frac{w l^2}{8} = \frac{26.5 \times 1.5 \times 5 \times 5}{8}$$

$$= 124.22 \text{ KN-m}$$

MR of singly reinforced limiting section

$$M_R = Q \cdot B \cdot d^2$$

$$= 2.76 \times 250 \times 350 \times 350 = 84525 \text{ KN-m}$$

$B.M > MR_1$, So we have a doubly reinforced sections

3. Area of steel (A_{st})

(a) $A_{st1} = \frac{MR_1}{0.87 f_y (d - 0.42 x_{u,lim})}$ (71)

$$= \frac{84.525 \times 10^6}{0.87 \times 415 (350 - 0.42 \times 0.48 \times 350)} = \frac{84.525 \times 10^6}{100891.812}$$

$$= 837.78 \text{ mm}^2$$

(b) A_{st2}

$$A_{st2} = \frac{B.M - MR_1}{0.87 f_y (d - d_c)}$$

$$= \frac{39.695 \times 10^6}{0.87 \times 415 (350 - 50)} = 366.48 \text{ mm}^2$$

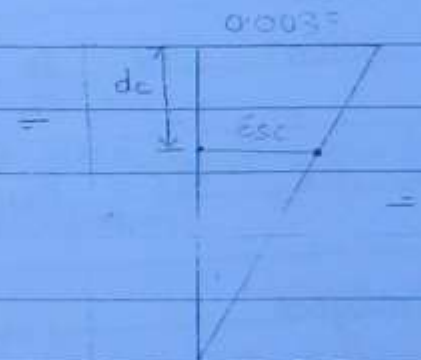
Total $A_{st} = A_{st1} + A_{st2} = 837.78 + 366.48 = 1204.26 \text{ mm}^2$

(c) $A_{sc} = \frac{B.M - MR_1}{(f_{sc} - 0.45 f_{ck}) \times (d - d_c)}$

Value of f_{sc} depends upon ϵ_{sc}

$$\epsilon_{sc} = \frac{x_{u,lim} - d_c}{x_{u,lim}} \times 0.0035$$

$$= \frac{0.48 \times 350 - 50}{0.48 \times 350} \times 0.0035 = 0.00246$$



f_{sc} Should be read for above values

Table is not given, So assuming $f_{sc} = 350 \text{ N/mm}^2$

$$A_{sc} = \frac{39.695 \times 10^6}{(350 - 0.45 \times 20) (350 - 50)}$$

$$= 388.02 \text{ mm}^2$$

T-BEAM / L-BEAM

DATE: 3/10/2011

Effective width of plange

1. Isolated T-beam / L-beam

T-beam $B_f = \frac{l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$ (72)

L-beam $B_f = \frac{0.5 l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$

2. Slab + Beam type

1. T-beam $B_f = \frac{l_0}{6} + b_w + 6d_f$

$B_f = \text{centre to centre letr adjacent slab}$
 $= b_w + \frac{l_1}{2} + \frac{l_2}{2}$

Whichever is smaller

L-beam $B_f = \frac{l_0}{12} + b_w + 3d_f$

OR $B_f = b_w + \frac{l_1}{2}$

Whichever is smaller

Analysis

1. Limiting depth of N.A

$x_{u \text{ lim}} = 0.53d$ — Fe 250

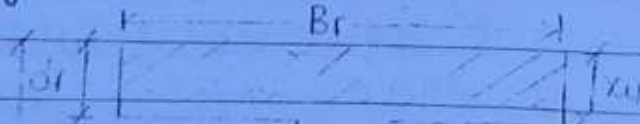
$0.48d$ — Fe 415

$0.46d$ — Fe 500

2. Actual depth of N.A & Moment of Resistance

ASE I When N.A lie within plange (i.e $x_u < d_f$)

0.45 f_{ck}



T (B_f)

AS
1007
K-bw-2

$$FES = 0.87 f_y A_{st}$$

$$F_{tc} < FES \rightarrow \text{unbalanced} \quad \text{PAGE NO. } \quad \text{DATE } \quad \text{for the function}$$

$$F_{tc} = FES = \frac{N \cdot A}{d}$$

It is similar case as of a Rectangular section of $(B \times d)$

$$C = 0.36 f_{ck} B_f \cdot x_u$$

$$T = 0.87 f_y A_{st}$$

$$L \cdot A = (d - 0.42 x_u)$$

(73)

Actual depth of N.A

$$C = T$$

$$0.36 f_{ck} B_f \cdot x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B_f}$$

Moment of Resistance

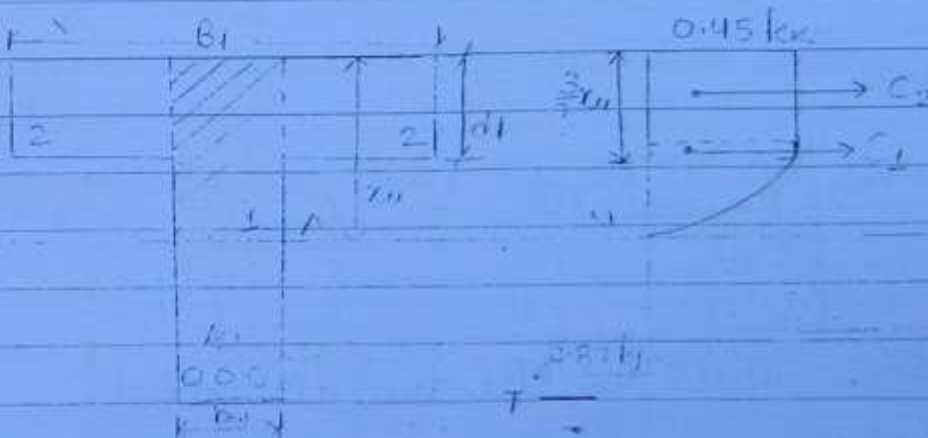
$$MR = C \cdot L \cdot A \quad \text{OR} \quad T \cdot L \cdot A$$

$$MR = 0.36 f_{ck} B_f \cdot x_u (d - 0.42 x_u) \quad \text{OR} \quad 0.87 f_y A_{st} (d - 0.42 x_u)$$

CASE II When N.A lie within web area

SUB CASE I : $x_u > d_f$ (N.A is within web area)

& $d_f < \frac{3}{7} x_u$ (Depth of flange is within rectangular portion of stress diagram)



Actual depth of N.A

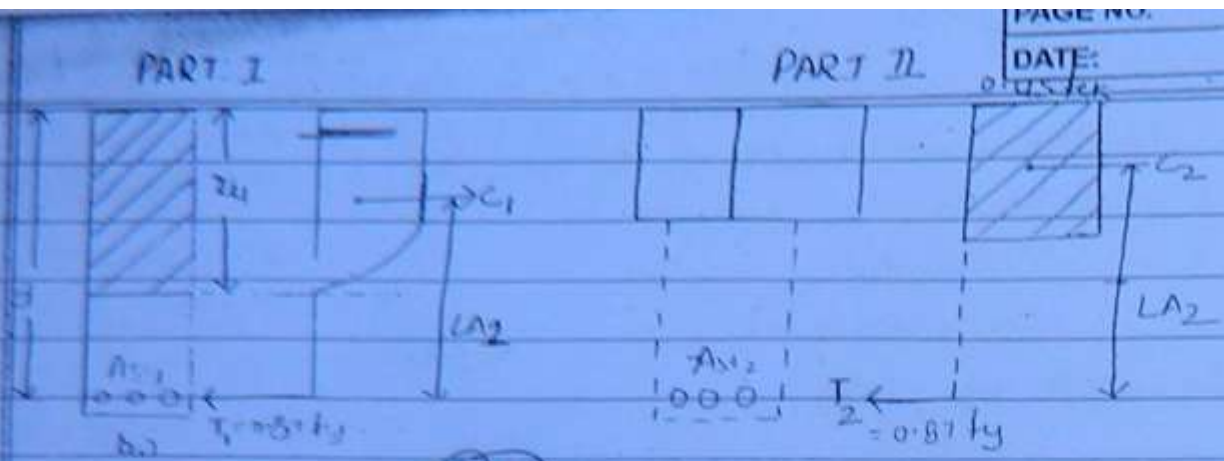
$$C_1 + C_2 = T$$

$$0.36 f_{ck} b_w \cdot x_u + (B_f - b_w) d_f 0.45 f_{ck} = 0.87 f_y A_{st}$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$$

Find $x_u = ?$

— (A)



$$C_1 = 0.36 f_{ck} \cdot b_w \cdot x_u$$

$$T_1 = 0.87 f_y \cdot A_{st1}$$

$$LA_1 = (d - 0.42 x_u)$$

$$C_2 = 0.45 f_{ck} (B_f - b_w) d_f$$

$$T_2 = 0.87 f_y A_{st2}$$

$$LA_2 = \left(d - \frac{d_f}{2} \right)$$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

Moment of Resistance.

on Compression Side $MR = C_1 \cdot LA_1 + C_2 \cdot LA_2$

$$MR = 0.36 f_{ck} \cdot b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2} \right)$$

on Tension Side $MR = T_1 \cdot LA_1 + T_2 \cdot LA_2$

$$MR = 0.87 f_y A_{st1} (d - 0.42 x_u) + 0.87 f_y A_{st2} \left(d - \frac{d_f}{2} \right)$$

Design formula

$$C_1 = T_1$$

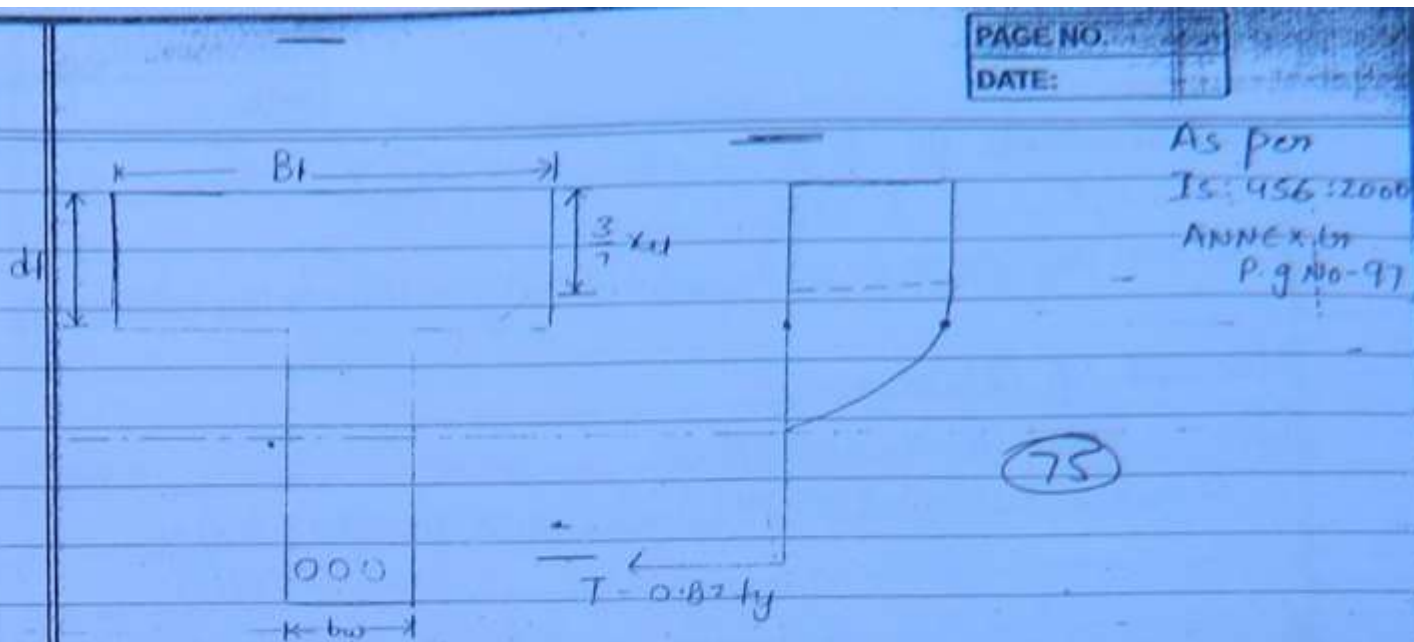
$$A_{st1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{0.87 f_y} \quad \text{--- 1}$$

again $C_2 = T_2$

we get $A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) d_f}{0.87 f_y} \quad \text{--- 2}$

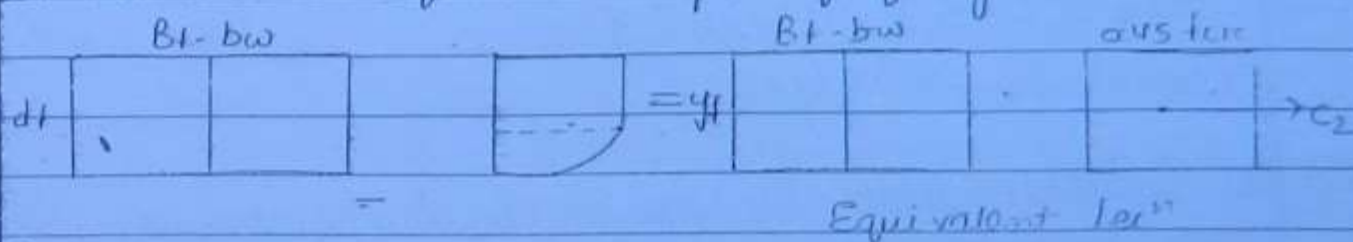
CASE II If $x_u > D_f$ (N.A is within web area)

but $d_f > \frac{3}{7} x_u$ (Depth of flange is more than the rectangular portion of stress diagram)



For flange area

Stress diagram is partly rectangular & partially parabolic.
In this case an equivalent depth of flange is considered

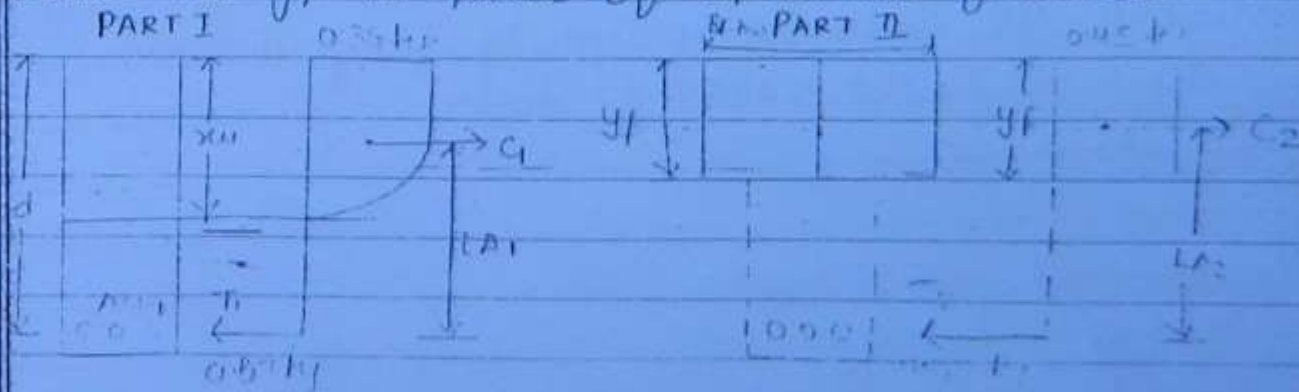


Depth of equivalent section

$$y_f = 0.15 x_u + 0.65 d_f \quad \text{--- A}$$

Stress in overall depth of equivalent section is considered $0.45 f_{ck}$

Now use y_f in place of d_f in all formula.



$$C_1 = 0.36 f_{ck} b_w \cdot x_u$$

$$T_1 = 0.87 f_y A_{st1}$$

$$I.A_1 = (d - 0.42 x_u)$$

$$C_2 = 0.45 f_{ck} (B_f - b_w) y_f$$

$$T_2 = 0.87 f_y A_{st2}$$

$$I.A_2 = (d - y_f/2)$$

DATE: _____

Actual depth of N.A

(76)

$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) y_f = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$$

Moment of Resistance

$$M_R = C_1 \times LA_1 + C_2 \times LA_2$$

$$= 0.36 f_{ck} x_u \cdot b_w (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d - y_f/2)$$

Design formula

$$A_{st1} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y}$$

$$A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$$

Ques- An Isolated T-beam of span 6.0 m simply supported
Width of beam flange = $B_f = 1500 \text{ mm}$

Overall depth $D = 800 \text{ mm}$

Effective Cover = 50 mm

Use M-25

Area of steel = 5-25 ϕ

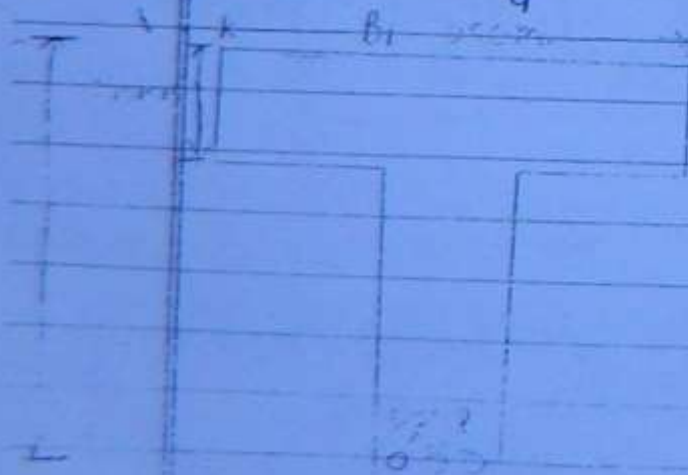
fc 415

Width of web = 300 mm

Calculate M_R of section if depth of flange is 100 mm

Solⁿ

$$A_{st} = 5 \times \frac{\pi}{4} \times 25^2 = 2454.37 \text{ mm}^2$$



DATE: _____

1 Eff. width of flange $B_f = \frac{l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$

$$B_f = \frac{6000}{\left(\frac{6000}{1500}\right) + 4} + 300 = 1050 \text{ mm} \quad (77)$$

2 Given $b_w = 300 \text{ mm}$; $D = 800 \text{ mm}$; $d = 750 \text{ mm}$
 $d_f = 100 \text{ mm}$

3 Limiting depth of N.A

$$x_{u \text{ lim}} = 0.48 \times d = 0.48 \times 750 = 360 \text{ mm}$$

4 Actual depth of Neutral axis

CASE 1 - Assume that N.A lie within flange ($x_u < d_f$)

$$0.36 f_{ck} B_f x_u = 0.87 f_y A_{st}$$

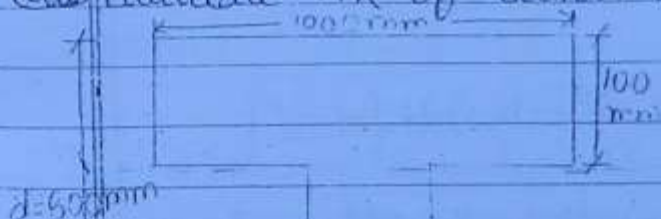
$$x_u = \frac{0.87 \times 415 \times 2454.37}{0.36 \times 25 \times 1050} = 93.77 \text{ mm}$$

So Assumption is correct

$x_u < x_{u \text{ lim}}$
Under reinforced
Section

$$\begin{aligned} M_R &= 0.36 f_{ck} x_u B_f \times (d - 0.42 x_u) \\ &= 0.36 \times 25 \times 93.77 \times 1050 (750 - 0.42 \times 93.77) \\ &= 629.70 \text{ kN-m} \end{aligned}$$

Ques Calculate M.R of Section as shown in fig.



Span of beam = 5.0 m
Des' beam point of zero
moment in the beam = 40 m

Soln Eff. width of flange

$$B_f = \frac{l_0}{\left(\frac{l_0}{B}\right) + 4} + b_w$$

$$= \frac{5000}{\left(\frac{5000}{1000}\right) + 4} + 300$$

$$= 800 \text{ mm}$$

limiting depth of N.A

$$x_{u\lim} = 0.42 \times d = 0.42 \times 800 = 384 \text{ mm}$$

Actual depth of N.A

1. Assuming $x_u < d_f$

$$= 0.36 f_{ck} \cdot x_u \cdot B_f = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 \times 415 \times 6 \times \frac{\pi}{4} \times 25^2}{0.36 \times 20 \times 800}$$

$$= 184.61 \text{ mm}$$

this assumption is not correct

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$
using d_f value we get

$$= 0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$$

$$= 0.36 \times 20 \times 300 \cdot x_u + 0.45 \times 20 (800 - 300) 100 = 0.87 \times 415 \times \frac{\pi}{4} \times 25^2 \times 6$$

$$x_u = \frac{613380.0259}{0.36 \times 20 \times 300}$$

$$x_u = 283.97 \text{ mm}$$

$$d_f < \frac{3}{7} \times 283.97$$

$$d_f < 121.70 \quad \text{Its OK}$$

$\left\{ \begin{array}{l} x_u < x_{u\lim} \\ x_u \text{ is used} \\ \text{(under reinforced section)} \end{array} \right.$

So Assumption is correct

$$M_R = \frac{0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (B_f - b_w) d_f}{(d - 0.42 x_u)} \left(d - \frac{d_f}{2} \right)$$

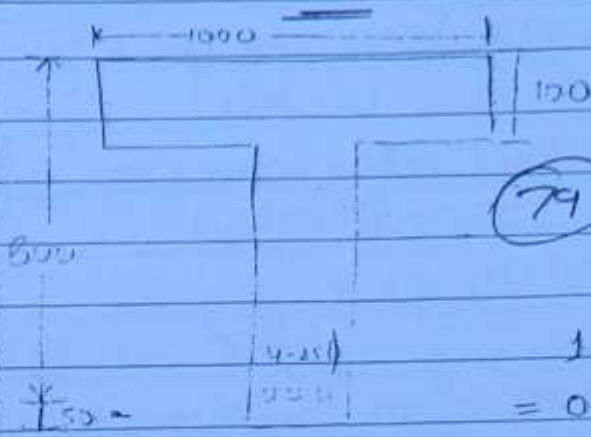
$$= \frac{0.36 \times 20 \times 300 \times 283.97 + 0.45 \times 20 (500 - 100) 100}{(800 - 100)} \left(\frac{800 - 100}{2} \right)$$

$$= 755.04 \text{ kN-m}$$

Ques- Calculate the M_R of the section shown in fig.
use M 20 / Fe 415

Span = 5m

$l_0 = 4\text{m}$



$$B_f = \frac{4000}{1000} + 4 = 800 \text{ mm}$$

(79) $x_{u\text{lim}} = 0.42 \times d = 384 \text{ mm}$

Actual depth of N.A

1. Assuming $x_u < d_f$

$$= 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 \times 415 \times 4 \times \pi \times 25^2}{0.36 \times 20 \times 800 \times 4}$$

$$x_u = 123.08 \text{ mm} < 100 \text{ mm}$$

So assumption is incorrect.

2. Assuming $x_u > d_f$ but $d_f < \frac{3}{7} x_u$

$$= 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) d_f = 0.87 f_y A_{st}$$

$$x_u = 119.87 \text{ mm} \rightarrow \text{this is correct}$$

$$\frac{3}{7} x_u = 51.37 \text{ mm}$$

$d_f > 51.37$ So assumption is incorrect

Now we know that $d_f > \frac{3}{7} x_u$ & also $x_u > d_f$

$$\text{Use } y_f = 0.15 x_u + 0.65 d_f = 0.15 x_u + 65$$

$$y_f = 0.15 \times 119.87 + 65 = 82.98$$

Actual depth of N.A

$$C_1 + C_2 = T_1 + T_2$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (B_f - b_w) y_f = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + 0.45 \times 20 \times (800 - 300) (0.15 x_u + 65) = 0.87 \times 415 \times 4 \times \pi \times 25^2$$

$$2160 x_u + 675 x_u + 292500 = 708920.0172$$

$$x_u = 146.88 \text{ mm} > d_f \text{ OK}$$

$$\frac{3}{7} x_u = \frac{3}{7} \times 146.88 = 62.95 < d_f (100 \text{ mm}) \text{ OK}$$

$$y_f = 0.15 \times 146.88 + 65 = 87.032 \text{ mm}$$

DATE: _____

$$\begin{aligned}
 MR &= 0.36 f_{ck} x_u b_w (d - 0.42 x_u) + 0.45 f_{yk} (B_f - b_w) y_f (d - y_f/2) \\
 &= 0.36 \times 20 \times 146.88 \times 300 (800 - 0.42 \times 146.88) + 0.45 \times 20 \\
 &\quad (800 - 300) 87.032 (800 - 87.032/2) \\
 &= ~~600000~~ 530.51 \text{ KN-m}
 \end{aligned}$$

(80)

Ques In design section must be given or we can assume the section as balanced section.

<u>Method- Step 1</u> If $x_u = d_f$, Calc^e M_R of the section $M_R = 0.36 f_{ck} B_f x_u (d - 0.42 x_u)$ $y_f = 0.36 f_{ck} B_f x_u (d - 0.42 d)$ or $B.M < M_{u1}$; the value of x_u is $0 < x_u < d_f$ using $B.M = M_R$ $= 0.36 f_{ck} B_f x_u (d - 0.42 x_u)$	Given - Size of beam B, B_f, b_w, d_f, d, D load or B.M design $B.M = B.M$ Grade of concrete & steel find out area of steel for req. B.M.
---	--

find out $x_u = ?$

calculate area of steel $A_{st} = \frac{B.M}{0.87 f_y (d - 0.42 x_u)}$

ep 2 If $B.M > M_{u1}$ then $x_u = \frac{7}{3} d_f$ & calculate M_R for this is equal to M_{u2}



It is case 2nd
So d_f is use

$$d_f = \frac{3 x_u}{7}$$

$$M_R = M_{u2} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) \frac{d}{2} (d - \frac{d}{2})$$

Calculate M_{u1im} [assuming $x_{u1im} > \frac{1}{3} d_f$] - (81)

So $d_f < \frac{3}{7} x_{u1im}$ Use d_f formula

Calculate M_{u1im} for x_{u1im}

$$M_{u1im} = 0.36 f_{ck} b_w x_{u1im} (d - 0.42 x_{u1im}) + 0.45 f_{ck} (B_f - b_w) \frac{d}{2} (d - \frac{d}{2})$$

If B.M is beam case 1 M_{u1} & case 2 M_{u2} then x_u is beam x_u & $\frac{1}{3} d_f$ So $x_u < \frac{1}{3} d_f$

i.e $d_f > \frac{3}{7} x_u$ So use y_f formula

If B.M is beam M_{u2} & M_{u1im} then x_u is beam $\frac{1}{3} d_f$ & x_{u1im} $x_u > \frac{1}{3} d_f$ [i.e $d_f < \frac{3}{7} x_u$]

So d_f will be Consider

p4 If B. $M_{u1} < BM < M_{u2} \rightarrow x_u < \frac{1}{3} d_f$

So $d_f > \frac{3}{7} x_u$ So it is a y_f formula

Equating B.M = MR

$$B.M = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f \frac{d}{2} (d - \frac{d}{2})$$

$$y_f = 0.15 x_u + 0.65 d_f$$

Get $x_u = ?$

Now find Out Area Of Steel

$$A_{st1} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y} ; A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y}$$

If $M_{u2} < B.M < M_{u1im} \rightarrow x_u > \frac{1}{3} d_f$ [i.e $d_f < \frac{3}{7} x_u$]

So D_f formula is used

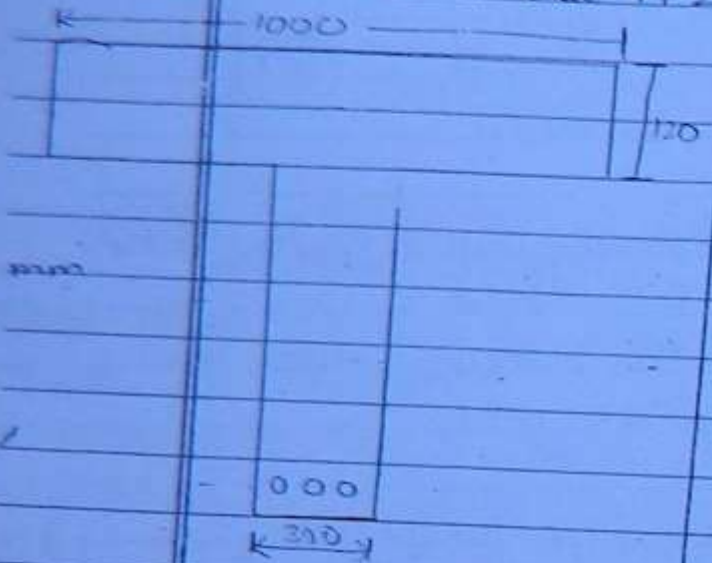
$$\text{Equating } B.M - M_R = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) \frac{d}{2} (d - \frac{d}{2})$$

Get $x_u = ?$

Also find Area Of Steel A_{st1} & A_{st2}

DATE: _____

Ques- Design a T-beam of following cross-section. For bending moment due to service load 320 kN-m
② 430 kN-m. Use M-20 / Fe-415



Given $B = 1000 \text{ mm}$

Span = 7.5 m

$l_o = 0.8 \text{ times Span} = 0.8 \times 7.5 = 6 \text{ m}$

$b_w = 300 \text{ mm}$

$d_f = 120 \text{ mm}$

$d = 600 \text{ mm}$

$$\text{Effective width } B_f = \frac{1000 \times 6000}{\frac{6000}{1000} + 4} + 300 = 932.17 \text{ mm} \approx 900 \text{ mm}$$

$$x_{u \text{ lim}} = 0.48 \times 600 = 288 \text{ mm}$$

Design B.M

$$\text{① } BM_1 = 1.5 \times 320 = 480 \text{ kN-m}$$

$$BM_2 = 1.5 \times 430 = 645 \text{ kN-m}$$

Step I When $x_u = d_f$ $x_u = 120 \text{ mm}$

$$MR = M_{u1} = 0.36 f_{ck} B_f x_u (d - 0.42 x_u)$$

$$M_{u1} = 0.36 \times 20 \times 900 \times 120 (600 - 0.42 \times 120)$$

$$M_{u1} = 427.36$$

$M_{u1} < BM_1$ So this is incorrect.

$$\text{II When } x_u = \frac{7}{3} d_f = 280 \text{ mm}$$

$$d_f < \frac{3}{7} x_u = d_f = 120 < \frac{3}{7} x_u = 120 \text{ mm}$$

So use d_f formula

$$M_{u2} = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) d_f \left(d - \frac{d_f}{2} \right)$$

$$M_{u2} = 0.36 \times 20 \times 300 \times 280 (600 - 0.42 \times 280) + 0.45 \times 20 (900 - 300) \times 120 (600 - 120/2)$$

$$= 641.67 \text{ kN-m}$$

$$M_{u2} < BM_1$$

$$M_{u2} < BM_2$$

(83)

III When $x_u = x_{uim} = 288 \text{ mm}$

$$\frac{3}{7} x_u = \frac{3}{7} \times 288 = 123.43 \text{ mm}$$

$df < \frac{3}{7} x_u$ df formula is use

$$M_{uim} = 0.36 \times 20 \times 300 \times 288 (600 - 0.42 \times 288) + 0.45 \times 20 (900 - 300) \times 120 (600 - 120/2) \times 1/10^6$$

$$= 647.9 \text{ kN-m}$$

① for BM_1 it is less M_{u1} & M_{u2} So calculate x_{u1}
So $x_u < \frac{7}{3} df$ So $df > \frac{3}{7} x_u$ (y_f formula)

② for BM_2 it is less M_{u2} & M_{uim} & calculate x_{u2}
 $x_u > \frac{7}{3} df$ So $df < \frac{3}{7} x_u$ (df formula)

for ① $B.M_1 = MR$

$$BM_1 = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) y_f (d - y_f/2)$$

$$y_f = 0.15 x_u + 0.65 df$$

$$480 \times 10^6 = 0.36 \times 20 \times 300 \times x_u (600 - 0.42 x_u) + 0.45 \times 20 (900 - 300) (0.15 x_u + 0.65 \times 120) (600 - 0.75 x_u - 39)$$

~~206000~~

$$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (0.15 x_u + 78) (561 - 0.08 x_u)$$

$$480 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 5400 (0.15 \times 561 x_u - 0.15 \times 0.08 x_u^2 + 43758 - 585 x_u)$$

$$129600 x_u - 907.2 x_u^2 + 454410 x_u - 60.75 x_u^2 + 236293200 - 31590 x_u - 480 \times 10^6 = 0$$

$$-967.95 x_u^2 + 552420 x_u - 243706800 = 0$$

$$x_u = 155.38$$

$$y_f = 0.15 \times 155.38 + 78$$

$$= 101.307 \text{ mm}$$

$$A_{st1} = \frac{0.36 f_{ck} b_w x_u}{0.87 f_y} = \frac{0.36 \times 20 \times 300 \times 155.38}{0.87 \times 415}$$

$$A_{st1} = 929.57 \text{ mm}^2$$

$$A_{st2} = \frac{0.45 f_{ck} (B_f - b_w) y_f}{0.87 f_y} = \frac{0.45 \times 20 (900 - 300) 101.307}{0.87 \times 415}$$

$$A_{st2} = 1515.185 \text{ mm}^2$$

For M_2 (df formula)

$$M_2 = MR$$

$$= 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) df \left(\frac{d - df}{2} \right)$$

$$= 0.36 \times 20 \times 300 x_u \times 600 - 0.36 \times 20 \times 300 \times 0.42 x_u^2 + 0.45 \times 20$$

$$(900 - 300) 120 \left(\frac{600 - 120}{2} \right)$$

$$645 \times 10^6 = 1296000 x_u - 907.2 x_u^2 + 349920000 = 0$$

$$-907.2 x_u^2 + 1296000 x_u + 349920000 = 0$$

$$x_u = \frac{232.24}{2}$$

$$A_{st1} = \frac{0.36 \times 20 \times 300 \times 232.24}{0.87 \times 415}$$

$$A_{st2} = 0.45 \times 20 (900 - 300)$$

$$-907.2 x_u^2 + 1296000 x_u - 29508 \times 10^4 = 0$$

$$x_u = \frac{-1296000 \pm \sqrt{(1296000)^2 - 4(907.2)(29508 \times 10^4)}}{-2 \times 907.2}$$

$$= \frac{-1296000 \pm 780215.3975}{-1814.4}$$

$$= 284.24 \text{ mm}$$

$$A_{st1} = \frac{0.36 \times 20 \times 300 \times 284.24}{0.87 \times 415}$$

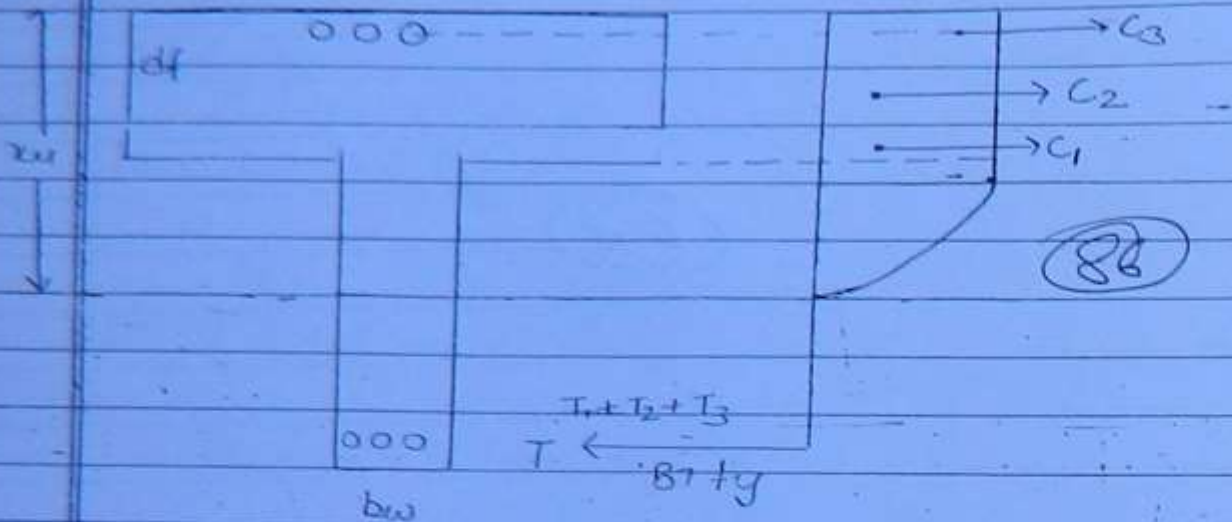
$$= 1700.48 \text{ mm}^2$$

$$A_{st2} = \frac{0.45 \times 20 (900 - 300) \times 120}{0.87 \times 415} = 1794.77 \text{ mm}^2$$

85

Doubly reinforced T-beam

($x_u < 8-x_u$)



When $x_u > d_f$

① $d_f < \frac{3}{7} x_u$ - d_f formula

② $d_f > \frac{3}{7} x_u$ - y_f formula

$$y_f = 0.15 x_u + 0.65 d_f$$

For actual depth of N.A

$$C_1 + C_2 + C_3 = T_1 + T_2 + T_3$$

$$\begin{aligned} 0.36 f_{ck} \cdot b_w \cdot x_u &= 0.87 f_y A_{s1} + \\ + 0.45 f_{ck} (B_f - b_w) (d_f \text{ or } y_f) &0.87 f_y A_{s2} + \\ + (f_{sc} - 0.45 f_{ck}) A_{sc} &0.87 f_y A_{s2} \end{aligned}$$

Moment of Resistance

$$\begin{aligned} M_R &= C_1 \times LA_1 + C_2 \times LA_2 + C_3 \times LA_3 \\ &= 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (B_f - b_w) (d_f \text{ or } y_f) \\ &\quad (d - \frac{y_f \text{ or } d_f}{2}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c) \end{aligned}$$

Area of Steel -

$$A_{s1} = \frac{0.36 f_{ck} \cdot b_w \cdot x_u}{0.87 f_y}$$

$$A_{s2} = \frac{(f_{sc} - 0.45 f_{ck}) \cdot A_{sc}}{0.87 f_y}$$

$$A_{s2} = \frac{0.45 f_{ck} (B_f - b_w) (y_f \text{ or } d_f)}{0.87 f_y}$$

005 Ques Doubly Reinforced T-beam by Working Stress Method.
Find the M.R of the hollow box beam

Use M-20 / $f_c = 115$
 $\sigma_{cbc} = 7$
 $\sigma_{st} = 230$
 $m = 13.33$ in tension
 $= 20$ in Compression

(81)

Solⁿ $d = 400 - 40 = 360 \text{ mm}$
 $A_{st} = A_{sc} = \frac{2 \times \pi \times 16^2}{4} = 402.12 \text{ mm}^2$
 $x_c = \frac{m_c}{1 + m_c} \times d = \frac{13.33 \times 7}{230 + 13.33 \times 7} \times 360$
 $x_c = 103.9 \text{ mm}$
 Actual depth of N.A - Assuming $x_u > d_f = 50 \text{ mm}$
 Equating moment of Area on both side of N.A
 $\frac{B \times x_a \cdot x_a}{2} - \frac{300 \times (x_a - 50)^2}{2} + (1.5m - 1) \times A_{sc} \times (x_a - 40) = m A_{st} (d - x_a)$
 $200 x_a^2 - 150 (x_a^2 - 100 x_a + 2500) + (1.5 \times 13.33 - 1) \times 402.12 (x_a - 40) = 13.33 \times 402.12 (360 - x_a)$
 $1.333 x_a^2 - x_a^2 + 100 x_a - 2500 + 50.92 - 2036.87 + 35.73 x_a - 12864.62 = 0$
 $x_a = 81.39 \text{ mm} > 50 \text{ mm}$
 assumption is correct
 Moment of Resistance = $C_1 L A_1 + C_2 L A_2 + C_3 L A_3$
 $M_R = C_1 \times L A_1 = 400 \times x_a$

DATE: _____



Stress - $\sigma_a < \sigma_c \rightarrow$ It is under-reinforced

$$\sigma_a < \sigma_{scb}$$

$$t = \sigma_{st} = 230 \text{ N/mm}^2$$

(88)

$$\frac{\sigma_a}{x_u} = \frac{t}{d - x_u} = \frac{\sigma_a}{81.39} = \frac{230}{360 - 81.39}$$

$$\sigma_a = 5.04 \text{ N/mm}^2$$

Concrete $\frac{C_1}{x_u - 50} = \frac{\sigma_a}{x_u} \rightarrow C_1 = \frac{5.04 (81.39 - 50)}{81.39}$

$$C_1 = 1.94 \text{ N/mm}^2$$

Steel $C' = \frac{81.39 - 40}{81.39} \times 5.04 = 2.56 \text{ N/mm}^2$

Moment of Resistance

$$C_1 = \frac{400 \times x_u \times \sigma_a}{2} = \frac{400 \times 81.39 \times 5.04}{2} = 82041.32 \text{ N}$$

$$y_1 = \frac{x_u}{3} = \frac{81.39}{3} = 27.13$$

$$L_1 = d - y_1 = 360 - 27.13 = 332.87$$

$$MR_1 = \frac{82041.32 \times 332.87 \times 1}{106} = (+) 27.31 \text{ kN-m}$$

(-ve) $C_2 = \frac{300 \times (x_u - 50) \times C_1}{2} = \frac{300 \times (81.39 - 50) \times 1.94}{2} = 9134.49$

$$y_2 = \frac{x_u - 50}{3} + 50 = \frac{10.46}{3} + 50 = 60.46$$

$$L_2 = d - y_2 = 360 - 60.46 = 299.54$$

$$MR_2 = (-) \frac{9134.49 \times 299.54 \times 1}{106} = -27.74 \text{ kN-m}$$

$$C_3 = (1.5m - 1) A_{sc} \times \sigma_c$$

$$(1.5 \times 13.33 - 1) (402.12) \times 2.56 = 19553.97 \text{ N}$$

$$L_3 = y_3 = 40$$

$$LA_3 = 360 - 40 = 320 \text{ mm}$$

$$MR_3 = \frac{19553.97 \times 320 \times 1}{10^6} = 6.26 \text{ kN-m}$$

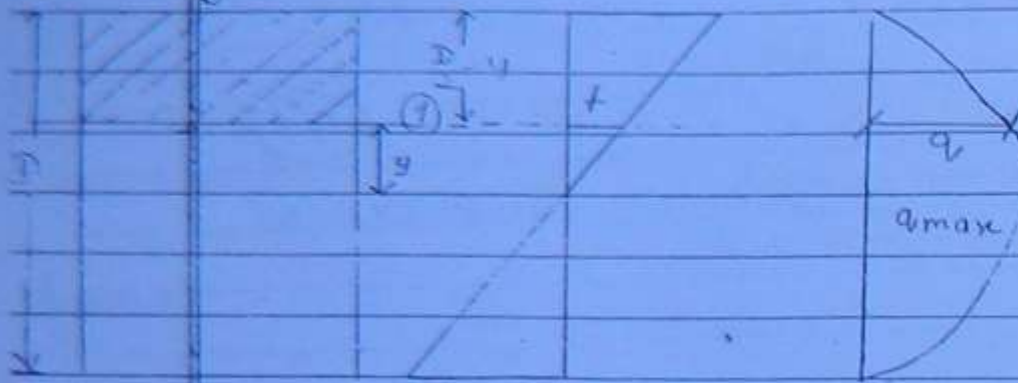
$$\text{Total MR} = MR_1 + MR_2 + MR_3$$

$$= 27.31 - 2.74 + 6.26 = 30.83 \text{ kN-m}$$

(89)

SHEAR

Shear Stress in a homogeneous section



Bending Section

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \rightarrow \text{Bending}$$

$$\text{bending stress} = f = \frac{M}{I} y$$

Shear Stress at secⁿ 1-1

$$q = \frac{V}{IB} (A\bar{y})$$

V = Shear force

I = M.I of sec. abt x-x

B = width of section at 1-1

$A\bar{y}$ = Moment of area above 1-1 abt N.A

$$A\bar{y} = B \left(\frac{D}{2} - y \right) \left[y + \frac{1}{2} \left(\frac{D}{2} - y \right) \right]$$

$$= B \left(\frac{D}{2} - y \right) \times \left(y + \frac{\frac{D}{2} - y}{2} \right)$$

$$= B \left(\frac{D}{2} - y \right) \left(\frac{\frac{D}{2} + y}{2} \right)$$

$$= \frac{1}{2} B \left(\frac{D}{2} - y \right) \left(\frac{D}{2} + y \right)$$

$$= \frac{B}{2} \left(\frac{D^2}{4} - y^2 \right)$$

$$q = \frac{V}{IB} \frac{B}{2} \left(\frac{D^2}{4} - y^2 \right)$$

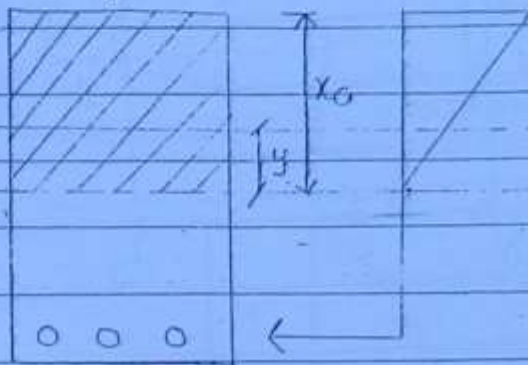
$$q = \frac{V}{2I} \left(\frac{D^2}{4} - y^2 \right)$$

Max. q at $y=0$ at N.A

$$q_{\max} = \frac{V}{2I} \left(\frac{D^2}{4} \right)$$

Shear Stress diagram for a R.C.C Beam.

(91)



It is possible with zero shear stress at top & max at N.A

Shear stress at any section

$$q = \frac{V}{I_{eq} B} (A \bar{y})$$

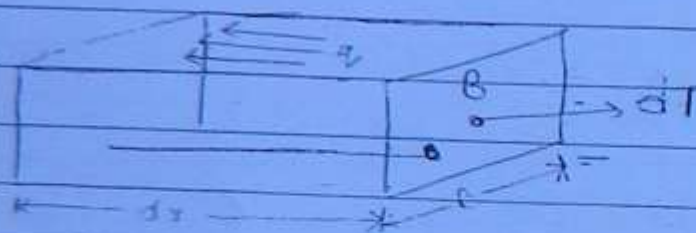
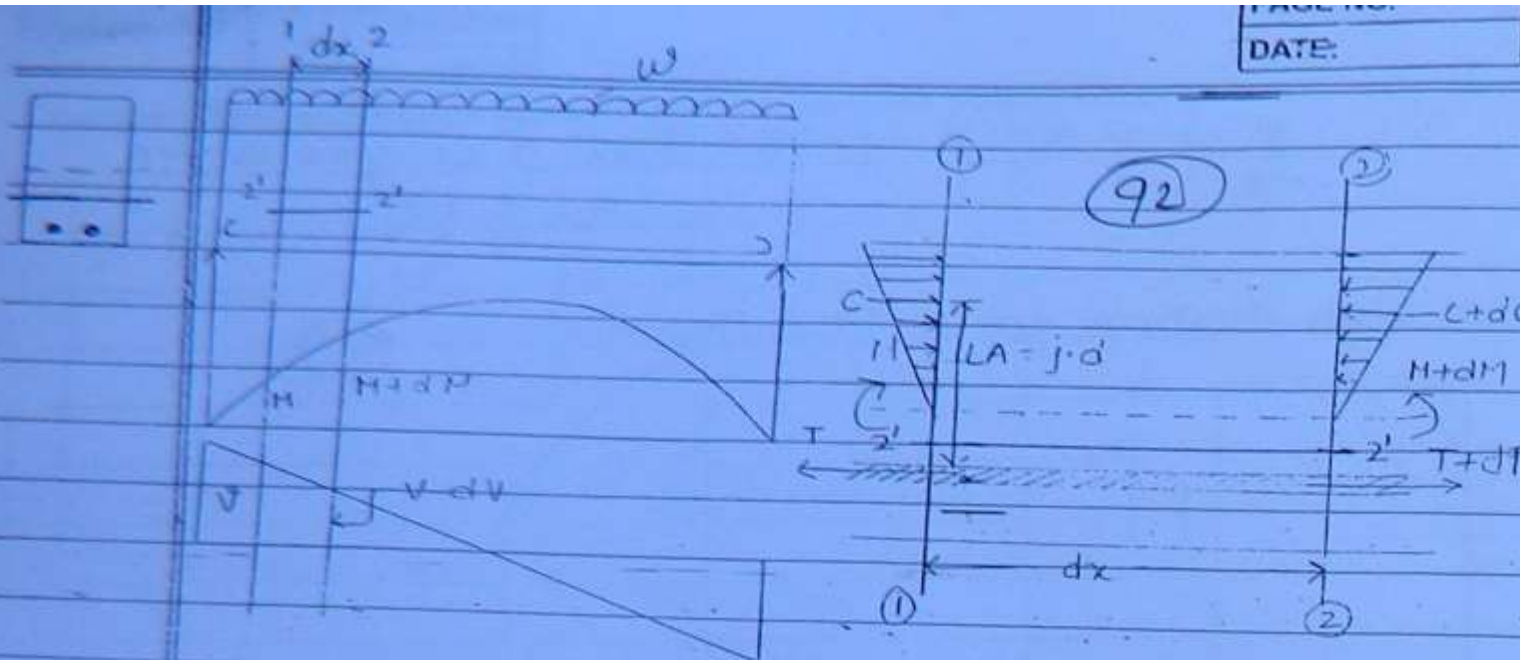
Where $A\bar{y} = \frac{B}{2} (x_0^2 - y^2)$ - ~~width~~

$$q = \frac{V}{I_{eq} B} \cdot \frac{B}{2} (x_0^2 - y^2) \Rightarrow q = \frac{V}{2I_{eq}} (x_0^2 - y^2)$$

Shear Stress above N.A

$$I_{eq} = \frac{B \cdot x_0^3}{3} + m A_{st} (d - x_0)^2$$

$$\begin{aligned} I_{base} &= \frac{B x_0^3}{12} + B x_0 \left(\frac{x_0}{2} \right)^2 \\ &= \frac{B x_0^3}{12} + \frac{B x_0^3}{4} = \frac{1+3}{12} B x_0^3 \\ &= \frac{B x_0^3}{3} \end{aligned}$$



Consider two section 1-1 & 2-2 in the beam where moments are M & $M + dM$. At Section 1-1

$$\text{Moment } M = C \cdot j \cdot d = T \cdot j \cdot d \quad \text{--- 1}$$

At section 2-2

$$\text{Moment } M + dM = (C + dC) \cdot j \cdot d = (T + dT) \cdot j \cdot d \quad \text{--- 2}$$

$$dM = (T + dT) \cdot j \cdot d - T \cdot j \cdot d$$

$$dM = dT \cdot j \cdot d$$

$$dT = \frac{dM}{j \cdot d}$$

This unbalanced dT forces in the reinforcement causes

Shear stress in concrete beam N.A & reinforcement

If q = Shear stress developed

$$\text{Resulting force} = q \cdot B \cdot dx \quad \text{--- (4)}$$

Equating eqⁿ (3) & (4)

$$q \cdot B \cdot dx = dT = \frac{dM}{j \cdot d}$$

DATE: _____

Shear Stress developed

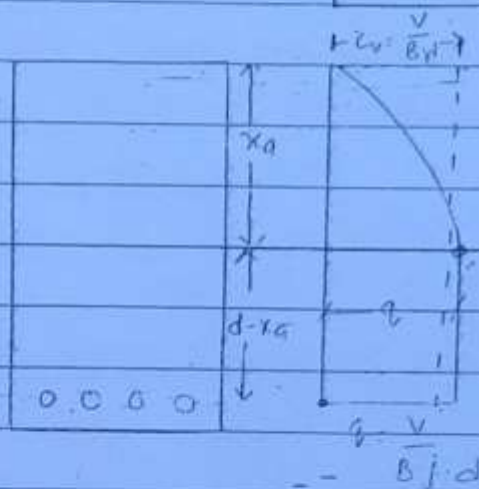
$$q = \frac{dM}{B \cdot dx \cdot j \cdot d} = \frac{V}{B \cdot j \cdot d}$$

$$q = \frac{V}{B \cdot j \cdot d} \quad \text{--- L.T.S. ---}$$

(93)

As per IS 456

$$q = \frac{V}{B \cdot d}$$



This shear stress $q = \frac{V}{B \cdot j \cdot d}$ is constant below N.A

Shear stress diagram is as shown in fig for a shear force = V

$$\begin{aligned} \text{Shear force } V &= (\text{width of section}) \times (\text{Area of shear stress}) \\ &= B \left(\frac{2}{3} x_a q + q (d - x_a) \right) \end{aligned}$$

$$= B \cdot q \left(\frac{2}{3} x_a + d - x_a \right)$$

$$= B \cdot q \left(\frac{d - x_a}{3} \right) = B \cdot q \cdot j \cdot d$$

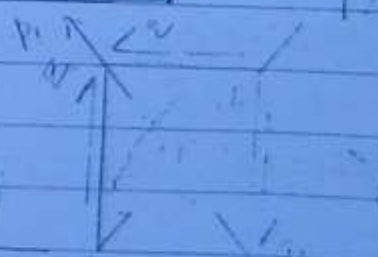
Shear value as per IS Code

$$\tau_v = \frac{V}{B \cdot d}$$

Effect of Shear

Point No. 4

$$f = \frac{M \cdot y}{I} \quad \parallel \quad q = \frac{V \cdot y}{I \cdot B} (A \cdot y)$$



Principal Stresses

$$p_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2} \quad (\text{tensile +ve})$$

$$p_2 = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2} \quad (\text{comp -ve})$$

Q3 $p_1 = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$
 $p_2 = -\frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$

Q2/1 $p_1 = q$
 $p_2 = -q$
 $\theta = 45^\circ$

is called diagonal tension

Design of beam for Shear (IS:456)

1. Shear force value is calculated
 Design S.F Should be calculated at distance from face of support

We can also design for max. S.F

DATE: _____

2. Nominal Shear Stress - It is the Shear Stress value developed in the Section (τ_v)

$$\tau_v = \frac{V}{B \cdot d} \quad \text{--- 1}$$

This $\tau_v \neq \tau_{max}$ in any case

(95)

3. Design Shear Strength of Concrete

(a) Without Shear reinforcement (τ_c)

→ Shear Strength of Concrete without Shear reinforcement (τ_c) depends upon

1. Grade of Concrete (TABLE - 23 - IS CODE - 456) 2. % Tension reinforcement

(b) With Shear reinforcement (τ_{cmax})
(TABLE - 20, 24)

	M15	M20	M25	M30	M35	M40
WSM	1.6	1.8	1.9	2.2	2.3	2.5
LSM	2.5	2.8	3.1	3.5	3.7	4.0

In any case $\tau_v < \tau_{max}$

If $\tau_v > \tau_{cmax} \rightarrow$ there is no other option then to revise the Section, Change the Size.

Min Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{f_y}$$

(26.5-1.6

Page No-48)

Design Step

1. Calculate $\tau_v = \frac{V}{B \cdot d}$

2. Calculate % of Steel $p = \frac{A_{st}}{B \cdot d} \times 100$

Find out τ_c from table

DATE: _____

If $\tau_v < \tau_c \rightarrow$ even if in this case - min shear reinforcement shall be provided

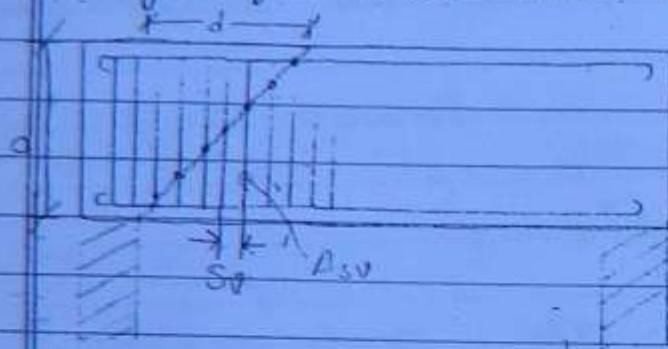
(9/6)

5. If $\tau_v > \tau_c$

Net Shear force V_s for design of shear reinforcement

$$\begin{aligned} V_s &= V - V_c \\ &= V - \tau_c \cdot B \cdot d \\ &= \tau_v B d - \tau_c \cdot B \cdot d \\ V_s &= (\tau_v - \tau_c) B \cdot d \end{aligned}$$

6. Design of vertical shear reinforcement



Shear strength of reinforcement

$$\begin{aligned} V_s &= n \times A_{s_v} \times \sigma_{s_v} \\ &= d \times A_{s_v} \times \sigma_{s_v} \end{aligned}$$

Spacing
of VSR

$$S_v = \frac{A_{s_v} \cdot \sigma_{s_v} \cdot d}{V_s}$$

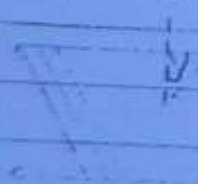
Working Stress Method

Spacing
of VSR

$$S_v = \frac{A_{s_v} (0.87 f_y) d}{V_s}$$

Limit State Method

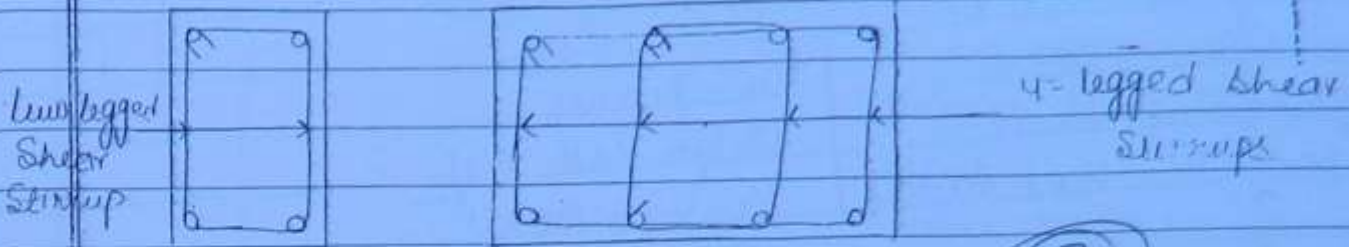
7. Design of Inclined Shear reinforcement



$$S_v = \frac{A_{s_v} \cdot \sigma_{s_v} \cdot d}{V} (\sin \alpha + \cos \alpha) \quad \text{W.S.M}$$

$$S_v = \frac{A_{s_v} (0.87 f_y) d}{V} (\sin \alpha + \cos \alpha) \quad \text{L.S.M}$$

Area of Steel (A_{sv}) at one section

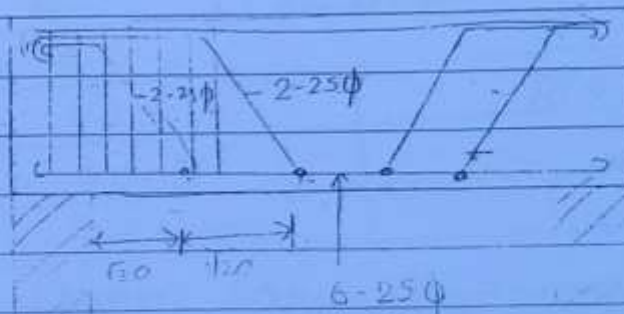


$$A_{sv} = n \times \frac{\pi}{4} \times \phi^2$$

n = no. of legs

(97)

B USE OF BENT UP BARS
(along vertical shear reinforcement)



a = lever arm = $j \cdot d$
1st
At location of Bent up bars = $\sqrt{2}a$
= $\sqrt{2} \cdot j \cdot d$ distance from support

2nd location at $\sqrt{2}j \cdot d$ distance from 1st location

Generally bars should not be bent beyond $\frac{\text{span}}{4}$ from support

Strength of bent up bars

$$V_{sb} = A_{sb} \times \sigma_{sv} \sin \alpha \rightarrow \text{WSM}$$

$$V_{sb} = A_{sb} \times (1.87 f_y) \sin \alpha \rightarrow \text{L.S.M}$$

Contribution of bent up bars should be kept limited to $\frac{V_s}{2}$

Vertical shear reinf. should be designed for atleast

① $V_s/2$ ② $(V_s - V_{sb})$ whichever is more

DATE: _____

Ex. $V = 400 \text{ kN}$

$\tau_c = ?$ $V_c = 100 \text{ kN}$

(98)

$V_s = 300 \text{ kN}$

$V_{sb} = 100 \text{ kN}$

$V_{sb} = 200 \text{ kN}$

Design vertical shear

Design V.S.R $V_s/2$

inf. for $(V_s - V_{sb}) = 200 \text{ kN}$

$V_s - V_{sb} = 100 \text{ kN}$

$V_s/2 = 150 \text{ kN} \checkmark$

Maximum Spacing of Shear reinforcement

(i) $0.75d$ — for vertical stirrups

(ii) d — for inclined stirrups at 45°

(iii) 300 mm — in any case

E-2009

Design shear reinforcement at Support if τ_c

p 1.25 1.50 1.75

τ_c 0.70 0.74 0.78

min. p

Sol. $B = 500 \text{ mm}$

$D = 1500 \text{ mm}$

$\bar{d} = 1400 \text{ mm}$

$A_{st} = 21 \times \frac{\pi}{4} \times 25^2 = 10308 \text{ mm}^2$

$A_{sc} = 4 \times \frac{\pi}{4} \times 25^2 = 2277 \text{ mm}^2$

Service O.D.L = 52.92 kN/m

Span = 18 m

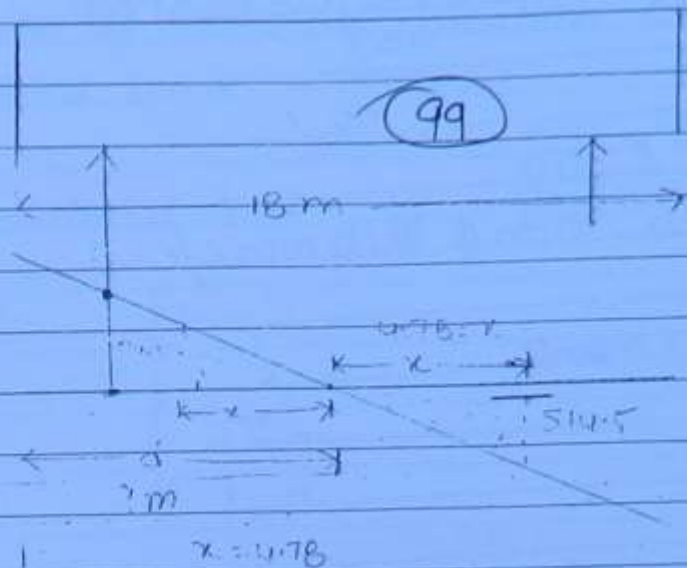
Soln Dead load = $25 \times 5 \times 1.5 = 18.75 \text{ kN/m}$

Total load = $52.92 + 18.75 = 71.67 \text{ kN/m}$

Factored load = $1.5 \times 71.67 = 107.505 \text{ kN/m}$

$V = \frac{kL}{2} = \frac{107.505 \times 18}{2} = 967.545 \text{ kN}$

SFD $\frac{x}{9} = \frac{514.5}{967.5}$ $x = 4.78 \text{ m}$



In dotted area there is no requirement to provide shear Reinforcement

→ Hence provide minimum Shear reinforcement.

Nominal Shear Stress $\tau_v = \frac{V}{B \cdot d} = \frac{967.50}{500 \times 1400} = 1.382 \text{ N/mm}^2$

% of Steel

$$P = \frac{A_{st}}{B \cdot d} \times 100$$

$$= \frac{10308}{500 \times 1400} \times 100 = 1.47$$

τ_c (from table by interpolation)

$$\tau_c = 0.7 + \left(\frac{0.74 - 0.7}{1.5 - 1.25} \right) (1.47 - 1.25)$$

$$= 0.7352 \text{ N/mm}^2$$

Design Shear Strength of Concrete

$$V_c = \tau_c \cdot B \cdot d = 0.735 \times 500 \times 1400 = 514.5 \text{ kN}$$

Shear reinforcement is required for

$$V_s = V - V_c$$

$$= 967.5 - 514.5$$

$$= 453 \text{ kN}$$

Let us provide 2-legged - 8mm ϕ Stirrups

$$A_{sv} = \frac{2 \times \pi \times 8^2}{4} = 100.53$$

DATE: _____

$$S_v = \frac{A_{sv} \times 0.87 f_y \times d}{V_s} = \frac{100.53 \times 0.87 \times 415 \times 1400}{453}$$

$$= 112.17 \text{ mm}$$

This value is less than 100 & 200

⇒ Hence, provide 2-legged 8mm ϕ @ 110 mm c/c

Check

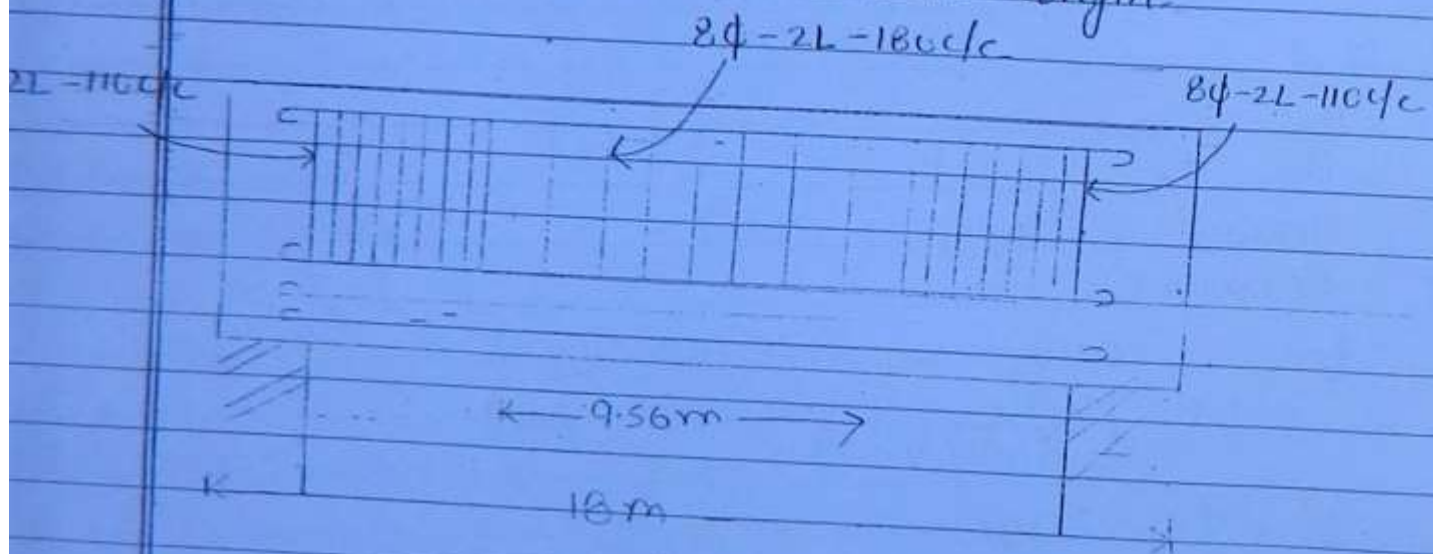
Min. Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq 0.4$$

$$B \cdot S_v \geq 0.87 f_y$$

$$S_v \leq 181.4 \text{ mm}$$

Provide 2 legged 8mm ϕ @ 180 mm c/c in central $(4.78 \times 4) = 9.56 \text{ m}$ length.



Maximum Spacing

$$1. \text{ For vertical stirrups } - 0.75 \times 1400 = 1050 \text{ mm}$$

$$2. = 300 \text{ mm} \Rightarrow \text{Safe}$$

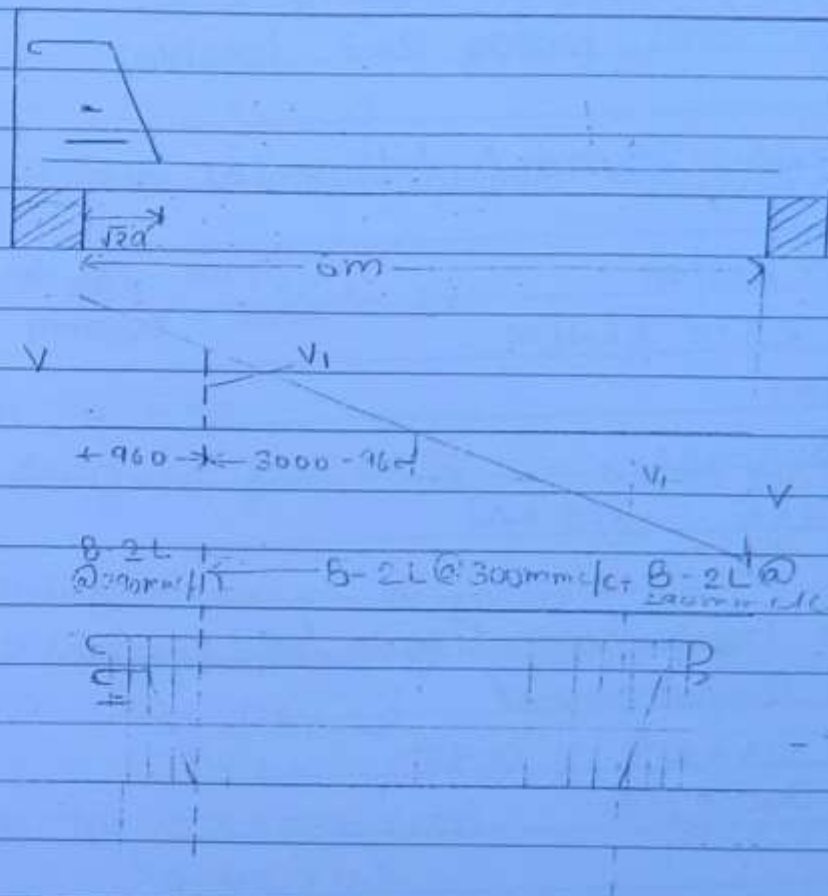
A simply reinforced beam of size 300 x 750 mm eff. depth is supported over a clear span of 6m & subjected to a U.D.L of 65 kN/m including self wt. If 80 nos of 25 mm ϕ has been provided as tension reinforcement. Design shear reinforcement using bent up bars.

Also at suitable location M-20/Fe-415 is used.

$$\text{Total load} = 1 \cdot L + D \cdot l$$

$$= 6.5 + 5 \cdot 6.25 = 70.625 \text{ kN/m} \quad - (101)$$

$$\text{factored load} = 1.5 \times 70.625 = 105.94 \text{ kN/m} \quad -$$



Location of bent up bars

1st location of bent up bars

$$= \sqrt{2} a = \sqrt{2} \cdot j \cdot d$$

$$j = \frac{1 - K}{3}$$

$$K = \frac{m c}{t + m c} = \frac{13 \times 7}{230 + 13 \times 7}$$

$$= 0.2835$$

$$j = \frac{1 - K}{3} = 0.905$$

$$= \sqrt{2} \times 0.905 \times 700$$

$$= 960 \text{ mm from Supports}$$

2nd location of bent up bars

$$2\sqrt{2}a = 1920 \text{ mm from support}$$

$$a = \frac{6000}{4} = 1500 \text{ mm}$$

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We cannot use the 2nd bent up bars

⇒ Bars cannot be bent from 2nd location

Let us bent 2 nos. - 25 mm ϕ from 1st location

2. Load / SF

$$\text{Live load} = 65 \text{ kN/m}$$

$$D.L = 8 \times 3 \times 25 = 6 \text{ kN/m}$$

$$\begin{aligned} 750 \text{ mm} - \text{eff depth} \\ \text{Total} = 800 \text{ mm} \end{aligned}$$

$$\text{Total} = 71 \text{ kN/m}$$

Max S.F at Support

$$V = \frac{wL}{2} = \frac{71 \times 6}{2} = 213 \text{ kN}$$

Design of Shear reinforcement with bent up bars
Nominal Shear Reinforcement

$$\tau_v = \frac{V}{b \cdot d} = \frac{213 \times 10^3}{300 \times 750} = 0.95$$

1. Of Steel

$$p = \frac{6 \times \frac{\pi}{4} \times 25^2}{300 \times 750} \times 100 = 1.311$$

$$\tau_c = 0.42 + 0.45 - 0.42 \times [1.31 - 1.25]$$

$$\tau_c = 0.427$$

$$V_c = 0.427 \times 300 \times 750 = 96.12 \text{ kN}$$

Shear reinforcement designed for

$$V_s = V - V_c$$

$$= 213 - 96.12$$

$$= 116.88 = 116.9 \text{ kN}$$

Strength of bent up bars

$$\begin{aligned}
 V_{sb} &= A_{sb} \times \sigma_{sb} \times \sin \alpha \\
 &= \left(2 \times 25^2 \times \frac{\pi}{4} \right) (230) \sin 45^\circ \quad (703) \\
 &= \frac{7070 \times 785 \times 2 \times 625 \times 230}{10^3} \\
 &= 159.56 \text{ kN}
 \end{aligned}$$

In this case vertical shear reinforcement is designed for

$$\frac{V_s}{2} = \frac{116.9}{2} = 58.45 \text{ kN}$$

$$V_s - V_{sb} = -ve$$

It is designed for $V_s/2 = 58.45 \text{ kN}$

Shear reinforcement required.

Use 2-legged 8mm ϕ vertical stirrups spacing

$$\begin{aligned}
 S_v &= \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s/2} \\
 &= \frac{100.53 \times 230 \times 150}{58.45 \times 10^3} = 296.69 \text{ mm}
 \end{aligned}$$

$\Rightarrow$ Provide 2-legged 8-mm dia @ 290 mm c/c

Shear force beyond bent up bars

$$\frac{V_1}{V} = \frac{3000 - 960}{300} \Rightarrow V_1 = 144.84 \text{ kN}$$

Design for this shear force

$$\begin{aligned}
 z_v &= \frac{V_1}{B \cdot d} = \frac{144.84 \times 10^3}{300 \times 750} \\
 &= 0.64
 \end{aligned}$$

% of steel

$$p = \frac{8 \times \pi/4 \times 25^2}{300 \times 750} \times 100 = 0.0114 \times 100 = 1.14\%$$

$$\tau_c \text{ bar } 1.74 \% = 0.47$$

$$V_c = \tau_c \cdot B \cdot d = 0.47 \times 300 \times 750 = 105.75 \text{ kN}$$

(104)

$$V_s = V - V_c = 144.84 - 105.75 = 39.09 \text{ kN}$$

Spacing of 2-legged 8mm dia

$$S_v = \frac{A_{sv} \cdot \sigma_{sv} \cdot d}{V_s}$$

$$= \frac{100.53 \times 230 \times 750}{39.09 \times 10^3}$$

$$= 443.63 \text{ mm}$$

Min Shear Reinforcement

$$\frac{A_{sv}}{B \cdot S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{100.53 \times 0.87 \times 415}{0.4 \times 300}$$

$$= 302.47 \text{ mm} > 300 \text{ mm}$$

Max. Spacing provided = 300 mm

$$S_v = 300 \text{ mm}$$

In Rectangular R.C Beam grade M-15 / Fe-415 size 250x500 effective. $V = 60 \text{ kN}$. Under Service Load Condition. (calculate Spacing of 8mm dia 2-legged stirrups. Steel-3-16)

P	0.25	0.5	0.75	1.00
τ_c	0.35	0.45	0.54	0.64

Soln

Load = 60 kN

$$D.L = 25 \times 25 \times 55 = 3.4375 \text{ kN}$$

$$\text{Total Load} = 63.4375 \text{ kN}$$

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = 0.72$$

$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \pi/4 \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.48 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25) = 0.4512$$

DATE: _____

$$\begin{aligned}V_c &= \tau_c \times B \cdot d \\&= 0.4512 \times 250 \times 500 \\&= 56.4 \text{ kN}\end{aligned}$$

(105)

$$V_s = V - V_c = 90 - 56.4 = 33.6 \text{ kN}$$

Spacing of 2 legged 8mm ϕ stirrups

$$S_v = \frac{100.53 \times 0.87 \times 415 \times 500}{33600}$$

$$= 546.12$$

$$\begin{aligned}A_{sv} &= 2 \times \frac{\pi}{4} \times 8^2 \\&= 100.53\end{aligned}$$

Minimum Shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{0.4}{0.87 f_y}$$

$$\begin{aligned}S_v &= \frac{0.87 \times 415 \times 100.53}{0.4 \times 250} \\&= 362.96 \text{ mm}\end{aligned}$$

Max. Spacing

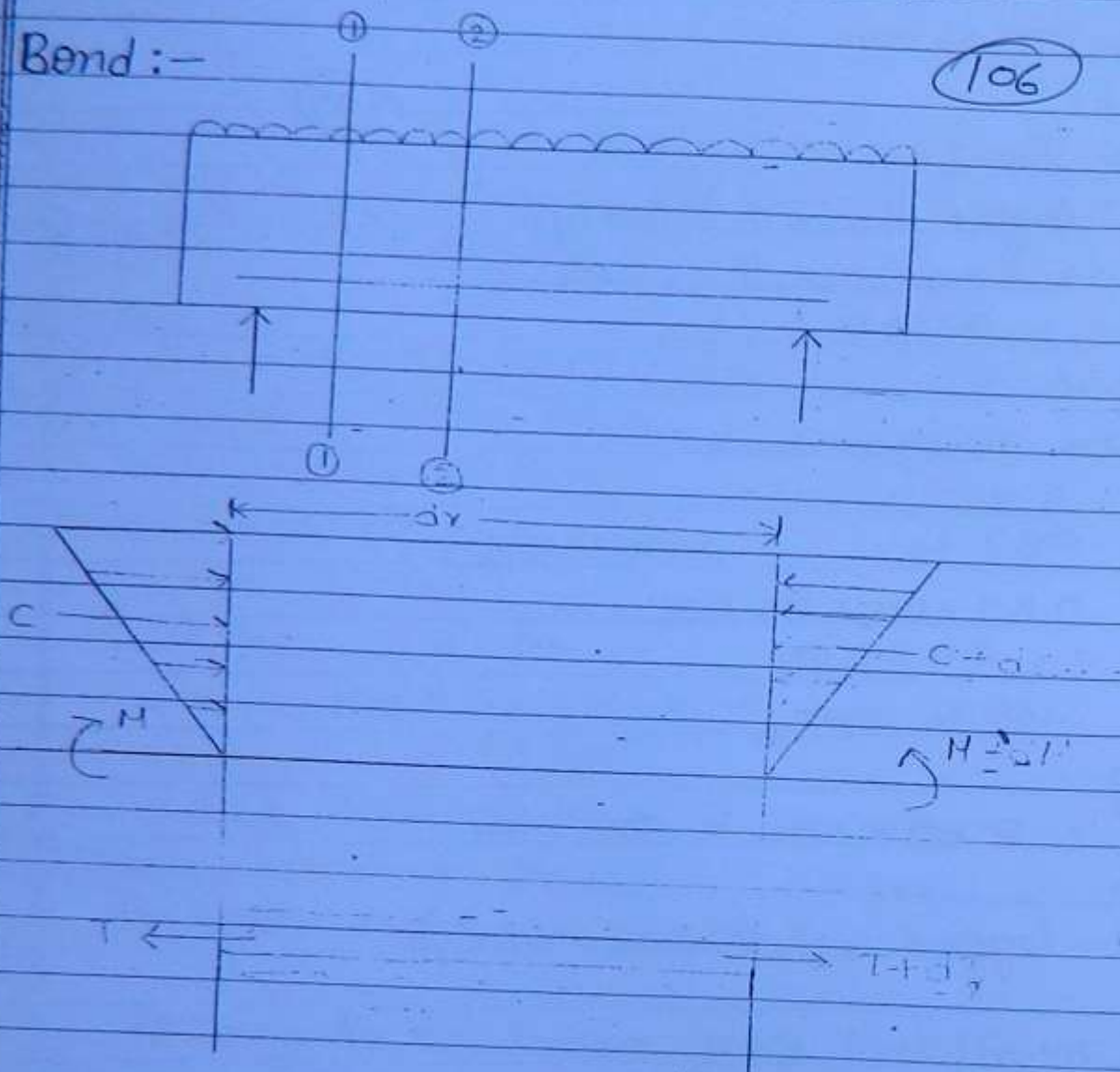
$$\begin{aligned}0.75 \times d &= 0.75 \times 500 = 375 \text{ mm} \\300 \text{ mm}\end{aligned}$$

provide 2-legged 8 ϕ @ 300 mm c/c

BOND / DEVELOPMENT LENGTH

Bond:-

(106)



In a Simply Supported beam / any other kind of beam subjected to B.M & S.F

Bond Stress (longitudinal shear stress developed betn Steel and Concrete) is developed betn Steel & Concrete in longitudinal dirⁿ due to unbalanced dT force as shown in fig.

Unbalanced force $dT =$ Bond force

$\Sigma O =$ Sum of all perimeter

$$d \times \Sigma O \times \tau_{bd}$$

$$\tau_{bd} = \frac{dT}{dx \Sigma o} \quad \text{--- (1)}$$

(107)

At section ①-①

$$M = C \cdot j \cdot d = T \cdot j \cdot d \quad \text{--- (2)}$$

At section 2-2

$$M + dM = (C + dC) \cdot j \cdot d = (T + dT) \cdot j \cdot d$$

3-2

$$\Rightarrow dM = dT \cdot j \cdot d$$

$$dT = \frac{dM}{j \cdot d} \quad \text{--- (4)}$$

Bond stress developed

$$\tau_{bd} = \frac{dM}{dx \Sigma o \cdot j \cdot d} = \frac{V}{\Sigma o \cdot j \cdot d}$$

Here

$$\Sigma o = n \times \pi \times \phi$$

↑
Developed value of bond stress due to Shear force

CHECK

Permissible values of Bond Stress, (Table-21 Page-80)

For mild steel

LISM

LSM → Page No-43

Reinfⁿ in tension

M-15	0.6	x
M-20	0.8	1.2
M-25	0.9	1.4
M-30	1.0	1.5
M-35	1.1	1.7
M-40	1.2	1.9
M-45	1.3	1.9
M-50	1.4	1.9

For HYSD bars ; values at given in table are increased by $\Rightarrow 60\%$ for both LSM & LSM.

For bars in Compression values in table are to be increased by 25% — LSM & LSM

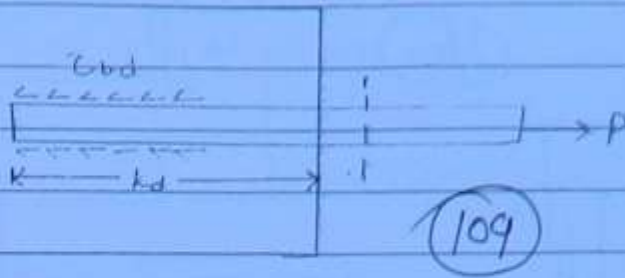
108

τ_{bd} Calculated $\neq \tau_{bd}$ permissible.

Ex - τ_{bd} for M-30 Concrete for LSM for Fe-415 bars in Compression

$$\tau_{bd} = 1.5 \times 1.6 \times 1.25 = 3$$

Development length — Min length of steel requirement required to be embedded in concrete so that strength of bond is equal to strength of reinforcement in tension



$$P = A_{st} \cdot \sigma_{st} = \text{Bond Strength}$$

$$A_{st} \cdot \sigma_{st} = \tau_{bd} \times \pi \times \phi \times L_d$$

$$\frac{\pi}{4} \times \phi^2 \times \sigma_{st} = \tau_{bd} \times \pi \times \phi \times L_d$$

$L_d = \frac{\phi \sigma_{st}}{4 \tau_{bd}}$	Working Stress method.
$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}}$	Limit State method.

Calculate development length req. for a steel reinforcement Fe-415 for tension and compression. Use M-25 & Use L.S.M.

Solⁿ For tension

$$\tau_{bd} = 1.4 \times 1.60 = 2.24$$

$$L_d = \frac{\phi (0.87 f_y)}{4 \tau_{bd}} = \frac{\phi (0.87 \times 415)}{4 \times 2.24} = 4.295 \phi$$

In compression

$$\tau_{bd} = 1.4 \times 1.6 \times 1.25 = 2.8 \text{ N/mm}^2$$

$$L_d = \frac{\phi (0.87 \times 415)}{4 \times 2.8} = 32.24 \phi$$

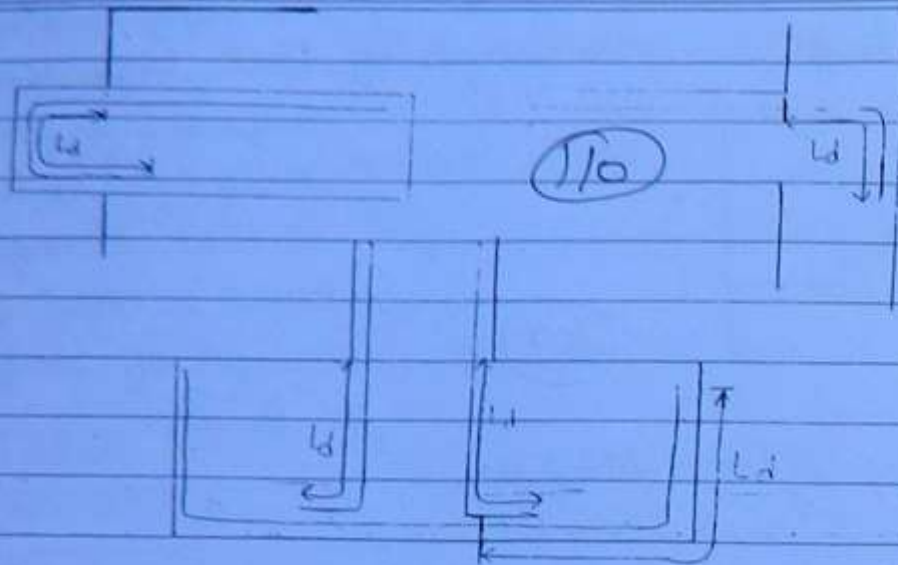
Use of development length

1. For lap length of steel in Slab/Beam/Column/foundation

$$\text{Lap length} = L_d$$

2. At corner in beam.

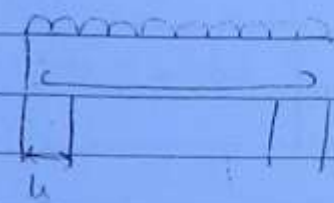
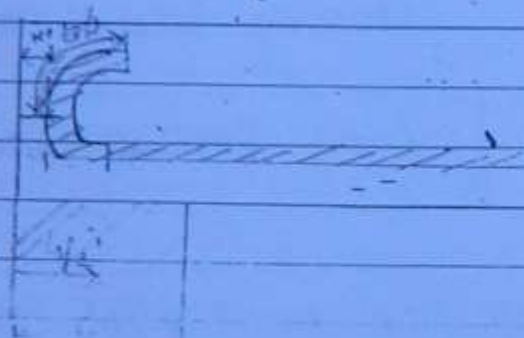
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1/10

Development length requirement At Simple Support

code 23.3)



positive moment reinforcement shall be limited to a dia such that

$$L_d \leq \frac{M_u}{V} + l_o$$

M_u = Moment of resistance of section assuming all reinforcement stressed upto σ_{st} or $0.87 f_y$

$$A_{st} \cdot \sigma_{st} \cdot (j \cdot d) \text{ --- WSM}$$

$$A_{st} \cdot (0.87 f_y) \cdot j \cdot d \text{ --- LSM}$$

V = Shear force at Support

l_o = length of reinf. beyond Centre of Support

$$l_o = \frac{l_s - x'}{2} + B \phi \text{ --- (for mild steel)}$$

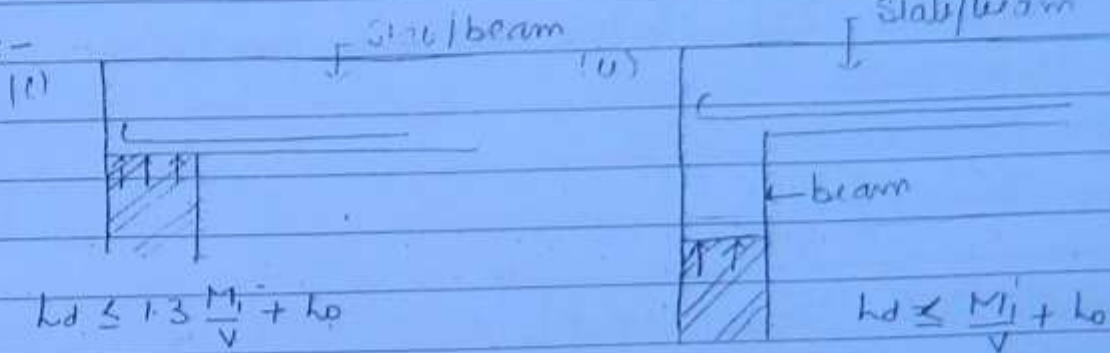
$$l_o = \frac{l_s - x'}{2} \text{ --- for HYSD Bars}$$

DATE: / /

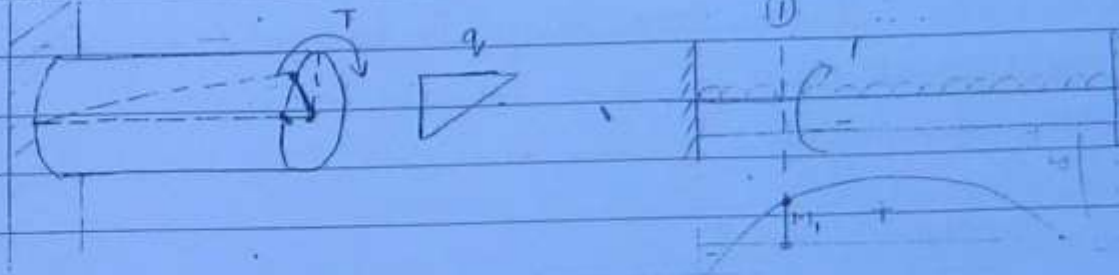
If the end of the reinforcement are combined by a compressive reaction.

$$L_d \leq 1.3 \frac{M_1}{V} + l_o \quad (11)$$

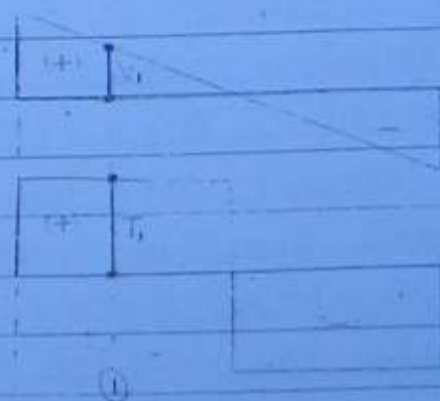
Example:-



TORSION



If a beam is subjected to BM-M
S.F = V & torsional moment T at
a particular cross-section



Design

(By working stress method)

1. Equivalent Shear

$$V_{eq} = V + 1.6 \frac{T}{B} \quad (1)$$

Nominal shear stress

$$\tau_{veq} = \frac{V_{eq}}{B \cdot d} \neq \tau_{cmax}$$

If $\tau_{veq} > \tau_{cmax}$ → change the size of beam

2. Equivalent Moment

$$M_{eq} = M + \frac{T}{1.7} \left(1 + \frac{D}{B} \right)$$

(112)

$$M_{eq} = M + M_T$$

CASE I Generally ($M_T < M$) In this case design only tension reinforcement

depth required

$$d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$$

Area of Steel (tension reinf.)

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d} = \frac{M_{eq}}{\sigma_{st} (d - x_a/3)}$$

CASE II If $M_T > M$ - In this case design in tension & compression reinforcement both.

Check $d = \sqrt{\frac{M_{eq}}{Q \cdot B}}$

$$A_{st} = \frac{M_{eq}}{\sigma_{st} \cdot j \cdot d}$$

Compression reinforcement is also provided for a moment $M_{e2} = (M_T - M)$

The moment M_{e2} being taken as acting in the opposite sense of moment M .

If $M = (+)$ ve B.M

Consider M_{e2} as (-)ve B.M

Compression steel

$$A_{sc} = A_{s2} = \frac{M_{e2}}{\sigma_{st} \cdot j \cdot d}$$

3. Design of Shear reinforcement

Spacing of shear reinforcement

$$A_{sv} = \frac{T_a S_v}{b d_1 (0.87 f_y) \sigma_{sv}} + \frac{V_a S_v}{2 \cdot S \cdot d_1 (0.87 f_y) \sigma_{sv}}$$

$$S_v = \frac{(0.87 f_y) A_{sv} \cdot \sigma_{sv} \cdot d_1}{\left[\frac{T}{b_1} + \frac{V}{2.5} \right]} \quad (113)$$

Spacing of shear reinforcement

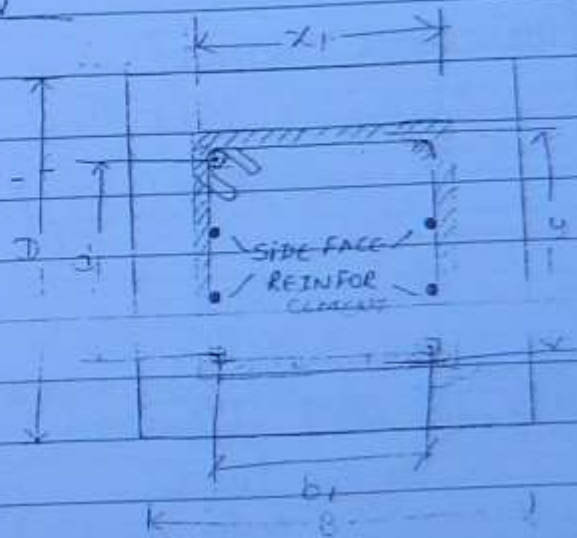
Max. Spacing of shear reinfⁿ (page No-48)

- 1 x_1
- 2 $\frac{x_1 + y_1}{4}$ $x, y \rightarrow$ short & long dim. of stirrups
- 3 300 mm

Min. Shear reinforcement in case of torsion - (Page No-47)
Total transverse reinforcement A_{sv} shall not be less than

$$A_{sv} \geq \frac{(T_{ve} - T_c) R_{sv}}{\sigma_{sv}}$$

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{(T_{ve} - T_c)}{\sigma_{sv}}$$



Side face Reinforcement

When depth of beam is more than

- 1 750 mm - When beam is not subjected to torsion
- 2 450 mm - When subjected to torsion

Side face reinfⁿ shall be provided equal to 0.1% of web area & distributed on both face

$$A_{sf} = \frac{0.1}{100} \times B \cdot D$$

Max. Spacing - keep Side face reinfⁿ = 300 mm
or web thickness $\leq$ even is less

Limit state method
factored values

$$B.M = M_u = 1.5M$$

$$T_u = 1.5T$$

$$V_u = 1.5V$$

(114)

Equivalent Shear

$$V_{ueq} = V_u + 1.6 \frac{T_u}{B}$$

Nominal Shear Stress $\tau_{ueq} = \frac{V_{ueq}}{B \cdot d} \nless \tau_{cmax}$

If $\tau_{ueq} > \tau_{cmax}$ revise the section

Equivalent moment

$$M_{ueq} = M_u + M_{Tu}$$

$$= M_u + \frac{T_u}{1.7} \left(1 + \frac{D}{B} \right)$$

$$d = \sqrt{\frac{M_{ueq}}{0.87 B}}$$

Area of Steel (tension)

$$A_{st} = \frac{M_{ueq}}{0.87 f_y \cdot j \cdot d}$$

$$= \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_{ueq}}{f_{ck} B d^2}} \right] B d$$

If $M_{Tu} > M_u \rightarrow$ provide Comp reinfⁿ for M_{ueq} ,

$$M_{ue2} = M_{Tu} - M_u$$

$$A_{sc} = A_{st2} = \frac{M_{ue2}}{0.87 f_y \cdot j \cdot d}$$

Shear reinfⁿ

$$S_v = \frac{A_{sv} (0.87 f_{ys}) d_1}{\left[\frac{T_u}{b_1} + \frac{V_u}{2.5} \right]}$$

Min Shear reinfⁿ

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{T_{ve} - \tau_c}{0.87 f_y}$$

(115)

Max. spacing

- ① x_1 ③ 300mm
② $\frac{x_1 + y_1}{4}$

Side face reinfⁿ is same as L.S.M

Ques Design a beam subjected to a following at a particular section (Use L.S.M) —

1. B.M = 300 kN-m

2. S.F = 200 kN-m

3. T.M = 80 kN-m

Use width of beam = 400mm

Use M-25 / Fe-415

Sol. Factored values

$M_u = 450$ kN-m

$T_u = 120$ kN-m

$V_u = 300$ kN-m

Equivalent Shear

$$V_{ueq} = V_u + \frac{1.6 T_u}{B} = 300 + \frac{1.6 \times 120}{400} = \frac{300 + 48}{780 \text{ kN}}$$

Assume depth = 600 mm (D) $d = 550$

Nominal Shear Stress = $\tau_{veq} = \frac{V_{ueq}}{B \cdot d}$

$$\Rightarrow \tau_{veq} = \frac{780 \times 10^3}{400 \times 550} = 3.54$$

τ_{cmax} M-25 = 3.10

$\tau_{veq} > \tau_{cmax}$ So revise the section

Assume depth = 900mm

$$\tau_{veq} = \frac{780 \times 10^3}{400 \times 850} = 2.29$$

Equivalent Moment $M_{ueq} = M_u + M_{res} \frac{T_u}{1.7} \left(\frac{1.7 D}{8} \right)$

$$= 450 + \frac{120}{1.7} \left(1 + \frac{900}{400} \right) = 450 + \frac{22}{22.441}$$

DATE: _____

$$M_{ueq} = 679.41 \text{ kN-m}$$

$$d = \sqrt{\frac{M_{ueq}}{Q \cdot B}}$$

$$d = \sqrt{\frac{679.41 \times 10^6}{1.38 \times 25 \times 400}}$$

$$d = 701.66 \text{ mm}$$

$$\text{Say } \phi = D = 750 \text{ mm}$$

$$\phi = 800 \text{ mm}$$

$$\tau_{veq} = \frac{V_{ueq}}{B \cdot D} = \frac{780 \times 10^3}{400 \times 700} = 2.8$$

Safe

$$M_{ueq} = 450 + \frac{120}{1.7} \left(1 + \frac{750}{400} \right)$$

$$= 450 + 202.94$$

$$M_{ueq} = 652.9 \text{ kN-m}$$

$$\text{Area of } d = \sqrt{\frac{652.9 \times 10^6}{1.38 \times 25 \times 400}} = 688 \text{ mm}$$

< 700 mm provided

$$\text{Size } B = 400 \text{ mm}$$

$$D = 750 \text{ mm}$$

$$d = 700 \text{ mm}$$

$$A_{st} = \frac{M_{ueq}}{87 f_y \cdot j \cdot d} = \frac{652.9 \times 10^6}{87 \times 415 \times 0.80 \times 700}$$

$$= 3229.4 \text{ mm}^2 - 25 \text{ mm } \phi \approx 7 \text{ nos.}$$

Shear reinforcement

Spacing

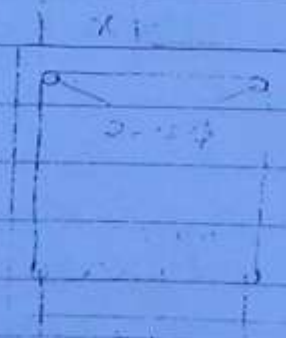
$$b_1 = 400 - 100 = 300$$

$$d_1 = 750 - 100 = 650$$

$$x_1 = 300 + 35 = 335$$

$$650 + 35 = 685$$

$$S_v = \frac{A_{sv} (0.87 \times f_y) \cdot d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{b_1} \right)}$$



DATE: / /

$$= \frac{2 \times \frac{\pi}{4} \times 10^2 \times 0.87 \times 415 \times 650}{\left(\frac{300 \times 10^3}{25} + \frac{120 \times 10^6}{300} \right)} = 70.9 \text{ mm}$$

(117)

Using 2 legged 12 ϕ bars

$$= 70.89 \times \frac{12^2}{10^2} = 102.08 \text{ mm}$$

Use 2 legged @ 100 mm c/c.

Min Shear Reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{ave} - \tau_c}{0.87 f_y}$$

$$\frac{2 \times \frac{\pi}{4} \times 12^2}{400 \times 100} \geq \frac{2.8 - 3.1}{0.87 \times 415}$$

In a Rectangular R.C Beam grade M-15 / $f_y = 415$ Size - 250 x 500 mm
effective $V = 60 \text{ kN}$. Under Service Load condition. Calculate
Spacing of 8mm dia 2-legged stirrups. Steel - 3-16 ϕ

P	0.25	0.5	0.75	1.00
τ_c	0.35	0.45	0.54	0.64

Solⁿ Load - 60 kN

$$D.L = 25 \times 25 \times 5.50 = 3.4375 \text{ kN}$$

$$\text{Total load} = 63.4375 \text{ kN}$$

$$\tau_v = \frac{V}{B \cdot d} = \frac{60 \times 10^3 \times 1.5}{250 \times 500} = 0.72$$

$$p = \frac{A_{st}}{250 \times 500} = \frac{3 \times \frac{\pi}{4} \times 16^2}{250 \times 500} \times 100 = 0.48$$

$$\tau_c = 0.35 + \frac{0.48 - 0.35}{0.50 - 0.25} \times (0.48 - 0.25)$$

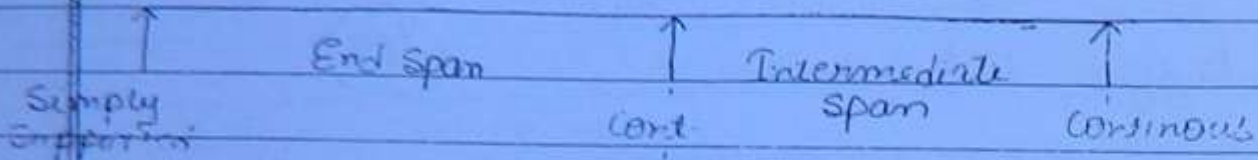
$$= 0.4512$$

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Bending moment and Shear force Coefficient for a Continuous beam

1. For bending moment

(1/8)

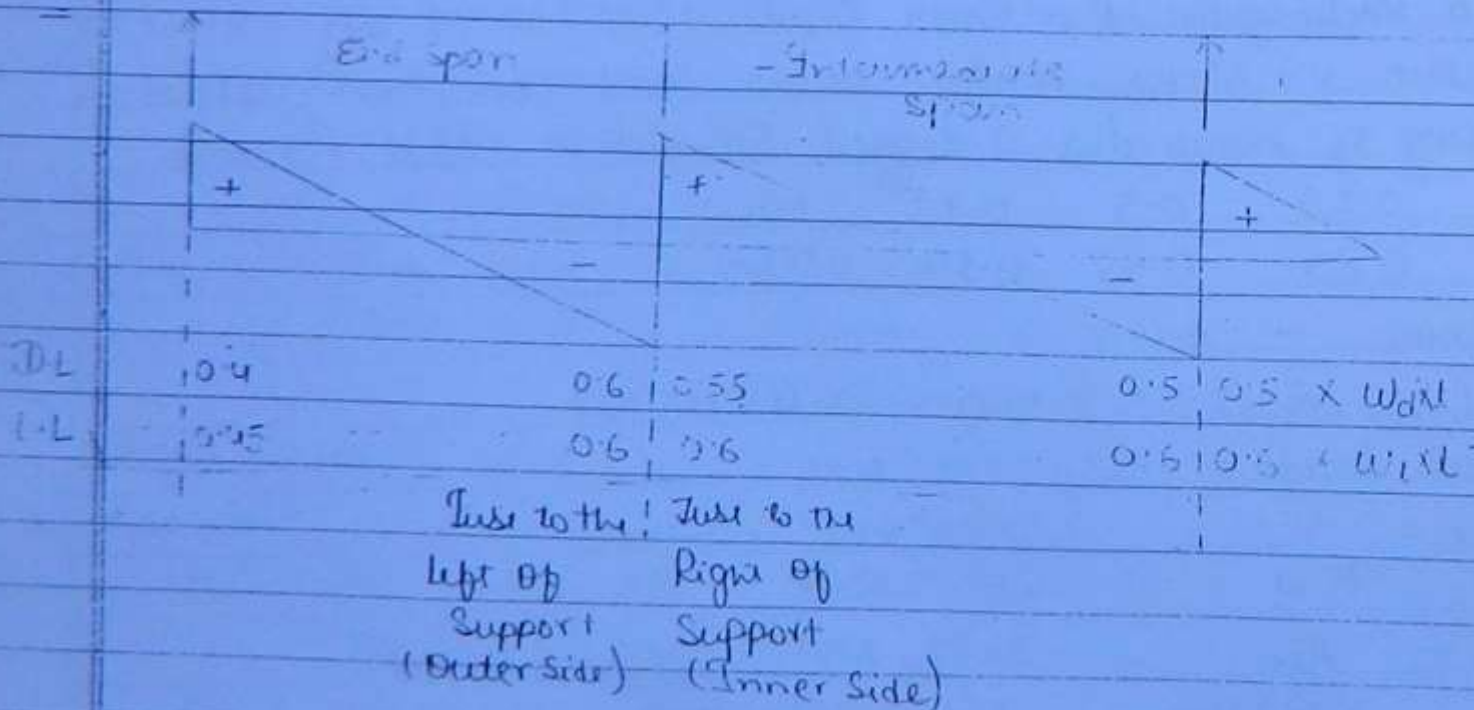


DL	$\frac{1}{12}$	$-\frac{1}{10}$	$\frac{1}{16}$	$-\frac{1}{12}$
I.L	$\frac{1}{10}$	$-\frac{1}{9}$	$\frac{1}{12}$	$-\frac{1}{9}$

B.M at any location = $K_d w_d l^2 + K_e w_e l^2$

At Support + = $-\frac{1}{10} w_d l^2 - \frac{1}{9} w_e l^2$

For Shear force (TABLE-13 IS: 456)



ES-1992

(a) 550 x 750 mm Overall depth

$$M_u = BM = 150 \text{ kN-m}$$

$$T_u = 50 \text{ kN-m}$$

Use M-15 / Fe-415

(1/9)

$\mu(1)$	z_c
0.25	0.35
0.50	0.46

Determine longitudinal reinforcement.

(b) If $V_u = 130 \text{ kN}$

For the beam in Ques above Determine the transverse reinforcement

Solⁿ Equivalent Shear

$$V_u = 0 ; T_u = 50 \text{ kN-m}$$

$$V_{ue} = V_u + \frac{1.6 T_u}{B} = 0 + \frac{1.6 \times 50}{0.55} = 145.45 \text{ kN}$$

$$z_{ve} = \frac{V_{ue}}{B d} = \frac{145.45 \times 10^3}{550 \times 700} = 0.38 < (z_{c \max})$$

Safe

Equivalent moment

$$M_{ue} = M_u + M_{Tu} = M_u + \frac{T_u}{1.7} \left(1 + \frac{D}{B} \right)$$

$$= 150 + \frac{50}{1.7} \left(1 + \frac{750}{550} \right)$$

$$150 + 69.52 = 219.52 \text{ kN-m}$$

$M_{Tu} < M_{ue}$ — Compression Steel not required

$$\text{Depth } d = \sqrt{\frac{M_{ue}}{Q \cdot B}} = \sqrt{\frac{219.52 \times 10^6}{1.38 \times 15 \times 550}} = 439.12 \text{ mm}$$

Available depth is = 700 mm OK.
So we need under reinforced section because depth available is more than required

(120)

$$\text{Area of Steel } A_{st} = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_{ue}}{f_{ck} B d^2}} \right] B d$$

$$A_{st} = \frac{0.5 \times 15}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 219.52 \times 10^6}{15 \times 550 \times 700^2}} \right] 550 \times 700$$

$$A_{st} = 931.3 \text{ mm}^2$$

$$\text{No. of 16mm dia bar} = \frac{931.3}{\frac{\pi}{4} \times 16^2} = 4.63$$

Say 5 No.

(ii) Solⁿ If Shear force $V_u = 130 \text{ kN}$
 $T_u = 50 \text{ kN-m}$

Equivalent Shear

$$V_{ue} = 130 + \frac{1.6 \times 50}{5.50} = 275.45$$

$$\tau_{vue} = \frac{V_{ue}}{b d} = \frac{275.45 \times 10^3}{550 \times 700} = 0.715 \text{ N/mm}^2$$

< τ_{cmax}
(2.5) - 4.15

OK
(Safe)

Design of Shear reinforcement

$$S_v = \frac{A_{sv} (0.87 f_y) \cdot d_1}{\left(\frac{V_u}{2.5} + \frac{T_u}{b_1} \right)}$$

Use 2-legged 8mm ϕ shear reinforcement

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

$$S_v = \frac{100.53 \times 87 \times 415 \times 450 \times 650}{\left(\frac{130 \times 10^3}{2.5} + \frac{50 \times 10^6}{450} \right)}$$

(121)

$$S_v = 144.64$$

provide 2-legged 8mm ϕ @ 140mm c/c

Check - Min shear reinforcement

$$\frac{A_{sv}}{B \cdot S_v} \geq \frac{\tau_{\text{true}} - \tau_c}{\cdot 87 f_y}$$

$$\% \text{ of steel } p = \frac{A_{st}}{B \cdot d} \times 100 = \frac{5 \times \frac{\pi}{4} \times 16^2}{5500 \times 700} \times 100$$

$$= 0.26\%$$

$$\tau_c = 0.35 + \frac{0.46 - 0.35}{0.25} \times (0.26 - 0.25)$$

$$= 0.3544$$

$$\frac{100.53}{550 \times S_v} \geq \frac{(0.715 - 0.3544)}{87 \times 415}$$

$$S_v \leq \frac{134.3678 \cdot 87 \times 415 \times 100.53}{139.3678 (550 (0.715 - 0.3544))} = 182 \text{ mm}$$

Max Spacing $x_1 = 450 + 16 + 8 = 474$
 $y_1 = 650 + 16 + 8 = 674$

① $x_1 = 474 \text{ mm}$

② $\frac{x_1 + y_1}{4} = \frac{474 + 674}{4} = 287$

③ 300 mm

Provide 2 legged - 8mm ϕ @ 140 mm c/c

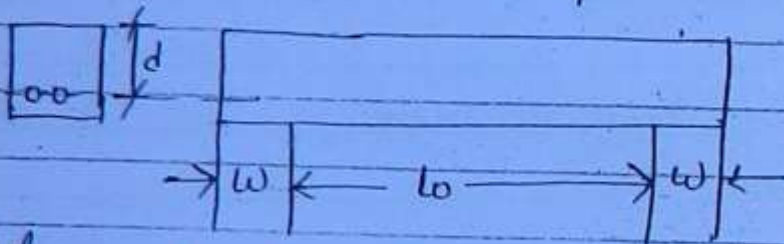
(122)

DESIGN OF BEAM AND SLAB (One way Slab)

Important I.S Provision :—

① Effective Span

(a) Simply supported beam / slab (22.2 - Page No-34/35)

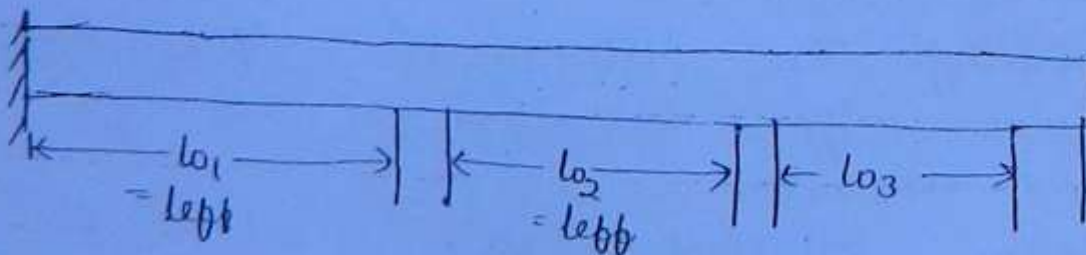


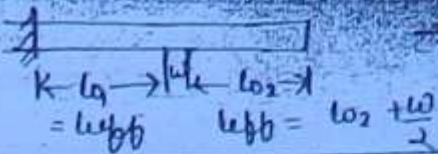
$$\left. \begin{aligned} l_{eff} &= l_0 + d \\ &= l_0 + w \end{aligned} \right\} \text{whichever is less}$$

(b) For a Continuous beam or Slab

1. If width of support is less than $\frac{1}{12}$ of clear span
 $\left(w < \frac{l_0}{12} \right)$ then l_{eff} span is same as that of simply supported

2. If $\left(w > \frac{l_0}{12} \right)$ then $\boxed{\text{Min}}$
 OR $w = 600 \text{ mm}$





① For one end fixed & other Continuous or Both end fixed Continuous (intermediate span)
 $l_{eff} = \text{Clear Span}$ (123)

2. With one end Continuous & other free (Simply supports)
 $l_{eff} = \text{Clear span} + \frac{l_1}{2} = l_0 + \frac{w}{2}$
 $l_{eff} = \text{Clear span} + \frac{d}{2} = l_0 + \frac{d}{2}$ } whichever is small.

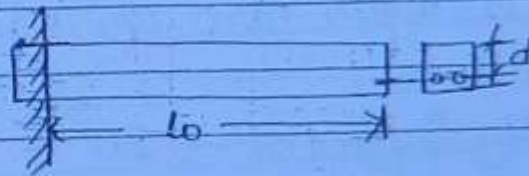
(c)

Cantilever

(i) with fixed end

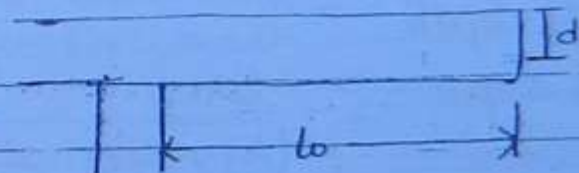
$$l_{eff} = l_0 + \frac{d}{2}$$

$d = \text{eff depth}$



(ii) with Continuous end

$$l_{eff} = l_0 + \frac{l_1}{2}$$



(d)

Span - c/c distance between beam/Column.

Control of deflection

(Important Criteria of limit state of serviceability)

(Page NO-31)

The deflection shall generally be limited to following-

(a) The final deflⁿ of all load including the effect of temp, creep, & shrinkage & measured from the as-cast level of the support of floor, roof & all

Horizontal members should not normally exceed $\text{Span}/250$

(intermediate deflⁿ) (124)

(b) The deflection including the effect of Temp, Creep & Shrinkage occurring after erection of partition & the application of finishes should not normally exceed $\text{span}/350$ or 20mm whichever is less

Check for deflection

→ Vertical deflⁿ limit may be generally be assumed satisfied if—

Span to depth ratio are not greater than

Cantilever	— 7	} (A) values (upto 10m) Span.
Simply Supported	— 20	
Continuous	— 26	

$$\frac{\text{Span}}{\text{Eff depth}} \leq (A) \text{ values}$$

$$\text{Eff depth} \geq \frac{\text{Span}}{(A) \text{ values}}$$

Except for Cantilever if span is more than 10m the (A) Value should be multiplied by $\frac{10}{\text{Span in metre}}$

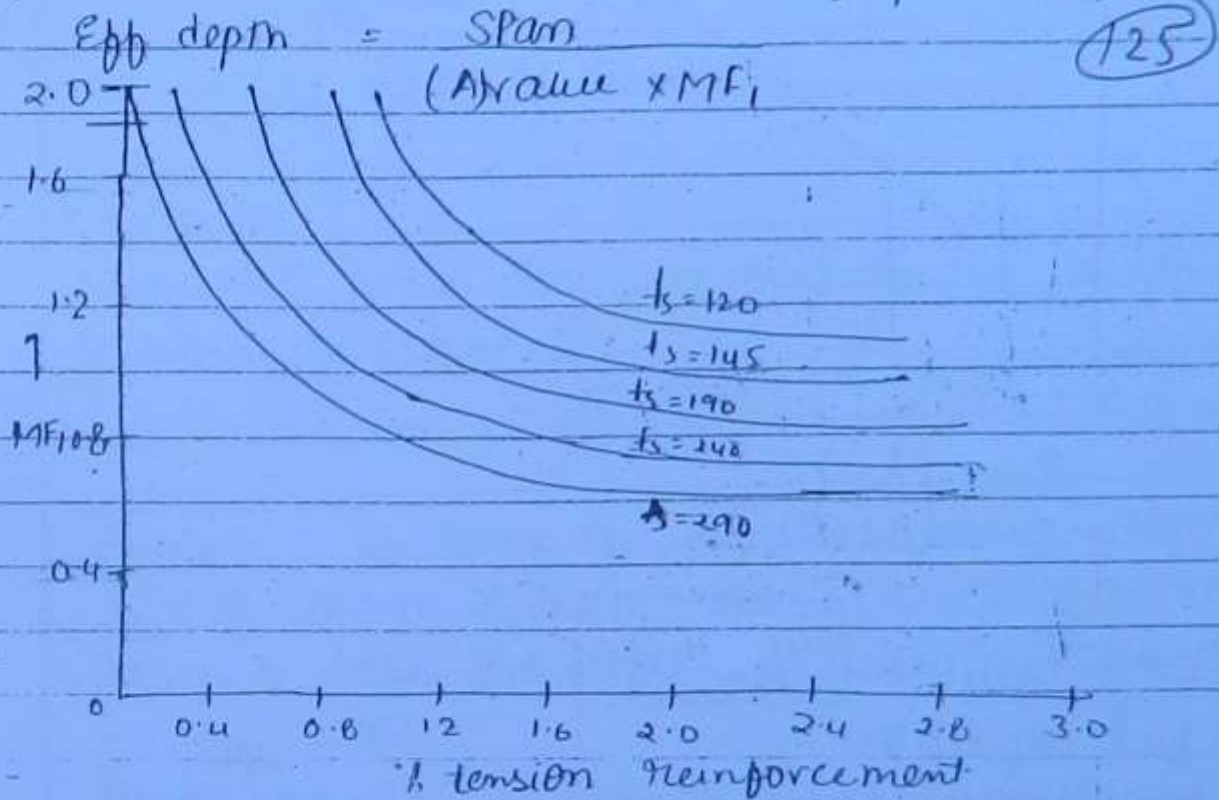
$$\text{Eff depth} = \frac{\text{Span}}{A \text{ value} \times \frac{10}{\text{Span (m)}}}$$

eg- For a 20m span S.S.B eff. depth req. will

$$\text{Eff} = \frac{20 \times 10^3}{20 \times \frac{10}{20}} = 2 \text{ m or } 2000 \text{ mm}$$

Modification factor (MF_1) based on tension reinf.

A value can be multiplied by a Modification factor MF_1 based on tension reinf provided.

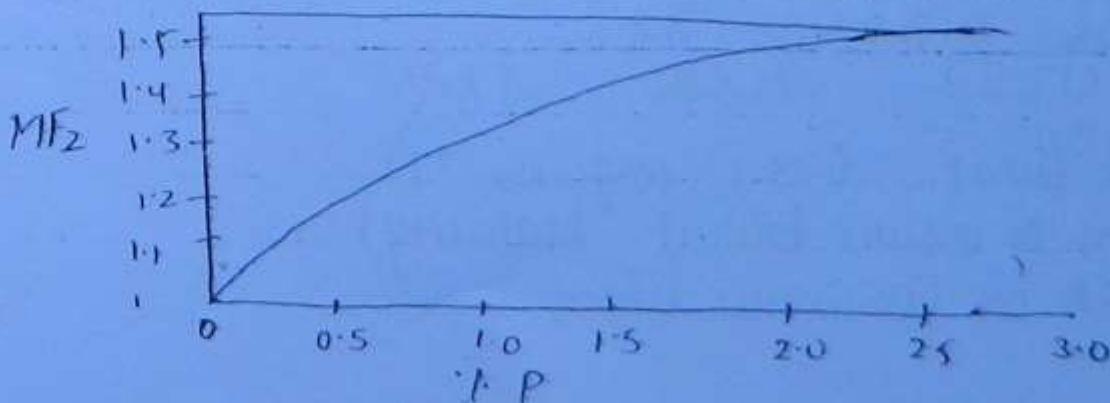


$$l_s = 0.58 l_y \frac{A_{st} \text{ reqd}}{A_{st} \text{ provided}}$$

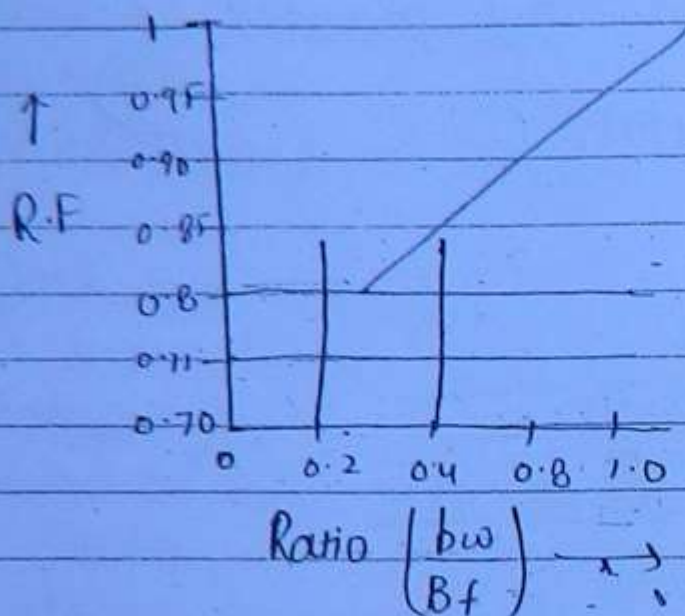
Modification factor (MF_2) based on Comp. reinforcement

Eff depth = Span

(A) value $\times MF_1 \times MF_2$



For planged beam (T-beam / L-beam)
Reduction factor is used.



Two way Slab

$$\boxed{l_y/l_x < 2}$$

Check for deflection

Only for shorter span < 3.5 m

(upto
(load ≤ 3 kN/m²))

Span to overall depth ratio

Simply Supported = 35

Continuous = 40

} for Mild Steel

For HYSD Bar of grade k-415 the values
shall be multiplied by 0.8

$$S.S.B = 0.8 \times 35 = 28$$

$$\text{Continuous} = 0.8 \times 40 = 32$$

Slenderness limit (23.3 - page No-39)

(For beams to ensure lateral stability) -

① For S.S.B or Continuous beam

Clear distⁿ betⁿ lateral restraint $\nless 60b \}$ or
 $\nless \frac{250b^2}{d} \}$ (small)

$b \rightarrow$ width

$d \rightarrow$ eff. depth of beam

(127)

Cantilever

Clear distⁿ from the free end of the Cantilever
 to lateral restraint $\nless 25b \}$ less
 $\nless \frac{100b^2}{d} \}$

Example - If span of Cantilever beam is 5000mm
 Size of beam 250x450 mm. Check whether
 beam is safe for deflⁿ & lateral stability
 $\rightarrow$ two way

Deflⁿ $\rightarrow$

$$\text{Eff depth} = \frac{\text{Span}}{7} = \frac{5000}{7} = 714.29 \text{ mm}$$

$$> 450 \text{ mm}$$

$$\text{Lateral Stability} = \frac{100 b^2}{d}$$

$$= \frac{100 \times 250^2}{400} = 15625 \text{ mm}$$

(Safe)

$$25 \times b = 25 \times 250 = 6250 \text{ mm}$$

$$> 5000 \text{ mm}$$

OK

Steel Reinforcement (Beams)

Min tension reinforcement

$$\frac{A_{st \min}}{bd} = \frac{0.85}{l_y}$$

(128)

Max tension reinfⁿ

$$= 4\% \text{ of total cross section area}$$

$$= 0.04 bD$$

Compression reinfⁿ

$$\text{Max Comp reinf}^n \neq 0.04 bD$$

Slab

Min reinforcement

Mild Steel $\rightarrow$ 0.15% of total cross section

$$\frac{0.15}{100} \times B \cdot D$$

$$\text{HYSD Bar} \rightarrow \frac{0.12}{100} \times B \cdot D$$

Max Spacing of bars

(i) for main reinfⁿ

$$= 3d \text{ or } 300 \text{ mm}$$

(ii) for distribution bars

$$= 5d \text{ or } 450 \text{ mm}$$

Max dia Min dia of bars for slab

$$\neq \frac{\text{Depth}}{8} = \frac{1}{8} \times \text{total thickness of slab}$$

If a slab is 75 mm thick

$$\text{Max dia} = \frac{75}{8} = \underline{\underline{9.375}}$$

Max 8mm ϕ bars is used.

129

130

DATE:

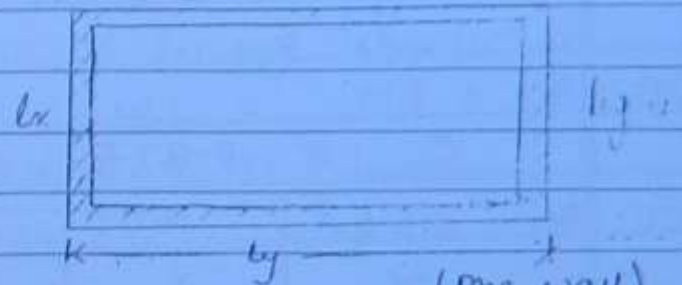
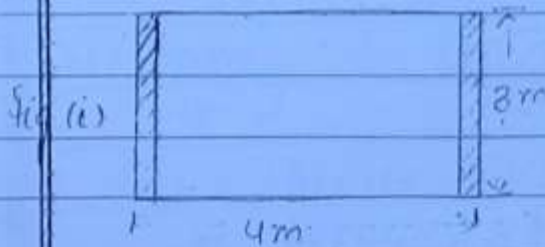
Design of one way slab ($l_y/l_x > 2$)

A slab is said one way only if :-

- (i) If the slab is supported only on two support (opposite) (Span ratio is not to be seen) (fig i)
- (ii) If the slab is supported on all four sides and span ratio $\frac{l_y}{l_x} > 2 \rightarrow$ One way slab

$\frac{l_y}{l_x} \leq 2 \rightarrow$ two way slab

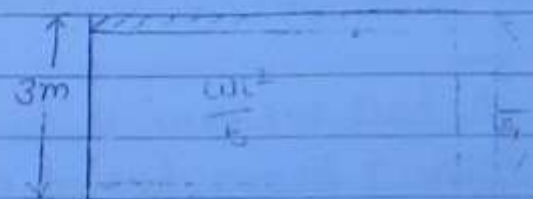
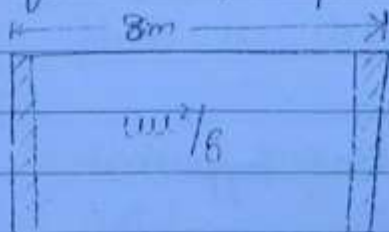
(13)



In fig (ii)

l_y = longer span & l_x = shorter span ($l_y/l_x > 2$)

- $\rightarrow$ In this case slab is designed for shorter span only
- $\rightarrow$ In longer span dirⁿ only distribution bars (min. steel reinforcement) is provided



Only shorter span is effective in design & design will be done for shorter span only due to above deflⁿ calculated. As we know moment is proportional to deflⁿ only so we in one way we design only for shorter span.

Design Steps

- (1) Load over
- $1\text{m} \times 1\text{m}$
- of Slab is calculated

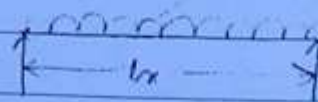
Dead load = $1 \times 1 \times 1 \times 25 = w_d$

Live load = $111 \times 1 \times 1 = w_l$

Floor finish = $\frac{1}{4} \times 1 \times 1 \times 24 = w_f$

Total load = w

- (2) Bending Moment =
- $B.M = \frac{w \cdot l_x^2}{B}$



- (3) Depth
- $d = \sqrt{\frac{B.M}{Q.B}}$
- $B = 1000\text{mm}$

- (4)
- $A_{st} = \frac{B.M}{\sigma_{st} \cdot j \cdot d} = \frac{M_u}{0.87 \cdot f_y \cdot j \cdot d}$

(132)

OR $A_{st} = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B \cdot d^2}} \right] \times B \cdot d$

(Useful for under reinforced section & also for limiting section)

- (5) Check for Shear
 (6) Check for Bend
 (7) Check for development length

:- 1984

QuesA hall $10 \times 40\text{m}$ is covered by R.C.C. slab supported over T-beam spaced at 4m intervalFloor finish = 60 kg/m^2 Live load = 200 kg/m^2

Use M-15 mild steel use

Design one of the intermediate slab

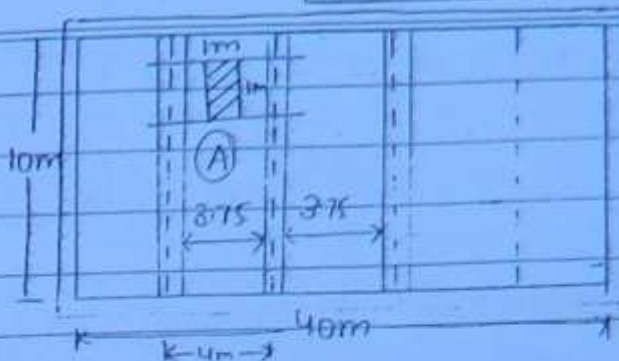
Solⁿ For one slabClear span $l_x = 3.75\text{ m}$ $l_y = 10\text{ m}$ Assume thickness of slab using defⁿ criteria

Eff^t depth = $\frac{\text{Span}}{26} = \frac{3750}{26} = 144\text{mm}$

DATE: _____

$$\text{Eff. Cover} = 25 \text{ mm} \\ = 169 \text{ mm}$$

$$\text{bay } D = 170 \text{ mm} \\ d = 145 \text{ mm}$$



Effective span

$$\text{Width} = l = 250 \text{ mm}$$

$$\text{Span} = \frac{3750}{12} = 312.5 \text{ mm}$$

133

$\left(\frac{l < \text{Span}}{12} \right) \Rightarrow$ left is same as simply supported

$$\text{left} = l_0 + d = 3.75 + 0.145 = 3.895 \text{ m} \quad \left. \begin{array}{l} \text{small} \\ \text{or } l_0 + w = 3.75 + 0.25 = 4.0 \text{ m} \end{array} \right\}$$

$$l_x = 3.895 \text{ m}$$

$$\text{Similarly } l_y = 10.145 \text{ m}$$

$$\text{Span ratio } \frac{l_y}{l_x} = \frac{10.145}{3.895} = 2.6 > 2$$

Hence it is one way slab

Slab shall be designed for $l_x = 3.895 \text{ m}$

Load

Consider 1m width of slab

Dead load

$$\text{Self wt} = 17 \times 1 \times 1 \times 25 = 4.25 \text{ kN/m}^2$$

$$\text{Floor finish} = 60 \times 10 \times 10^{-3} \times 1 \times 1 = 0.60 \text{ kN/m}^2$$

Live load

$$W_L = 2.0 \times 1 \times 1 = 2.0 \text{ kN/m}^2$$

Bending moment (from Gnom coefficient table)

$$\text{B.M} = \frac{W_d \times l}{16} l_x^2 + \frac{W_L \times l}{12} l_x^2$$

$$M(+ve) = \frac{1}{16} \times 4.85 \times 3.895^2 + \frac{1}{12} \times 2 \times 3.895^2$$

$$M(+ve) = 113 \text{ kN}$$

$$(-ve) B.M = -\frac{1}{10} W_d l x^2 - \frac{1}{9} w_u l x^2$$

$$= -\frac{1}{10} \times 4.85 \times 3.895^2 - \frac{1}{9} \times 2 \times 3.895^2$$

$$(-ve) B.M = 10.73 \text{ kN} \quad (134)$$

$$\text{Depth } d = \sqrt{\frac{B.M}{Q.B}}$$

$$d = \sqrt{\frac{10.73 \times 10^6}{0.87 \times 1000}}$$

$$d = 111 \text{ mm} < 145 \text{ mm}$$

(safe)

$$k = \frac{m_c}{1 + m_c}$$

$$= \frac{19 \times 5}{140 + 19 \times 5}$$

$$= 0.404$$

$$j = 1 - \frac{k}{3} = 0.865$$

$$Q = \frac{1}{2} c \cdot j \cdot k$$

$$= \frac{1}{2} \times 5 \times 0.404 \times 0.865$$

$$= 0.87$$

$$A_{st} = \frac{B.M}{\sigma_{st} \cdot j \cdot d}$$

$$+ve) B.M = \frac{7.13 \times 10^6}{140 \times 0.865 \times 145} = 406 \text{ mm}^2$$

Use 10mm ϕ bars spacing

$$S = \frac{1000}{406} \times \frac{\pi}{4} \times (10)^2 = 193 \text{ mm}$$

provide 10mm ϕ bars @ 190 mm c/c

$$(-ve) B.M = \frac{10.73 \times 10^6}{140 \times 0.865 \times 145} = 611 \text{ mm}^2$$

Spacing of 10 mm ϕ bars

$$S = \frac{1000}{611} \times \frac{\pi}{4} \times 10^2 = 128 \text{ mm}$$

provide 10mm ϕ bars @ 125 mm c/c

Check for shear

$$SF = 0.55 W_d \cdot l_x + 0.60 W_u \cdot l_x$$

$$= 0.55 \times 4.85 \times 3.895 + 0.60 \times 2 \times 3.895$$

$$V = 15.06 \text{ kN}$$

$$\tau_v = \frac{V}{b \cdot d} = \frac{15.06 \times 10^3}{1000 \times 145} = 0.104$$

$$\tau_{c \min} = 0.18 \quad (\tau_v < \tau_c) \text{ Safe}$$

No shear reinforcement is required.

(135)

Check for bond.

For (+ve) reinforcement 10mm ϕ bars @ 190 c/c

S.F at Support + $V = 15.06 \text{ kN}$

Bond stress developed

$$\tau_{bd} (\text{developed}) = \frac{V}{\sum o \cdot j \cdot d} = \frac{15.06 \times 10^3}{\left(\frac{1000}{190} \right) \times \pi \times 10 \times 0.865 \times 145}$$

$$\tau_{bd} = 0.726 \text{ N/mm}^2$$

$$\sum o = n \times \pi \phi$$

$$\tau_{bd} (\text{permissible}) = 0.6$$

$\tau_{bd} > \tau_{bd} (\text{permi}) \rightarrow \text{failed}$

Change dia of bars provided

Use 8mm dia bars for +ve reinforcement

$$\text{Spacing} = 190 \times \frac{8^2}{10^2} = 121.16$$

Provide 8mm ϕ bars @ 120mm c/c

$$\tau_{bd} \text{ developed} = \frac{15.06 \times 10^3}{\left(\frac{1000}{120} \right) \times \pi \times 8 \times 0.865 \times 145} = 0.57 < 0.6 \quad (\text{Safe})$$

Distribution bars

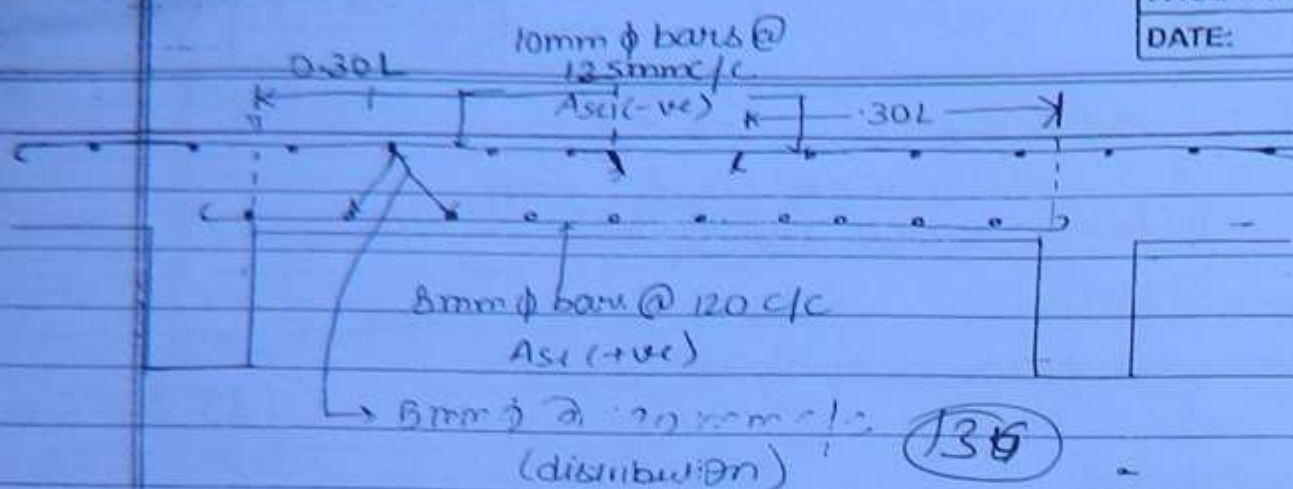
= 0.15% of total cross section area

$$\approx \frac{0.15}{100} \times B \times D$$

$$\approx \frac{0.15}{100} \times 1000 \times 170 = 255 \text{ mm}^2$$

$$\text{Spacing of 8mm } \phi \text{ bars} = \frac{1000 \times \pi \times 8^2}{4 \times 255} = 197.12 \text{ mm}$$

Provide 8mm ϕ bars @ 190 mm c/c



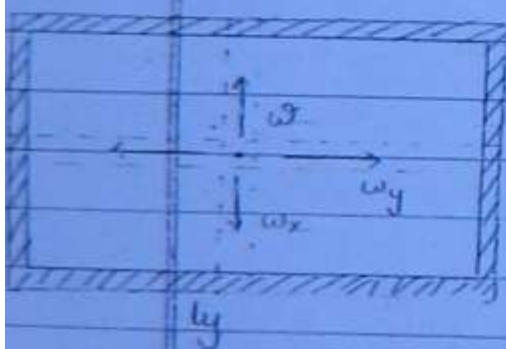
Design of two way slab ($l_y/l_x \leq 2$)

- A Slab is said to be two way only if -
1. Slab should be supported on all four sides
 2. Span Ratio ≤ 2

Design

1. Rankine grashoff Method
(Simply supported slab)

With corner not held down position



(corners are free to lift in upward dirⁿ)
No negative B.M will generate (pure Simply supported case)

If w = total load

$$w = w_x + w_y \quad \text{--- 1}$$

$$\text{defl}^n \quad \delta_x = \frac{5}{384} \frac{w_x l_x^4}{EI}$$

$$\delta_y = \frac{5}{384} \frac{w_y l_y^4}{EI}$$

$$\text{In every case } \delta_x = \delta_y = \frac{5}{384} \frac{w_x l_x^4}{EI} = \frac{5}{384} \frac{w_y l_y^4}{EI}$$

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$$\frac{w_x}{w_y} = \left(\frac{l_y}{l_x} \right)^4 = n^4$$

$$\boxed{w_x = n^4 w_y} \quad \text{--- (2)}$$

(137)

Total load $w = n^4 \cdot w_y + w_y$
 $(1 + n^4) w_y$

$$w_y = \left(\frac{1}{1 + n^4} \right) w \quad \text{--- (3)}$$

$$w_x = n^4 w_y = \left(\frac{n^4}{1 + n^4} \right) w \quad \text{--- (4)}$$

BM in x-dirⁿ $\Rightarrow M_x = w_x \cdot \frac{l_x^2}{8} = \left(\frac{n^4}{1 + n^4} \right) w \cdot \frac{l_x^2}{8}$

$$M_y = w_y \cdot \frac{l_y^2}{8} = \left(\frac{1}{1 + n^4} \right) w \cdot \frac{l_y^2}{8}$$

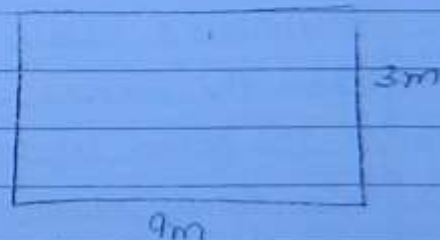
This is as per Rankine Grasshoff theory

Eg. $w_x = \left(\frac{n^4}{1 + n^4} \right) w$

$n = 3 = l_y / l_x$ (One way slab)

$$w_x = \left(\frac{3^4}{1 + 3^4} \right) w = 0.99w$$

$$w_y = \left(\frac{1}{1 + 3^4} \right) w = 0.01w$$



Q. 2

Simply supported slab (When corners held down position)
 (Negative B.M will develop at support and corner)
 Method

- (i) Pigeaud's method
- (ii) Marcus Method
- (iii) IS Code Method

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1. Pigeaud's Method

$$M_x = \frac{\eta'_x \cdot W_x \cdot l_x^2}{8}$$

$$M_y = \frac{\eta'_y \cdot W_y \cdot l_y^2}{8}$$

(138)

here η'_x & η'_y are Coefficients values are read from table.

2. Marcus Method (Modification over Rankine grasshoff Method) Marcus Correction

$$C = \left(1 - \frac{5 \eta^2}{6 (1 + \eta^4)} \right) \text{ is applied}$$

$$M_x = C \cdot W_x \cdot \frac{l_x^2}{8}$$

$$M_y = C \cdot W_y \cdot \frac{l_y^2}{8}$$

3. I-S Code Method (ANNEX D)

Restrained Slabs

→ When the Corners of slab are prevented from lifting & corners held down position)

2x B.M per unit width

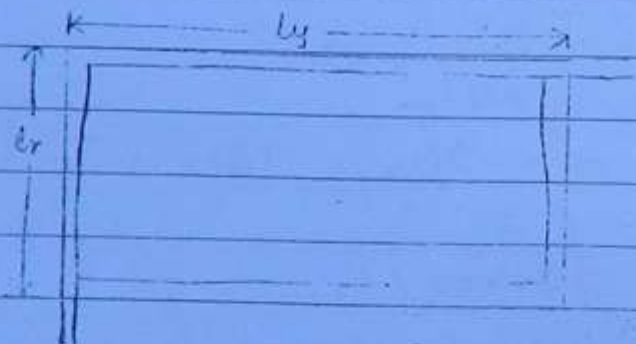
$$M_x = \alpha_x w l_x^2$$

$$M_y = \alpha_y w l_y^2$$

α_x & α_y are Coefficient based on Support Condition. (TABLE-26)

l_x = Shorter Span

w = total load on slab per m^2 area



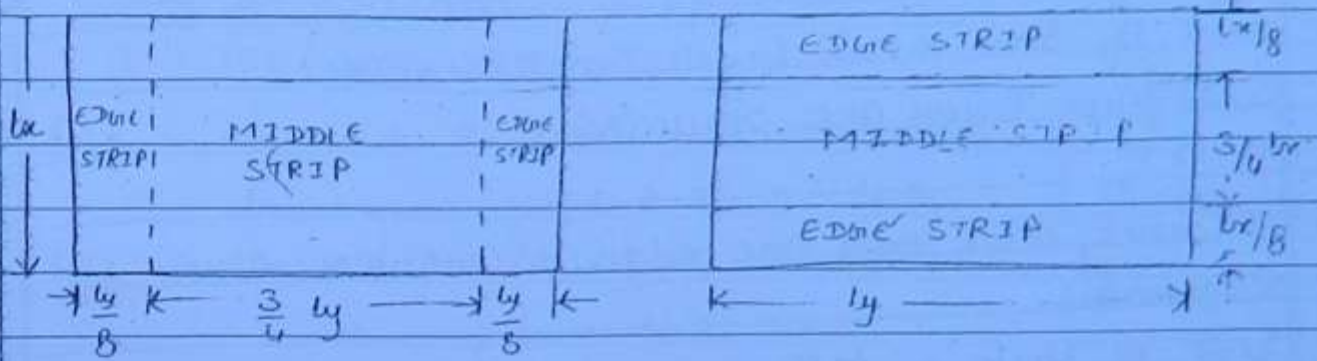
Four B.M are found

$M_x (+ve)$, $M_x (-ve)$, $M_y (+ve)$

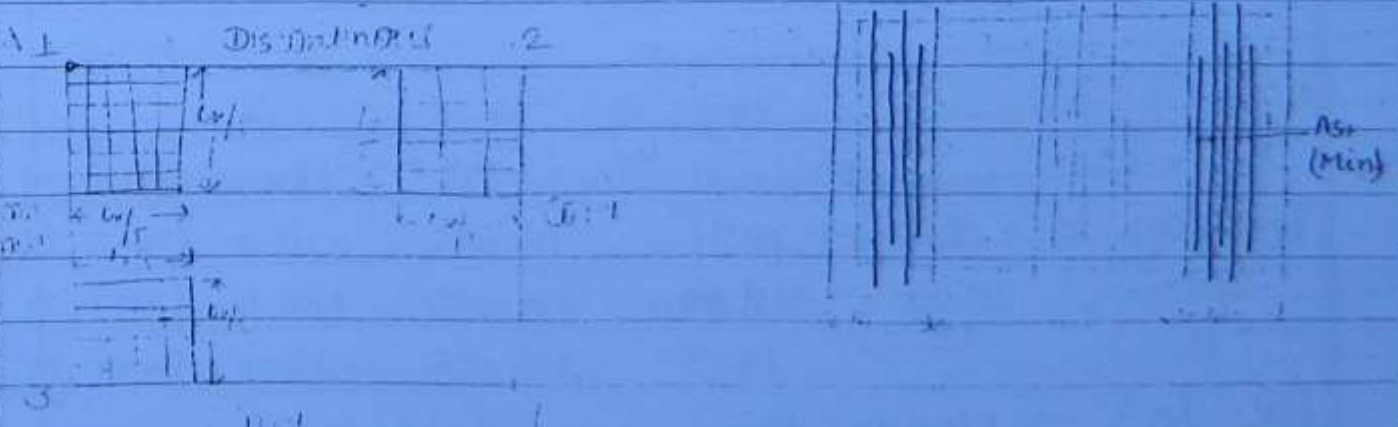
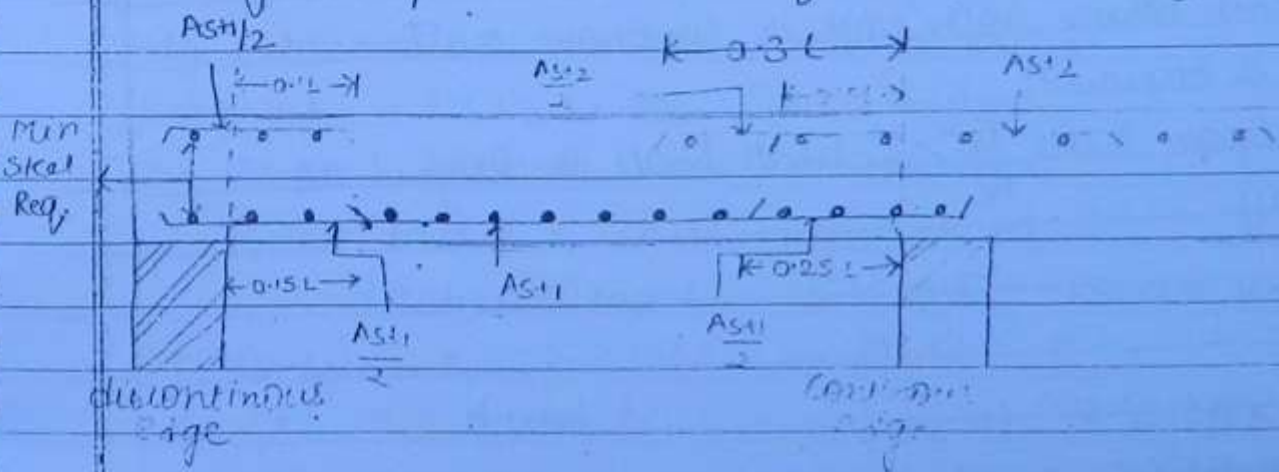
$M_y (-ve)$

4m	4	3	3	4
3m	2	1	3	2
4m	4	4	3	7
6m			5m	

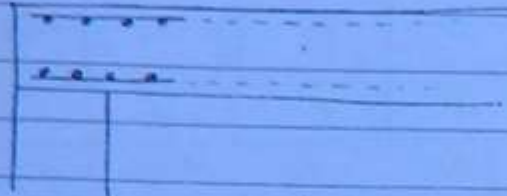
139



Moments M_x & M_y are applicable for middle strip only.
For edge strip min Steel reinfⁿ required shall be provided.



- Torsion reinforcement shall be provided at corner where two edges are discontinuous (Simply supported)



(140)

- Size of Mesh = $\frac{l_x}{5}$
Area of Steel in each layer = $\frac{3}{4} \times A_{stx} (+ve)$
→ Such four layers are provided

- At corner when only one edge is continuous other discontinuous

Size of Mesh = $\frac{l_x}{5}$
Area of Steel = $\frac{3}{8} \times A_{stx} (+ve)$

- At corner where both edge is continuous no tension "stitch" is required.

- When span ratio $\frac{l_y}{l_x} > 2$ shall be designed as one way.

Ques Design a continuous slab of 4.5m x 6m. The slab is simply supported on all four sides. The effective span is 4.5m. The slab is to be designed as two way slab. The slab is to be designed as two way slab. The slab is to be designed as two way slab.

Sol:- $l_x = 4.5m$; $l_y = 6$ $\frac{l_y}{l_x} = 1.3 < 2$ (two way)

Eff Span $l_1 = 3000mm$

$\frac{Span}{12} = \frac{4500}{12} = 375mm$

$\left(\frac{l_y}{l_x} < 2 \right) \rightarrow$ Effective span as per S.S. case

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Assume thickness of slab as per deflection criteria

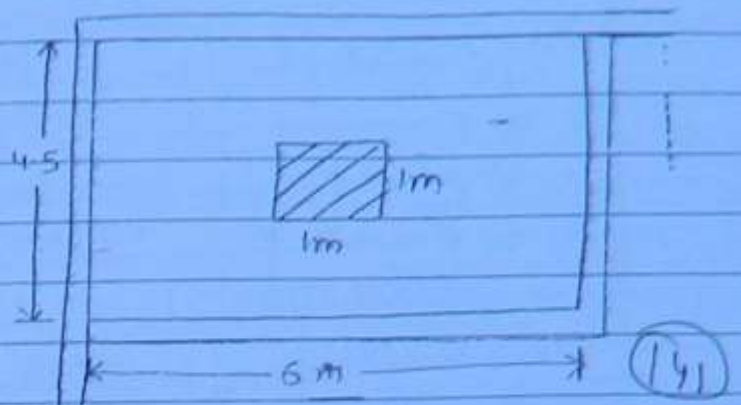
Continuous = $0.8 \times 40 = 32$

Overall depth = $\frac{l_x}{32} = \frac{4500}{32}$

$\Phi = 140.625$

Say $\Phi = 150 \text{ mm}$

$d = 150 - 30 = 120 \text{ mm}$



$l_{eff} = l_{x0} + d = 4.50 + 0.12 = 4.62 \text{ m}$

$l_{x0} + H = 4.50 + 0.30 = 4.8$

left

4.62 m

is consider

$l_{eff} = l_{y0} + d = 6.0 + 0.12 = 6.12 \text{ m} = l_y$

$l_x = 4.62 \text{ m}$

Span ratio $\frac{l_y}{l_x} = \frac{6.12}{4.62} = 1.32 < 2$ (Two way slab)

Load Calculation

Self wt = $0.15 \times 1 \times 1 \times 25 = 3.75 \text{ kN/m}^2$

Floor finish = $1 \times 1 \times 1 \times 200 = 2 \text{ kN/m}^2$

Line load = $1 \times 1 \times 4 = 4 \text{ kN/m}^2$

Total load = $8.75 \text{ kN/m}^2 = w$

factored load = $8.75 \times 1.5 = 13.125 \text{ kN/m}^2$

B.M Coefficient

$\alpha_x (+) = 0.050$

$\alpha_y (+) = 0.035$

$\alpha_x (-) = 0.066$

$\alpha_y (-) = 0.047$

M_x (F)

	α	$w \cdot l_x^2$	M	d
$M_x (+ve)$	0.05	280.145	14.0	Req. Provision
$M_x (-ve)$	0.066	280.145	18.49	Provide 120mm
$M_y (+ve)$	0.047	280.145	13.17	
$M_y (-ve)$	0.035	280.145	13.17	

Depth $d = \sqrt{\frac{M_u}{0.87 f_y}} = \sqrt{\frac{18.49 \times 10^6}{0.87 \times 250 \times 1000}} = 81.85 < 120 \text{ mm}$ (Safe)

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→ A_{st}

$$A_{st1} = \frac{0.5 f_{yk}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{yk} B d^2}} \right] \times B d$$

$$= \frac{0.5 \times 20}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 14.01 \times 10^6}{20 \times 1000 \times 120^2}} \right] 1000 \times 120$$

$$= 344.0 \text{ mm}^2$$

$$A_{st2} = \frac{0.5 \times 20}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 18.4 \times 10^6}{20 \times 1000 \times 120^2}} \right] 1000 \times 120$$

$$A_{st2} = 464 \text{ mm}^2$$

$$A_{st3} = 236 \text{ mm}^2$$

$$A_{st4} = 322 \text{ mm}^2$$

(142)

Spacing of 10 mm ϕ bars

$$\frac{1000}{A_{st}} \times \frac{\pi}{4} \times 10^2 = \frac{78539.82}{A_{st}}$$

x (+) $S_1 \rightarrow 228 \text{ mm} \rightarrow 10 \text{ mm } \phi @ 228 \text{ mm c/c}$

x (-) $S_2 \rightarrow 169 \text{ mm} \rightarrow 10 \text{ mm } \phi @ 160 \text{ mm c/c}$

y (+) $S_3 \rightarrow \begin{cases} 10 \text{ mm } \phi - 332 \\ 8 \text{ mm } \phi - 213 \end{cases} \rightarrow 8 \text{ mm } \phi @ 210 \text{ mm c/c}$

y (-) $S_4 \rightarrow \begin{cases} 10 \text{ mm } \phi - 244 \\ 8 \text{ mm } \phi - 156 \end{cases} \rightarrow 8 \text{ mm } \phi @ 150 \text{ mm c/c}$

Torsion reinforcement

At corner (1)

$$\text{Size of Mesh} = \frac{b}{5} = \frac{4620}{5}$$

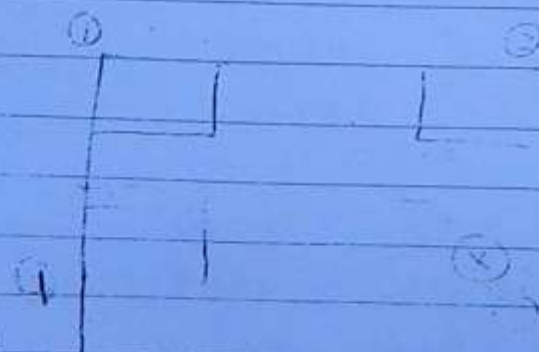
$$= 924 \text{ mm}$$

$$\text{Area of Steel} = \frac{3}{4} \times A_{st1} (+ve)$$

$$= \frac{3}{4} \times 344 = 258 \text{ mm}^2$$

spacing (10 mm ϕ)

$$S = \frac{1000}{258} \times \frac{\pi}{4} \times 10^2 = 300 \text{ mm}$$



8mm — Spacing = 194mm

Torsion reinfⁿ at Corner 1

$M_x(+)$ — Provide 10mm ϕ @ 220 c/c

$M_x(-)$ — 10mm ϕ @ 160 c/c

$M_y(+)$ — 8mm ϕ @ 105 c/c

$M_y(-)$ — 8mm ϕ @ 150 mm c/c

Corner 2/3

10mm ϕ @ 320 mm c/c

10mm ϕ @ 320 mm c/c

8mm ϕ @ 210 mm c/c

8mm ϕ @ 300 mm c/c

At Corner ②/③

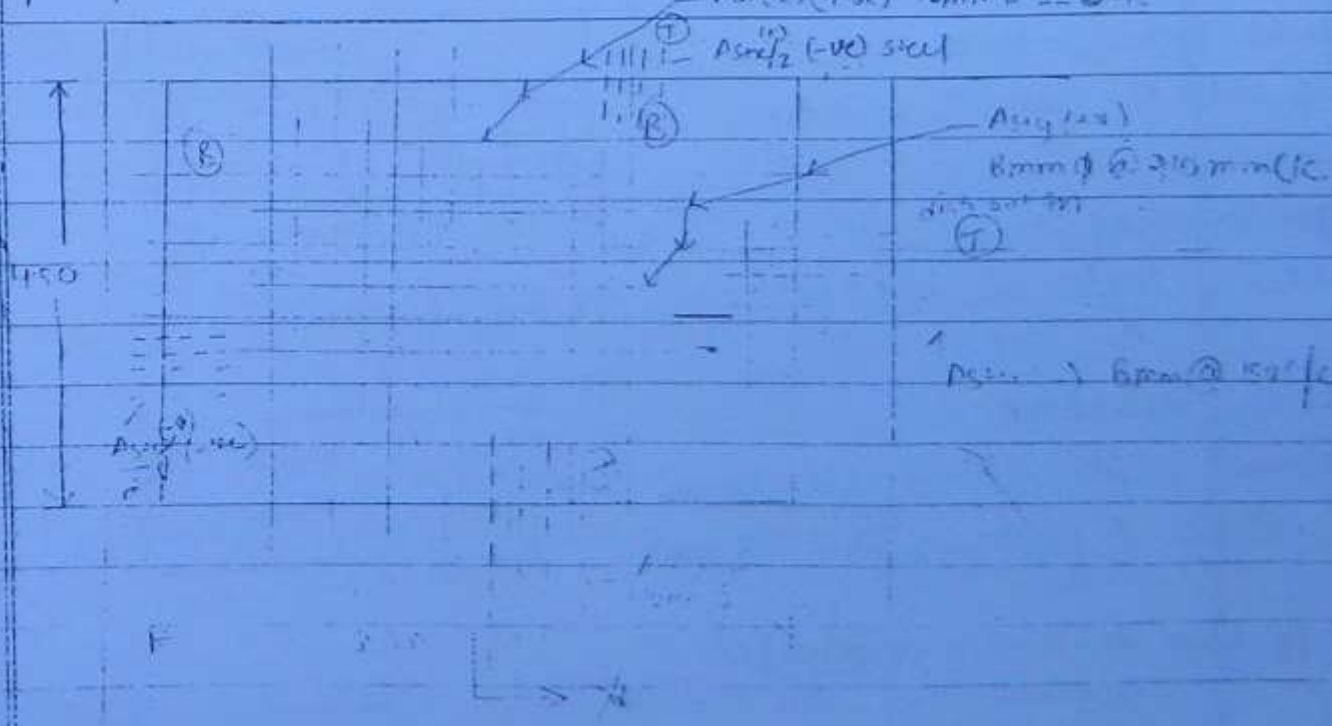
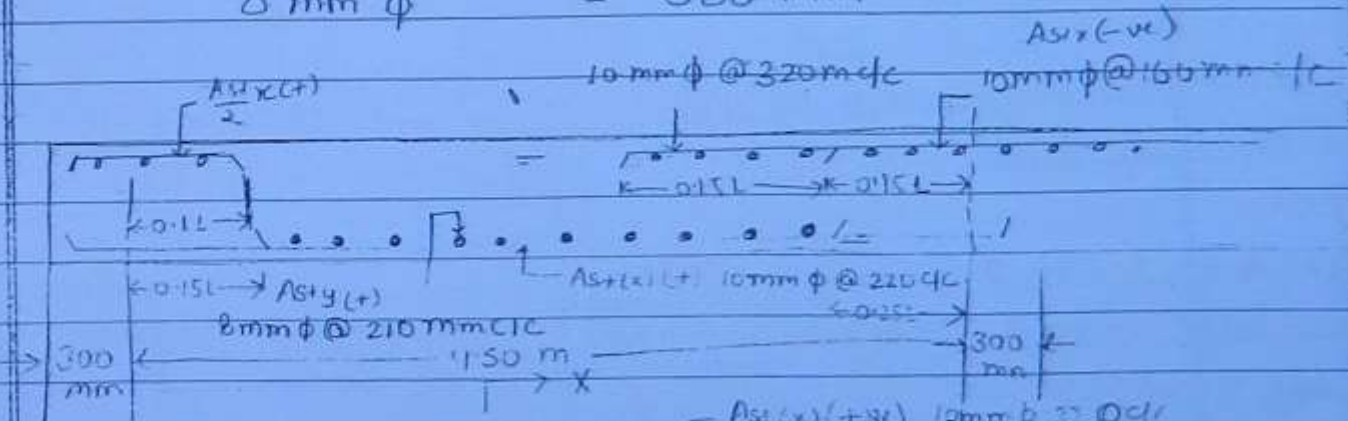
(143)

Size of Mesh = $l_x/5 = 924 \text{ mm}$

Area of Steel = $\frac{3}{8} \times A_{x+y} (+ve) = \frac{3}{8} \times 344 = 129 \text{ mm}^2$

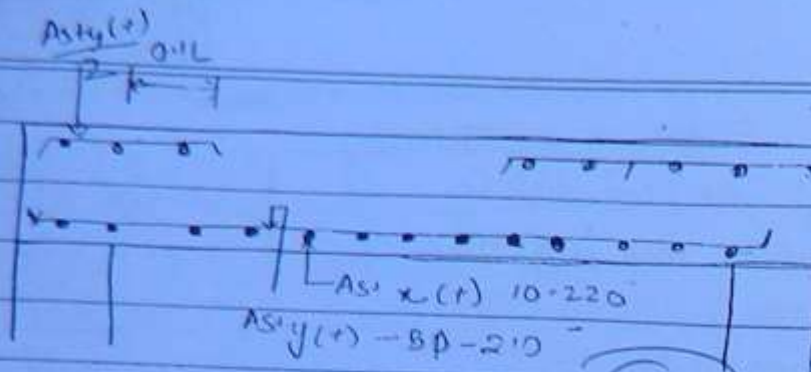
Spacing 10mm ϕ = 600 mm

8mm ϕ = 388 mm



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A rectangular slab 2m x 3m is simply supported along shorter edge but that clear dist bet support is 2.7m. The slab is 15 cm thick. Use 16mm ϕ @ 25mm/c along longer edge & 10mm ϕ @ 25mm/c along shorter edge. Use M-15 / M15 / M15 / M15 is used. Permissible steel Bending - 10, Stress - 3.0 N/mm² in concrete. Calculate max load carrying capacity of slab.

Let us consider 1m width of slab

It is one way slab

Span

$$l_{eff} = l_o + d = 2.7 + 0.15 = 2.825 \text{ m}$$

$$l_o + d = 2.7 + 0.15 = 2.85 \text{ m}$$

$$l_{eff} = 2.825$$

Consider w kN/m is total load (i.e. self wt) the beam can carry.

Check for bending :-

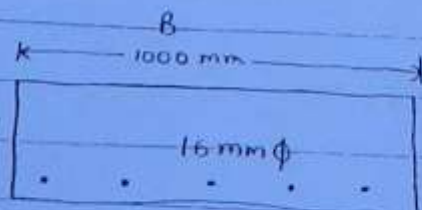
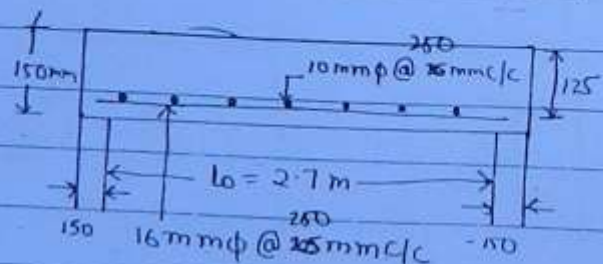
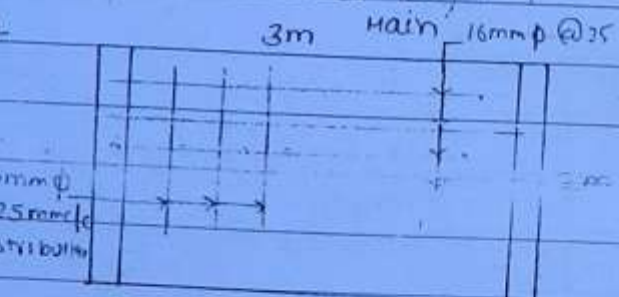
$$B.M = \frac{wl^2}{8} = \frac{w \times 2.825^2}{8} = 9976.48 \text{ KN-m}$$

Requirement of Resistance

$$b = 1000 \text{ mm}; d = 125 \text{ mm}; \bar{x} = 150 \text{ mm}$$

$$A_{st} = \frac{1000}{250} \times \frac{\pi}{4} (16)^2 = 804.25 \text{ mm}^2$$

4 Nos of bars of 16mm ϕ is provided.



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$$x_c = \frac{m_c}{t + m_c} \times d = \frac{19 \times 5}{140 + 19 \times 5} = 50.53 \text{ mm}$$

$$B \cdot \frac{x_a^2}{2} = m \cdot A_{st}(d - x_a) \Rightarrow \frac{1000 \cdot x_a^2}{2} = 19 \times 804.25(125 - x_a)$$

$$500x_a^2 = 1910093.75 - 1910093.75x_a$$

$$500x_a^2 + 1910093.75 - 1910093.75 = 0$$

$$x_a = 48.38 \text{ mm}$$

(145)

$x_a < x_c$ So it is under reinforced beam

$\sigma_c < \sigma_{cbc}$ & $\sigma_{st} = t_a$

$$M_R = t_a \cdot A_{st} \left(d - \frac{x_a}{3} \right) = 140 \times 804.25 \left(125 - \frac{48.38}{3} \right) \times \frac{1}{10^6}$$

$$= 12.26 \text{ kN-m}$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \Rightarrow C_a = \frac{x_a \cdot t_a}{(d - x_a)m} = \frac{48.38 \times 140}{(125 - 48.38)19} = 4.653 \text{ N/mm}^2$$

$$M_R = B \cdot \frac{x_a}{2} \cdot \frac{C_a}{3} \left(d - \frac{x_a}{3} \right) = 1000 \times \frac{48.38}{2} \times \frac{4.653}{3} \left(125 - \frac{48.38}{3} \right) \times \frac{1}{10^6}$$

$$= 12.26$$

Equating B.M = M.R

$$0.9976 \omega = M_R 12.26$$

$$\omega = 12.29 \text{ kN/m}$$

Check for Shear

$$V = \frac{\omega l}{2} = \frac{\omega \times 2.825}{2} = 1.4125 \omega$$

$$\tau_v = \frac{V}{Bd} = \frac{1.4125 \times 10^3 \omega}{1000 \times 125} = 0.0113 \omega$$

$$\tau_v = \tau_c \Rightarrow 0.0113 \omega = 0.30$$

$$\omega = 26.55 \text{ kN/m}$$

Check for Bond

$$\tau_{bd} (\text{developed}) = \frac{V}{\sum o_j d} = \frac{1.4125 \omega \times 10^3}{4 \times 16 \times \pi \times 865 \times 125}$$

$$= 0.06497 \omega$$

$$\tau_{bd} \text{ permissible} = 0.6 \text{ N/mm}^2 \Rightarrow \tau_{bd} \text{ developed}$$

$$\tau_{bd}(\text{developed}) = \tau_{bd}(\text{permissible})$$

$$0.06497w = 0.6$$

$$w = 9.235 \text{ kN/m}$$

$$\text{Self wt} = 0.15 \times 1 \times 1 \times 25 = 3.75 \text{ kN/m}$$

$$\text{Live load that can be provided} = 5.485 \text{ kN/m}$$

Over the slab.

(146)

Ques Design a slab simply supported over a span of $3.5 \text{ m} \times 5 \text{ m}$.
Clear span the slab is loaded with live load of 4 kN/m^2 .
Floor finishes 50mm thick c-c flooring. Slab is prevented from lifting at corner & edges. Use any method. Use M-25 / Fe 415 W.S.M
Use Marcus Method

Soln $l_x = l_{\text{clear}} + d = 3.5 + 0.10 = 3.6$

$$l_{x0} + l_1 = 3.5 + 0.25 = 3.75$$

Assume depth

$$\text{Overall depth} = \frac{\text{Span}}{0.8 \times 35}$$

$$= \frac{3500}{0.8 \times 35} = 1250 \text{ mm}$$

$$D = 125 \text{ mm}$$

$$d = 100 \text{ mm} = 0.10 \text{ m}$$

$$\text{Assume } l_x = 3.6 \text{ m}$$

$$l_y = 5 + 0.1 = 5.1 \text{ m}$$

$$l_y = 1.416 < 2 \text{ two way}$$

l_x

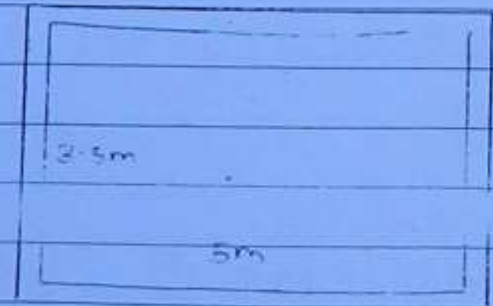
load

$$\text{Self wt} = 0.125 \times 1 \times 1 \times 25 = 3.125$$

$$\text{Live load} = 4 \times 1 \times 1 = 4.00 \text{ kN/m}$$

$$\text{Floor finish} = 0.05 \times 1 \times 1 \times 24 = 1.2$$

$$8.325 \text{ kN/m}$$



B.M =

$$M_x = \left(1 - \frac{5}{6} \frac{y^2}{1+y^4}\right) \left(w \cdot \frac{lx^2}{8}\right) \left(\frac{y^4}{1+y^4}\right)$$

$$= \left(1 - \frac{5}{6} \frac{1.42^2}{1+1.42^4}\right) \left(\frac{1.42^4}{1+1.42^4}\right) \left(\frac{8.325 \times 3.6^2}{8}\right)$$

$$= 7.23 \text{ kN-m}$$

(147)

$$M_y = \left(1 - \frac{5}{6} \frac{y^2}{1+y^4}\right) \left(\frac{1}{1+y^4}\right) \left(w \cdot \frac{ly^2}{8}\right)$$

$$= \left(1 - \frac{5}{6} \frac{1.42^2}{1+1.42^4}\right) \left(\frac{1}{1+1.42^4}\right) \left(\frac{8.325 \times 5.1^2}{8}\right)$$

$$= 3.57 \text{ kN-m}$$

Depth of slab $d = \sqrt{\frac{M}{Q \cdot B}}$

$$d = \sqrt{\frac{7.23 \times 10^6}{1.11 \times 1000}} = 80.70 \text{ mm}$$

< 100 mm (Safe)
OK

$$K = \frac{m_c}{t + m_c} = \frac{11 \times 8.5}{230 + 11 \times 8.5} = 0.289$$

$$j = 0.904$$

$$Q = \frac{1}{2} c \cdot j \cdot K = 1.11$$

Let us use $d = 100 \text{ mm}$
 $D = 125 \text{ mm}$

$$\text{Area of steel } A_{s(x)} = \frac{M}{\sigma_s \cdot j \cdot d} = \frac{7.23 \times 10^6}{230 \times 0.904 \times 100} = 348 \text{ mm}^2$$

$$A_{s(y)} = \frac{3.57 \times 10^6}{230 \times 0.904 \times 100} = 172 \text{ mm}^2$$

Use 10 mm ϕ bars

$$\text{Spacing } (x) = \frac{1000}{348} \times \frac{\pi}{4} \times 10^2 = 225 \text{ mm}$$

$$8 \text{ mm } \phi \text{ Spacing} = \frac{1000}{172} \times \frac{\pi}{4} \times 8^2 = 144 \text{ mm}$$

Use 8 mm ϕ @ 140 mm/c

$$\text{Spacing } (y) = \frac{1000}{172} \times \frac{\pi}{4} \times 8^2 = 292 \text{ mm}$$

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12% of BD

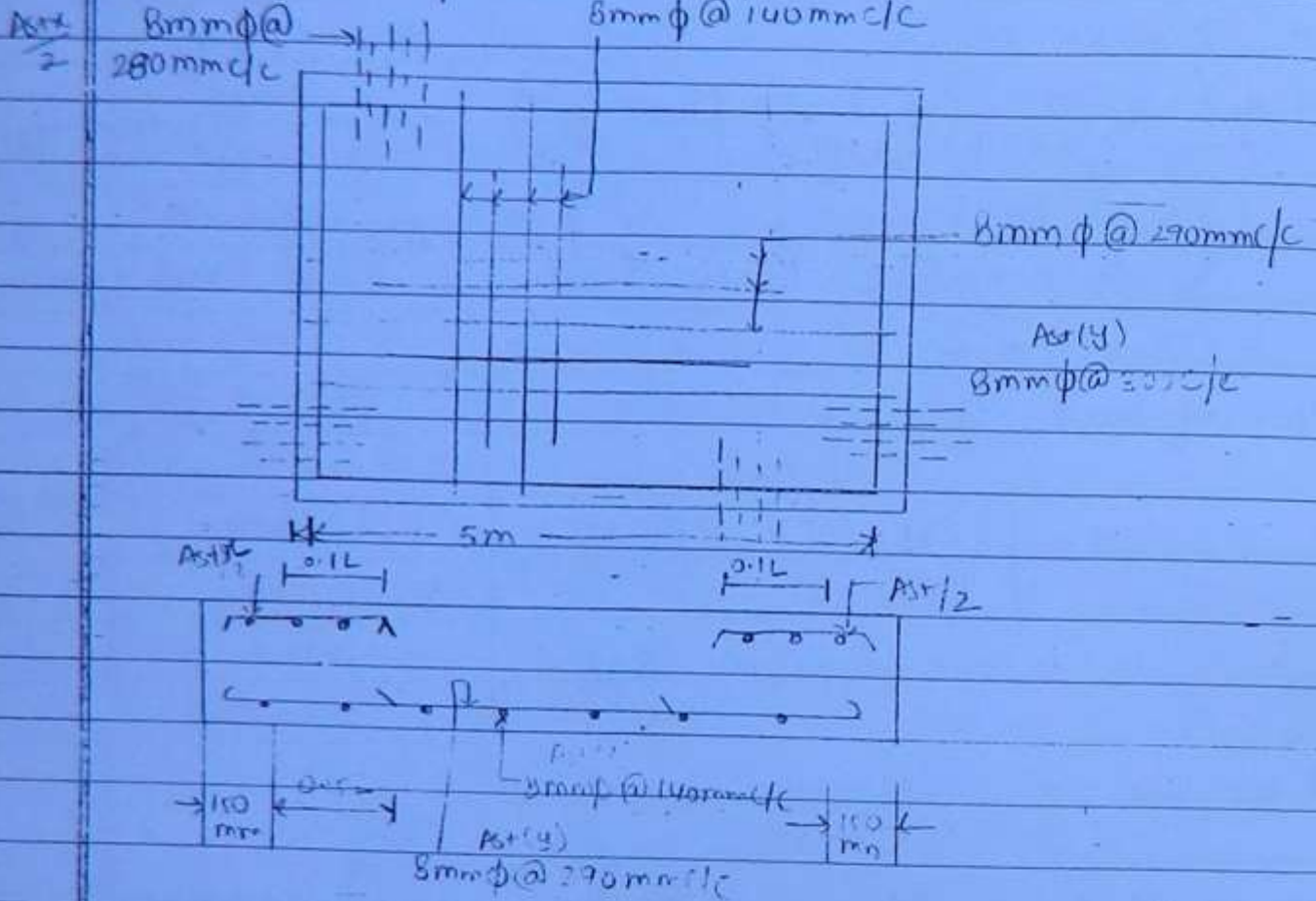
$$\text{Min Steel} = \frac{12}{100} \times 1000 \times 125$$

$$= 150 \text{ mm}^2 \text{ OK}$$

Max Spacing = 300mm

provide 8mm ϕ @ 290mm/c

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COLUMNS

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Working Stress Method

→ long & short Column

$$\text{Slenderness Ratio} = \frac{e_{bb} \cdot \text{length}}{\text{least lateral dimension}}$$

$$\frac{e_{bb}}{B} < 12 \quad (\text{Short Column})$$

$$\frac{e_{bb}}{B} > 12 \quad (\text{long Column})$$

(149)

Load bearing Capacity of a Column

$$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc} \quad \text{--- Short Column}$$

$$A_c = (A - A_{sc})$$

For long Column

Load Carrying Capacity

$$P = C_r [A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}]$$

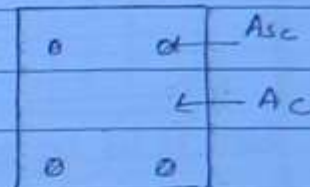
Where

$$C_r = \left(1.25 - \frac{e_{bb}}{48B} \right) \quad \text{Reduction Coefficient}$$

B = least lateral dimension

$$C_r = \left(1.25 - \frac{e_{bb}}{160 i_{min}} \right)$$

$$i_{min} = \text{least radius of gyration} = \sqrt{\frac{I}{A}}$$

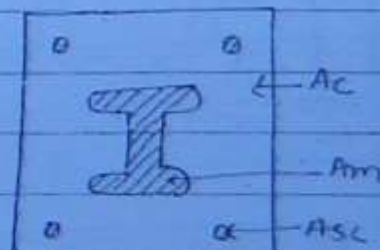


For a Composite Column

$$A_c = (A - A_{sc} - A_m)$$

Load Carrying Capacity

$$P = C_r [A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc} + A_m \cdot \sigma_{mc}]$$



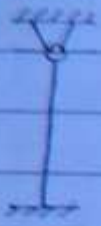
Effective length (TABLE 2B)

Fixed - effectively held in position & restrained against Rotation

$$\text{Theoretical} = 0.50L - \frac{L}{2}$$

$$\text{Recommended} = 0.65L$$

- 2 Effectively held in position but not restrained against Rotation (One end hinge & other end is fixed)



Theoretical - $0.70L$ $\frac{4}{\sqrt{2}}$

Recommended - $0.80L$

(150)

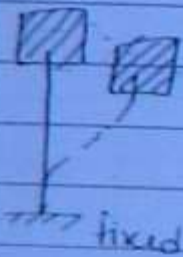
- 3 Held in position but not restrained against Rotation (Both ends are hinge)



Theoretical - $1.0L$

Recommended = $1.0L$

- 4 Not held in position but restrained against Rotation



Theoretical $1.0L$

Recommended $1.2L$

- 5 Not held in position & partially restrained against Rotation



Theoretical - —

Recommended - $1.50L$

6



Theoretical - $2.0L$

Recommended - $2.0L$

7



Theoretical 2-0L
Recommended 2-0L

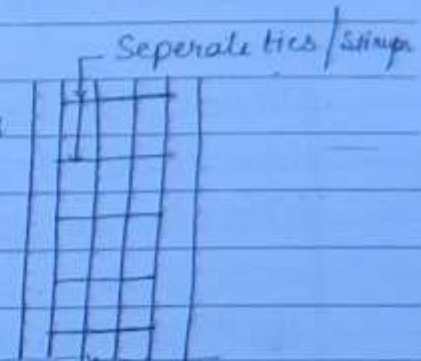
(157)

Design Of Circular Columns

(a) (With Seperate Stirrups)

Design Same as for Sample R.C.C Column

$$P = G (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$$



(b) With helical Reinforcement (Tie)

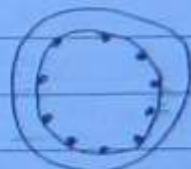
Load Carrying capacity

$$P = 1.05 G (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$$

For helical Reinforcement following Criteria should be satisfied

page no 71

$$0.36 \frac{f_{ck}}{f_y} \left[\frac{A_g}{A_c} - 1 \right] \leq \frac{V_h}{V_c} \quad A$$



Where

$$A_g = \text{Gross area} = \frac{\pi}{4} (D_g)^2$$

D_g = Gross diameter

A_c = Core area

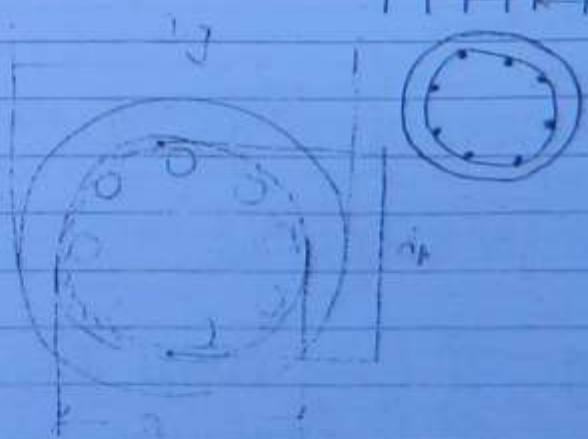
$$D_c = D_g - 2 \times \text{clear cover}$$

$$A_c = \frac{\pi}{4} D_c^2$$

V_c = Volume of core area for 1m length of column

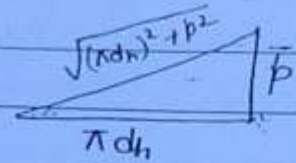
$$\frac{\pi}{4} (D_c)^2 \times 1000$$

V_h = Volume of helical reinfⁿ in 1 unit length of column
= (No. of turns) $\times$ length in 1 turn $\times$ c/s area of helix



$$V_h = \left(\frac{1000}{p} \right) \times (\pi d_h) \times \left(\frac{\pi}{4} \times \phi_h^2 \right)$$

Where $d_h = d_c - \phi_h$
Length in one turn
 $= \sqrt{(\pi d_h)^2 + p^2}$



ϕ_h = diameter of helix.

(152)

Other T.S Recommendation

Min percentage of steel = 0.8% of total area

Max percentage of steel = 6% of total c/s area
(If bars are not lapped)

If bars are lapped = 4.0% of total area

Minimum no. of bar - 4 - Rectangular
6 - Circular

Min dia of bars - 12mm dia of bars

Max dis betm main bars (ie longitudinal)
= 300mm

Design of transverse reinforcement (Tie)

Diameter

(i) $\frac{\phi_{main}}{4}$ } whichever is greater

(ii) 6mm

ϕ_{main} for larger dia

Spacing - ① least lateral dimension
② $16 \phi_{main}$ (min dia)
③ 300mm

Pitch - Pitch of helical reinforcement

① $\nless 75$ mm

② $\nless d_c/6$

③ $\nless 3\phi$

Design a Column to Carry a load of 1000 kN Column is 12m in length and restrained at both end use - M20 & Fe-415 Design a Square Column.

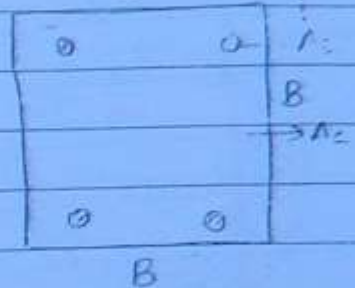
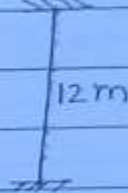
$l = 12\text{m}$

Effective length of Column

$l_{eff} = 0.65l$

$= 0.65 \times 12 = 7.80\text{m}$

(153)



Assume steel 1% of Total C/s area

$A_{sc} = 0.01A$

$A_c = A - A_{sc} = B^2 - 0.01B^2 = 0.99B^2$

Design of Column

Assuming Short Column

$P = A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}$

$1000 \times 10^3 = 190 \cdot 0.01B^2 + 0.99B^2 \times 5$

$B = 382\text{mm}$

Slenderness Ratio

$\frac{l_{eff}}{B} = \frac{7800}{382} = 20.4 > 12$

It is a long Column

For long Column

$P = C_m (A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc})$

$1000 \times 10^3 = \left(1.25 - \frac{l_{eff}}{48B}\right) (0.01B^2 \times 190 + 0.99B^2 \times 5)$

$1000 \times 10^3 = \left(1.25 - \frac{7800}{18336}\right) (0.01B^2 \times 190 + 0.99B^2 \times 5)$

$= 6.85 (1.25B^2 - 162.5B)$

$1.25B^2 - 162.5B - 145985.4 = 0$

$B = 412\text{mm}$

Use $B = 420\text{mm}$

$A_{st} = 0.01 \times 412^2 \rightarrow 0.01 \times 420^2 = 1764\text{mm}^2$

No. of 16mm dia bars = 8.7

Use 4-20mm ϕ = 1256.64

Grade σ_{cbc} σ_{cc}

M-15 5 - 4

M-20 7 - 5

M-25 8.5 - 6

M-30 10 - 8

M-35 11.5 - 9

M-40 13 - 10

Steel σ_{sc} σ_{sc}

Fe-250 130 140

Fe-415 190 230

$$4-12\phi = 452.37$$

$$\text{total} = 1709.03 \text{ mm}^2$$

Stirrups

① Diameter

$$\frac{\phi_{\text{main}}}{4} = \frac{20}{4} = 5 \text{ mm} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{use } 6 \text{ mm}$$

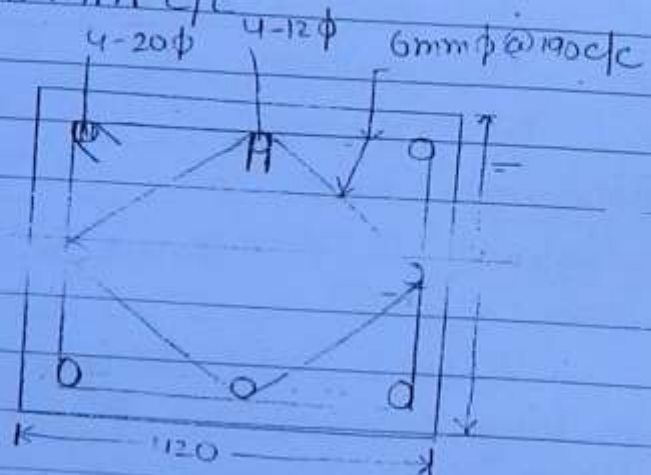
② 6 mm

Spacing

① least lateral dimension = 420 mm

② $16 \times \phi_{\text{main}} (\text{min dia}) = 16 \times 12 = 192 \text{ mm}$

③ 300 mm

provide 6 mm ϕ bars @ 190 mm c/c

Design a short circular column of 600 mm diameter for a load of ²⁵⁰⁰2500 kN using helical reinforcement use M-25 & Fe-415. Calculate main bars, size & pitch of helical ties.

① Design a short column — $D = 600 \text{ mm}$

$$P = (A_{sc} \cdot \sigma_{sc} + A_c \cdot \sigma_{cc}) 1.05$$

$$2500 \times 10^3 = 1.05 (A_{sc} \cdot 190 + \left(\frac{\pi}{4} \times 600^2 - A_{sc} \right) 6)$$

$$2380952.381 = 190 A_{sc} + 1696460.033 - 6 A_{sc}$$

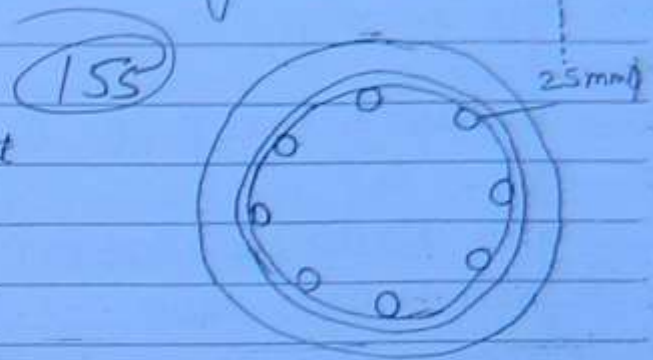
$$684492.348 = 184 A_{sc}$$

$$A_{sc} = 3720 \text{ mm}^2$$

DATE: _____

No. of 25 mm ϕ bars required

$$n = \frac{3720}{\frac{\pi}{4} \times 25^2} = 7.57 \text{ say } 8 \text{ Nos.}$$



Design of helical reinforcement

$$0.36 \frac{f_{ck}}{f_y} \left(\frac{A_g}{A_c} - 1 \right) \leq \frac{V_h}{V_c}$$

Where $D_g = 600 \text{ mm}$

$$A_g = \frac{\pi}{4} (600)^2 = 282743.33 \text{ mm}^2$$

Core diameter $D_c = D_g - 2 \times \text{Clear Cover}$

$$600 - 2 \times 40 = 520 \text{ mm}$$

$$A_c = \frac{\pi}{4} D_c^2 = \frac{\pi}{4} \times 520^2 = 212371.66 \text{ mm}^2$$

V_c for 1 m length

$$= A_c \times 1000$$

$$= 212371663.4 \text{ mm}^3$$

V_h = Volume of helical reinf for 1 m length of column

Assuming dia of helical reinf ϕ_h

$$d_h = d_c - 2\phi_h = 520 - 8 = 512 \text{ mm}$$

$$V_h = \left(\frac{1000}{P} \right) \left(\frac{\pi \times d_h}{4} \right) \left(\frac{\pi \times \phi_h^2}{4} \times 8 \right)$$

$$V_h = 10 \frac{80851799.25}{P}$$

$$0.36 \times \frac{25}{415} (0.331) \leq \frac{80851799.25}{P \times 212371663.4}$$

$$7.186 \times 10^{-3} = 0.3807$$

$$P = \frac{52.98 \text{ mm}}{P}$$

DATE: _____

$$p \neq 75 \text{ mm OK}$$

$$p \neq \frac{d_e}{6} = \frac{520}{6} = 86.67 \text{ mm}$$

$$p \neq 3\phi_h = 3 \times 8 = 24 \text{ mm}$$

provide 8mm ϕ @ 50 mm c/c 156

IS Recommendation

① Slenderness ratio (Page NO-42)

(a) Unsupported length between end restraint $\neq 60$ times of least lateral dimension

Max. Slenderness ratio limit = 60

(b) If one end of column is unrestrained its unsupported length $l \neq \frac{100B^2}{D}$

② Minimum eccentricity

All columns shall be designed for a minimum eccentricity of = $\frac{\text{unsupported length of column}}{500} + \frac{\text{lateral dimension}}{30}$

Subjected to minimum of 20 mm

$$e_{\min} = \frac{l}{500} + \frac{B \text{ or } D}{30} \text{ OR } 20 \text{ mm}$$

Whichever is greater.

→ Combined bending & direct stress

P = Axial compressive load

e_x = eccentricity about Y-Y

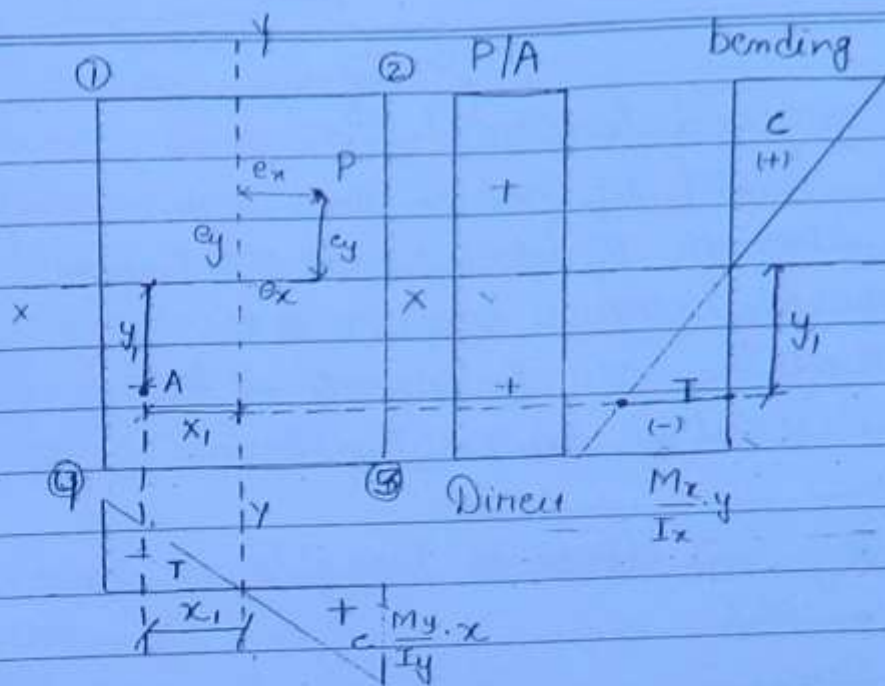
e_y = " " " X-X

M_x = Moment about X-X → $M_x = P \cdot e_y$

M_y → $P \cdot e_x$

Stresses

① Direct stress = $p_o = \frac{P}{A}$



(157)

(ii) Bending stress due to M_x

$$p_{bx} = + \frac{M_x \cdot y}{I_x}$$

(iii) Bending stress due to M_y

$$p_{by} = \pm \frac{M_y \cdot x}{I_y}$$

Final stresses at any point on the column section

$$p_o + p_{bx} + p_{by} = + \frac{P}{A} \pm \frac{M_x \cdot y}{I_x} \pm \frac{M_y \cdot x}{I_y}$$

At A point in fig. (x_1, y_1)

$$\text{final stress} = + \frac{P}{A} - \frac{M_x \cdot y_1}{I_x} - \frac{M_y \cdot x_1}{I_y}$$

Stress at four corners of column

$$\text{Corner 1} = \frac{P}{A} + \frac{M_x}{I_x} \left(\frac{D}{2} \right) - \frac{M_y}{I_y} \left(\frac{B}{2} \right)$$

$$2 = \frac{P}{A} + \frac{M_x}{I_x} \left(\frac{D}{2} \right) + \frac{M_y}{I_y} \left(\frac{B}{2} \right)$$

$$3 = \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2} \right) + \frac{M_y}{I_y} \left(\frac{B}{2} \right)$$

$$4 = \frac{P}{A} - \frac{M_x}{I_x} \left(\frac{D}{2} \right) - \frac{M_y}{I_y} \left(\frac{B}{2} \right)$$

Design Criteria

1. Design based on uncracked section (158)

→ If tensile stresses develop in concrete are within permissible tensile strength of concrete, Area in tension need not to be ignored. Design is based on total c/s area, without ignoring the area of concrete in tension. This design principle is called design-based on uncracked section.

In this case following eqⁿ should be satisfied.

$$\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{cbc, cal}}{\sigma_{cbc}} \leq 1$$

if satisfied, column is safe as uncracked section.

here $\sigma_{cc, cal}$ = Direct stress developed = $p_o = \frac{P}{A}$

$\sigma_{cbc, cal}$ = Max stress developed in concrete in bending compression at point no 2.

$$= \frac{M_x (D)}{I_x} + \frac{M_y (B)}{I_y}$$

6, 8, 9, 10 σ_{cc} - Max. permissible stress in direct compression
 8, 5, 10, 11, 13 σ_{cbc} - Max. permissible stress in bending compression

Design based on Cracked Section

If the Criteria $\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{cbc, cal}}{\sigma_{cbc}} \leq 1$ is not satisfied

the Column section should be checked as cracked secⁿ

- ① Ignoring the area of concrete in tension
- ② Tensile stresses are taken by steel alone.

Actual stresses developed in the section are calculated. If the stresses are less than permissible limits Column is considered safe.

DATE: _____

Different Cases :-

- ① Small eccentricity case ($e < D/4$) (159)
Column is mostly designed as uncracked secⁿ
- ② Medium eccentricity case ($e > D/4$) but ($e < 1.5D$)
(Complicated case) but design acc. to cracked secⁿ
- ③ Long eccentricity case ($e > 1.5D$)
Column is design as cracked secⁿ due to high moment

→ Small eccentricity (Design based on uncracked secⁿ)

→ Uniaxial bending
(A symmetrical secⁿ)

Load = P

Eccentricity abt X-X = e

Moment abt X-X = $M_x = P \cdot e$

If Column is safe as uncracked secⁿ

$$\frac{\sigma_{cc \text{ cal}}}{\sigma_{cc}} + \frac{\sigma_{cbc \text{ cal}}}{\sigma_{cbc}} \leq 1$$

$$\sigma_{cc \text{ cal}} = \frac{P}{A_{eq}} =$$

Where A_{eq} = Equivalent Area of Column secⁿ

$$= A_c + (1.5m - 1) A_{sc}$$

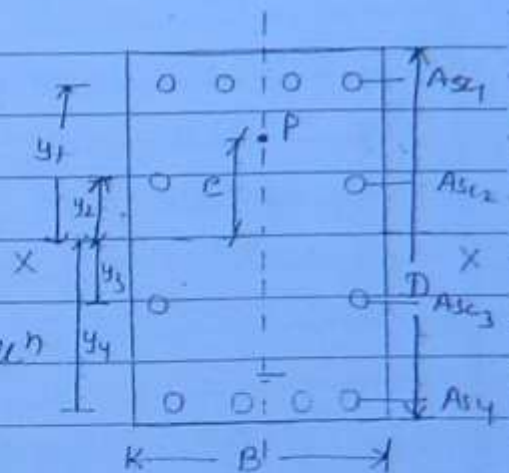
$$A_{eq} = BD + (1.5m - 1) (A_{sc1} + A_{sc2} + A_{sc3} + A_{sc4})$$

$$\sigma_{cbc \text{ cal}} = \frac{M_x \left(\frac{D}{2} \right)}{I_{eq}} \quad \text{(at top stress)}$$

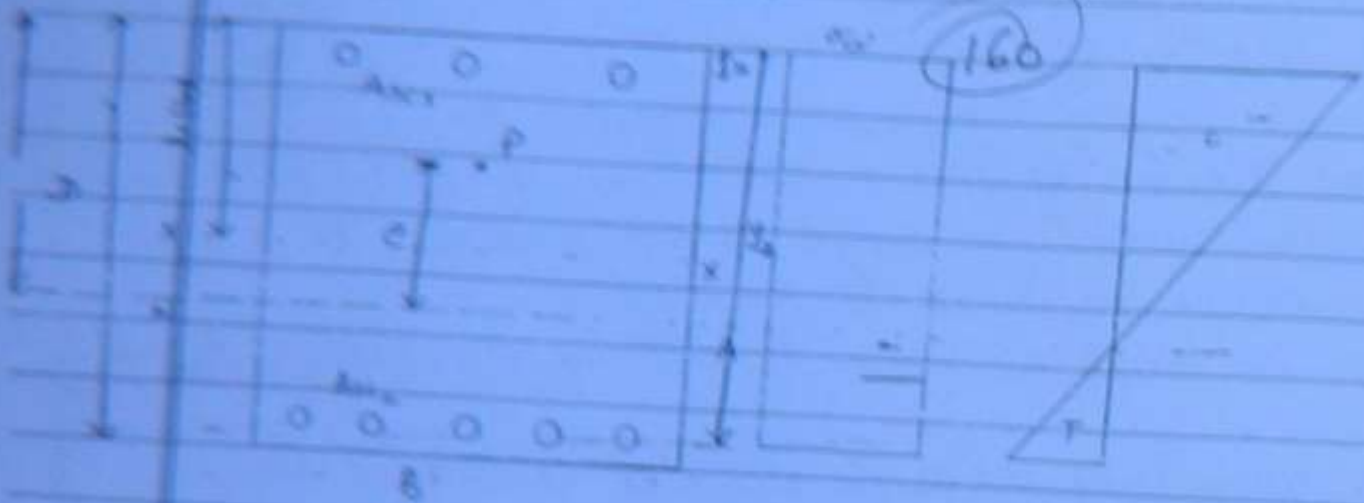
Where $I_{eq} = \frac{BD^3}{12} + (1.5m - 1) \left[A_{sc1} x_1^2 + A_{sc2} x_2^2 + A_{sc3} x_3^2 + A_{sc4} x_4^2 \right]$
(Concrete)

$$I_{eq} = \frac{BD^3}{12} + (1.5m - 1) \left[\sum A_{sc} \cdot y^2 \right]$$

put these values in eqⁿ. if eqⁿ get satisfied then the column section is safe otherwise failed.



For unsymmetrical section



If a load P is acting at an eccentricity e from $x-x$ axis

$$A_{eq} = B \cdot D + (1.5m-1) (A_{sc1} + A_{sc2})$$

Cent of Section (N.A) from Top

$$\bar{y} = \frac{B \cdot D \cdot \frac{D}{2} + (1.5m-1) (A_{sc1} \cdot y_1 + A_{sc2} \cdot y_2)}{A_{eq}}$$

$$A_{eq} =$$

$$= \frac{B \cdot D^2}{2} + (1.5m-1) (A_{sc1} \cdot y_1 + A_{sc2} \cdot y_2)$$

$$B \cdot D + (1.5m-1) (A_{sc1} + A_{sc2})$$

Net eccentricity $e_{net} = e + (\bar{y} - \frac{D}{2})$

Moment $= P \cdot e_{net} = M$

Equivalent moment of Inertia

$$I_{eq} = \frac{B \cdot D^3}{12} + B \cdot D \left(\bar{y} - \frac{D}{2} \right)^2 + (1.5m-1) [A_{sc1} (\bar{y} - y_1)^2 + A_{sc2} (\bar{y} + y_2)^2]$$

$$\sigma_{c,cal} = \frac{P}{A_{eq}}$$

$$\sigma_{c,cal} = \frac{M}{I_{eq}} \bar{y} = \frac{P \cdot e_{net}}{I_{eq}} \times \bar{y}$$

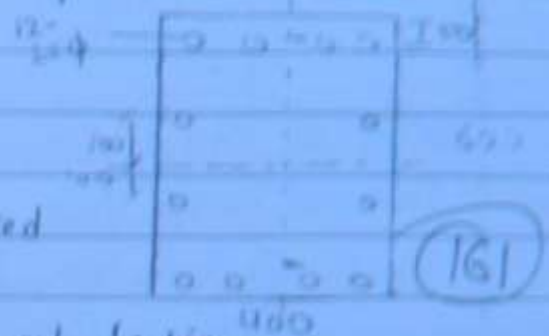
$$\text{Check } \frac{\sigma_{c,cal}}{\sigma_{c,lim}} + \frac{\sigma_{m,cal}}{\sigma_{m,lim}} \leq 1$$

Hence safe

DATE: _____

A Column Secⁿ of size 400x600 mm is reinforced with bars of 25 mmφ as shown in fig. Use M25 / Fe-415. Find out

- (i) Max. eccentricity about X-X s.t. a load can be applied so that there is no tension in the section.



- (ii) Max. value of load that can be applied at eccentricity calculated above.

Column should be safe as uncracked section.

Solⁿ Uniaxial bending case

$$A_{eq} = BD + (1.5m-1) [2A_{sc}]$$

$$= 400 \times 600 + (1.5 \times 11 - 1) (12 \times \frac{\pi}{4} \times 25^2)$$

$$= 331362.5 \text{ mm}^2$$

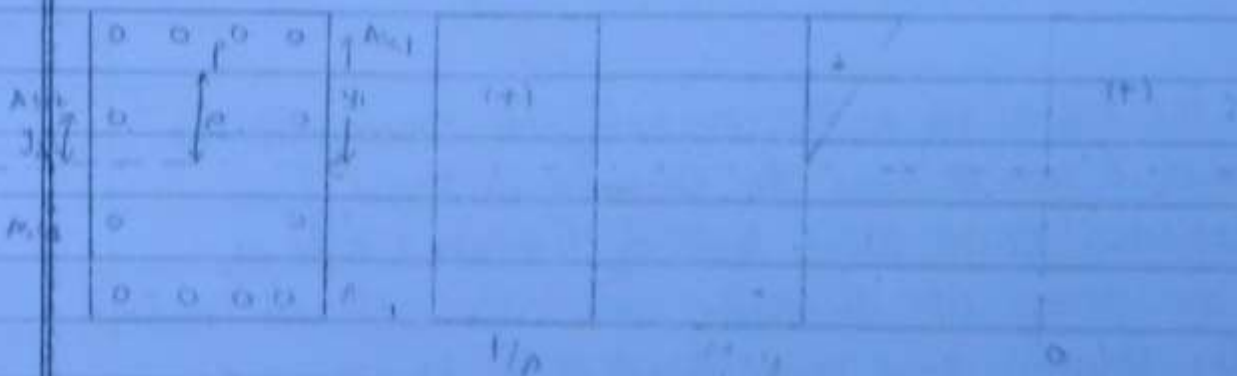
N.A is at X-X

Moment of Inertia

$$I_{eq} = \frac{BD^3}{12} + (1.5m-1) [A_{sc1} y_1^2 + A_{sc2} y_2^2]$$

$$= \frac{400 \times 600^3}{12} + (1.5 \times 11 - 1) \left[2 \times 4 \times \frac{\pi}{4} \times 25^2 \times 250^2 + 2 \times 2 \times \frac{\pi}{4} \times 25^2 \times 100^2 \right]$$

$$= 1.13086 \times 10^{10}$$



If there is no tension, stress at bottom = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} \cdot y = 0$$

$$\frac{P}{A_{eq}} - \frac{P \cdot e}{I_{eq}} \cdot y = 0 \Rightarrow \frac{P}{A_{eq}} = \frac{P \cdot e \cdot y}{I_{eq}}$$

DATE: _____

Max. eccentricity

$$e_{max} = \frac{I_{eq}}{A_{eq} \cdot y} = \frac{1.13086 \times 10^{10}}{331302.5 \times 300} = 113.78 \text{ mm}$$

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value of load.

Criteria req. for uncracked Secⁿ

$$\frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \leq 1$$

$$\sigma_{cc,cal} = \frac{P}{A_{eq}} \quad || \quad \sigma_{cbc,cal} = \frac{M}{I_{eq}} \bar{y} = \frac{P}{A_{eq}}$$

$$\sigma_{cc} = 6$$

$$\sigma_{cbc} = 8.5$$

$$\frac{P/A_{eq}}{6} + \frac{M/I_{eq} \cdot \bar{y}}{8.5} = 1$$

$$\frac{P/A_{eq}}{6} + \frac{P/A_{eq}}{8.5} = 1$$

$$\frac{P}{A_{eq}} \left(\frac{1}{6} + \frac{1}{8.5} \right) = 1$$

$$P = \frac{A_{eq}}{\left(\frac{1}{6} + \frac{1}{8.5} \right)} = \frac{331302.5}{\frac{1}{6} + \frac{1}{8.5}}$$

$$= 1165.271 \text{ kN}$$

Q2

Ques Check the safety of a RCC rectangular column 250x350 mm reinf $\bar{c}$ 4-16 ϕ bars on each of its short face and subjected to an axial load of 450 kN & B.M of 15 kN-m acting abt its major axis.

Can the column any more load if so how much?
M20 conc, $H=19$. As per code the section is safe

$$\text{if } \frac{\sigma_{cc,cal}}{\sigma_{cc}} + \frac{\sigma_{cbc,cal}}{\sigma_{cbc}} \leq 1$$

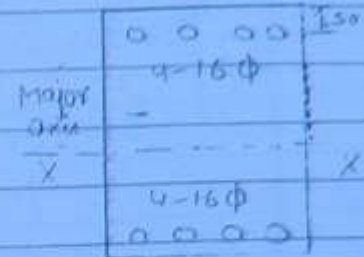
DATE: _____

Sol: $P = 450 \text{ kN}$ & $M = 15 \text{ kN-m}$

$$A_{eq} = BD + (1.5m - 1) \sum A_{sc}$$

$$= 250 \times 350 + (1.5 \times 19 - 1) \times 4 \times \frac{\pi}{4} \times 16^2$$

$$= 131733.6 \text{ mm}^2$$



$$I_{eq} = \frac{BD^3}{12} + (1.5m - 1) \left\{ 2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2 \right\}$$

$$= \frac{250 \times 350^3}{12} + (1.5 \times 19 - 1) \left\{ 2 \times \frac{\pi}{4} \times 4 \times 16^2 \times 125^2 \right\}$$

$$= 1584.38 \times 10^6 \text{ mm}^4$$

$$\sigma_{cc, cal} = \frac{P}{A_{eq}} = \frac{450 \times 10^3}{131733.6} = 3.416 \text{ N/mm}^2$$

$$\sigma_{cbc, cal} = \frac{M \cdot y}{I} = \frac{15 \times 10^6}{1584.38 \times 10^6} \times \left(\frac{350}{2} \right)$$

$$= 1.657 \text{ N/mm}^2$$

$\sigma_{cc} = 5$ $\sigma_{cbc} = 7$

Check $\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{cbc, cal}}{\sigma_{cbc}} = \frac{3.416}{5} + \frac{1.657}{7}$

$$= 0.92 < 1$$

Column is safe

If moment is kept same

$$\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{cbc, cal}}{\sigma_{cbc}} = 1$$

$$P / \frac{131733.6}{5} + \frac{1.657}{7} = 1$$

$$P = 502752 \text{ N} = 502.75 \text{ kN}$$

Extra load = $502.752 - 450$

$$= 52.75 \text{ kN}$$

If eccentricity value is kept same $e = \frac{M}{P} = \frac{15 \times 10^6}{450 \times 10^3}$

$$= 33.33 \text{ mm}$$

$$\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{cbc, cal}}{\sigma_{cbc}} = 1$$

DATE: _____

$$\frac{P}{A_{eq} \cdot \sigma_{cc}} + \frac{P \cdot e \cdot y}{I_{eq} \cdot \sigma_{cbc}} = 1$$

$$P \left[\frac{1}{131733.6 \times 5} + \frac{33.33 \times 350/2}{1584.38 \times 10^6 \times 7} \right] = 1$$

$$P = 489.21 \text{ kN}$$

$$\text{Extra load} = 39.21 \text{ kN}$$

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For a Column Section Shown in fig

(i) Find out max. eccentricity value abt x-x so that no tension has developed in the section

(ii) Find out max. load value that can be applied on column at eccentricity as calculated above. Column should be safe as uncracked beam. Use M-20/Fe-415

Secⁿ properties

$$= BD + (1.5m - 1) \Sigma A_{sc}$$

$$= 300 \times 500 + (1.5 \times 13 - 1) \times [6 \times \pi \times 25^2 + 4 \times \pi \times 20^2]$$

$$= 227734.8 \text{ mm}^2$$

A_{eq}

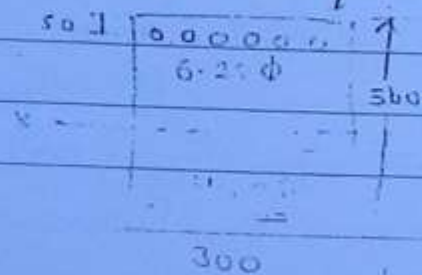
$$= \frac{300 \times 500 \times 500/2 + (1.5 + 13 - 1) [2945.2 \times 50 + 1256.64 \times 450]}{227734.8}$$

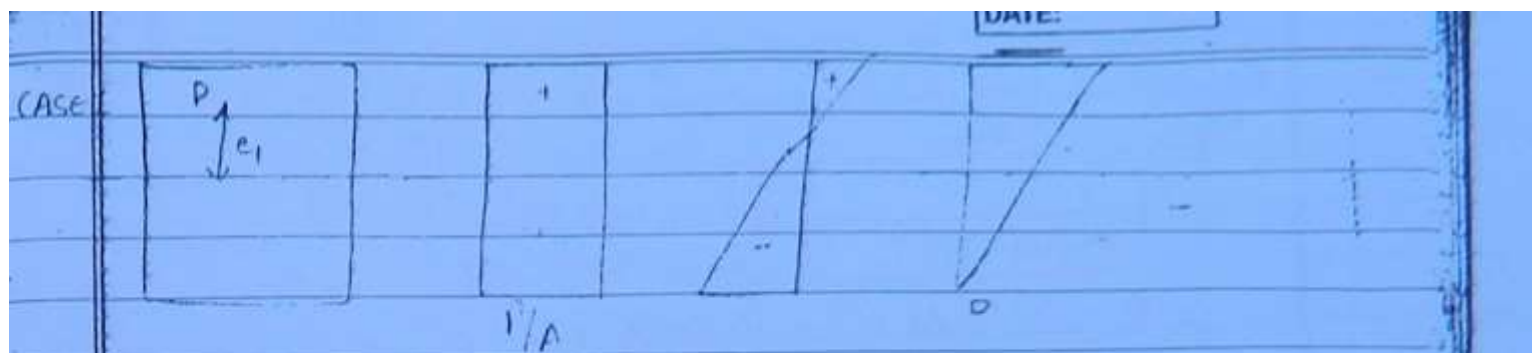
$$= 222.56 \text{ mm}$$

$$I_{eq} = \frac{BD^3}{12} + BD \left(\bar{y} - \frac{D}{2} \right)^2 + (1.5m - 1) [A_{sc1} (\bar{y} - y_1)^2 + A_{sc2} (\bar{y} - y_2)^2]$$

$$= \frac{300 \times 500^3}{12} + 300 \times 500 (222.56 - 250)^2 + (1.5 \times 13 - 1) [2945.2 (222.56 - 50)^2 + 1256.64 (222.56 - 450)^2]$$

$$= 6062.9 \times 10^6 \text{ mm}^4$$





If e is above N.A. Stress at bottom = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_B = 0$$

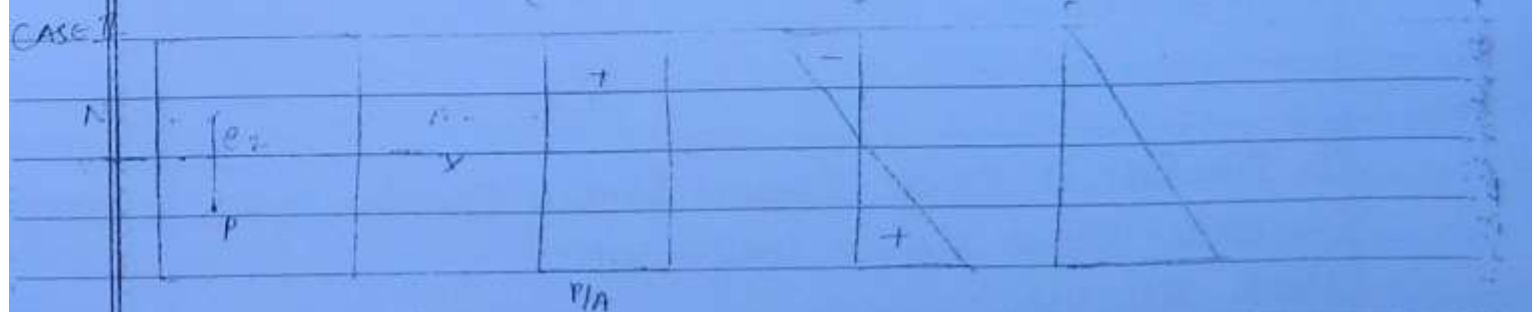
$$\frac{P}{A_{eq}} = \frac{P \cdot e_1}{I_{eq}} y_B \Rightarrow e_1 = \frac{I_{eq}}{A_{eq} \cdot y_B}$$

$$e_1 = \frac{6062.9 \times 10^6}{227734.78 \times (500 - 222.56)} = 95.96 \text{ mm}$$

(165)

Eccentricity about x-x = $e_1 + \left(\frac{D}{2} - \bar{y} \right)$

$$= 95.96 + \left(\frac{500}{2} - 222.56 \right) = 123.4 \text{ mm}$$



If e is below N.A. Stress at top = 0

$$\frac{P}{A_{eq}} - \frac{M}{I_{eq}} y_t = 0$$

$$\frac{P}{A_{eq}} = \frac{P \cdot e_2}{I_{eq}} y_t \Rightarrow e_2 = \frac{I_{eq}}{A_{eq} \cdot y_t}$$

$$e_2 = \frac{6062.9 \times 10^6}{227734.78 \times 222.56} = 119.62 \text{ mm}$$

Eccentricity abt x-x = $119.62 + (250 - 222.56) = 92.10 \text{ mm}$

In first case Eccentricity about x-x is Max.

$$e_{nn} = 123.4 \text{ mm} ; e_{net} = 95.96 \text{ mm}$$

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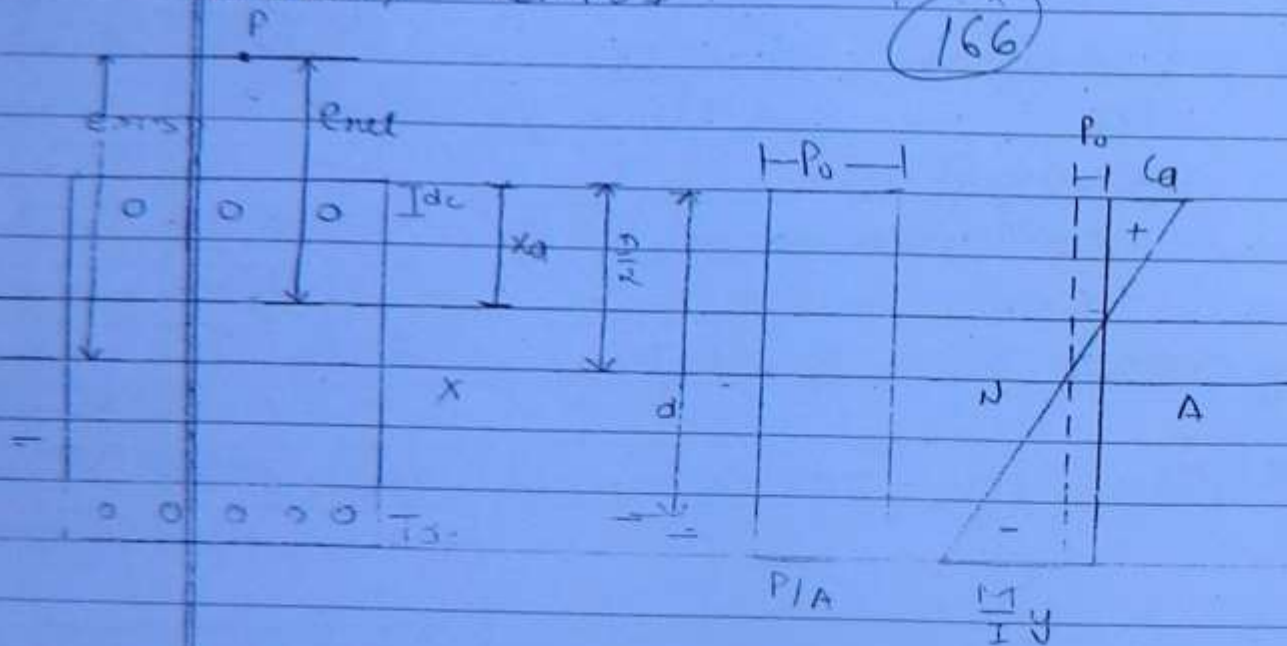
$$\frac{\sigma_{cc, cal}}{\sigma_{cc}} + \frac{\sigma_{tnc, cal}}{\sigma_{tnc}} = 1 \rightarrow \frac{P}{A_{eq} \cdot \sigma_{cc}} + \frac{P \cdot e_{net} \cdot y_t}{I_{eq} \cdot \sigma_{tnc}} = 1$$

$$P \left[\frac{1}{227734.8 \times 5} + \frac{95.56 \times 222.56}{6062.9 \times 10^6 \times 7} \right] = 1$$

$$P = 724985 N = 724.985 kN$$

- LARGE ECCENTRICITY CASE (Design based on Cracked Section) $e > 1.5D$

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In this case a large tensile stresses are developed so area below N.A is neglected.

1. Depth of N.A

Equating moment of area on both side of N.A

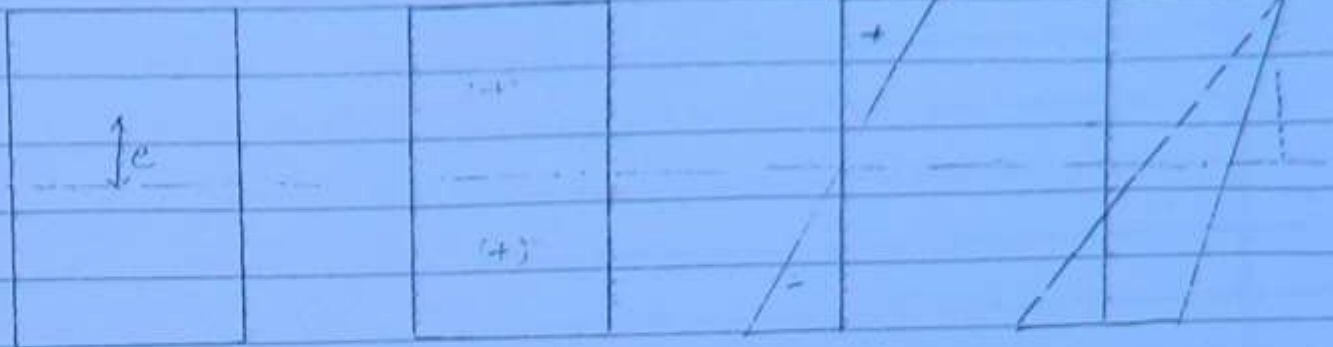
$$B \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d) = m A_{st} (d - x_a) \quad \text{--- ①}$$

2. Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a \right)$$

The Column is subjected to a load P and Moment $P \cdot e_{net}$

DATE:

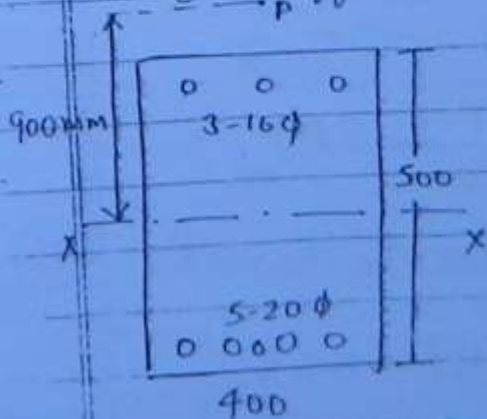


167

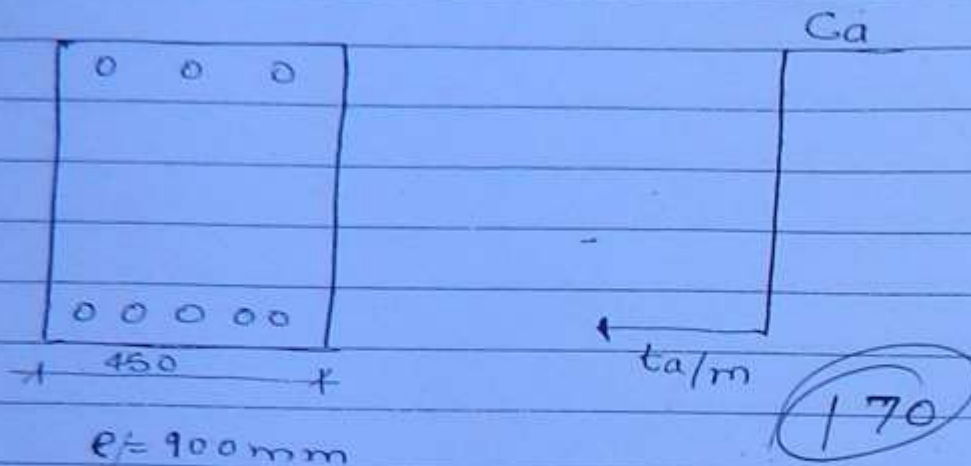
168

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Ques A Column Secⁿ 400x500mm size is subjected to a load of 80kN at an eccentricity of 900mm. Use M-20/Fe-415
 $m=13$. Effective Colⁿ = 50mm



Find out actual stress developed in the section. Check the safety of section. 169



$$1.5D = 1.5 \times 500 = 750 \text{ mm}$$

$$e > 1.5D$$

This is a large eccentricity. Let us check the section as cracked section.

1. Depth of N.A.

$$B \cdot \frac{x_a^2}{2} + (1.5m - 1) A_{sc} (x_a - d_c) = m A_{st} \cdot (d - x_a)$$

$$400 \cdot \frac{x_a^2}{2} + (1.5 \times 13 - 1) \times$$

② Net eccentricity

$$e_{net} = e - \left(\frac{D}{2} - x_a \right)$$

$$= 900 - (250 - 155.5) = 805.5 \text{ mm}$$

$$(> 1.5D)_{OK}$$

(3). Moment

$$B.m. = P \cdot e_{net} = 80 \times \frac{805.5}{1000} = 64.44 \text{ kN-m}$$

④ If actual stress developed in concrete due to moment $P \cdot e_{net} = C_a$

Stress in con in comp ~ steel C'

$$C' = \frac{x_a - d_c}{x_a} \times C_a = \frac{155.5 - 50}{155.5} \times C_a$$

$$= 0.678 C_a$$

(1.71)

⑤ Equating $B.M. = M \cdot R$

$$84.44 \times 10^6 = B \cdot x_a \cdot \frac{C_a}{2} \left(d - \frac{x_a}{3} \right) + (1.5m - 1) A_{sc} \cdot C' (d - d_c)$$

$$= 400 \times 155.55 \times \frac{C_a}{2} \left(450 - \frac{155.5}{3} \right) + (1.5 \times 13 - 1) \times 603.2 \times 0.678 C_a \times (450 - 50)$$

$$C_a = 4.18 \text{ N/mm}^2$$

$$C' = 0.678 \times C_a = 2.84 \text{ N/mm}^2$$

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a} \quad \parallel \quad t_a = \frac{(d - x_a) m C_a}{x_a}$$

$$= \frac{(450 - 155.5) \times 13 \times 4.18}{155.55}$$

$$t_a = 102.91 \text{ N/mm}^2$$

⑥ Direct stresses $\Rightarrow$

$$p_o = \frac{P}{A_{eqv}}$$

$$= \frac{80 \times 10^3}{8 \times 2a + (1.5m - 1) A_{sc} + m A_{st}}$$

$$= \frac{80 \times 10^3}{400 \times 155.5 + (1.5 \times 13 - 1) 603.2 + 13 \times 1570.8}$$

$$= 0.85 \text{ N/mm}^2$$

Final stresses

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(i) In concrete $= p_o + c_a = 0.85 + 4.18$
 $= 5.03 \text{ N/mm}^2$
 $5.03 < 7 \text{ N/mm}^2$ (OK safe)

In steel (comp.)

$$= 1.5m (p_o + c') = 1.5 \times 13 \times (0.85 + 2.84) = 71.955$$

$$71.955 < \frac{170}{230} \text{ OK}$$

In tensile steel

$$t_a - m p_o = 102.91 - 13 \times 0.85 = 91.86 < 230 \text{ OK}$$

Question: A circular column section of dia 600mm, has 12 nos. 16mm ϕ bars. It is subjected to a load of 1200 kN and B.m. of 150 kN-m. M25 / Fc 415.

Check whether column section is safe as uncracked section.

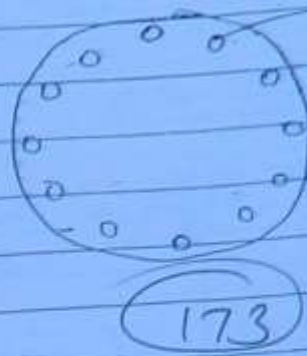
Ans: The section to be safe as uncracked section

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$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \leq 1$$

(1) σ_{cc}' for $P = 1200 \text{ kN}$

$$\sigma_{cc}' = \frac{P}{A_{eqv.}}$$



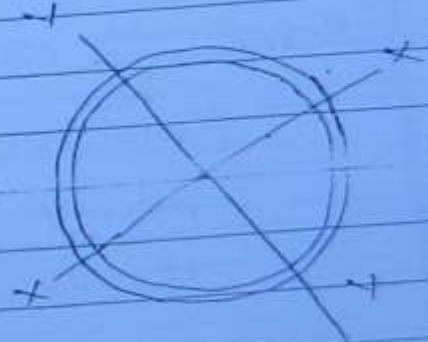
$$A_{eqv.} = \frac{\pi}{4} (D)^2 + (1.5m - 1) \times A_{sc}$$

$$= \frac{\pi}{4} (600)^2 + (1.5m - 1) \times 12 \times \frac{\pi}{4} \times (16)^2$$

$$= 320140.85$$

$$\sigma_{cc}' = \frac{1200 \times 10^3}{320140.85} = 3.75 \text{ N/mm}^2$$

(2) $\sigma_{cbc}' = \frac{M \times y}{I_{eqv.}}$



For $I_{eqv.}$

For steel

Consider steel equally distributed over a circular surf.

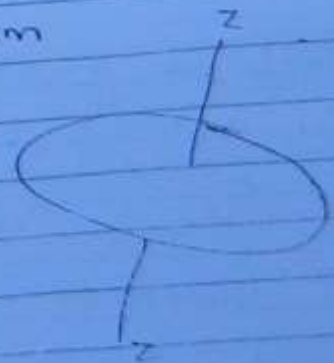
$$R = 300 - 50 = 250 \text{ mm}$$

M.I. about Z-Z axis

$$I_{zz} = A R^2$$

$$= 18 \times \frac{\pi}{4} \times (16)^2 \times (250)^2$$

$$150796447.4$$



Applying perpendicular axis theorem

$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = 2 I_0$$

$$I_D = \frac{I_{zz}}{2} = \frac{150796447.4}{2} = 75398223.69 \text{ mm}^4$$

Total M.I. of column about Diameter

$$= \frac{\pi}{64} (D^4) + (1.5\text{m} - 1) \times I_{\text{steel}} \quad (174)$$

$$= \frac{\pi}{64} (600)^4 + (1.5 \times 11 - 1) \times 75398223.69$$

$$= 7530.4 \times 10^6 \text{ mm}^4$$

$$\sigma_{cbc}' = \frac{M \times y}{I_{eq}}$$

$$= \frac{150 \times 10^6}{7530.4 \times 10^6} \times 300$$

$$\sigma_{cbc}' = 5.97 \text{ N/mm}^2$$

Check

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} = \frac{3.75}{6} + \frac{5.97}{8.5}$$

$$= 1.33 \quad \text{failed.}$$

Question A column of size $300 \times 400 \text{ mm}^2$ is subjected to ① Load $P = 400 \text{ kN}$

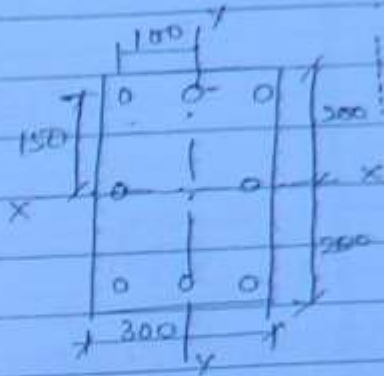
② $BM_{xx} = 30 \text{ kN-m}$

③ $BM_{yy} = 40 \text{ kN-m}$

Bars: $20 \text{ mm } \phi$ Bars provided

Check whether column is safe as uncracked section

M-20, $E_c = 415$.



175

Sol: Symmetrical section

① $A_{eqv} = 300 \times 400 + (1.5 \times m - 1) \times A_{sc}$

$= 300 \times 400 + (1.5 \times 13 - 1) \times 8 \times \frac{\pi}{4} \times (20 \text{ mm})^2$

$A_{eqv} = 166495.57 \text{ mm}^2$

② $I_{eqv} = \frac{BD^3}{12} + (1.5m - 1) \times \left(\frac{6 \times \pi}{4} \times 20^2 \times 150^2 \right)$

$= \frac{300 \times 400^3}{12} + 784612765.2$

$= 2384.61 \times 10^6 \text{ mm}^4$

③ $I_{eqvyy} = \frac{400 \times 300^3}{12} + (1.5 \times 13 - 1) \times \left(\frac{6 \times \pi}{4} \times 20^2 \times 100^2 \right)$

$= 1248.7 \times 10^6$

④ $p_0 = \frac{P}{A_{eqv}} = \frac{400 \times 10^3}{166495.6} = 2.40 \text{ N/mm}^2$

⑤ $\sigma_{cbc} = \frac{M_x \times y}{I_{eqvxx}} + \frac{M_y \times x}{I_{eqvyy}}$

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$$= \frac{30 \times 10^6}{2384.6 \times 10^6} \times 200 + \frac{40 \times 10^6}{1248.7 \times 10^6} \times 120$$

$$= 7.32 \text{ N/mm}^2$$

check.

$$\frac{\sigma_{cc}'}{\sigma_{cc}} + \frac{\sigma_{cbc}'}{\sigma_{cbc}} \leq 1$$

(176)

$$\frac{2.40}{5} + \frac{7.32}{7.0} = 1.52 > 1 \quad \text{failed} \quad \therefore$$

Column Design By LSM

⑧ All column should be designed for min eccentricity of

$$\left(\frac{l}{500} + \frac{D}{30} \right) \text{ or } 20 \text{ mm whichever is less.}$$

l = Unsupported length of column

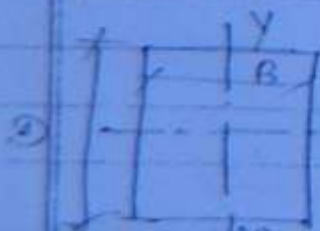
D = Lateral dimension.

About x-x

D = Lateral Dimension



About y-y



B = Lateral Dimension

(-)+ Assumption

(+) Assumption
All assumption for beam/slab are
also valid for column.

① Max^m Comp. strain in concrete

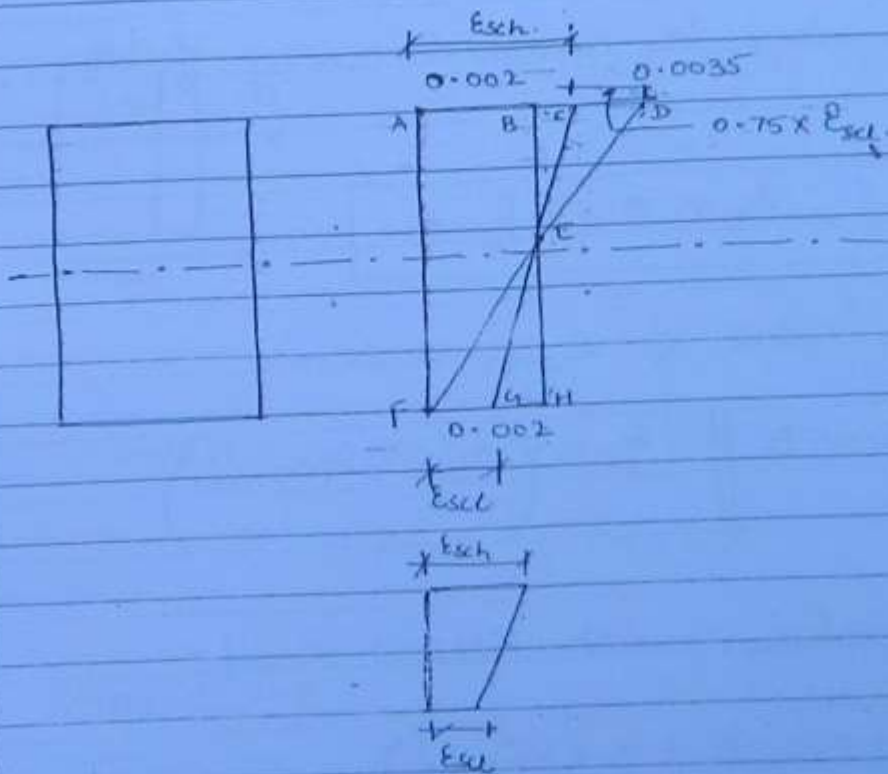
In axial comp. = 0.002

(In bending comp. = 0.0035)

677

(5) The max^m comp. strain at highly compressed fibre in concrete, subjected to axial comp. and when there is no tension in the section

$\epsilon = 0.0035 - 0.75 \times$ strain at least comp fibre.



$$\text{Ratio } \frac{BE}{EH} = \frac{BD}{FH} = \frac{0.0015}{0.002} = 0.75$$

$$\frac{BE}{EH} = \frac{EC}{EG} = \frac{CD}{FG} = 0.75$$

$$CO = 0.75 FGr \text{ and } 0.75 E_{scd}$$

So strain at highly comp. fibre

$$\epsilon_c = 0.0035 - \epsilon_d$$

$$\epsilon_{sch} = 0.0035 - 0.75 \times \epsilon_{sc}$$

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(ff) Design of Short Axially Loaded Column

→ All columns should be designed for min. eccentricity value.

except if $[e_{min} \leq 0.05D]$.

D = Lateral Dimension

→ The column may be designed using following equation

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

The minimum value of $e_{min} = 20 \text{ mm}$.

$$20 \leq 0.05D \quad || \quad D \geq \left(\frac{20}{0.05} = 400 \text{ mm} \right)$$

For Circular column with Helical reinf.

$$P_u = 1.05 (0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$

and for helical Reinf

$$\frac{0.36 f_{ck}}{f_y} \left(\frac{A_g}{A_c} - 1 \right) \leq \frac{V_h}{V_c}$$

$$f_y \neq 415 \text{ N/mm}^2$$

Question
ES 2009

DATE: _____

Design the

A reinforced concrete column of size 460×600 mm having effective length of 3.6 m is to be designed using limit state method to support an axial service load of 2500 kN. Use M20 / Fe 415 steel.

Soln

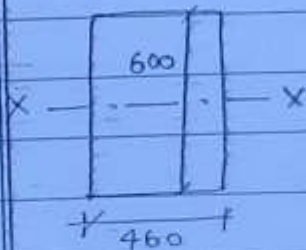
$l_{eff} = 3.6 \text{ m}$

(179)

① Slenderness ratio $= \frac{l_{eff}}{B} = \frac{3600}{460} = 7.8 < 12$

It is a short column

② Min. eccentricity about XX



Lateral dimension

$$e_{min} = \frac{l}{500} + \frac{D}{30}$$

$$= \frac{3600}{500} + \frac{600}{30} = 7.2 + 20 = 27.2 > 20 \text{ mm}$$

$$0.05D = 0.05 \times 600 = 30 \text{ mm}$$

$$e_{min} < 0.05D$$

③ About Y-Y



$$e_{min} = \frac{3600}{500} + \frac{460}{30}$$

$$= 7.2 + 15.33 = 22.53 \text{ mm}$$

$$> 20 \text{ mm}$$

$$0.05B = 0.05 \times 460 = 23 \text{ mm}$$

So we have $e_{min} > 0.05D$

Now we can use the equation

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y \cdot A_{sc}$$

$$1.5 \times 2500 \times 10^3 = 0.4 \times 20 \times (A - A_{sc}) + 0.65 \times 415 \times A_{sc}$$

$$= 2208000 - 8A_{sc} + 269.75A_{sc}$$

$$A_{sc} = 5710 \text{ mm}^2$$

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$$\text{No. of } 25 \text{ mm } \phi \text{ Bar} = \frac{5710}{\frac{\pi \times (25)^2}{4}} = 12 \text{ Nos}$$

Stirrups

①. $\text{Dia} = \frac{\phi_{\max}}{4} = \frac{25}{4} = 6.25 \text{ mm}$

Use 8 mm ϕ Bar

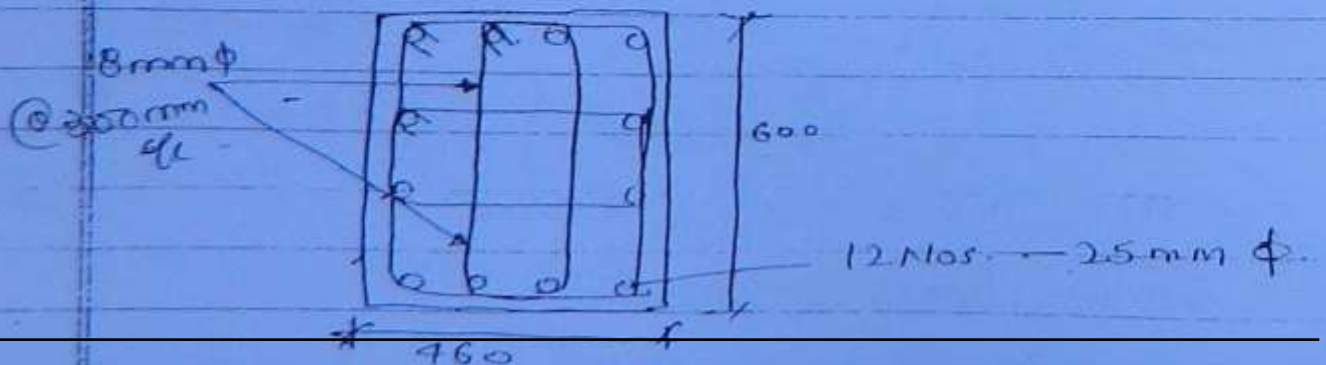
② Spacing.

1. LLD = 460 mm

2. $16 \phi_{\max} = 16 \times 25 = 400$

3. 300 mm

Provide 8 mm ϕ @ 300 mm c/c



Design of Column Subjected to axial load + Bending (uniaxial bending)
Using interaction Chart

Load = P_u

Moment = M_u

Size of Column = B x D

Find out area of Steel

$A_{sc} = ?$

$\frac{P_u}{f_{ck} B D}$

$\frac{M_u}{f_{ck} B D^2}$

Steps

1. Find out $\frac{P_u}{f_{ck} B D}$ = y-value

2. Find out $\frac{M_u}{f_{ck} B D^2}$ = x-value

(18)

Based on x & y value

read $\frac{P}{f_{ck}}$ from interaction chart

$\frac{P}{f_{ck}} = K$

$$P = K \cdot f_{ck} = \frac{A_{sc} \times 100}{B D}$$

Area of Steel $A_{sc} = \frac{P \times B D}{100}$

Stirrups - Same as previous

Dia - $\frac{\Phi}{4}$ or 6mm

Spacing - 11D, 16 ϕ , 300mm

For a column of size B x D

& fixed area of Steel (A_{sc})

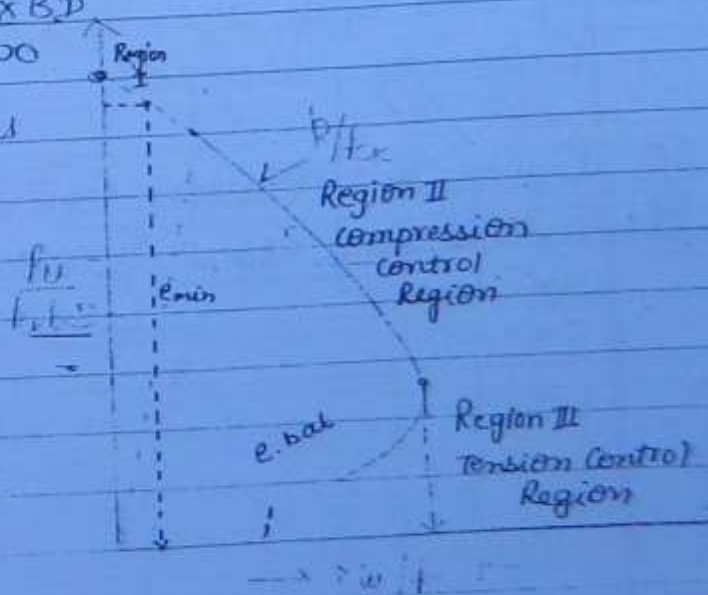
$\frac{P}{f_{ck}}$ = One value, we get

One line on interaction chart

1. P_{uz} = Max. load carrying capacity of column

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

Also called as ultimate load carrying capacity.



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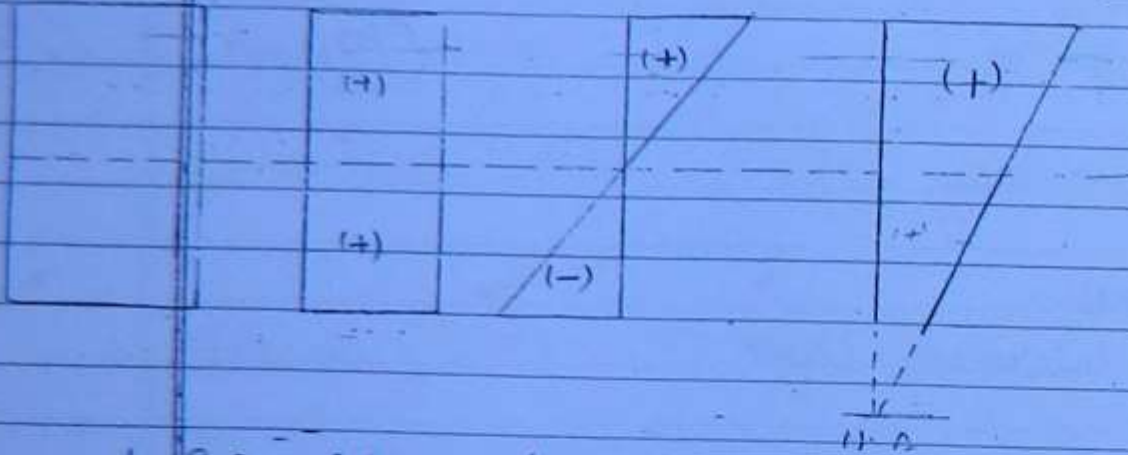
② Region I - Min. eccentricity value. or region Column is subjected to less than min eccentricity value.

Effect of P_u is more as compared to effect of M_u .

→ Column is subjected to compressive stress only.

③ Region II -

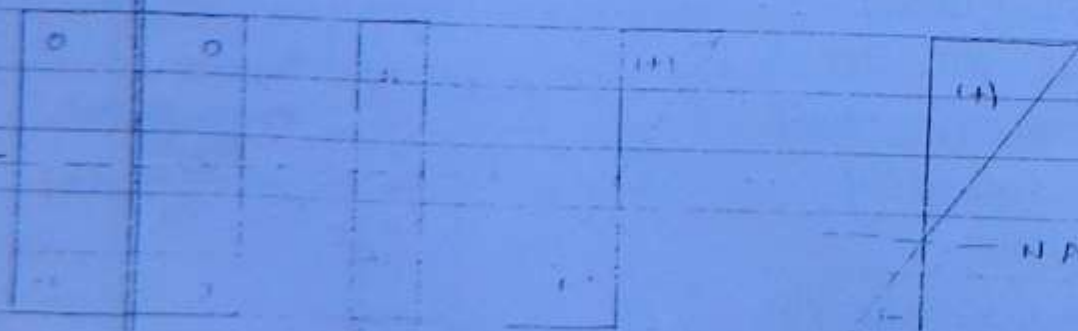
(182)



1. Entire Column section is in compression
2. N.A. lie Outside the Column section
3. $\epsilon_{sc} = 0.0035 - 0.75 \epsilon_{sc}$
4. Effect of load & moment are equally high
5. If load value are ^{reduced} ~~considered~~ moment carrying capacity increased

④ Region III - Tension Control Region

(1) In this case effect of moment is more as load values are less



2. Tensile stresses are developed in the section
3. With reduction of load even moment carrying capacity also reduced due to this reason curve takes a inward direction
4. Maximum strain = 0.0035
5. N.A lies within the section

(183)

Design of column subjected to combined axial load & Biaxial bending.

If Column = B x D is to be designed for

1. Load $\rightarrow P_u$
2. Moment $\rightarrow M_{ux}$ abt X-X
3. Moment $\rightarrow M_{uy}$ abt Y-Y
4. f_{ck} / f_y are used
5. Using interaction chart - only for uniaxial bending.

Steps (i) - Consider a percentage of steel = $p\%$

(ii) Find out P & P_u & $\frac{d_o}{D}$
(About X-X) f_{ck} $f_{ck} B \cdot D$ D

Read the appropriate Chart No. & find out

$$\frac{M_u}{f_{ck} B D^2} = K_1$$

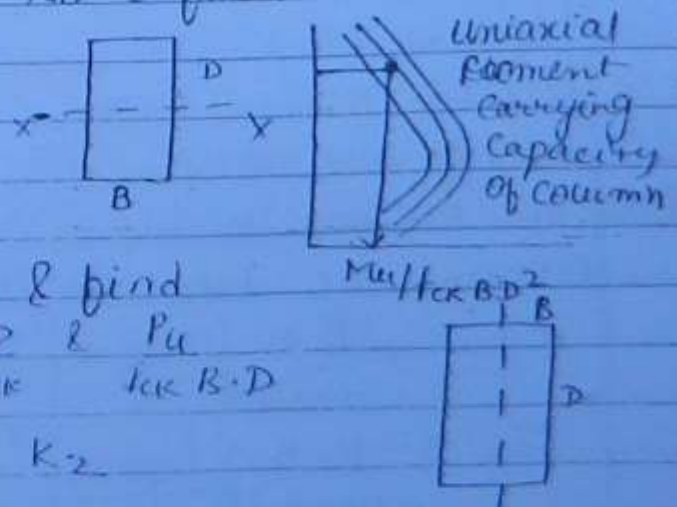
$$(i) M_{ux} = K_1 \cdot f_{ck} B D^2$$

Design =

About Y-Y - Check for $\frac{d_o}{D}$ & find out value from chart $\frac{P}{f_{ck} B \cdot D}$ & $\frac{P_u}{f_{ck} B \cdot D}$

Read values of $\frac{M_u}{f_{ck} B D^2} = K_2$

$M_{uy} = K_2 f_{ck} B D^2$ - Uniaxial Moment carrying capacity of column abt Y-axis



The Column shall be safe if

$$\left[\frac{M_{ux}}{M_{uxl}} \right]^{\lambda_n} + \left[\frac{M_{uy}}{M_{uyl}} \right]^{\lambda_n} \leq 1.0$$

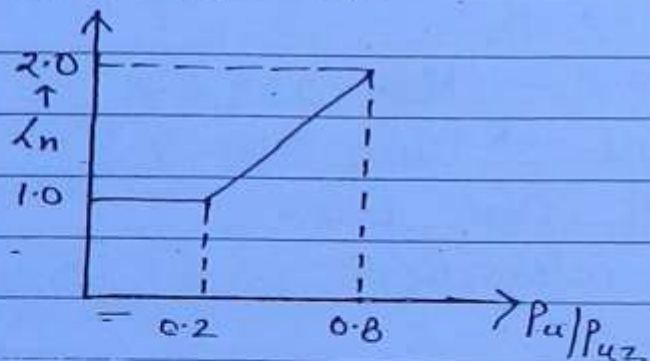
Where λ_n = an exponent $\leq$ value depends upon P_u / P_{uz}

$$P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y \cdot A_{sc}$$

$\frac{P_u}{P_{uz}}$ 0.2 to 0.8 the value of λ_n = 1.0 to 2.0

for value less than 0.2 $\lambda_n = 1.0$

for value greater than 0.8 $\lambda_n = 2.0$



(184)

Design of ~~totu~~ long columns

If a column is subjected to

- (i) Load P_u
- (ii) Moment M_{ux}
- (iii) Moment M_{uy}
- (iv) Size of column B x D
- (v) f_{ck} / f_y are used
- (vi) To consider slenderness effect, some additional moment are added to above design moment.

$$M_{ux} = \frac{P_u D}{2000} \left\{ \frac{L_{effx}}{D} \right\}^2$$

$$M_{uy} = \frac{P_u b}{2000} \left\{ \frac{L_{effy}}{b} \right\}^2$$

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Now design the Column for

- (i) P_u
- (ii) $(M_{ux} + M_{ax})$
- (iii) $(M_{uy} + M_{ay})$

Ques Design a Column of size 400×600 for following data
Concrete - M15 & Steel - Fe415
load $P_u = 1600 \text{ kN}$

$M_{ux} = 120 \text{ kN-m}$; $M_{uy} = 90 \text{ kN-m}$

Reinfⁿ are distributed equally on four sides.

Solⁿ Moment due to min eccentricity

$$e_{min} = 20 \text{ mm}$$

$$M_{umin} = \frac{P_u \cdot e_{min}}{1000} = \frac{1600 \times 20}{1000} = 32 \text{ kN}$$

$M_{ux} > M_{umin}$ & $M_{uy} > M_{umin}$ OK.

① Let us assume a percent of Steel

$$p = 1.4\%$$

$$\Rightarrow \frac{p}{f_{ck}} = \frac{1.4}{15} = 0.093$$

$$\frac{P_u}{f_{ck} B D} = \frac{1600 \times 10^3}{15 \times 400 \times 600} = 0.444$$

$$\text{About X-X} \rightarrow \frac{d'}{D} = \frac{50}{600} = 0.083$$

Use 0.10 & Chart no 44. SP-16

$$\frac{M_{ux}}{f_{ck} B D^2} = \frac{120 \times 10^6}{15 \times 400 \times 600^2} = 0.1 = K_1$$

$$M_{ux1} = 0.1 f_{ck} B D^2$$

$$0.1 \times 15 \times 400 \times 600^2 = 216 \text{ kN-m}$$

$$\text{About Y-Y} \rightarrow \frac{d'}{B} = \frac{50}{400} =$$

Chart no- 45 for value of $\frac{d'}{B} = 0.15$

DATE: _____

$$M_{uy} = 0.10 = k_2$$

$$f_{ck} DB^2$$

$$M_{uy1} = 0.10 \times 15 \times 400^2 \times 600$$

$$144 \text{ KN-m}$$

$$P_{u2} = 0.45 f_{ck} A_c + 0.75 f_{ck} A_{sc}$$

$$= 0.45 \times 15 \times (400 \times 600 + 0.75 \times 15 \times 1.4 \times 400 \times 600)$$

$$P_{u2} = 2643.12 \text{ KN}$$

$$\frac{P_u}{P_{u2}} = \frac{1600}{2643.12} = 0.605$$

$\frac{P_u}{P_{u2}}$	α_n	
0.2	1.0	(186)
0.8	2.0	

for 0.605

$$\alpha_n = 1.0 + \frac{2-1}{0.8-0.2} (0.605-0.2)$$

$$= 1.575$$

Check $\left(\frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1.0$

$$\left(\frac{80}{216} \right)^{1.575} + \left(\frac{90}{144} \right)^{1.575}$$

$$0.376 + 0.455$$

$$0.83 \leq 1.0$$

Safe

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Foundation

→ Types

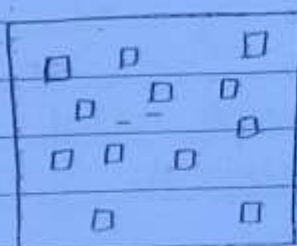
(a) Isolated footing (b) Combined footing (c) Strip footing

imp



(d) Raft

(e) Pile foundation



underreamed

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Isolated Footing Foundation

- For a Rectangular footing of uniform thickness
- Load from column = P kx & by are given
- Size of Column = $a \times b$
- Bearing Capacity of Soil = q_0

Step 1 Size of foundation

Load from Column = P

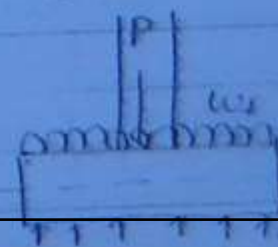
(actual 15-20%)

Weight of foundation = $0.10 P$

(10% of load)

Total load $P_T = 1.10 P$

Area of foundation $\rightarrow A = \frac{P_T}{q_0}$



Find out length $L \times B$

$$A = L \times B$$

Pressure due to weight of foundation

$$w_f = \frac{0.1P}{A}$$

Net soil pressure for c foundation is designed

$$w_o = \frac{q_o - w_f}{A} = \frac{P_f - 0.1P}{A}$$

$$w_o = \frac{P}{A}$$

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For L.S.M it will be the factored.

$$w_{uo} = \frac{1.5P}{A}$$

Step 2 Check for Bending moment

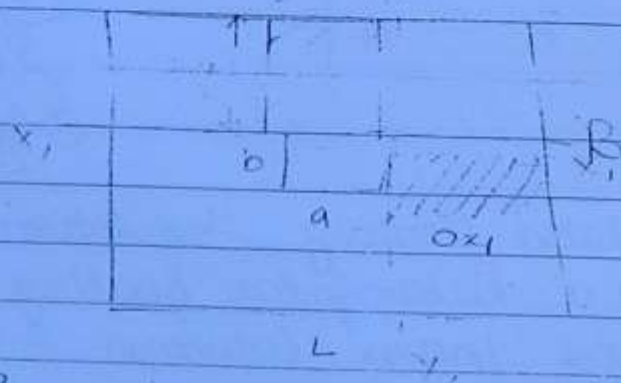
Critical Secn for B.M

→ At the face of column

Let $x_1 - x_1$

Consider 1m width of foundation

$$\text{Overhang } O_{y_1} = \left(\frac{B-b}{2} \right)$$



$$B.M \text{ at } x_1 x_1 = \frac{w_o \times 1 \times O_{y_1}^2}{2}$$

$$= \frac{w_o}{2} \left(\frac{B-b}{2} \right)^2 = \left[\frac{w_o (B-b)^2}{8} \right]$$

$$B.M \text{ at } y_1 y_1 = \frac{w_o \times 1 \times O_{x_1}^2}{2}$$

$$= \frac{w_o}{2} \left(\frac{L-a}{2} \right)^2 = \left[\frac{w_o (L-a)^2}{8} \right]$$

Depth of foundation $d = \sqrt{\frac{(M_x \text{ or } M_y)_{\text{greater}}}{Q \cdot B}}$

Step 3 Check for one way Shear
Critical Secⁿ for one way Shear
(Single Shear)

Max Shear force shall be at $\frac{l}{2}$

Where Overhang is maximum.

$$\text{Overhang } O_x = \left[\left(\frac{l-a}{2} \right) - d \right]$$

Max Shear force

$$V_y = w_o \times l \times O_x$$

$$w_o \times \left[\left(\frac{l-a}{2} \right) - d \right]$$

Nominal Shear Stress

$$\tau_v = \frac{V_y}{Bd} = \frac{V_y}{1000d} \leq \tau_{min}$$

$$\leq K \cdot \tau_c$$

K = factor depending on Slab thickness

B.S.2:1 Page No-84

overall depth of slab	300 or more	275	250	225	200	175	150 or less
K	1.0	1.05	1.10	1.15	1.20	1.25	1.30

Step-4 Check for punching Shear failure or two way shear

→ Net punching force

$$= P = w_o \times (a+d) \times (b+d)$$

Net punching Shear Stress developed

= Net punching force

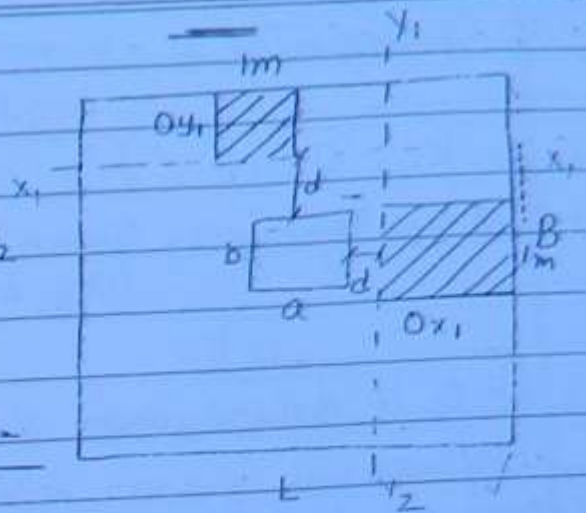
Resisting area

$$= P_o = w_o (a+b)(b+d)$$

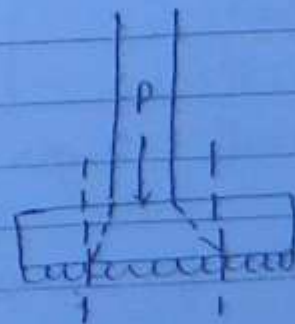
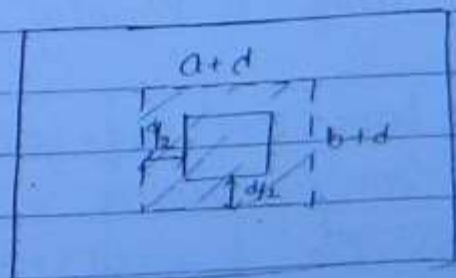
Perimeter $\times$ depth

$$= \frac{P - w_o (a+d)(b+d)}{2[(a+d) + (b+d)] \times d}$$

$$\leq C_{vp} (\text{permissible})$$



(189)



12% HSSD Bar
(Cover = 50mm)

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geno-59
$$Z_{up} (\text{permissible}) = 0.25 \sqrt{f_{ck}} \times K_s \rightarrow \text{L.S.M}$$

$$= 0.16 \sqrt{f_{ck}} \times K_s \quad \text{K.S.M}$$

$$K_s = 0.5 + \beta_c \neq 1$$

$$\beta_c = \frac{b}{a} = \frac{\text{Short Side}}{\text{Long Side}}$$

(190)

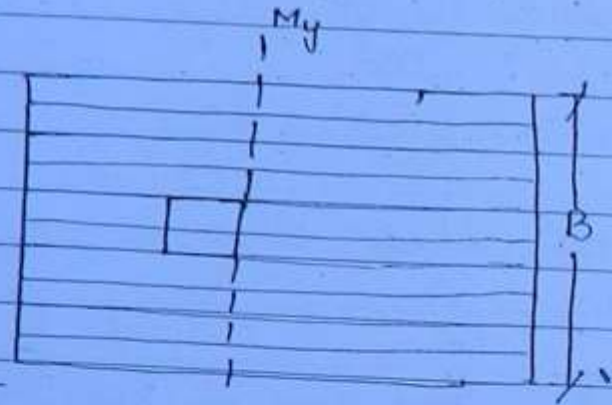
Step 5 Depth is decided, max. value required from above three Criteria.

Area of Steel

(a) Longer bars are req. for M_y . For 1m width

$$A_{st} = \frac{M_y}{\sigma_{st} \cdot j \cdot d} \quad \text{K.S.M}$$

$$= \frac{M_y}{0.87 f_y j \cdot d} \quad \text{L.S.M}$$



$$= 0.5 f_{ck} \left[1 - \sqrt{1 - \frac{4.6 M_{uwy}}{f_{ck} \cdot B d^2}} \right] B d \quad \text{L.S.M}$$

So for total B width Area of Steel = $B \cdot A_{st}$

OR No. of bars = $B \times (\text{No. of bars req. for } A_{st})$

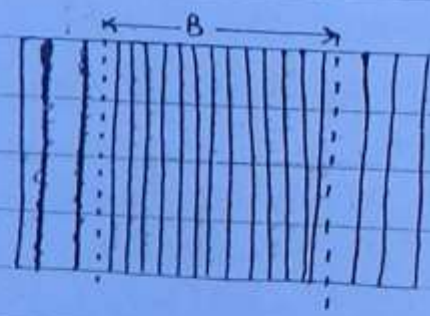
"This reinf" is equally distributed over 'B' width.

(b) Short bars are req. for M_x . For 1m width

$$A_{st} = \frac{M_x}{\sigma_{st} \cdot j \cdot d} \quad \text{L.S.M}$$

$$\sigma_{st} = 0.87 f_y \quad \text{L.S.M}$$

So for total L length area of Steel = $L \cdot A_{st}$



no. for bars for 1m width $n_1 = \frac{A_{st}}{\frac{\pi}{4} \phi^2}$

No. for bars for L width $n_1 = L \times n_1$

No. of bars provided for in Central Band of width B is

$$n_c = n_T \left(\frac{2}{1 + L/B} \right)$$

Remaining bars are distributed equally on two side bands.

(91)

ES-1993

Ques- A rectangular RC column of size 60×40 cm carries a load of 80 tonnes. Design a rectangular footing. use M-15 / fc-250

$$q_o = 20 \text{ t/m}^2$$

$$a = 600 \text{ mm}$$

$$b = 400 \text{ mm}$$

$$80 \text{ T} = \frac{800 \times 1000 \times 10}{1000} = 800 \text{ kN}$$

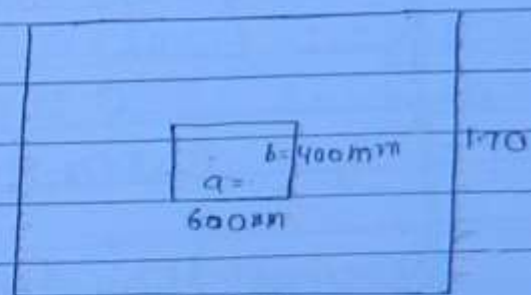
$$P = 800 \text{ kN}$$

$$q_o = 200 \text{ kN/m}^2 \quad (1.5 \times P)$$

$$\text{Total load } P_T = 880 \text{ kN}$$

$$\text{Area of foundation} = \frac{880 \times 1.5}{200 \times 1.5}$$

$$= 4.4 \text{ m}^2$$



$$\text{Size of foundation} = 1.5B^2 = 4.4$$

(LXB)

$$B = 1.71 \text{ m} \text{ \& } L = 2.57 \text{ m}$$

$$B = 1.70 \text{ m}$$

$$L = 2.60$$

$$w_o = \frac{1.5P}{A} \quad (\text{Net Soil pressure})$$

$$A = 4.42 \text{ m}^2$$

$$= \frac{1.5 \times 800}{4.42} = 271.5 \text{ kN/m}^2$$

Check for B.M

At the face of Column abt $X-X$

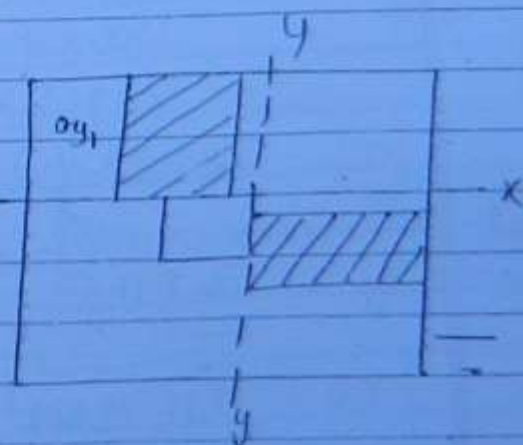
$$O_{y1} = \frac{1.70 - 0.4}{2} = 0.65$$

$$\text{B.M at } X-X = \frac{271.5 \times 0.65^2}{2}$$

$$57.35 \text{ kN-m}$$

$$\text{B.M at } Y-Y = \frac{271.5 \times \left(\frac{2.60 - 0.6}{2} \right)^2}{2}$$

$$= 135.75 \text{ kN-m}$$



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Depth of foundation $d = \sqrt{\frac{M_y}{Q \cdot B}}$

$$d = \sqrt{\frac{135.75 \times 10^6}{2.22 \times 1000}} = 247.29 \text{ mm}$$

Say = 250 mm

$$Q = 0.36 f_{ck} \frac{x_{u \text{ lim}}}{d} (1 - 0.42 \frac{x_{u \text{ lim}}}{d})$$

$$= 0.36 \times 15 \times 0.53 (1 - 0.42 \times 0.53) = 2.22$$

Check for One way shear

at Critical Secⁿ

Max. S.F. shall be at V-V where overhang is max

$$O_x = \left[\left(\frac{2.60 - 0.6}{2} \right) - 0.250 \right]$$

$$= 0.75 \text{ m}$$

$$V_y = w_o \times O_x \times l = 271.5 \times 0.75$$

$$= 203.625 \text{ kN}$$

Nominal Shear Stress

$$\tau_v = \frac{V_y}{Bd} = \frac{203.625 \times 10^3}{1000 \times 250} = 0.8145 < \tau_{\text{min}}$$

Depth reqⁿ

$$0.28 = \frac{203.625 \times 10^3}{1000 \times d}$$

$$= 727.23 \text{ Say } =$$

$$d = \frac{727.23 + 250}{2} = 488.61 \text{ Say } 500$$

$$V_y = w_o \times 0.5 = 271.5 \times 0.75$$

$$= 135.75 \text{ kN}$$

$$\tau_v = \frac{135.75 \times 10^3}{1000 \times 500} = 0.2715 < 0.28 (\tau_{\text{min}})$$

Hence Safe.

Check for two way shear or punching
Net punching force = $P - w_o (a + d)(b + d)$

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$$= 1.5 \times 10^3$$

$$= 800 - 271.5 (0.6 + 0.5) (0.4 + 0.5) \times 10^3$$

$$= 931215 \text{ N/m}^2$$

Resisting Area

$$= 2[(a+d) + (b+d)] \times d$$

$$= 2[1.1 + 0.9] \times 0.5$$

$$= 0.99 \text{ m}^2 = 2 \times 10^6$$

$$= 990000 = 990 \times 10^6 \text{ mm}^2$$

τ_{vp} (developed)

$$= 0.465 \text{ N/mm}^2$$

(193)

$$\tau_{vp}(\text{permissible}) = 0.25 \sqrt{f_{ck}} \times K_s$$

$$K_s = \left(\frac{0.5 + b}{a} \right) = \left(\frac{0.5 + 400}{600} \right) = 1.67 > 1.0$$

So take 1.0

$$\tau_{vp}(\text{permissible}) = 0.25 \sqrt{15} \times 1.0$$

$$= 0.968 \text{ N/mm}^2$$

$> \tau_{vp}(\text{developed})$ - Safe

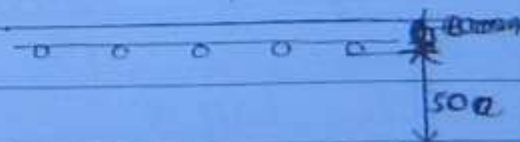
Depth required = 500 mm

Ebb Cover + 80 mm

Total depth = 580 mm

Ebb Cover = 50 + 16 + 8

$$= 74 \text{ mm}$$



Area of Steel

longer bar (M_y)

$$A_{st} = \frac{0.5 \times 15}{250} \left[1 - \sqrt{1 - \frac{4.6 \times 135.75 \times 10^6}{15 \times 1000 \times 500^2}} \right] 1000 \times 500$$

$$= 1305.75 \text{ mm}^2$$

Use 16mm ϕ bars. No. of bars

$$= \frac{1305}{\frac{\pi}{4} 16^2} = 6.49 \text{ No}$$

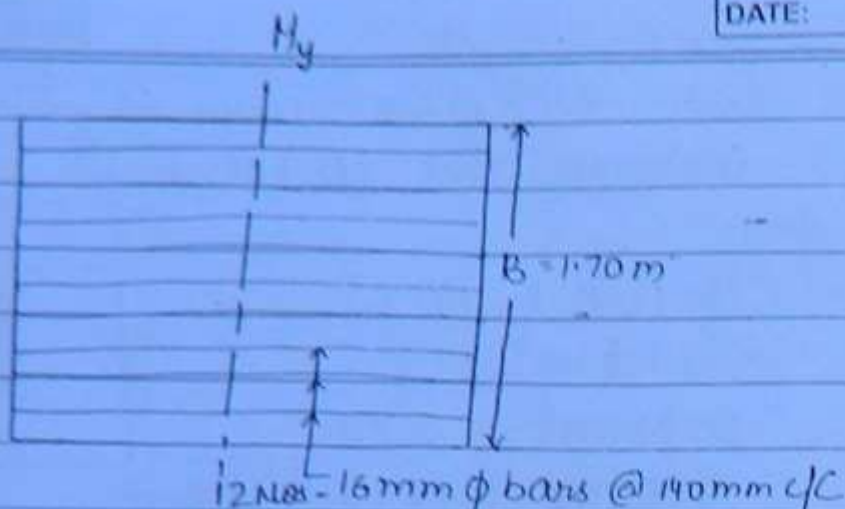
$\frac{\pi}{4} 16^2$

Total bars req. for B width $n_1 = 6.49 \times 1.7$

$$= 11.03$$

Say 12 Nos. of 16mm ϕ bars

$$\text{Spacing} = \frac{1700}{2} = 141 \text{ mm}$$



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Shorter Side

$$A_{st} = \frac{0.5 \times 15}{250} \left[1 - \sqrt{\frac{4.6 \times 53.35 \times 10^6}{15 \times 1000 \times 500}} \right] 1000 \times 500$$

$$537.24 \text{ mm}^2$$

$$\text{No. of bars } 10 \text{ mm } \phi = \frac{537.24}{\frac{\pi}{4} \times 10^2} = 6.84$$

$$\text{Total NO} = 2.6 \times 6.84 = 17.784$$

Say 18 Nos of 10 mm ϕ bars

$$\text{Spacing} = \frac{2600}{18} = 144.44 \text{ mm}$$

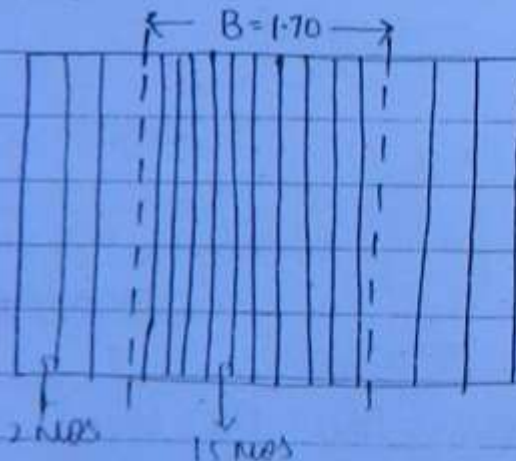
initial bond width

$$n_c = 14.22 \text{ Say } 15$$

$$\text{Spacing} = \frac{1700}{15} = 113$$

No. of bars in side band

$$= \frac{18 - 15}{2} = 1.5 = 2 \text{ nos.}$$



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2 Nos - 10mm ϕ

15 Nos 16mm ϕ bars

16mm ϕ bars - 12 Nos

1.70

2 Nos - 10mm ϕ bars

1.70

195

2 Nos - 10mm ϕ bars

2.60

12 Nos - 16mm ϕ bars

15 Nos - 10mm ϕ bars

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Sloped Isolated footing

1. Size of foundation - Same

$$P + 0.10P = P_T$$

$$A = \frac{P_T}{\sigma_0}$$

$$w_0 = \frac{P}{A}$$

$$B = \sqrt{A}$$

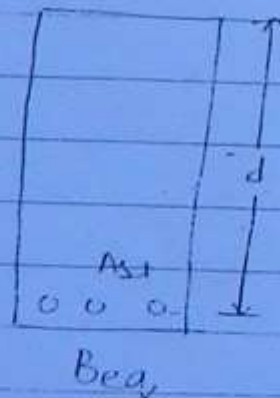
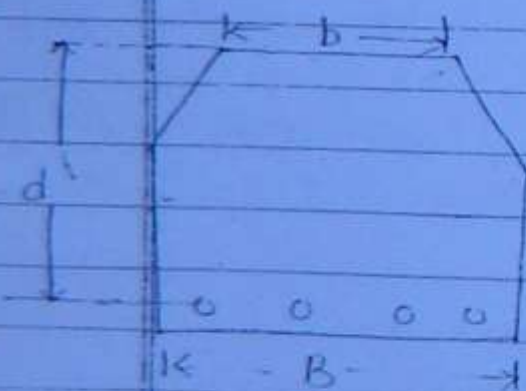
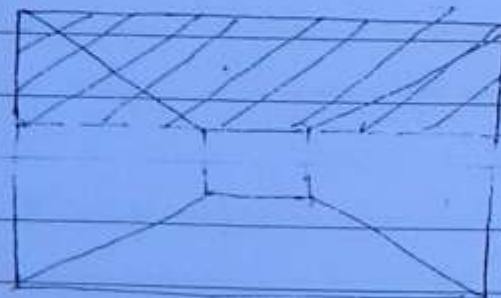
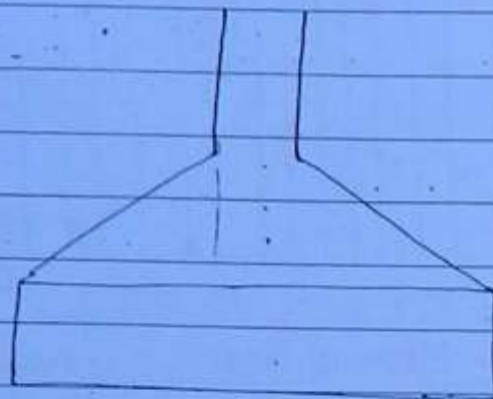
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Bending moment

Critical secⁿ at face of column

$$B.M = w_0 B \times \left(\frac{B-b}{2} \right) \left(\frac{B+b}{4} \right)$$

$$= w_0 B \times \frac{(B-b)^2}{8}$$



Width of equivalent secⁿ

$$B_{eq} = b + \frac{1}{6}(B-b)$$

Depth of secⁿ req.

$$d = \sqrt{\frac{BM}{Q.Beq}}$$

Check for one way shear

$$V = w_0 \times B \times \left[\frac{B-b}{2} - d \right]$$

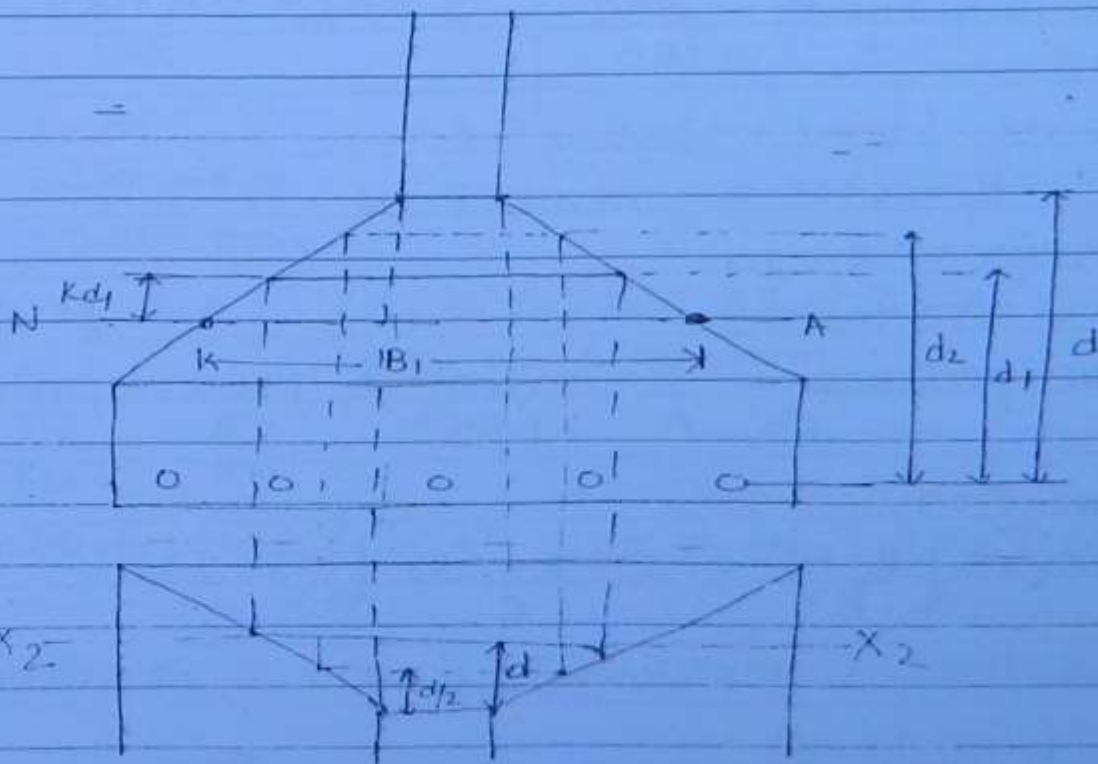
Nominal Shear Stress $\tau_v = \frac{V}{B_1 d_1}$

(197)

here

B_1 = width of footing at critical secⁿ or N.A

d_1 = depth of footing at critical secⁿ.



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Punching Shear

$$\text{Punching Shear force} = P - w_o(a+d)(b+d)$$

$$\text{Punching Shear Stress} = \frac{P - w_o(a+d)(b+d)}{2[(a+d)+(b+d)] \times d_2} \leq \tau_{cp \text{ permissible}}$$

Area of Steel - Same.

2001

Ques

Design a Sloped Square footing for RC Column, $300 \times 300 \text{ mm}$ size, carrying an axial load of 320 kN . The safe bearing capacity of soil is 150 kN/sqm . Use M-20 / Fe-415
Use L.S.M

1. Size of foundation

$$\text{Load from Column} = 320 \text{ kN}$$

$$\text{Foundation weight} = 32 \text{ kN}$$

$$352 \text{ kN}$$

$$\text{Area required} = \frac{352}{150} = 2.35 \text{ m}^2$$

Size of Square footing

$$B = \sqrt{2.35} = 1.55 \times 1.55$$

$$\text{Area} = 2.4 \text{ m}^2$$

$$B = 1550 \text{ mm}$$

$$\text{Net Soil pressure} = \frac{P}{w_o A} = \frac{320}{1.55^2} = 133.2 \text{ kN/m}^2$$

Check for B.M.

$$B.M = w_o \times B \left(\frac{B-b}{8} \right)^2 = \frac{133.2 \times 1.55 (1.55 - 0.3)^2}{8}$$

$$= 40.32 \text{ kN-m}$$

$$B_{eq} = \frac{b+1}{8} (B-b)$$

$$= \frac{300+1}{8} (1550 - 300) = 456.25 \text{ mm}$$

$$\text{Depth required } d = \sqrt{\frac{B.M}{F_y B_{eq}}}$$

$$\sqrt{\frac{B.M}{F_y B_{eq}}}$$

$$d = \sqrt{\frac{40.32 \times 10^6}{0.898 \times 456.25}}$$

$$d = 313.7 \text{ mm}$$

$$\text{Say } d = 350 \text{ mm}$$

$$K = m_c = 13.7$$

$$t + m_c = 230 + 13.7$$

$$= 0.2835$$

$$j = 1 - \frac{K}{3} = 0.9055$$

$$Q = \frac{1}{2} c_j \cdot K = \frac{1}{2} \times 17.1 \times 0.9055$$

$$Q = 0.898$$

Check for one way shear

Total shear force

$$= w_0 \cdot B \cdot \left[\frac{B - b - d}{2} \right]$$

$$= 133.2 \times 1.55 \left[\frac{1.55 - 0.3 - 0.35}{2} \right]$$

$$= 56.78 \text{ kN}$$

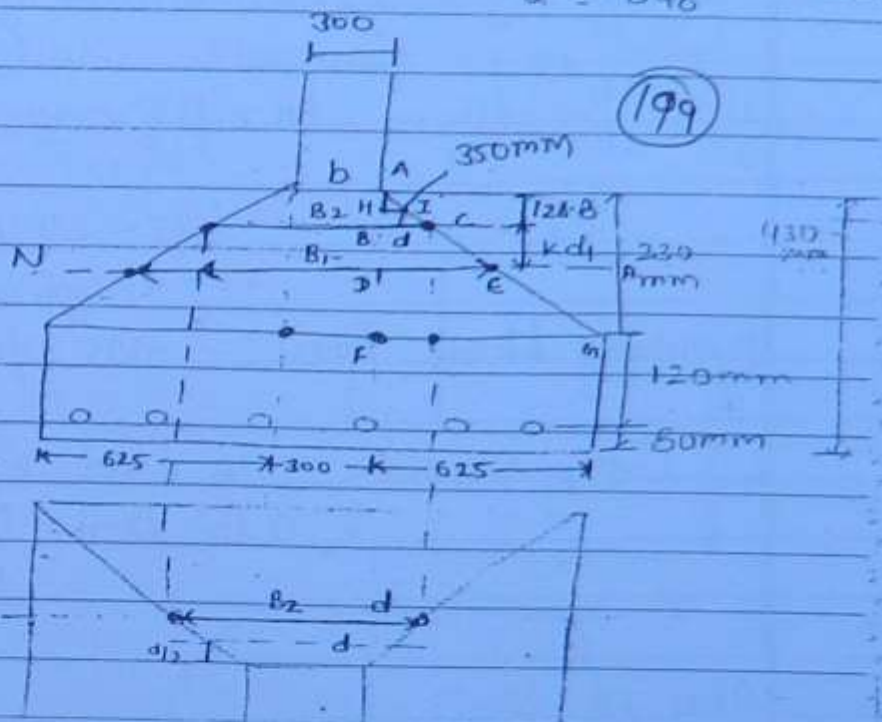
$$AB = \frac{AF = 230}{350 \quad 625}$$

$$AB = \frac{230 \times 350}{625} = 128.8$$

$$d_1 = d - AB$$

$$= 350 - 128.8$$

$$= 221.2 \text{ mm}$$



Depth of N.A $Kd_1 = 0.2835 \times 221.2$

$$BD = 62.71 \text{ mm}$$

$$\frac{DE}{FG} = \frac{AD}{AF}$$

$$DE = \frac{128.8 + 62.71}{230} \times 625 = 520.4 \text{ mm}$$

$$\text{Width } B_1 = 30 + 2 \times 520.4 \text{ mm} = 1340.8 \text{ mm}$$

Check for single shear

$$\tau_v = \frac{V}{B_d} = \frac{56.78 \times 10^3}{1340.8 \times 221.2 \text{ mm}} = 0.19 \leq \tau_{\text{min}} = 0.19$$

(safe)

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Check for two way shear (Punching Shear)

$$\text{Punching Shear Stress} = \frac{[320 - 133.2 \cdot (.3 + .35) \cdot (.3 + .35)] \times 10^3}{10^6 \times 2 \cdot [(.3 + .35) + (.3 + .35)] \cdot 285.6}$$

$$H_I = \frac{d}{2} = \frac{350}{2} = 175 \text{ mm}$$

(200)

$$\frac{A_H}{A_F} = \frac{H_I}{F_u} = A_H = \frac{175}{625} \times 230 = 64.4 \text{ mm}$$

$$d_2 = d - A_H = 350 - 64.4 \text{ mm} = 285.6 \text{ mm}$$

$$\text{Punching Shear Stress} = .355 \text{ N/mm}^2 < \tau_{vp} \text{ permitt}$$

$$\tau_{vp} \text{ permitt} = 0.16 \sqrt{f_{uc}} \cdot k_1$$

$$= 0.16 \sqrt{20} = 0.7155$$

Safe

Area of Steel

$$A_{st} = \frac{40.32 \times 10^6}{230 \times 9055 \times 350} = \text{A.M.}$$

$$= 553.14 \text{ mm}^2$$

$$\text{No. of } 10 \text{ mm } \phi \text{ bars} = \frac{553.14}{\pi/4 \times 10^2} = 7.04$$

Provide 8 Nos 10 mm ϕ bars.

Circular footing

Ques. Design a circular Column of 500mm dia having a load of 1200 kN. Bearing Capacity of soil is 150 kN/m². Use M-25 / Fe 415 Steel. Use 1.5m

Ans. Size of foundation

$$\text{Load from Column} = 1200 \text{ kN}$$

$$\text{Weight of foundation} = 120 \text{ kN}$$

$$P_T = 1320 \text{ kN}$$

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$$\text{Area required} = \frac{1320}{150} = 8.8 \text{ m}^2$$

$$\text{Dia of footing } D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 8.8}{\pi}} = 3.35 \text{ m}$$

$$\text{Provide } D = 3400 \text{ mm} \quad (201)$$

$$\text{Radius} = 1700 \text{ mm} = 1.70$$

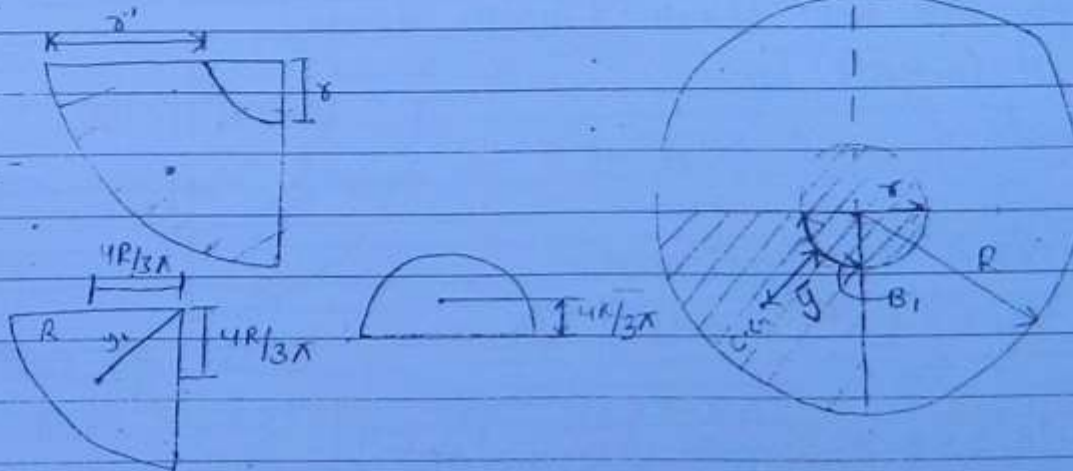
$$\text{Net Soil pressure} = \frac{1200 \times 1.5}{W_u \times \pi \times 1.7^2} = 139.198 \cdot 26 \text{ kN/m}^2$$

Check for B.M

Critical section is at face of Column

$$B.M = W_u \times \left(\frac{\pi R^2 - \pi r^2}{4} \right) \bar{y}$$

$\bar{y}$ value



Distance of C.G. from Centre

$$y_1 = \sqrt{2} \times \frac{4R}{3\pi} = 0.6R$$

Distance of C.G. of Shaded portion from Centre

$$= \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{\frac{\pi r^2}{4} \times 0.6R - \frac{\pi R^2}{4} \times 0.6R}{\frac{\pi r^2}{4} - \frac{\pi R^2}{4}}$$

$$= 0.6(R^3 - r^3) / (R^2 - r^2)$$

Distance of C.G. from face of column

$$\bar{y} = 0.6(R^3 - r^3) / (R^2 - r^2)$$

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$$B.M = W_u \left(\frac{\pi R^2 - \pi r^2}{4} \right) \left[\frac{0.6(R^3 - r^3)}{R^2 - r^2} - r \right]$$

$$= 198.26 \times \frac{\pi}{4} (1.70^2 - 0.25^2) \times \left[\frac{0.6(1.70^3 - 0.25^3)}{1.70^2 - 0.25^2} - 0.25 \right]$$

$$= 347.46 \text{ kN-m}$$

Depth req. = $\sqrt{\frac{B.M}{Q.B_1}}$ (202)

Width $B_1 = \frac{2\pi r}{4} = \frac{2 \times \pi \times 0.25}{4} = 392.7 \text{ mm}$

$d = \sqrt{\frac{347.46 \times 10^6}{1.38 \times 25 \times 392.7}} = 506.42 \text{ mm}$

Say $= 510 \text{ mm} = d$

Check for one way shear - (Single Shear)

Width at critical section

$B_2 = \frac{2\pi(r+d)}{4} = \frac{\pi(250+510)}{2}$

$= 1193.8 \text{ mm}$ ~~596.9 mm~~

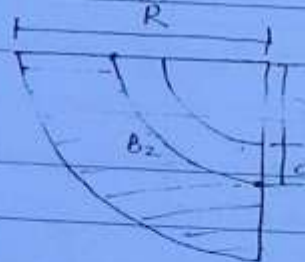
$V_u = \frac{W_u \times \pi}{4} [R^2 - (r+d)^2] \times 10^3$

$= 198.25 \times \frac{\pi}{4} [1.70^2 - (0.25+0.51)^2]$

$= 360.25 \text{ kN}$

$\tau_{vu} = \frac{V_u}{B_1 d} = \frac{360.25 \times 10^3}{1193.8 \times 510}$

$0.32 > 0.29$ fail



Depth req.

$d = \sqrt{\frac{V_u}{\tau_{vu} B_1}} = \sqrt{\frac{360.25 \times 10^3}{0.29 \times 1193.8}} = 1040.6 \text{ mm}$

Say $d = 1040.6 \text{ mm}$ $\approx 1045 \text{ mm}$ Say 1045 mm

$$V_u = 198.25 \times \frac{\pi}{4} [1.70^2 - (0.25 + 0.8)^2]$$

$$= 278.32 \text{ kN}$$

$$B_2 = \frac{\pi(250 + 800)}{2} = 1649.34 \text{ mm}$$

$$\tau_{vu} = \frac{278.32 \times 10^3}{1649.34 \times 800} = 0.21 < 0.29 \text{ (safe)}$$

way

(203)

Check for two-way shear (Punching shear)

Net punching shear stress

$$P_u = W_u \pi \left(r + \frac{d}{2} \right)^2$$

$$2\pi \left(r + \frac{d}{2} \right) \times d$$

$$1.5 \times 1200 \times 10^3 - 198.25 \times \pi \left(0.25 + \frac{0.8}{2} \right)^2 \times 10^3$$

$$2 \times \pi \left(250 + \frac{800}{2} \right) 800$$

$$= 0.47 \text{ N/mm}^2$$

permissible $\tau_{vp,pc}$

$$= 1.25 \text{ N/mm}^2$$

Safe

Area of Steel

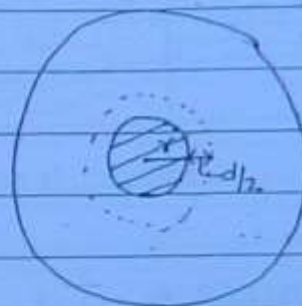
$$A_{st} = \frac{0.5 \times 25}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 347.46 \times 10^6}{25 \times 392.7 \times 800^2}} \right] \times 392.7 \times 800$$

$$= 1291.7 \text{ mm}^2$$

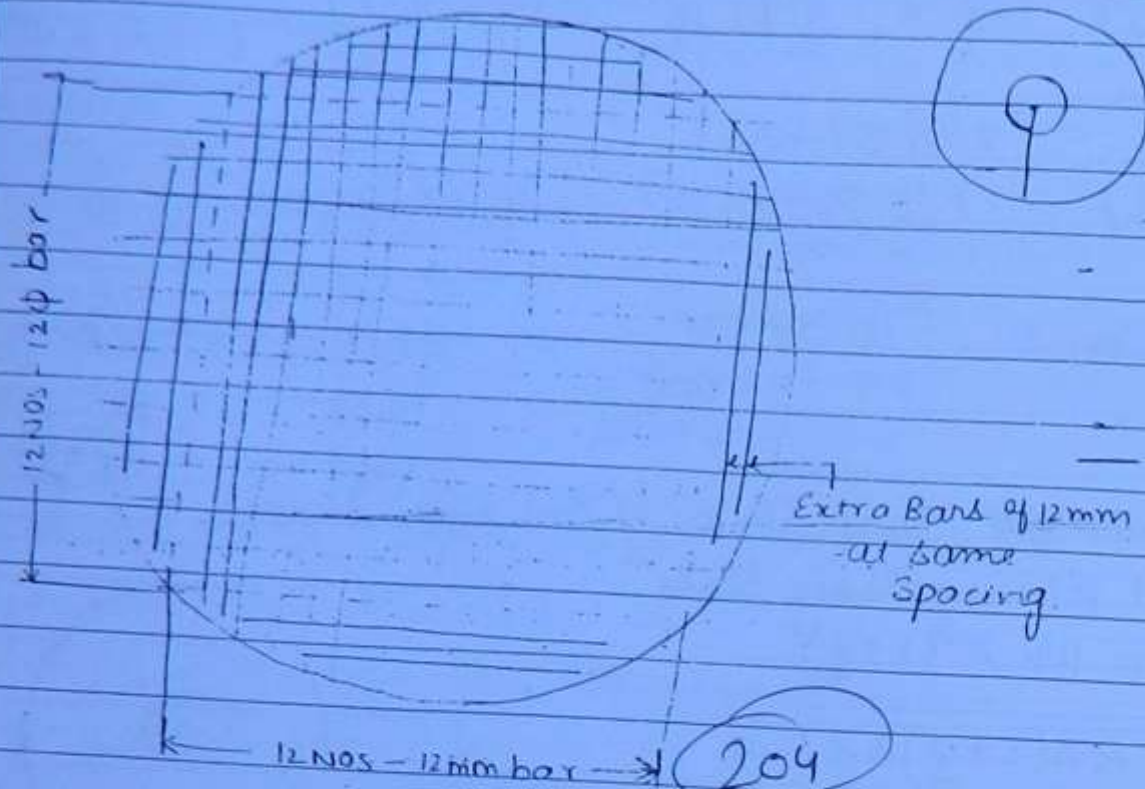
$$\text{No. of } 12 \text{ mm } \phi \text{ bars} = \frac{1291.7 \times 4}{\pi \times 10^2} = 11.42$$

Say 12 Nos.

provide 12 nos of 12mm ϕ bars.



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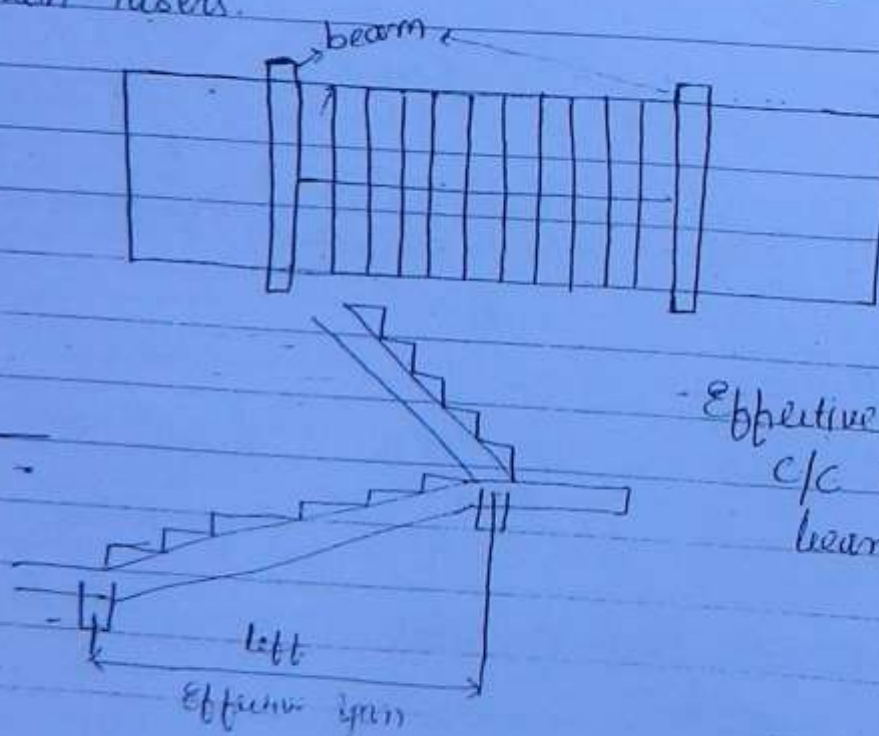
31/10/2011

Staircase:-

IS code provision:- (As per 33.1 / P-63)

Effective span of staircase

- (a) When supported at top and bottom runs spanning parallel with risers.

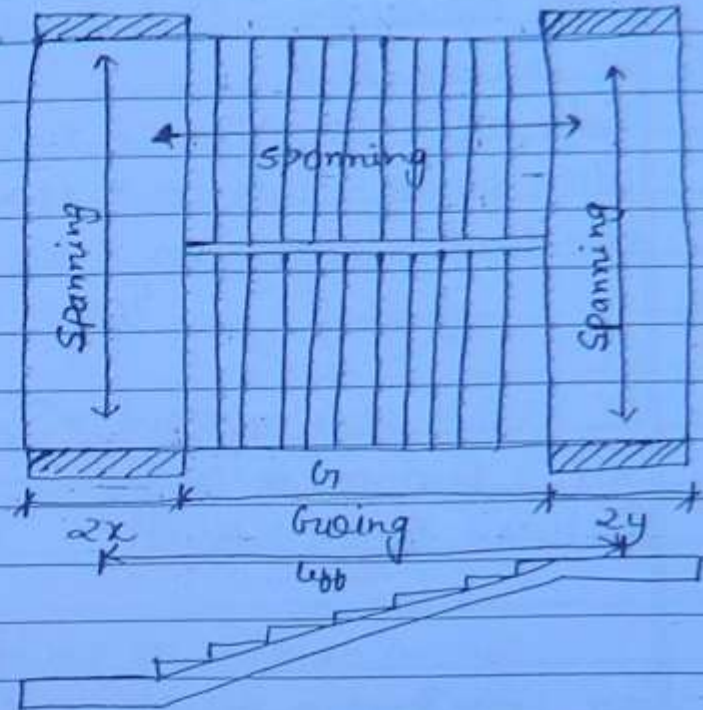


Effective span =
c/c distance betⁿ
beams

- 1b) Landings are supported on side beam walls and waist slab are supported on landings.

Eff Span = $l_{eff} = 205$
 $G_1 + x + y$

Where value of x or y $\neq 1m$



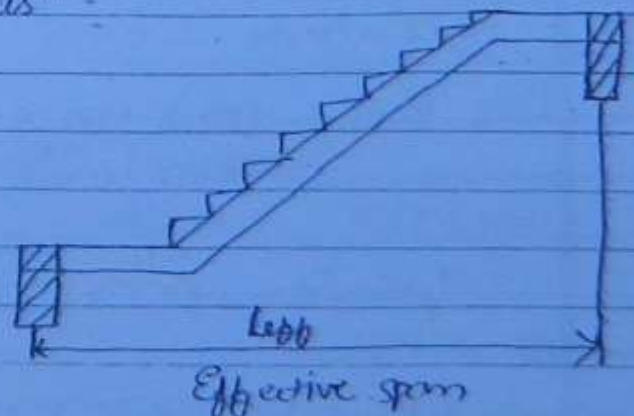
From Code.

x	y	Eff Span
$< 1m$	$< 1m$	$G_1 + x + y$
$\geq 1m$	$< 1m$	$G_1 + 1 + y$
$< 1m$	$\geq 1m$	$G_1 + x + 1$
$\geq 1m$	$\geq 1m$	$G_1 + 1 + 1$

G_1 = going (horizontal length of waist slab)

- 1c) When supported on two beams provided at end landing slab is also spanning in same direction as of waist slab.

Eff span = Centre to centre disⁿ between beams.



Load Calculation:- For 1m width of stair

1. At landing Slab

$$\text{Self wt} = 1\text{m} \times 1\text{m} \times t \times 25 = ?$$

$$\text{Floor finish} = 1\text{m} \times 1\text{m} \times t_f \times 24 = ?$$

$$\text{Live load} = 1\text{m} \times 1\text{m} \times w_l = ?$$

$$\text{Total load} = w_1$$

2. At waist slab portion (266)

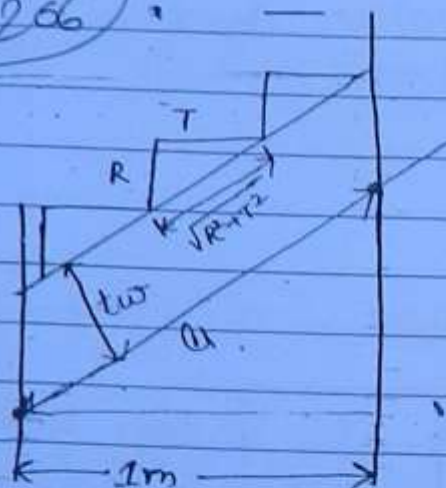
For T horizontal length

Inclined length is

$$\sqrt{R^2 + T^2}$$

For 1m horizontal length
inclined length is

$$= \frac{\sqrt{R^2 + T^2}}{T} \times 1\text{m} = l_1$$



Wt - waist Slab

$$= l_1 \times 1\text{m} \times t_w \times 25$$

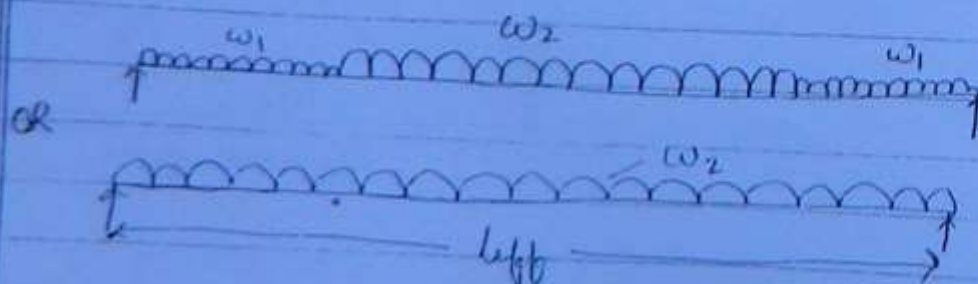
$$\text{Self wt} = \frac{\sqrt{R^2 + T^2}}{T} \times 1\text{m} \times 1\text{m} \times t_w \times 25$$

$$\text{Self wt. Steps} = \left(\frac{1}{T} \right) \times \frac{1}{2} \times R \times T \times 1\text{m} \times 25$$

No. of Steps
if T is metre

$$\text{Live load} = 1\text{m} \times 1\text{m} \times w_l$$

$$\text{Total load} = w_2$$



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$$B.M = \frac{w_2 \times l_{eff}^2}{8}$$

$$d = \sqrt{\frac{B.M}{Q.B}}$$

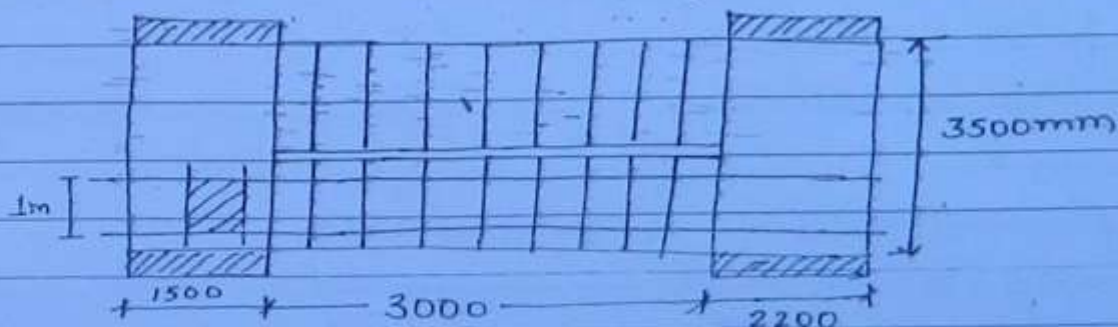
$$A_{st} = \frac{B.M}{\sigma_{st} \cdot j \cdot d}$$

$$= \frac{B.M}{\sigma_{st} \cdot j \cdot d}$$

(207)

$$\text{or } A_{st} = \frac{0.5 f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck} B d^2}} \right] B d$$

Ques Design a dog-legged staircase as shown in fig.
Live load = 5 kN/m^2 & Floor finish = 1 kN/m^2



Rise $r = 160 \text{ mm}$

Treads = 300 mm

Use - M-20 / Fe-415

Use L.S.M.

Solⁿ Effective span :- $l_{eff} = \frac{1500 + 3000 + 2200}{2} = 4850$
 $0.75 + 3.0 + 1 = 4.75 \text{ m}$

Load :-

(Thickness of slab (using deflⁿ Criteria))

$$\text{thickness} = \frac{\text{Span}}{20} = \frac{4750}{20} = 237.5 \text{ mm} + 30 \text{ mm} = 267.5 \text{ mm}$$

Use - 270 mm as thickness

DATE: _____

Landing Slab

Self wt - $1\text{m} \times 1\text{m} \times 0.27 \times 25 = 6.75$

Floor finish = $1 \times 1 \text{m} \times 1 = 1.0$

live load = $1\text{m} \times 1\text{m} \times 5 = 5$

Total = 12.75 KN/m^2

Wrist Slab

(208)

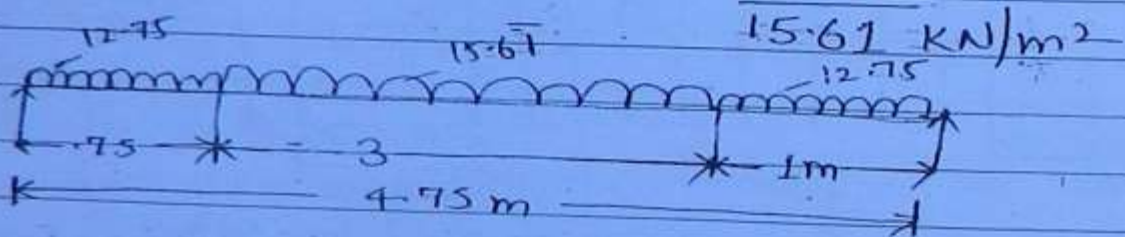
Self wt = $\frac{\sqrt{R^2 + T^2}}{T} \times 1 \times 0.27 \times 25 =$

$1.13 \times 1 \times 0.27 \times 25 = 7.61 \text{ KN/m}^2$

Self wt of Steps = $\left(\frac{1}{T}\right) \times \frac{1}{2} \times R \times T \times 1\text{m} \times 25$
 $= \left(\frac{1000}{300}\right) \times \frac{1}{2} \times 0.16 \times 0.30 \times 1\text{m} \times 25 = 2.0$

Floor finish = $1 \times 1 \times 1 = 1.0$

Live load = $1 \times 1 \times 5 = 5.0$



Max. B.M (Considering same load on entire span)

B.M = $\frac{15.61 \times 4.75^2}{8} = 44.03 \text{ KN-m}$

Factored = $1.5 \times \text{B.M} = 66.038 \text{ KN-m}$

$d = \sqrt{\frac{\text{B.M}}{Q \cdot B}} = \sqrt{\frac{66.04 \times 10^6}{1.38 \times 20 \times 1000}}$

$d = 154.68 \text{ mm} \nless 240 \text{ mm}$
 Safe

Use $\phi = 270 \text{ mm}$

$d = 240 \text{ mm}$

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$$A_{st} = \frac{0.5 \times 20}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 66.04 \times 10^6}{1000 \times 240^2 \times 20}} \right] 1000 \times 240^2$$

$$= 820.75 \text{ mm}^2$$

Use 16mm dia bars

(209)

$$\text{Spacing} = \frac{1000}{820.75} \times \frac{\pi}{4} \times 16^2 = 244.9 \text{ mm}$$

Use 12mm

$$\text{Spacing} = \frac{1000}{820.75} \times \frac{\pi}{4} \times 12^2 = 137 \text{ mm}$$

Use 12mm ϕ bars @ 130 mm c/c

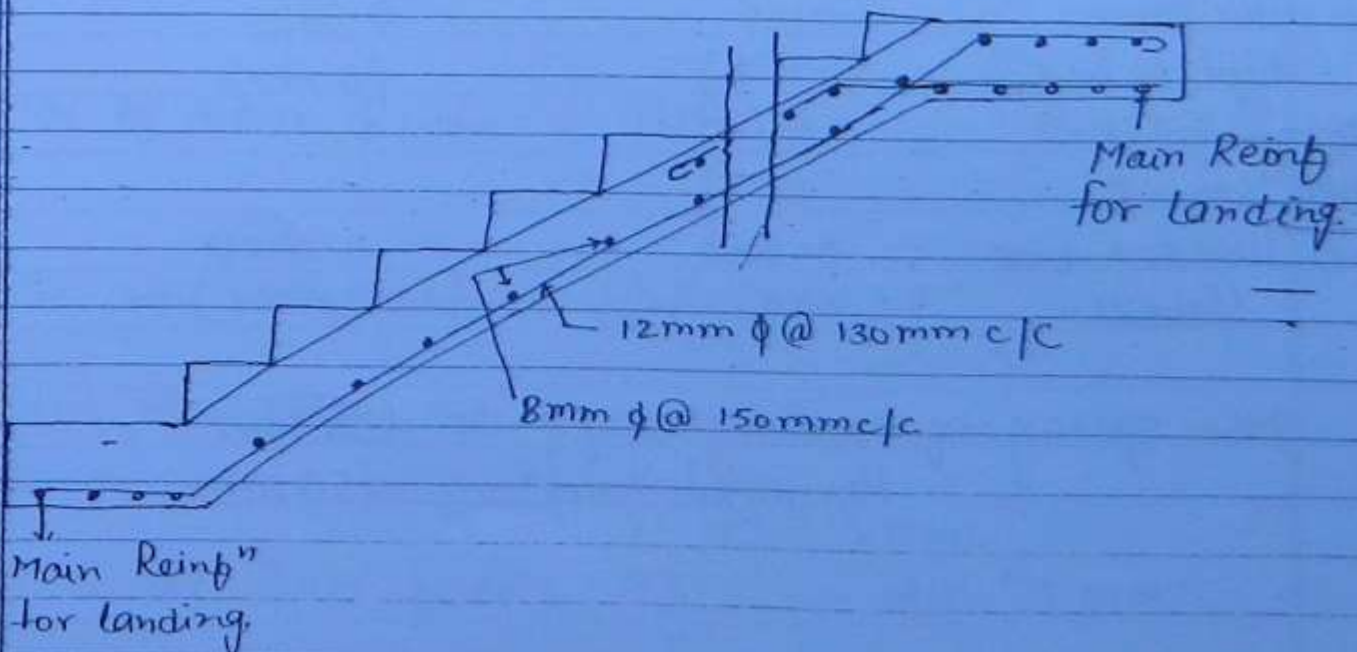
Distribution bars

$$= \frac{0.12}{100} \times 1000 \times 270 = 324 \text{ mm}^2$$

$$\text{Spacing of } 8\text{-mm } \phi \text{ bars} = \frac{1000}{324} \times \frac{\pi}{4} \times 8^2$$

$$= 155 \text{ mm}$$

Use 8mm ϕ bars @ 150 mm c/c



DATE: _____

→ Design of wall (As per 32/P-61)
designed same as column

32.3.3 → Max. eff. height to thickness ratio
 $\frac{H_{eff}}{t} \neq 30$

(210)

Effective height

(a) Restrained against rotation at both end

(i) By floor = $0.75 H_w$

(ii) By intersecting walls = $0.75 L_1$

(b) When not Restrained against rotation

(i) By floor = $1.0 H_w$

By intersecting walls = $1.0 L_1$

32.5 - Min Reinfⁿ required -

Horizontal & Vertical both reinfⁿ are provided.

Vertical Reinfⁿ

Horizontal Reinfⁿ

(i) Deformed
bars
 $\neq 16\text{mm}$

0.0012
(0.12%)

0.0020
(0.20%)

(ii) For other
type of bars
& greater

0.0015
(0.15%)

0.0025
(0.25%)

Mild Steel
than 16mm ϕ)

(iii) Welded wire
fabrics

0.0012
(0.12%)

0.0020
(0.20%)

Max. Spacing of vertical reinfⁿ = 450 mm
Horizontal reinfⁿ = 450 mm

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Ques A reinf concrete wall 175 mm thickness & 3.2 m effective height is needed for a compressive load of 1000 kN/m. Design the wall. Use - M20 / Mild steel reinf.

$P = 1000 \text{ kN}$ for 1m length of wall

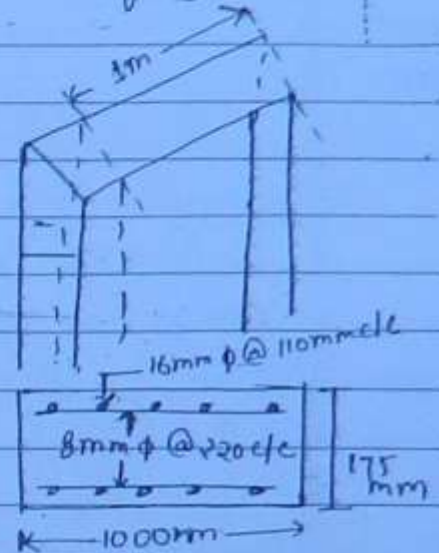
Area = $1000 \times 175 \text{ mm}$

$A_{sc} =$

(211)

$$\frac{H_{eff}}{t} = \frac{3200}{175} = 18.29 \geq 12$$

Design as long column.
Using K.S.M



$$P = C_r (A_c \cdot \sigma_{cc} + A_{sc} \cdot \sigma_{sc})$$

$$\left(1.25 - \frac{1000}{488}\right) [(1000 \times 175 - A_{sc}) \times 4 + A_{sc} \times 130]$$

$$= 1000 \times 10^3 = \left(1.25 - \frac{3200}{48 \times 1000 \times 175}\right) [1000 \times 175 \times 4 - 4A_{sc} + 130A_{sc}]$$

On each face $A_{sc} = \frac{3576.86}{2} = 1788.43$

No. of 16mm ϕ bars = $\frac{1788.43}{\frac{\pi}{4} \times 16^2} = 8.89$ say 9 Nos.

Min vertical reinf = $\frac{1.5 \times 1000 \times 175}{100} = 262.5 \text{ mm}^2$ OK

Min Horizontal reinf = $\frac{0.25 \times 1000 \times 175}{100} = 437.5 \text{ mm}^2$

On each face = $\frac{437.5}{2} = 218.75 \text{ mm}^2$

Spacing of 8mm ϕ bars

$$= \frac{1000}{218.75} \times \frac{\pi}{4} \times 8^2 = 229\text{mm}$$

Tuesday
11/2011

2/2

PRESTRESSED CONCRETE

Concept:- If a steel reinforcement provided in a concrete is applied a force and is anchored after getting an elongation of sl. Now force is removed

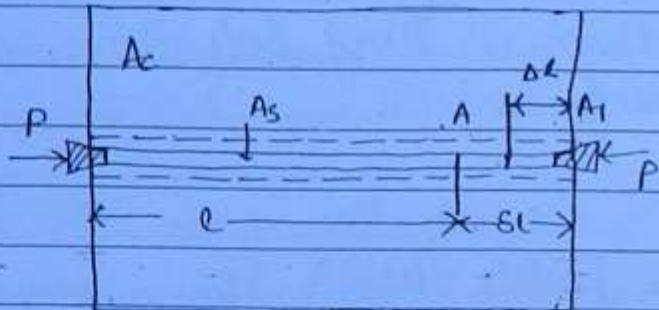
Strain in steel

$$\Rightarrow e = \frac{\Delta l}{l}$$

Stress developed in steel

$$\Rightarrow p_s = e \cdot E_s$$

$$\text{Then } p_s = \frac{\Delta l}{l} E_s$$



If cross-section area of steel = A_s

Force developed due to sl deformation

$$P_s = p_s \times A_s = \frac{\Delta l}{l} E_s \cdot A_s$$

This force $P_s = P$ = Force applied on concrete

It is called prestressing force

P_s = Tensile in steel

P = Compressive in concrete

$$\text{Stress developed in concrete} = p_c = \frac{P}{A_c}$$

If due to any reason elongation provided is reduced, say by Δl . It will cause loss of prestress

If loss of elongation = Δl

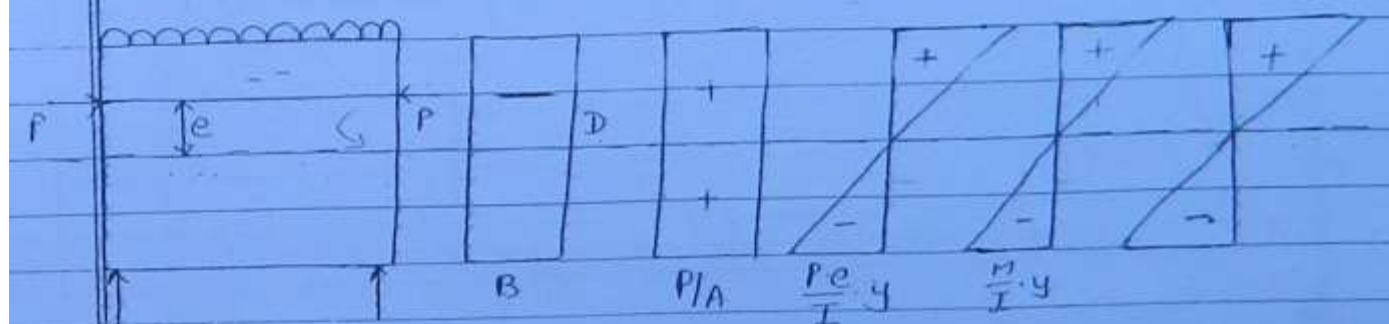
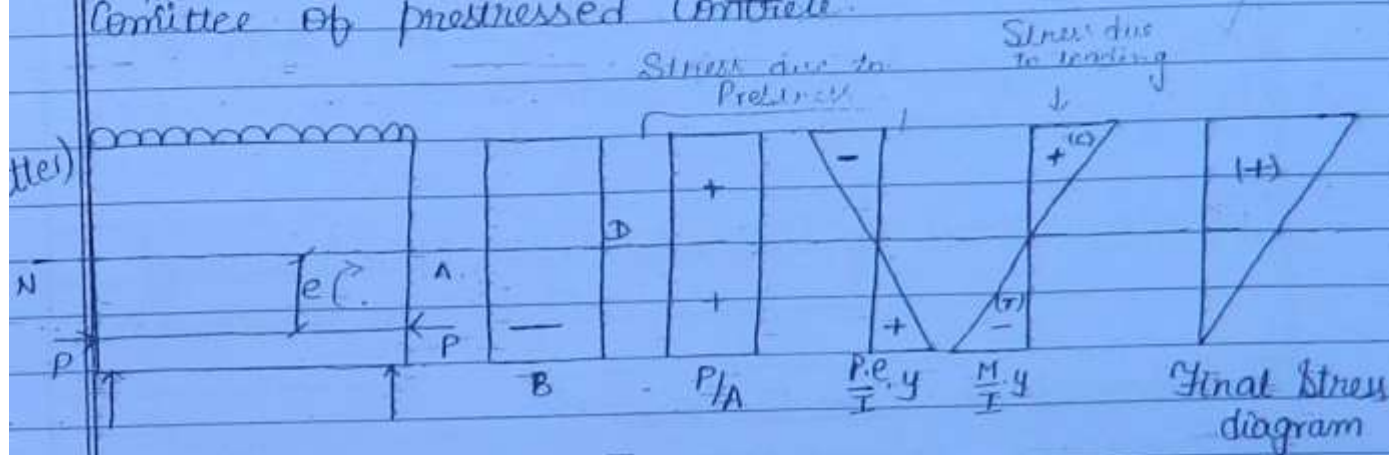
$$\text{loss of strain} = \frac{\Delta l}{l}$$

$$\text{loss of stress or prestress} = \frac{\Delta l}{l} E_s$$

$$\text{Net (Final) Stress available} = \frac{S_1 - A_1}{e} E_s$$

(213)

Definition - "Prestress Concrete is one in which there have been introduced internal stresses of such magnitude and distribution, that stresses caused due to external loading are counterbalance upto a desired degree" As per A.C.I. Committee of prestressed Concrete.



Necessity of high strength Steel and Concrete

⑧ Steel - In case of steel, losses are very high. Only due to Creep & shrinkage loss of strain = 0.0008.

$$\begin{aligned} \text{loss of stress due to Creep and shrinkage} &= 0.0008 \times E_s \\ &= 0.0008 \times 2 \times 10^5 \\ &= 160 \text{ N/mm}^2 \end{aligned}$$

Total losses may ^{not} be in the range of 200 to 300 N/mm²

→ The losses should be more than 10 to 20% of initial stresses provided in steel.

DATE: _____

→ Initial stress required —

1000 N/mm^2 to 2000 N/mm^2

Due to above reason we need a very high strength steel for PSC.

(214)

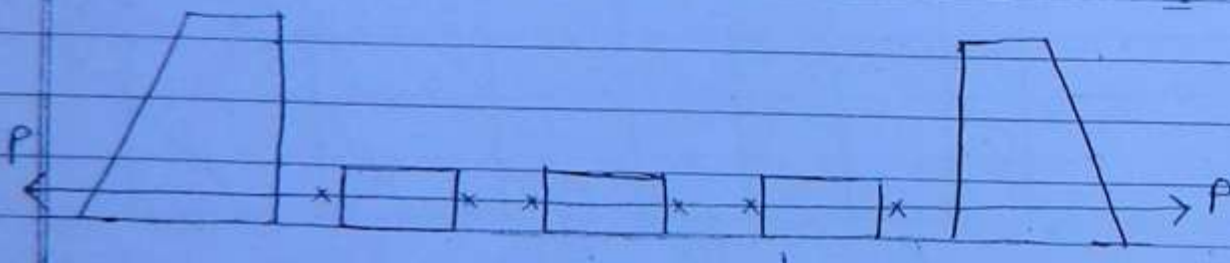
⑦ Concrete —

1. To bear high bearing stresses developed at ends.
2. In case of pre tensioned members to bear high bond stress developed betⁿ steel and concrete.
3. To reduce creep and shrinkage losses.

→ Minimum grade of concrete — M-30 (Post tensioned)
— M-40 (Pre tensioned)

Pretensioned and post tensioned

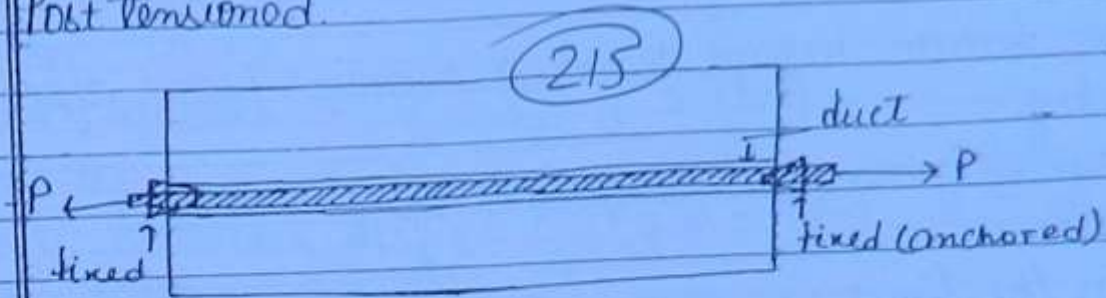
⇒ Pretensioned — (Hoyer Method or long line Method)



Properties

- (i) Steel is in direct contact with concrete
- (ii) Steel is tensioned before casting of concrete
- (iii) After setting of concrete steel reinforcement are cut
- (iv) Prestressing force is applied due to bond action betⁿ steel and concrete.

→ Post tensioned.



• Properties

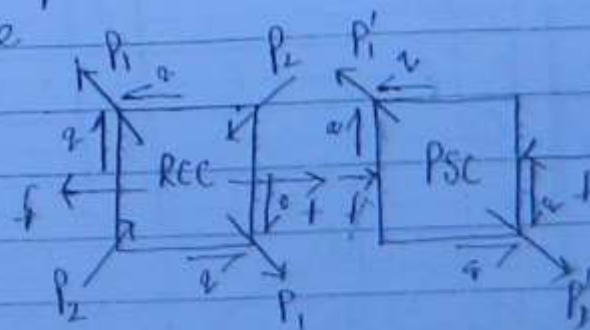
- (i) Concrete is casted before prestressing of steel.
- (ii) A duct is left in concrete during casting.
- (iii) After setting of concrete, steel reinforcement (tendons) are inserted in duct and tensioned either from one end or both ends, and anchored.
- (iv) Steel and concrete are not in direct contact.

→ Advantage of using prestressed concrete

- (i) Cross-section of concrete is utilised more effectively & entire cross-section is in compression (minor tensile stresses can be ignored).
- (ii) In case of PSC, lighter c/s are required. Saving material in supporting structure and foundation due to lighter weight of superstructure.
- (iii) Improve shearing resistance.

$$p_1 = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$

$$p'_1 = -\frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2}$$



Assumption

1. Plane section before bending remains plane after bending.

→ Strain diagram is uniform (216)

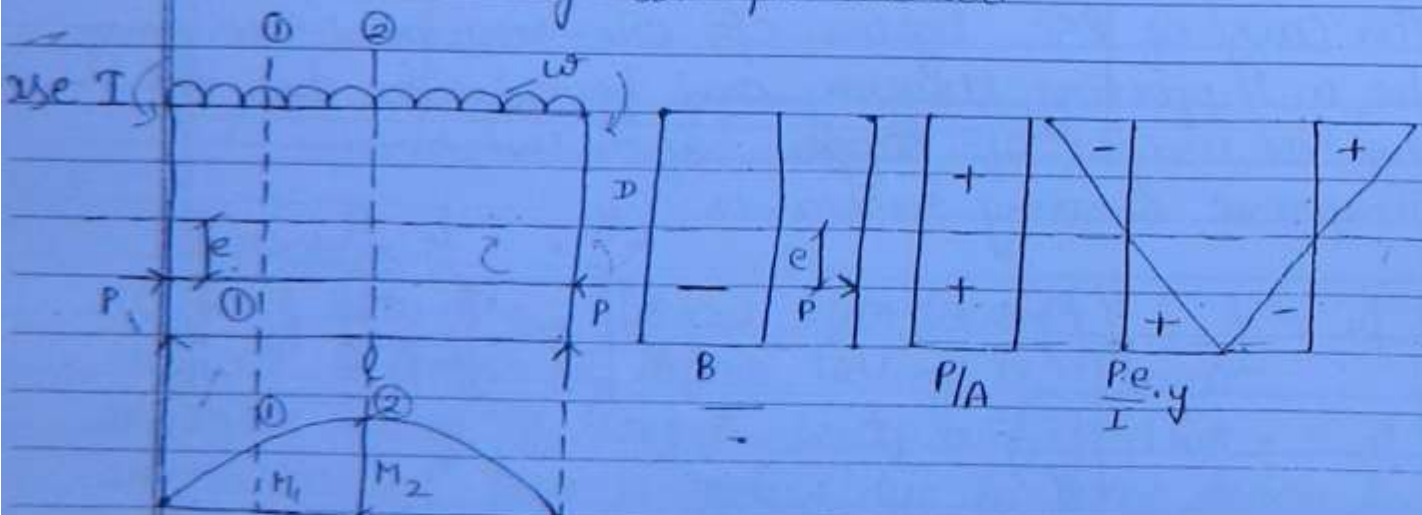
2. Within the limit of deformation hooke's law is valid applicable to Concrete & Steel

3. The stresses in reinforcement doesn't change along the length of reinforcement, stress changes takes place only for concrete.

Variation of stress in reinfⁿ due to change in external loading is ignored.

Basic Concepts

1. Stress Concept method
2. Strength Concept method
3. Load balancing concept method

Stresses developed

1. Direct stress due to $P = + P/A = P_0$
2. Bending stresses due to $P_e = P_{br} = \pm \frac{P_e \cdot y}{I}$

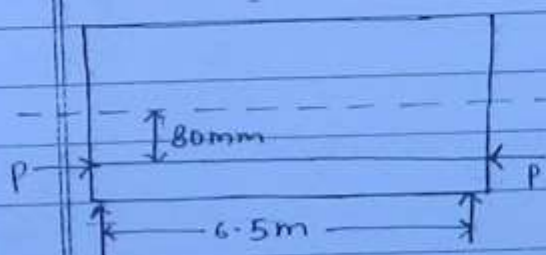
Moment due to load at $\bar{AT} = + \frac{M \cdot y}{I}$
Secⁿ ①-①

→ Final stresses = $\frac{P}{A} - \frac{P \cdot e \cdot y}{I} + \frac{M \cdot y}{I}$
At top

Final stresses = $\frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M \cdot y}{I}$ (217)
at bottom

Final stresses = $\frac{P}{A} + \frac{P \cdot e \cdot y}{I} + \frac{M \cdot y}{I}$

A prestress concrete beam 250mm x 300mm is prestress by 14 wires of 5 mmφ (straight cable) provided at an eccentricity of 80mm. Initial prestress = 1500 N/mm². If span is 6.5m & beam is subjected to live load of 8 kN/m. Calculate max. stress developed in beam at mid span at end. ^{losses} ~~losses~~ are considered.



$P = A_s \times P_s = \frac{14 \times \pi}{4} \times 5^2 \times 1500 = 413.33 \text{ kN}$

$e = 80 \text{ mm}$

loads - D.L = $0.25 \times 0.30 \times 1 \times 25$

L.L = 8 kN/m

total = 9.875 kN/m

Max. B.M = $\frac{wL^2}{8} = m_2 = \frac{9.875 \times 6.5^2}{8} = 52.15 \text{ kN-m}$

Area = $250 \times 300 = 75 \times 10^3 \text{ mm}^2$

$I = \frac{250 \times 300^3}{12} = 5625 \times 10^5 \text{ mm}^4$

$y = \frac{300}{2} = 150$

$y_t = 150 \text{ mm}$

$y_b = 150 \text{ mm}$

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At ends

$$\text{Stress at top} = \frac{P}{A} - \frac{P \cdot e \cdot y}{I}$$

$$\frac{412.33}{75 \times 10^3} - \frac{412.33 \times 80}{5625 \times 10^5} \times 150$$

$$= -3.296 \text{ N/mm}^2$$

218

$$\text{Stress at bottom} = \frac{P}{A} + \frac{P \cdot e \cdot y}{I}$$

$$\frac{412.33}{75 \times 10^3} + \frac{412.33 \times 80}{5625 \times 10^5} \times 150 = 14.296 \text{ N/mm}^2$$

Stress at mid span at top

$$= \frac{P}{A} - \frac{P \cdot e \cdot y}{I} + \frac{M_2 \cdot y}{I}$$

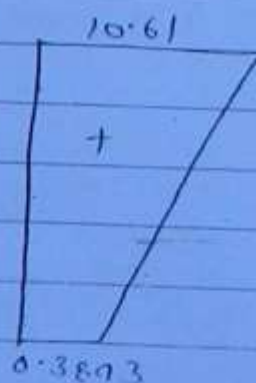
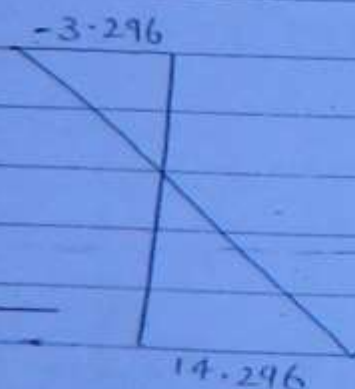
$$-3.296 + \frac{52.15 \times 10^6}{5625 \times 10^5} \times 150 = 10.61 \text{ N/mm}^2$$

At bot

$$\text{At bottom} = \frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M_2 \cdot y}{I}$$

$$14.296 - 13.90 = 0.3893$$

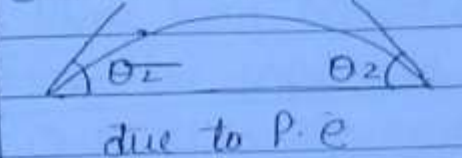
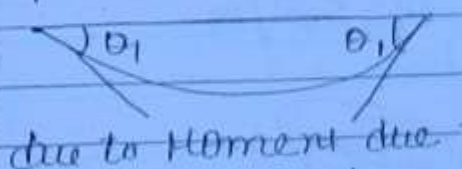
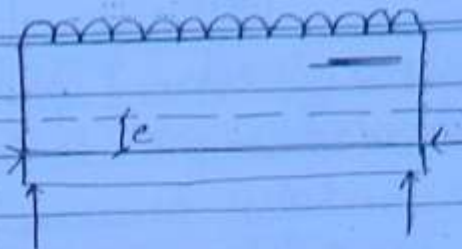
At end



(#) Effect of loading on stresses in tendons

Slope of beam at end due to U.D.L = θ

$$\theta_1 = \frac{wL^3}{24EI}$$



(219)

Slope of beam at ends due to moment $P.e$

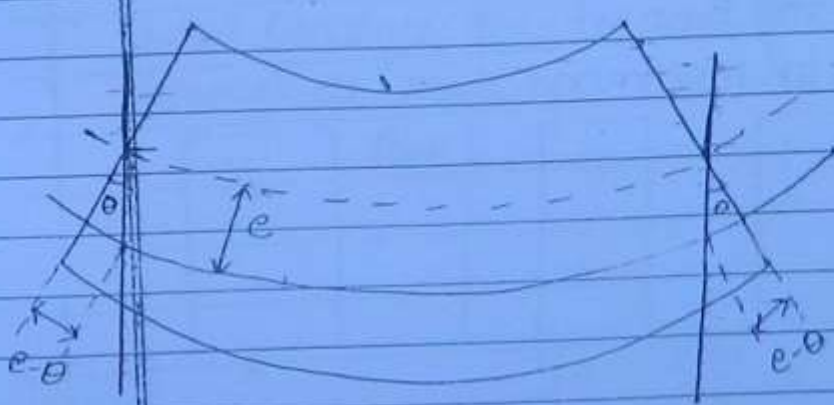
$$\theta_2 = \frac{P.e.l}{2EI}$$

(using mmT area method)

Net Slope $\theta = \theta_1 - \theta_2$

$$\theta = \frac{wl^3}{24EI} - \frac{P.e.l}{2EI}$$

Net Slope due to combined effect



total increase in length
= $2e\theta$

Total increase in length of steel π/t (tendons)

$$\Delta l = (+) 2.e.\theta$$

gain in stress due to Δl increase in elongation

$$\text{stress increases} = \frac{\Delta l}{l} \times E_s$$

DATE: _____

Ques

A PSC beam of span 6m has a width of 150 mm & depth = 300 mm. Initial stress = 1000 N/mm². Cable is at constant $e = 50$ mm. $A_s = 100 \text{ mm}^2$. Find % increase in stress in wires if beam supports live load of 5 kN/m. Unit wt. of concrete = 24 kN/m³.

$$E_c = 36 \text{ kN/mm}^2$$

$$E_s = 210 \text{ kN/mm}^2$$

220

Soln

$$\text{load} - D.L = 0.15 \times 0.30 \times 1 \times 24 = 1.08$$

$$L.L = 5 \text{ kN/m}$$

$$\text{total} = 6.08 \text{ kN/m}$$

$$P = A_s \times P_s = 100 \times 1000$$

$$e = 50$$

$$\text{Slope due to load} = \frac{wl^3}{24EI}$$

$$A = 150 \times 300 = 45 \times 10^3 \text{ mm}^2$$

$$I = \frac{150 \times 300^3}{12}$$

$$\frac{wl^3}{24EI} =$$

$$= 0.0045$$

$$\text{Slope due to moment} = \frac{P_e \cdot l}{2EI}$$

$$= 1.2345 \times 10^{-3}$$

$$\text{Net slope (sagging)} \theta = \theta_1 - \theta_2$$

$$= 3.269 \times 10^{-3}$$

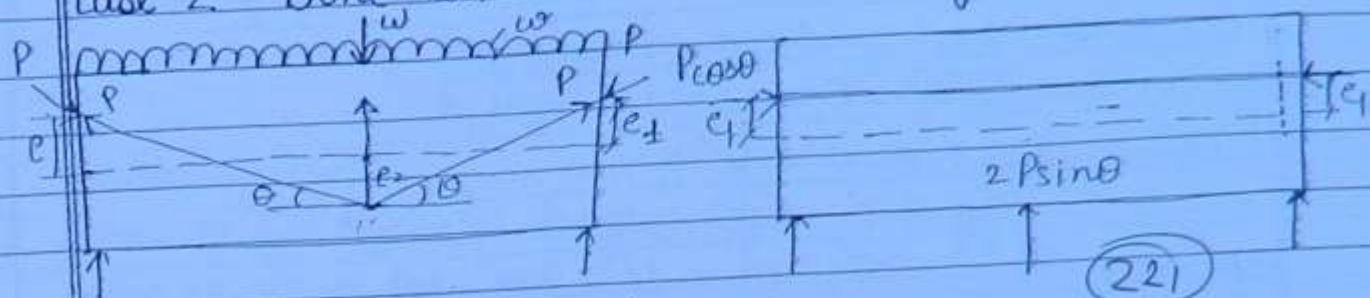
$$\text{Gain in strength} = \frac{\Delta l}{l} \times E_s$$

$$= \frac{2 \cdot e \cdot \theta}{l} \times E_s$$

$$= 11.4415 \text{ N/mm}^2$$

$$\% \text{ increase} = \frac{11.44}{1000} \times 100 = 1.144$$

Case-2. Bent tendons (load balancing concept method)



Using load balancing Concept.

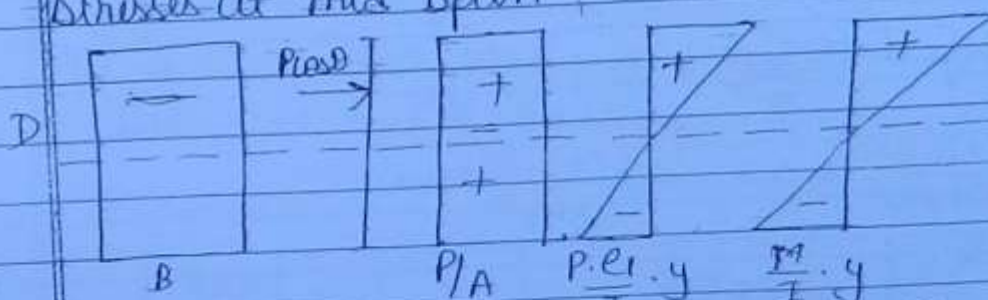
1. Make the profile of cable straight from the point where it is at end

2. Upward balancing load due to profile of cable (Point load) = $2P \sin \theta$ ($\uparrow$)

downward load = kl

Net point load = $(kl - 2P \sin \theta)$

Now Analyse the beam for equivalent beam shown stresses at mid span:



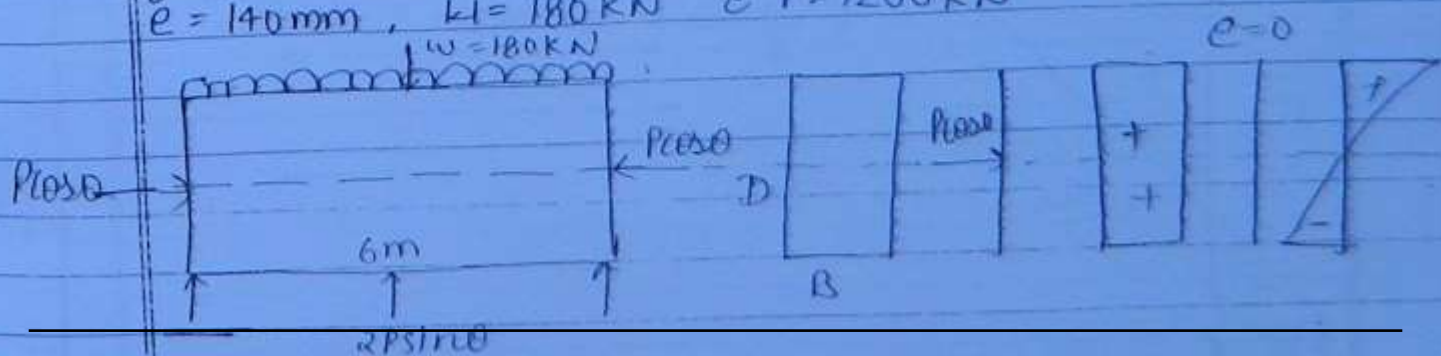
Where moment $M = \frac{w \cdot l^2}{8} + \frac{P \sin \theta \cdot l}{4}$

Final Stresses = $\frac{P}{A} + \frac{P \cdot e_1 \cdot y}{I} + \frac{M \cdot y}{I}$

Es-2005
Ques

A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at mid span considering self wt also.

$e = 140 \text{ mm}$, $kl = 180 \text{ kN}$ & $P = 1200 \text{ kN}$



DATE: _____

$$P = 1200 \text{ kN}$$

$$\theta = \tan^{-1} \frac{140}{3000} = 0.0467 \text{ radian}$$

$$\theta = 2^\circ 40' 18.71''$$

$$P \cos \theta = 1200 \times \cos 2^\circ 40' 18.71''$$

$$= 1198.99 \approx 1200 \text{ kN}$$

(222)

$$\text{Upward force} = 2P \sin \theta = 111.878 \text{ kN}$$

$$\text{net point load} = w - 2P \sin \theta = 180 - 111.88$$

$$= 68.12 \text{ kN}$$

$$\text{Self wt} = 0.4 \times 0.6 \times 1 \times 25 = 6 \text{ kN/m} = w$$

$$\text{B.M at mid span} = \frac{(w - 2P \sin \theta) L^2}{8} + \frac{wL^2}{8}$$

$$\frac{68.12 \times 6^2}{8} + \frac{6 \times 6^2}{8}$$

$$= 129.18 \text{ kN-m}$$

$$A = 400 \times 600 = 24 \times 10^4 \text{ mm}^2$$

$$I = \frac{400 \times 600^3}{12} = 72 \times 10^8 \text{ mm}^4$$

$$\text{Stresses at top} = \frac{P \cos \theta}{A} + 0 + \frac{M \cdot y}{I}$$

$$= \frac{1200 \times 10^3}{24 \times 10^4} + 0 + \frac{129.18 \times 10^6}{72 \times 10^8} \times 300$$

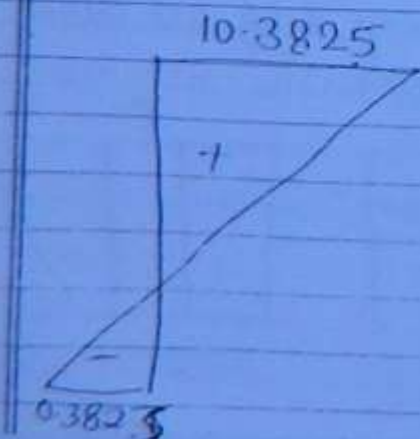
$$5 + 5.3825$$

$$= 10.3825 \text{ N/mm}^2$$

$$\text{Stresses at bottom} = \frac{1200 \times 10^3}{24 \times 10^4} - \frac{M \cdot y}{I}$$

$$5 - 5.3825$$

$$= -0.3825 \text{ N/mm}^2$$



Tendon of parabolic profile

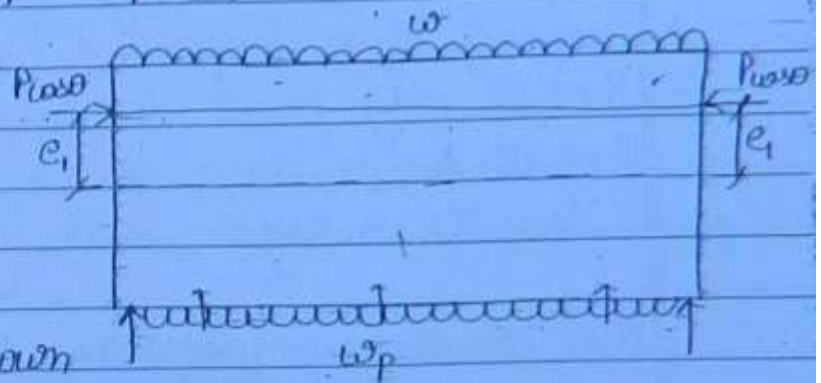
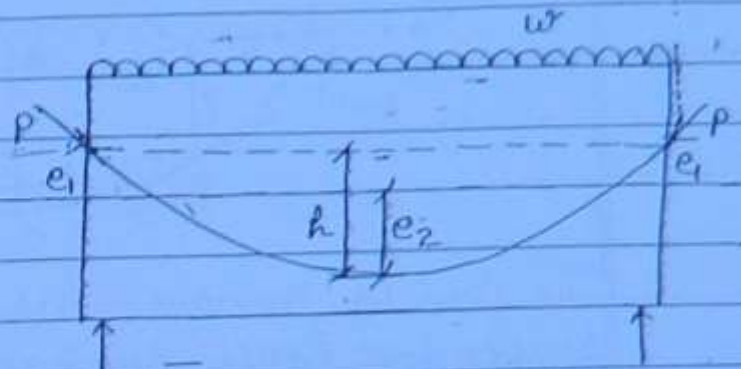
→ (Load balancing concept)

In this case 'due to load' balancing :-

1. Make profile of cable straight from pt. where the cable lie at ends
2. An upward balancing load (U.d.l) may be applied

$$\text{upward } w_p = \frac{8Ph}{l^2}$$

U.d.l

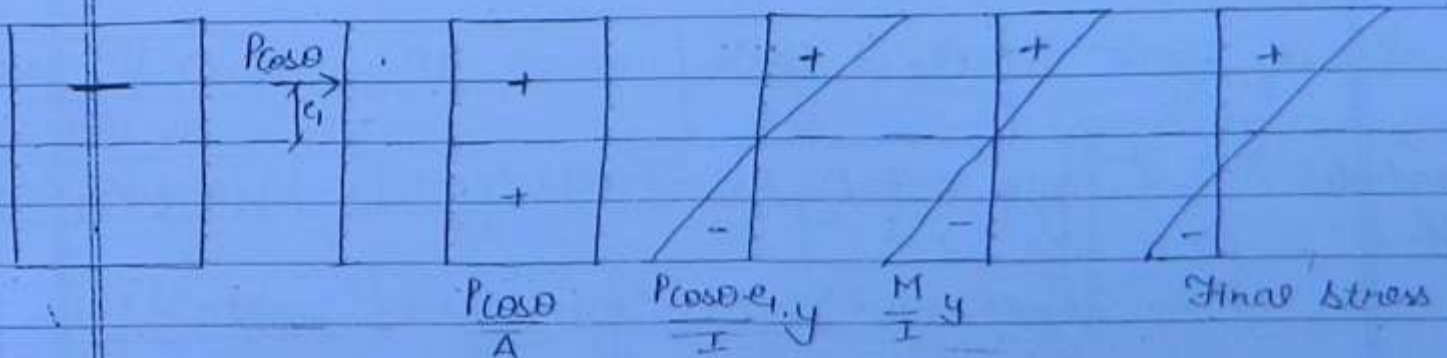


Not equivalent beam as shown in fig. can be analysed.

$$\text{Net U.d.l} = (w - w_p)$$

$$\text{B.M} = M = \frac{(w - w_p)l^2}{8}$$

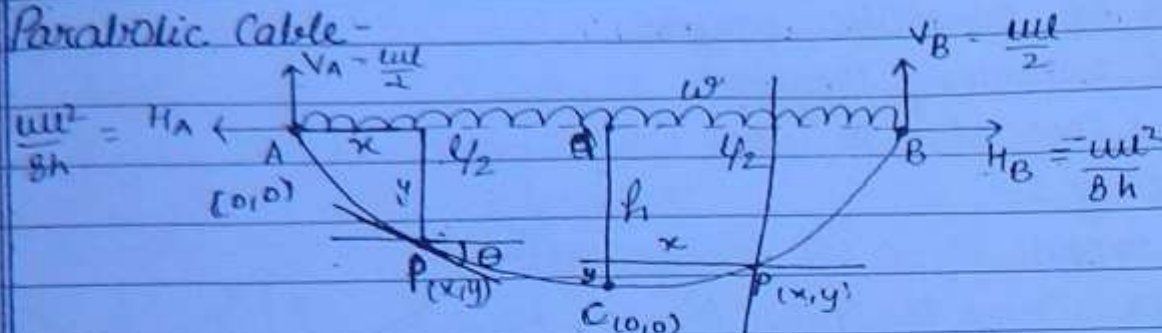
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Final Stresses

$$= \frac{P\cos\theta}{A} + \frac{P\cos\theta \cdot e_1}{I} y + \frac{M}{I} y$$

Parabolic Cable -



The Case of parabolic cable is similar to the Case of a suspended cable, subjected to U.d.l. (w)

→ Property of a suspended cable.

∴ B.M at any point in suspended cable = 0.

B.M at C = 0

$$V_A = V_B = \frac{wl}{2}$$

224

$$\text{B.M at C} = \frac{wl^2}{8} + H_B \cdot h - V_B \times \frac{l}{2} = 0$$

$$\frac{wl^2}{8} + H_B \cdot h - \frac{wl^2}{4} = 0$$

$$H_B \cdot h = \frac{wl^2}{8}$$

$$H_B = \frac{wl^2}{8h}$$

For parabolic cable $H_B = P \cos \theta \approx P$

$$w = w_p$$

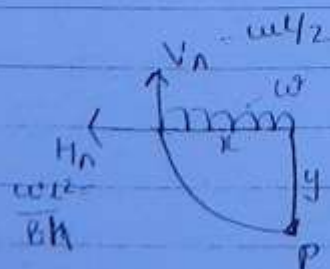
$$P = \frac{w_p l^2}{8h}$$

$$w_p = \frac{8Ph}{l^2}$$

Equation of parabolic tendon

Case 1 Origin is at ends.

B.M at P = 0



$$\frac{w \cdot x \cdot x}{2} + H_A \cdot y - V_A \cdot x = 0$$

$$\frac{wx^2}{2} + \frac{wl^2}{8h} y - \frac{wlx}{2} = 0$$

$$\frac{wl^2}{8h} y = \frac{wlx}{2} - \frac{wx^2}{2}$$

$$\frac{l^2}{4h} y = lx - x^2$$

$$\frac{l^2}{4h} y = x(l-x)$$

(225)

$$y = \frac{4hx(l-x)}{l^2}$$

Eq. of parabolic Cable
(also called as deflection eqⁿ)

$$\tan \theta = \frac{dy}{dx} = \frac{4h(l-2x)}{l^2}$$

Slope equation

Slope at ends at $x=0$ or $x=l$

$$\frac{dy}{dx} = \tan \theta = \frac{4h}{l} = 0 \quad \text{Slope at end}$$

If origin is at centre say C (0,0) then $V_A = \frac{wl}{2}$

B.M at P = 0

$$\frac{w}{2} \left(\frac{l}{2} - x \right) \left(\frac{l}{2} - x \right) + \frac{wl^2}{8h} (h-y) - \frac{wl}{2} \left(\frac{l}{2} - x \right) = 0$$

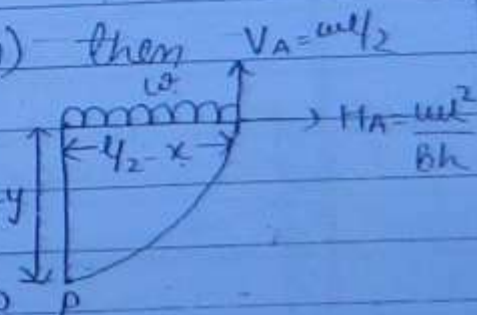
$$\frac{w}{2} \left(\frac{l^2}{4} - lx + x^2 \right) + \frac{wl^2}{8} - \frac{wl^2}{8h} y - \frac{wl^2}{4} + \frac{wlx}{2} = 0$$

$$\frac{wl^2}{8} - \frac{wlx}{2} + \frac{wx^2}{2} + \frac{wl^2}{8} - \frac{wl^2}{8h} y - \frac{wl^2}{4} + \frac{wlx}{2} = 0$$

$$\frac{wx^2}{2} = \frac{wl^2}{8h} y$$

$$\frac{4x^2 h}{l^2} = y$$

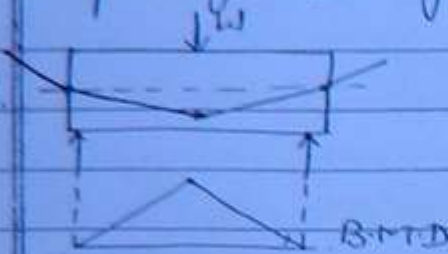
Eq. of Cable if origin is at Centre



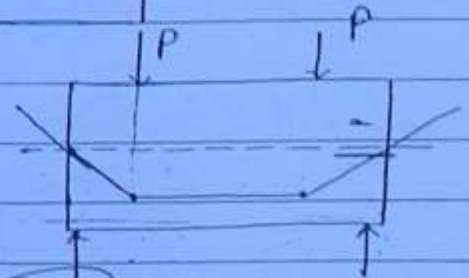
DATE: _____

- Shape of a load balancing cable.
⇒ Shape of a load balancing cable is similar to the shape of bending moment diagram (B.M.D)

Case 1.



$$w = 2P \sin \theta$$

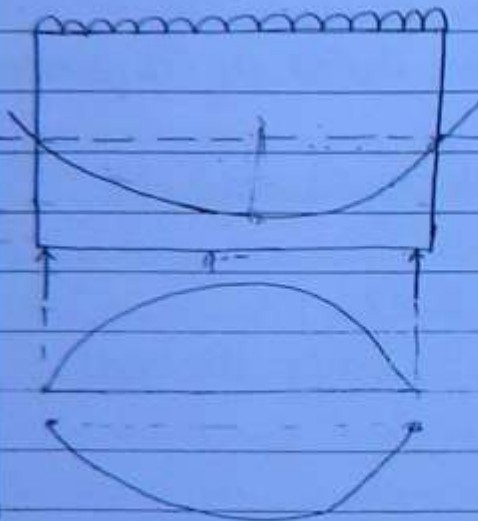


Mirror image of B.M.D

Shape of cable

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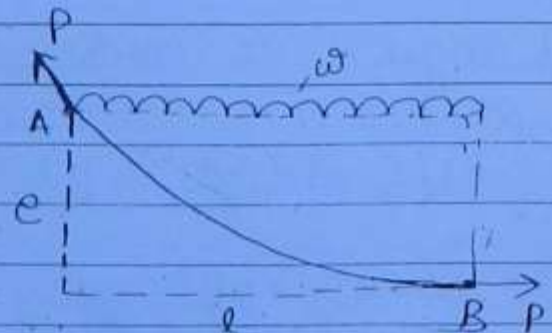
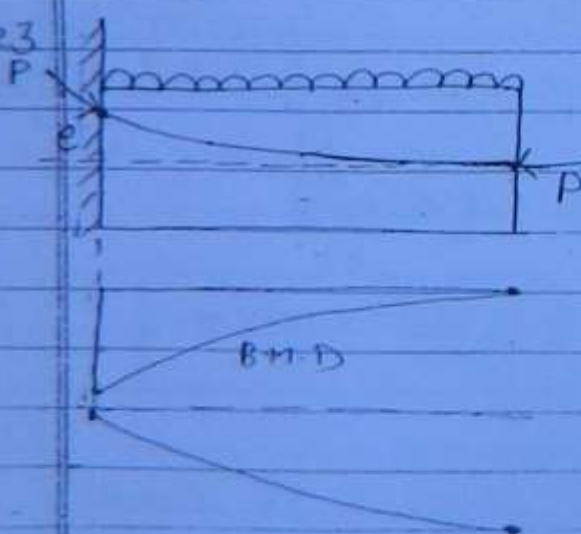
Case 2.



$$w_p = \frac{8Ph}{l^2}$$

$$h = \frac{w_p l^2}{8P}$$

Case 3.



$$B.M. \text{ at } A = 0$$

$$P \cdot e = \frac{w l^2}{2}$$

$$e = \frac{w l^2}{2P}$$

DATE: _____

Ques

Determine the profile of a load balancing cable for a cantilever beam of the span 4m carrying an U.D.L of 12 kN/m . Prestressing force is 1500 kN .

Sol

For a cantilever shape is shown on left hand side page. Shape of cable is mirror image of B.M.D as shown.
eccentricity at support = $e = \frac{wL^2}{2P}$

$$e = \frac{12 \times 4^2}{2 \times 1500} = 0.064 \text{ m}$$

$$e = 64 \text{ mm}$$

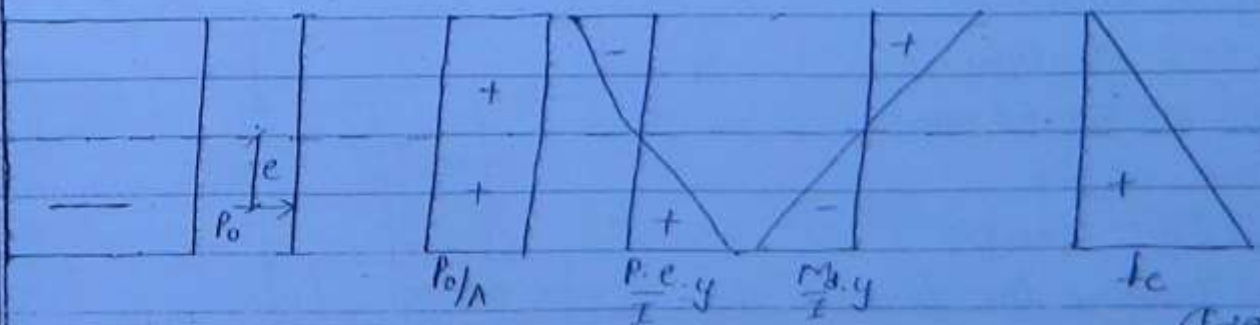
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Stresses in a beam at different stage of loading

1. At transfer
2. At final stage of loading

→ At transfer:- Just after transfer of prestressing force in the beam.

1. Losses of prestress are not considered.
2. Live load is not considered.
3. Dead load may be considered (if not supported).
4. Dead load not to be considered if beam is still on its prestressing bed.



Ideal
Stress diagram
at transfer

→ At final stage

1. Losses are to be considered

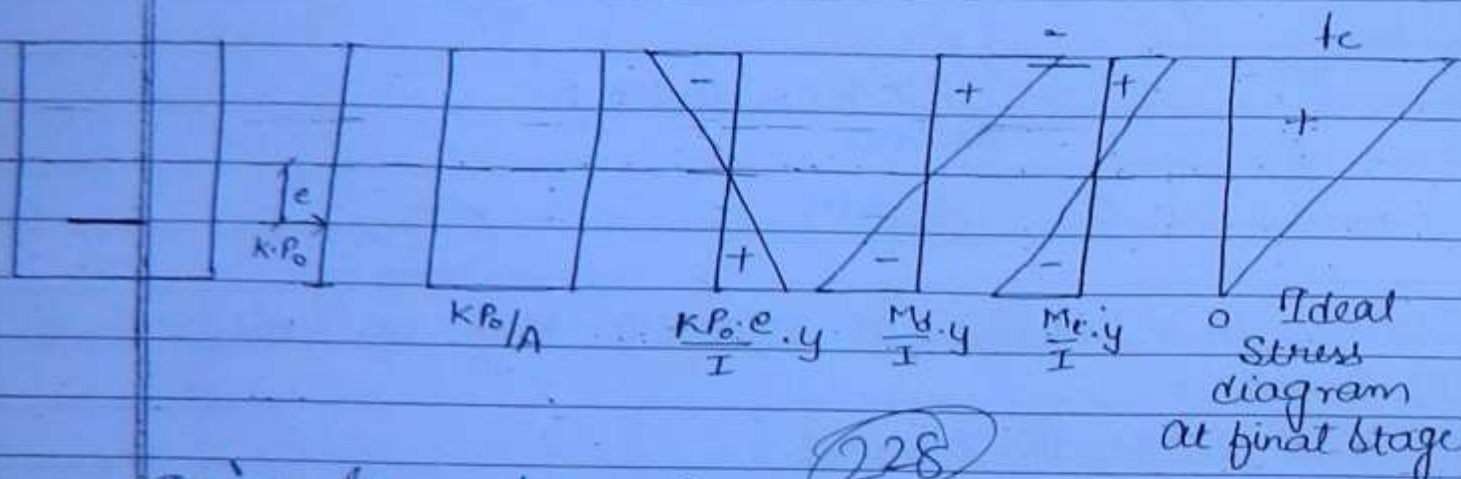
$$\text{Final } P = k P_0$$

$$K = \text{loss factor } (1 - \text{loss}\%)$$

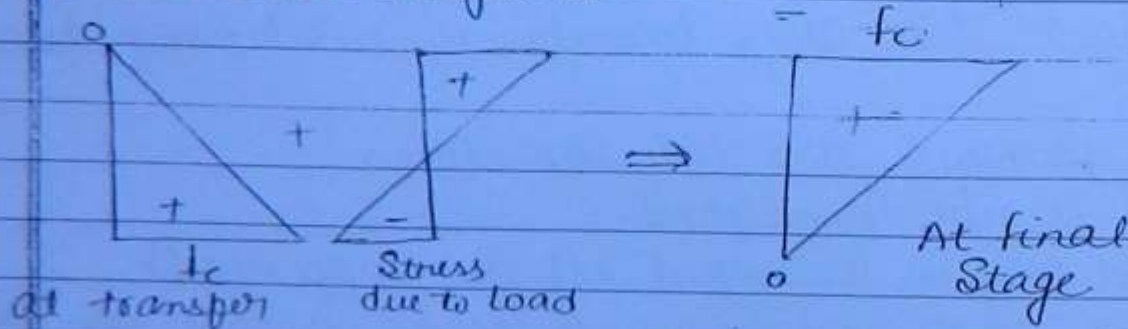
if loss = 20%

$$K = 0.80$$

2. All ~~loss~~ ^{loads} are to be considered.



Ideal Stress diagram



Ques

A post tensioned prestress Concrete is as shown in fig.

Span = 20m

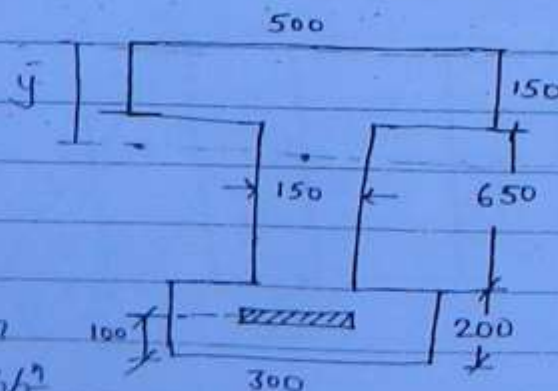
$$p = 1100 \text{ N/mm}^2$$

$$A_s = 1385.44 \text{ mm}^2$$

loss = 15%

$$U.d.l = 12 \text{ kN/m}$$

Determine the stresses in beam at mid span at diff² stage of loading



Sol. $A = 500 \times 150 + 650 \times 150 + 200 \times 300 = 232500 \text{ mm}^2$
 $\bar{y} = \frac{500 \times 150 \times 75 + (150 \times 650 \times 475) + (300 \times 200 \times 900)}{232500}$

$\bar{y} = y_t = 455.65 \text{ mm}$

$y_b = 1000 - 455.65 = 544.35 \text{ mm}$

$$I = \left[\frac{500 \times 150^3}{12} + 500 \times 150 \times (455.65 - 75)^2 + \frac{150 \times 650^3}{12} + 150 \times 650 \times (455.65 - 475)^2 + \frac{300 \times 200^3}{12} + 300 \times 200 \times (455.65 - 900)^2 \right]$$

$I = 2.6524 \times 10^{10} \text{ mm}^4$

(229)

Dead load = $\frac{232500}{10^6} \times 25 = 5.8125 \text{ kN/m}$

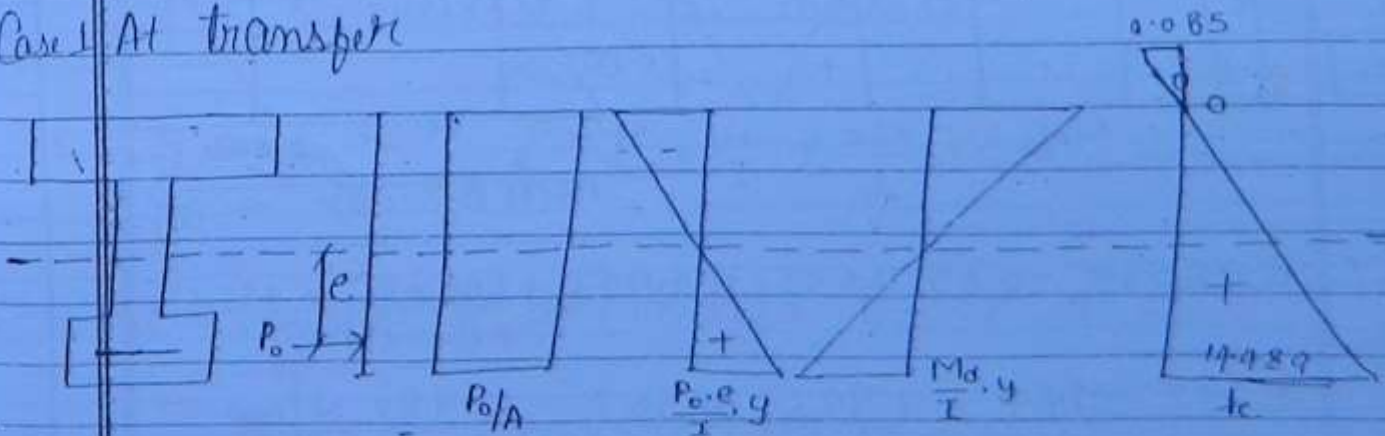
$M_d = \frac{w_d l^2}{8} = \frac{5.8125 \times 20^2}{8} = 290.625 \text{ kN-m}$

Live load = 12 kN/m

$M_l = \frac{12 \times 20^2}{8} = 600 \text{ kN-m}$

$e = y_b - 100$
 $= 444.35 \text{ mm}$

Case 1 At transfer



Prestressing force $P_0 = A_s \times p_s = 1385.44 \times 1100$
 $= 1523984 \text{ N}$

Stress at top

$$= \frac{P_0}{A} - \frac{P_0 \cdot e \cdot y_t}{I} + \frac{M_d \cdot y_t}{I}$$

$$= \frac{1523984}{232500} - \frac{1523984 \times 44.35}{2.6524 \times 10^{10}} \times 455.65 + \frac{290.625 \times 44.35}{2.6524 \times 10^{10}}$$

$$= 6.555 - 11.633 + 4.993 = -0.085 \text{ N/mm}^2$$

Stress at bottom

$$= 6.555 + 11.633 - 4.993 = 600$$

$$= \frac{P_0}{A} + \frac{P_0 \cdot e \cdot y_b}{I} - \frac{M_d \cdot y_b}{I}$$

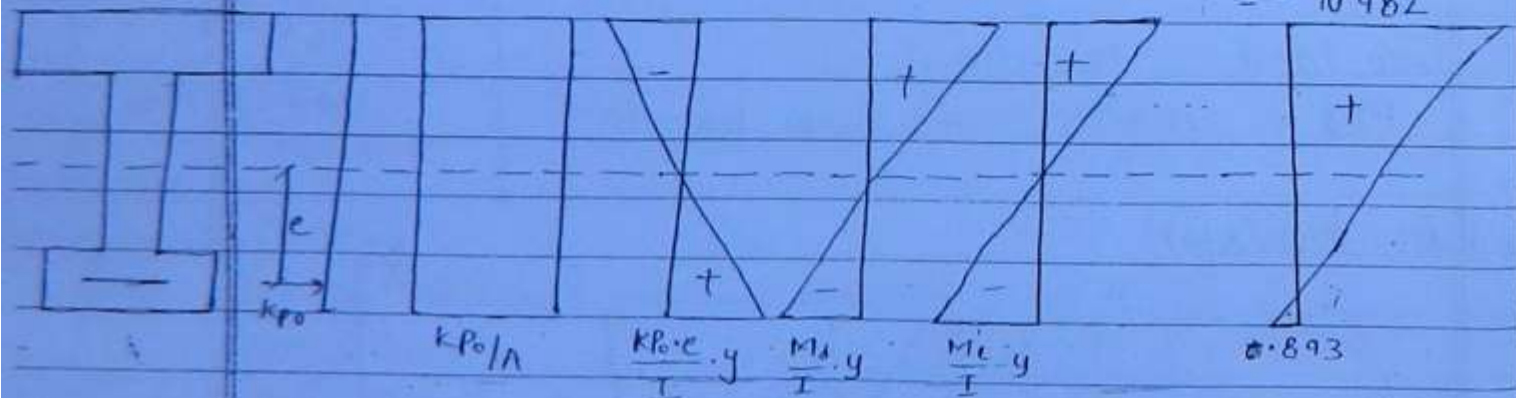
(230)

$$= 6.555 + 11.633 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65}$$

$$= 6.555 + 13.898 - 5.964 = 14.489 \text{ N/mm}^2$$

Final stage

$$\text{Final } P = K P_0 = 0.85 \times 1523984 = 1295386.4$$



$$\text{Stress at top} = 0.85 \times 6.555 - 0.85 \times 11.6333 + 4.993 + \frac{600 \times 10^6}{2.6524 \times 10^{10}} \times 455.65$$

$$= 5.572 - 9.89 + 4.993 + 10.307 = 10.982 \text{ N/mm}^2$$

Stress at bottom

$$= 5.572 + 9.89 \times \frac{544.35}{455.65} - 4.993 \times \frac{544.35}{455.65} - 10.307 \times \frac{544.35}{455.65}$$

$$= -0.893 \text{ N/mm}^2$$

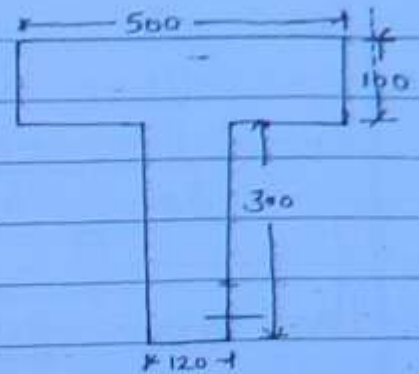
Ques A prestress concrete beam as shown in fig is to be designed for 15 kN/m live load, over a span of 6m. If stress in concrete should not exceed.

In compression = 15 N/mm^2

In tension = 0

at any stage.

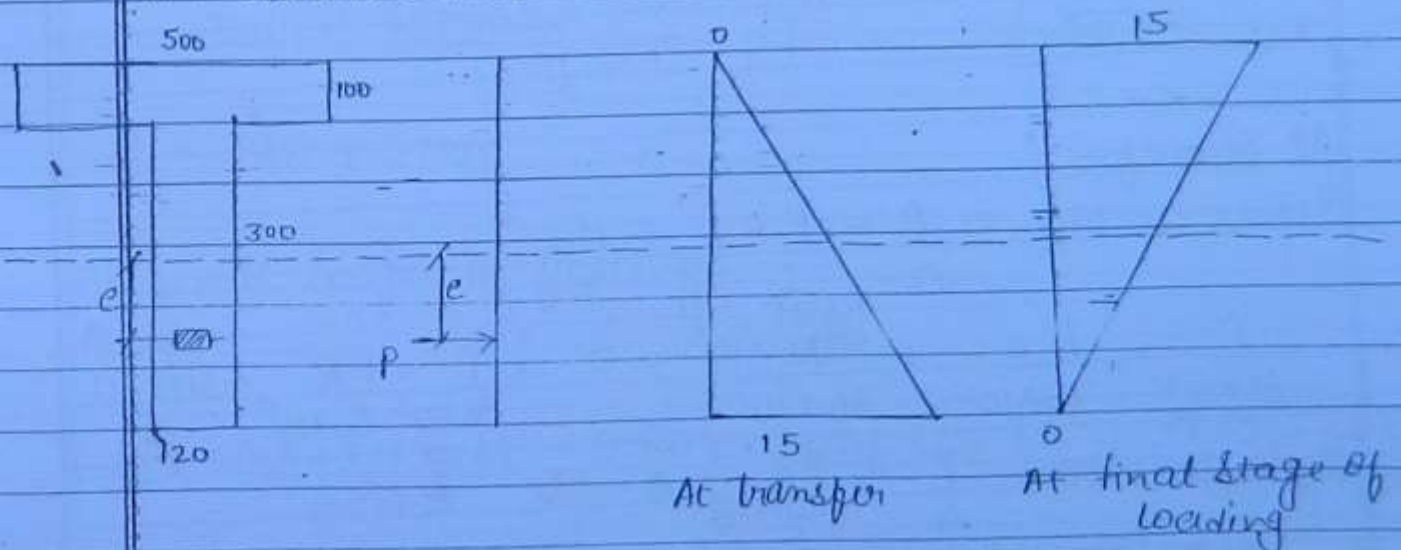
Find out prestressing force & eccentricity at which P should be applied.



(231)

Solⁿ Properties of Secⁿ

$$A = 500 \times 100 + 300 \times 120 = 86 \times 10^3 \text{ mm}^2$$



$$\bar{y} = \frac{500 \times 100 \times 50 + 300 \times 120 \times 250}{86 \times 10^3}$$

$$\bar{y} = 133.72 = y_t$$

$$y_b = 266.28 \text{ mm}$$

$$I = \frac{500 \times 100^3}{12} + 500 \times 100 (133.72 - 50)^2 +$$

$$\frac{120 \times 300^3}{12} + 120 \times 300 (133.72 - 250)^2$$

$$= 1.15 \times 10^9 \text{ mm}^4$$

$$\frac{M}{I} y = \frac{M}{Z}$$

$$Z_t = \frac{I}{y_t} = \frac{1.15 \times 10^9}{133.72} = 86 \times 10^5$$

$$Z_b = \frac{I}{y_b} = \frac{1.15 \times 10^9}{266.28} = 43.19 \times 10^5$$

$$\text{Dead load} = \frac{86 \times 10^3}{10^6} \times 1 \times 25 = 2.15 \text{ kN/m}$$

$$M_d = \frac{w L^2}{8} = \frac{2.15 \times 6^2}{8} = 9.675 \text{ kN}\cdot\text{m}$$

$$\text{live load} = 12$$

$$M_L = \frac{12 \times 6^2}{8} = 49.5 \text{ kN}\cdot\text{m}$$

At transfer

$$\text{Stress at top} = \frac{P}{A} - \frac{P \cdot e}{I} y_t + \frac{M_d}{Z_t} = 0$$

$$\frac{P}{86 \times 10^3} - \frac{P \cdot e}{86 \times 10^5} + \frac{9.675 \times 10^6}{86 \times 10^5} = 0 \Rightarrow \frac{P}{86 \times 10^3} - \frac{P \cdot e}{86 \times 10^5} + 1.125 = 0 \quad \text{--- (1)}$$

$$\text{Stress at bottom} \frac{P}{A} + \frac{P \cdot e}{I} y_b - \frac{M_d}{Z_b} = 15$$

$$\frac{P}{86 \times 10^3} + \frac{P \cdot e}{86 \times 10^5} - \frac{9.675 \times 10^6}{86 \times 10^5} = \frac{P}{86 \times 10^3} + \frac{P \cdot e}{86 \times 10^5} - 1.125 = 15$$

$$\frac{P}{86 \times 10^3} + \frac{P \cdot e}{86 \times 10^5} = 16.125 \quad \text{--- (2)}$$

Multiply eqn (1) by 86×10^5

$$100P - P \cdot e + 9.675 \times 10^6 = 0$$

Multiply eqn (2)

$$50.22P + P \cdot e - 9.675 \times 10^6 = 15 \times 43.19 \times 10^5$$

$$\text{Eq (1)} \quad 100P - P \cdot e + 9.675 \times 10^6$$

$$\text{Eq (2)} \quad 50.22P + P \cdot e - 9.675 \times 10^6 = 15 \times 43.19 \times 10^5$$

DATE: _____

$$\text{eq (1)} \rightarrow \frac{P}{86 \times 10^3} - \frac{P \cdot e}{86 \times 10^5} + 1.125 = 0$$

$$\text{eq (2)} \rightarrow \frac{P}{86 \times 10^3} + \frac{P \cdot e}{86 \times 10^5} - 16.125 = 0$$

Add both eqⁿ

$$\frac{P}{86 \times 10^3} + \frac{P}{86 \times 10^3} + 1.125 - 16.125 = 0$$

$$2/326 \times 10^{-5} P = 15$$

$$P = 431265 \text{ N}$$

$$e = 100P + 9.675 \times 10^6$$

$$= \frac{100 \times 431265 + 9.675 \times 10^6}{431265}$$

(233)

$$e = 122.43 \text{ mm}$$

At final stage of loading

$$\text{At top} = \frac{K P}{A} - \frac{K P e}{Z_t} + \frac{M_d + M_e}{Z_t}$$

$$= \frac{0.80 \times 431265}{86 \times 10^3} - \frac{0.80 \times 431265 \times 122.43}{86 \times 10^5} + \frac{(9.675 + 49.5)}{4386 \times 10^5}$$

$$= 4.01 - 4.911 + 6.88 = 5.979 < 15 \text{ OK}$$

$$\text{At bottom} = \frac{K P}{A} + \frac{K P e}{Z_b} - \frac{M_d + M_e}{Z_t}$$

$$4.01 + \frac{0.80 \times 431265 \times 122.43}{4319 \times 10^5} - \frac{(9.675 + 49.5)}{4319 \times 10^5}$$

$$4.01 + 9.78 - 13.70 = 0.09 > 0$$

Hence safe

$$P = 431265 \text{ N}$$

$$e = 122.43 \text{ mm}$$

DATE: _____

Cracking moment

The value of moment at which visible tensile cracks are developed in the section is called Cracking moment.

If tensile stress developed due to B.M, increases the tensile strength of concrete, the corresponding B.M is called Cracking moment.

If losses are not considered then

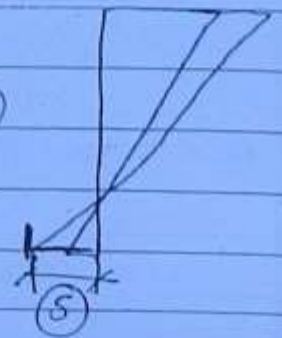
At bottom
$$\frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M \cdot y}{I} > (-f_t)$$

If losses are considered

$$\frac{kP}{A} + \frac{kP \cdot e \cdot y}{I} - \frac{M \cdot y}{I} > (-f_t)$$

f_t is tensile strength of concrete.

234



ES-1990

Ques A rectangular concrete beam $150 \times 300 \text{ mm}^2$ is prestressed by cable carrying $P = 225 \text{ kN}$ & $e = 50 \text{ mm}$

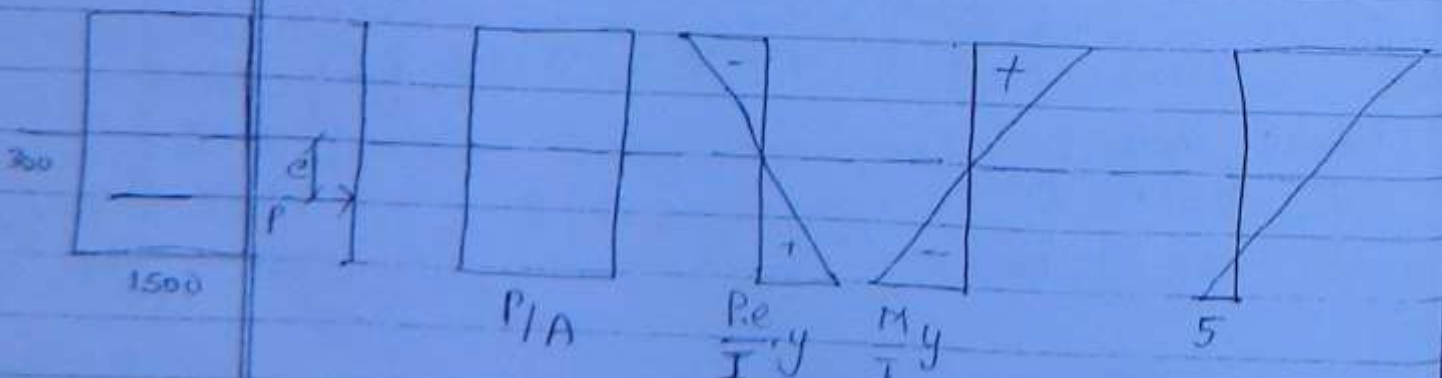
The beam supports a u.d.l 92 kN/m including

(+) Centric self wt of beam

Span = 5 m

If modulus of rupture = 5 N/mm^2 (tensile stress)
Calculate load factor against cracking.

Soln



Stress at bottom of beam

$$\frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M \cdot y}{I} = -5$$

$$\frac{225 \times 10^3}{45 \times 10^3} + \frac{225 \times 10^3 \times 50}{3375 \times 10^5} \times 150 - \frac{M \times 150}{3375 \times 10^5} = -5$$

$$5 + 5 + 0.5 = \frac{M \times 150}{3375 \times 10^5}$$

$$M = 33.75 \times 10^6 \text{ N-mm Cracking Moment}$$

$$\text{Cracking load} = \frac{w_c l^2}{8}$$

$$33.75 = \frac{w_c \times 52}{8}$$

(235)

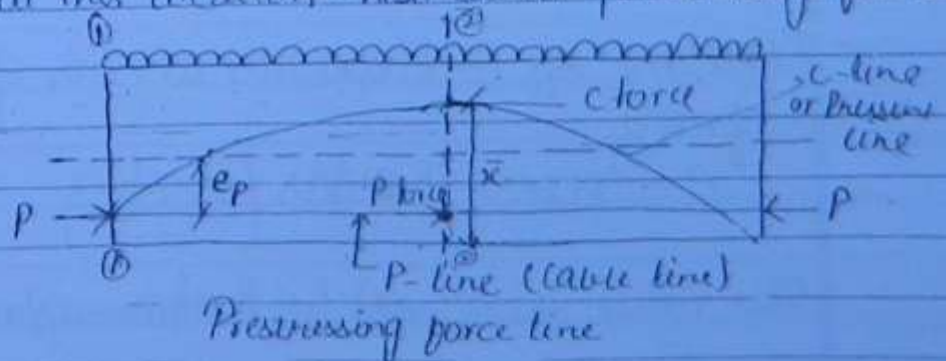
$$w_c = 10.8 \text{ kN/m}$$

$$\text{Service load or working load} = 7.2 \text{ kN/m}$$

$$\begin{aligned} \text{load factor} &= \frac{\text{cracking load}}{\text{working load}} = \frac{10.8}{7.2} \\ &= 1.5 \end{aligned}$$

imp # P-line / C-line
(cable line) (Pressure line)

P-line — is nothing but location of cable itself. Prestressing force is acting at this location. Also called prestressing force line.

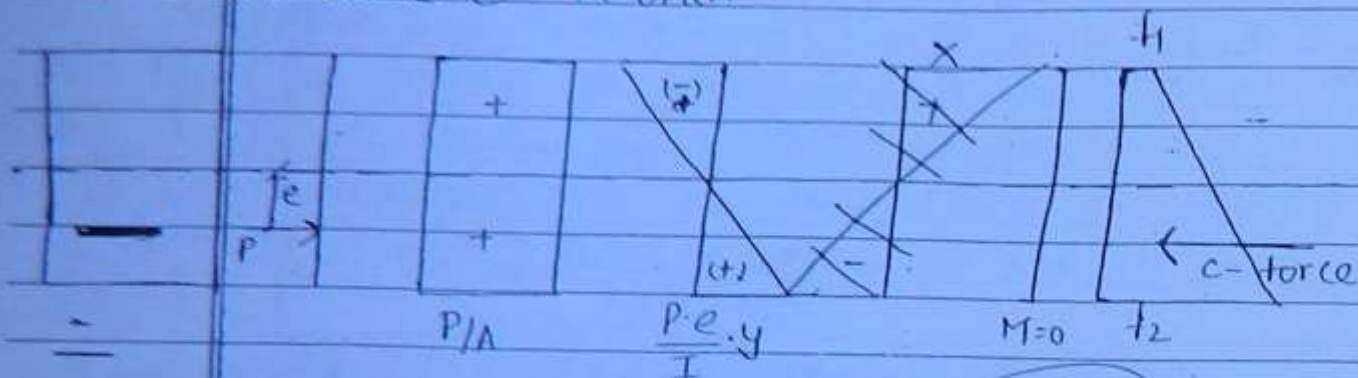


C-line — (Thrust line or Pressure line)

It is the location of Resultant thrust force due to final stress diagramme due to all loads. Pressure line is locus of Resultant force of stress diagramme in a beam.

DATE: _____

At Sec (1)-(1) At end.



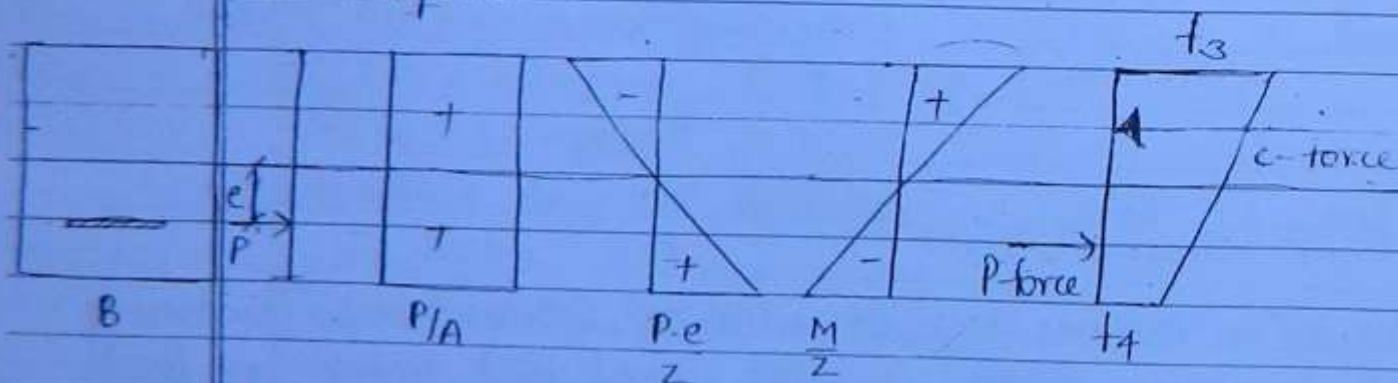
At end where $M=0$

(236)

Resultant C-force always acts at same location of P-force

$$C\text{-force} = B \times D \times \left(\frac{t_1 + t_2}{2} \right) = P\text{-force}$$

At Mid span



Resultant of C-force act at C.G. of the final stress diagram.

$$C\text{-force} = B \times D \times \left(\frac{t_3 + t_4}{2} \right)$$

This value is equal to P-force always.

At all Cross-section the value of C-force = P-force

$$\Rightarrow t_1 + t_2 = t_3 + t_4$$

it means Sum of stress at top & bottom is always constant.
At all section

Shift of c-line w.r.t P-line ($\bar{x}$)

It is always equal to $\bar{x} = \frac{M}{P}$
P-force / c-force

$$\bar{x} = \frac{M}{P}$$

Shape of c-line is same as the shape of B.M.D.

At each c/s -

$$\text{Moment } M = \text{c-force} \times \bar{x} \\ = \text{P-force} \times \bar{x}$$

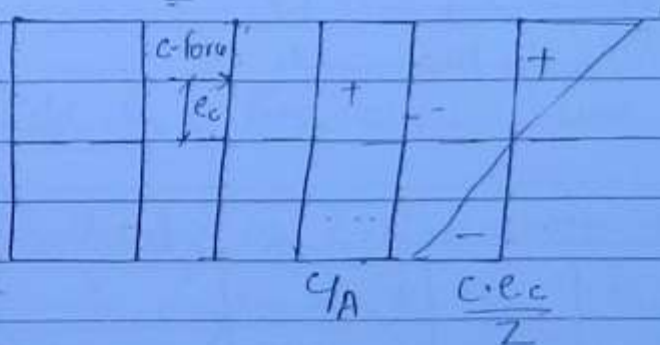
c-force or Pressure line follows the path similar to the B.M.D.

At any cross-section eccentricity of E line

$$e_c = \bar{x} - e_p$$

237

At mid span



$$\text{Final Stress} = \frac{C}{A} + \frac{C \cdot e_c}{Z} \quad (\text{top})$$

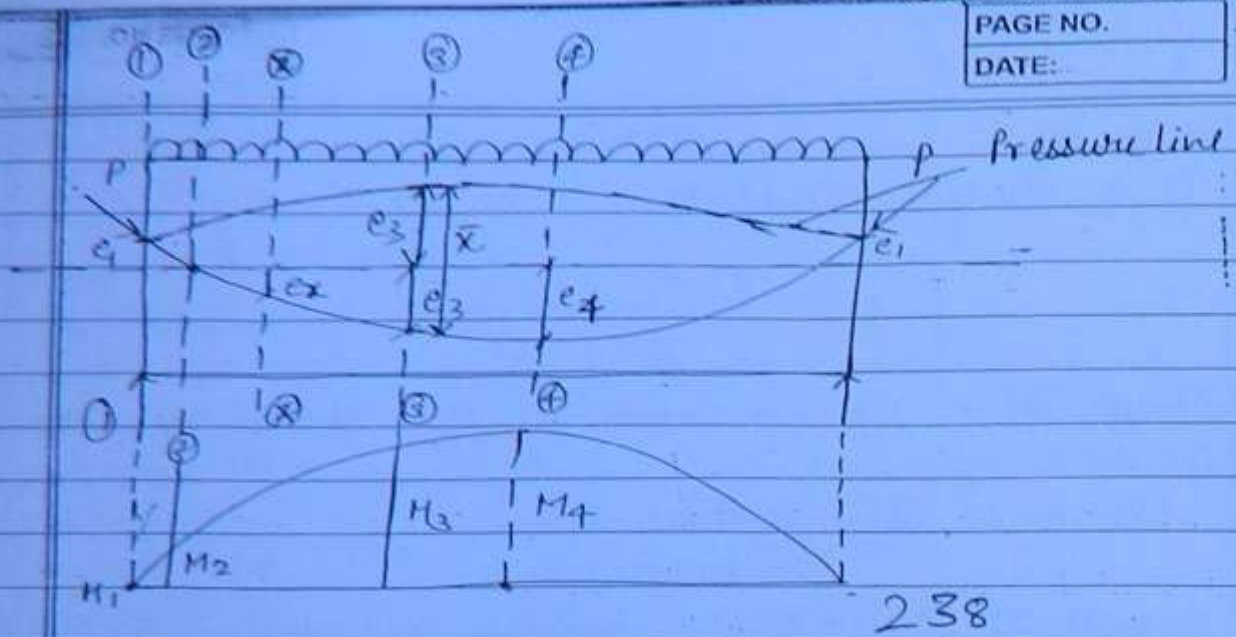
$$= \frac{C}{A} - \frac{C \cdot e_c}{Z} \quad \text{Bottom}$$

No moment is required to be considered.

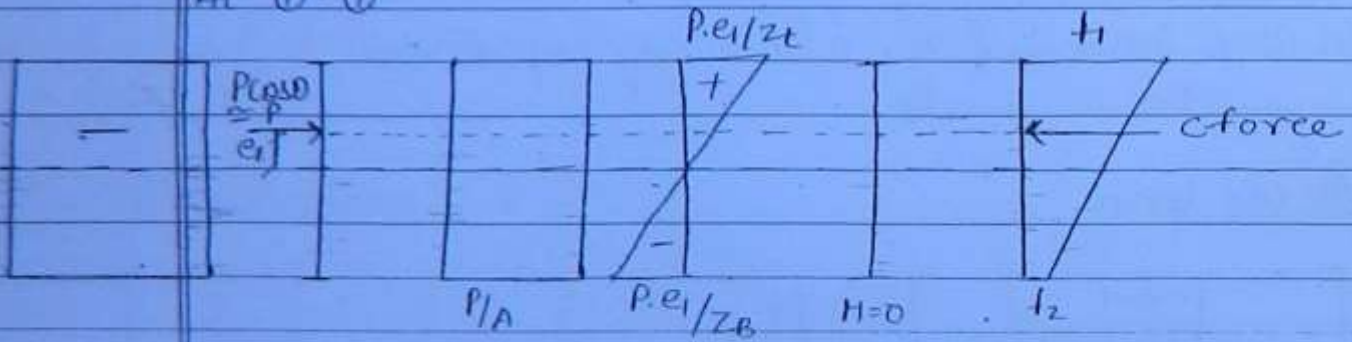
Stress

(#) Strength Concept Method. (P-force Method)

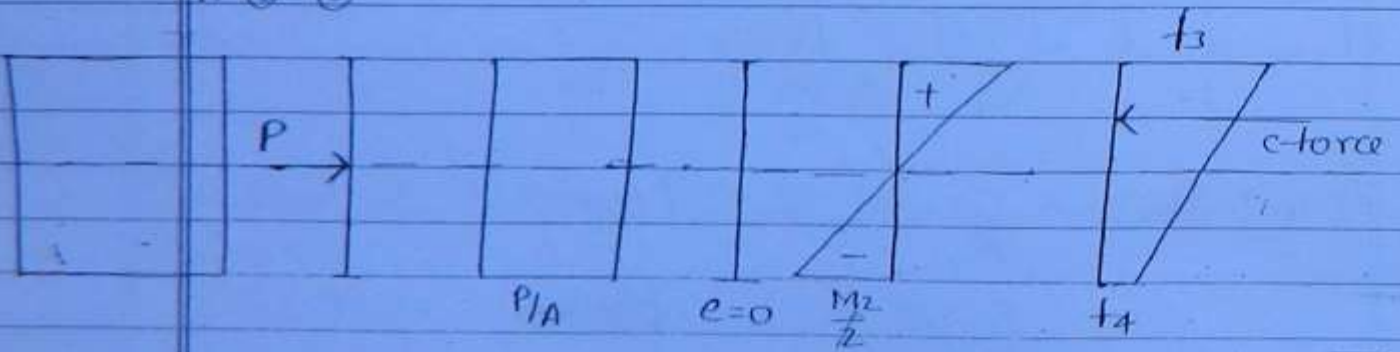
→ In this method stresses are found using actual location of prestressing force (P-force) without changing any value.



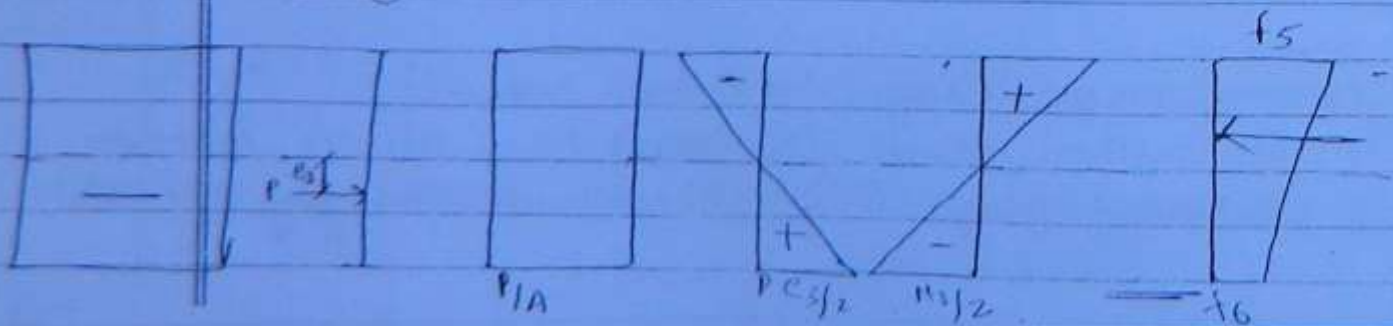
At ① - ①



At ② - ②



At ③ - ③



$$\text{Final Stress} = \frac{f_s}{I_o} \frac{P}{A} \mp \frac{P \cdot e_3}{Z} + \frac{M_3}{Z}$$

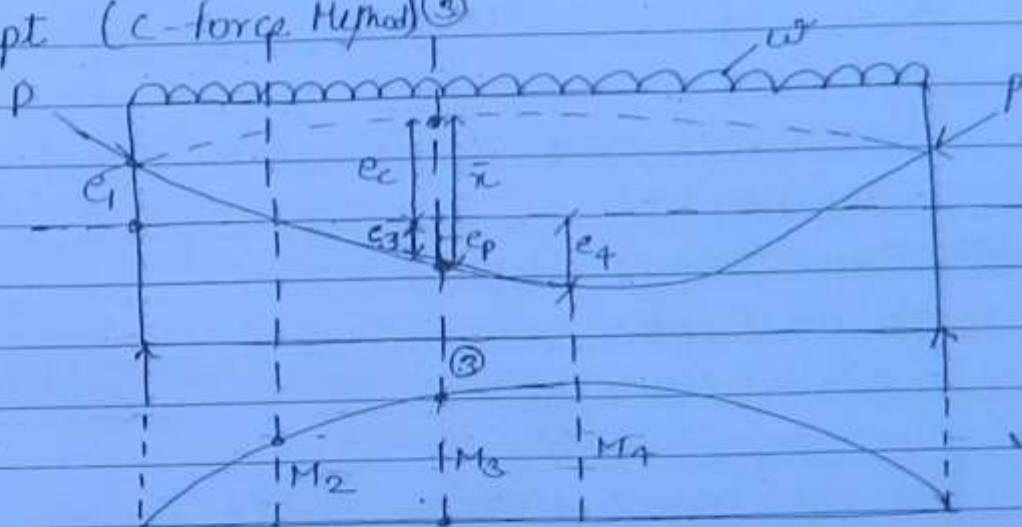
In this $\bar{x} = \frac{M_3}{P}$

$$e_c = \bar{x} - e_3$$

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5-NOV-2011

Strength Concept (C-force Method) ③



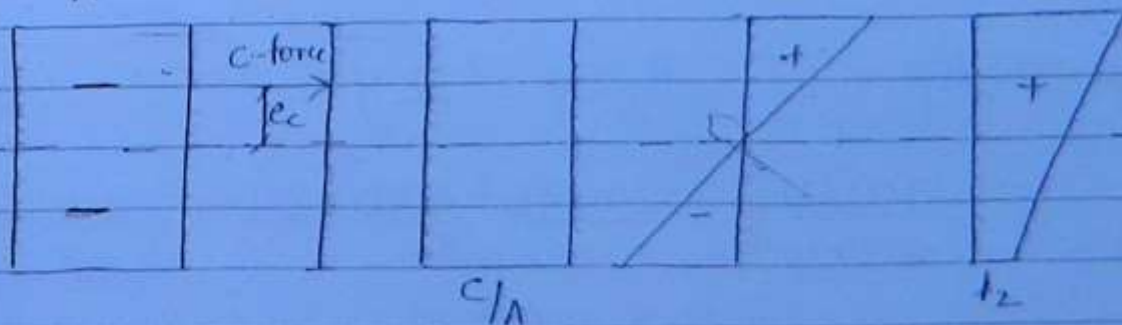
Shift of C line w.r.t P-line $\bar{x} = \frac{M_3}{P}$

Eccentricity of C-line w.r.t N.A

$$e_c = \bar{x} - e_p = \bar{x} - e_3$$

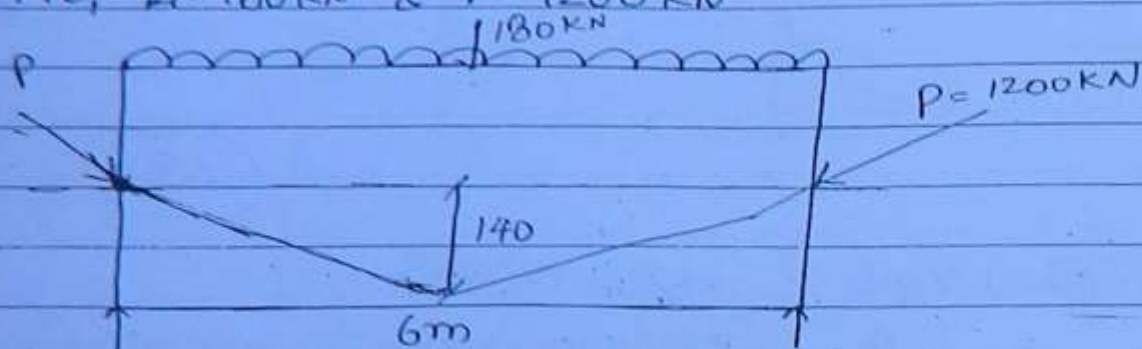
Stress in section

$$\frac{C}{A} \pm \frac{C \cdot e_c \cdot y}{I}$$



In this case, moment is not considered effect of moment is already considered in e_c .

2005
Ques A PSC beam 400x600 has span 6m. Calculate extreme fibre stress at midspan considering self wt. also.
 $e = 140$, $W = 180 \text{ kN}$ & $P = 1200 \text{ kN}$



$$\text{Dead load} = 0.4 \times 0.6 \times 1 \times 25 = 6 \text{ kN/m}$$

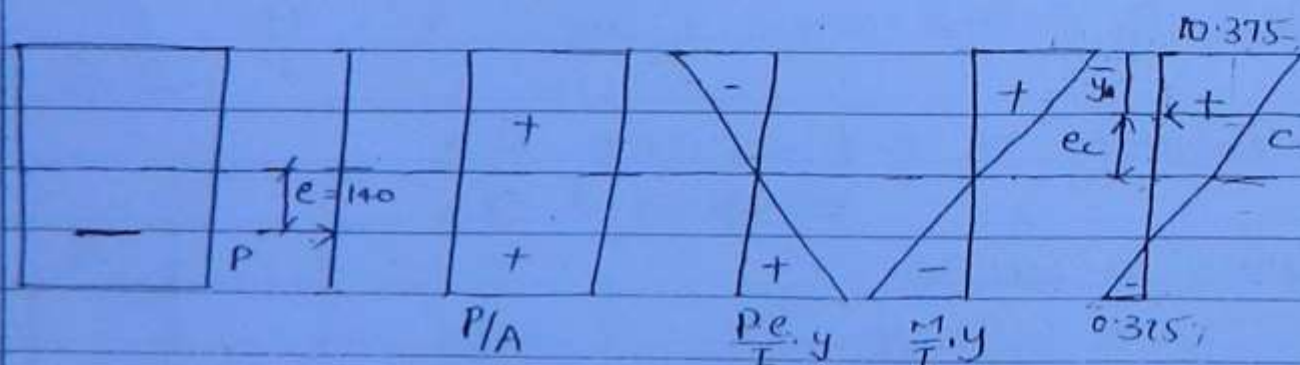
$$W = 180 \text{ kN}$$

(a) Stress Concept Method

At mid span

$$B.M = \frac{Wl}{4} + \frac{wl^2}{8}$$

$$= \frac{180 \times 6}{4} + \frac{6 \times 6^2}{8} = 297 \text{ kN-m}$$



$$\text{Stress at top} = \frac{P}{A} - \frac{P.e.y}{I} + \frac{M.y}{I}$$

$$= \frac{1200 \times 10^3}{24 \times 10^4} - \frac{1200 \times 10^3 \times 140 \times 300}{72 \times 10^8} + \frac{297 \times 10^6 \times 300}{72 \times 10^8}$$

$$5 - 7 + 12.375$$

$$\text{Stress at bottom} = 5 + 7 - 12.375$$

$$= -0.375$$

DATE: / /

C-force

$$C = 400 \times 600 \left(\frac{10.375 - 0.375}{2} \right)$$

$$C = 1200,000 \text{ N} \Rightarrow 1200 \text{ kN} = P_{\text{force}}$$

$$G = \bar{y} = \frac{a+2b}{a+b} \times \frac{h}{3}$$

$$= \frac{10.375 + 2(-0.375)}{10.375 + (-0.375)} \times \frac{600}{3}$$

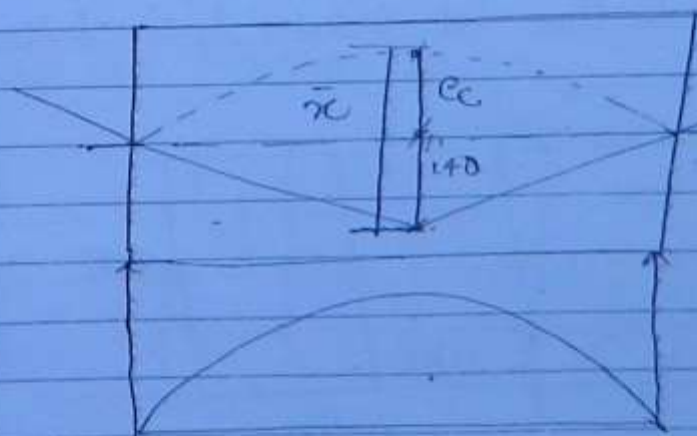
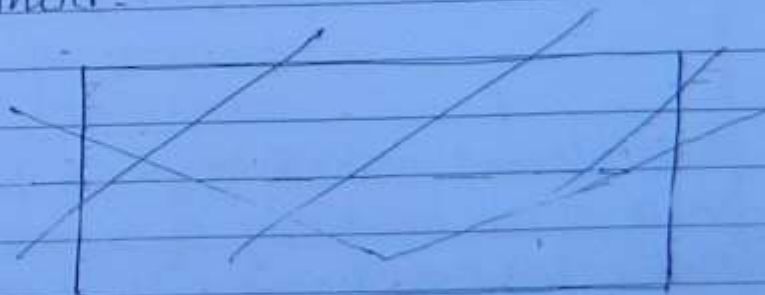
$$= \cancel{208.40} 192.5 \text{ mm}$$

(241)

Eccentricity of C-force.

$$e_c = \text{---} \text{---} \text{---} 300 - 192.5 = 107.5$$

Strength Concept Method.



Shift of C-line
w.r.t to P-line

$$\bar{x} = \frac{M}{P} = \frac{247 \times 10^6}{1200 \times 10^3}$$

$$= 247.5$$

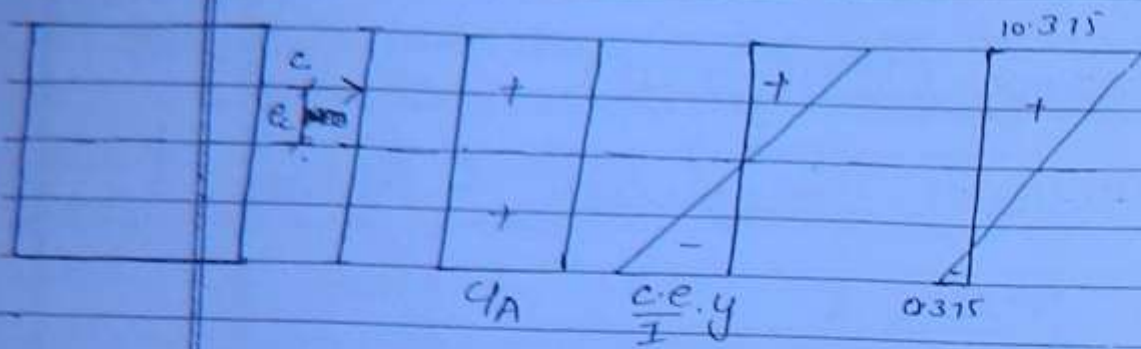
Eccentricity of
C-line w.r.t to N-A

$$e_c = \bar{x} - e_p$$

$$247.5 - 140$$

$$= 107.5 \text{ mm}$$

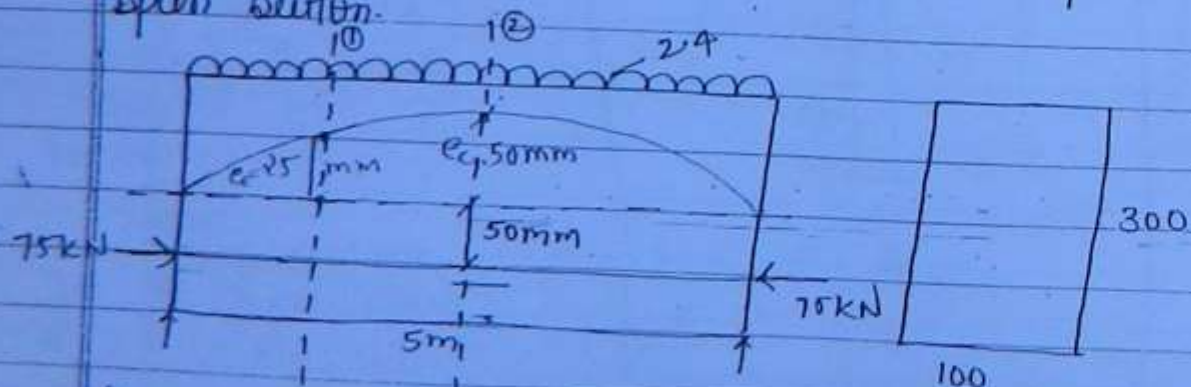
(above N-A)



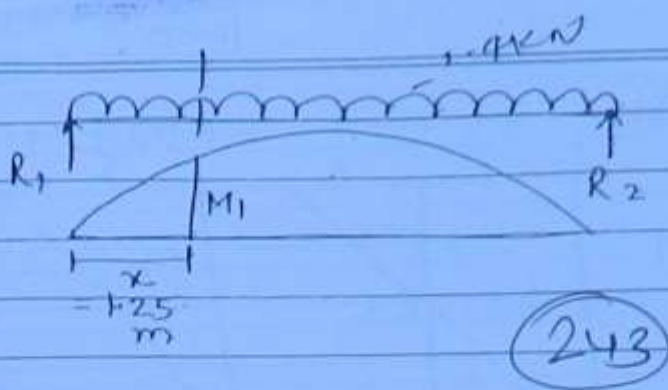
$$\begin{aligned} \text{Stress} &= \frac{c}{A} + \frac{c.e.y}{I} \quad (242) \\ &= \frac{1200 \times 10^3}{24 \times 10^4} + \frac{1200 \times 10^3 \times 107.5 \times 300}{72 \times 10^8} \\ &= 5 - 5.375 \\ &= 10.375 \text{ N/mm}^2 \\ \text{At bottom} &= 5 + 5.375 \\ &= -0.375 \text{ N/mm}^2 \end{aligned}$$

ES-1991
Ques

A PSC. beam $100 \times 300 \text{ mm}^2$ is prestressed with a straight cable at an eccentricity of 50 mm, $P = 75 \text{ kN}$, span = 5m, Total load on beam including self wt = 2.4 kN/m . Determine the pressure line at Quarter span & Mid span Section.



$$\begin{aligned} \text{Area} &= 3 \times 10^4 \text{ mm}^2 \\ I &= 225 \times 10^6 \text{ mm}^4 \\ w &= 2.4 \text{ kN/m} \\ \text{B.M at mid span} &= \frac{2.4 \times 5^2}{8} = 7.5 \text{ kN-m} \end{aligned}$$



$$R_1 = R_2 = \frac{2.4 \times 5}{2} = 6 \text{ kN}$$

$$M_1 = R_1 \times 2.5 - \frac{w \times 2.5^2}{2} = 6 \times 2.5 - \frac{2.4 \times 2.5^2}{2} = 5.625 \text{ kN-m}$$

Strength Concept Method

Shift of c-line w.r.t to P-force line

$$\bar{x} = \frac{M \text{ at quarter span}}{P} = \frac{5.625 \times 10^6}{75 \times 10^3} = 75 \text{ mm}$$

Eccentricity of c-line w.r.t to N.A

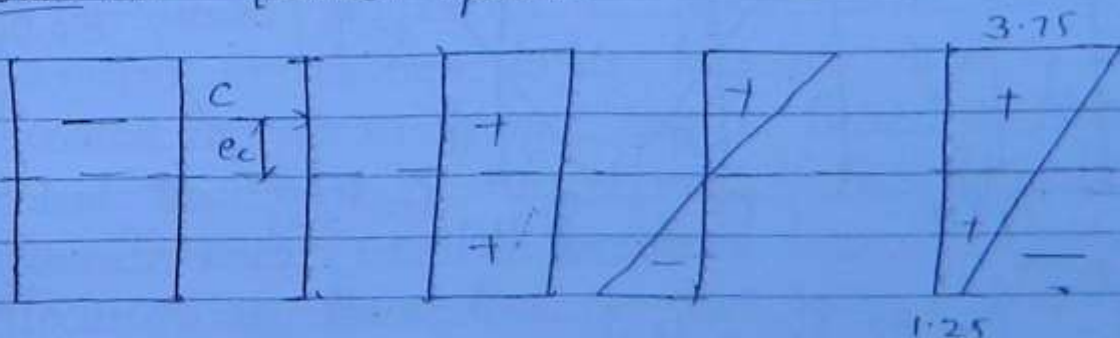
$$e_c = \bar{x} - e_p = 75 - 50 = 25 \text{ mm (above N.A)}$$

Shift of c-line At mid span

$$\bar{x} = \frac{H \text{ mid span}}{P} = \frac{7.5 \times 10^6}{75 \times 10^3} = 100 \text{ mm}$$

$$\text{Eccentricity} = e_c = \bar{x} - e_p = 100 - 50 = 50 \text{ mm}$$

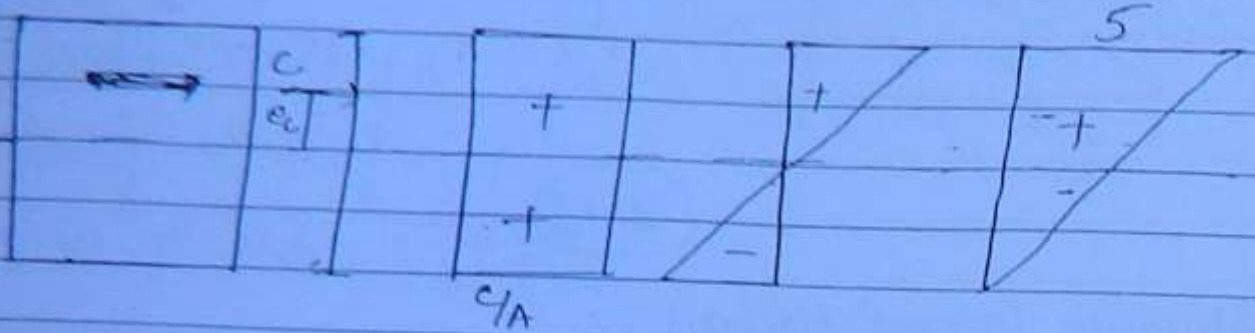
Stress At Quarter span.



$$\begin{aligned} \text{At top} &= \frac{C}{A} + \frac{C e_c y}{I} = \frac{75 \times 10^3}{3 \times 10^4} + \frac{75 \times 10^3 \times 25 \times 150}{225 \times 10^6} \\ &= 2.5 + 1.25 \\ &= 3.75 \end{aligned}$$

$$\text{At bottom} = 2.5 - 1.25 = 1.25$$

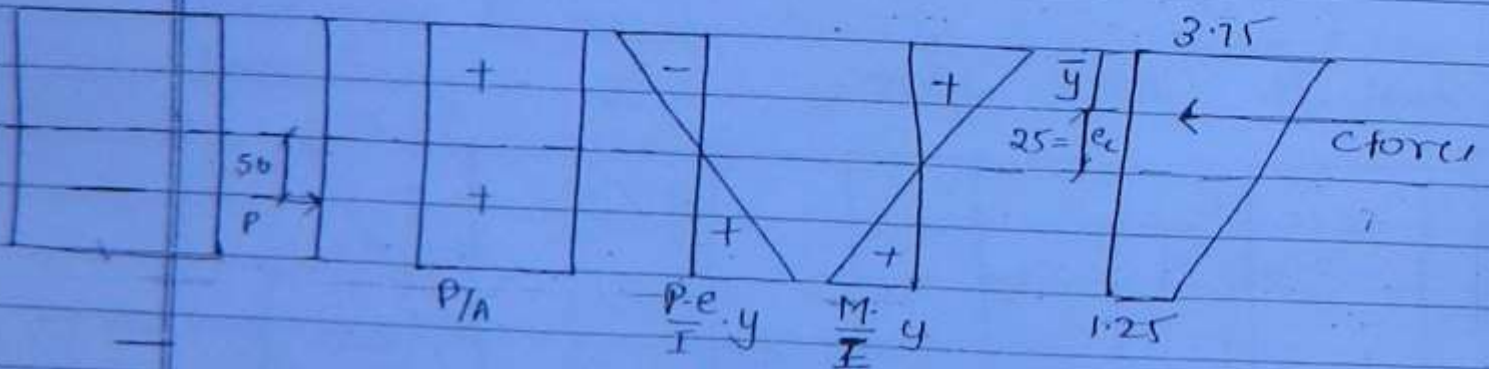
At mid span Stresses



$$\begin{aligned}
 \text{At top} &= \frac{C}{A} + \frac{C \cdot e \cdot y}{I} \\
 &= \frac{75 \times 10^3}{3 \times 10^4} + \frac{75 \times 10^3 \times 50}{225 \times 10^6} \times 150 \\
 &= 2.5 + 2.5 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 \text{At bottom} &= 2.5 - 2.5 = 0 \quad (2.48) \\
 &= 0
 \end{aligned}$$

(b) Stress Concept Method
At Quarter span
B.M = 5.625 KN-m



$$\begin{aligned}
 \text{At top} &= \frac{P}{A} - \frac{P \cdot e \cdot y}{I} + \frac{M \cdot y}{I} \\
 &= \frac{75 \times 10^3}{3 \times 10^4} - \frac{75 \times 10^3 \times 50}{225 \times 10^6} \times 150 + \frac{5.625 \times 10^6}{225 \times 10^6} \times 150 \\
 &= 2.5 - 2.5 + 3.75 \\
 &= 3.75 \text{ N/mm}^2
 \end{aligned}$$

at bottom $\frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M \cdot y}{I}$
 $= 2.5 + 2.5 - 3.75$
 $= 1.25 \text{ N/mm}^2$

Shift $\bar{x} = \frac{M}{P} = \frac{5.625 \times 10^6}{75 \times 10^3} = 75 \text{ mm}$

$e_c = 75 - 50 = 25 \text{ mm}$

(245)

C.G. = $\frac{a+2b}{a+b} \times \frac{h}{3} = \frac{3.75 + 2(1.25)}{3.75 + 1.25} \times \frac{300}{3}$
 $= 125 \text{ mm}$

C-force = $B \times D \times \left(\frac{f_1 + f_2}{2} \right)$

$= 100 \times 300 \left(\frac{5}{2} \right)$

$= 75 \text{ kN} = P \text{ force}$

$e_c = 150 - 125 = 25 \text{ mm}$

2006-ES

Ques

A PSC beam $300 \times 600 \text{ mm}$ span = 12m carries a v.d.l 12 kN/m in addition to its self wt. Area of steel is 2000 mm^2 located at 175mm from soffit. Cable is straight initially stress to a level of $800 \text{ N/mm}^2 = P_0$. It is bonded to concrete (i.e. Pre-tensioned beam). Determine the location of thrust line at mid span & ^{end} quarter span

$E_s = 6$

E_c

Sol. Prestress force = $P_0 \times A_s = 2000 \times 800 \times 10^{-3}$
 $= 1600 \text{ kN}$

$$\text{B.M at Mid span} = \frac{12 \times 12^2}{8} = \frac{144}{8}$$

$$= 216 \text{ KN-m}$$

$$A = 300 \times 600 = 18 \times 10^4 \text{ mm}^2$$

$$I = \frac{300 \times 600^3}{12} = 54 \times 10^8$$

At end Moment = 0

Shift of C-line wrt to P-line

$$\bar{x} = \frac{M}{P} = 0$$

(246)

Eccentricity of C-line = $e_c = \bar{x} - e_p$



$$e_c = 0 - 125$$

$$= -125$$

125 mm below N.A

$$\text{Stress} = \frac{C}{A} \mp \frac{C \cdot e_c \cdot y}{I}$$

$$= \frac{1600 \times 10^3}{18 \times 10^4} \mp \frac{1600 \times 10^3 \times 125}{54 \times 10^8} \times 300$$

$$8.889 \mp 11.11$$

$$\text{At Top} = -2.227 \text{ N/mm}^2$$

$$\text{At bottom} = 19.999 \text{ N/mm}^2$$

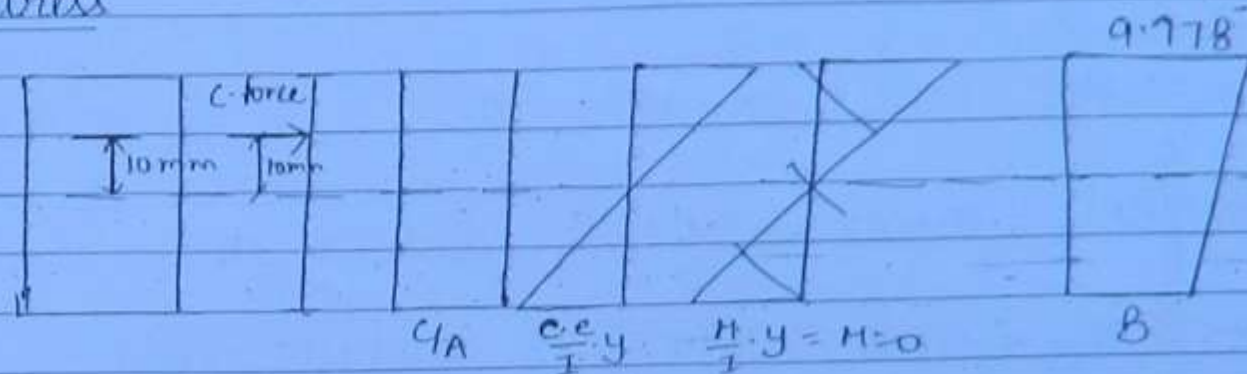
$$f_1 + f_2 = 17.78 \text{ N/mm}^2$$

b). At Mid span

$$\text{Shift } \bar{x} = \frac{M}{P} = 135 \text{ mm}$$

$$\begin{aligned}\text{Eccentricity} &= e_c = \bar{x} - e_p \\ &= 135 - 125 \\ e_c &= 10\end{aligned}$$

Stress



$$\text{Stresses} = \frac{C}{A} + \frac{C \cdot e \cdot y}{I} + \frac{H \cdot y}{I}$$

(247)

$$= 8.889 + \frac{1600 \times 10^3 \times 10}{54 \times 10^8} \times 300 + \frac{216 \times 10^6}{54 \times 10^8} \times 300$$

$$8.889 + 0.889 = 9.778$$

At top = 9.778

At bottom = 8

Ques A psc beam is as shown in fig. (3) (4)

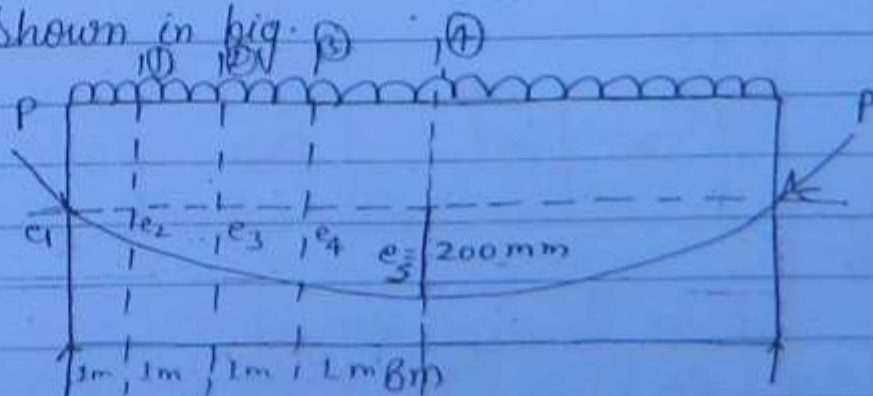
$$w = 30 \text{ kN/m}$$

Excluding self wt.

$$\text{Size of beam} = 400 \times 750 \text{ mm}$$

$$e = 200 \text{ mm}$$

$$P = 1500 \text{ kN}$$



Show the position of thrust line at every 1m distance & find out stresses at top & bottom.

$$\text{Sol}^n \quad A = 3 \times 10^5 \text{ mm}^2$$

$$I = 1.41 \times 10^{10} \text{ mm}^4$$

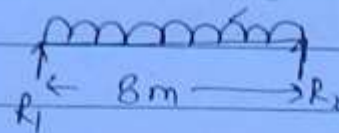
$$\text{Self wt} = 4 \times 75 \times 1 \times 25 = 75 \text{ kN/m}$$

$$\text{total } w = 37.5 \text{ kN/m}$$

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At C.O.D

$$R_1 = R_2 = 150 = \frac{37.5 \times 8}{2}$$



$$B.M = 150 \times 1 - \frac{37.5 \times 1^2}{2}$$

$$1m \quad B.M_1 = 131.25 \text{ kN-m} = M_1$$

$$2m \quad B.M_2 = 150 \times 2 - \frac{37.5 \times 2^2}{2} = 225 \text{ kN-m}$$

$$3m \quad B.M_3 = 150 \times 3 - \frac{37.5 \times 3^2}{2} = 281.25 \text{ kN-m}$$

$$4m \quad B.M_4 = \frac{37.5 \times 8^2}{8} = 300$$

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~~At C.O.D~~ Eccentricity max eccentricity

$$y = \frac{4b \times (l-x)}{l^2}$$

$$= \frac{4 \times 200 \times (8000-x)}{8000^2} = \frac{x(8000-x)}{80000}$$

y = 375

	1m	2m	3m
x	875	150	187.5

Stress Concept Method

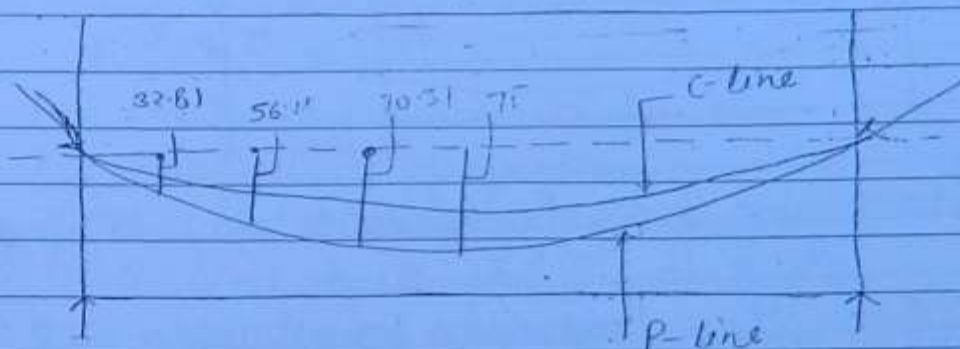
Point	B.M	e_p	P/A	$\frac{P \cdot e \cdot y}{I}$	$\frac{M \cdot y}{I}$	Stress at top	Stress at Bottom	$\bar{y}$	$\bar{y} - 375$ e_c
0m	0	0	5	± 0	± 0	5	5	562.5	
1m	131.25	875	5	± 3.49	± 3.49	5	5	562.5	
2m	225	150	5	± 5.98	± 6	5	5	562.5	
3m	281.25	187.5	5	± 7.5	± 7.5	5	5	562.5	
4m	300	200	5	± 8.0	± 8.0	5	5	562.5	

$P = 2400 \text{ kN}$

PAGE NO.

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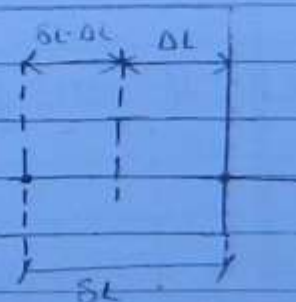
Point	BM	e_p	P/A	$\frac{P \cdot e \cdot y}{I}$	$\frac{M \cdot y}{I}$	Stress at top	Stress at bottom	$\bar{y}$	e_c
0 m	0	0	± 8	∓ 0	± 0	8	8	375	375
1 m	13125	87.5	± 8	∓ 5.6	± 3.5	5.9	10.1	407.8	327.8
2 m	225	150	± 8	∓ 9.6	± 6	4.4	11.6	431.25	291.25
3 m	28125	187.5	± 8	∓ 12.0	± 7.5	3.5	12.5	445.31	275.31
4 m	300	200	± 8	∓ 12.8	± 8	3.2	12.8	450	275



Below
N.A

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Losses (Loss of Prestress)



If an initial elongation of Sl is provided

$$p_0 = \frac{Sl}{l} \times E_s$$

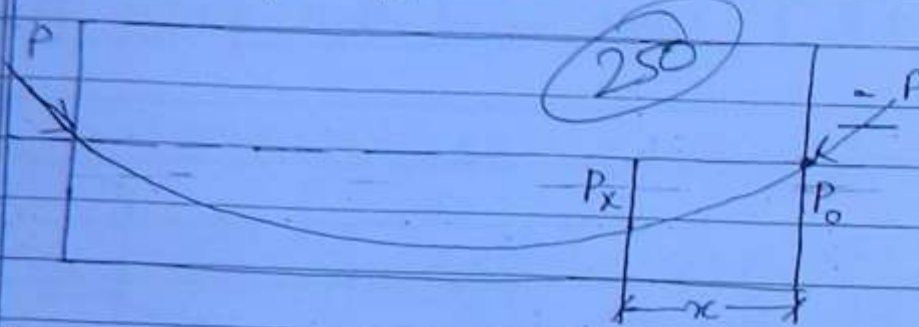
If Δl = loss of elongation provided (due to any reason)
Loss of Prestress

$$p_{loss} = \frac{\Delta l}{l} \times E_s$$

Final stress is due to $(Sl - \Delta l)$ if an initial elongation of Sl is provided
 $p_{final} = \frac{Sl - \Delta l}{l} \times E_s$

Type of losses

1. Loss due to friction (During tensioning process)
(Due to combined effect of length & curvature)
- Only for post tensioned beam or member
- Due to length effect



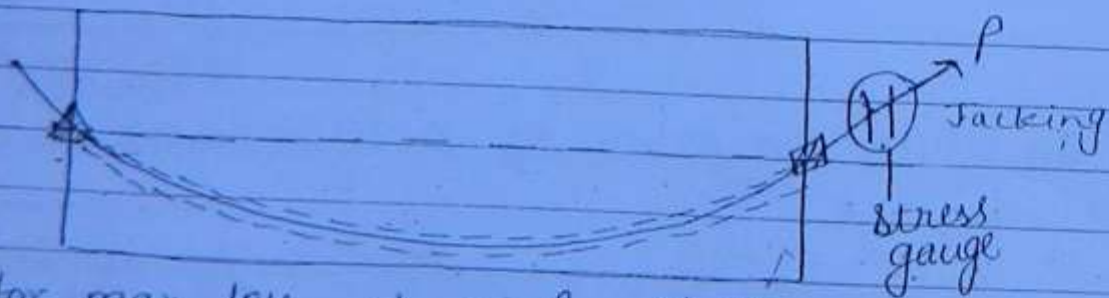
Prestressing force P_x available at x distⁿ from free end

$$P_x = P_0 \cdot e^{-Kx}$$

Where P_0 = Prestressing force at free end.
 x = distance

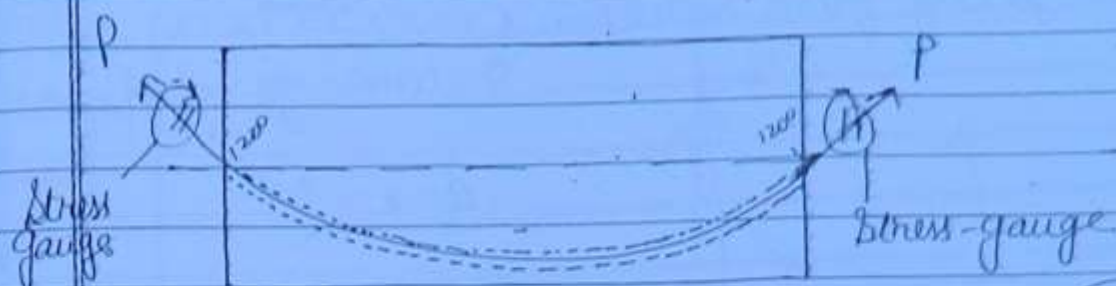
K = coefficient correction factor a constant
= $(15 \times 10^{-4} \text{ to } 50 \times 10^{-4})$ per meter

Case 1 - If jacking is done from one end.



For max loss $\Rightarrow x = l$ at other end.

Case 2 When jacking is at both ends
For max loss $x = \frac{l}{2}$



(257)

→ Due to Curvature effect.

Prestressing force at x dist from free end.

$$P_x = P_0 e^{-\mu x}$$

Where P_0 = Prestressing force at free end.

μ = Coefficient of friction

= 0.55 - Steel is moving over concrete

→ 0.35 - Steel duct tendon is moving on steel duct

→ 0.25 - Steel tendon are moving over lead.

Δ = Change in gradient b/w two pt. of consideration

→ Combined effect of length & Curvature

$$P_x = P_0 \cdot e^{-\mu x - \mu \Delta}$$

$$P_x = P_0 (1 - (\mu x + \mu \Delta))$$

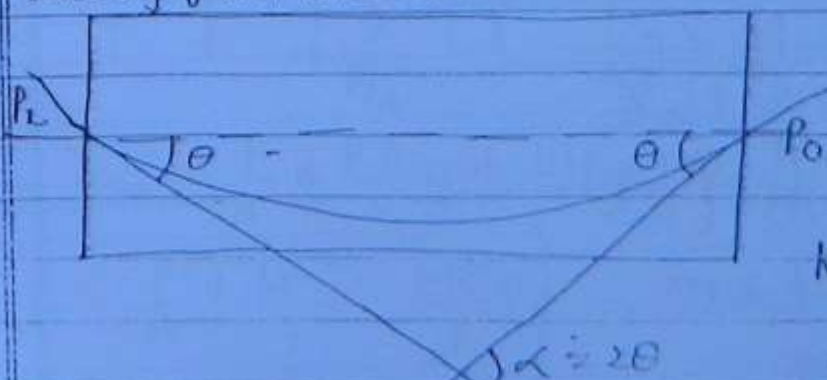
Loss of prestress.

$$= P_0 - P_x$$

$$= P_0 - P_0 (1 - \mu x + \mu \Delta)$$

$$= P_0 (\mu x + \mu \Delta) \text{ - same as recommended by IS code.}$$

Value of Δ - Eq. of Parabolic cable = $y = \frac{4h}{l^2} x(1-x)$
Jacking from one end.



$$\text{Slope} = \frac{dy}{dx} = \frac{4h}{l} (1-2x)$$

at $x=0$

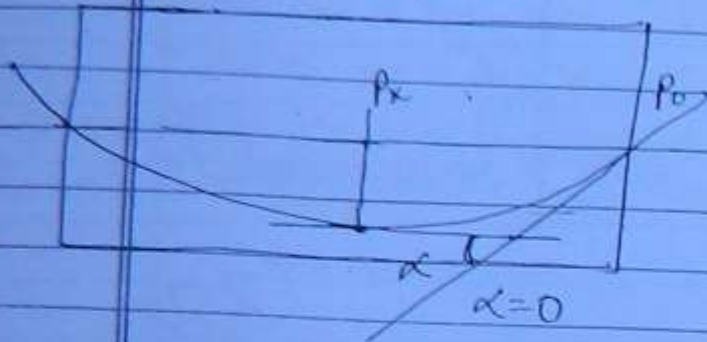
$$\theta = \tan \theta = \frac{4h}{l} \text{ radians}$$

here change in gradient

$$\Delta = 2\theta = \frac{8h}{l} \text{ radians}$$

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When jacking from both end.



$$\theta = \tan \theta = \frac{dy}{dx} = \frac{4h}{l^2} (l - 2x)$$

$$\text{At } x = \frac{l}{2}$$

$$\alpha = \theta = \frac{4h}{l} \text{ radian}$$

2. Loss due to Anchorage slip, deformation etc. (Post tensioned)
(during Anchoring process)

If Δl = loss of elongation

$$\text{loss} = \frac{\Delta l}{l} \times E_s$$

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3. Loss occurring subsequently.

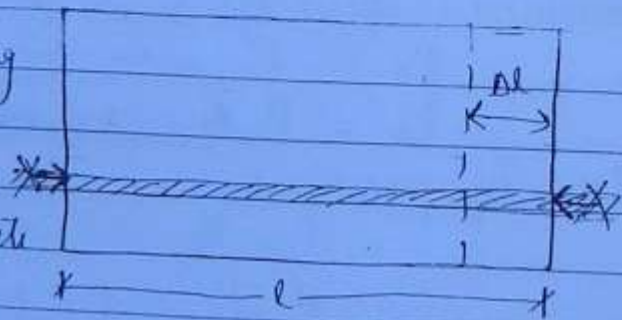
(a) Due to Elastic Shortening

→ For pretensioned Members.

(a) Just after cutting the reinforcement prestressing force is applied on Concrete.

Elastic Shortening of Concrete due to P compressive force

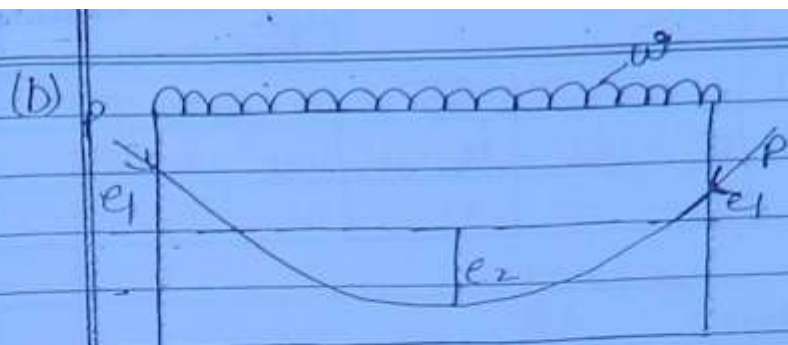
$$\Delta l = \frac{Pl}{A_c E_c}$$



The same deformation will be in steel also
Loss of Prestress in Steel = $\frac{\Delta l}{l} \times E_s$

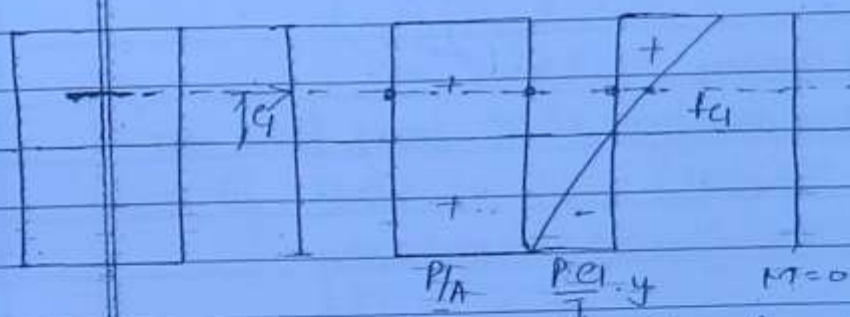
$$\begin{aligned} \Delta l &= \frac{Pl}{A_c E_c} \cdot \frac{E_s}{l} \\ &= \frac{P}{A_c} \left(\frac{E_s}{E_c} \right) \end{aligned}$$

Loss of prestress = $l \times m$
due to Elastic Shortening



Stress in concrete at the level of steel is found at end & at mid span.

→ At end.

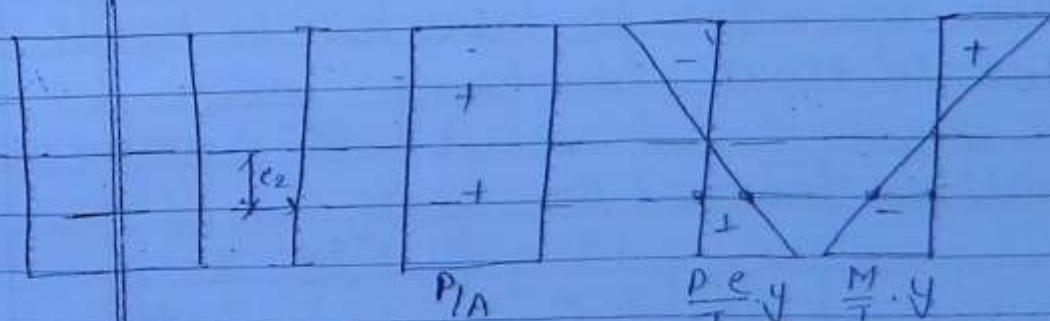


Stress in concrete at the level of steel

$$f_{c1} = \frac{P}{A} + \frac{P \cdot e_1 \cdot y}{I} = \frac{P}{A} + \frac{P \cdot e_1 \cdot e_1}{I}$$

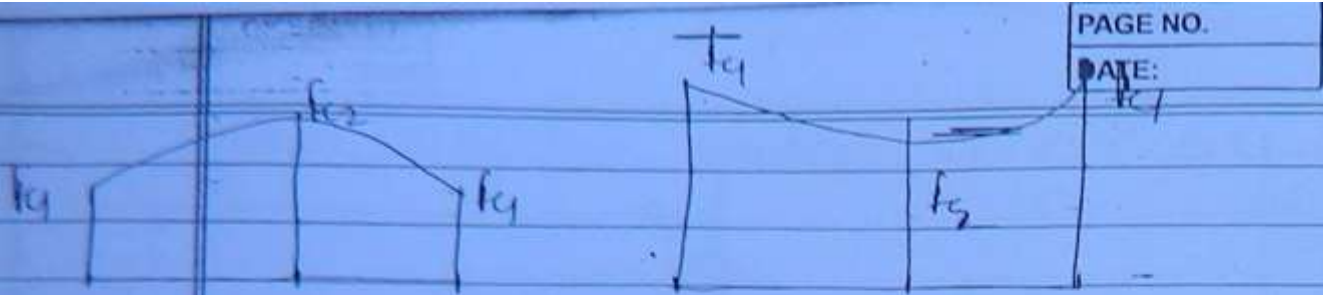
$$f_{c1} = \frac{P}{A} + \frac{P \cdot e_1^2}{I}$$

At Mid Span



$$f_{c1} = \frac{P}{A} + \frac{P \cdot e_1 \cdot y}{I} + \frac{M \cdot y}{I} \Rightarrow \frac{P}{A} + \frac{P \cdot e_2 \cdot y}{I} - \frac{M \cdot y}{I}$$

$$f_{c1} = \frac{P}{A} + \frac{P \cdot e_2^2}{I} - \frac{M \cdot e_2}{I}$$



→ Average f_c is Calculated.

① $f_c = \frac{f_{t1} + f_{t2}}{2}$ ← B.C Punmia

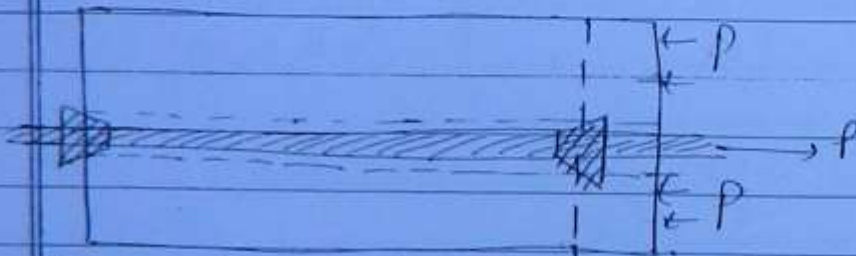
② $f_c = f_{t1} + \frac{2}{3}(f_{c2} - f_{t1})$ ← K. Raju

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→ For post tensioned beam.

Case 1. If all tendons are tensioned at same time or there is only one tendon.

Loss of stress due to Elastic Shortening
= Zero.



In this case Elastic Shortening take place before anchoring the reinforcement so there will be no further Elastic Shortening after Anchoring. So there will be no loss in steel due to Elastic Shortening.

Case 2. If diff. wire are tensioned one by one.



If Reinfⁿ 1 is anchored 1 & 4 this is anchored last
 Finally loss in steel. NO - 4 = 0
 " " " - 3 = Due to prestressing force P_4

$$= \frac{\Delta l_4}{l} \times E_s$$

$$= \frac{P_4 \cdot l}{A_c E_c \times l} \times E_s = f_{c4} \cdot m$$

loss of stress in wire (2) - Due to prestressing force P_3 & P_4

$$= \frac{\Delta l_3 + \Delta l_4}{l} \times E_s$$

$$\frac{P_3 + P_4}{A_c} \times m = (f_{c3} + f_{c4}) m$$

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loss of stress in wire (1)

$$= \frac{\Delta l_2 + \Delta l_3 + \Delta l_4}{l} \times E_s$$

$$= (f_{c2} + f_{c3} + f_{c4}) m =$$

loss due to Creep of Concrete

$$\text{Loss of Stress} = \text{Creep strain} \times E_s$$

$$\text{Creep Coefficient} = \frac{\text{Creep strain}}{\text{Elastic strain}}$$

$$\text{Creep strain} = \frac{\text{Creep Coefficient} \times \text{Elastic strain}}{\phi \times \frac{s l}{l}}$$

$$\text{loss of stress due to Creep} = \phi \times \frac{s l}{l} \times E_s$$

$$= \phi \times \frac{P \cdot l}{A_c \cdot E_c \cdot l} \times E_s$$

$$= \phi \left(\frac{P}{A_c} \right) \left(\frac{E_s}{E_c} \right)$$

$$= \phi \cdot m \cdot f_c$$

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Creep Coefficient

$\phi =$	7 days	28 days	1 year
$\phi =$	2.2	1.6	1.1

Loss due to Creep of Steel (Relaxation of Steel)

loss = 1 to 5% of p_o

Generally 2 to 3% of p_o is considered.

Loss due to Shrinkage of Concrete.

loss of stress = Shrinkage Strain $\times E_s$

(i) Pretensioned PSC Member

$$= 3 \times 10^{-4} \times E_s$$

(ii) Post tensioned PSC Member

$$= \frac{2 \times 10^{-4}}{\log_{10}(T+2)} \times E_s$$

T = Age of loading in days.

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TYPE OF LOSS	PRETENSIONED	POST-TENSIONED
→ During tensioning (due to friction) length & curvature	—	$P_o (Kx + \mu L)$
→ During Anchoring due to Anchorage slip	—	$\frac{\Delta L}{L} \times E_s$
Due to Elastic Shortening	$(m \times f_c)$	① If one & all tendon tensioned at same time = 0 ② Subsequent tensioning $= \sum \frac{S_i L}{L} \times E_s = \sum \frac{P_i}{A} \times m$
Due to Creep of Concrete	$\phi \cdot m \cdot f_c$	$\phi \cdot m \cdot f_c$
Due to Creep of Steel	1 to 5%	1 to 5%
Due to Shrinkage of Concrete	$3 \times 10^{-4} \times E_s$	$\frac{2 \times 10^{-4}}{\log_{10}(T+2)} \times E_s$

Total loss of Prestress

TYPE OF LOSS	PRETENSIONED	POST TENSIONED
1. Elastic Shortening	3%	1%
2. Creep of Concrete	6%	5%
3. Creep of Steel	2%	3%
4. Shrinkage of Concrete	7%	6%
	18%	15%

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ES-2001

Ques

A Simply supported Post tensioned Concrete beam span 15m has rectangular C/s 300x600 mm. The prestress force is 1150 kN & zero eccentricity at end & at mid span is 200 mm. Cable is parabolic. Friction Coeffⁿ = 0.15 per 100m LK & $\mu = 0.35$. Determine the loss due to friction at centre of beam.

Solⁿ

$$P_0 = 1150 \text{ kN}$$

$$K = 0.15 \text{ per mebu} \times \frac{1}{100}$$

$$= 0.0015 \text{ per mebu}$$

$$\mu = 0.35$$

$$L = 15 \text{ m}$$

Value of α

$$\alpha = 0$$

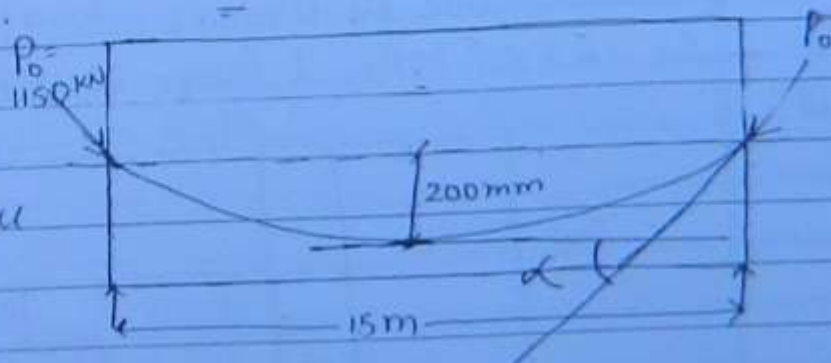
Eqⁿ of Cable. $y = \frac{4h}{l^2} x(l-x)$

$$\frac{dy}{dx} = \frac{4h}{l^2} (l-2x)$$

Slope at end at $x=0$

$$\theta = \tan \theta = \frac{dy}{dx} = \frac{4h}{l}$$

$$\Rightarrow \alpha = \theta = \frac{4 \times 200}{15000} = 0.5333 \text{ radian}$$



$$\theta = 3^\circ 30' 10.55''$$

$$= \frac{3^\circ 30' 10.38''}{180} \times \pi = 0.053328 \text{ radians}$$

loss of prestress due to friction

$$= P_0 (kx + \mu \theta)$$

$$= 1150 (0.0015 \times 7.5 + 0.35 \times 0.0533)$$

$$= 34.4 \text{ kN}$$

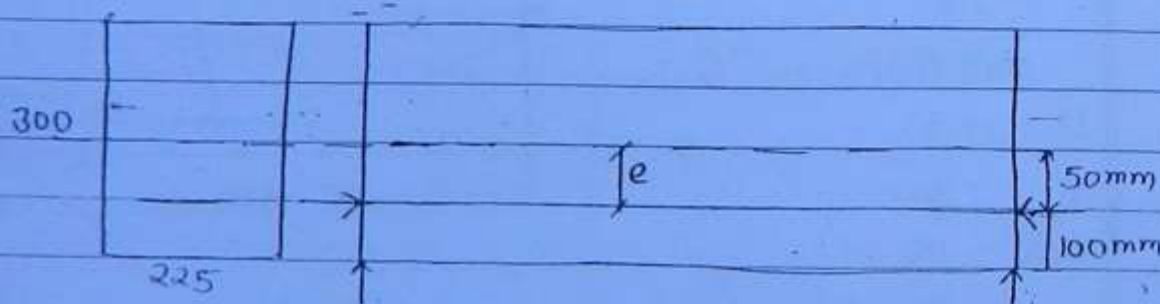
$$\% \text{ loss} = \frac{34.4}{1150} \times 100 = 2.99\%$$

5-2003

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Ques A Pretensioned beam of size $225 \times 300 \text{ mm}$ is prestressed by 12 wires of $5 \text{ mm } \phi$ initially prestressed at 1100 MPa . The centroid of prestressing wire is located at 100 mm from bottom. Estimate the loss of prestress due to all reasons. Grade of concrete - M-40, Relaxation of steel = 5%.
 $E_s = 2 \times 10^5 \text{ MPa}$ & Creep Coefficient = 1.6

Sol.



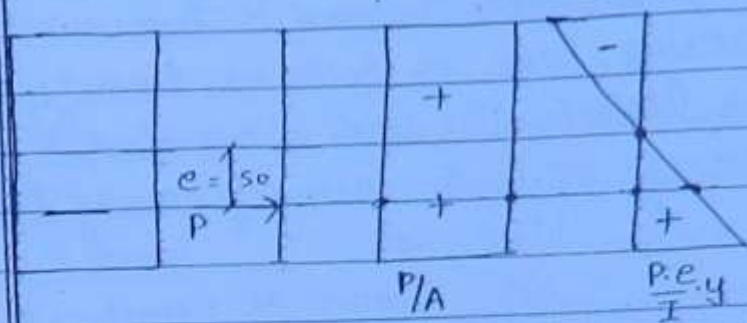
$$A_s = \frac{12 \times \pi \times 5^2}{4} = 235.62 \text{ mm}^2$$

$$P_0 = A_s \times f_p \Rightarrow 235.62 \times 1100$$

$$= 259.18 \text{ kN}$$

$$P_0 = 1100 \text{ N/mm}^2 \text{ (initially Prestress)}$$

1. loss due to elastic shortening = $m \times f_c$



Stress in Concrete At the level of Steel -

$$= \frac{P}{A} + \frac{P.e.y}{I} = \frac{259.18 \times 10^3}{300 \times 225} + \frac{259.18 \times 10^3 \times 50}{50625 \times 10^4}$$

$$f_c = 3.84 + 1.28$$

$$f_c = 5.12 \text{ N/mm}^2$$

$$\text{loss} = \frac{m \times f_c}{E_s \times E_c}$$

$$\frac{2 \times 10^5}{5000 \sqrt{40}} \times 5.12$$

$$= 32.38 \text{ N/mm}^2$$

$$\% \text{ loss} = \frac{32.38}{1100} \times 100 = 2.94\%$$

$$\begin{aligned} \text{Creep of Concrete} &= \phi \cdot m \cdot f_c \\ &= 1.6 \times 32.38 \\ \text{loss} &= 51.81 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Creep of Steel} &= 5\% \text{ of } P_0 \\ &= \frac{5}{100} \times 1100 = 55 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Shrinkage of Concrete} &= 3 \times 10^{-4} \times E_s \\ &= 3 \times 10^{-4} \times 2 \times 10^5 \\ &= 60 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total loss} &= 32.38 + 51.81 + 55 + 60 = 199.19 \\ \% \text{ loss} &= \frac{199.19}{1100} \times 100 = 18.11\% \end{aligned}$$

ES 1993 Ques A Post tensioned, Prestressed Concrete beam is tensioned initially by a Prestressing force of 1458 kN transferred at 28 days strength of concrete. The cable is parabolic with an eccentricity of 520 mm at Centre. Using following data

$$A_c = 2.42 \times 10^5 \text{ mm}^2$$

$$A_s = 1385 \text{ mm}^2$$

$$I = 5.3 \times 10^{10} \text{ mm}^4$$

$$f_s = 1059 \text{ N/mm}^2 \text{ at transfer}$$

$$E_s = 2.1 \times 10^5 \text{ N/mm}^2$$

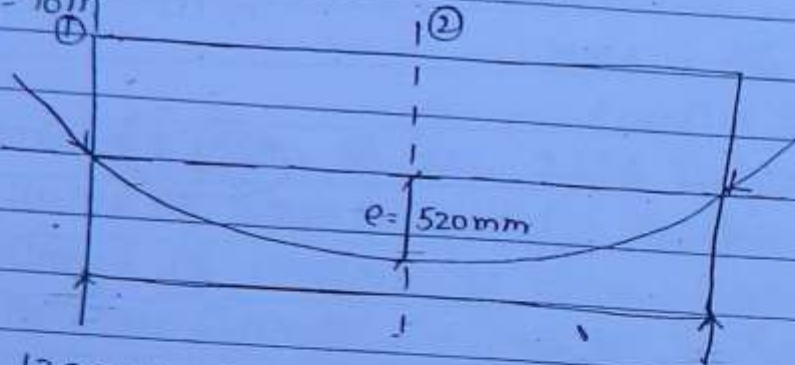
$$E_c = 0.382 \times 10^5 \text{ N/mm}^2$$

$$\text{Anchorage slip} = 2.5 \text{ mm}$$

$$k = 0.0015 / \text{metre}, \mu = 0.25$$

Calculate all losses.

$$\text{Span} = 16 \text{ m}$$



$$P = 1385 \times 1059 = 1466.7 \text{ kN}$$

1. Loss due to friction = $P_0 (Kx + \mu \alpha)$

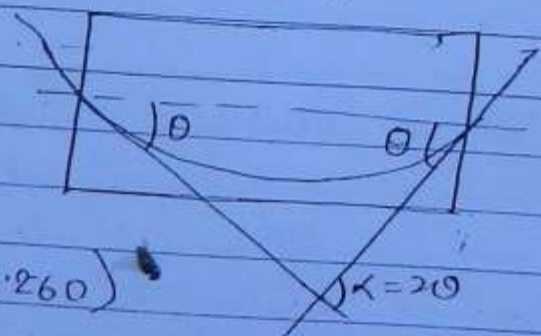
Assuming jacking is done from one end only. Max loss will be at other end

$$\alpha = 2\theta = \frac{8h}{l}$$

$$= \frac{8 \times 520}{16 \times 10^3} = 0.260 \text{ radian}$$

$$\text{loss} = \frac{1059}{16 \times 10^3} (0.0015 \times 16 + 0.25 \times 0.260)$$

$$= 94.25 \text{ N/mm}^2$$



2. Loss due to Anchorage slip = $\frac{\Delta l}{l} \times E_s$

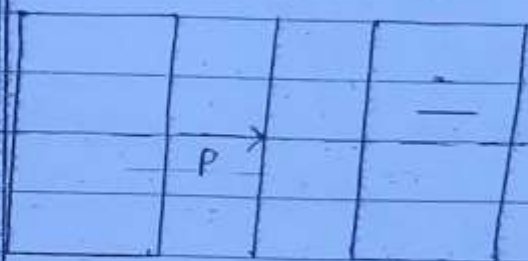
$$= \frac{2.5}{16 \times 10^3} \times 2.1 \times 10^5$$

$$= 32.81 \text{ N/mm}^2$$

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3. loss due to elastic shortening of concrete = 0

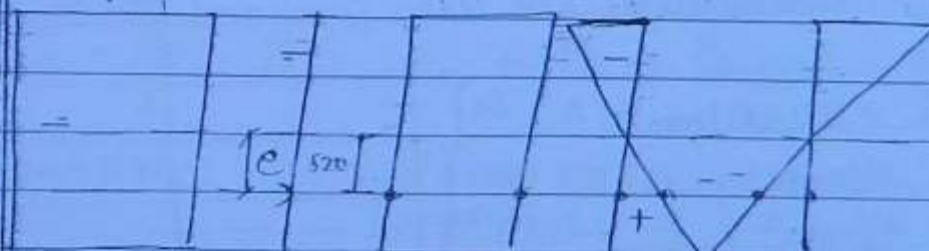
4. loss due to Creep of concrete = $\phi \cdot m \cdot f_c$
Stress in concrete at the level of steel.
At ends



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$$f_{c1} = \frac{P}{A} = \frac{1466.7 \times 10^3}{2.42 \times 10^5} = 6.06 \text{ N/mm}^2$$

At mid span



$$M_d = \frac{w_d \times l^2}{8}$$

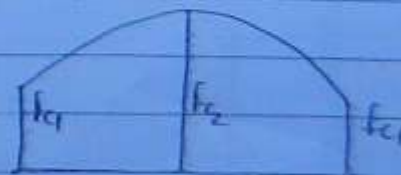
$$w_d = \frac{2.42 \times 10^5}{10^6} \times 1 \times 25 = 6.05$$

$$M_d = 193.6 \text{ kN-m}$$

$$f_{c2} = \frac{P}{A} + \frac{P \cdot e^2}{I} - \frac{M \cdot y}{I} = \frac{1466.7 \times 10^3}{2.42 \times 10^5} + \frac{1466.7 \times 10^3 \times 520^2}{5.3 \times 10^{10}} - \frac{193.6 \times 10^6 \times 520}{5.3 \times 10^{10}}$$

$$f_{c2} = 6.06 + 7.48 - 1.899 = 11.641$$

$$f_c = \frac{f_{c1} + f_{c2}}{2} = 8.85 \text{ N/mm}^2$$



$$E_s = m = 5.497$$

ϕ

$$\text{loss} = \phi \times m \times f_c = 1.6 \times 5.497 \times 8.85$$

$$\phi = 1.6 \text{ at 28 days.} = 77.84 \text{ N/mm}^2$$

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5. loss due to Creep of Steel = 1 to 5% of P_0
 $= 3\% \text{ of } P_0$
 $= \frac{3}{100} \times 1059 = 31.77 \text{ N/mm}^2$

6. loss due to shrinkage $= \frac{2 \times 10^{-4}}{\log_{10}(f+2)} E_s$
 $= \frac{2 \times 10^{-4}}{\log_{10}(28+2)} \times 2.1 \times 10^5$
 $= 28.43 \text{ N/mm}^2$

Total loss = 265.101 N/mm²

1. loss = $\frac{265.101}{1059} \times 100$
 $= 25\%$

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Ques A PSC beam (Pretensioned) has been provided tendons as shown in fig

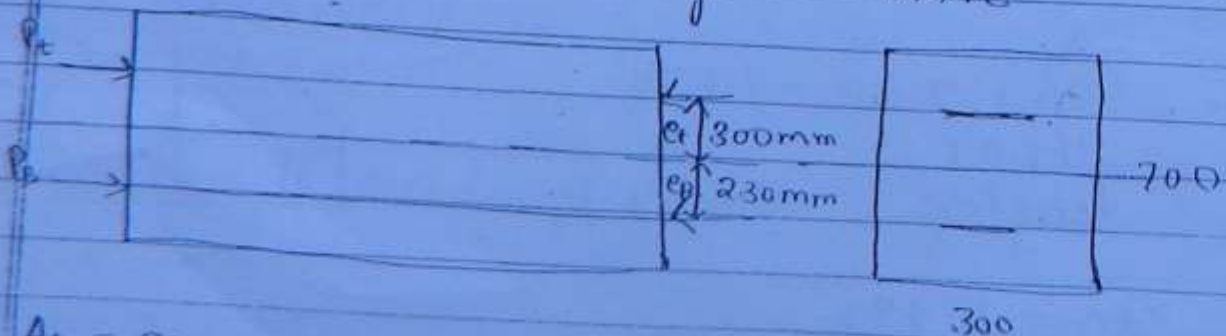
Initial Prestress = 1200 N/mm²

$E_s = 210 \times 10^3 \text{ N/mm}^2$

$E_c = 36 \times 10^3 \text{ N/mm}^2$

Calculate loss in wires due to elastic shortening. The Member is still supported on its prestressing bed.

Sol. loss due elastic shortening = $m \times f_c$



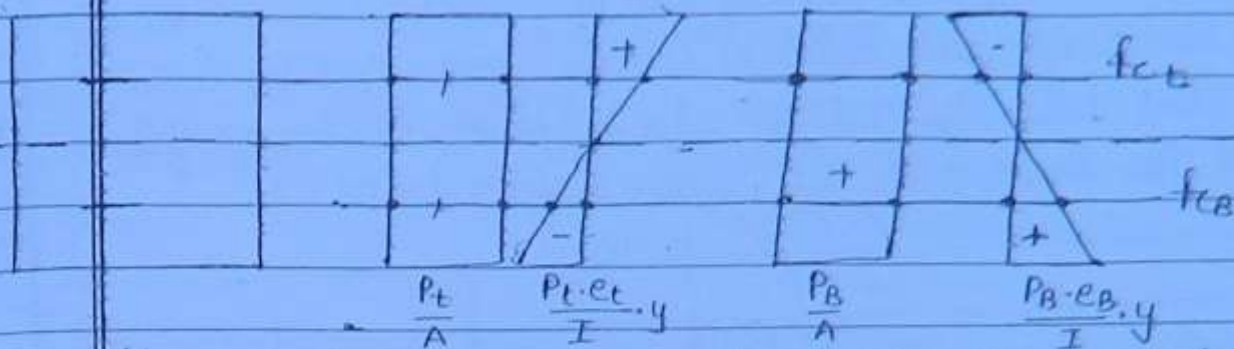
$A_c = 314.16$

$P_t = 1200 \times 314.16 = 377 \text{ kN}$

$A_b = 30 \times \frac{\pi}{4} \times 6^2 = 848.23$

$$P_B = 1017.9 \text{ kN}$$

$$I = 8575 \times 10^6 \text{ mm}^4$$



Stress in concrete at the level of steel at top

$$f_{ct} = \frac{P_t}{A} + \frac{P_t \cdot e_t \cdot y}{I} + \frac{P_B}{A} - \frac{P_B \cdot e_B \cdot y}{I}$$

$$= \frac{377 \times 10^3}{21 \times 10^4} + \frac{377 \times 10^3 \times 300^2}{8575 \times 10^6} + \frac{1017.9 \times 10^3}{21 \times 10^4} - \frac{1017.9 \times 10^3 \times 230^2}{8575 \times 10^6}$$

$$= 1.795 + 3.96 + 4.85 - 8.19$$

$$= 2.415 \text{ N/mm}^2$$

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$$f_{cb} = \frac{P_t}{A} - \frac{P_t \cdot e_t \cdot e_B}{I} + \frac{P_B}{A} + \frac{P_B \cdot e_B^2}{I}$$

$$= 1.795 - \frac{377 \times 10^3 \times 300 \times 230}{8575 \times 10^6} + 4.85 + \frac{1017.9 \times 10^3 \times 230^2}{8575 \times 10^6}$$

$$= 1.795 - 3.03 + 4.85 + 6.28$$

$$= 9.895 \text{ N/mm}^2$$

loss of stress in top steel = $m \cdot f_{ct}$

$$= \frac{E_s}{E_c} \times f_{ct}$$

$$= 5.83 \times 2.415 = 14.08 \text{ N/mm}^2$$

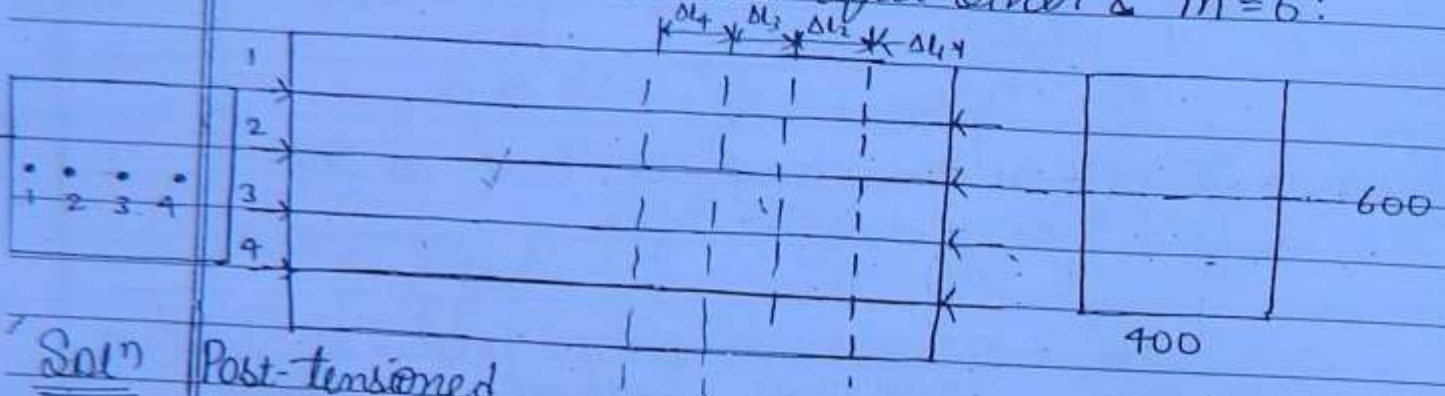
loss of stress in bottom = $m \cdot f_{cb}$

$$= 5.83 \times 9.895$$

$$= 57.68 \text{ N/mm}^2$$

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Ques - A Post-tensioned Concrete beam 20 m long has c/s $400 \times 600 \text{ mm}^2$ is prestressed by 4-cable of 300 mm^2 each provided without any eccentricity. Each tendon is initially prestressed to 1800 N/mm^2 . Find out loss of prestress due to elastic shortening of concrete. Wires are tensioned one after other & $m=6$.



Solⁿ

Post-tensioned

$$A_s = 300 \text{ mm}^2 \text{ each}$$

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

$$P_s = 1800 \text{ N/mm}^2$$

$$E_c =$$

$$P = 300 \times 1800 = 540 \text{ kN}$$

$$\Delta L_1 = \Delta L_2 = \Delta L_3 = \Delta L_4$$

$$\Delta L = \frac{Pl}{A_c E_c} = \frac{540 \times 10^3 \times 20 \times 10^3 \times 6}{400 \times 600 \times 2 \times 10^5} = 1.35 \text{ mm}$$

$$\Delta \sigma \text{ of stress due to one wire} = \frac{\Delta L}{l} \times E_s = \frac{1.35}{20 \times 10^3} \times 2 \times 10^5 = 13.5 \text{ N/mm}^2$$

$$\text{loss in steel due to wire no 4} = 0$$

$$\text{due to wire no. 3} = \frac{\Delta L_4 \times E_s}{l}$$

$$= 13.5 \text{ N/mm}^2$$

$$\text{due to wire no. 2} = \frac{\Delta L_3 + \Delta L_4 \times E_s}{l}$$

$$= \frac{1.35 \times 2}{20 \times 10^3} \times 2 \times 10^5 =$$

$$= 27 \text{ N/mm}^2$$

$$\text{due to wire no 1} = \frac{\Delta L_2 + \Delta L_3 + \Delta L_4 \times E_s}{l}$$

$$= \frac{3 \times 1.35}{20 \times 10^3} \times 2 \times 10^5 = 40.5 \text{ N/mm}^2$$

- (b) If it is desired that a final stress of 1800 N/mm^2 is maintained after losses how much initial stress should be given to diffⁿ wires.

Solⁿ Wire no-4 $\rightarrow$ loss of stress = 0

$$\text{Initial stress} = 1800 \text{ N/mm}^2$$

" " 3 $\rightarrow$ loss = 13.5 N/mm^2

$$\therefore \quad \sigma_v = \frac{300 \times 1800}{400 \times 600} \times 6 = 13.5 \text{ N/mm}^2$$

$$\text{Initial stress} = 1800 + 13.5 = 1813.5 \text{ N/mm}^2$$

wire no-2 $\rightarrow$ loss due to P_3 & P_4

$$\frac{P_3 + P_4}{A_c} \times m = \text{265}$$

$$= \frac{300 \times 1813.5 + 300 \times 1800}{400 \times 600} \times 6 = 27.10 \text{ N/mm}^2$$

$$\text{Initial stress} = 1800 + 27.10 = 1827.10 \text{ N/mm}^2$$

wire no-1 - loss due to P_2 & P_3 & P_4

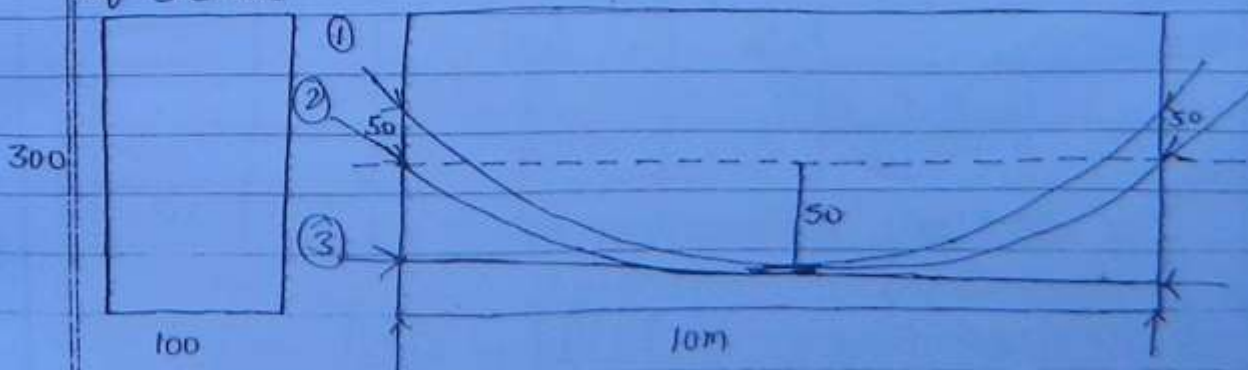
$$= \frac{P_2 + P_3 + P_4}{A_c} \times m$$

$$= \frac{(1827.10 + 1813.5 + 1800) \times 300}{400 \times 600} \times 6$$

$$= 40.8 \text{ N/mm}^2$$

$$\text{Initial stress} = 1840.8 \text{ N/mm}^2$$

Ques A Post tensioned Concrete beam is stressed by successive tensioning of 3 cables $m=6$



c/s of each cable = 200 mm^2 , Initial stress in each cable = 1200 N/mm^2

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Estimate % loss of stress in each cable due to plastic shortening only (Neglect self wt of & live load)

Solⁿ

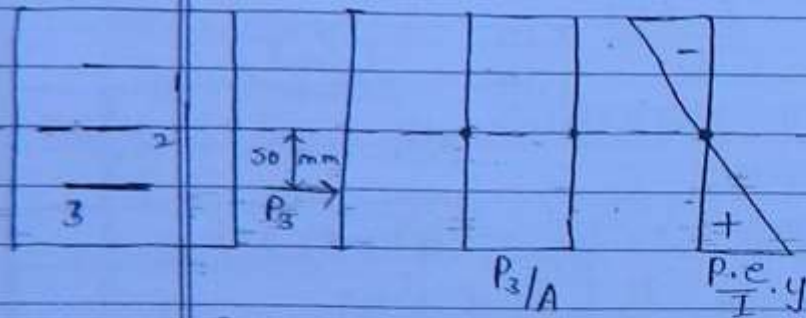
wire no-3 - loss of stress = 0

wire no-2 - due to wire no-3 loss will occur.

$$= \frac{\Delta l_3}{L} \times E_s = \frac{P_3 \cdot L}{A_c \cdot E_c \cdot L} E_s$$

$$= \frac{P_3}{A_c} \cdot \frac{E_s}{E_c} = f_{c3} \times m$$

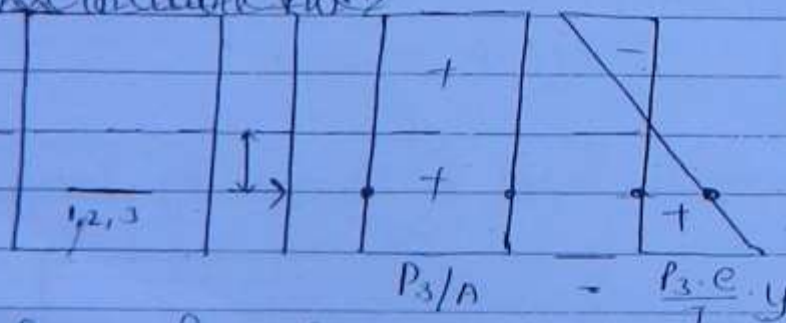
At end - stress in concrete at the location of wire no-2 due to prestressing of wire no-3



$$f_{c1} = \frac{P_3}{A}$$

$$= \frac{200 \times 1200}{300 \times 100} = 8 \text{ N/mm}^2$$

Midspan



Stress in concrete at the level of wire no-2 due to prestressing force P_3

$$f_{c2} = \frac{P_3}{A} + \frac{P_3 \cdot e \cdot y}{I}$$

$$= \frac{200 \times 1200}{300 \times 100} + \frac{200 \times 1200 \times 50^2}{225 \times 10^6}$$

$$= 8 + 2.67$$

$$= 10.67 \text{ N/mm}^2$$

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$$\text{loss} = \text{Average stress} = 8 + 10.67 - f_1 + \frac{2}{3}(f_2 - f_1)$$

$$= 8 + \frac{2}{3}(10.67 - 8)$$

$$f_{c3} = 9.78 \text{ N/mm}^2$$

(2.67)

$$\text{loss of prestress} = 9.78 \times 6$$

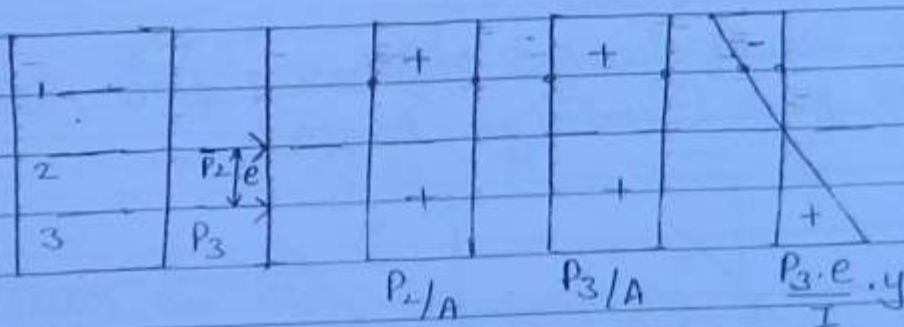
$$= 58.68 \text{ N/mm}^2$$

Wire No. 1 - due to wire no 2 & 3 loss will occur

$$\text{loss} = (f_{c3} + f_{c2}) m$$

→ Stress in Concrete at the location of wire no. 1 due to force in wire no. 3 & 2

At end

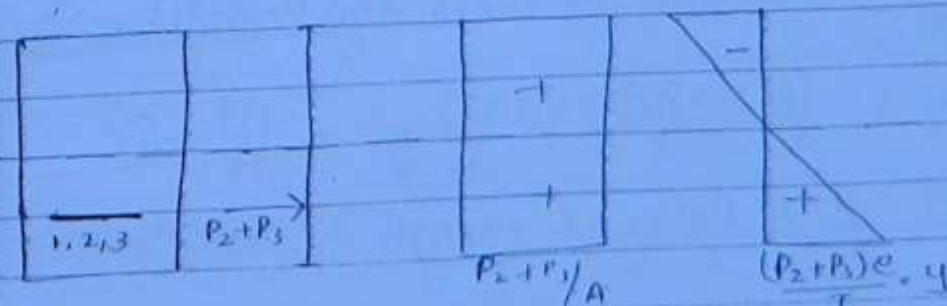


$$f_1 = \frac{P_2}{A} + \frac{P_3}{A} - \frac{P_3 \cdot e \cdot y}{I}$$

$$= \frac{200 \times 1200}{100 \times 300} + \frac{200 \times 1200}{300 \times 100} - \frac{200 \times 1200 \times 50}{225 \times 10^6} \times 50$$

$$f_{c1} = 8 + 8 - 2.67 = 13.33$$

At mid span



$$f_{c2} = \frac{P_2 + P_3}{A} + \frac{(P_2 + P_3) \cdot e \cdot y}{I} = \frac{2(200 \times 1200)}{100 \times 300} + \frac{2(200 \times 1200) \times 50}{225 \times 10^6}$$

$$= 8 \times 2 + 5.33 = 21.33 \text{ N/mm}^2$$

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$$\text{Average} = 13.33 + \frac{2}{3} (21.33 - 13.33) \\ = 18.66 \text{ N/mm}^2$$

$$\text{Loss} = 18.66 \times 6 \\ = 111.98 \text{ N/mm}^2$$

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Design of Prestressed beam

Design of Rectangular beam (Symmetrical Secⁿ)

TYPE I- Rectangular Secⁿ: for a given value of B.M

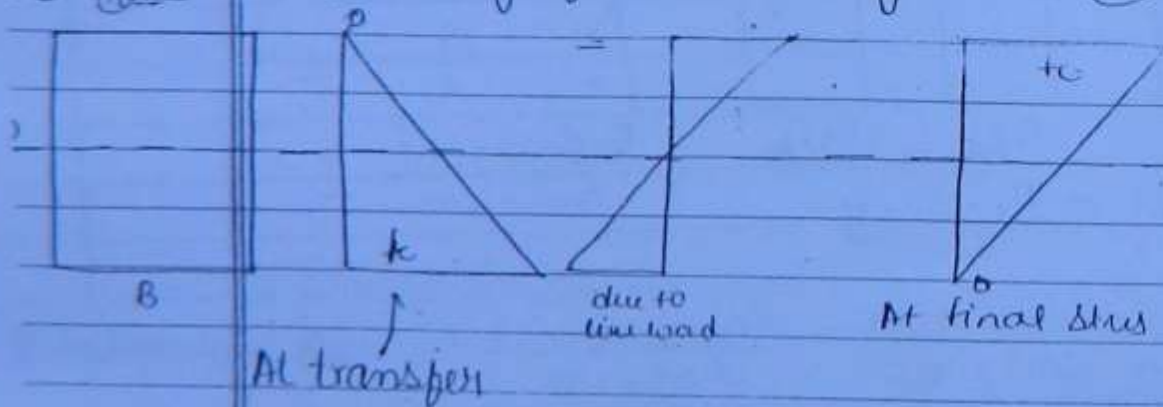
Given values -

① Permissible stresses in concrete in compression f_c
in tension = 0

2. Permissible stress of steel = f_s

As of steel = 0 in this case

To find out size of beam, prestressing force, location of prestressing force (eccentricity), Area (A_s)



Design formula are based on ideal stress diagram as shown in fig.

$$\text{At transfer (top)} = \frac{P}{A} - \frac{P \cdot e \cdot y}{I} + \frac{M_d \cdot y}{I} = 0 \quad \text{--- (1)}$$

$$\text{Bottom} = \frac{P}{A} + \frac{P \cdot e \cdot y}{I} - \frac{M_d \cdot y}{I} = f_c \quad \text{--- (2)}$$

$$\text{At final stage (top)} = \frac{P}{A} - \frac{P \cdot e}{Z} + \frac{(M_d + M_c)}{Z} = f_c \quad \text{--- (3)}$$

$$-bottom = \frac{P}{A} + \frac{P \cdot e}{Z} - \frac{(M_d + M_e)}{Z} = 0 \quad \text{--- (1)}$$

From (3) --- (1)

$$\frac{M_e}{Z} = f_c \quad \text{--- I}$$

(Section Modulus) $Z = M_e / f_c$

(264)

decide B & D so that $Z = \frac{BD^2}{6}$

$$A = B \times D$$

$$\text{From (1)} \quad \frac{P \cdot e}{Z} = \frac{P}{A} + \frac{M_d}{Z}$$

$$\text{From (4)} \quad \frac{P \cdot e}{Z} = \frac{P}{A} + \frac{M_d}{Z} + \frac{M_e}{Z}$$

$$2 \frac{P \cdot e}{Z} = \frac{2M_d}{Z} + \frac{M_e}{Z}$$

$$e = \frac{2M_d + M_e}{2P} \quad \text{--- II}$$

From (3) + (4)

$$\frac{2P}{A} = f_c$$

$$\boxed{P = \frac{A \cdot f_c}{2}} \quad \text{--- III}$$

Design Step

1. Find out live load Moment

$$M_e = \frac{w \cdot l^2}{8}$$

2. $Z = \frac{M_e}{f_c}$ (Section Modulus)

3. Decide Size of beam so that $Z = \frac{BD^2}{6}$

4. Calculate Area $B \times D$

5. Calculate $P = \frac{A \times f_c}{2}$

6. Calculate Area of Steel $= A_s = \frac{P}{f_s}$

7. Calculate dead load

$$\Rightarrow w_d = B \times d \times 1 \times 25 =$$

$$M_d = \frac{w_d l^2}{8}$$

B. Eccentricity $e = \frac{2M_d + M_e}{2P}$

(270)

5-1998
Ques Design a prestressed concrete slab span in 12m, carrying an imposed load of 20 kN/m^2 , M-40 concrete & steel with ultimate strength of 1600 N/mm^2 are available. Permissible stresses in concrete in comp $= 14 \text{ N/mm}^2$ tension $= 0$. Neglect the loss in prestress. Calc of 12 wires of $5 \text{ mm } \phi$ are there carrying an eff. prestress $= 225 \text{ kN}$ each

Sol: $w_i = 20 \text{ kN/m}^2$

$$M_i = \frac{w_i l^2}{8} = \frac{20 \times 12^2}{8} = 360 \text{ kN-m}$$

Section Modulus $Z = \frac{M_i}{f_c} = \frac{360 \times 10^6}{14}$

$$Z = 25714285.71$$

$$\frac{BD^2}{6} = 25714285.71$$

$$BD^2 = 154285714.3$$

Width of slab $= 1000 = B$

$$D^2 = 154285.7143$$

$$D = 392.79 \text{ mm} \approx 392.8 \text{ mm}$$

say $D = 400 \text{ mm}$

DATE: _____

$$\text{Area} = 400 \times 1000 = 4 \times 10^5 \text{ mm}^2$$

$$\text{Prestressing force} = \frac{4 \times 10^5 \times 14}{2} = 2800 \text{ kN}$$

$$\text{Area of Steel } A_s = \frac{2800 \times 10^3}{1600} = 1750 \text{ mm}^2$$

$$\text{No. of Cables req. (of 12 wires of 5mm dia)} = \frac{2800}{225} = 12.44 \rightarrow 13 \text{ nos. Cable}$$

$$\text{Total No. of wire} = 13 \times 12 = 156$$

$$\text{Area of Steel } A_s = \frac{13 \times A}{4}$$

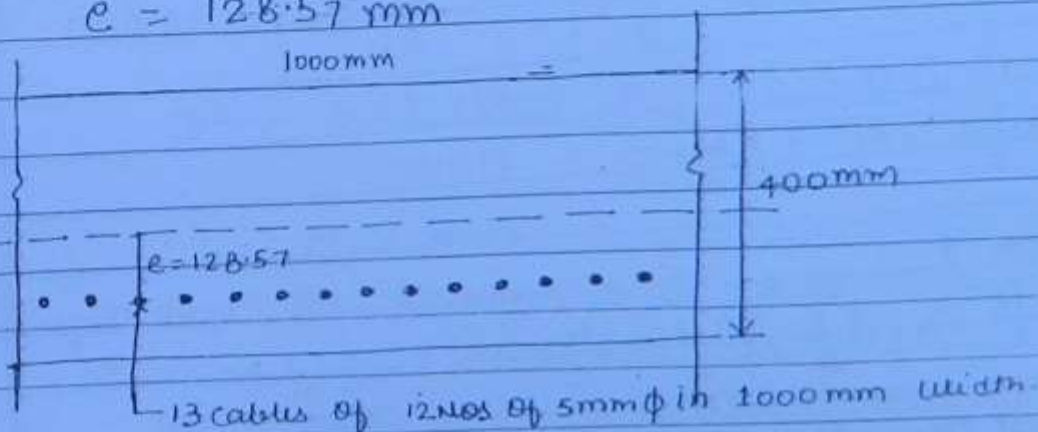
(271)

$$\text{Dead load} = 1 \times 4 \times 1 \times 25 = 10 \text{ kN/m}^2$$

$$M_d = \frac{10 \times 12^2}{8} = 180 \text{ kN-m}$$

$$e = \frac{2 \times 180 \times 10^6 + 360 \times 10^6}{2 \times 2800 \times 10^3}$$

$$e = 128.57 \text{ mm}$$



Case 2 If losses are considered.

Permissible stress in concrete

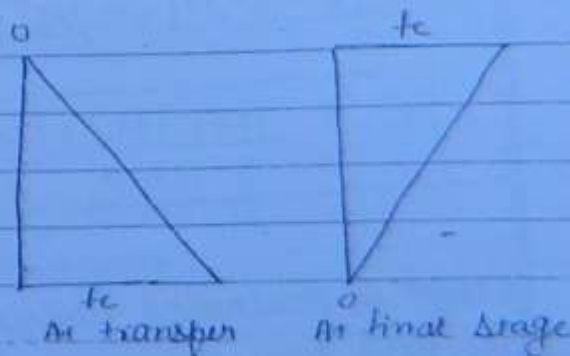
in compression = f_c

in tension = 0

loss factor = K

At transfer

$$\text{At top} = \frac{P}{A} - \frac{P e}{Z} + \frac{M_d}{Z} \leq 0 \quad \text{--- (1)}$$



$$\text{At bottom} = \frac{P}{A} + \frac{P \cdot e}{Z} - \frac{M_d}{Z} = f_c \quad \text{--- (2)}$$

At final stage of loading

$$\text{At top} = \frac{K P}{A} - \frac{K P e}{Z} + \frac{M_d}{Z} + \frac{M_e}{Z} = f_c \quad \text{--- (3)}$$

$$\text{At bottom} \quad \frac{K P}{A} + \frac{K P e}{Z} - \frac{M_d}{Z} - \frac{M_e}{Z} = 0 \quad \text{--- (4)}$$

From (3) - (1) × K

$$\frac{M_d}{Z} + \frac{M_e}{Z} - \frac{K \cdot M_d}{Z} = f_c$$

$$\frac{(1-K) M_d}{Z} + \frac{M_e}{Z} = f_c$$

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Section Modulus	$\frac{(1-K) M_d + M_e}{f_c} = Z$	--- (A)
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From (3) + 4

$$\frac{2 K P}{A} = f_c$$

$P = \frac{A \cdot f_c}{2 K}$	--- (B)
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From (1) × K

$$\frac{K \cdot P e}{Z} = \frac{K P}{A} + \frac{K M_d}{Z}$$

From 4

$$\frac{K \cdot P e}{Z} = -\frac{K P}{A} + \frac{M_d}{Z} + \frac{M_e}{Z}$$

Adding these two we get

$$-\frac{2 K P e}{Z} = \frac{(1+K) M_d}{Z} + \frac{M_e}{Z}$$

$e = \frac{(1+K) M_d + M_e}{2 K P}$	--- (C)
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Ques

Design a prestressed concrete beam for a span of 20m, live load over beam is 35 kN/m. Permissible stress in concrete in compression = 18 N/mm² & in tension = 0

Permissible stress in steel = 1600 N/mm^2

Loss of prestress = 15%

Design a rectangular beam & show the prestressing force & its location.

Sol:- Live load = 35 kN/m

$$M_L = \frac{35 \times 20^2}{8} = 1750 \text{ kN-m}$$

Let us assume c/s of rectangular beam

Size of beam $400 \times 600 \text{ mm}$

$$w_d = 4 \times 6 \times 1 \times 25 = 6 \text{ kN/m}$$

$$M_d = \frac{6 \times 20^2}{8} = 300 \text{ kN-m}$$

$$\text{NAW } Z = \frac{(1 - 0.85) 300 \times 10^6 + 1750 \times 10^6}{18}$$

$$Z = 99722222.22$$

$$\frac{BD^2}{6} = Z \quad \text{If } B = 400$$

$$D^2 = \frac{6 \times 99722222.22}{400}$$

$$D = 1223 \text{ mm} \quad 1050$$

Keep a size $B = 600 \text{ mm}$ & $D = 1050 \text{ mm}$

$$w_d = 6 \times 1.05 \times 1 \times 25 = 15.75 \text{ kN/m}$$

$$M_d = \frac{15.75 \times 20^2}{8} = 787.5 \text{ kN-m}$$

$$Z = \frac{(1 - 0.85) 787.5 \times 10^6 + 1750 \times 10^6}{18}$$

$$Z = 103784722.2$$

$$D = 1018 \text{ mm} \quad \& \quad B = 600 \text{ mm}$$

Size provided = $600 \times 1050 \text{ mm}$

$$\text{Area} = 63 \times 10^4 \text{ mm}^2$$

$$\text{prestressing force } P = \frac{A f_c}{2k} = \frac{63 \times 10^4 \times 18}{2 \times 0.85} = 6670.59 \text{ kN}$$

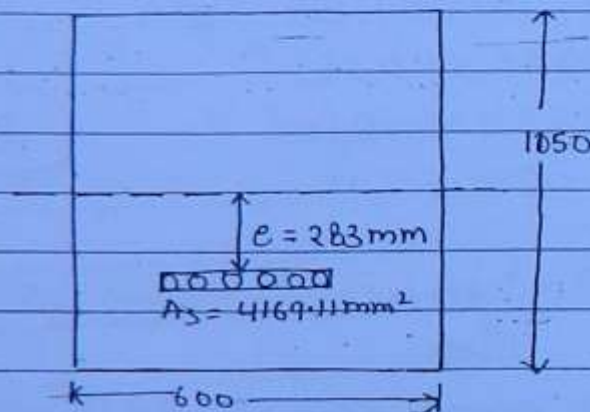
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$$\text{Area of Steel} = \frac{P}{f_s} = \frac{6670.6 \times 10^3}{1600} = 4169.11$$

$$e = \frac{(1+k) M_d + M_e}{2kP}$$

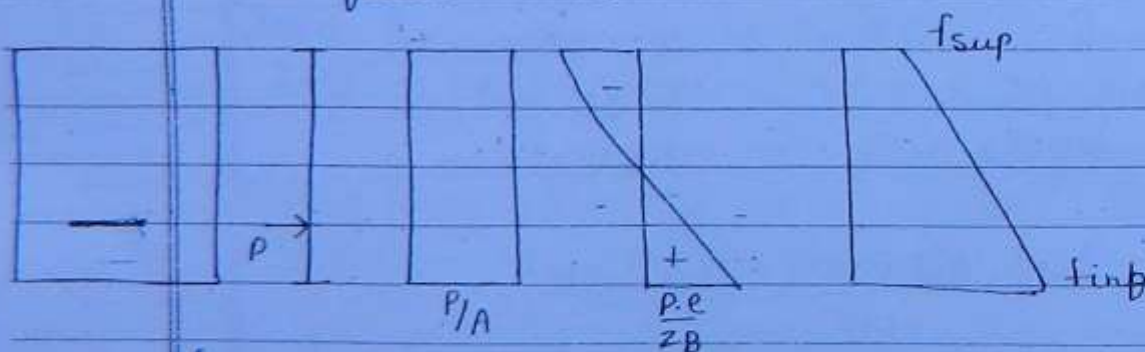
$$= \frac{(1+0.85) 787.5 \times 10^6 + 300 \times 10^6}{2 \times 0.85 \times 6670.6 \times 10^3}$$

$$e = 283 \text{ mm}$$



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Case 3: If permissible stresses in concrete
in compression = f_c
in tension = $-f_t$
loss factor = k



$$f_{sup} = \frac{P}{A} - \frac{P \cdot e}{Z_t}$$

$$f_{inf} = \frac{P}{A} + \frac{P \cdot e}{Z_b}$$

Design Steps

1. Assume a section

Calculate w_d & M_d

$$2. M_e = \frac{w_d \cdot l^2}{8}$$

3. Section Modulus required

$$Z = \frac{(1-K)M_d + M_e}{(k \cdot f_c - f_c)}$$

$$4. f_{sup} = \left(f_t - \frac{M_d}{Z_t} \right) = \left(f_t - \frac{M_d}{Z} \right)$$

$$5. f_{inf} = \frac{f_c}{K} + \frac{M_d + M_e}{K Z_B}$$

6. Prestressing force

$$P = \frac{A (f_{inf} \cdot Z_B + f_{sup} \cdot Z_t)}{Z_B + Z_t}$$

$$\text{If } Z_B = Z_t = Z$$

$$\text{then } P = \frac{A (f_{inf} + f_{sup})}{2}$$

$$7. \text{Eccentricity} = e = \frac{Z_t \cdot Z_B (f_{inf} - f_{sup})}{A (f_{sup} Z_t + f_{inf} Z_B)}$$

$$\text{If } Z_t = Z_B = Z$$

$$e = \frac{Z (f_{inf} - f_{sup})}{A (f_{sup} + f_{inf})}$$

ES-2007
Ques

A Post-tensioned PSC Beam of rect. secⁿ 240 mm wide is to be designed for a live load of 25 kN/m over an effective span of 12 m. Permissible stresses in concrete in compression = 17 N/mm² & tension = 1.45 N/mm². Loss = 15%. Calculate min. possible depth of beam P & e

Sol Assume size of beam

$$B = 240 \text{ mm}$$

$$D = 600 \text{ mm}$$

(276)

$$\text{dead load} = 24 \times 6 \times 1 \times 25 = 3.6 \text{ kN/m}$$

$$M_d = \frac{3.6 \times 12^2}{8} = 64.8 \text{ kN-m}$$

$$M_e = \frac{25 \times 12^2}{8} = 450 \text{ kN-m}$$

$$Z = \frac{(1-R) M_d + M_e}{(k f_c - f_t)}$$

$$= \frac{(1-0.85) 64.8 \times 10^6 + 450 \times 10^6}{(0.85 \times 17 + 1.4)}$$

$$= 29004416.4$$

$$\frac{B D^2}{6} = 851.53 \text{ mm}$$

$$\text{Assume } D = 860 \text{ mm}$$

$$w_d = 24 \times 86 \times 1 \times 25 = 5.16$$

$$M_d = 92.88 \text{ kN-m}$$

$$Z = \frac{(1-0.85) 92.88 \times 10^6 + 450 \times 10^6}{(0.85 \times 17 + 1.4)}$$

$$Z = 29270157.73$$

$$D = 855 \text{ mm} = \text{Min depth req.} = 855 \text{ mm}$$

$$\text{Say } D = 860 \text{ mm} = \text{Min depth req.}$$

$$\text{Size of beam} = 240 \times 860 \text{ mm}^2$$

$$4. f_{\text{sup}} = \left(f_t - \frac{M_d}{Z} \right)$$

$$Z = 29584 \times 10^3$$

$$= -1.4 - \frac{92.88 \times 10^6}{29270157.73}$$

$$= \frac{240 \times 860^2}{6}$$

$$f_{\text{sup}} = -4.54 \text{ N/mm}^2$$

