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Compressive Strength of Concrete :

It is defined as characteristic Compressive strength of 150mm Cubes of the age of 28 days $\rightarrow$ M20 grade

M $\rightarrow$ Mix proportion

20 $\rightarrow$ characteristic Compressive strength of Concrete in MPa

1. Ordinary Concrete - M₁₀ to M₂₀
2. Standard Concrete - M₂₅ to M₅₅
3. High strength Concrete - M₆₀ to M₈₀

Mix proportions:

M₅ - 1 : 5 : 10

M_{7.5} - 1 : 4 : 8

M₁₀ - 1 : 3 : 6

M₁₅ - 1 : 2 : 4

M₂₀ - 1 : 1½ : 3

Steel Reinforcement: Steel bars are essentially used in tension zone of flexural members to resist the tensile stresses.

Why steel is used as reinforcement?

- Its tensile strength is very high.
- It can develop good bond with concrete.
- The thermal coefficient value is more.
- It is easily available.

Reinforcement serves the following functions:

1. To resist the bending tension.
2. To increase the load carrying capacity.
3. To resist the diagonal tension due to shear.
4. To reduce the shrinkage of concrete.
5. To resist wide cracks & spiral cracks.

Types of reinforcement:

1. Mild steel & medium tensile steel.
2. High yield strength deformed steel bars (HYSD).
3. Steel wire fabrics.
4. Structural steel.

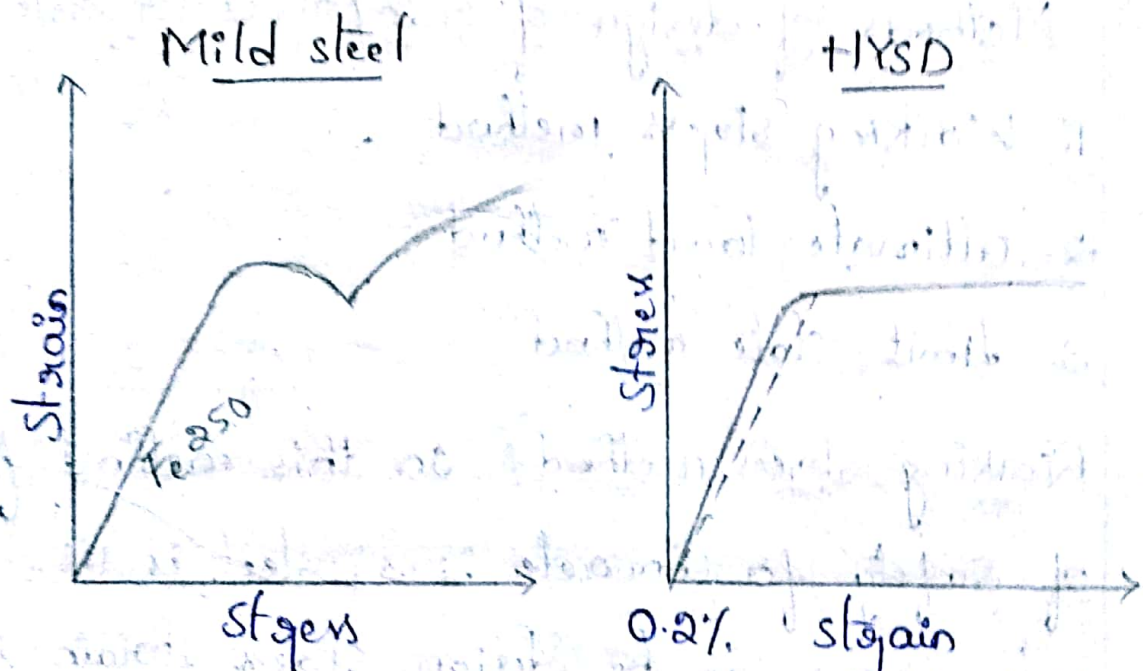
These are plain bars of grade Fe 250 where,

Fe $\rightarrow$ Ferricous

250 $\rightarrow$ yield strength (or) yield stress

HYSD bars:

They are having high yield strength of grade Fe 415 or Fe 500.



- $\rightarrow$ Yield stress is given by 0.2% of proof stress
- $\rightarrow$ The young's modulus of all types of steel are $2 \times 10^5 \text{ N/mm}^2$.
- $\rightarrow$ Unit wt. of steel is 78.5 kN/m^3

Types of loads:

1. Dead load - Self wt (Constant)
2. Live load - moving (Assume).
3. Wind load
4. Earthquake load
5. Snow load

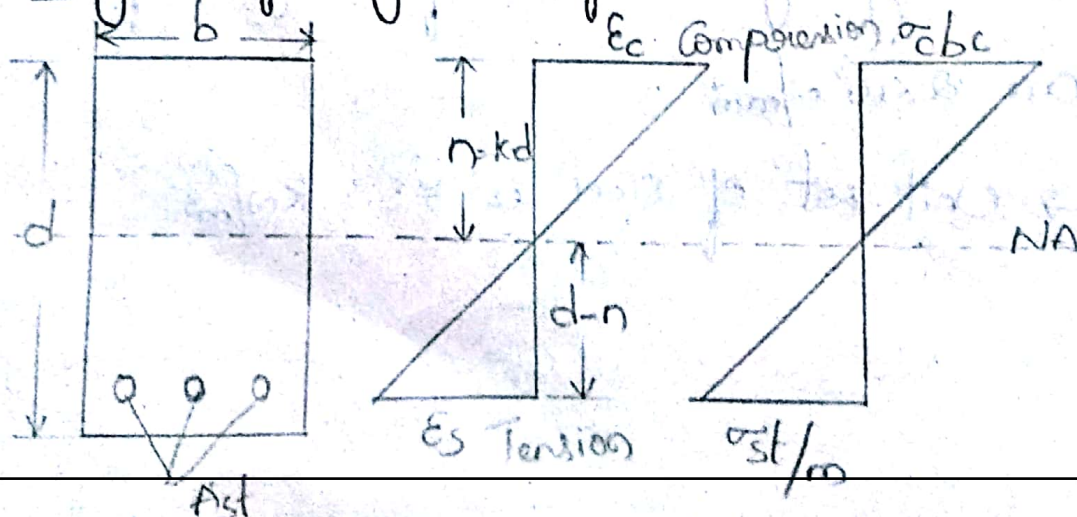
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Methods of design of reinforced concrete:

1. Working stress method
2. ultimate load method
3. Limit state method

Working stress method: In this method factor of safety for concrete is 3, steel is 1.5. In this only beams can be design. stress strain is linear.

Analysis of single reinforced sections:



Reinforcement bars are provided only on tension side in the beam section. It is called as single reinforced beam sections.

When the stresses developed in the concrete section are known,

Let,

σ_{bc} = Compressive stress in concrete due to bending.

σ_{st} = Tensile stress in steel

A_{st} = Area of tension steel

ϵ_c = Maximum strain in concrete.

ϵ_s = Maximum strain at the centroid of steel.

d = Effective depth.

b = width of the member.

n = Depth of neutral axis = $K \cdot d$

K = Neutral axis depth factor.

m = Modular ratio = $280 / 3\sigma_{bc}$

Since the strains in concrete & steel are proportional to their distances from neutral axis,

$$\frac{\epsilon_c}{\epsilon_s} = \frac{n}{d-n}$$

$$\frac{d-n}{n} = \frac{\epsilon_s}{\epsilon_c}$$

$$\left(\frac{d}{n}\right) - 1 = \frac{\sigma_{st}}{\epsilon_s} \times \frac{\epsilon_c}{\sigma_{cbc}}$$

$$= \frac{\sigma_{st}}{\sigma_{cbc}} \cdot \frac{1}{m}$$

$$\left[\because \epsilon_s = \frac{\sigma_{st}}{\epsilon_s}, \right. \\ \left. \epsilon_c = \frac{\sigma_{cbc}}{\epsilon_c} \right]$$

$$\left[\because m = \frac{\epsilon_s}{\epsilon_c} \right]$$

$$\Rightarrow \left(\frac{d}{n}\right) - 1 = \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}$$

↓
modular ratio

$$\Rightarrow \frac{d}{n} = 1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}$$

$$\boxed{m = \frac{280}{3\sigma_{cbc}}}$$

$$n = \left[\frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}} \right] d$$

$$n = k \cdot d$$

$$\text{where } k = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

the dimensions of the beam & the reinforcements are given.

The depth of neutral axis can be obtained by considering the equilibrium of internal forces of compression & tension

Force of Compression = Force of tension

$$\begin{aligned}\text{Force of Compression } C &= \text{Average stress} \times \text{area of beam in compression} \\ &= \frac{\sigma_{cbc}}{2} \times b \times n\end{aligned}$$

Force of tension T = permissible stress $\times$ Area of steel.

$$= \sigma_{st} \times A_{st}$$

$$= m \cdot \sigma_{cbc} \cdot \left(\frac{d-n}{n}\right) \times A_{st}$$

$$\Rightarrow \frac{\sigma_{cbc}}{2} \times b \times n = m \cdot \sigma_{cbc} \cdot \left(\frac{d-n}{n}\right) \cdot A_{st}$$

$$\Rightarrow \boxed{\frac{bn^2}{2} = m \cdot A_{st} (d-n)}$$

$$\Rightarrow \frac{bn^2}{2} \neq m \cdot A_{st} (d-n) \text{ also}$$

$$\Rightarrow \frac{bn^2}{2} + m \cdot A_{st} n - m \cdot A_{st} d = 0$$

By solving the above quadratic equation,

$$n = \frac{-m \cdot Ast + \sqrt{(m \cdot Ast)^2 - 2 \cdot b \cdot m \cdot Ast \cdot d}}{b}$$

Lever arm:

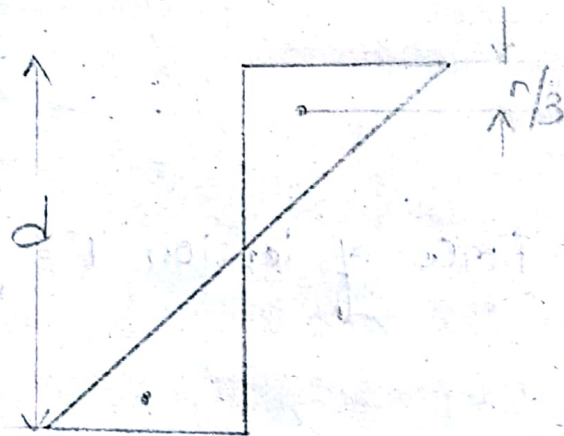
The forces of Compression & tension form a Couple. The distance between the lines of action of Compression & tension forces is called as lever arm.

$$z = d - \frac{n}{3}$$

$$= d - \frac{k \cdot d}{3}$$

$$z = d \left(1 - \frac{k}{3}\right)$$

$$z = j \cdot d$$



∴ Lever arm factor $j = 1 - \frac{k}{3}$

Moment of resistance:

Moment of resistance = Total Compression or tension × lever arm

Total Compressive force $C = \frac{1}{2} \times \sigma_{cb} \times b \times n$

Moment of resistance (M_R) = C. 2

$$= \frac{1}{2} \cdot \sigma_{cbc} \times b \times n \times (j \cdot d)$$

$$= \frac{\sigma_{cbc}}{2} \times b \times k d \times j d$$

$$= \frac{1}{2} k j \cdot \sigma_{cbc} \cdot b d^2$$

$$\therefore \boxed{M_R = Q \cdot b d^2}$$

Q = moment of resistance Constant

$\therefore m, k, j, Q$ are also design Constants.

Types of section:

1. Balanced section.
2. Under reinforced section.
3. Over reinforced section

Balanced Section:

A reinforced concrete section in which steel & concrete reach their maximum allowable stresses simultaneously is called balanced or critical section. Neutral axis corresponding to balanced section is called as critical neutral axis.

The percentage steel corresponding to balanced section is called critical percentage of steel.

Under reinforced Section:

A section in which the area of steel reinforcement provided is less than that is required for a balanced section is called as under reinforced section.

$$\text{Moment of resistance} = \sigma_{st} \cdot A_{st} \left(d - \frac{n_a}{3} \right)$$

Over reinforced Section:

A section in which the area of steel reinforcement provided is more than that is required for a balanced section is called as ~~under~~ over reinforced section.

$$MR = \frac{1}{2} \sigma_{cbc} \times b \times n_a \left(d - \frac{n_a}{3} \right)$$

- A reinforced concrete beam 250mm wide, 475mm overall depth is reinforced with 3 bars of 16mm at an effective cover of 50mm using M20 grade concrete & Fe 415 steel. Find the depth of NA.

$$b = 850 \text{ mm}$$

$$d = 475 - 50$$

$$= 425 \text{ mm}$$

$$A_{st} = 3 \times \frac{\pi}{4} (16)^2$$

$$A_{st} = 603.19 \text{ mm}^2$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2 \quad [\because \text{From table no: 21 for } M_{20} \text{ grade}]$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

[\because From table no: 22]

$\frac{bn^2}{2}$ Depth of ^{actual} neutral axis (n_a):-

$$\frac{bn^2}{2} = m \cdot A_{st} (d - n)$$

$$\frac{850 \times n^2}{2} = 13.33 \times 603.19 (425 - n)$$

$$n^2 = 64.32 (425 - n)$$

$$n^2 - 27337.78 + 64.32n = 0$$

$$\therefore \boxed{n_a = 136.28 \text{ mm}}$$

2. A reinforced concrete beam of 300mm wide by 550mm overall depth is reinforced with 4 bars

of 20mm ϕ at an effective depth of 50mm

Using M20 grade concrete & Fe 415 steel. Estimate the moment of resistance of the section.

Sol

Given data,

$$b = 300\text{mm}$$

$$D = 550\text{mm}$$

$$\text{Effective depth } d = 550 - 50 \\ = 500\text{mm}$$

$$\text{Area of tension steel } A_{st} = 4 \times \frac{\pi}{4} (20)^2 \\ = 1256.6\text{mm}^2$$

$$\sigma_{st} = 230\text{N/mm}^2$$

$$\sigma_{cbc} = 7\text{N/mm}^2$$

Design Constant for M20 & Fe 415

$$m = \frac{280}{3\sigma_{cbc}} = 13.33$$

Critical depth of neutral axis,

$$n_c = k \cdot d$$

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m \times \sigma_{cbc}}}$$

$$= \frac{1}{1 + \frac{230}{13.33 \times 7}}$$

$$\therefore n_c = k \cdot d$$

$$= 0.289 \times 500$$

$$\therefore \boxed{n_c = 144.5 \text{ mm}}$$

Depth of actual neutral axis (n_a):

$$\frac{bn^2}{2} = m \cdot A_{st} (d - n_a)$$

$$\frac{300n^2}{2} = 13.33 \times 1256.6 (500 - n_a)$$

$$\boxed{n_a = 187 \text{ mm}}$$

$n_a < n_c \rightarrow$ under reinforced

$n_a > n_c \rightarrow$ Over reinforced

$$\boxed{n_a > n_c}$$

$\therefore$ The section is over reinforced.

Moment of resistance (MR): $\frac{1}{2} \sigma_{bc} \times b \times n_a (d - \frac{n_a}{3})$

$$\Rightarrow \frac{1}{2} \times 7 \times 300 \times 187 (500 - \frac{187}{3})$$

$$\therefore \boxed{MR = 85.9 \text{ kN-m}}$$

A reinforced concrete beam 300mm x 600mm, over all depth is reinforced with 4 bars of 20mm at an effective cover of 50mm. what uniformly distributed load of this beam can carry excluding self wt, over a s.s span of 5m. Assume M25 grade & Fe 415 steel.

Sol

Given data,

$$b = 300\text{mm}$$

$$D = 600\text{mm}$$

$$\text{Effective Cover } d = 600 - 50 \\ = 550\text{mm}$$

$$\text{Area of tension steel } A_{st} = 4 \times \frac{\pi}{4} (20)^2 \\ = 1256.6\text{mm}^2$$

$$\sigma_{st} = 230\text{N/mm}^2 \quad (\because \text{From table 22})$$

$$\sigma_{bc} = 8.5\text{N/mm}^2 \quad (\because \text{from Pg: no 81 Table 21})$$

Design Constant for M25 grade & Fe 415

$$m = \frac{280}{3\sigma_{bc}} = \frac{280}{3 \times 8.5} = 10.98$$

Critical depth of neutral axis,

$$n_c = k \cdot d$$

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{bc}}} = \frac{1}{1 + \frac{230}{10.98 \times 8.5}}$$

$$n_c = 0.28 \times 550$$

$$n_c = 158.76 \text{ mm}$$

Depth of actual neutral axis (n_a):

$$\frac{bn^2}{2} = m \cdot A_{st}(d - n_a)$$

$$\frac{300n^2}{2} = 10.98 \times 1256.6(550 - n_a)$$

$$n_a = 183.59 \text{ mm}$$

$$n_a > n_c$$

$\therefore$ The section is over reinforced section.

$$M_R = \frac{1}{2} \times \sigma_{cbc} \times b \times n_a \left(d - \frac{n_a}{3}\right)$$

$$= \frac{1}{2} \times 8.5 \times 300 \times 183.59 \left(550 - \frac{183.59}{3}\right)$$

$$M_R = 114.42 \text{ kN-m}$$

Maximum load:

Let w kN/m be the uniformly distributed load beam can carry.

$$\text{Max BM} = \frac{wl^2}{8} \Rightarrow \frac{w \times 5^2}{8} = 3.125w$$

$$114.42 = 3.125W$$

$$W = 36.51 \text{ [including self wt]}$$

$$\begin{aligned} \text{Self wt. of the beam} &= 0.3 \times 0.6 \times 1 \times 25 \\ &= 4.5 \text{ kN/m} \end{aligned}$$

$$\begin{aligned} \text{Uniformly distributed load excluding self wt.} \\ &= 36.51 - 4.5 \\ &= \underline{32.01 \text{ kN/m}} \end{aligned}$$

4. Design a RCC beam 230mm wide to resist a BM of 30kN-m. use M_{20} & Fe 415.

sol

Given data,

$$b = 230 \text{ mm}$$

$$BM = 30 \text{ kN-m}$$

$$\sigma_{st} = 230 \text{ N/mm}^2$$

$$\sigma_{cbc} = 7 \text{ N/mm}^2$$

Design Constants for M_{20} grade & Fe 415 steel

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$$

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}} = \frac{1}{1 + \frac{230}{13.33 \times 7}} = 0.289$$

$$j = 1 - \frac{k}{3}$$
$$= 0.904$$

$$Q = \frac{1}{2} \sigma_{cbc} \times k \times j$$
$$= \frac{1}{2} \times 7 \times 0.289 \times 0.904$$
$$= 0.914$$

Depth required,

$$MR = Qbd^2$$

$$d = \sqrt{\frac{BM}{Q \times b}} \quad [\because MR = \max. BM]$$
$$= \sqrt{\frac{30 \times 10^6}{0.914 \times 230}}$$

$$d = 377.8 \text{ mm} \approx 380 \text{ mm}$$

Area of tensile steel A_{st} :

$$MR = A_{st} \cdot \sigma_{st} \times j \cdot d$$

$$30 \times 10^6 = A_{st} \times 230 \times 0.904 \times 380$$

$$30 \times 10^6 = 79009.6 A_{st}$$

$$A_{st} = 379.7 \text{ mm}$$

$$D = 380 + 40$$

$$= 420 \text{ mm } [\because \text{Assume } 40 \text{ mm}]$$

Let,

Assume 12mm ϕ bars to find out the no. of bars.

$$A_{st} = \text{No. of bars} \times \frac{\pi}{4} (12)^2$$

$$379.9 = 113.09 \times \text{No. of bars}$$

$$\text{No. of bars} = \frac{379.9}{113.09}$$

$$= 3.35 \approx 4 \text{ bars}$$

$$\therefore \text{No. of bars} = \underline{\underline{4}}$$

17/17

5. Double reinforced beam is to be design having an overall c/s dimensions of 250mm x 400mm with an effective span of 4m. The beam has to support and uniformly distributed dead load of 2.5 kN/m together with a live load of 8.0 kN/m in addition to its self wt adopting M20 grade Concrete & Fe 415. Design suitable reinforcement in the beam

Effective span $L = 4\text{m}$
Breadth of beam $b = 250\text{mm}$

Overall depth $D = 400\text{mm}$

Dead load = 2.5 kN/m

Live load = 20 kN/m

M₂₀ grade Concrete & Fe 415 HYSD bars

$$\sigma_{cbc} = 7\text{ N/mm}^2$$

$$Q = 0.91$$

$$j = 0.90$$

$$\sigma_{st} = 230\text{ N/mm}^2$$

$$m = \frac{250}{3\sigma_{cbc}} = \frac{250}{3 \times 7} = 11.90$$

$$Q = \frac{1}{2} \times \sigma_{cbc} \times k \times j$$

~~$$Q = \frac{1}{2} \times 7 \times k \times j$$~~

$$k = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}} = \frac{1}{1 + \frac{230}{11.90 \times 7}}$$

$$k = 0.289$$

$$j = 1 - \frac{k}{3}$$

$$= 1 - \frac{0.289}{3}$$

$$Q = \frac{1}{2} \times 3 \times 0.289 \times 0.904$$

$$= 0.914$$

Depth required

~~MR~~

Loads:

Self wt. of beam = $0.25 \times 0.4 \times 25$

$$W = 2.5 \text{ kN/m}$$

Dead Load = 2.5

Live Load = 20.00

total load $W = 22.5 \text{ kN/m}$

Adopt an effective Cover of 50mm

Effective depth = $400 - 50$

$$d = 350 \text{ mm}$$

Bending moments & shear force

$$M = \frac{wl^2}{8} = \frac{22.5 \times 4^2}{8} = 50 \text{ kN-m}$$

Depth required.

$$MR = Qbd^2$$

$$= 0.914 \times 0.25 \times (3.5 \times 10^4)^2$$

$M_R = 27.8 \text{ kN-m} \rightarrow M_1$
 M_1 greater than M . So, it is design by doubly reinforced section.

$$\begin{aligned}\text{Balance moment } M_2 &= M - M_1 \\ &= 50 - 27.8 \\ &= 22.2 \text{ kN-m}\end{aligned}$$

Tension steel required for balanced singly reinforced section,

$$\begin{aligned}A_{st1} &= \frac{M_1}{\sigma_{st} \cdot j \cdot d} \Rightarrow \frac{27.8 \times 10^6}{230 \times 0.904 \times 350} \\ &= 352 \text{ mm}^2\end{aligned}$$

Additional tension steel for balanced moment,

$$\begin{aligned}A_{st2} &= \frac{M_2}{\sigma_{st}(d - d_c)} \\ &= \frac{22.2 \times 10^6}{230 \times (350 - 50)} \\ &= 321.7 \approx 322 \text{ mm}^2\end{aligned}$$

Total tensile steel bars,

$$A_{st} = A_{st1} + A_{st2}$$

$$= 703 \text{ mm}^2$$

Provide 4 bars of 16mm diameter

Compression reinforcement,

$$A_{sc} = \frac{m \cdot A_{st2} (d - n_c)}{(1.5m - 1)(n_c - d_c)}$$



$$n_c = 0.289 d$$

$$= 0.289 \times 350$$

$$= 101.15$$

$$A_{sc} = \frac{13.33 \times 309 (350 - 101.15)}{(1.5 \times 13.33 - 1) (101.15 - 50)}$$

$$= 1098.3 \text{ mm}^2$$

Assume 16mm ϕ bars to find out the no of bars

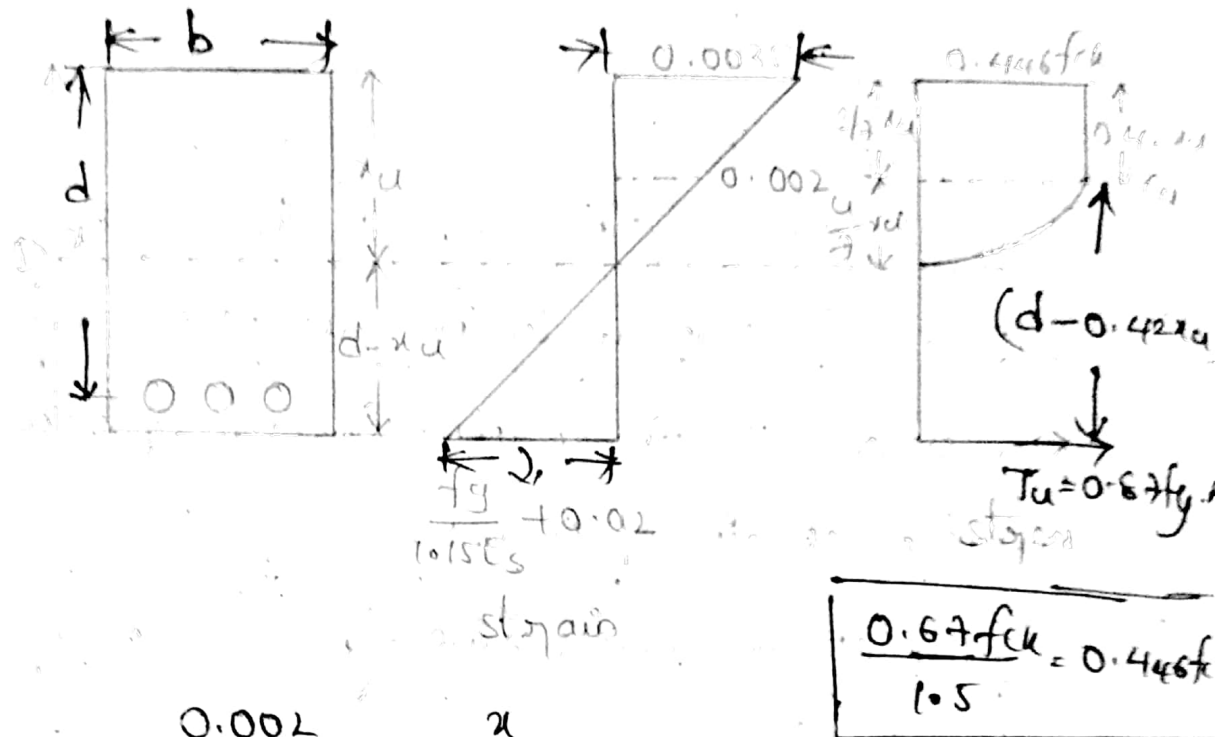
$$A_{sc} = \text{No. of bars} \times \frac{\pi}{4} (16)^2$$

$$1098.3 = \text{No. of bars} \times 201.06$$

$$\text{No. of bars} = 5.46 \approx 6 \text{ bars}$$

8, 12, 20, 16, 18, 22, 24, 25, 32, 36 - ϕ

Limit state Method



$$\frac{0.002}{0.0035} = \frac{x}{x_u}$$

$$x = \frac{4}{7} x_u$$

Area of concrete = parabolic + rectangle

$$= \frac{2}{3} b x + \frac{3}{7} x_u \times 0.446 f_{ck}$$

$$= \left(\frac{2}{3} \times \frac{4}{7} x_u \times 0.446 f_{ck} \right)$$

$$+ \frac{3}{7} x_u \times 0.446 f_{ck}$$

$$= 0.36 f_{ck} \cdot x_u$$

Distance of Centroid of stress block from the top fibre:

$$x_c = \left(\frac{3}{7} x_u \times 0.446 f_{ck} \right) \left(\frac{1}{2} \times \frac{3}{7} x_u \right) +$$

$$\boxed{x_c = 0.4274}$$

Stress Block parameters:

Stress-strain Curve at the crushing of concrete is assumed to be parabolic shape upto 0.002 strain & then constant upto the maximum strain of 0.0035.

The strain varies linearly across the depth of the section.

4/7/17 Depth of Neutral axis (x_u):

Force of Compression = Force of Tension

$$0.36 f_{ck} b \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b}}$$

Moment = Force $\times$ distance

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

Moment = Stress $\times$ Area $\times$ distance

(i) For balanced reinforced section,

$$x_u = x_{u\max}$$

In this concrete & steel both are failed

So, find the moment capacity

$$M_R = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

→ for concrete

for steel,

$$M_R = 0.87 f_y A_{st} (d - 0.42 x_u)$$

(ii) For under reinforced section

$$x_u < x_{u\max}$$

$$M_R = 0.87 f_y A_{st} (d - 0.42 x_u)$$

Steel fails in this section

(iii) for over reinforced section ($x_u > x_{u\max}$)

On this section concrete fails.

So, the moment of resistance is,

$$M_R = 0.36 f_{ck} \cdot x_{u\max} \cdot b (d - 0.42 x_{u\max})$$

x_u = Actual depth of Neutral axis,

$x_{u\max}$ = Max. depth of Neutral axis.

⇒ From pg. 70 in Code book:

f_y	$x_{u\max}/d$
250	0.53
415	0.48
500	0.46

for Fe 250 steel, $x_{u\max} = 0.53d$

for Fe 415 steel, $x_{u\max} = 0.48d$

for Fe 500 steel, $x_{u\max} = 0.46d$

Limiting value of M_R ($M_{u, \text{limit}}$)

Since the maximum depth of NA is limited to avoid brittle failure, the maximum value of moment of resistance is also limited.

$M_{u, \text{lim}}$

$$M_R = 0.36 f_{ck} \times b \times x_u (d - 0.42 x_u)$$

for Fe 250 steel, $x_{u \text{ max}} = 0.53d$ [from Code book]

$M_{u, \text{lim}}$

$$M_R = 0.36 f_{ck} \times b (0.53d) (d - (0.42 \times 0.53d))$$

$$\therefore \boxed{M_{u, \text{lim}} = 0.148 f_{ck} b d^2}$$

for Fe 415 steel, $x_{u \text{ max}} = 0.48d$ [from Code book].

$M_{u, \text{lim}}$

$$M_R = 0.36 f_{ck} \times b \times x_u (d - 0.42 x_u)$$

$$= 0.36 f_{ck} \times b \times 0.48d (d - 0.42(0.48d))$$

$$\boxed{M_{u, \text{lim}} = 0.138 f_{ck} b d^2}$$

for Fe 500 steel, $x_{u \text{ max}} = 0.46d$

$$M_{u, \text{lim}} = 0.36 f_{ck} \times b \times 0.46d (d - 0.42(0.46d))$$

$$= 0.133 f_{ck} b d^2$$

Beam of $230\text{mm} \times 480\text{mm}$ effective depth reinforced with 3 bars of 20mm diameter. Concrete is of M_{20} grade & $F_e 415$ steel

sol

Given data,

$$b = 230\text{mm}$$

$$d = 480\text{mm}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$f_y = 415\text{N/mm}^2$$

$$A_{st} = \frac{\pi}{4} (20)^2 \times 3$$

$$A_{st} = 942.47\text{mm}^2$$

Depth of Neutral axis:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 942.47}{0.36 \times 20 \times 230}$$

$$x_u = 205.48\text{mm}$$

$$x_{u\text{max}} = 0.48d \text{ [for } F_e 415 \text{ steel]}$$

$$= 0.48(480)$$

$$\therefore x_u < x_{u \max}$$

So, the section is under reinforced.

Moment of resistance:

$$M_R = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 942.47 (480 - 0.42 (205.48))$$

$$M_R = 133.967 \times 10^6 \text{ N-mm}$$

$$M_R = 133.97 \times 10^6 \text{ N-mm}$$

$$= 133.97 \text{ kN-m}$$

2. Find the ultimate moment of resistance of singly reinforced beam 200mm x 400mm effective depth reinforced with 5 bars of 20mm ϕ . Concrete is of M20 grade & steel Fe 250.

Sol

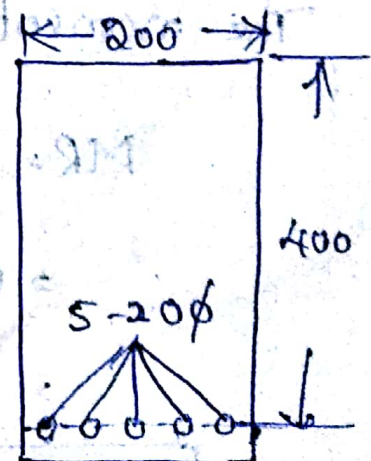
Given data,

$$b = 200 \text{ mm}$$

$$d = 400 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$



$$A_{st} = \frac{5 \times \pi (20)^2}{4}$$

$$A_{st} = 1570.79 \text{ mm}^2$$

Depth of neutral axis:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 250 \times 1570.79}{0.36 \times 20 \times 200}$$

$$x_u = 237.25 \text{ mm}$$

$$x_{u, \max} = 0.53 d \quad [\text{for Fe 250 steel}]$$

$$= 0.53 \times 400$$

$$x_{u, \max} = 212 \text{ mm}$$

$$\therefore x_u > x_{u, \max}$$

So, the section is over reinforced

The moment of resistance is:

$$M_R = 0.36 f_{ck} x_{u, \max} b (d - 0.42 x_{u, \max})$$

$$= 0.36 \times 20 \times 212 \times 200 (400 - (0.42 \times 212))$$

$$= 94.92 \times 10^6 \text{ N-mm}$$

$$M_R = 94.92 \text{ kN-m}$$

To determine the Ast dimensions of the beam
& MR is given

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 f_y A_{st} \left(d - 0.42 \left(\frac{0.87 f_y A_{st}}{0.36 f_{ck} b} \right) \right)$$

$$\therefore M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

3. A reinforced concrete beam has a section of 200 x 500mm overall. It is subjected to a factored moment of 80 kN-m. Design the reinforcement using Fe 250 steel & M20 grade concrete. use effective ^{cover} of 50mm

Sol

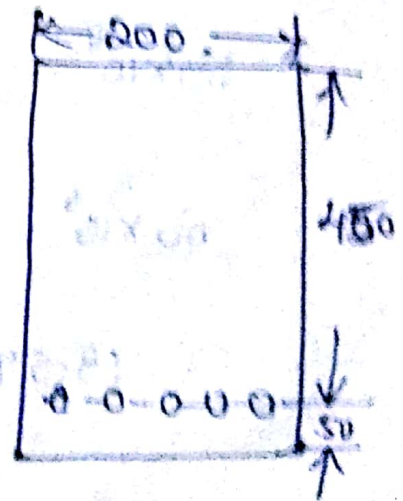
Given data,

$$b = 200 \text{ mm}$$

$$d = 500 - 50$$

$$= 450 \text{ mm}$$

$$M_u = 80 \text{ kN-m}$$



$M_u < M_{u \text{ limit}} \rightarrow$ Single reinforced beam

$$f_y = 250 \text{ N/mm}^2$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$M_{u, \text{limit}} = 0.148 f_{ck} b d^2$$

$$= 0.148 \times 20 \times 200 \times 450^2$$

$$= 119.88 \times 10^6 \text{ N-mm}$$

$$= 119.88 \text{ kN-m}$$

$$\therefore M_u < M_{u, \text{limit}} \text{ (single reinforced)}$$

$$\therefore M_u = 0.87 f_y A_{st} x d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$80 \times 10^6 = 0.87 \times 250 \times A_{st} \times 450 \left[1 - \frac{250 \times A_{st}}{20 \times 200 \times 450} \right]$$

$$80 \times 10^6 = 97875 A_{st} \left[1 - 1.388 \times 10^{-4} A_{st} \right]$$

$$80 \times 10^6 = 97875 A_{st} - 13.593 A_{st}^2$$

$$13.593 A_{st}^2 - 97875 A_{st} + 80 \times 10^6 = 0$$

$$A_{st} = 940.2 \text{ mm}^2$$

Provide 20mm ϕ for finding no. of bars

$$940.2 = \text{No. of bars} \times 314.15$$

$$314.15 \times \text{No. of bars} = 940.2$$

$$\text{No. of bars} = 2.99 \approx 3$$

$$\therefore \boxed{\text{No. of bars} = 3}$$

Design
General Considerations for beam:

1. Effective span: The effective span of S.S. beam shall be taken as clear span plus effective depth of the beam (or) Centre to Centre distance b/w the supports whichever is less

2. Limiting stiffness: $\left(\frac{l}{d}\right)$ ratio

Cantilever - 7

l = Effective span

Simply supported - 20

d = Effective depth

Continuous - 26

3. Minimum reinforcement = $\frac{A_{st\min}}{bd} = \frac{0.85}{f_y}$

4. Maximum reinforcement = $0.04bd$

forced concrete beam section
200 x 450 mm is reinforced with 4 bars of 20 mm ϕ with an effective cover of 40 mm. The beam is S.S over a span of 4 m. Find the safe uniform distributed load the beam can carry. Use M20 grade concrete & Fe 415 steel

Sol

Given data:

$$b = 200 \text{ mm}$$

$$d = 450 - 40$$

$$d = 410 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} (20)^2$$
$$= 1256.6 \text{ mm}^2$$

Depth of Neutral axis:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 1256.6}{0.36 \times 20 \times 200}$$

$$= 315.1 \text{ mm}$$

$$x_u = 315.1 \text{ mm}$$

$$x_{u \max} = 0.48d \quad [\text{for Fe 415 steel}]$$

$$= 0.48 \times 410$$

$$= 196.8 \text{ mm}$$

$$x_{u \max} = 196.8 \text{ mm}$$

$$\therefore x_u > x_{u \max}$$

$\therefore$ So, the Section is Over reinforced.

Moment of resistance: (MR)

$$MR = 0.36 f_{ck} x_{u \max} \times b (d - 0.42 x_{u \max})$$

$$= 0.36 \times 20 \times 196.8 \times 200 (410 - 0.42 (196.8))$$

$$= 92.76 \times 10^5 \text{ N-mm}$$

$$= 92.76 \text{ kN-m}$$

$$M_u = 92.76 \text{ kN-m}$$

Safe load:

$$BM = \frac{wl^2}{8} = \frac{10(4)^2}{8} = 20$$

$$\text{Equate } BM = M_u$$

$$20 = 92.76$$

$$w = 46.38 \text{ kN/m}$$

Safe working load $w = \frac{\text{Bending moment (u.u.)}}{\text{load factor}}$

$$= \frac{46.38}{1.5}$$

$$= 30.92 \text{ kN/m (including self wt)}$$

Self wt of the beam $= 0.2 \times 0.45 \times 25$

$$= 2.25 \text{ kN/m}$$

$\therefore$ Net Super imposed load $= 30.92 - 2.25$

$$= 28.67 \text{ kN/m}$$

7/17

General Design requirements for beams:

5. Spacing of bars: The horizontal distance between two parallel main reinforcement bars shall usually be ^{not} less than the greatest of the following.

- (a) Diameter of the bar if the diameters are equal.
- (b) Diameter of the largest bar if the bars are unequal.
- (c) 5mm more than the normal max. size of the aggregate.

Design procedure: (l, f_{ck}, f_y, b)

1. Assuming $\frac{l}{d}$ ratio (10 to 15)

Assume $b = 230\text{mm}$

2. Effective span :

- a) clear span + d
 b) clear span + $\frac{b}{2} + \frac{b}{2}$ } max min

3. Calculation of load & BM

Self weight = volume of beam $\times$ density of concrete

$$= b \times d \times l \times 25$$

$$BM = \frac{wl^2}{8}$$

factored BM, $M_u = 1.5 \times BM$

$$R_{cc} = 25 \text{ kN/m}^3$$

$$P_{cc} = 24 \text{ kN/m}^3$$

4. Check of depth required,

M_{ulimit} is equating to M_u ($M_u = M_{ulim}$)

for Fe 415, $M_{ulim} = 0.138 f_{ck} b d^2$

$$d = \sqrt{\frac{M_{ulimit}}{0.138 f_{ck} \cdot b}}$$

5. Calculated area of reinforcement

$$M_u = 0.87 f_y A_{st} x d \left(1 - \frac{f_y A_{st}}{f_{ck} b x d} \right)$$

$$A_{st} = ?$$

$$A_{stmin} < A_{st} < A_{stmax}$$

6. check for deflection,

5. Design a rectangular S-S reinforced concrete beam over a clear span of 4m. If the super imposed load is 20 kN/m and support width is 300mm each. Use M20 and Fe 415 steel. check for deflection also.

sol

Given data,

$$\text{Clear span } L = 4\text{m} \Rightarrow 4000\text{mm}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$f_y = 415\text{N/mm}^2$$

$$b = 300\text{mm}$$

Super imposed load = 20 kN/m
 (i) Assuming $\frac{l}{d} = 12$ based on

$$\text{Stiffness } d = \frac{4000}{12}$$

$$\frac{4000}{d} = 12$$

$$12d = 4000$$

$$d = \frac{4000}{12}$$

$$d = 333.33 \text{ mm} \approx 350 \text{ mm}$$

$$\boxed{d = 350 \text{ mm}}$$

Effective Cover assuming = 50 mm

$$\text{Overall depth} = 50 + 350$$

$$\boxed{D = 400 \text{ mm}}$$

(ii) Effective Span:

It is the least of the following

$$(i) \text{ Centre to Centre} = \text{clear span} + \frac{b}{2} + \frac{b}{2}$$

$$= 4000 + \frac{300}{2} + \frac{300}{2}$$

$$= 4300 \text{ mm} \Rightarrow 4.3 \text{ m}$$

$$(ii) \text{ Clear span} + d = 4000 + 350$$

$$= 4350 \text{ mm}$$

$$= 4.35 \text{ m}$$

$\therefore$ Hence effective span $l = 4.3 \text{ m}$

(iii) Calculation of loads & BM

$$= 20 + 3 \text{ kN/m}$$

$$BM = \frac{wl^2}{8}$$

$$\text{Super imposed load} = 20 \text{ kN/m}$$

$$\therefore \text{Total load} = 20 + 3$$

$$= 23 \text{ kN/m}$$

$$\therefore \text{Total load} = 23 \text{ kN/m}$$

$$BM = \frac{23 \times 4.3^2}{8}$$

$$\therefore \boxed{BM = 53.15 \text{ kN-m}}$$

$$\text{factored BM, } = 1.5 \times BM$$

$$M_u = 1.5 \times BM$$

$$= 1.5 \times 53.15$$

$$\boxed{M_u = 79.74 \text{ kN-m}}$$

$$0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 300 \times 350^2$$

step 4: check of depth required,

$$M_u = M_{u \text{ limit}}$$

$$\text{for Fe 415, } M_{u \text{ limit}} = 0.138 f_{ck} b d^2$$

$$d = \sqrt{\frac{M_{u \text{ limit}}}{0.138 f_{ck} b}}$$

$$\sqrt{\frac{79.74 \times 10^6}{0.138 \times 20 \times 300}}$$

$$\boxed{d = 310.32 \text{ mm}} < d \text{ provided } (d = 350 \text{ mm})$$

hence provided depth is adequate

17 step 5: Area of reinforcement,

$$M_u = 0.87 f_y A_{st} \times d \left(1 - \frac{f_y A_{st}}{f_{ck} b \times d} \right)$$

$$79.74 \times 10^6 = 0.87 \times 415 \times A_{st} \times 350 \left[1 - \frac{415 A_{st}}{20 \times 300 \times 350} \right]$$

$$79.74 \times 10^6 = 126.36 \times 10^3 A_{st} \left[1 - 1.976 \times 10^{-4} A_{st} \right]$$

$$79.74 \times 10^6 = 126.36 \times 10^3 A_{st} - 24.97 A_{st}^2$$

$$24.97 A_{st}^2 - 126.36 \times 10^3 A_{st} + 79.74 \times 10^6 = 0$$

$$\boxed{A_{st} = 735.8 \text{ mm}^2}$$

Provide 4 bars of 16mm ϕ for finding no. of bars

$$A_{st} = \text{No. of bars} \times \frac{\pi}{4} (16)^2$$

$$735.8 = \text{No. of bars} \times 201.06$$

$$201.06 \text{ No. of bars} = 735.8$$

$$\therefore \text{No. of bars} = \frac{3.65}{0.25} = 14.6 \approx 15$$

$$(15 \times 0.25) = 3.75 \text{ m} \approx 3.65 \text{ m}$$

$$\therefore \text{No. of bars} = 15$$

Minimum reinforcement: (pg: 47)

$$\Rightarrow \frac{A_{stmin}}{bd} = \frac{0.85}{f_y}$$

$$A_{stmin} = \frac{0.85 bd}{f_y}$$

$$= \frac{0.85 \times 300 \times 350}{415}$$

$$\therefore A_{stmin} = 215.06 \text{ mm}^2$$

Maximum reinforcement:

$$A_{stmax} = 0.04 \times bD$$

$$= 0.04 \times 300 \times 350$$

$$A_{stmax} = 4200 \text{ mm}^2$$

$$A_{st} \text{ provided} = \frac{\pi}{4} (16)^2 \times 4$$

$$= 804.24 \text{ mm}^2$$

6. Check for deflection: (stiffness)

For S.S beam basic value $\left(\frac{l}{d} = 20\right)$

in code book, pg: no-38

Modification factor for tension reinforcement,

$$f_s = 0.58 f_y \times \frac{\text{Area of c/s of } A_{st} \text{ required}}{\text{Area of c/s of } A_{st} \text{ provided}}$$

$$= 0.58 \times 415 \times \frac{735.8}{804.24}$$

$$\boxed{f_s = 220.22 \text{ N/mm}^2}$$

$$\% \text{ of steel} = \frac{A_{st} \text{ provided}}{bd} \times 100$$

$$= \frac{804.24}{300 \times 350} \times 100$$

$$= 0.76\%$$

From Fig. 4 of IS: 456, modification

$$\text{factor} = 1.15$$

$$\frac{l}{d} \text{ provided } = \frac{4300}{350}$$

$$(max \text{ limit}) = 12.28 < 23 \quad \text{OK}$$

∴ hence deflection control is safe.

6. Design a rectangular beam for effective span 4m. which is subjected to dead load of 15 kN/m and a live load of 12 kN/m. use M15 & Fe 250 grade steel. Assume $b = 250 \text{ mm}$

sol

Given data,

Effective Clear Span $l = 4 \text{ m} \Rightarrow 4000 \text{ mm}$

$$f_{ck} = 15 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$b = 250 \text{ mm}$$

$$\text{Dead load} = 15 \text{ kN/m}$$

$$\text{Live load} = 12 \text{ kN/m}$$

(i) Assuming $\frac{l}{d} = \frac{10}{9}$ based on

$$d = 400 \text{ mm}$$

Assuming effective cover = 50 mm

$$\text{Overall depth} = 50 + 400$$

$$D = 450 \text{ mm}$$

(ii) Effective span $l = 4000 \text{ mm}$

(iii) Calculation of loads & BM

$$\begin{aligned} \text{Self wt} &= 0.25 \times 0.45 \times 1 \times 25 \\ &= 2.8125 \text{ kN/m} \end{aligned}$$

BM =

$$\text{Dead load} = 15 \text{ kN/m}$$

$$\text{Live load} = 12 \text{ kN/m}$$

$$\text{Total load} = 15 + 12 + 2.8125$$

$$= 29.81 \text{ kN/m}$$

$$\therefore \text{Total load} = 29.81 \text{ kN/m}$$

$$\text{BM} = \frac{29.81 \times 4^2}{2}$$

$$\text{BM} = 59.62 \text{ kN/m}$$

$$M_u = 1.5 \times BM$$

$$= 1.5 \times 59.62$$

$$\therefore M_u = 89.43 \text{ kN-m}$$

Step 4: Depth required,

$$M_u = M_{u \text{ limit}}$$

for $F_y 250$ steel, $M_{u \text{ limit}} = 0.148 f_{ck} b d^2$

$$d = \sqrt{\frac{M_u}{0.148 f_{ck} b}}$$

$$= \sqrt{\frac{89.43 \times 10^6}{0.148 \times 15 \times 250}}$$

$$d = 401.4 \text{ mm} > d = 400$$

from Code book procedure $\frac{l}{d}$ ratio (10 for)

Take, $\frac{l}{d} \text{ ratio} = 9$

So, procedure will be redesign

(i) Assuming $\frac{l}{d} = 9$ based on
stiffness $d = \frac{4000}{9}$

$$d = 450 \text{ mm}$$

Assuming effective Cover = 50 mm

$$D = 450 + 50$$

$$= 500 \text{ mm}$$

$M_u < M_{u \text{ limit}} - \text{Singly}$

$$D = 500 \text{ mm}$$

$M_u > M_{u \text{ limit}} - \text{Doubly reinforced beam}$

(ii) Effective span $l = 4000 \text{ mm}$

(iii) Calculation of loads & BM:

$$\text{Self wt} = 0.25 \times 0.5 \times 1 \times 25$$

$$= 3.125 \text{ kN/m}$$

$$\text{Dead load} = 15 \text{ kN/m}$$

$$\text{Live load} = 12 \text{ kN/m}$$

$$\text{Total load} = 3.125 + 15 + 12$$

$$= 30.125 \text{ kN/m}$$

$$BM = \frac{30.125 \times 4^2}{8}$$

$$BM = 60.25 \text{ kNm}$$

Factored Bending moment,

$$M_u = 1.5 \times 60.25$$

$$\therefore M_u = 90.375 \text{ kNm}$$

Step 4: Depth required,

$$M_u = M_{u \text{ limit}}$$

for f_e 250 steel, $M_{u \text{ limit}} = 0.148 f_{ck} b d^2$

$$d = \sqrt{\frac{M_{u \text{ limit}}}{f_{ck} \times 0.148 \times b}}$$

$$= \sqrt{\frac{90.375 \times 10^6}{15 \times 0.148 \times 250}}$$

$$d = 4103.53 \text{ mm} < 4500 \text{ mm}$$

Hence provided depth is adequate.

Step 5: Area of reinforcement,

$$M_u = 0.87 f_y A_{st} \times d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b \cdot d} \right]$$

$$= 0.87 \times 250 \times A_{st} \times 450 \left[1 - \frac{250 A_{st}}{15 \times 250 \times 450} \right]$$

$$= 97.875 \times 10^3 A_{st} - 14.5 A_{st}^2 = 0$$

$$14.5 A_{st}^2 - 97.875 \times 10^3 A_{st} + 90.375 \times 10^6$$

Provide 16mm ϕ for finding no. of bars

$$A_{st} = \text{No. of bars} \times \frac{\pi}{4} (16)^2$$

$$201.06 \text{ No. of bars} = 1103.9$$

$$\text{No. of bars} = \frac{1103.9}{201.06}$$

$$= 5.49 \approx 6 \text{ bars}$$

$$\boxed{\text{No. of bars} = 6}$$

$$A_{st} \text{ provided} = \frac{\pi}{4} (16)^2 \times 6$$

$$= 201.06 \times 6$$

$$= 1206.37 \text{ mm}^2$$

$$\boxed{A_{st} \text{ provided} = 1206.37 \text{ mm}^2}$$

6. Check for deflection: (Stiffness)

For s.s beam basic value $\left(\frac{l}{d} = 20\right)$

on Code book pg: no-38

Modification for tension reinforcement,

$$f_s = 0.58 f_y \times \frac{\text{Area of c/s of } A_{st} \text{ required}}{\text{Area of c/s of } A_{st} \text{ provided}}$$

$$f_s = \frac{1103.9}{1206.37}$$

$$f_s = 138.68 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{A_{st \text{ provided}}}{bd} \times 100$$

$$= \frac{1206.37}{250 \times 450} \times 100$$

$$= 1.072\%$$

From fig. 4 of IS: 456, modification factor = 1.5

$$\text{Max. permitted } \frac{l}{d} \text{ ratio} = 1.5 \times 20 = 30$$

$$\frac{l}{d} \text{ provided} = \frac{4000}{400} = 10 < 30$$

∴ Hence deflection control is safe.

Min reinforcement: step 5:-

$$\frac{A_{st \text{ min}}}{bd} = \frac{0.85}{f_y}$$

$$A_{st \text{ min}} = \frac{0.85 \times 250 \times 400}{250}$$

$$A_{st \text{ min}} = 340 \text{ mm}^2$$

$$A_{st \max} = 0.04 \times bD$$

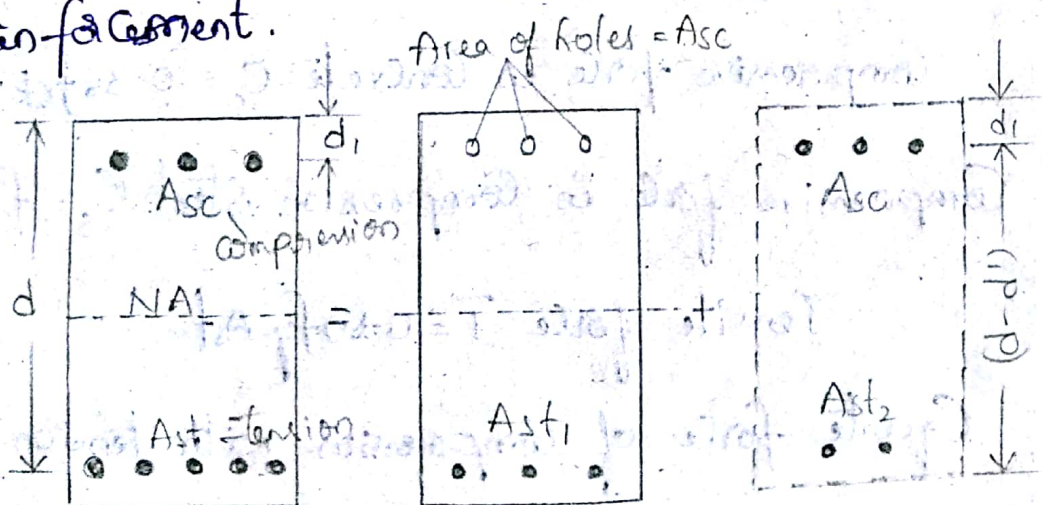
$$= 0.04 \times 250 \times 450$$

$$A_{st \max} = 4500 \text{ mm}^2$$

15/3/17

Doubly reinforced beams:

Beams which are reinforced in both compression & tension sides are called as doubly reinforced beam. These beams are generally provided when the dimensions of the beam are restricted & it is required to resist moment higher than the limiting moment of resistance of a singly reinforced section. The additional moment of resistance required can be obtained by providing compression reinforcement & additional tension reinforcement.



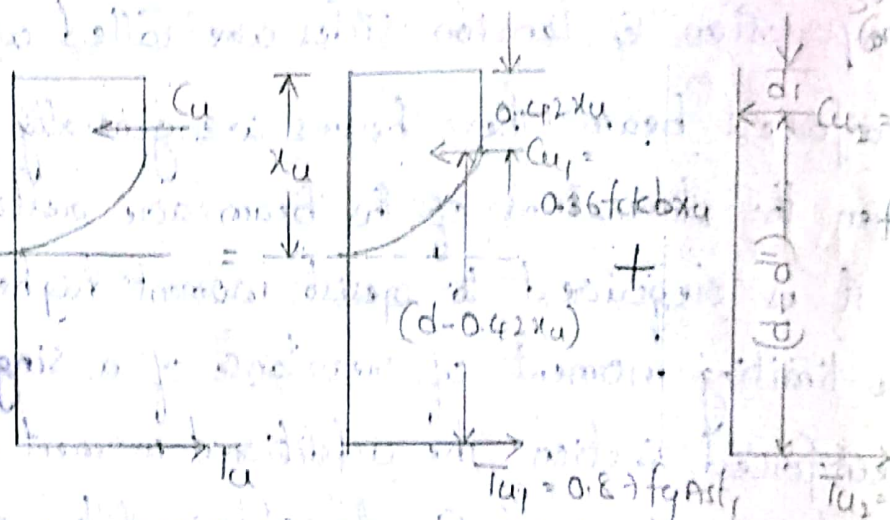
Section Subjected
to moment M_u

Section-I: Resisting
moment $M_{u1} = M_{u \lim}$

Section-II: Resisting
balance moment
 $M_{u2} = M_u - M_{u \lim}$

This doubly reinforced section can be considered to be composed of two sections as given below:
(a). A singly reinforced section with $M_{u,lim}$
(b) A section with compression steel & additional tension steel to resist additional moment

$$M_{u2} = M_u - M_{u,lim} \text{ i.e., a steel beam without concrete.}$$



1. Depth of Neutral axis :

$$\text{Compressive force in concrete } C_c = 0.36f_{ck} \cdot b \cdot x_u$$

$$\text{Compressive force in compression steel } C_s = f_{sc} \cdot A_s$$

$$\text{Tensile force } T = 0.87f_y \cdot A_{st}$$

Equate force of compression with tension

$$C_c + C_s = T$$

$$x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} b}$$

17/7/17

2. ultimate moment of resistance:

The ultimate moment of resistance of doubly reinforced is given by,

$$M_u = \overset{\text{concrete}}{M_{u1}} + \overset{\text{steel}}{M_{u2}}$$

$$= 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

when,

$x_u > x_{u, \max}$, x_u is limited to $x_{u, \max}$.

$$M_u = 0.36 f_{ck} b x_{u, \max} (d - 0.42 x_{u, \max}) + f_{sc} A_{sc} (d - d')$$

3. Area of Compression steel:

Additional moment of resistance M_{u2}

$$M_{u2} = f_{sc} A_{sc} (d - d')$$

$$A_{sc} = \frac{M_{u2}}{f_{sc} (d - d')}$$

the maximum area of compression reinforcement shall not exceed 0.04BD i.e., 4% of gross area.

4. Area of Tension steel: The limiting moment of resistance of singly reinforced section is given by,

$$M_{u,lim} = 0.87 f_y \cdot A_{st1} (d - 0.42 x_{u,max})$$

$$A_{st1} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$$

Additional area of tensile steel (A_{st2}) can be calculated by equating the compressive force in compression steel & tensile force in additional tension steel.

$$0.87 f_y \cdot A_{st2} = f_{sc} \cdot A_{sc}$$

$$A_{st2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}$$

A_{st2} can also be calculated by using,

$$M_{u2} = 0.87 f_y \cdot A_{st2} (d - d')$$

$$A_{st2} = \frac{M_{u2}}{0.87 f_y (d - d')}$$

Stress in Compression steel (f_{sc}) based on d'/d

Table 3.5:

Stress in Compression steel f_{sc} N/mm² in doubly reinforced beam with cold worked bars.

(Table-F in SP16) when $d'/d < 0.2$

Grade of steel	d'/d			
	0.05	0.10	0.15	0.20
Fe 415	355	353	342	329
Fe 500	424	412	395	370

For $d'/d < 0.2$, f_{sc} for mild steel is $0.87f_y$.

7. Calculate the ultimate moment of resistance of an R.C beam of rectangular section 300mm wide & 400mm deep. Area of steel consists of 6 Nos 18 ϕ in tension side & 2 Nos 18 ϕ in Compression side. Assume steel of grade Fe 415 & Concrete of grade M₂₀ & an effective cover 35mm on both sides.

Sol

Given data,

$$b = 300\text{mm}$$

$$d = 400 - 35$$

$$= 365\text{mm}$$

$$f_{ck} = 80 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 6 \times \frac{\pi}{4} (12)^2 = 1526.8 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} (12)^2 = 508.9 \text{ mm}^2$$

1. Stress in Compression steel:

$$\frac{d'}{d} = \frac{35}{365} = 0.09$$

$$\frac{d'}{d} = 0.09 < 0.2$$

from ~~Code~~ ^{Text} book table 3.5.

$$0.05 - 355$$

$$0.10 - 353$$

$$0.09 - ?$$

$$\Rightarrow 355 - \left(\frac{355 - 353}{0.10 - 0.05} \right) (0.09 - 0.05)$$

$$f_{sc} = 353.4 \text{ N/mm}^2$$

2. Depth of Neutral Axis:

$$x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} \times b}$$

$$0.36 \times 20 \times 300$$

$$x_u = 171.94 \text{ mm}$$

Limiting depth of neutral axis:-

$$x_{u\max} = 0.48d$$

$$= 0.48 \times 365$$

$$= 175.2 \text{ mm}$$

$$\therefore x_u < x_{u\max}$$

So, the section is under reinforced.

3. Moment of resistance:-

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$\Rightarrow 0.36 \times 20 \times 300 \times 171.94 (365 - (0.42 \times 171.94))$$

$$+ (353.4 \times 508.91) (365 - 35)$$

$$\Rightarrow 167.62 \times 10^6 \text{ N-mm}$$

$$M_u = 167.62 \times 10^6 \text{ N-mm}$$

$$= 167.62 \text{ kN-m}$$

8. A rectangular beam is 500 mm deep it is reinforced beam & bars of 20 mm in compression with an effective cover of 50 mm. Determine the area of tension reinforcement to make the beam section fully effective. Also find the moment of resistance of Concrete & Fe 415 steel.

Sol

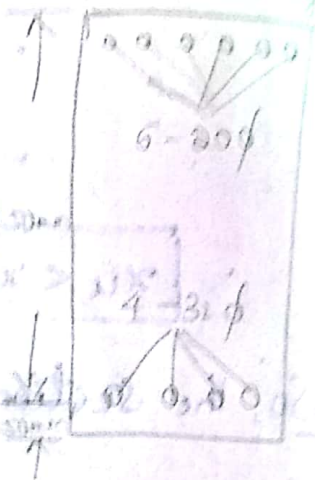
Given data,

$$b = 200 \text{ mm}$$

$$d = 500 - 50$$

$$= 450 \text{ mm}$$

$$D = 500 \text{ mm}$$



$$A_{sc} = 6 \times \frac{\pi}{4} (20)^2 \Rightarrow 1884.95 \text{ mm}^2$$

$$(1) \text{ } f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$(2) \text{ } d' = 50 \text{ mm}$$

$$d' = 50 \text{ mm}$$

$$(3) \text{ } f_{ck} = 20 \text{ N/mm}^2$$

A_{st} is provided based on M_{ulim} :-

$$M_{ulim} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 200 \times 450^2$$

$$= 11178 \times 10^3 \text{ N-mm}$$

$$= 111.78 \text{ kN-m}$$

$$A_{st1} = 0.87 f_y (d - 0.42 x_{u, \max})$$

~~111.78~~

$$x_{u, \max} = 0.48 d$$

$$= 0.48 \times 450$$

$$= 216 \text{ mm}$$

$$\therefore A_{st1} = \frac{111.78 \times 10^6}{0.87 \times 415 (450 - 0.42 (216))}$$

$$A_{st1} = 861.71 \text{ mm}^2$$

$$A_{st2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}$$

for f_{sc} value, $d' = 50$

$$\frac{d'}{d} = \frac{50}{450} = 0.11$$

from table,

$$0.10 - 353$$

$$0.15 - 342$$

$$f_{sc} = 353 - \left(\frac{353 - 342}{0.15 - 0.10} \right) (0.11 - 0.10)$$

$$f_{sc} = 350.8 \text{ N/mm}^2$$

$$= \frac{350.8 \times 1889.95}{0.87 \times 415}$$

$$= 1831.43 \text{ mm}$$

$$\boxed{A_{st2} = 1831.43 \text{ mm}^2}$$

$$A_{st} = A_{st1} + A_{st2}$$

$$= 861.71 + 1831.43$$

$$= 2693.14 \text{ mm}^2$$

$$\therefore \boxed{A_{st} = 2693.14 \text{ mm}^2}$$

$$\mu_a = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} \times b}$$

$$= \frac{0.87 \times 415 \times 2693.14 - (350.8 \times 1889.95)}{0.36 \times 20 \times 200}$$

$$\mu_a = 216.05 \text{ mm}$$

$$\therefore \mu_a = \mu_{a \max}$$

So, the section is balanced section.

$$\begin{aligned}
 &= 0.36 \times 20 \times 200 \times 216 (450 - 0.42(216)) \\
 &\quad + (350.8 \times 1884.95) (450 - 50) \\
 &= 376.24 \times 10^6 \text{ N-mm}
 \end{aligned}$$

$$M_u = 376.24 \text{ kN-m}$$

9. Determine the main tensile & Compression reinforcement required for a rectangular beam with a following data. 1. Overall size of the beam

$$= 250 \times 550 \text{ mm}$$

2. Concrete grade = M20

3. $F_y = 415 \text{ steel}$

4. Factored moment = 200 kN-m

5. Effective Cover = 50 mm

sol

Given data,

$$b = 250 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$D = 550 \text{ mm}$$

$$d' = 50 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$M_u = 200 \text{ kN-m}$$

1. Limiting moment of resistance of the given section as singly reinforced section:

$$M_{u,lim} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 20 \times 250 \times 500^2$$

$$= 172.5 \times 10^6 \text{ N-mm}$$

$$M_{u,lim} = 172.5 \text{ kN-m}$$

$$\therefore M_u > M_{u,lim}$$

So, the section should be designed as doubly reinforced section.

2. Area of tension steel corresponding to $M_{u,lim}$

$$A_{st1} = \frac{0.87 M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$$

$$x_{u,max} = 0.48 d$$

$$= 0.48 \times 500$$

$$= 240 \text{ mm}$$

$$A_{st1} = \frac{172.5 \times 10^6}{0.87 \times 415 (500 - (0.42 \times 240))}$$

$$A_{st1} = 1196.82 \text{ mm}^2$$

$$3. M_{u2} = M_u - M_{u1im}$$

$$= 200 - 172.5$$

$$M_{u2} = 27.5 \text{ kN-m}$$

$$M_{u2} = f_{sc} \cdot A_{sc} (d - d')$$

$$\frac{d'}{d} = \frac{50}{500} = 0.1$$

from table 3.5

$$0.10 - 353$$

$$\text{So, } f_{sc} = 353 \text{ N/mm}^2$$

$$M_{u2} = 353 \times A_{sc} (500 - 50)$$

$$27.5 \times 10^6 = 158.85 \times 10^3 A_{sc}$$

$$158.85 \times 10^3 A_{sc} = 27.5 \times 10^6$$

$$A_{sc} = \frac{27.5 \times 10^6}{158.85 \times 10^3}$$

$$173.11 \text{ mm}^2$$

$$A_{sc} = 173.11 \text{ mm}^2 \text{ required}$$

$$A_{st2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}$$

$$= \frac{353 \times 173.1}{0.87 \times 415}$$

$$A_{st2} = 169.24 \text{ mm}^2$$

∴ Total tension steel,

$$A_{st} = A_{st1} + A_{st2}$$

$$= 1196.82 + 169.24$$

$$A_{st} = 1366.06 \text{ mm}^2 \text{ required}$$

Assume 20mm ϕ to finding no. of bars

$$A_{st} = \text{No. of bars} \times \frac{\pi}{4} (20)^2$$

$$1366.06 = \text{No. of bars} \times 314.15$$

$$314.15 \text{ No. of bars} = 1366.06$$

$$\text{No. of bars} = 4.3 \approx 5 \text{ bars}$$

$$A_{st} = 5 \times \frac{\pi}{4} (20)^2$$

$$= 1570.7 \text{ mm}^2$$

Provide 12mm ϕ to find out no. of bars in Compression

$$A_{sc} = \text{No. of bars} \times \frac{\pi}{4} (12)^2$$

$$173.11 = 113.09 \text{ No. of bars}$$

$$\text{No. of bars} = 1.53 \approx 2$$

$$\therefore \text{No. of bars} = 2$$

$$A_{sc} = 2 \times \frac{\pi}{4} (12)^2$$

$$= 226.18 \text{ mm}^2$$

$$\therefore \text{Provided } A_{sc} = 226.18 \text{ mm}^2$$

Design of shear: [Pg: No - 72, 73]

τ_v = nominal shear stress

τ_c = shear strength resistance by Concrete [from table No: 19]

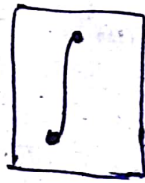
τ_{max} = Max. shear stress [Table no: 20]

$$\tau_v = \frac{V}{bd} ; V = \text{shear force}$$

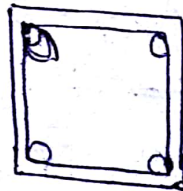
$\tau_v > \tau_c$ - provided shear reinforcement

The inclined shear crack starts at the bottom near the support and extends toward compression zone. The shear reinforcement can be provided in any of the following forms.

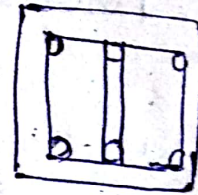
- (a). Vertical stirrups.
- (b). Bent up bars along with stirrups.
- (c) Inclined stirrups.



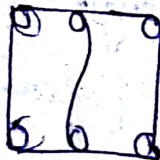
One legged stirrup



Two legged stirrup



Four legged stirrup



Three legged stirrup.

$\tau_v \rightarrow$ minimum reinforcement is to be provided.
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from code book - Table no:19 - τ_c values.

τ_{cmax} - Table no:20.

10. A simply supported R.C.C beam $250\text{mm} \times 450\text{mm}$ (effective) is reinforced with 4 nos of $18\text{mm } \phi$ bars. Design the shear reinforcement if M_{20} grade concrete & Fe 415 steel is used & beam is subjected to a force of 150kN at service load.

Sol

Given data,

$$b = 250\text{mm}$$

$$d = 450\text{mm}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$f_y = 415\text{N/mm}^2$$

$$\text{Shear force} = 150\text{kN}$$

$$\text{Factored shear force } V_u = 1.5 \times 150 = 225\text{kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{225 \times 10^3}{250 \times 450} = 2\text{N/mm}^2$$

from code book table no:19

$$= 1017.8 \text{ mm}^2$$

$$\% \text{ of steel} = \frac{1017.8}{250 \times 450} \times 100$$

$$= 0.9047$$

from table no:19

$$0.75 - 0.56$$

$$1.00 - 0.62$$

$$0.9047 - ?$$

$$\tau_c = 0.56 + \left(\frac{0.56 - 0.62}{1.00 - 0.75} \right) (0.9047 - 0.75)$$

$$\tau_c = 0.52 \text{ N/mm}^2, \tau_{cmax} = 2.8 \text{ N/mm}^2$$

$$\boxed{\tau_v > \tau_c}$$

Shear reinforcement has to be designed.

$$\tau_{cmax} = 2.8 \text{ N/mm}^2$$

(ii) Design of shear reinforcement:

$$V_{uc} = \tau_c \cdot bd \Rightarrow 0.52 \times (250 \times 450)$$

$$= 58500 \text{ N} \Rightarrow 58.5 \text{ kN}$$

$$V_{us} = V_u - V_{uc}$$

$$= 225 - 58.5 \Rightarrow 166.5 \text{ kN}$$

$$A_{sv} = 2 \times \frac{\pi}{4} (10)^2$$

$$= 157.07 \text{ mm}^2$$

Spacing of stirrups:

$$1. \cancel{0.75 \times 450} + 0.75 \times 450 = 337.5 \text{ mm}$$

2. 300 mm which ever is less.

3. for vertical stirrups:

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 157.07 \times 450}{166.5}$$

$$S_v = 153.56 \times 10^3 \text{ m}$$

$$\boxed{S_v = 153.56 \text{ mm}} \approx 165 \text{ mm}$$

from code book pg: 46.

26.5.1.6 - min shear reinforcement

$$\frac{A_{sv}}{b S_v} \geq \frac{0.4}{0.87 f_y}$$

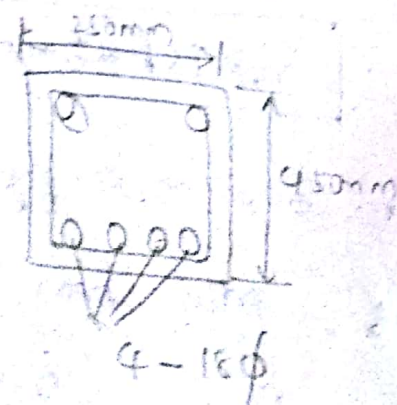
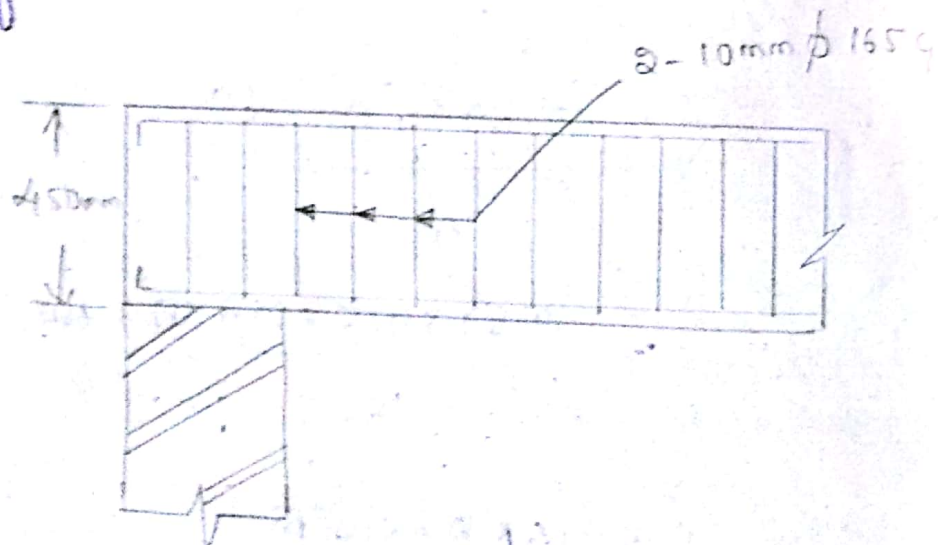
$$\frac{A_{sv}}{b \cdot s_v} = \frac{0.87}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y \cdot A_{sv}}{0.4 \times b}$$

$$= \frac{0.87 \times 415 \times 157.07}{0.4 \times 250}$$

$$S_v = 567.10 \text{ mm}$$

Provide 10mm ϕ 2-legged stirrups with a spacing of 165mm



11. A S.S R.C.C beam of $200\text{mm} \times 400\text{mm}$ (effective) is reinforced with 4 bars of 22mm ϕ on tension side the beam is carrying a udl of 10 kN/m over a clear span of 8m . Design the shear reinforcement by using M_{20} grade concrete & $Fe\ 415$ steel.

sol

Given data,

$$b = 200\text{mm}$$

$$d = 400\text{mm}$$

$$\text{load} = 10\text{ kN/m}$$

$$l = 8\text{m}$$

$$f_{ck} = 20\text{ N/mm}^2$$

$$f_y = 415\text{ N/mm}^2$$

$$D = 400 + 50$$

$$= 450\text{mm}$$

$$\text{Self wt} = 25 \times 0.45 \times 0.2$$

$$= 2.25\text{ kN/m}$$

$$\text{Total load} = 2.25 + 10$$

$$= 12.25\text{ kN/m}$$

$$\text{Factored load } w_u = 12.25 \times 1.5$$

$$= 18.375\text{ kN/m}$$

$$= \frac{18.375 \times 8.2}{2}$$

$$V_u = 75.33 \text{ kN}$$

$$\text{Effective span} = 8 + \frac{0.2}{2} + \frac{0.2}{2}$$

$$l = 8.2 \text{ m}$$

1. Normal shear stress:

$$\tau_v = \frac{V_u}{bd} \Rightarrow \frac{75.33 \times 10^3}{200 \times 400}$$

$$\tau_v = 0.942 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} (22)^2$$

$$A_{st} = 1520.5 \text{ mm}^2$$

$$\% \text{ of steel} = \frac{1520.5}{200 \times 400} \times 100$$

$$= 1.9\%$$

from table no:19

$$1.75 \rightarrow 0.75$$

$$2.00 \rightarrow 0.79$$

$$1.9 \rightarrow ?$$

$$\tau_c = 0.75 \left(\frac{0.79 - 0.75}{2.00 - 1.75} \right) (1.9) = 0.774 \text{ N/mm}^2$$

$$\tau_c = 0.774 \text{ N/mm}^2$$

$$\tau_{cmax} = 2.8 \text{ N/mm}^2$$

$$\tau_v > \tau_c$$

Shear reinforcement has to be designed

(i) Design of shear reinforcement:

$$V_{uc} = \tau_c \cdot b \cdot d \Rightarrow 0.774$$

$$= 0.774 \times 200 \times 400$$

$$= 16920 \text{ N } 61920 \text{ N}$$

$$V_{uc} = 61.92 \text{ kN}$$

$$V_{us} = V_u - V_{uc}$$

$$= 75.33 - 61.92$$

$$= 13.41 \text{ kN}$$

$$V_{us} = 13.41 \text{ kN}$$

(ii) Design of vertical stirrups,

Spacing of 2-legged comm stirrups.

$$= 157.07 \text{ mm}^2$$

Spacing of stirrups:

$$1. \quad 0.75d \Rightarrow 0.75 \times 400$$

$$= 300.0 \text{ mm}$$

2. 300mm which ever is less

3. For Vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 415 \times 157.07 \times 400}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 157.07 \times 400}{13.41}$$

$$= 1691.57 \times 10^3 \text{ mm}$$

$$= 1691.57 \text{ mm} \approx 1700 \text{ mm}$$

from Code book pg: 48

26.5.6 - Minimum shear reinforcement.

$$\frac{A_{sv}}{b_s v} \geq \frac{0.4}{0.87 f_y}$$

$$A_{sv} = \frac{\pi}{4} (10)^2$$

$$= 157.07 \text{ mm}^2$$

Spacing of stirrups:

$$1. 0.75d \Rightarrow 0.75 \times 400$$

$$= 300.0 \text{ mm}$$

2. 300mm which ever is less

3. For Vertical stirrups,

$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 415 \times 157.07 \times 400}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 157.07 \times 400}{13.41}$$

$$= 1691.57 \times 10^3 \text{ mm}$$

$$= 1691.57 \text{ mm} \approx 1700 \text{ mm}$$

from Code book pg: 48

26.5.5.6 - Minimum shear reinforcement

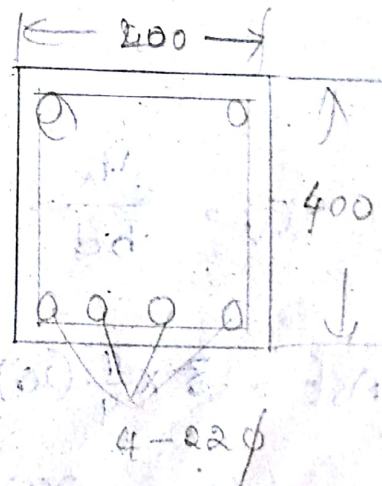
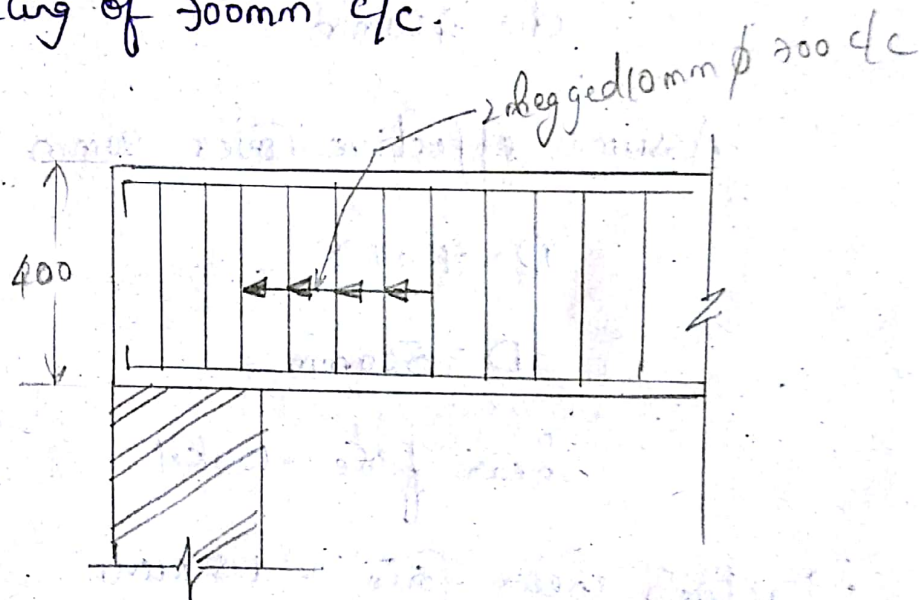
$$\frac{A_{sv}}{b_s v} \geq \frac{0.4}{0.87 f_y}$$

$$\frac{0.87 \times 915 \times 157.07}{0.4 \times 200}$$

$$2708.88 \text{ mm} \approx 700 \text{ mm}$$

$$S_v = 700 \text{ mm}$$

Hence provide 2-legged - 10mm ϕ stirrups with a spacing of 700mm c/c.



A R.C.C beam has an effective depth of 450mm & breadth of 300mm it contains of 20mm ϕ bars. Mild steel out of wh 2 bars are curtailed at a section where force is 100kN at service load. Design reinforcement if the concrete is M20

Sol

Given data,

$$b = 300\text{mm}$$

$$d = 450\text{mm}$$

Assume effective cover = 50mm

$$D = 450 + 50$$

$$D = 500\text{mm}$$

$$\text{Shear force} = 100\text{kN}$$

$$\text{Factored shear force} = 1.5 \times 100$$

$$V_u = 150\text{kN}$$

$$\tau_v = \frac{V_u}{bd} = \frac{150 \times 10^3}{300 \times 450} = 1.11\text{N/mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} (20)^2 \rightarrow 942.47\text{mm}^2$$

$$\begin{aligned} \% \text{ of steel} &= \frac{942.47}{300 \times 450} \times 100 \\ &= 0.69\% \end{aligned}$$

$$0.50 \rightarrow 0.48$$

$$0.75 \rightarrow 0.56$$

$$0.698 \rightarrow ?$$

$$\tau_c = 0.48 + \left(\frac{0.56 - 0.48}{0.75 - 0.50} \right) (0.698 - 0.50)$$

$$\tau_c = 0.543 \text{ N/mm}^2$$

$$\tau_{max} = 2.8 \text{ N/mm}^2$$

$$\tau_v > \tau_c$$

Shear reinforcement has to be designed.

(ii) Design of shear reinforcement:

$$V_{uc} = \tau_c \times b \times d \Rightarrow 0.543 \times 300 \times 450$$

$$V_{uc} = 73305 \text{ N}$$

$$V_{uc} = 73.305 \text{ kN}$$

$$V_{us} = V_u - V_{uc}$$

$$= 150 - 73.305$$

$$V_{us} = 76.695 \text{ kN}$$

(ii) Design of vertical stirrups

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spacing of 2-legged 8mm stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} (8)^2 \Rightarrow 100.53 \text{ mm}^2$$

Spacing of stirrups:

$$(i) 0.75 \times d \Rightarrow 0.75 \times 400 = 337.5 \text{ mm}$$

(ii) 300mm which ever is less

(iii) for vertical stirrups

$$V_{us} = \frac{0.87 f_y A_{sv} \cdot d}{S_v}$$

$$S_v = \frac{0.87 f_y A_{sv} \cdot d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times 100.53 \times 400}{76.695}$$

$$= 128.29 \times 10^3 \text{ mm}$$

$$S_v = 128.29 \text{ mm}$$

$$S_v = 130 \text{ mm}$$

from code book pg-48

26.5.1.6 - Min. shear reinforcement

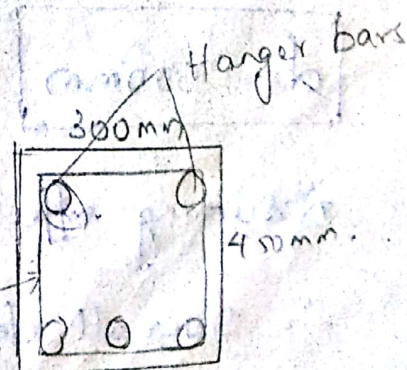
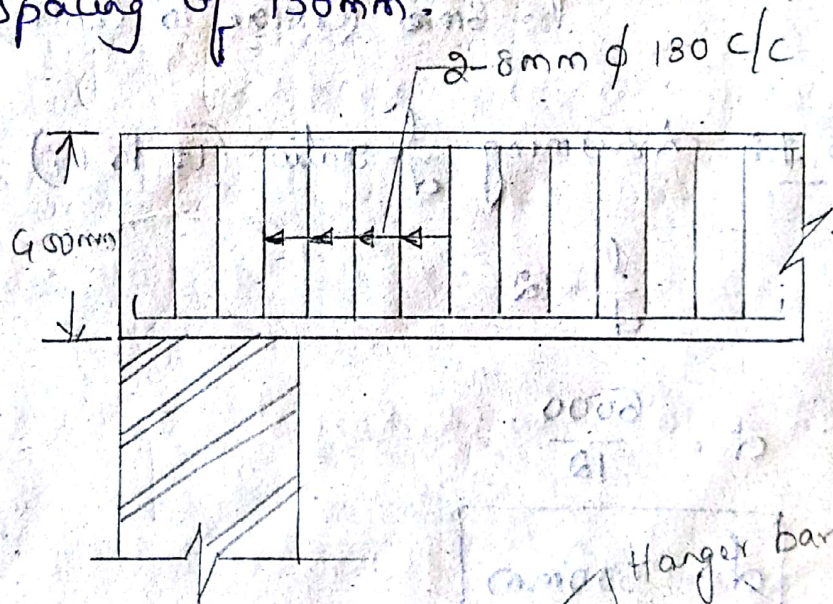
$$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y \cdot A_{sv}}{0.4 \times b}$$

$$= \frac{0.87 \times 250 \times 100.53}{0.4 \times 300}$$

$$S_v = 182.21 \text{ mm}$$

Provide 8mm ϕ of 2-legged stirrups with
of spacing of 130mm.



8mm ϕ
2-legged stirrups
at 130mm c/c
at support

26/11/13

A rectangular concrete beam is s.s on two masonry walls 230mm ^{thick}, 6mm apart (Centre to Centre) the beam is carrying an imposed load 15kN/m. Design the beam with all necessary checks. Use M25 concrete & Fe 415 steel. sketch the details of reinforcement

sol

Given data,

$$b = 230\text{mm}$$

$$\text{imposed load} = 15\text{ kN/m}$$

$$f_y = 415\text{ N/mm}^2$$

$$f_{ck} = 25\text{ N/mm}^2$$

$$l = 6\text{m (Centre to Centre)}$$

Step 1: Assuming $\frac{l}{d}$ ratio (10 to 15)

$$\frac{l}{d} = 12$$

$$d = \frac{6000}{12}$$

$$d = 500\text{mm}$$

Assuming effective cover = 50mm

$$\therefore \text{Overall depth } D = 500 + 50 = 550\text{mm}$$

(ii) Calculation of loads & BM

$$\text{Self wt} = 0.23 \times 0.55 \times 1 \times 25$$

$$= 3.1625 \text{ kN/m}$$

$$\text{Imposed load} = 15 \text{ kN/m}$$

$$\text{Total load} = 15 + 3.1625$$

$$= 18.1625 \text{ kN/m}$$

$$\text{BM} = \frac{wl^2}{8} \Rightarrow \frac{18.1625 \times 6^2}{8}$$

$$\boxed{\text{BM} = 81.73 \text{ kN/m}}$$

factored bending moment,

$$M_u = 1.5 \times \text{BM}$$

$$= 1.5 \times 81.73$$

$$\boxed{M_u = 122.6 \text{ kN/m}}$$

Step 4:- Depth required,

$$M_u = M_{u \text{ limit}}$$

$$\text{for } f_{ek} 415 \text{ steel, } M_{u \text{ limit}} = 0.138 f_{ek} b d^2$$

$$d = \sqrt{\frac{M_{u \text{ limit}}}{0.138 f_{ek} \cdot b}} = \sqrt{\frac{122.6 \times 10^6}{0.138 \times 25 \times 230}}$$

Hence provided depth is adequate.

$$\begin{aligned} M_{ulim} &= 0.138 f_{ck} b d^2 \\ &= 0.138 \times 25 \times 230 \times 500^2 \\ &= 198.37 \text{ kN-m} \end{aligned}$$

$$M_u < M_{ulim}$$

$\therefore$ So, the section is singly reinforced.

Step 5:- Area of reinforcement,

$$M_u = 0.87 f_y A_{st} x d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$122.6 \times 10^6 = 0.87 \times 415 \times A_{st} \times 500 \left[1 - \frac{415 A_{st}}{25 \times 230 \times 500} \right]$$

$$122.6 \times 10^6 = 180.525 \times 10^3 A_{st} \left[1 - 1.443 \times 10^{-9} A_{st} \right]$$

$$122.6 \times 10^6 = 180.525 \times 10^3 A_{st} - 26.058 A_{st}^2$$

$$26.058 A_{st}^2 - 180.525 \times 10^3 A_{st} + 122.6 \times 10^6 = 0$$

$$A_{st} = 763.2 \text{ mm}^2$$

$$A_{st} = \text{No. of bars} \times \frac{\pi}{4} (16)^2$$

$$763.21 = \frac{201.06}{113.09} \text{ No. of bars}$$

$$\frac{201.06}{113.09} \text{ No. of bars} = 763.21$$

18
16
20

$$\text{No. of bars} = 3.79 \approx 4 \text{ bars}$$

$$\therefore \text{No. of bars} = 4$$

$$A_{st} \text{ required} = 763.21 \text{ mm}^2$$

$$\begin{aligned} A_{st} \text{ provided} &= 4 \times \frac{\pi}{4} (16)^2 \\ &= 804.24 \text{ mm}^2 \end{aligned}$$

Min. reinforcement:

$$\Rightarrow \frac{A_{st \min}}{bd} = \frac{0.85}{f_y}$$

$$\begin{aligned} A_{st \min} &= \frac{0.85 bd}{f_y} \\ &= \frac{0.85 \times 230 \times 500}{415} \end{aligned}$$

$$\therefore \boxed{A_{st \min} = 235.5 \text{ mm}^2}$$

Max. reinforcement:

$$A_{st \max} = 0.04 \times bD$$

$$= 5060$$

$$A_{st \max} = 5060 \text{ mm}^2$$

Step 5:- Check for deflection (stiffness)

For s.s beam basic value $\left(\frac{l}{d} \geq 20\right)$

$$\tau_v < \tau_{c \max}$$

$\tau_v < \tau_c$ - min. reinforcement

$\tau_v > \tau_c$ & $\tau_v < \tau_{c \max}$ - design design
shear reinforcement

$\tau_v > \tau_c$ & $\tau_v > \tau_{c \max}$ - redesign the
beam.

Step 6:- Check for shear reinforcement

$$\tau_v = \frac{V_u}{bd} = \frac{81.73}{230 \times 500} \Rightarrow 7.106 \times 10^{-4}$$

$$\tau_v = 7.106 \times 10^{-4} \text{ N/mm}^2$$

$$\tau_{c \max} = 3.1 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{A_{st}}{bd} \times 100$$

$$= 0.699$$

from code book table no:19, τ_c value
for 0.699

$$0.50 - 0.49$$

$$0.75 - 0.57$$

$$0.699 - ?$$

$$\tau_c = 0.49 + \left(\frac{0.57 - 0.49}{0.75 - 0.50} \right) (0.699 - 0.50)$$

$$= 0.55$$

$$\tau_c = 0.55 \text{ N/mm}^2$$

$$\tau_{c_{\max}} = 3.1 \text{ N/mm}^2 \text{ from table no:20}$$

$$\tau_c < \tau_{c_{\max}}$$

$\tau_v > \tau_c$ & $\tau_v < \tau_{c_{\max}} \rightarrow$ So design
shear reinforcement.

Design of shear reinforcement:

stirrups 220 mm c/c.

check for deflection:-

for s.s beam basic value ($\frac{l}{d} = 20$)

in Code book, pg. no - 38

modification factor for tension reinforcement

$$f_s = 0.58 f_y \times \frac{\text{Area of c/s of Ast required}}{\text{Area of c/s of Ast provided}}$$

$$= 0.58 \times 415 \times \frac{763.21}{804.29}$$

$$f_s = 228.42 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{\text{Ast provided}}{bd} \times 100$$

$$= \frac{804.29}{230 \times 500} \times 100$$

$$= 0.699\%$$

from fig. 4 of Is: 456, modification factor = 1.2

Max. permitted $\frac{l}{d}$ ratio = $\frac{1.2}{1.25} \times 20$

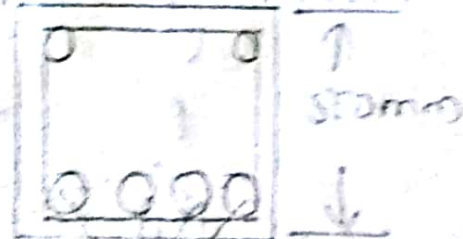
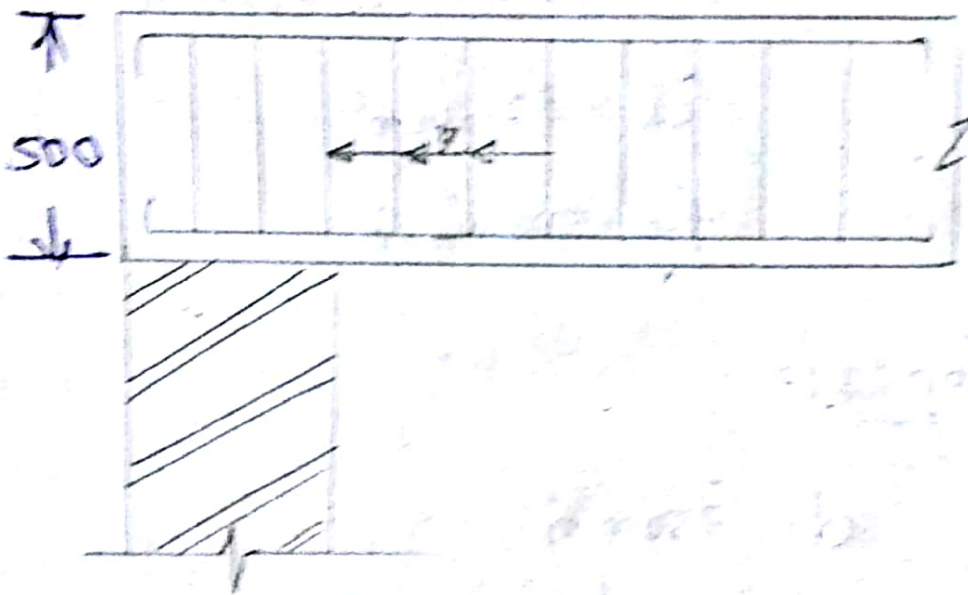
$$= 24$$

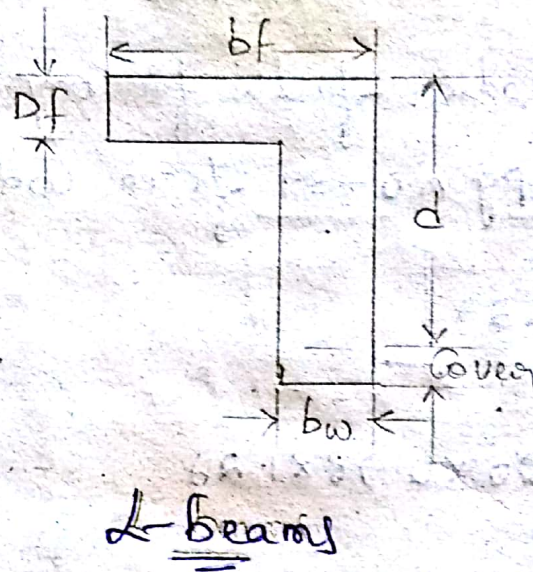
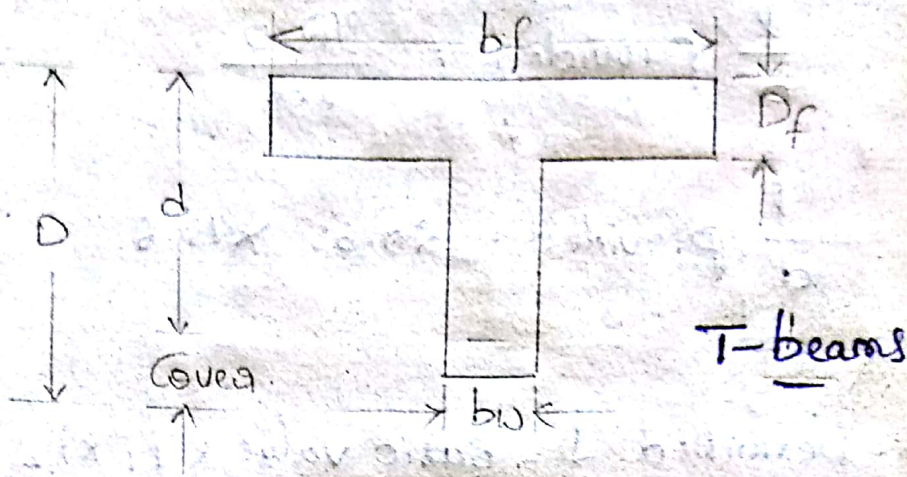
$$\frac{l}{d} \text{ provided} = \frac{6000}{500}$$

$$\frac{l}{d} \text{ provided} = 12 < 24$$

$\therefore$ Hence deflection control is safe.

6mm ϕ 2-legged @ 220 mm





For T-beams:

$$b_f = \frac{l_o}{6} + b_w + 6D_f$$

For L-beams:

$$b_f = \frac{l_o}{12} + b_w + 3D_f$$

→ When the slab occurs on both the sides of the beam (intermediate beams), the beam is known as T-beam.

When the slab is only on one side of the beam (end beams), the beam is known as L-beam

Advantages of T-beams:

1. As the slab being monolithic with the beam is also compressed & shares the compressive force with the beam, which significantly increases the moment of resistance of the beam.
2. As most of the compressive force is shared by the flange, the depth of the beam required is less and hence the maximum deflections are also less.

For isolated beams, the effective width shall be obtained as given below but in no case greater than the actual width (b)

$$\text{For T-beams } b_f = \frac{l_o}{\left(\frac{l_o}{b} + 4\right)} + b_w$$

$$\text{For L-beams } b_f = \frac{0.5 l_o}{\left(\frac{l_o}{b} + 4\right)} + b_w$$

where,

b_f = effective width of flange

l_0 = distance b/w points of zero moment in the beam.

b_w = breadth of web.

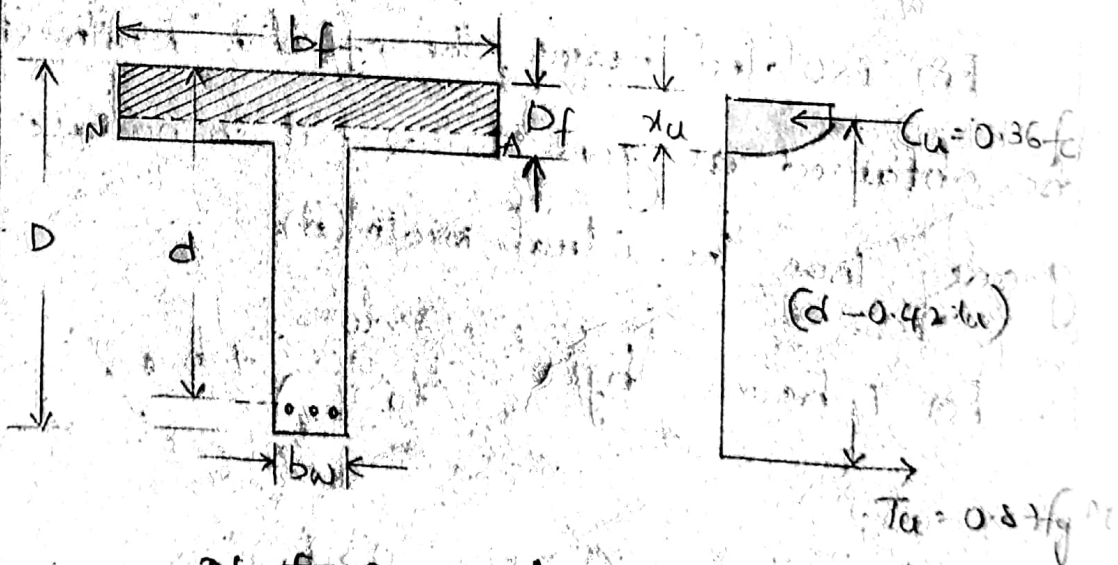
D_f = thickness of flange.

b = Actual width of the flange.

x_1, x_2 = half the clear distance b/w two adjacent beams.

Analysis of T-beams:-

Case 1: Neutral axis is with in the flange ($x_u \leq D_f$)



Neutral axis lying inside the flange.

When the NA falls with in the flange as shown, the T-beam can be treated as a normal

rectangular beam of width b_f & depth d . The moment of resistance of the section can be calculated by the same procedure as that of rectangular section of width b_f & depth d .

$$C = 0.36 f_{ck} \cdot b_f \cdot x_u$$

$$T = 0.87 f_y \cdot A_{st}$$

1. Depth of Neutral axis

$$C = T$$

$$0.36 f_{ck} \cdot b_f \cdot x_u = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y \cdot A_{st}}{0.36 f_{ck} \cdot b_f}$$

2. Moment of resistance:

$$M_u = 0.36 f_{ck} b_f x_u (d - 0.42 x_u) \text{ --- Concrete}$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u) \text{ --- Steel}$$

Case II: Neutral axis is below the flange ($x_u > D_f$)
i.e., in the web and $D_f/d \leq 0.2$

1. Find the moment of resistance of T-beam

Section having $b_w = 300$, $b_f = 1650\text{mm}$, $D_f = 150\text{mm}$,
 $d = 550\text{mm}$. The reinforcement consists of 6 bars
 of 20mm - Use M_{20} Concrete & Fe 415 steel.

Sol

Given data,

$$b_w = 300\text{mm}$$

$$b_f = 1650\text{mm}$$

$$D_f = 150\text{mm}$$

$$d = 550\text{mm}$$

$$\text{No. of bars} = 6$$

$$\text{Dia} = 20\text{mm}$$

$$f_{ck} = 20\text{N/mm}^2$$

$$f_y = 415\text{N/mm}^2$$

$$A_{st} = \left(\frac{\pi}{4} \times (20)^2 \times 6 \right) = 1884.9\text{mm}^2$$

Step 1:-

Assuming depth of N-A is in the flange

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 1884.9}{0.36 \times 20 \times 1650}$$

$$= 194.5\text{mm}$$

$$x_u, \max = 0.48d \Rightarrow 0.48 \times 550$$

$$= 264 \text{ mm}$$

$$x_u < x_{u, \max}$$

So, the beam is under reinforced section

Moment of resistance:

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1884.9 (550 - 0.42 (57.28))$$

$$= 357.92 \times 10^6 \text{ N-mm}$$

$$M_u = 357.92 \text{ kN-m}$$

2/8/17

2. Find the moment of resistance of a T-beam having $b_w = 300 \text{ mm}$, $b_f = 1650 \text{ mm}$, $D_f = 120 \text{ mm}$, $d = 510 \text{ mm}$. The reinforcement consists of 4 bars of $25 \text{ mm } \phi$. use M_{20} concrete & Fe 415 steel.

2/8

Given data,

$$b_w = 300 \text{ mm}$$

$$b_f = 1650 \text{ mm}$$

$$D_f = 160 \text{ mm}$$

$$\text{www.FirstRanker.com}$$

$$d = 510 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 4 \times \frac{\pi}{4} (25)^2 \rightarrow 1963.49 \text{ mm}^2$$

Step 1:-

Assuming depth of NA is in the flange

$$\lambda_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B_f}$$

$$= \frac{0.87 \times 415 \times 1963.49}{0.36 \times 20 \times 160}$$

$$\lambda_u = 59.67 \text{ mm} \leq D_f$$

$$\lambda_{u \max} = 0.48 d \Rightarrow 0.48 (510)$$

$$= 244.8 \text{ mm}$$

$$\lambda_u < \lambda_{u \max}$$

So, the beam is under reinforced section.

Moment of resistance:

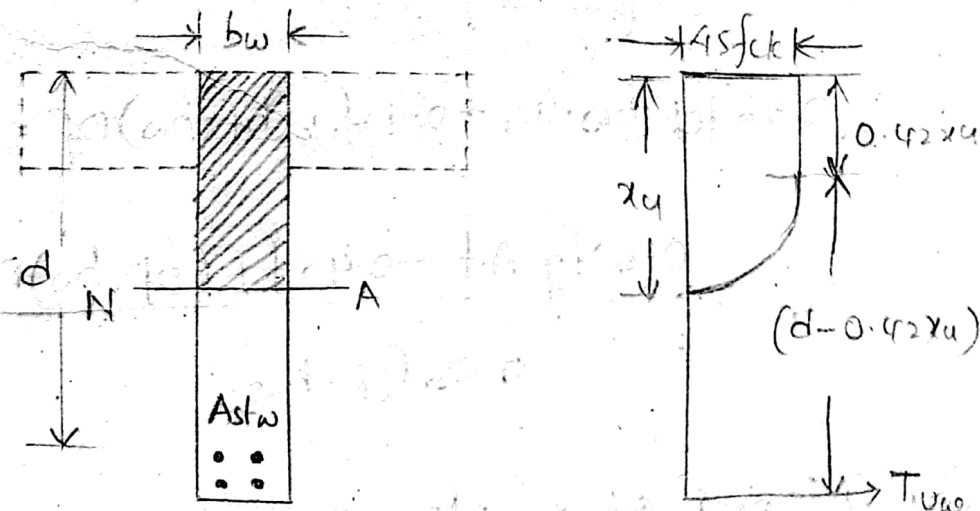
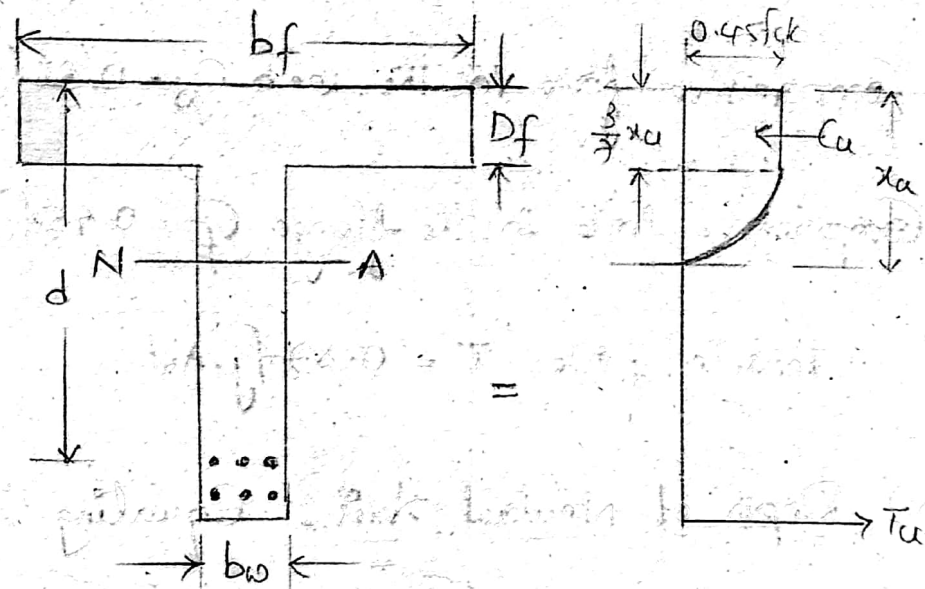
$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u) \quad \text{www.FirstRanker.com}$$

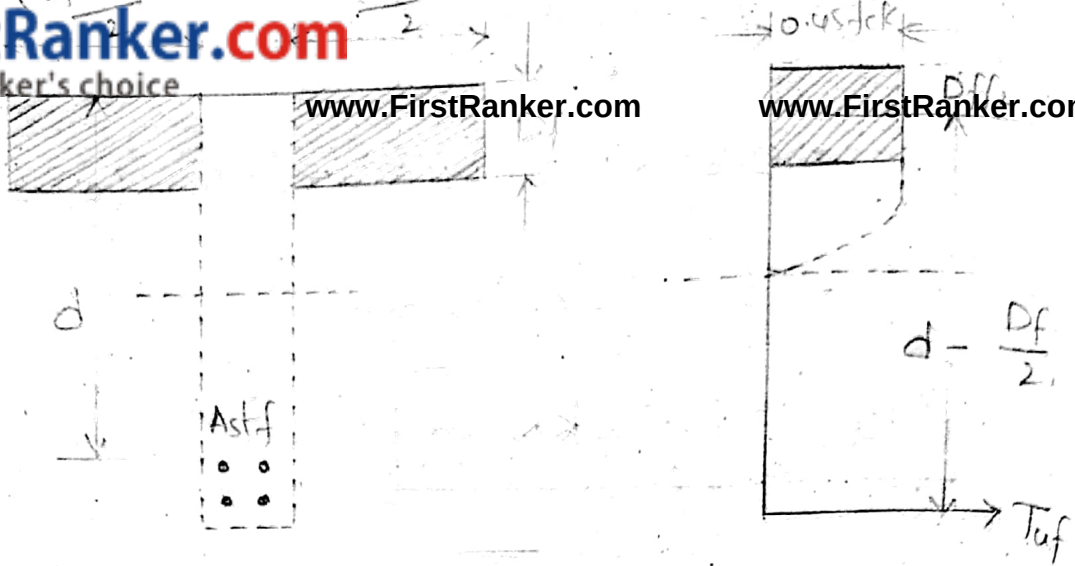
$$= 0.87 \times 415 \times 1963.49 (510 - (0.42 \times 59.6))$$

$$= 343.78 \times 10^6 \text{ N-mm}$$

$$M_u = 343.78 \text{ kN-m}$$

Case 2: Neutral Axis is below the flange
($x_u > D_f$) i.e., in the web & $D_f/d \leq 0.2$





Neutral Axis is below the flange & $\frac{D_f}{d} \leq 2$

Compressive force in the web $C_w = 0.36 f_{ck} \cdot b_w \cdot x_u$

Compressive force in the flange $C_f = 0.45 f_{ck} (b_f - b_w) D_f$

Tensile force $T = 0.87 f_y \cdot A_{st}$

(a). Depth of Neutral Axis: Equating Compression & tension force.

$$0.36 f_{ck} \cdot b_w \cdot x_u + 0.45 f_{ck} (b_f - b_w) D_f = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y \cdot A_{st} - 0.45 f_{ck} (b_f - b_w) D_f}{0.36 f_{ck} \cdot b_w}$$

(b). Moment of Resistance

$$M_u = M_{u, \text{web}} + M_{u, \text{flange}}$$

$$M_u = C_w (d - 0.42x_u) + C_f \left(d - \frac{D_f}{2} \right)$$

Substituting the values of C_w & C_f

$$M_u = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) D_f \left[d - \frac{D_f}{2} \right]$$

3. Calculate the moment of resistance of the T-beam with the following data.

Width of the flange = 800 mm = b_f

Thickness of slab = 110 mm = D_f

Width of rib = 300 mm = b_w

Effective depth = 600 mm = d

Area of tension steel = 2500 mm² = A_{st}

Characteristic strength of concrete = 20 N/mm² = f_{ck}

Characteristic strength of steel = 415 N/mm² = f_y

Step 1: Assuming x_u is within the flange:

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 2500}{0.36 \times 20 \times 800}$$

$x_u = 110 \text{ mm} > D_f$
 $\therefore$ Our assumption is wrong. So, the N.A lies in the web.

$$\frac{D_f}{d} = \frac{110}{600} = 0.18 < 0.2$$

So, it is Case - 2

Step 2:

Actual depth of N.A

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w) D_f}{0.36 \times f_{ck} \times b_w}$$

$$= \frac{0.87 \times 415 \times 2500 - 0.45 \times 20 (800 - 300) 110}{0.36 \times 20 \times 300}$$

$$x_u = 188.72 \text{ mm}$$

$$x_{u \max} = 0.48 d$$

$$= 0.48 \times 600$$

$$= 288$$

$$x_u < x_{u \max}$$

$\therefore$ The section is under reinforced section.

$$M_u = 0.36 f_{ck} \cdot b_w \cdot x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) b_f \left(d - \frac{D_f}{2} \right)$$

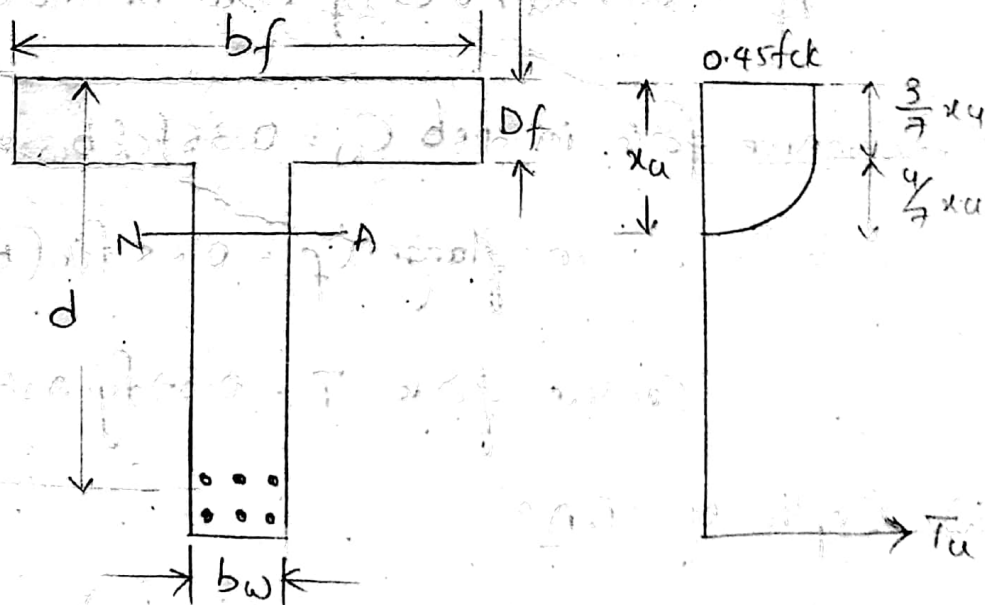
$$\Rightarrow 0.36 \times 20 \times 300 \times 188.72 (600 - (0.42 \times 188.72)) + 0.45 \times 20 (800 - 300) 110 \times \left(600 - \frac{110}{2} \right)$$

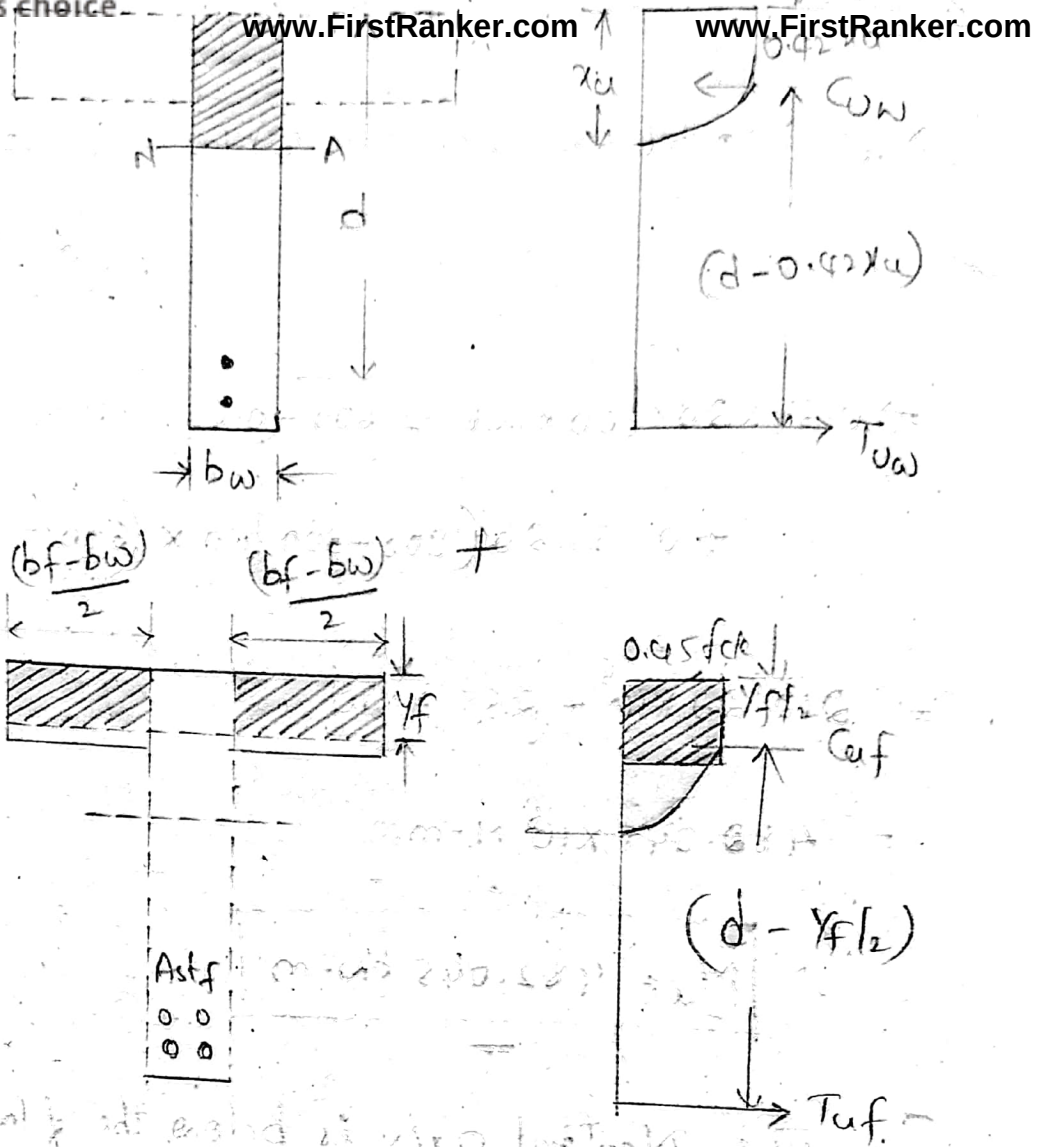
$$\Rightarrow 212.27 \times 10^6 + 269.7 \times 10^6$$

$$= 482.045 \times 10^6 \text{ N-mm}$$

$$\therefore \boxed{M_u = 482.045 \text{ kN-m}}$$

Case III: Neutral axis is below the flange ($x_u > D_f$) i.e., in the web & $D_f/d > 0.2$





Modified thickness of flange = y_f

$$y_f = 0.15 x_u + 0.65 D_f, \text{ but not more than } D_f$$

Compressive force in web $C_w = 0.36 f_{ck} b_w x_u$

" " in flange $C_f = 0.45 f_{ck} (b_f - b_w) y_f$

Tensile force $T = 0.87 f_y A_{st}$

(a). Depth of N.A:

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w) y_f}{0.36 f_{ck} b_w}$$

(b) Moment of resistance:

$$M_u = C_w (d - 0.42 x_u) + C_f \left(d - \frac{y_f}{2} \right)$$

$$M_u = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) y_f \left(d - \frac{y_f}{2} \right)$$

Types of footings:-

1. Isolated footing
2. Combined footing.
3. Strap footing.
4. Raft or mat footing.

Design procedure:-

1. Size of the footings:

Area of the footing required,

$$A = \frac{1.1P}{\text{SBC of soil}}$$

where P = working load

SBC = safe bearing capacity.

2. Determine the upward soil reaction for the factored load:

$$\begin{aligned} q_u &= \frac{P_u}{A} \\ &= \frac{1.5P}{A} \end{aligned}$$

3. Determine the min. depth required to resist

Bending moment:

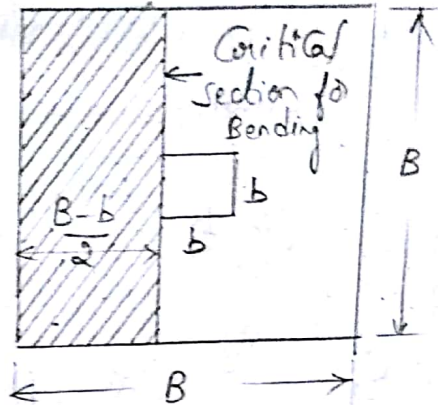
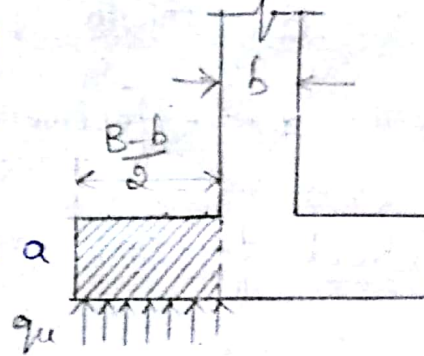
Projection of the footing

$$= \frac{(B-b)}{2}$$

The B.M about x-x is (as a cantilever slab, $\frac{wl^2}{2}$).

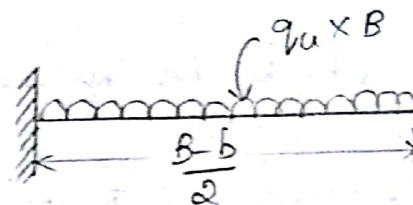
$$M_u = \frac{q_u \cdot B \left(\frac{B-b}{2} \right)^2}{2}$$

$$= \frac{q_u \cdot B(B-b)^2}{8}$$



where q_u = upward soil pressure

B = width of footing,
 b = width of column.



4. Determine the Area of reinforcement required in width B using,

$$M_u = 0.87 \times f_y \times A_{st} \times d \left[1 - \frac{f_y \cdot A_{st}}{f_{ck} \cdot B \cdot d} \right]$$

using the bars of diameter not less than 10mm, find the spacing of bars.

$$\text{Spacing} = \frac{B \cdot a_{st}}{A_{st}}$$

where, a_{st} = area of bar used

A_{st} = total area of steel required.

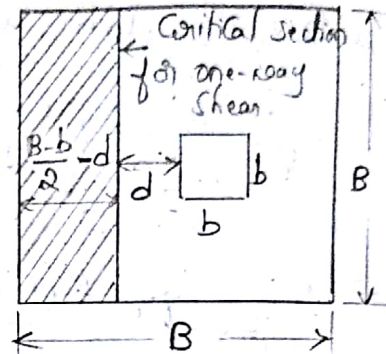
B = width of the footing

d = effective depth.

5. check for one way shear:

V_u = Soil pressure from the shaded area.

$$= q_u \cdot B \left[\frac{B-b}{2} - d \right]$$



$$\tau_v = \frac{V_u}{Bd} < \tau_c, \text{ permissible shear stress in Concrete.}$$

6. check for Two way shear: (Punching shear).

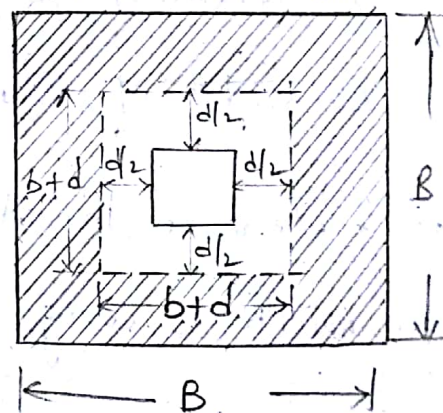
Perimetre of the punching

$$\text{Area} = 4(b+d)$$

Area of Concrete resisting

Punching ~~area~~ ^{force} = perimetre of punching $\times$ depth

$$A = 4(b+d)d$$



Force of punching $S = q_u \times \text{shaded Area}$

$$= q_u [B^2 - (b+d)^2]$$

Punching shear stress,

$$\tau_p = \frac{S}{A} < \text{permissible value.}$$

Permissible value of punching shear stress is,

$$\tau_p = 0.85 \sqrt{f_{ck}}$$

& check for Bond Length:

$$L_d = \frac{0.87 f_y \phi}{4 \cdot \tau_{bd}}$$

Design of stairs

(1) Assuming $\frac{L}{B}$ ratio (20 to 25).

2. Effective span:

x	y	span in m
$< 1m$	$< 1m$	$G+x+y$
$< 1m$	$> 1m$	$G+x+l$
$> 1m$	$< 1m$	$G+y+l$
$> 1m$	$> 1m$	$G+l+l$

3. Loads on stairs:

live load if crowded - 5 kN/m^2

if not " - 3 kN/m^2

Dead load:

thickness of waist slab = D

(a). weight of waist slab per unit horizontal area,

$$w_1 = \frac{D \sqrt{R^2 + T^2}}{T} \times 25$$

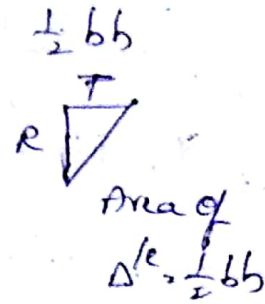
$$= D \sqrt{1 + \left(\frac{R}{T}\right)^2} \times 25$$

(b) vol. of steps per unit horizontal Area

$$= \frac{\frac{1}{2} \times R \times T \times 25}{8}$$

$$= \frac{1}{2} R \times 25$$

where R in metre



Finishing load (0.5 to 1 kn/m²) may be added to the above values.

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4. Determine the max. bending moment,

$$M_u = \frac{w_u l^2}{8}$$

5. Determine the min. depth required to resist the bending moment by equating,

$$M_u = M_{u\lim} = k \cdot f_{ck} \cdot b d^2$$

b = 1000mm, k = 0.138 for Fe 415 steel & 0.148 for mild steel.

Provided depth should be more than this value. Otherwise increase the depth.

6. Calculate the area of steel per metre width of slab by using,

6. Given

$$M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b \cdot d} \right]$$

7. Find the spacing of bars using,

$$S = \frac{1000 \times a_{st}}{A_{st}}$$

where,

a_{st} = area of bar used.

A_{st} = total area of steel required.

Spacing should not be more than $3d$ or 300mm whichever ever is less.

8. providing distribution reinforcement perpendicular to the span direction at 0.12% (for HYSD bars) of gross c/c area & find the spacing of these bars. if mild steel bars are used, provide 0.15% of gross area as distribution steel.

4. Design a doglegged staircase for a building in which the height of floor is 3.3m . Adopt rise & tread of each step are 150mm & 225mm respectively. The stair hall is $2.5\text{m} \times 4.5\text{m}$. Live load may be taken as 3kN/m^2 . Use M20 grade concrete & Fe 415 grade steel. Assume the stairs are supported on 230mm walls at the ends of

Outer edge of landing slab.

1. proportioning of stairs:

Dimensions of stair hall = $2.5\text{m} \times 4.5\text{m}$

Height of the floor = 3.3m

$$\text{Height of one flight} = \frac{3.3}{2} \Rightarrow 1.65\text{m} \\ \Rightarrow 1650\text{mm}$$

$$\text{Rise, } R = 150\text{mm}$$

$$\text{Tread, } T = 225\text{mm}$$

$$\text{No. of rises} = \frac{\text{Height of one flight}}{\text{Rise}}$$

$$\Rightarrow \frac{1650}{150}$$

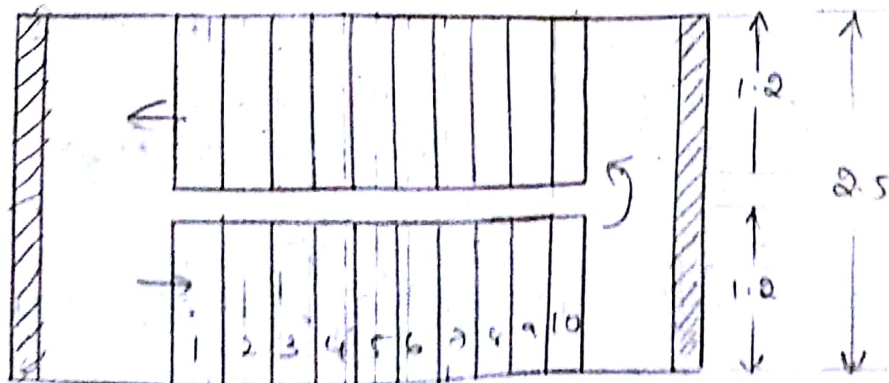
$$= 11$$

$$\text{Hence, no. of treads} = 11 - 1 \Rightarrow 10$$

$$\text{Adopt width of stair} = 1.2\text{m}$$

$$\text{For 10 treads, the length required} = 10 \times 0.25 \\ = 2.5\text{m}$$

$$\text{width of landing} = \left(\frac{4.5 - 2.5}{2} \right) \Rightarrow 1.125\text{m}$$



2. Effective span

As the stair slab is spanning longitudinally,

Effective span = Centre to centre distance of walls.

$$= 4.5 + 0.23$$

$$l = 4.73 \text{ m}$$

3. Thickness of slab:

Assume effective depth,

$$d = \frac{\text{Span}}{25} \Rightarrow \frac{4.73}{25} = 189.2 \text{ mm}$$

$$d = 190 \text{ mm}$$

$$D = 220 \text{ mm}$$

4. Loads: Load per metre horizontal width of stairs are as follows.

$$\text{wt. of waist slab} = D \sqrt{1 + \left(\frac{R}{T}\right)^2} \times 25$$

$$= 0.22 \sqrt{1 + \left(\frac{0.15}{0.225}\right)^2} \times 25$$

$$= 6.61 \text{ kN/m}^2$$

$$\text{wt. of steps} = \left(\frac{1}{2} R\right) \times 25 \Rightarrow R \times \frac{25}{2}$$

$$\Rightarrow 1.875 \text{ kN/m}^2$$

$$\text{live load} = 3 \text{ kN/m}^2$$

$$\text{Floor finish} = 0.6 \text{ kN/m}^2$$

$$\text{Total load} = 12.1 \text{ kN/m}^2$$

$$\text{Factored load } w_u = 1.5 \times 12.1$$

$$= 18.15 \text{ kN/m}^2$$

5. Factored BM:

$$M_u = \frac{w_u l^2}{8} \Rightarrow \frac{18.15 \times (4.73)^2}{8}$$

$$M_u = 50.76 \text{ kN-m}$$

$$= 50.76 \times 10^6 \text{ N-mm}$$

6. Min. depth required:

$$M_u = 0.138 f_{ck} b d^2$$

$$50.76 \times 10^6 = 0.138 \times 20 \times 1000 \times d^2$$

$$d = \sqrt{\frac{50.76 \times 10^6}{0.138 \times 20 \times 1000}}$$

$$d = 135.6 \text{ mm} < 190 \text{ mm, provided depth}$$

hence provided depth is adequate

7. Tension reinforcement:

$$M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$50.76 \times 10^6 = 0.87 \times 415 \times A_{st} \times 190 \left[1 - \frac{415 A_{st}}{20 \times 1000 \times 190} \right]$$

$$50.76 \times 10^6 = 68.6 \times 10^3 A_{st} - 7.49 A_{st}^2$$

$$7.49 A_{st}^2 - 68.6 \times 10^3 A_{st} + 50.76 \times 10^6 = 0$$

$$\boxed{A_{st} = 812 \text{ mm}^2}$$

using 12mm ϕ bars, spacing of bars,

$$S = \frac{A_{st}}{A_{bar}} \times 1000$$

$$\Rightarrow \frac{\pi}{4} \times 12^2 \times \frac{1000}{812}$$

$$= 139.3 \text{ mm}$$

provide 12mm bars @ 130mm c/c.

8. Distribution reinforcement:

$$A_{st} = 0.12\% \text{ of gross area}$$

$$= 0.12 \times 1000 \times \frac{220}{100}$$

$$\boxed{A_{st} = 264 \text{ mm}^2}$$

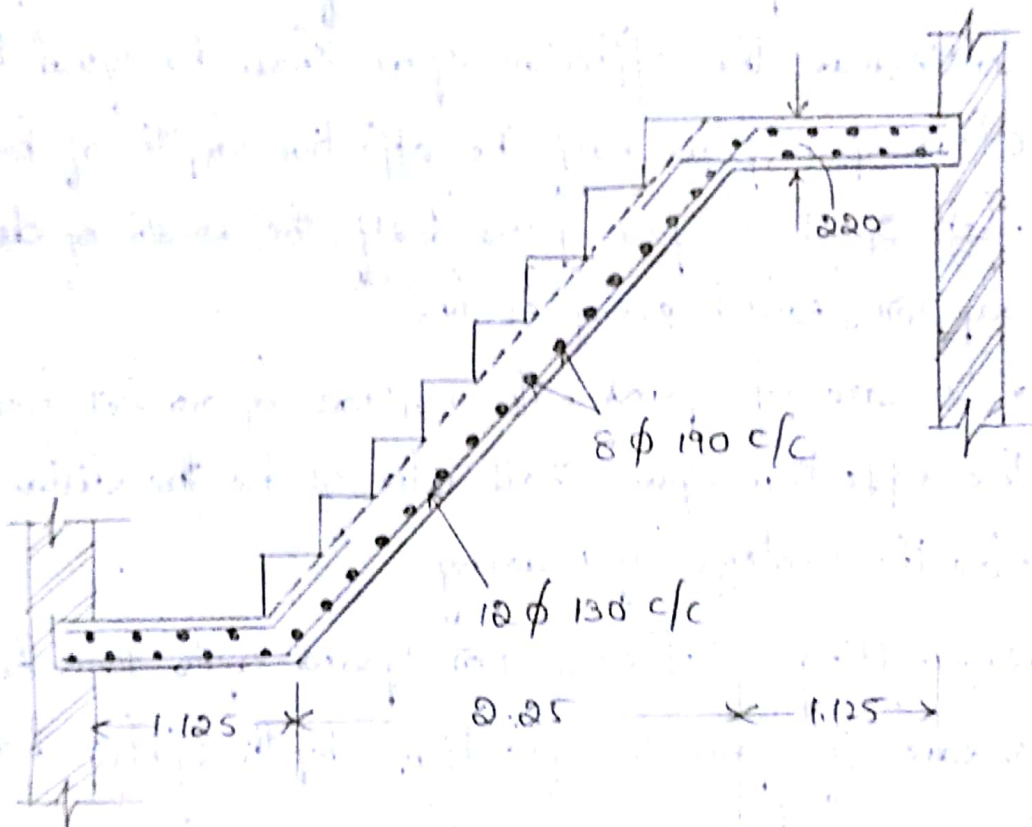
using 8mm bars, spacing

$$S = \frac{\pi}{4} \times 6 \times \frac{100}{264}$$

$$= 190.4 \text{ mm}$$

Hence, provide 8mm bar @ 190 mm c/c

The details of reinforcement are shown in fig 4.21



19/9/17

Continuous Beams :-

1. Effective span :- If the width of the support is less than $\frac{1}{12}$ of clear span, the effective span shall be as per simply supported beam.

- (i) width of support $< \frac{1}{12}$ of clear span
- (ii) width of support $> \frac{1}{12}$ of clear span (or) 600mm

which ever is less, the effective span shall be as follows.

- For end spans with one end free & the other continuous or for intermediate spans, the effective span shall be the clear span b/w the supports.
- For end span with one end free & the other continuous, the effective span shall be equal to the clear span plus half the effective depth of beam/slab & clear span plus half the width of continuous support, whichever ever is less.
- In case of spans with rollers & rocket bearings, the effective span shall always be the distance b/w the centers of bearing.

2. Limiting stiffness: For spans upto 10m, the basic value of span to effective depth ratio should not exceed 26. Depending up on the area & type of tension steel the span to depth ratio may be multiplied by the modification factors.

In general, Continuous beams carry heavy load & consequently the span/depth ratio recommended in practical design is normally in between 15 to 20.

22/9/17
3. Bending moments & shear forces it needs rigorous structural analysis to get the design

moments & shear forces. However IS: 456-2000 permits use of design coefficients shown in table 41 & 42 (Table 12 & 13 of IS: 456-2000) subjected to the following conditions.

- There are three or more spans.
- Spans don't differ by 15% of the longest.
- Loads are predominantly uniformly distributed loads.

4. Reinforcements

Same rules apply as for simply supported beams/slabs.

1. Mid span reinforcement
2. Support reinforcement.

5. Design a singly reinforced Continuous RC rectangular beam for flexure for the following conditions. use M20 grade Concrete & Fe 415 steel.

No. of spans = 3

Clear distance b/w supports = 3600mm

width of the support = 300mm

Imposed load (not fixed) = 5 kN/m^2

Imposed load (fixed) = 4.5 kN/m^2 (excluding self wt).

Partial fixity may be expected at the ends continuous edge.

Given data,

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$b = 300 \text{ mm}$$

$$\text{width} = 300 \text{ mm}$$

$$\text{No. of spans} = 3$$

$$\text{Clear span } L = 3.6 \text{ m}$$

1. Depth of the beam:

Selecting the depth in range of $\frac{l}{15}$ to $\frac{l}{20}$ based on stiffness

Assume $\frac{l}{d}$ ratio as 15

$$\frac{l}{d} = 15$$

$$d = \frac{3600}{15}$$

$$= 240 \text{ mm}$$

Adopt $d = 250 \text{ mm}$

Assume effective cover = 50 mm

$$D = 250 + 50$$

$$\boxed{D = 300 \text{ mm}}$$

2. Effective Span:

$$(i) \text{ clear span} + d = 3.6 + 0.25$$

$$= 3.85 \text{ m}$$

(ii) clear span $+ \frac{b}{2} + \frac{b}{2}$

$$= 3.6 + \frac{0.23}{2} + \frac{0.23}{2}$$

$$= 3.85 \text{ m}$$

Effective span $l = 3.85 \text{ m}$

3. Loads:-

Self wt. of the beam = ~~$0.2 \times 0.25 \times 25$~~ $0.3 \times 0.3 \times 25$
 $= 2.25 \text{ kN/m}^2$

Imposed load, fixed = 7.5 kN/m^2

Total load fixed = 9.75 kN/m^2

Imposed load, not fixed = 5 kN/m^2

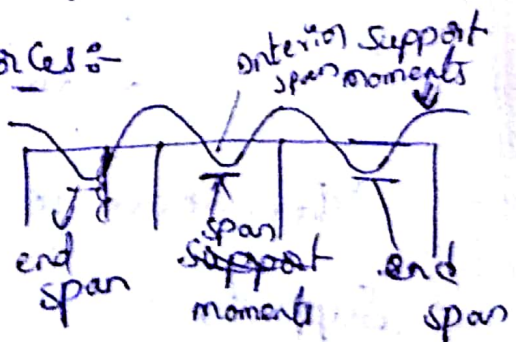
Factored loads:

factored fixed load $w_{ud} = 1.5 \times 9.75$
 $= 14.63 \text{ kN/m}$

not fixed $w_{ul} = 1.5 \times 5$
 $= 7.5 \text{ kN/m}$

4. Bending moments & shear forces:-

span moments: (at the middle of end span).



$$M_u = \left(\frac{1}{12} \right) \times 14.63 \times 3.85^2 + \left(\frac{1}{10} \right) \times 7.5 \times 3.85^2$$

$$= 18.07 + 11.12$$

$$= 29.2 \text{ kN-m}$$

At the middle of interior spans.

$$M_u = \left(\frac{1}{16}\right) \times 14.63 \times 3.85^2 + \left(\frac{1}{12}\right) \times 7.5 \times 3.85^2$$

$$= 13.55 + 9.26$$

$$= 22.81 \text{ kN-m}$$

Support moment:- (At support next to the end Support).

Max

$$M_u = \left(\frac{1}{10}\right) \times 14.63 \times 3.85^2 + \left(\frac{1}{9}\right) \times 7.5 \times 3.85^2$$

$$= 21.68 + 12.35$$

$$= 34.03 \text{ kN-m}$$

At other interior Support,

$$M_u = \left(\frac{1}{12}\right) \times 14.63 \times 3.85^2 + \left(\frac{1}{9}\right) \times 7.5 \times 3.85^2$$

$$= 18.07 + 12.35$$

$$= 30.42 \text{ kN-m}$$

Max. shear force at support next to the end Support,

$$V_{u, \text{max}} = (0.6 \times 14.63 \times 3.85) + (0.6 \times 7.5 \times 3.85)$$

$$= 63.27 \text{ kN}$$

4. Depth required:-

$$M_u = 0.138 f_{ck} b d^2$$

$$34.03 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{34.03 \times 10^6}{0.138 \times 20 \times 300}}$$

$$d = 202.72 < 250 \text{ mm}$$

Hence provided depth is adequate.

5. Reinforcement at supports:-

$$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} b d} \right]$$

$$34.03 \times 10^6 = 0.87 \times 415 \times A_{st} \times 250 \left[1 - \frac{415 A_{st}}{20 \times 300 \times 250} \right]$$

$$34.03 \times 10^6 = 90.26 \times 10^3 A_{st} - 24.97 A_{st}^2$$

$$24.97 A_{st}^2 - 90.26 \times 10^3 A_{st} + 34.03 \times 10^6 = 0$$

$$A_{st} = 427.6 \text{ mm}^2$$

Provide, 12mm ϕ to finding no. of bars

$$\text{No. of bars} = \frac{A_{st}}{\frac{\pi}{4} (12)^2}$$

$$= \frac{427.6}{\frac{\pi}{4} (12)^2}$$

$$= 3.28 \sim 4 \text{ bars}$$

hence provide 4 bars of 12mm ϕ

$$A_{st} \text{ provided} = 4 \times \frac{\pi}{4} (12)^2$$

$$= 452.4 \text{ mm}^2$$

6. Reinforcement at mid spans:-

$$M_u = 0.87 \cdot f_y \cdot A_{st} \cdot x_d \left[1 - \frac{f_y \cdot A_{st}}{f_{ck} \cdot b \cdot d} \right]$$

$$29.2 \times 10^6 = 0.87 \times 415 \times A_{st} \times 250 \left[1 - \frac{415 A_{st}}{20 \times 300 \times 250} \right]$$

$$29.2 \times 10^6 = 90.26 \times 10^3 - 24.97 A_{st}^2$$

$$24.97 A_{st}^2 - 90.26 \times 10^3 + 29.2 \times 10^6 = 0$$

$$A_{st} = 359.2 \text{ mm}^2$$

Assume 12mm ϕ bars,

$$\text{No. of bars} = \frac{359.2}{\frac{\pi}{4} (12)^2} \Rightarrow 3.17 \approx 3 \text{ bars.}$$

hence provide 3 bars of 12mm ϕ ,

$$A_{st} \text{ provided} = 3 \times \frac{\pi}{4} (12)^2$$

$$= 339.3 \text{ mm}^2$$

7. Design of shear reinforcement:-

$$\tau_v = \frac{V_u}{b d} = \frac{63.27 \times 10^3}{300 \times 250} = 0.84 \text{ N/mm}^2$$

$$\text{percentage of steel } P_t = \frac{A_t}{bd} \times 100$$

$$= \frac{628.3 \times 100}{300 \times 200} = 1.04\%$$

Referring to the table -19 of IS: 456, shear strength of concrete is

$$1.00 - 0.62$$

$$1.25 - 0.67$$

$$1.04 - ?$$

$$\tau_c = 0.67 + \left(\frac{0.67 - 0.62}{1.25 - 1.00} \right) (1.04 - 1.00) \Rightarrow 0.67 \text{ N/mm}^2$$

max. shear stress in concrete τ_c max from table 20 of IS 456 (Page NO: 73)

$$\tau_{c \text{ max}} = 2.8 \text{ N/mm}^2$$

As $\tau_v > \tau_c$, shear reinforcement has to be designed
shear resistance of concrete $V_{uc} = \tau_c \cdot bd$

$$= 0.67 \times 300 \times 200$$

$$= 40200 \text{ N}$$

$$= 40.2 \text{ kN}$$

shear to be resisted by shear reinforcement
(vertical stirrups)

$$V_{us} = V_u - V_{uc}$$

$$= 49.15 - 40.2$$

$$= 8.95 \text{ kN}$$

using 6mm, 2 legged Fe 415 steel

$$A_{sv} = 2 \times \frac{\pi}{4} (6)^2 \Rightarrow 56.55 \text{ mm}^2$$

$$\text{Spacing } S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} \Rightarrow \frac{0.87 \times 415 \times 56.55 \times 250}{89.50} \\ = 456.25 \text{ mm}$$

max. allowed spacing $0.75d \Rightarrow 0.75 \times 200 \Rightarrow 150 \text{ mm}$

a 300mm which ever is less.

Hence, provide 2 legged 6mm stirrups @ 150mm c/c

check for deflection: (Def. Stiffness)

$$\frac{l}{d} = 26 \text{ (for continuous)}$$

$$\% \text{ of steel @ mid span} = \frac{603.18 \times 100}{300 \times 200} = 1.00$$

$$f_s = 0.58 \times 415 \times \frac{456.68}{603.18} \Rightarrow 161.99 \text{ N/mm}^2$$

from fig. 4 of Is: 456

$$\text{modification factor} = 1.3$$

$$\text{max. permitted } \frac{l}{d} \text{ ratio} = 1.3 \times 26 \\ = 33.8$$

$$\frac{l}{d} \text{ provided} = \frac{3800}{200}$$

$$= 19 < 33.8$$

$\therefore$ Hence deflection control is safe.

Class - 23

Limit state of collapse: compression.

Axial moment

Biaxial moment

Based on reinforcement

Tied column.

Spiral column.

Composite column.



Based on the loading.

i) Axially loaded.

ii) Eccentrically loaded

1) Ecc. " " In one axis.

" " " " double axis.



iii) Based on slenderness ratio

1) Short column.

$$\frac{l}{b} \leq 12.$$

ii) long or slender column. $\rightarrow \frac{l}{b} > 12.$

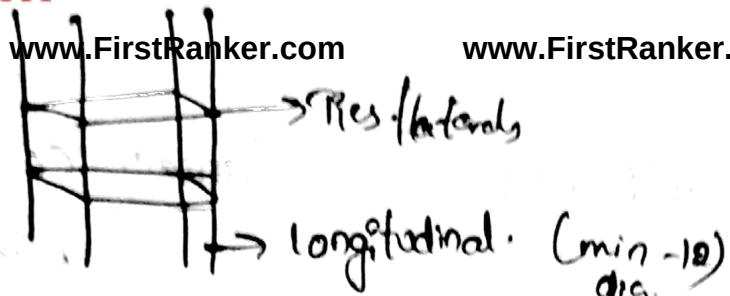
Columns

Form

1) Circular

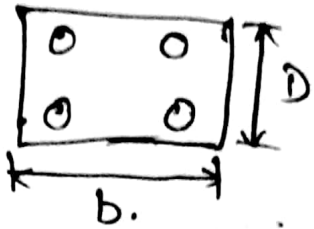
2) Rectangular

3) Square.



$$P = \frac{A_{st}}{bD} \times 100$$

$D = \text{gross area. (total)}$



$$l_e < 3b$$

25.12

it is column if not pedestal.

$$e = \frac{l}{500} + \frac{b}{30}$$

Pg. 41.

25.4

subjected to min of 20mm

$$e_{min} = 20mm$$

Pg. 20-48, 26.5.3., 26.5.3.1

↓
Columns

↓
longitudinal reinforcement

0.8% to 6%

for circular - min 6 bars

Sler - min 4 bars.

Pg NO - 49 : c) Pitch & diameter of the longitudinal bars

s_{max} i) b

ii) 16ϕ times the small diameter of longitudinal bar

iii) 300 mm.

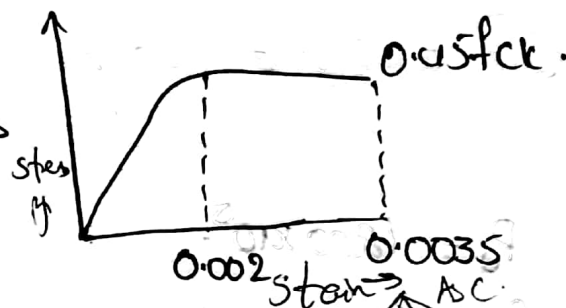
which ever is less.

2) diameter:-

$$d = \frac{1}{4} \times \phi \rightarrow \text{largest longitudinal bar dia.} \\ < 16 \text{ mm.}$$

Pg NO - 70

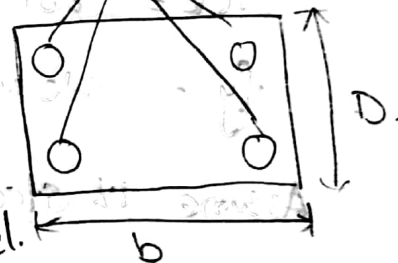
axial compression is 0.002.



CLS of the column.

$$A_c = (bD) A_{sc} - A_{sc}$$

P_u = load carried by concrete +
load carried by steel.

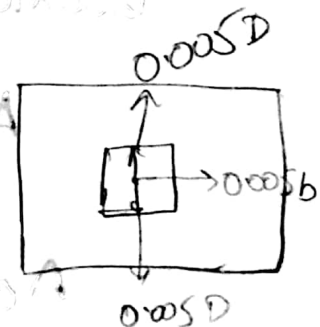


$$= A_c (0.45 f_{ck}) + A_{sc} (0.75 f_y)$$

$$= 0.9 (0.45 f_{ck} A_c + 0.75 f_y A_{sc})$$

$$= 0.405 f_{ck} A_c + 0.675 f_y A_{sc}$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$



$$P_0 = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 12^2 = 452.38$$

$$f_{ck} = 20 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2$$

$$A_c = (950 \times 300) - 452.38$$

$$= 74547.62$$

$$P_0 = 0.4 \times 20 \times 74547.62 + 0.67 \times 415 \times 452.38$$

$$= 596 \text{ kN} + 125 \text{ kN}$$

$$= 722.165 \text{ kN}$$

Design a square column for the design load of 1200 kN
Use M20 concrete, Fe415 steel.

$$P_0 = 1200 \times 10^3$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Assume 1% steel $A_{sc} = 1\% \text{ steel}$

$$= 0.01 A_c$$

$$1200 \times 10^3 = 0.4 \times 20 \times A_c + 0.67 \times 415 \times 0.01 A_c$$

$$A_c = \frac{1200 \times 10^3}{10.78} = 111312 \text{ mm}^2$$

$$A_c = 111312 \text{ mm}^2$$

$$b = \sqrt{111312}$$

for square.

$$= 333.6$$

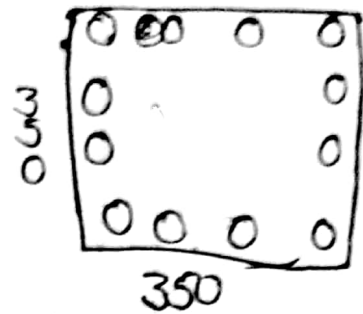
∴ provide $350 \times 350 \text{ mm}$.

$$A_{sc} = 0.01 \times (350 \times 350)^2$$

$$= 1225 \text{ mm}^2;$$

$$12 \Phi = 10.83.$$

$$16 \Phi = 6.09.$$



Provide 12 - 12Φ as longitudinal reinforcement

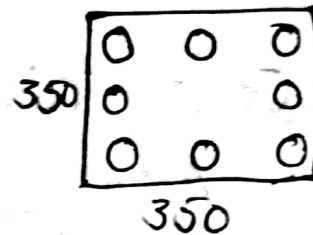
Provide 8 - 16Φ as longitudinal rft.

$$P = \frac{A_{sc}}{bd} \times 100.$$

$$= \frac{8 \times \frac{\pi}{4} \times 16^2}{350 \times 350} \times 100$$

$$= 1.3 \%$$

Hence Ok.



Lateral ties:-

$$\text{Diameter } d = \frac{1}{4} \times 16$$

$$= 4 \text{ mm}$$

Hence provide 6 mm ϕ (mrs) bar.

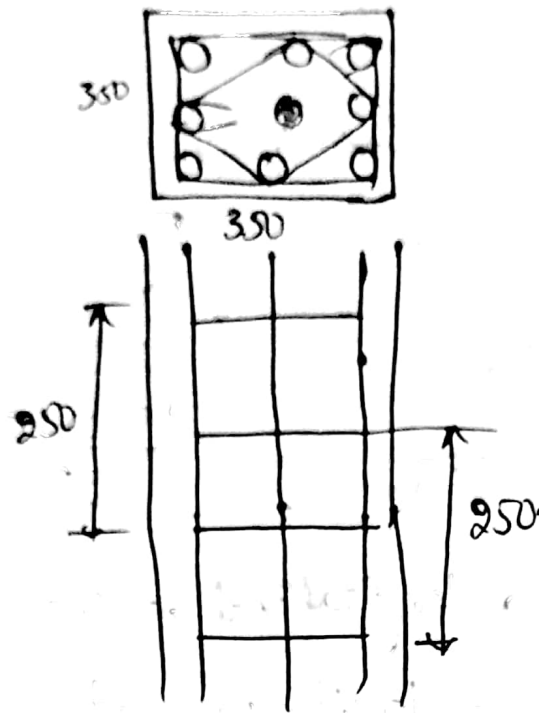
spacing. least of

350 mm

$$16 \Phi = 16 \times 16 = 256 \text{ mm}$$

300 mm.

provide 6 mm ϕ @ 250 mm c/c.



Center Columns:- P_u

Axial :- P_u, M_u

Biaxial :- P_u, M_u, M_y

Design a ^{short} axially loaded tied column pinned at both ends with an unsupported length of 3.5 mts. for carrying a characteristic load of 1500 kN. Use M20, Fe 415 steel.

Given data;

$$f_{ck} = 20 \text{ N/mm}^2$$

$$P = 1500 \text{ kN.}$$

$$f_y = 415 \text{ N/mm}^2.$$

$$l = 3500 \text{ mm}$$

Pinned - pinned.

$$\text{Design load } P_0 = 1.5 \times 1500 = 2250 \text{ kN.}$$

$$\text{Eff. length } l_e = l = 3500 \text{ mm.}$$

$$\text{For short column } \frac{l_e}{D} = 12.$$

$$\frac{3500}{12} = D.$$

$$D = 291.67 \text{ mm.}$$

$$\text{Assume col. section } 300 \times 300 \text{ mm}^2.$$

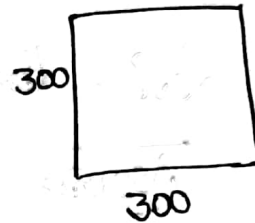
$$e_{\min} = \frac{l}{500} + \frac{D}{30}.$$

$$= \frac{3500}{500} + \frac{300}{30}$$

$$= 17 < 20 \text{ mm.}$$

$$\frac{l}{D} = \frac{3500}{300}$$

$$= 11.67 < 12.$$



$$A_g = 300 \times 300 = 90000 \text{ mm}^2.$$

$$\text{Area of steel } A_{sc} = ?$$

$$A_c = A_g - A_{sc} = 90000 - A_{sc}.$$

$$P_0 = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}.$$

$$2250 \times 10^3 = 0.4 \times 20 \times (90000 - A_{sc}) + 0.67$$

$$2250 \times 10^3 = 720000 - 8A_{sc} + 278.05 A_{sc}.$$

$$= 720000 - 270.05 A_{sc}.$$

$$A_{sc} = 5665.61 \text{ mm}^2$$

$$\% \text{ steel} = \frac{A_{sc}}{bd} \times 100 = \frac{5665.61}{300 \times 300} \times 100$$

$$= 6.29\%$$

$$A_{sc} = 5665.61 \text{ mm}^2$$

25 ϕ .

$$A_n = \frac{5665.61}{490.8}$$

$$= 11.54 \text{ bar} \approx 12 \text{ bars}$$

12/9/015

Revise the section

140

Assume $375 \times 375 \text{ mm}$

$$\frac{l_e}{D} = \frac{3500}{375} = 9.33 < 12$$

140625

$$A_g = 375 \times 375 = 140625 \text{ mm}^2$$

$$A_c = A_g - A_{sc} = 140625 - A_{sc}$$

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$2250 \times 10^3 = 0.4 \times 20 \times (140625 - A_{sc}) + 0.67 \times 415 \times A_{sc}$$

$$A_{sc} = 4165.89 \text{ mm}^2$$

$$\% \text{ of steel } p = \frac{A_{sc}}{bd} \times 100$$

$$= \frac{4165.89}{375 \times 375} \times 100 = 2.96\%$$

$$A_{sc} = 4165.89 \text{ mm}^2$$

Assume 25ϕ $A_{\phi} = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$

$$\eta = \frac{416589}{490.87}$$

$$= 8.48$$

Provide $10 \times 25 \phi$

$$\% \text{ steel provided} = \frac{10 \times \frac{\pi}{4} \times 25^2}{375 \times 375} \times 100 = 3.49\%$$

Lateral ties:

$$d = \frac{\phi}{4} = \frac{25}{4} = 6.25 \text{ mm}$$

Provide $8 \text{ mm } \phi$.

Spacing:

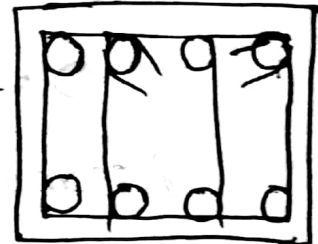
i) 375 mm

ii) $16 \phi = 16 \times 25 = 400 \text{ mm}$

iii) 300 mm

Ties $8 \phi @ 300 \text{ c/c}$

375



① Design a circular column to carry an axial load of 1000 kN using lateral ties. Use M_{20} , $Fe415$ steel.

$$P = 1000 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$\text{Design load } P_0 = 1.5 \times P = 1.5 \times 1000 = 1500 \text{ kN}$$

Assume 16 Φ $A\phi = 201.06$

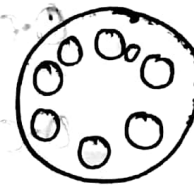
$$\eta = \frac{1452.20}{201.06} = 7.22 \text{ bars.}$$

provide 8 bars 16 Φ .

$$\text{ASC provided} = 1608.49$$

$$\text{ASC provided} = \frac{1608.49}{\pi \times D^2} \times 100$$

$$= 11.67$$



lateral ties;

$$\text{Dia } d = \frac{\phi}{4} = \frac{16}{4} = 4 \text{ mm.}$$

provide 6mm ϕ (M.S).

spacing

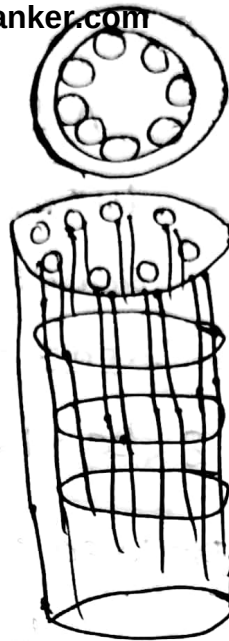
$$\text{i) } 430 \text{ mm}$$

$$\text{ii) } 16 \times 16 = 256 \text{ mm.}$$

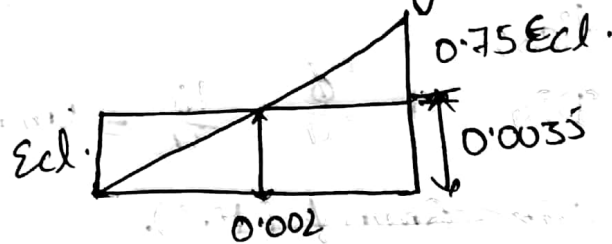
$$\text{iii) } 300 \text{ mm.}$$

provide 6mm ϕ @ 250 mm c/c.

(4)



0.002 for axial load.
0.0035 for bending.



D

$$\epsilon_{cl} = 0.0035 - 0.75 \epsilon_{cl}$$

Design of column carrying axial load & moment. (P_0, M_0).

$M_0 = 0$ then it is a axially loaded column.

$P_0 = 0$ then pure bending case.

P_0 & M_0 are present for uniaxial column.

Design a column for the following data.

Axial load $P_0 = 1200 \text{ kN}$.

B.M $M_0 = 250 \text{ kN-m}$.

Unsupported length $l = 3200 \text{ mm}$.

Clear cover 40 mm .

pinned-pinned case.

Given data.

$$P_0 = 1200 \text{ kN}$$

$$M_0 = 250 \text{ kN-m}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

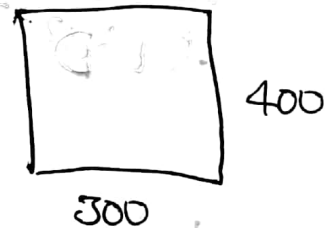
$$C.C = 40 \text{ mm}$$

$$l = 3200 \text{ mm}$$

Pinned-pinned.

$$\text{Effective length } l_e = l = 3200 \text{ mm}$$

$$e = \frac{l}{500} + \frac{D}{30}$$



$$= \frac{3200}{500} + \frac{100}{30}$$

$$= 19.73 < 20 \text{ mm.}$$

$$e_{min} = 20 \text{ mm.}$$

$$M_0 = P_0 \times e \Rightarrow e = \frac{M_0}{P_0} = \frac{250 \times 10^6}{1200 \times 10^3}$$

$$= 208.33$$

$$e > e_{min}$$

11/9/15

Design for bending.

$$\frac{P_0}{f_{ck} b D} = \frac{1200 \times 10^3}{20 \times 300 \times 400}$$

$$= 0.5$$

$$\frac{M_0}{f_{ck} b D^2} = \frac{250 \times 10^6}{20 \times 300 \times 400^2}$$

$$= 0.5 \times 0.26$$

$$d' = 40 \text{ mm} + \frac{\phi}{2}$$

$$= 50 \text{ mm}$$

$$\frac{d'}{D} = \frac{50}{400} = 0.125$$

From Sp-16, Chart 33, $f_y = 415 \text{ N/mm}^2$

$$\frac{P}{f_{ck}} = \frac{0.24}{20} \times 20$$

$$= 4.8$$

$$p = 4.8\%$$

$$A_{sc} = \frac{4.8}{100} \times 300 \times 400$$

$$= 5760 \text{ mm}^2$$

$$20 \Phi \quad A_{\phi} = 314 \text{ mm}^2$$

$$= \frac{5760}{314} = 18.34 \text{ bars}$$

Provide 20 bars

$$25 \Phi$$

$$= \frac{5760}{25}$$

$$= 12 \text{ bars.}$$

$$= 300 - 40 - 40 - \frac{25}{2} - \frac{25}{2}$$

$$= 195$$

$$= \frac{195}{5} = 39 \text{ mm.}$$

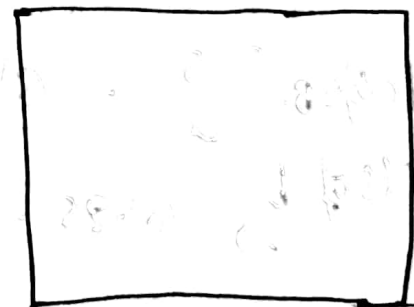
$$\Rightarrow 39 - \frac{25}{2} - \frac{25}{2}$$

$$= 14 \text{ mm} < 25 \text{ mm.}$$

Revise the section;

$$\frac{P_u}{f_{ck} b d} = \frac{1200 \times 10^3}{20 \times 350 \times 450} = 0.38$$

$$\frac{M_u}{f_{ck} b d^2} = \frac{200 \times 10^6}{20 \times 350 \times 450^2} = 0.176$$



$$d' = 40 + \frac{95}{2}$$

$$= 52.5$$

$$\frac{d'}{D} = 0.15$$

Chart-33, Sp-16

$$\frac{P}{f_{ck}} = 0.135$$

$$P = 2.7\%$$

$$A_{sc} = \frac{2.7}{100} \times 350 \times 480$$

$$= 4252.5 \text{ mm}^2$$

$$\text{Assume } 25 \Phi = A \Phi = 490.87 \text{ mm}^2$$

$$n = 8.66$$

$$10 \times 25 \Phi$$

Detailing:-

C/c of outer bars.

$$350 - 40 - 40 - \frac{25}{2} - \frac{25}{2}$$

$$= 245$$

$$C/c \text{ of I.B. } \frac{245}{4} = 61.25 \text{ mm.}$$

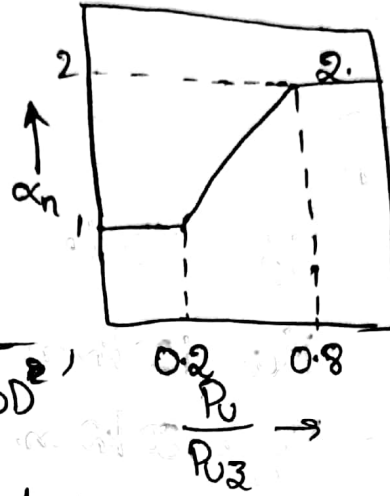
$$C/c \text{ of } b \Rightarrow 61.25 - \frac{25}{2} - \frac{25}{2}$$

$$= 36.25 \text{ mm.}$$

Biaxial columns

Procedure: P_u, M_{ux}, M_{uy} .

1) Assume cross section. (b, D)
Assume % of steel (ρ).



Step-2 Find $P_u / f_{ck} b D^2$, $\frac{M_{ux}}{f_{ck} b D^2} \rightarrow$ & find M_{ux} .

Step-3 Find $\frac{P}{f_{ck}} ; \frac{M_{uy}}{f_{ck} b D^2} \rightarrow M_{uy}$.

$$\left(\frac{M_{ux}}{M_{ux1}} \right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}} \right)^{\alpha_n} \leq 1.$$

Rectangular column with biaxial bending.

Determine the rft to be provided in a column subjected to biaxial bending with the following data

Size of column 400x500 mm.

Concrete mix M20.

characteristic strength of rft 415 N/mm²

Factored load = 1200 kN.

Factored moment acting // to the larger dimension 100 kN-m.

Factored moment acting // to the shorter direction
= 80 kN-m

clear cover = 40 mm.

Given;

$$P_u = 1200 \text{ kN}$$

$$M_{ux} = 100 \text{ kN-m}$$

$$M_{uy} = 80 \text{ kN-m}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Sized column = 400 x 500 mm.

Assume % of steel as $P = 1.6\%$
Moment carrying capacity

$$\frac{P}{f_{ck}} = \frac{1.6}{20} = 0.08$$

$$\frac{P_u}{f_{ck} b D} = \frac{1200 \times 10^3}{20 \times 400 \times 500}$$

$$= 0.3$$

chart

$$\frac{d'}{D} = \frac{52.5}{500} = 0.105$$

Chart - 44

$$\frac{M_{ux1}}{f_{ck} b D^2} = \frac{M_{ux}}{20 \times 400 \times 500^2} = 0.105$$

$$= 0.12$$

$$= 5.15 \times 10^{-4} M_{ux1}$$

$$M_{ux1} = 240 \text{ kN-m}$$

$$\frac{d'}{b} = \frac{52.5}{400} = 0.13125$$

$$\frac{M_{uy1}}{f_{ck} b b^2} = \frac{M_{uy1}}{20 \times 500 \times 400^2}$$

chart 45

$$\frac{M_{uy1}}{f_{ck} b b^2} = 0.11$$

$$M_{uy1} = 176 \text{ kN-m.}$$

$$\left(\frac{M_{ux1}}{M_{ux1}} \right)^{\alpha_n} + \left(\frac{M_{uy1}}{M_{uy1}} \right)^{\alpha_n} \leq 1$$

$$\alpha_n = \frac{P_u}{P_2}$$

$$P_2 = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}$$

$$A_{sc} = \frac{1.6}{100} \times 400 \times 500$$

$$= 3200 \text{ mm}^2$$

$$A_g = 400 \times 500 = 200000 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= 200000 - 3200$$

$$= 196800$$

$$P_{u2} = 0.45 \times 20 \times 196800 + 0.75 \times 415 \times 3200$$

$$= 2767200$$

$$P_u/P_{u2} = \frac{1200 \times 10^3}{2767200}$$

$$= 0.4336$$

$$= \left(\frac{100 \times 10^6}{24 \times 10^6} \right)^{1.28} + \left(\frac{8 \times 10^6}{176 \times 10^6} \right)^{1.38}$$

$$= 0.334 + 0.336$$

$$= 0.67 \leq 1, \text{ Hence ok.}$$

Assume 25 Φ .

$$= \frac{3000}{490.87}$$

$$= 6.519$$

Provide 8 x 25 Φ .

$$\text{Lateral ties} = \frac{\Phi}{4} = \frac{25}{4} = 6.25$$

Provide 8 mm bars.

Spacing / pitch.

$$\text{i) } 16 \Phi = 16 \times 25 = 400$$

iii) 300 mm

Provide 8 Φ @ 300 mm c/c.

Circular column with
Design a reinforced column 400mm square
to carry an ultimate load of 1000kN and an
eccentricity of 160mm. Use M20, Fe250

Sol $P_u = 1000 \text{ kN}$

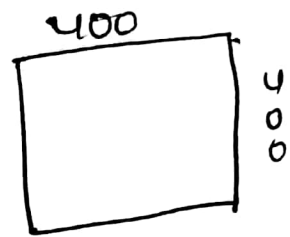
$e = 160 \text{ mm}$

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 250 \text{ N/mm}^2$

$b = 400 \text{ mm}$

$D = 400 \text{ mm}$



$M_u = P_u \times e = 160 \text{ kN-m}$

Assume % of steel = 1.6%

$$\frac{P}{f_{ck}} = \frac{1.6}{20} = 0.08$$

$$\frac{P_u}{f_{ck} b D} = \frac{1000 \times 10^3}{20 \times 400 \times 400} = 0.3125$$

$$\frac{d'}{d} = \frac{50}{400} = 0.125$$

$$\frac{P \cdot M_u}{f_{ck} b D^2} = \frac{160 \times 10^6}{20 \times 400 \times 400^2} = 0.125$$

Design a short circular column of 500mm dia with the following data.

Factored load = 800kN.

Factored moment = 162.5 kN-m.

Provide Hoop reinforcement. Take M20, Fe415 steel.

$$D = 500 \text{ mm.}$$

$$P_u = 800 \text{ kN.}$$

$$M_u = 162.5 \text{ kN-m.}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2.$$

$$d' = 40 + \frac{20}{2} = 50 \text{ mm (Assume } 20\Phi)$$

$$\frac{d'}{D} = \frac{50}{500} = 0.1$$

$$\frac{P_u}{f_{ck} D^2} = \frac{800 \times 10^3}{20 \times 500^2} = 0.16.$$

$$\frac{M_u}{f_{ck} D^3} = \frac{162.5 \times 10^3}{20 \times 500^3} = 0.065.$$

Chart-56 $\frac{P}{f_{ck}} = 0.05.$

$$P = 0.05 \times 20 = 1\%.$$

$$P = 1\%.$$

$$A_{sc} = \frac{1}{100} \times \frac{\pi \times 500^2}{4}$$

$$= 1963 \text{ mm}^2.$$

$$20\phi = A\phi = 314 \text{ mm}^2$$

$$n = 6.25$$

$$7 \times 20\phi$$

lateral ties

$$d = \frac{20}{4} = 5 \text{ mm}$$

provide 6ϕ

spacing $\leq 500 \text{ mm}$

$$\text{i) } 16 \times 20 = 400 \text{ mm}$$

$$\text{iii) } 300 \text{ mm}$$

provide $6 \text{ mm}\phi @ 300 \text{ mm c/c}$.

$$b = 250 \text{ mm}$$

$$P_u = 600 \text{ kN-m}$$

$$M_{ux} = 60 \text{ kN-m}$$

$$M_{uy} = 40 \text{ kN-m}$$

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 30 \text{ N/mm}^2$$

Assume $P = 3\%$

$$\frac{P}{f_{ck}} = \frac{3}{30} = 0.1$$

$$\frac{P_u}{f_{ck} b D} = \frac{600 \times 10^3}{30 \times 250 \times 250}$$

$$= 0.32$$

$$\frac{M_{u1}}{f_{ck} b D^2} = ? = 0.11$$

$$40 + \frac{20}{2} = 50$$

$$\frac{d'}{D} = \frac{50}{250} = 0.2$$

$$M_{u1} = 51.5625 \text{ kN-m.}$$

Assume $P = 0.42$

$$\frac{P}{f_{ct}} = \frac{0.42}{30} = 0.14$$

$$\frac{M_{u1}}{f_{ck} b D^2} = 0.14$$

$$M_{u1} = 65.625 \text{ kN-m.}$$

$$M_{u1} = 65.625 \text{ kN-m.}$$

$$A_g = 250 \times 250 = 62500 \text{ mm}^2$$

$$A_{sc} = \frac{P_b D}{100} = 2625 \text{ mm}^2$$

$$A_c = 59875 \text{ mm}^2$$

$$P_{02} = 0.45 f_{ck} A_c + 0.75 \times f_y \times A_{sc}$$

$$= 1625343.75$$

$$\frac{P_0}{P_2} = \frac{600 \times 10^3}{1625343.75}$$

$$= 0.369$$

$$0.2 \rightarrow 1$$

$$0.8 \rightarrow 2$$

$$0.369 \rightarrow ?$$

$$1 + \frac{(2-1)}{(0.8-0.2)} (0.369-0.2)$$

$$= 1.281$$

$$= \left(\frac{60}{65.625} \right)^{1.28} + \left(\frac{40}{65.625} \right)^{1.28}$$

$$= 0.891 + 0.53$$

$$= 1.42.$$

$$M_{uac} = 0.16 \times 30 \times 30^3$$

$$= 75 \text{ kN-m.}$$

$$A_g = 3000$$

$$A_c = 59500$$

$$A_g = 62500.$$

$$P_{0t} = 1737000$$

$$P_{0n} = 0.345.$$

$$0.34$$

$$1 + \frac{(0.8 - 1)}{(0.8 - 0.2)} (0.34 - 0.2)$$

$$= 1.23.$$

$$= 0.75 + 0.46.$$

$$1.21.$$

$$5.4$$

$$\frac{P}{P_{ct}} = 0.018$$

$$0.17$$

$$79.6887500$$

$$3375$$

$$59125$$

$$P_{02} = 1848656.25$$

$$P_0 = 0.324$$

$$1 + (0$$

$$\cancel{1.015} 1.206$$

$$\left(\frac{60}{79.688} \right)^{1.206} + \left(\frac{40}{79.688} \right)^{1.206}$$

$$\cancel{0.741} 0.711 + 0.435$$

$$= \cancel{1.039} 1.14$$

$$6\% = 0.02$$

$$0.19 \times 89062500$$

$$= 3750$$

$$58750$$

$$P_2 = 1960312.5$$

$$= 0.306.$$

$$1.16.$$

$$= \left(\frac{60}{89.0625} \right)^{1.16} + \left(\frac{40}{89.0625} \right)^{1.16}.$$

$$= 0.622 + 0.32$$

$$= 0.9.$$

Spiral column:-

Design a short circular column 500 dia

factored load = 800 kN

factored moment = 162.5 kN-m

Provide Helical Hoop reinforcement. Take M20, Fe415 steel.

$$P_u = 800 \text{ kN}$$

$$M_u = 162.5 \text{ kN-m.}$$

$$f_{ck} = 20 \text{ N/mm}^2.$$

$$f_y = 415 \text{ N/mm}^2.$$

$$\left(\frac{P_u}{f_{ck} D^2} \right)_{\text{Hoop}} = 0.16.$$

$$\left(\frac{P_u}{f_{ck} D^2} \right)_{\text{Helical}} = 0.152.$$

$$\left(\frac{M_u}{f_{ck} D^3} \right)_{Hoop} = 0.065$$

$$\left(\frac{M_u}{f_{ck} D^3} \right)_{Helical} = 0.061$$

$$\frac{P}{f_{ck}} = 0.04 \Rightarrow P = 0.04 \times 20 = 0.8\%$$

$$\therefore \text{Area of steel} = \frac{P}{100} \times \frac{\pi}{4} \times D^2 = 1570 \text{ mm}^2$$

Assume 20 Φ .

Provide 6 x 20 TMT.

$$P = \frac{6 \times \frac{\pi}{4} \times 20^2}{\frac{\pi}{4} \times 500^2} \times 100$$

$$= 0.96\%$$

Let us use 8mm dia mild steel bar for helical reinforcement.

$$\therefore \text{Core dia; } D_c = 500 - 2 \times 40 + 2 \times 8 = 438 \text{ mm}$$

$$\therefore \text{Core area} = \frac{\pi}{4} \times D_c^2$$

$$V_c = A_c = 149301 \text{ mm}^2$$

$$\frac{A_g}{A_c} = \frac{\frac{\pi}{4} \times 500^2}{149301} = 1.3151$$



$$500 - 2 \times 40 + 2 \times 8$$

$$0.36 \left(\frac{A_g}{A_c} - 1 \right) \frac{f_{ck}}{f_{yh}}$$

$$0.36 \left(1.315 - 1 \right) \frac{20}{250}$$

$$= 0.00907$$

Dia. of core upto center of helics.



$$500 - 2 \times 40 + 8 = 428 \text{ mm.}$$

Let the pitch of the spiral be s mm
volume of spiral for 1mm length of column.

$$V_H = \frac{\pi d}{s} \times \frac{\pi}{4} \times s^2$$

$$= \frac{\pi \times 428}{s} \times \frac{\pi}{4} \times s^2$$

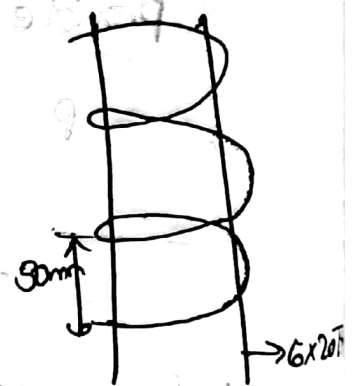
$$\frac{V_H}{V_C}$$

$$= \frac{67587}{s}$$

$$\frac{V_H}{V_C} = \frac{67587}{s}$$

$$\frac{V_H}{V_C} = \frac{67587}{s}$$

$$= \frac{0.452}{s}$$



$$\frac{V_H}{V_C} = 0.36 \left(\frac{A_g}{A_c} - 1 \right) \frac{f_{ck}}{f_{yh}} = 0.00907$$

$$\frac{0.452}{s} = 0.00907 \Rightarrow s = 50 \text{ mm}$$

Pitch should not more than

$$i) 75 \text{ mm.}$$

$$ii) \frac{1}{6} D_c = \frac{1}{6} \times 436$$

$$= 72.67$$

Pitch should not be less than $= 3 \times \phi_s$
 $= 3 \times 8 = 24 \text{ mm}$

Hence pitch 50mm is 'ok'

Long columns:-

$$M_{\max} = \frac{P_0 D}{2000} \left(\frac{le_x}{D} \right)^2$$

we have

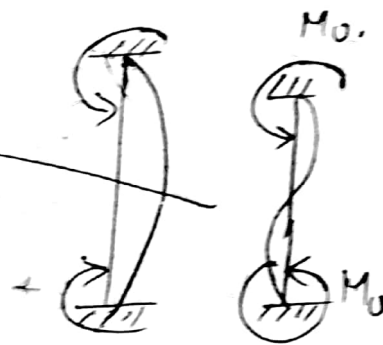
$$M_{\max} = P_0 e_{\max}$$

$$\therefore e_{\max} = \frac{D}{2000} \left(\frac{le_x}{D} \right)^2$$

$$M_{\max} = \frac{P_0 b}{2000} \left(\frac{le_y}{b} \right)^2$$

$$M_{\max} = 0.6 M_2 + 0.4 M_1$$

$$P_0 = \left(k_1 + k_2 + \frac{P}{f_{ck}} \right) f_{ck} b D$$



Design a slender braced circular column under uniaxial bending with the following data.

Size of column - 400mm.

M20 concrete

Fe 415 steel.

Effective length 6m, unsupported length 7m.

Factored load $P_0 = 1200 \text{ kN}$.

A moment $M_{\max} = 75 \text{ kN-m}$ at top & 50 kN-m at bottom.
 Column is bent in single curve.

$$\frac{l_{\text{eff}}}{D} = \frac{6000}{400} = 15 > 12.$$

Hence it is long column.

Additional moments

$$\begin{aligned} e_{\text{ax}} &= \frac{D}{2000} \left(\frac{l_{\text{eff}}}{D} \right)^2 \\ &= \frac{400}{2000} (15)^2 \\ &= 45 \text{ mm.} \end{aligned}$$

$$\begin{aligned} M_{\text{ax}} &= P_u \times e_{\text{ax}} \\ &= 1800 \times \frac{45}{1000} \\ &= 81 \text{ kN-m.} \end{aligned}$$

Calc. of k_a

$$k_a = \frac{P_{u2} - P_u}{P_{u2} - P_b}$$

$$P_b = \left(k_1 + k_2 \frac{P}{P_{ck}} \right) f_{ck} D^2$$

$$P_u = 2.5 \%$$

$$\frac{d'}{D} = \frac{50}{400} = 0.125$$

$$\text{Table 60; } k_1 = 0.155$$

$$k_2 = 0.266$$

$$= \left(0.155 + 0.266 \times \frac{2.5}{20} \right) 20 \times 400^2$$

$$= 1747800 = 601 \text{ kN.}$$

$$k = \frac{P_{u2} - P_0}{P_{u2} - P_0} \leq 1$$

$$A_g = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 400^2 = 125663.7 \text{ mm}^2$$

$$A_s = \frac{P}{100} \times \frac{\pi}{4} \times D^2 = \frac{2.5}{100} \times \frac{\pi}{4} \times 400^2 = 3141.59 \text{ mm}^2$$

$$A_c = 122522.1 \text{ mm}^2$$

$$P_{u2} = 0.45 f_{ck} A_c + 0.75 f_y A_s$$

$$= 0.45 \times 20 \times 122522.1 + 0.75 \times 415 \times 3141.59$$

$$= 2080518.854 \text{ N-m}$$

$$k = \frac{2080518.854 - 1200 \times 10^3}{2080518.854 - 60 \times 10^3} = 0.596 < 1$$

Assume 20R.

$$= \frac{3141.59}{314} = 10.005 \text{ bar}$$

provide 10x20R.

∴ Reduced Additional moment.

$$M'_a = k M_a$$

$$= 0.59 \times 54$$

$$= 31.86 \text{ kN-m}$$

Slabs:- Flexure, shear.

$$M = \frac{wle^2}{8} = \frac{Wle}{8}$$

$$V = \frac{wle}{2} = \frac{W}{2}$$

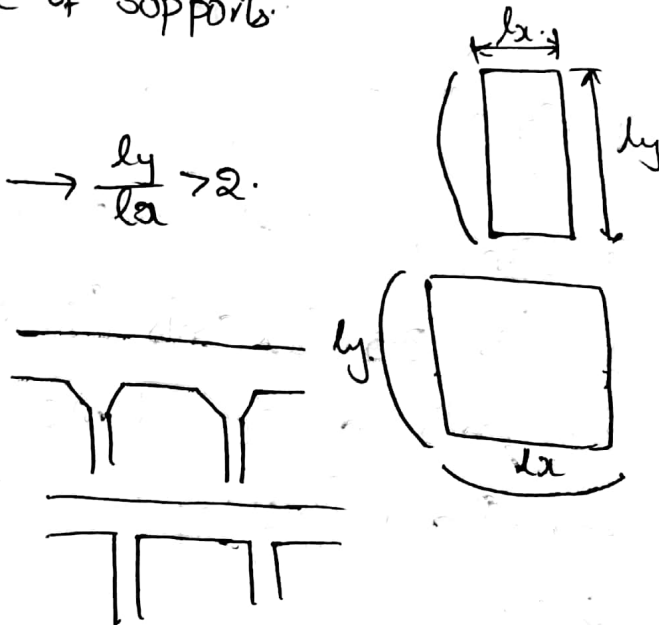
$$W = wle$$

Left is the least of

- i) clearspan + d.
- ii) C/c of supports.

Slab types:-

- 1) One-way slab $\rightarrow \frac{l_y}{l_x} > 2$.
- 2) Two-way slab.
- 3) Flat slab
- 4) Flat plate.



IS 875 (Part 182).

Finishes and Partitions 1.5 kN/m^2 .

For roofs:-

1.5 kN/m^2 with access.

0.75 kN/m^2 with out access

for floors:-

2 kN/m^2 for residential buildings.

3 kN/m^2 for office building.

Effective span

Leff small of

i) c/c of supports.

ii) $C.s + d$.

minimum $\%$ for slabs $\sim 0.12\%$

Minimum $\%$ of steel:-

0.12% for HYSD

0.15% for mild steel.

Design of One-way slab:-

Design a simply supported R.C.C. slab for a hall
a hall $3.5\text{ m} \times 8\text{ m}$ (inside dimensions) with 25
mm walls all around. Assume a live load of
 4 kN/m^2 and finishing load 1 kN/m^2 . Use M20 & Fe415
steel.

Step 1 Given data:-

Hall = $3.5\text{ m} \times 8\text{ m}$

width of masonry = 250 mm

live load = 4 kN/m^2

Finishing = 1 kN/m^2

$f_{ck} = 20\text{ N/mm}^2$

$f_y = 415\text{ N/mm}^2$

Step 1 Cal. of factored loads.

$$\text{Assume } \frac{l}{d} = \frac{3500}{25} = 140 \text{ mm.}$$

$$d = \frac{l}{25}$$

$$\text{Overall depth } D = d + c.c. + \frac{\phi}{2}$$
$$= 140 + 15 + \frac{10}{2}$$

$$D = 160 \text{ mm.}$$

Dead load;

$$\text{i) Slab} = 25 \times 0.16 = 4 \text{ kN/m}^2.$$

$$\text{ii) Finish} = \frac{1 \text{ kN/m}^2}{\text{---}}$$

$$\text{Total dead load} = 5 \text{ kN/m}^2.$$

$$\text{Live load} = 4 \text{ kN/m}^2.$$

$$\text{Total load } W = D.L + L.L = 5 + 4 = 9 \text{ kN/m}^2.$$

$$\text{Factored load } w_u = 1.5 \times W$$

$$= 1.5 \times 9 = 13.5 \text{ kN/m}^2.$$

Step 2 Calc. of effective span.

l_{eff} is the least of

$$\text{i) c/c of supports.}$$

$$= 3.5 + 0.25$$

$$= 3.75 \text{ m.}$$

$$\text{ii) } c.s + d = 3.5 + 0.14$$

$$= 3.64 \text{ m.}$$

Step 3

Factored BM & SF.

$$\begin{aligned}\text{Factored BM } (M_0) &= \frac{w l_{\text{eff}}^2}{8} \\ &= \frac{13.5 \times 3.6^2}{8} \\ &= 22.35 \text{ kNm}\end{aligned}$$

$$\begin{aligned}\text{Factored SF } (V_0) &= + \frac{w l_{\text{eff}}}{2} \\ &= 24.5 \text{ kN}.\end{aligned}$$

Step 4 Check for depth.

$$M_0 = 0.138 f_{ck} b d^2$$

$$\begin{aligned}d &= \sqrt{\frac{M_0}{0.138 f_{ck} b}} \\ &= \sqrt{\frac{22.35 \times 10^6}{0.138 \times 20 \times 1000}} \\ &= 90 \text{ mm} < 140 \text{ mm}.\end{aligned}$$

Step 5 Calc. of steel area.

$$\frac{x}{d} = 1.2 - \sqrt{1.44 - \frac{6.6 M_0}{f_{ck} b d^2}}$$

$$\begin{aligned}\frac{6.6 M_0}{f_{ck} b d^2} &= \frac{6.6 \times 22.35 \times 10^6}{20 \times 1000 \times 140^2} \\ &= 0.376\end{aligned}$$

$$\frac{x}{d} = 0.168$$

$$\frac{x}{d} = 0.168 < \left(\frac{x_{\text{max}}}{d} = 0.48 \right)$$

Hence it is under reinforced section.

$$\text{lever arm } z = d(1 - 0.42 \frac{d}{d})$$

$$= 140(1 - 0.42 \times 0.168)$$

$$= 130.19 \text{ mm.}$$

$$\text{Area of steel } A_{st} = \frac{M_u}{0.87 f_y z}$$

$$= 475.7 \text{ mm}^2.$$

$$\text{Assume 10TMT } A\phi = \frac{\pi}{4} \times 10^2 = 78.53 \text{ mm}^2.$$

$$\text{No of bars } n = \frac{A_{st}}{A\phi}$$

$$= \frac{475.7}{78.53}$$

$$= 6.05.$$

$$\text{Spacing } s = \frac{1000}{n}$$

$$= \frac{1000}{6.05}$$

$$= 165.28 \text{ mm.}$$

$$\text{Max. spacing (S}_{max}) \leq d$$

$$= 3d = 3 \times 140 = 420 \text{ mm.}$$

$$\text{ii) } 300 \text{ mm}$$

$$S_{max} = 300 \text{ mm}$$

Provide 10TMT @ 160 mm c/c.

Step-1 Check for deflection

For simply supported case basic value $\frac{l}{d} = 20$.
% of tensile steel at midspan.

$$A_{st \text{ provided}} = \frac{1000}{s} \cdot \frac{\pi}{4} d^2$$

$$= \frac{1000}{180} \times \frac{\pi}{4} \times 10^2$$

$$= 490.8$$

$$= 490.8 \text{ mm}^2$$

$$A_{st \text{ req}} = 475 \text{ mm}^2$$

$$f_s = 0.58 f_y \cdot \frac{A_{st \text{ req}}}{A_{st \text{ provided}}}$$

$$= 0.58 \times 415 \times \frac{475}{490.8}$$

$$= 232.9 \text{ mm}^2$$

$$\text{Mod: \% of steel} = \frac{5490.8}{1000 \times 100} \times 100 = 0.35$$

$$\frac{l}{d} = 20 \times 1$$

$$\frac{l}{d} = \frac{3500}{140} = 25$$

Hence deflection limit exceeds. So change the spacing of the main steel. ie, Assume $s = 150 \text{ mm}$ instead of 160 mm .

$$f_s = A_{st \text{ provided}} = \frac{1000}{150} \times \frac{\pi}{4} \times 10^2$$

$$= 523.5 \text{ mm}^2$$

$$0.58 \times 415 \times \frac{475}{523.5} = 1.55$$

$$\frac{l}{d} = 20 \times 1.55$$

$$= 31.$$

$$\left(\frac{l}{d}\right)_{\text{provided}} = \frac{3500}{140} = 25$$

hence ok.

Step-8 Check for shear (optional).

$$V_u = 24.5 \text{ kN.}$$

$$f_v = \frac{V_u}{bd} = \frac{24.5 \times 10^3}{1000 \times 140} = 0.175 \text{ N/mm}^2$$

Calc. of f_c .

$$50\% \quad P_f = \frac{0.37}{2} = 0.185\% \quad (\text{at supports})$$

$$\text{from Table 19 } f_c = 0.285 \text{ N/mm}^2$$

$f_c > f_v$ Hence ok.

Step-9 Summary of design.

Depth of slab $D = 160 \text{ mm.}$

Cover = 15 mm.

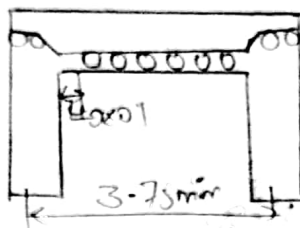
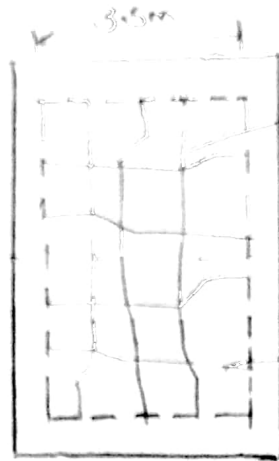
Concrete = M20.

Steel Fe415.

Primary 10TMT @ 150 mm c/c

8 TMT @ 260 mm c/c

Step-10 Detailing:-



Moments developed in Two way slabs (simply supports case).

$$\Delta_{max}^{shorter} = \frac{5}{384} \frac{q_x l_x^4}{EI}$$

$$\Delta_{max}^{longer} = \frac{5}{384} \frac{q_y l_y^4}{EI}$$

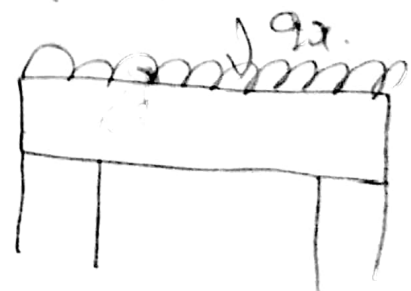
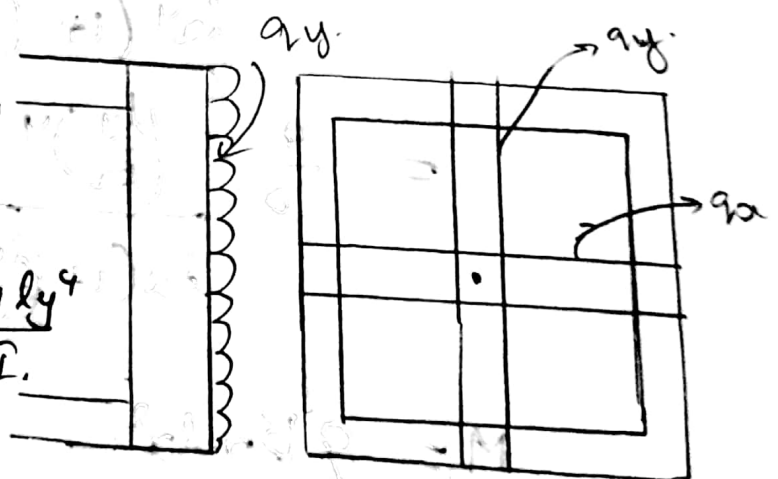
$$\frac{5}{384} \frac{q_x l_x^4}{EI} = \frac{5}{384} \frac{q_y l_y^4}{EI}$$

$$q_x = q_y \left(\frac{l_y}{l_x} \right)^4$$

$$q_y = q_x \left(\frac{l_x}{l_y} \right)^4$$

$$q = q_x + q_y$$

$$q = q_x + q_x \left(\frac{l_x}{l_y} \right)^4 + q_x$$



$$q_a = \frac{q}{\left(\frac{I_a}{I_y}\right)^4 + 1}$$

$$q_a = \frac{q \times I_y^4}{I_a^4 + I_y^4}$$

$$q_y = \frac{q I_a^4}{I_a^4 + I_y^4}$$

$$M_a = \frac{q_a I_a^2}{8}$$

$$= \frac{q I_y^4}{8(I_a^4 + I_y^4)} \times I_a^2$$

$$= \frac{q}{8} \frac{I_y^4}{I_a^4 \left(1 + \left(\frac{I_y}{I_a}\right)^4\right)} I_a^2$$

$$= \frac{q}{8} \frac{\left(\frac{I_y}{I_a}\right)^4}{1 + \left(\frac{I_y}{I_a}\right)^4}$$

$$M_a = q B_a \cdot I_a^2$$

$$B_a = \frac{\left(\frac{I_y}{I_a}\right)^4}{8 \left(1 + \left(\frac{I_y}{I_a}\right)^4\right)}$$

$$M_y = \frac{q_y l_y^2}{8}$$

$$= \frac{q l_x^4}{8 l_x^4 + l_y^4} l_y^2$$

$$= q \cdot \frac{l_x^4}{8 (l_x^4 (1 + (\frac{l_y}{l_x})^4))} l_y^2$$

$$= \frac{q}{8 (1 + (\frac{l_y}{l_x})^4)} l_y^2$$

$$= \frac{q}{8 (1 + (\frac{l_y}{l_x})^4)} \cdot \frac{l_y^2}{l_x^2} \times l_x^2$$

$$M_y = q B_y l_x^2$$

$$B_y = \frac{(\frac{l_y}{l_x})^2}{8 (1 + (\frac{l_y}{l_x})^4)}$$

$$d = \frac{\text{span}}{25 \text{ to } 30}$$

Design of simply supported two-way slab:

Design a reinforced concrete slab 5.5 m x 4 m.

Simply supported on all the four sides it has to carry a characteristic live load of 8 kN/m^2 M25, Fe415 exposure condition mild.

Given data.

Room Dimensions : 4 m x 5.5 m

Concrete $f_{ck} = 25 \text{ N/mm}^2$

Steel $f_y = 415 \text{ N/mm}^2$

Table 16 mild exposure = 20 mm

Calc. of Depth of the slab.

$$l_y = 5.5 \text{ m}$$

$$l_x = 4 \text{ m}$$

$$\frac{l_y}{l_x} = \frac{5.5}{4} = 1.375 < 2.$$

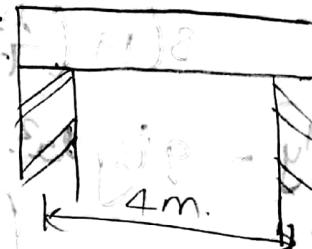
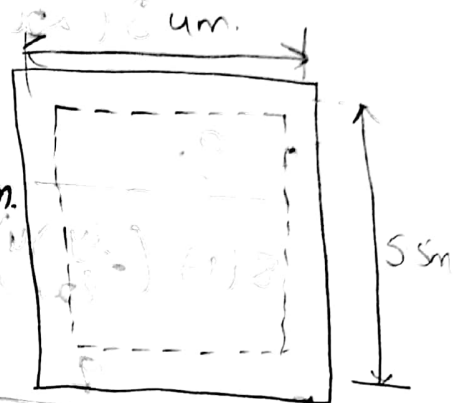
Hence it is two way slab.

$$l_x = 4000 \text{ mm}$$

$$\text{Total depth } D = \frac{l_x}{24} = \frac{4000}{24}$$

$$= 166.67$$

$$= 166 \text{ mm}$$



Take $D = 170 \text{ mm}$.

$$\begin{aligned}\text{Effective depth } d &= D - CC - \frac{\phi}{2} \\ &= 170 - 20 - \frac{10}{2} \quad (10 \text{ MT bars}) \\ &= 145 \text{ mm}.\end{aligned}$$

Step-2 Design load.

i, Dead load =

$$1) \text{ Self weight of slab} = 0.17 \times 25 = 4.25 \text{ kN/m}^2$$

$$2) \text{ Finishing load (25mm thick)} = 0.5 \text{ kN/m}^2$$

$$\frac{25}{1000} \times 25$$

$$3) \text{ Plastering load (6mm)} = \frac{6}{1000} \times u = 0.144 \text{ kN/m}^2$$

$$\text{Total dead load} = 4.894 \text{ kN/m}^2$$

$$\text{ii) Live load} = 8 \text{ kN/m}^2.$$

$$\text{Total load} = DL + LL$$

$$= 4.89 + 8$$

$$w = 12.89 \text{ kN/m}^2$$

$$\begin{aligned}\text{Design load} &= 1.5w = 1.5 \times 12.89 \\ &= 19.341 \text{ kN/m}^2\end{aligned}$$

Step-3 Design moment and S.F.

$$w_u = 19.341 \text{ kN/m}^2$$

$$\text{Moment} = l_x = 4000$$

$$l_y = 5500 \text{ mm}.$$

$$\frac{l_y}{l_x} = 1.375$$

Table 21;

$$\frac{l_y}{l_x} = 1.3 \quad \alpha_x = 0.093.$$

$$\frac{l_y}{l_x} = 1.4 \quad \alpha_x = 0.099$$

$$0.093 + \frac{(0.099 - 0.093)}{(1.4 - 1.375)} (1.375 - 1.3)$$

$$= 0.0975$$

$$\frac{l_y}{l_x} = 1.3 \quad 0.055$$

$$\frac{l_y}{l_x} = 1.4 \quad 0.051.$$

$$\alpha_y = 0.052.$$

$$M_x = \alpha_x w l_x^2.$$

$$= 0.0975 \times 19.341 \times u^2$$

$$= 30.17 \text{ kN-m.}$$

$$M_y = \alpha_y w l_x^2 = 0.052 \times 19.341 \times u^2$$

$$= 16.09 \text{ kN-m.}$$

Step 4 Check for depth.

$$M_{max} = 30.17 \text{ kN-m.}$$

$$\text{For Fe415 } M_0 = 0.138 f_{ck} b d^2$$

$$d = \sqrt{\frac{M_0}{0.138 f_{ck} b}} = \sqrt{\frac{30.17 \times 10^6}{0.138 \times 25 \times 100}}$$

$$= 93 \text{ mm.}$$

hence ok.

step-5 Primary steel.

$$M_u = 30.17 \text{ kN-m.}$$

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{b d f_{ck}}\right)$$

$$\frac{x}{d} = 1.2 - \sqrt{1.44 - \frac{6.6 M_u}{f_{ck} b d^2}}$$

$$= 1.2 - \sqrt{1.44 - \frac{6.6 M_u}{f_{ck} b d^2}}$$

$$\frac{6.6 M_u}{f_{ck} b d^2} = 0.378.$$

$$\frac{x}{d} = 0.169 < 0.48$$

It is a under reinforced.

$$\text{lever arm} = 145 (1 - 0.416 \times 0.169)$$

$$= 134.8 \text{ mm.}$$

Area of steel.

$$\frac{M_u}{0.87 f_y} = \frac{30.17 \times 10^6}{0.87 \times 415 \times 134.8}$$

$$= 620 \text{ mm}^2.$$

10T11 bar

$$A_\phi = \frac{\pi}{4} \times 10^2 = 78.5 \text{ mm}^2$$

$$n = \frac{A_{st}}{A_\phi} = \frac{620}{78.5} = 7.89$$

$$\text{Spacing } s = \frac{1000}{n} = \frac{1000}{7.89} = 126 \text{ mm.}$$

max. spacing is the least of

i) $3d = 3 \times 145 = 435 \text{ mm.}$

ii) 300 mm.

$S_{\text{max}} = 300 \text{ mm.}$

10 TMT @ 120 mm c/c.

$A_{\text{st provided}} = \frac{1000}{S} A \phi$

$= \frac{1000}{120} \times \frac{\pi}{4} \times 10^2 = 654 \text{ mm}^2.$

% of steel = $\frac{A_{\text{st}}}{b t} \times 100$

$= \frac{654}{100 \times 145} \times 100 = 0.45 \%$

Step 6 Secondary steel.

$M_{\text{max}} = 1.2 - \sqrt{1.44 - \frac{6.6196}{f_c k b d^2}}$

$\frac{a}{d} = 0.08 < 0.08.$

Hence. Ok.

clear arm $z = d \left(1 - 0.416 \times \frac{a}{d} \right)$
 $= 318.3 \text{ mm.}$

Assume 107 MPa.

$A \phi = 788$

$n = \frac{A_{\text{st}}}{A \phi} = 1.05$

$$S = \frac{1000}{n} = 246 \text{ mm}$$

$$(d = 135 \text{ mm})$$

$$\text{Smaad } 4) 3d = 3 \times 145 \\ = 435 \text{ mm.}$$

$$\text{i) } 450 \text{ mm.}$$

Provide 10 TMT @ 220 mm c/c.

$$\text{Ast provide} = \frac{1000}{S} A\phi \\ = \frac{1000}{220} \times 78.5 \\ = 356.9 \text{ mm}^2.$$

$$\% \text{ of steel} = \frac{356.9}{1000 \times 145} \times 100 = 0.24\% \\ \left(\frac{356.9}{1000 \times 135} \times 100 \right) = 0.26$$

Check for deflection.

For simply supported case $\frac{l}{d} = 20$.

$$M_{25} f_s = 0.58 \times 0.58 \times 145 \times \frac{620}{654} \\ = 228.18 \text{ N/mm}^2$$

Modification factor = 1.5.

$$\left(\frac{l}{d} \right)_{\text{max}} = 20 \times 1.5 = 30$$

$$\left(\frac{l}{d} \right)_{\text{provide}} = \frac{1000}{145} = 27.6 < 30$$

Hence OK.

Summary :-

Depth of slab = 170 mm.
C.C = 20 mm.

Primary 10 TMT @ 120 mm c/c.

Secondary 10 TMT @ 220 mm c/c.