

SYLLABUS:**UNIT I**

Three Hinged Arches: Elastic theory of arches – Eddy's theorem – Determination of horizontal thrust, bending moment, normal thrust and radial shear – effect of temperature.

Two Hinged Arches: Determination of horizontal thrust, bending moment, normal thrust and radial shear – Rib shortening and temperature stresses, tied arches – fixed arches – (No analytical question).

UNIT-II

Lateral Load Analysis Using Approximate Methods: application to building frames. (i) Portal method (ii) Cantilever method.

UNIT – III

Cable Structures And Suspension Bridges: Introduction, characteristics of cable, analysis of cables subjected to concentrated and uniformly distributed loads, anchor cable, temperature stresses, analysis of simple suspension bridge, three hinged and two hinged stiffening girder suspension bridges.

UNIT – IV

Moment Distribution Method: Stiffness and carry over factors – Distribution factors – Analysis of continuous beams with and without sinking of supports – Portal frames – including Sway-Substitute frame analysis by two cycle.

UNIT – V

Kani's Method: Analysis of continuous beams – including settlement of supports and single bay portal frames with and without side sway.

UNI – VI

Introduction to Matrix Methods:

Flexibility methods: Introduction, application to continuous beams (maximum of two unknowns) including support settlements.

Stiffness method: Introduction, application to continuous beams (maximum of two unknowns) including support settlements

THREE HINGED ARCHES

Generally Arches are also known as "Curved Beams."

There are 3 types of Arches. They are

- (1) Parabolic Arch
- (2) Circular Arch
- (3) Segmental Arch

Depending up on the end conditions the Arches are classified as follows:

- (1) Fixed Arches
- (2) 2 (Two) Hinged Arches
- (3) Three hinged Arches

$$\text{Radial shear} = V \cos \theta - H \sin \theta$$

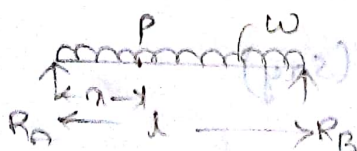
$$\text{Normal thrust} = V \sin \theta + H \cos \theta$$

$$y = \frac{4b}{l^2} x(l-x)$$

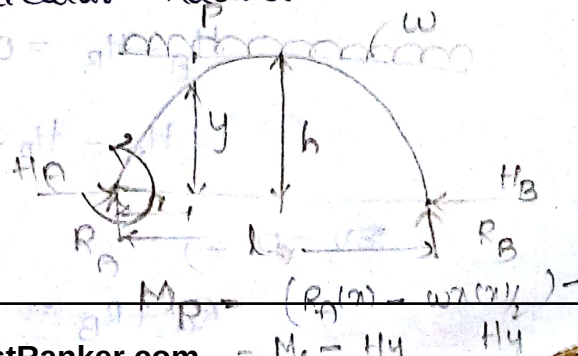
$$\text{ellipse} \div \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{Semi circular} \div x^2 + y^2 = r^2$$

Axis is perpendicular Radial shear



$$M_s = R_A(x) - wx \cdot x/2$$



$$M_p = (R_A(x) - wx \cdot x/2) - H_A y$$

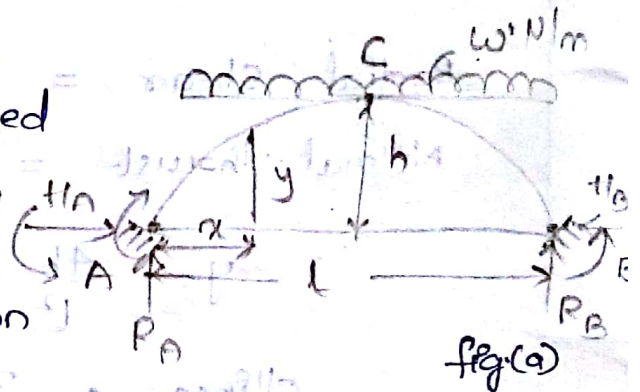
An arch is a curved beam in which horizontal movement at the support is wholly or partially prevented. Hence there will be a horizontal thrust induced at the supports.

An arch hinged at both supports and having third hinge (may) anywhere in the rib is called a "three hinged arch."

In three hinged arches, at each support there will be two reaction components, therefore, there will be 4 unknowns.

* Three hinged Parabolic Arches subjected to udl (uniformly Distributed Load) : throughout the span :

Consider a 3-hinged parabolic arch of span 'L' central rise 'h' as shown in figure (a)



Reactions:

$$\sum H = 0 \Rightarrow$$

$$H_A - H_B = 0$$

$$H_A = H_B = H \quad (\text{say})$$

$$\sum V = 0 \Rightarrow$$

$$R_A + R_B = wL$$

$$-R_B(l) + w(l)(l/2) = 0$$

$$R_B(l) = wl/2$$

$$\therefore R_B = wl/2$$

$$R_A = wl - wl/2$$

$$= \frac{2wl - wl}{2}$$

$$\therefore R_A = wl/2$$

$$\Sigma M_c = 0 \Rightarrow \text{(left of section)}$$

$$R_A(l/2) - H(h) - w(l/2) \times (l/4) = 0$$

$$\frac{wl}{2}(l/2) - H(h) - \frac{wl^2}{8} = 0$$

$$\frac{wl^2}{4} - \frac{wl^2}{8} = H(h)$$

$$H(h) = \frac{wl^2}{8}$$

$$\therefore H = \frac{wl^2}{8h}$$

$$H_A = H_B = H = \frac{wl^2}{8h}$$

Bending moment at any point 'p':

$$M_p = R_A(x) - H(y) - w(x)(x/2)$$

$$= \frac{wl}{2}(x) - \frac{wl^2}{8h}(y) - \frac{wx^2}{2}$$

$$= \frac{wl}{2}(x) - \frac{wl^2}{8h} \left(\frac{4h}{x^2} x(1-x) \right) - \frac{wx^2}{2}$$

$$= \frac{wlx}{2} - \frac{wx}{2}(1-x) - \frac{wx^2}{2}$$

$$= \frac{wlx}{2} - \frac{wx}{2} + \frac{wx^2}{2} - \frac{wx^2}{2} = 0$$

Conclusion:- Hence, the bending moment at any point

is zero, in the 3-hinged parabolic Arches.

Radial shear at any point (P):

$$S_p = V \cos \theta - H \sin \theta$$

Here, V = vertical force upto point 'P'

$$= R_A - wx$$

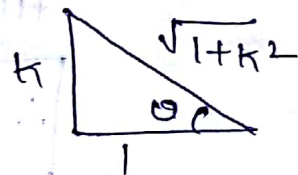
$$V = \frac{wl}{2} - wx$$

$$H = \frac{wl^2}{8h}$$

we know $y = \frac{4h}{l^2} (lx - x^2)$

$$\frac{dy}{dx} = \frac{4h}{l^2} (1 - 2x) = k$$

$$\sin \theta = \frac{k}{\sqrt{1+k^2}}$$



$$\cos \theta = \frac{1}{\sqrt{1+k^2}}$$

$$S_p = \frac{wl}{2} - wx \left(\frac{1}{\sqrt{1+k^2}} \right) - \frac{wl^2}{8h} \left(\frac{k}{\sqrt{1+k^2}} \right)$$

$$= \frac{1}{\sqrt{1+k^2}} \left(\frac{wl}{2} - wx - \frac{wl^2}{8h} \left(\frac{4h}{l^2} (1-2x) \right) \right)$$

$$= \frac{1}{\sqrt{1+k^2}} \left(\frac{wl}{2} - wx - \frac{w}{2} (1-2x) \right)$$

$$= \frac{1}{\sqrt{1+k^2}} \left(\frac{wl}{2} - wx - \frac{wl}{2} + wx \right)$$

$$\therefore S_p = 0$$

shear at point is equal to 'zero.'

Normal Thrust at any point (p):-

$$\begin{aligned} N_p &= V \sin \theta + H \cos \theta \\ &= \left(\frac{wl}{2} - wx \right) \left(\frac{k}{\sqrt{1+k^2}} \right) + \frac{wl^2}{8h} \left(\frac{1}{\sqrt{1+k^2}} \right) \\ &= \frac{1}{\sqrt{1+k^2}} \left(\left(\frac{wl}{2} - wx \right) \frac{4h}{l} (1-2x) + \frac{wl^2}{8h} \right) \\ \therefore N_p &= \frac{1}{\sqrt{1+k^2}} \left(\left(\frac{wl}{2} - wx \right) k + \frac{wl^2}{8h} \right) \end{aligned}$$

(*) Analyse the three hinged parabolic arch as shown in figure and find the radial shear and normal thrust, and bending moment at quarter span from left support.

Sol. — Reactions:-

$$\sum H = 0$$

$$H_A = H_B = H$$

$$\sum V = 0$$

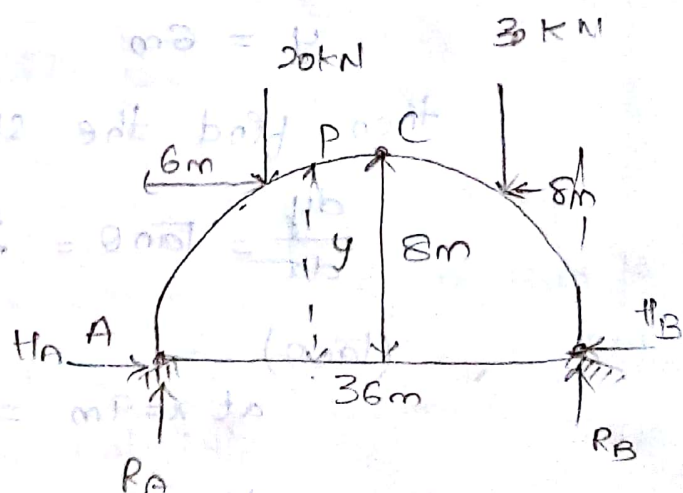
$$R_A + R_B = 20 + 30$$

$$= 50 \text{ kN}$$

$$\sum M_A = 0$$

$$-R_B(36) + 20 \times 6 + 30 \times 28 = 0$$

$$R_B = 26.67 \text{ kN}$$



$$R_A = 50.26 \text{ kN}$$

$$\therefore R_A = 23.33 \text{ kN}$$

$$\sum M_C = 0 \quad (\text{left of the span})$$

$$R_A(18) - H_A(8) - 20 \times 12 = 0$$

$$23.33 \times 18 - 20 \times 12 = 8H$$

$$H = 22.49 \text{ kN}$$

Bending moment, radial shear and normal thrust at $x = 9 \text{ m}$ (quarter span).

For parabolic arch

$$y = \frac{4h}{l^2} x(1-x)$$

Here, $x = 9 \text{ m}$

$h = \text{central rise}$

$l = \text{total span}$

$$y = \frac{4 \times 8}{36^2} \times 9(36-9)$$

$$y = 6 \text{ m}$$

then find the slope (θ)

$$\frac{dy}{dx} = \tan \theta = \frac{4h}{l^2} (1-2x)$$

($\tan \theta$)

$$\text{at } x = 9 \text{ m} = \frac{4 \times 8}{36^2} (36 - (2 \times 9))$$

$$\tan \theta = 0.45$$

$$\theta = \tan^{-1}(0.45)$$

$$\theta = 24.22^\circ$$

$$\cos \theta = 0.91$$

Bending moment at $x=9\text{ m}$ (quarter span)

$$\begin{aligned} \text{BM}_{x=9\text{ m}} &= R_A(9) - 20 \times 3 - H_A \times y_{x=9\text{ m}} \\ &= 23.33(9) - 20 \times 3 - 22.49 \times 6 \\ &= 15.03 \text{ kNm} \end{aligned}$$

Radial shear:-

$$S_p = V \cos \theta - H \sin \theta$$

V = vertical forces at left side

$$V = R_A - 20$$

$$V = 23.33 - 20$$

$$V = 3.33$$

$$\therefore S_p = 3.33 \times 0.91 - 22.49 \times 0.409$$

$$S_p = -6.168 \text{ kN}$$

Normal thrust:-

$$N_p = V \sin \theta + H \cos \theta$$

$$= 3.33 \times 0.409 + 22.49 \times 0.91$$

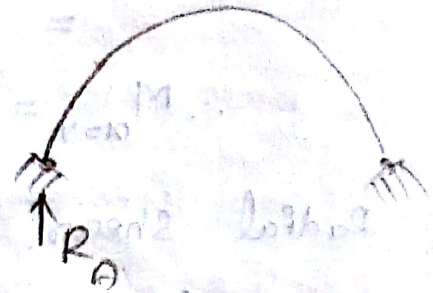
$$\therefore N_p = 21.82 \text{ kN}$$

Number of unknowns = 4

(R_A, H_A, R_B, H_B)

Number of Equations = 3

$$(\sum X=0, \sum H=0, \sum M=0)$$



$$= 4 - 3$$

$$= 1$$

$\therefore$ degree of static indeterminacy = 1

However, the fourth equation can be developed from the deflection criteria. i.e., the partial derivative of strain energy with respect to H_A (or) H_B will be given deflection in the direction of H_A (or) H_B .

So, if the supports don't yield $\frac{\partial U}{\partial H_A} = 0$

$$\frac{\partial U}{\partial H_B} = 0.$$

If the supports yield by ' Δ ', then,

$$\frac{\partial U}{\partial H_A} = -\Delta.$$

* If the supports don't yield :-

$$\begin{aligned} \text{we know } U &= \int \frac{M^2 ds}{2EI} \quad (M = M_s - H_y) \\ &= \int \frac{(M_s - H_y)^2 ds}{2EI} \end{aligned}$$

$$\frac{\partial U}{\partial H} = \frac{\partial U}{\partial H} = \frac{\partial U}{\partial H} = 0$$

$$\frac{\partial U}{\partial H} = \frac{\partial}{\partial H} \left(\int \frac{(M_s - Hy)^2}{2EI} ds \right) = 0$$

$$\frac{\partial}{\partial H} \left(\int \frac{(M_s - Hy)^2}{2EI} ds \right) = 0$$

$$\int \frac{2(M_s - Hy)(0 - y)}{2EI} ds = 0$$

$$- \int \frac{M_s y ds}{EI} + \int \frac{Hy^2}{EI} ds = 0$$

$$H \int \frac{y^2}{EI} ds = \int \frac{M_s y ds}{EI}$$

$$H = \frac{\int \frac{M_s y ds}{EI}}{\int \frac{y^2}{EI} ds}$$

$$= \frac{\frac{1}{EI} \int M_s y ds}{\frac{1}{EI} \int y^2 ds}$$

$$\therefore H = \frac{\int M_s y ds}{\int y^2 ds}$$

(ds = change of arch length)

* If the supports yield by ' Δ ' :-

we k/r

$$U = \int \frac{M^2 ds}{2EI}$$

$$U = \int \frac{(M_s - Hy)^2 ds}{2EI}$$

1 differentiate partially with respect to 'H'

$$(\because H_A = H_B = H)$$

$$\frac{\partial U}{\partial H} = \frac{\partial}{\partial H} \left(\int \frac{(M_s - Hy) ds}{2EI} \right) = -\Delta$$

$$\frac{\partial}{\partial H} \left(\int \frac{(M_s - Hy) ds}{2EI} \right) = -\Delta$$

$$\int \frac{2(M_s - Hy)(0 - y) ds}{2EI} = -\Delta$$

$$-\int \frac{M_s y ds}{EI} + \int \frac{Hy^2}{EI} ds = -\Delta$$

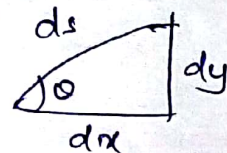
$$H \int \frac{y^2}{EI} ds = \int \frac{M_s y ds}{EI} - \Delta$$

$$\therefore H = \frac{\int \frac{M_s y ds}{EI} - \Delta}{\int \frac{y^2}{EI} ds}$$

$$H = \frac{\int M_s y ds}{\int y^2 ds}$$

$$= \frac{\int M y dx \sec \theta}{\int y^2 dx \sec \theta}$$

$$H = \frac{\int M y dx}{\int y^2 dx}$$



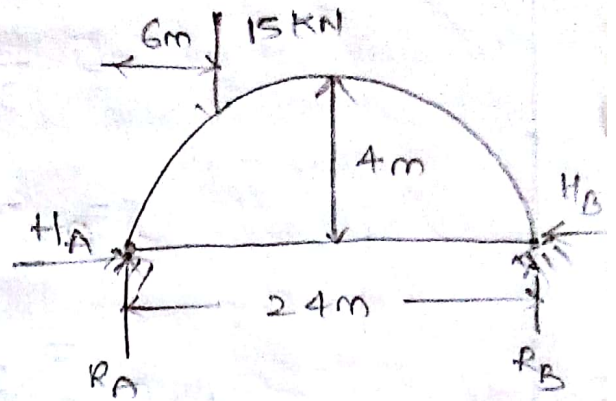
$$\cos \theta = \frac{dx}{ds}$$

$$ds = \frac{dx}{\cos \theta}$$

$$ds = dx \sec \theta$$

Find the horizontal thrust of two hinged parabolic arch as shown in figure. where I_c is moment of inertia at crown.

Sol: Calculation of Reactions :-



$$\sum H = 0 \Rightarrow H_A = H_B = H$$

$$\sum V = 0 \Rightarrow R_A + R_B = 15$$

$$\sum M_A = 0 \Rightarrow$$

$$-R_B(24) + (15 \times 6) = 0$$

$$R_B(24) = 15 \times 6$$

$$R_B = \frac{15 \times 6}{24} = 3.75 \text{ kN}$$

$$\therefore R_A = 15 - 3.75 = 11.25 \text{ kN}$$

we know horizontal thrust

$$H = \frac{\int M_s y dx}{\int y^2 dx}$$

For parabolic arch,

$$y = \frac{4h}{l^2} x(1-x)$$

$$= \frac{4 \times 4}{24^2} x(24-x)$$

$$= \frac{4 \times 4}{24 \times 24} x(24-x)$$

$$y = \frac{x}{36} (24-x)$$

$$\begin{aligned}
 \int_0^{24} y^2 dx &= \int_0^{24} \left(\frac{x}{36} (24-x) \right)^2 dx \\
 &= \frac{1}{36^2} \int_0^{24} x^2 (24-x)^2 dx \\
 &= \frac{1}{36^2} \int_0^{24} x^2 (24^2 + x^2 - 48x) dx \\
 &= \frac{1}{36^2} \left[\int_0^{24} 576x^2 dx + \int_0^{24} x^4 dx - \int_0^{24} 48x^3 dx \right] \\
 &= \frac{1}{36^2} \left(576 \left(\frac{x^3}{3} \right)_0^{24} + \left(\frac{x^5}{5} \right)_0^{24} - 48 \left(\frac{x^4}{4} \right)_0^{24} \right) \\
 &= \frac{1}{1296} \left(576 \left(\frac{24^3}{3} \right) + \left(\frac{24^5}{5} \right) - 48 \left(\frac{24^4}{4} \right) \right) \\
 &= \frac{1}{1296} (2654208 + 1592524.8 - 3981312) \\
 &= \frac{1}{1296} (822804.8) \\
 &= 634.80 \quad 204.80
 \end{aligned}$$

$$\therefore \int y^2 dx = 204.80$$

$$\therefore H = \frac{\int M_s y dx}{\int y^2 dx}$$

$$\int M_s y dx = \int_0^6 (R_A x) \frac{x}{36} (24-x) dx + \int_6^{24} (R_B(x)) \frac{x}{36} (24-x) dx$$

$$\int M_s y dx = \int_0^6 11.25(x) \left(\frac{x}{36} (24-x) \right) dx + \int_6^{18} 3.75(x) \left(\frac{x}{36} (24-x) \right) dx$$

$$= \frac{11.25}{36} \int_0^6 x^2 (24-x) dx + \frac{3.75}{36} \int_6^{18} x^2 (24-x) dx$$

$$= \frac{11.25}{36} \int_0^6 (24x^2 - x^3) dx + \frac{3.75}{36} \int_6^{18} (24x^2 - x^3) dx$$

$$= \frac{11.25}{36} \left(\frac{24(6^3)}{3} - \frac{6^4}{4} \right) + \frac{3.75}{36} \left(\frac{24(18^3)}{3} - \frac{18^4}{4} \right)$$

$$= \frac{11.25}{36} (1728 - 324) + \frac{3.75}{36} (46656 - 26244)$$

$$= 438.75 + 2126.25$$

$$= 2565 \text{ kN.m}$$

$$\therefore H = \frac{\int M_s y dx}{\int y^2 dx}$$

$$= \frac{2565}{204.80}$$

$$\therefore H = 12.52 \text{ kN}$$

* Effect of Rib shortening

In the analysis of 3-hinged arch the effect of rib shortening due to horizontal thrust was not considered. But effect of normal thrust in the arch is too shortened the rib of the arch, and thus release the path of the horizontal thrust.

Here,

$$N = V \sin \theta + H \cos \theta$$

$$\text{Bending moment } (M) = M_s - Hy$$

Strain energy due to normal thrust

$$(N) = U_1 = \int \frac{N^2 ds}{2AE}$$

strain energy due to

$$\text{Bending moment } (U_2) = \int \frac{M^2 ds}{2EI}$$

$$U = U_1 + U_2$$

$$\therefore \text{Total energy } (U) = \int \frac{N^2 ds}{2AE} + \int \frac{M^2 ds}{2EI}$$

* If the supports ~~don't~~ yield: $\frac{\partial U}{\partial H} = -\delta$

$$\frac{\partial U}{\partial H} = -\delta$$

$$\frac{\partial}{\partial H} \left(\int \frac{N^2 ds}{2AE} + \int \frac{M^2 ds}{2EI} \right) = -\delta$$

$$\Rightarrow \frac{\partial}{\partial H} \left(\int \frac{(V \sin \theta + H \cos \theta)^2 ds}{2AE} + \int \frac{(M_s - Hy)^2 ds}{2EI} \right) = -\delta$$

$$\Rightarrow \int \frac{2(V \sin \theta + H \cos \theta) \cos \theta ds}{2AE} + \int \frac{2(M_s - Hy)(-y) ds}{2EI} = -\delta$$

$$\Rightarrow \int \frac{V \sin \theta \cos \theta ds}{AE} + \int \frac{H \cos^2 \theta ds}{AE} - \int \frac{M_s y ds}{EI}$$

$$+ \int \frac{Hy^2}{EI} ds = -\delta$$

$$\Rightarrow H \left(\int \frac{\cos^2 \theta ds}{AE} + \int \frac{y^2}{EI} ds \right) = \int \frac{M_s y ds}{EI} - \int \frac{V \sin \theta \cos \theta ds}{AE} - \delta$$

$$\therefore H = \frac{\int \frac{M_s y ds}{EI} - \int \frac{V \sin \theta \cos \theta ds}{AE} - \delta}{\int \frac{y^2}{EI} ds + \int \frac{\cos^2 \theta ds}{AE}}$$

$$I = I_0 \sec \theta$$

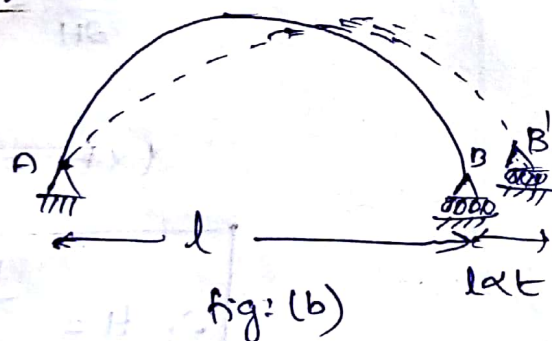
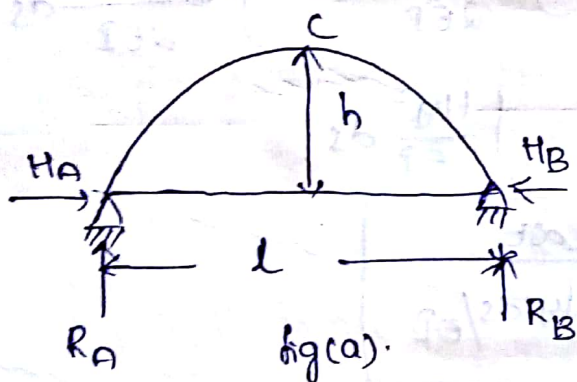
$$H = \frac{\int \frac{M_s y dx}{EI_0} - \delta}{\int \frac{y^2 dx}{EI_0} + \int \frac{\cos^2 \theta dx \sec \theta}{AE}}$$

$$\therefore H = \frac{\int \frac{M_s y dx}{EI_0} - \delta}{\int \frac{y^2 dx}{EI_0} + \frac{l}{AE}}$$

$$\cos \theta = 1$$

$$\int \frac{dx}{AE}$$

* Effect of Temperature Change:-



Consider a 2 hinged arch of span 'l' as shown in figure (a).

Let the temperature of the arch be raised by $t^\circ C$. If the hinged 'B' is replaced by a roller as shown in figure (b)

Let the end 'B' moves to position B'

So, $BB' = \Delta t$

Let 'H' be the force required to bring back B' into B. It means that 'H' is the horizontal thrust in the arch.

$$\therefore \frac{\partial U}{\partial H} = BB' = L \alpha t$$

But, $U = \int \frac{M^2 ds}{2EI}$

$$U = \int \frac{(M_s - Hy)^2 ds}{2EI}$$

Here, $M_s = 0$

$$U = \int \frac{(0 - Hy)^2 ds}{2EI}$$

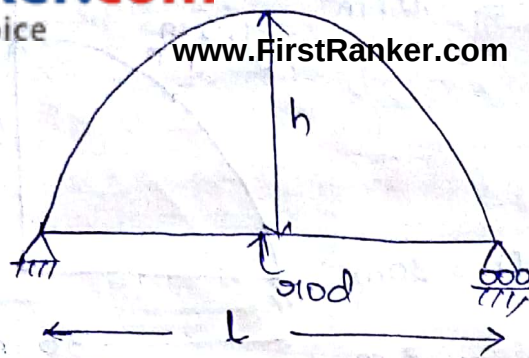
$$\frac{\partial U}{\partial H} = \int \frac{\cancel{Hy}^2}{2EI} ds \quad \int \frac{2Hy \cdot y}{2EI} ds$$

$$L \alpha t = \int \frac{Hy^2}{EI} ds$$

$$\therefore H = \frac{L \alpha t}{\int y^2 ds / EI}$$

Tied Arches:-

In a two hinged Arch some times, one end is hinged and other end is roller with two supports connected by a rod as shown in figure.



$A_t = \text{area of tie rod}$

$E_t = \text{Young's modulus of tie rod}$

A ~~tie~~ arch allows a structure to behave as a rigid unit.

From the formula of rib shortening and supports yield.

$$H = \frac{\int \frac{M_s y dx}{EI_0} - \delta}{\int \frac{y^2 dx}{EI_0} + \frac{L}{AE}}$$

$$= \frac{\int \frac{M_s y dx}{EI_0} - \frac{H L}{A_t E_t}}{\int \frac{y^2 dx}{EI_0} + \frac{L}{AE}}$$

$$\therefore H = \frac{\int \frac{M_s y dx}{EI_0}}{\int \frac{y^2 dx}{EI_0} + \frac{L}{AE} + \frac{L}{A_t E_t}}$$

- (1) A two hinged parabolic arch of span 30 m central rise 3 m. The horizontal is taken by tie rod. Calculate the bending moment at quarter span due to a crown load of 40 kN. allow for the extension for the tie rod & rib shortening. Area of rib at crown = 240 cm². E for arch 15 kN/mm²; E for tie rod = 200 kN/mm²; I = 18000 cm⁴

Sol:- Given data:-

span of arch (l) = 30m

central rise = 3m

crow load = 10 kN (or) $40 \times 10^3 \text{ N}$

$$\begin{aligned} \text{Area of rib at crown} &= 240 \text{ cm}^2 \\ &= 240 \times (100)^2 \\ &= 0.024 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} 'E' \text{ for arch} &= 15 \text{ kN/mm}^2 \\ &= 15 \times 10^6 \text{ kN/m}^2 \end{aligned}$$

$$\begin{aligned} 'E_t' \text{ for tie rod} &= 200 \text{ kN/mm}^2 \\ &= 200 \times 10^6 \text{ kN/m}^2 \end{aligned}$$

$$I = 18000 \text{ sec}^2 \text{ cm}^4$$

$$I = I_0 \text{ sec}^2$$

$$\begin{aligned} I &= I_0 = 18000 \text{ cm}^4 \\ &= 18000 / (100)^4 \end{aligned}$$

$$I_0 = 1.8 \times 10^{-4} \text{ m}^4$$

$$\text{Area of tie rod } (A_t) = 10 \text{ cm}^2$$

$$= 10 / (100)^2$$

$$\therefore A_t = 1 \times 10^{-3} \text{ m}^2$$

$$H = \frac{\int \frac{M_s y dx}{EI_0} - \delta}{\int \frac{y^2 dx}{EI_0} + \frac{l}{AE} + \frac{l}{A_t E_t}} + \frac{l \alpha t}{\int \frac{y^2 dx}{EI_0}}$$

$\Delta l =$

$$\int \frac{y^2 dx}{EI_0} + \frac{l}{AE} + \frac{l}{A_t E_t}$$

Reactions:-

$$R_A = R_B = \frac{\text{Total load}}{2}$$

$$= 40/2 = 20$$

$$\therefore R_A = R_B = 20 \text{ kN}$$

Equation of parabola $y = \frac{4h}{l^2} x(1-x)$

$$= \frac{4 \times 20}{30 \times 30} x(30-x)$$

$$y = \frac{x}{75} (30-x)$$

$$\int \frac{y^2 dx}{EI_0} = \frac{1}{EI_0} \int_0^{30} \left(\frac{x}{75} (30-x) \right)^2 dx$$

$$= \frac{1}{75^2 EI_0} \int_0^{30} x^2 (900 + x^2 - 60x) dx$$

$$= \frac{1}{5625 EI_0} \left[\int_0^{30} 900x^2 dx + \int_0^{30} x^4 dx - \int_0^{30} 60x^3 dx \right]$$

$$= \frac{1}{5625 EI_0} \left[\left(\frac{900x^3}{3} \right)_0^{30} + \left(\frac{x^5}{5} \right)_0^{30} - \left(\frac{60x^4}{4} \right)_0^{30} \right]$$

$$= \frac{1}{5625 EI_0} \left(\frac{900(30^3)}{3} + \frac{30^5}{5} - \frac{60 \times 30^4}{4} \right)$$

$$\frac{144}{EI_0}$$

$$= \frac{1}{5625 EI_0} (8100,000 + 4860000 - 12150000)$$

$$= \frac{144}{EI_0}$$

$$= \frac{144}{15 \times 10^6 \times 1.8 \times 10^4}$$

$$= \frac{144}{2700}$$

$$\therefore \int \frac{y^2 dx}{EI_0} = 0.0533$$

$$\left(\frac{l}{AE} \right) = \frac{30}{0.024 \times 15 \times 10^6}$$

$$= \frac{30}{360000}$$

$$= 8.33 \times 10^{-5}$$

$$\frac{l}{AE EI} = \frac{30}{1 \times 10^{-3} \times 200 \times 10^6}$$

$$= \frac{30}{200,000}$$

$$= 1.5 \times 10^{-4}$$

$$\int \frac{M_s y dx}{EI_0} = \frac{1}{EI_0} \int M_s y dx$$

$$= \frac{1}{EI_0} \left[\int_0^{15} R_A x \frac{1}{75} x (30-x) dx + \int_0^{15} R_B x \frac{1}{75} x (30-x) dx \right]$$

$$= \frac{2}{EI_0} \int_0^{15} R_A x \frac{1}{x} x (30-x) dx$$

$$= \frac{2 \times 20}{75 EI_0} \int_0^{15} x^2 (30-x) dx$$

$$= \frac{40}{75 EI_0} \left(\int_0^{15} 30x^2 dx - \int_0^{15} x^3 dx \right)$$

$$= \frac{40}{75 EI_0} \left(\frac{30 \times 15^3}{3} - \frac{x \cdot 15^4}{4} \right)$$

$$= \frac{40}{75 EI_0} (33750 - 12656.25)$$

$$= \frac{843750}{75 \times 2700}$$

$$\therefore \int \frac{M_s y dx}{EI_0} = 4.16$$

$$\int \frac{M_s y dx}{EI_0}$$

$$\therefore H =$$

$$\int \frac{y^2 dx}{EI_0} + \frac{L}{AE} + \frac{L}{A_t E_t}$$

$$= \frac{4.16}{0.0533 + (8.33 \times 10^5) + (1.5 \times 10^4)}$$

$$\boxed{\therefore H = 77.70 \text{ kN}}$$

Bending moment at 'P' :-

$$M_p = R_A \times 7.5 - H y_{x=7.5}$$

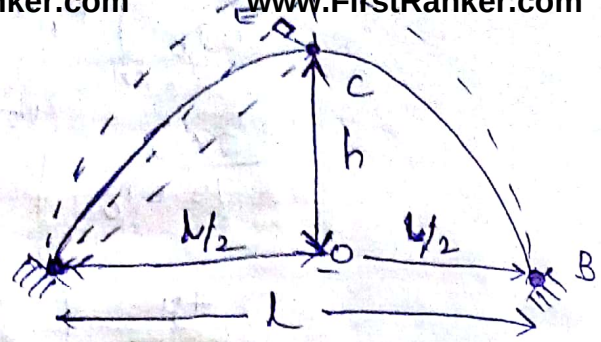
$$= 20 \times 7.5 - 77.70 \left(\frac{7.5}{75} (30 - 7.5) \right)$$

$$= (20 \times 7.5) - 174.825$$

$$= -24.825 \text{ kNm} \approx 25 \text{ kNm}$$

Temperature Effect on Three-Hinged Arch

Figure shows a 3 hinged parabolic arch of span 'l' and central rise 'h'.



Let the arch be subjected to a rise of temperature $t^{\circ}\text{C}$, the rise of temperature increases the length of arch. Since the ends A & B do not move, and hinge 'C' are not connected to any permanent object. So, the crown hinge rise from c to D. So, AD represents new position of AC.

Change in length of the arch = $l\alpha t$

Here,

α = Coefficient of linear expansion.

l = original length.

$$\text{Arc AD} - \text{Arc AC} = l\alpha t$$

$$= \text{Arc AC} \alpha t$$

$$\text{Arc AD} = \text{Arc AC} \alpha t + \text{Arc AC}$$

$$= \text{Arc AC} (1 + \alpha t)$$

Let us make the assumption,

Arc length = chord length.

Increase in length of AD :

$$AD = AC (1 + \alpha t)$$

let CE be perpendicular to 'AD'. CE is very small AC & AE is equal

$$\begin{aligned}\therefore ED &= AD - AE \\ &= AD - AC \\ &= AC(1 + \alpha t) - AC \\ &= AC + AC\alpha t - AC\end{aligned}$$

$$\boxed{\therefore ED = AC\alpha t}$$

$$\cos \angle EDC = \frac{ED}{CD}$$

$$CD = \frac{ED}{\cos \angle EDC}$$

$$= ED \sec \angle EDC$$

Assuming $\angle EDC = \angle ACO$

$$\therefore CD = ED \sec \angle ACO$$

$$\sec \angle ACO = \frac{AC}{OC}$$

$$CD = ED \sec \angle ACO$$

$$= AC\alpha t \times \frac{AC}{OC}$$

$$= \frac{AC^2}{OC} \alpha t$$

$$= \frac{\frac{l^2}{4} + b^2}{h} \alpha t$$

$$\therefore \delta = \frac{l^2 + 4b^2}{4h} \alpha t$$

2) A 3 hinged parabolic arch of span 30 m; raise ^{6m} is subjected to a raise of temperature 40°C . determine change in raise. Take $\alpha = 12 \times 10^{-6}$

Sol:- Given data:-

$$\text{Span of arch } (l) = 30 \text{ m}$$

$$\text{raise of arch } (h) = 6 \text{ m}$$

$$\text{temperature } (\Delta t) = 40^{\circ}\text{C}$$

$$\text{Coefficient of linear expansion } (\alpha) = 12 \times 10^{-6}$$

$$\therefore \delta = \frac{l^2 + 4h^2}{4h} \alpha t$$

$$= \frac{(30)^2 + (4 \times 6^2)}{4 \times 6} \times 12 \times 10^{-6} \times 40$$

$$\therefore \delta = 0.0208 \text{ m}$$

Lateral Load Analysis

These are 2 methods for Lateral load Analysis. They are:

- (1) Portal Method
- (2) Cantilever Method

Portal Method: (Assumptions)

- 1) Point of contraflexure occurs at the middle of the all the members i.e., Beams & Columns.
- 2) Horizontal shear taken by each interior is doubled than that of ^{is} taken by exterior Column

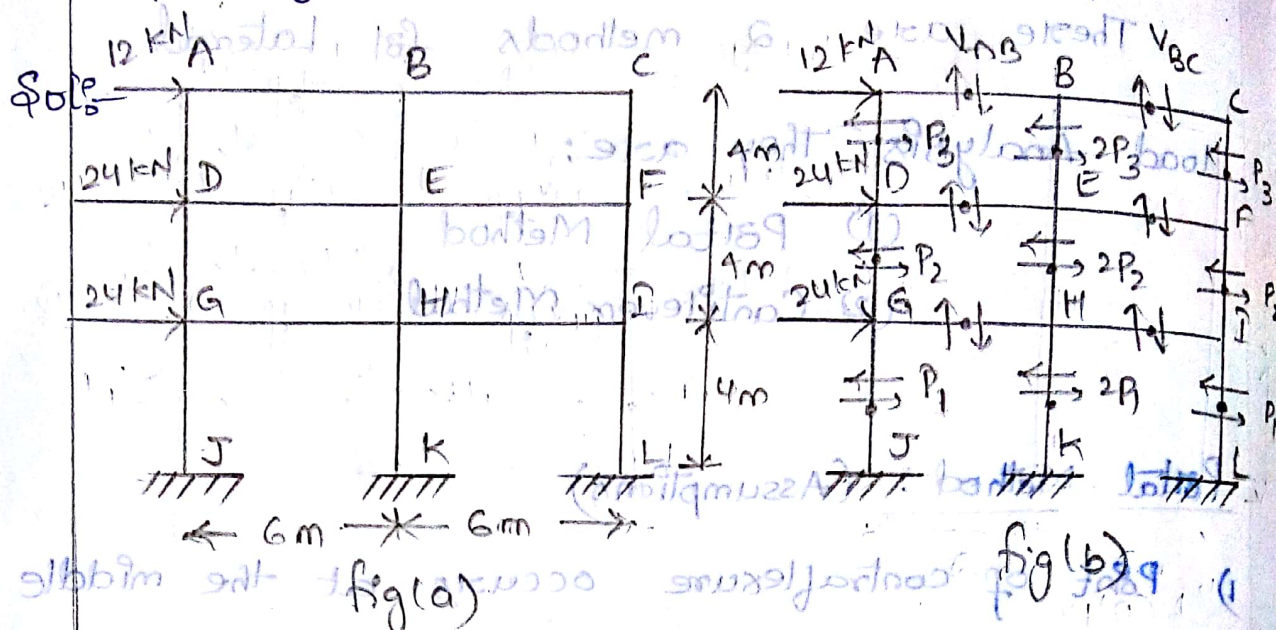
$$\text{Column moment} = \text{Column shear} \times \frac{\text{height of Column}}{2}$$

$$\text{Beam moment} = \frac{\text{Beam shear} \times \text{Span}}{2}$$

Sum of moments in the Column
= Sum of moments in Beam at any joint

$$\Rightarrow \sum M_c = \sum M_b$$

The figure shown below. The wind loads transfer to joints A, D & G are 12 kN, 24 kN and 24 kN respectively. Analyse the Frame by Portal Method



Let the horizontal shear taken by exterior columns P_3 , P_2 and P_1 in top, middle and bottom stories respectively.

According to Assumption made in this method the horizontal shear in interior columns be $2P_3$, $2P_2$ and $2P_1$ in top, middle and bottom stories respectively. (as shown in figure-B)

Step 1:- Calculation of shear in columns.

Top storey:

$$P_3 + 2P_3 + P_3 = 12 \text{ kN}$$

$$4P_3 = 12$$

$$P_3 = 12/4 = 3 \text{ kN}$$

$$P_3 = 3 \text{ kN}$$

$$P_2 + 2P_2 + P_2 = 24 + 12$$

$$4P_2 = 36$$

$$P_2 = 36/4$$

$$\boxed{\therefore P_2 = 9 \text{ kN}}$$

Bottom Storey:-

$$P_1 + 2P_1 + P_1 = 24 + 24 + 12$$

$$4P_1 = 60$$

$$P_1 = 60/4 = \boxed{\therefore P_1 = 15 \text{ kN}}$$

Step 2 :- calculation of Moments in columns.

$$\text{Column moment} = \frac{\text{Column shear} \times \text{height of column}}{2}$$

$$M_{AD} = P_3 \times h_{3/2}$$

$$= 3 \times 4/2 = 6 \text{ kNm}$$

$$M_{DG} = P_2 \times h_{2/2} = \frac{9 \times 4}{2} = 18 \text{ kNm}$$

$$M_{GJ} = P_1 \times h_{1/2} = \frac{15 \times 4}{2} = 30 \text{ kNm}$$

$$M_{BE} = 2P_3 \times h_{3/2} = \frac{2 \times 3 \times 4}{2} = 12 \text{ kNm}$$

$$M_{EH} = 2P_2 \times h_{2/2} = \frac{2 \times 9 \times 4}{2} = 36 \text{ kNm}$$

$$M_{HK} = 2P_1 \times h_{1/2} = \frac{2 \times 15 \times 4}{2} = 60 \text{ kNm}$$

$$M_{CF} = P_3 \times h_{3/2} = \frac{3 \times 4}{2} = 6 \text{ kNm}$$

$$M_{DL} = P_1 \times h/2 = 15 \times 4/2 = 30 \text{ kNm}$$

Step: 3 Calculation of shear in Beams:

At any joint sum of Beam moments are equal to sum of column moments.

Joint 'A' :-

$$M_{AB} = M_{AD}$$

$$M_{AB} = 6 \text{ kNm}$$

$$\text{Beam moment} = \frac{\text{Beam shear} \times \text{Span}}{2}$$

$$V_{AB} \times \frac{L_1}{2} = 6$$

$$V_{AB} \times \frac{6}{2} = 6$$

$$\therefore V_{AB} = \frac{6 \times 2}{6} = 2 \text{ kN}$$

Joint 'B' :-

$$M_{AB} + M_{BC} = M_{BE}$$

$$6 + M_{BC} = 12$$

$$M_{BC} = 12 - 6$$

$$V_{BC} \times \frac{L_2}{2} = 6$$

$$V_{BC} = \frac{6 \times 2}{6}$$

$$\therefore V_{BC} = 2 \text{ kN}$$

$$M_{AD} + M_{DG} = M_{DE}$$

$$6 + 18 = M_{DE}$$

$$M_{DE} = 24$$

$$V_{DE} \times \frac{1}{2} = 24$$

$$V_{DE} = \frac{24 \times 2}{0.3}$$

$$\therefore V_{DE} = 8 \text{ kN}$$

Joint 'E':-

$$M_{BE} + M_{EH} = M_{DE} + M_{EF}$$

$$M_{DE} + M_{EF} = 12 + 36$$

$$V_{DE} \times \frac{1}{2} + V_{EF} \times \frac{1}{2} = 48$$

$$V_{EF} \times \frac{6}{2} = 48 - \frac{8 \times 0.3}{2}$$

$$= 48 - 24$$

$$V_{EF} = \frac{24 \times 2}{0.3}$$

$$\therefore V_{EF} = 8 \text{ kN}$$

Joint 'G':-

$$M_{DG} + M_{GT} = M_{GH}$$

$$M_{GH} = 18 + 30$$

$$V_{GH} \times \frac{1}{2} = 48$$

$$V_{GH} = \frac{48 \times 2}{0.3}$$

$$\therefore V_{GH} = 16 \text{ kN}$$

Joint H

$$M_{HE} + M_{HK} = M_{GH} + M_{HI}$$

$$M_{GH} + M_{HI} = 36 + 60$$

$$48 + M_{HI} = 96$$

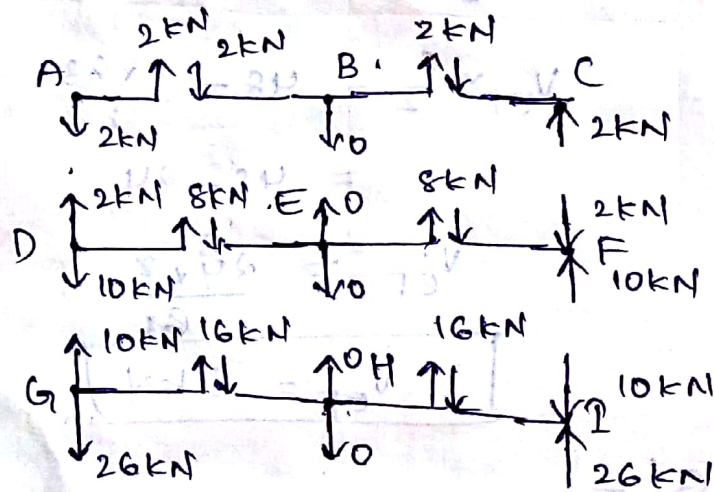
$$M_{HI} = 96 - 48$$

$$V_{HI} \times \frac{12}{2} = 48$$

$$V_{HI} = \frac{48 \times 2}{12}$$

$$V_{HI} = 16 \text{ kN}$$

Step 4 :- Calculation of vertical loads in columns:



Force in AD = 2 kN (comp)

Force in BE = 0

Force in CF = 2 kN (tension)

Force in DG = 10 kN (comp)

Force in EH = 0

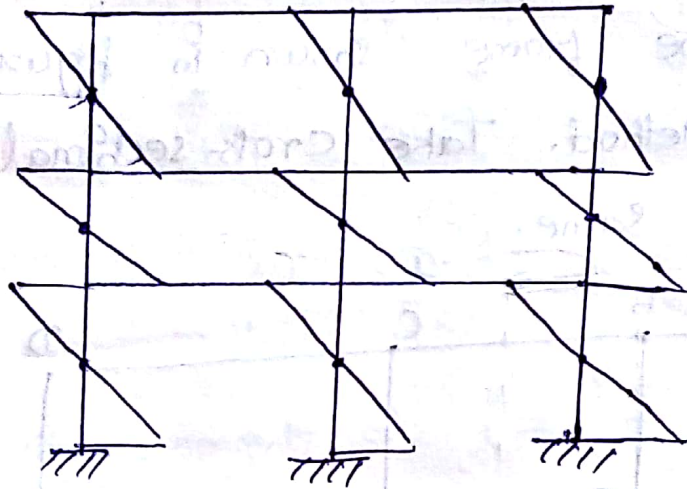
Force in FI = 10 kN (tension)

Force in HK = 0

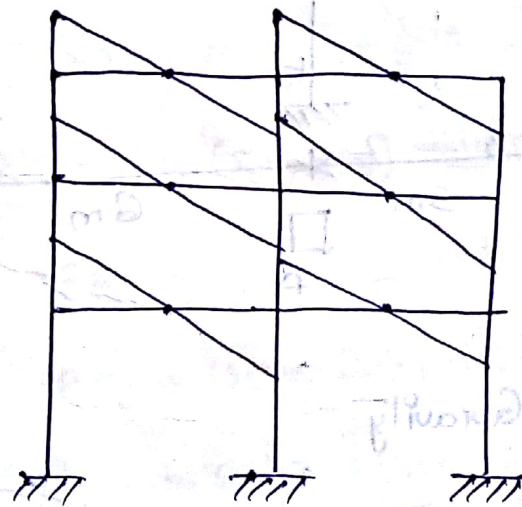
Force in IL = 26 kN (tension)

Step 5:- Bending moment diagrams.

(a) Bending moment diagrams for columns.

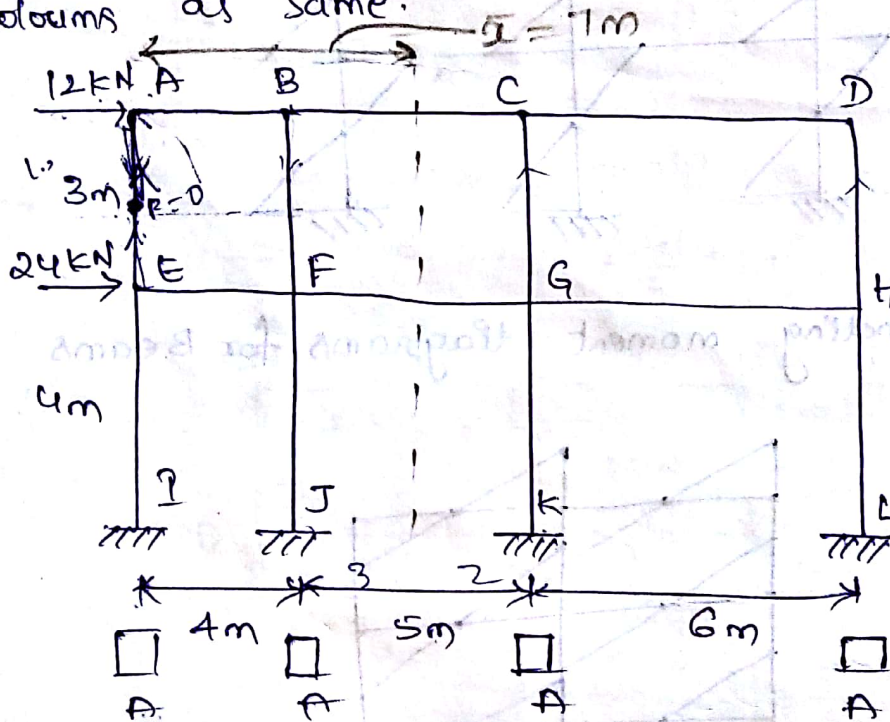


(b) Bending moment diagrams for Beams



- (1) Point of contraflexure occurs at the middle of all the members i.e., beams and columns.
- (2) The intensity of axial force in each column of storey is proportional to the horizontal distance of that column from C.G. of all columns of the storey.

(1) Analyse the frame shown in figure by Cantilever method. Take cross-sectional area of all columns as same.



Sol:-

Center of Gravity

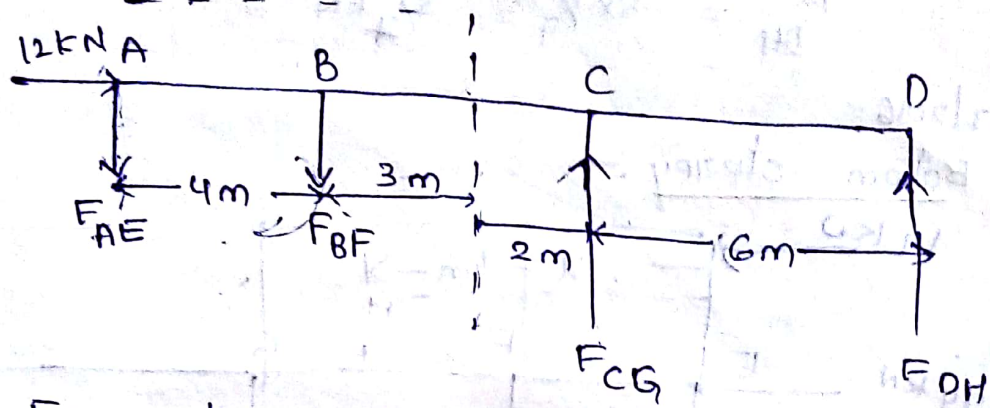
$$\bar{x} = \frac{\sum Ax}{\sum A}$$

$$= \frac{A \times 15 + A \times 9 + A \times 4 + A \times 0}{A + A + A + A}$$

$$= \frac{28A}{4A}$$

$$\therefore \bar{x} = 7m$$

Top storey:



$$F_{AE} = k_1 y_1 = k_1 \times 7$$

$$F_{BF} = k_1 y_2 = k_1 (8)$$

$$F_{CG} = k_1 y_3 = k_1 (2)$$

$$F_{DH} = k_1 y_4 = k_1 (8)$$

Bending moment at P = 0

$$M_P = 0$$

$$-12 \times 1.5 - F_{BF} \times 4 + F_{CG} \times 9 + F_{DH} \times 15 = 0$$

$$F_{CG} \times 9 + F_{DH} \times 15 = 12 \times 1.5 + F_{BF} \times 4$$

$$(2k_1 \times 9) + (8k_1 \times 15) = 18 + (3k_1) \times 4$$

$$18k_1 + 120k_1 = 18 + 12k_1$$

$$18k_1 + 120k_1 - 12k_1 = 18$$

$$126k_1 = 18$$

$$k_1 = \frac{18}{126} = \frac{1}{7}$$

$$k_1 = \frac{1}{7}$$

$$F_{AE} = \frac{1}{7} \times 7 = 1 \text{ kN}$$

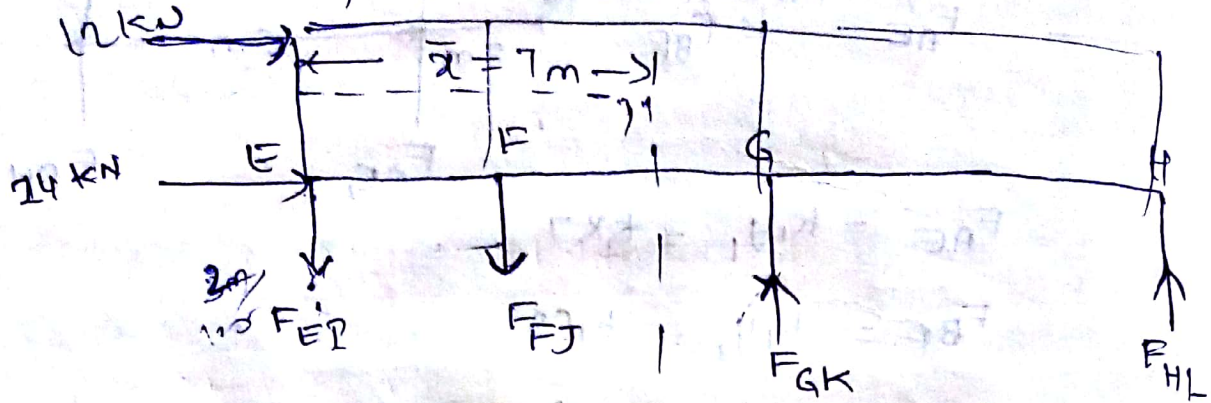
$$F_{BF} = 3 \times \frac{1}{7} = \frac{3}{7} \text{ kN}$$

$$F_{CG} = 2 \times \frac{1}{7} = \frac{2}{7} \text{ kN}$$

$$F_{DH} = 8 \times \frac{1}{7} = \frac{8}{7} \text{ kN}$$

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Bottom storey :-



$$F_{EI} = k_2 \times 7$$

$$F_{FJ} = k_2 \times 3$$

$$F_{GK} = k_2 \times 2$$

$$F_{HL} = k_2 \times 8$$

Bending moment at 'Q' = 0

$$M_Q = 0$$

$$12 \times 5 + 24 \times 2 + F_{FJ} \times 4 - F_{GK} \times 9 - F_{HL} \times 15 = 0$$

$$60 + 48 + (k_2 \times 3 \times 4) - (k_2 \times 2 \times 9) - (k_2 \times 8 \times 15) = 0$$

$$60 + 48 + 12k_2 - 18k_2 - 120k_2 = 0$$

$$120k_2 + 18k_2 - 12k_2 = 108$$

$$126k_2 = 108$$

$$k_2 = \frac{108}{126} = \frac{6}{7}$$

$$F_{EI} = k_2 \times 7 = \frac{6}{7} \times 7 = 6 \text{ kN}$$

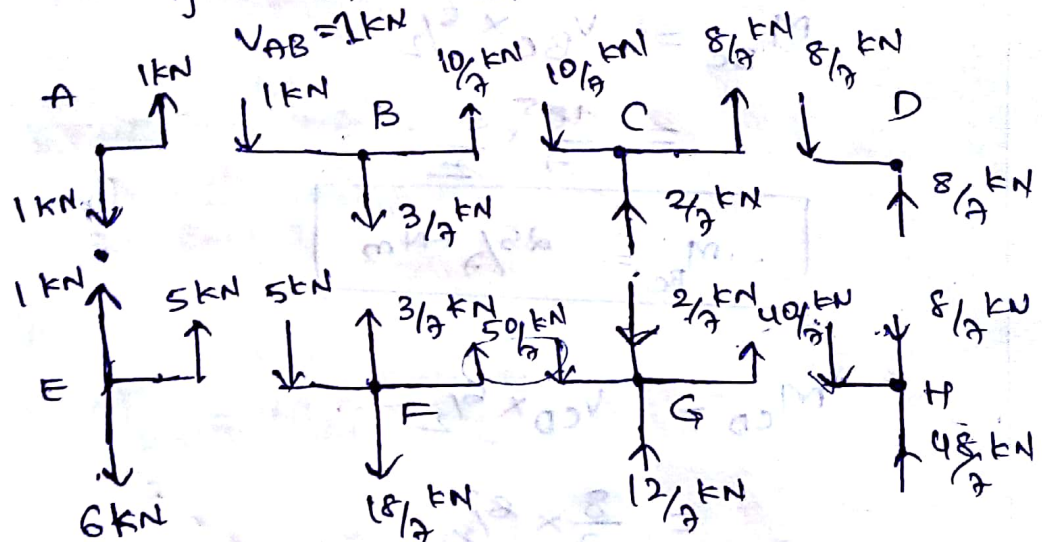
$$F_{FJ} = k_2 \times 3 = \frac{6}{7} \times 3 = \frac{18}{7} \text{ kN}$$

$$F_{GK} = k_2 \times 2 = \frac{6}{7} \times 2 = \frac{12}{7} \text{ kN}$$

$$F_{HL} = k_2 \times 8 = \frac{6}{7} \times 8 = \frac{48}{7} \text{ kN}$$

Step 2 :-

Calculation of beam shear:



Shear in $V_{AB} = 1 \text{ kN}$

$$V_{BC} = \frac{10}{7} \text{ kN}$$

$$V_{CD} = \frac{8}{7} \text{ kN}$$

$$V_{EF} = 5 \text{ kN}$$

$$V_{FG} = \frac{39}{7} \text{ kN}$$

$$V_{GH} = \frac{40}{7} \text{ kN}$$

Beam moment $\times$ Span

$$\text{Beam moment} = \frac{\quad}{2}$$

$$M_{AB} = V_{AB} \times 4/2$$

$$= 1 \times 4/2$$

$$= \frac{4}{2}$$

$$\therefore M_{AB} = 2 \text{ kNm}$$

$$M_{BC} = V_{BC} \times 5/2$$

$$= \frac{105}{7} \times \frac{5}{2}$$

$$\therefore M_{BC} = 25/2 \text{ kNm}$$

$$M_{CD} = V_{CD} \times 6/2$$

$$= \frac{8}{7} \times 6/2$$

$$M_{CD} = 24/7 \text{ kNm}$$

$$M_{EF} = V_{EF} \times 4/2$$

$$= 5 \times 4/2$$

$$= 20/2$$

$$\therefore M_{EF} = 10 \text{ kNm}$$

$$M_{FG} = V_{FG} \times 5/2$$

$$= \frac{50}{7} \times 5/2$$

$$M_{FG} = 125/7 \text{ kNm}$$

$$= \frac{40}{2} \times \frac{3}{2}$$

$$M_{GH} = 120 \text{ kNm}$$

Step 4: Calculation of moments in columns:-

At any joint moments in columns =
moments in Beams.

At joint 'A'

$$M_{AE} = M_{AB} = 2 \text{ kNm}$$

At joint 'B'

$$M_{BF} = M_{AB} + M_{BC}$$

$$= 2 + 25/2$$

$$M_{BF} = \frac{39}{2} \text{ kNm}$$

At joint 'C':-

$$M_{CG} = M_{BC} + M_{CD}$$

$$= \frac{25}{2} + \frac{24}{2}$$

$$= \frac{49}{2} = 7 \text{ kNm}$$

At joint 'D':-

$$M_{DH} = M_{CD} = \frac{24}{2} \text{ kNm}$$

$$M_{AE} + M_{ED} = M_{EF}$$

$$2 + M_{ED} = 10$$

$$M_{ED} = 10 - 2$$

$$M_{ED} = 8 \text{ kNm}$$

At joint 'F': -

$$M_{BF} + M_{FJ} = M_{EF} + M_{FG}$$

$$M_{FJ} = 10 + \frac{125}{7} - \frac{39}{7}$$

$$= \frac{70 + 125 - 39}{7}$$

$$M_{FJ} = \frac{156}{7} \text{ kNm}$$

At joint 'G': -

$$M_{CG} + M_{GK} = M_{FG} + M_{GH}$$

$$M_{GK} = M_{FG} + M_{GH} - M_{CG}$$

$$= \frac{125}{7} + \frac{120}{7} - 7$$

$$= \frac{125 + 120 \times 7 - 7 \times 7}{7}$$

$$= \frac{7 \times 120}{7}$$

$$= \frac{125 + 840 - 49}{7}$$

$$M_{GK} = \frac{916}{7} = 130.857 \text{ kNm}$$

$$M_{HL} + M_{DH} = M_{GH}$$

$$M_{HL} = \frac{120}{7} - \frac{24}{7}$$

$$M_{HL} = \frac{120-24}{7} = \frac{96}{7} \text{ kNm.}$$

Step 5:-

calculations of column shear.

$$\text{Column moment} = \text{column shear} \times \text{height} / 2$$

$$\text{Column shear} = \frac{\text{column moment} \times 2}{\text{height}}$$

$$V_{AE} = \frac{M_{AE} \times 2}{3}$$

$$= \frac{2 \times 2}{3}$$

$$= 4/3 \text{ kN}$$

$$V_{BF} = M_{BF} \times 2 / \text{height}$$

$$= \frac{39}{7} \times \frac{2}{3}$$

$$= \frac{78}{21}$$

$$= 26/7 \text{ kN}$$

$$V_{CG} = M_{CG} \times \frac{2}{3}$$

$$= \frac{7 \times 2}{3}$$

$$V_{CG} = 14/3 \text{ kN}$$

$$V_{DH} = M_{DH} \times \frac{2}{h}$$

$$= \frac{248}{7} \times \frac{2}{3}$$

$$V_{DH} = 16 \frac{1}{7} \text{ kN}$$

$$V_{EI} = M_{EI} \times \frac{2}{h}$$

$$= \frac{28 \times 2}{7}$$

$$V_{EI} = 4 \text{ kN}$$

$$V_{FJ} = M_{FJ} \times \frac{2}{h}$$

$$= \frac{156}{7} \times \frac{2}{4}$$

$$V_{FJ} = 18 \frac{1}{7} \text{ kN}$$

$$V_{GK} = M_{GK} \times \frac{2}{h}$$

$$= \frac{196}{7} \times \frac{2}{4}$$

$$V_{GK} = 14 \text{ kN}$$

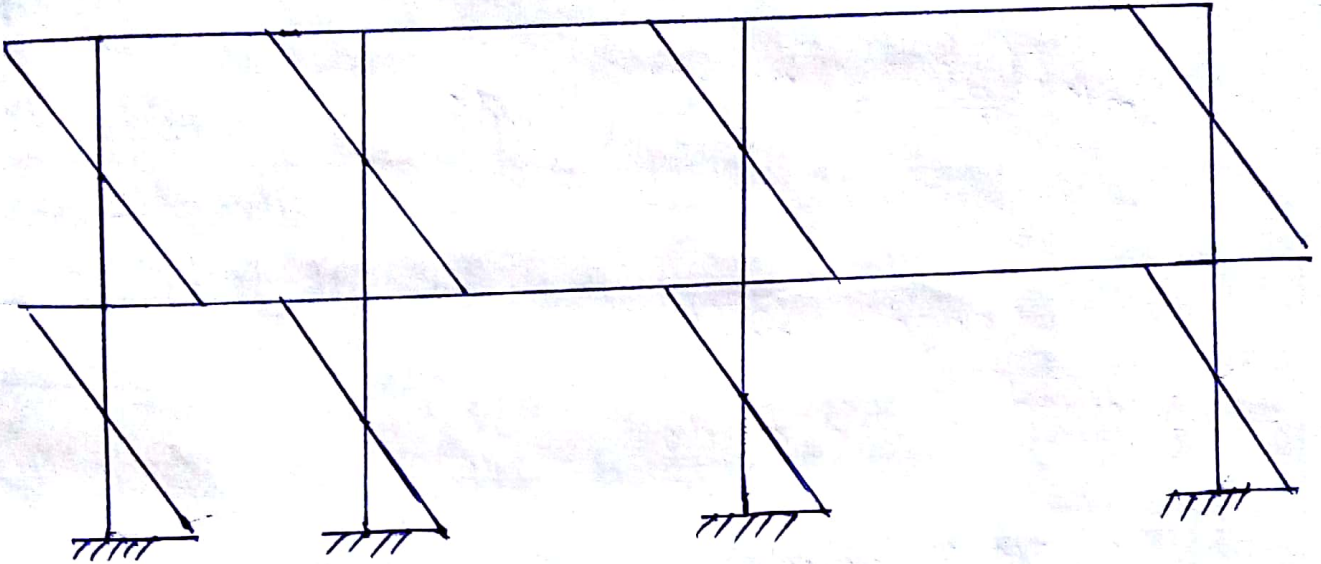
$$V_{HL} = M_{HL} \times \frac{2}{h}$$

$$= \frac{96}{7} \times \frac{2}{4}$$

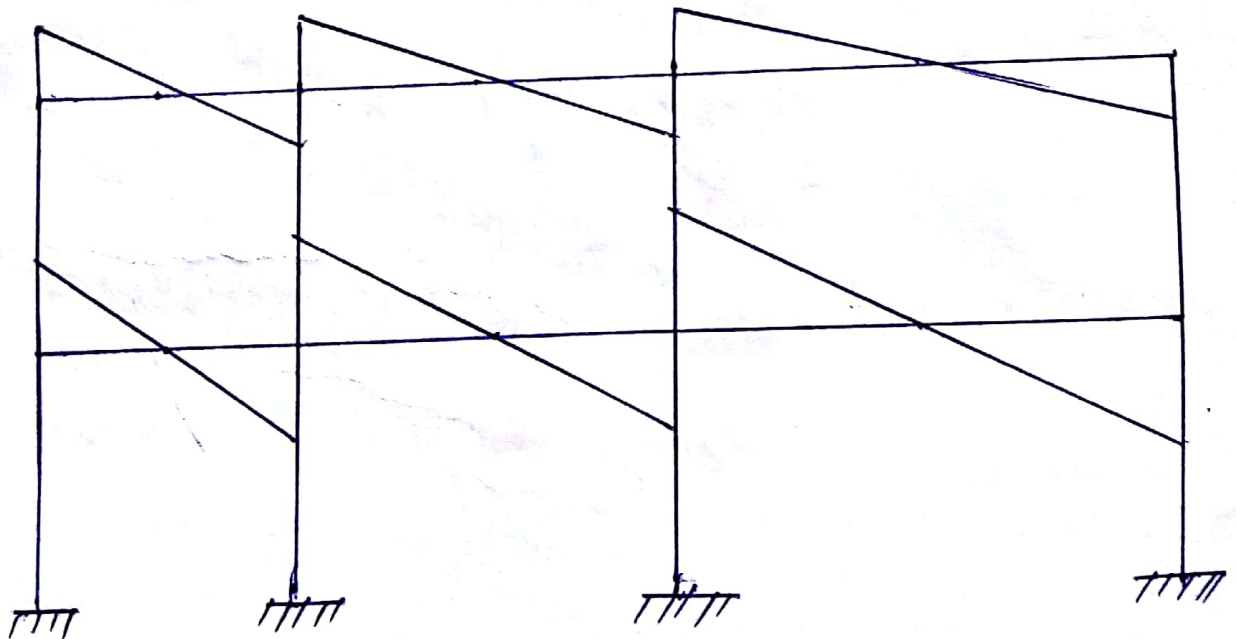
$$V_{HL} = 48 \frac{1}{7} \text{ kN}$$

Bending moment diagrams.

(a) Bending moment diagrams for columns.



(b) Bending moment diagrams for beams:-



Cable Structures And Suspension Bridges.

Cables are commonly used to support Suspension Bridges, Chimneys etc,

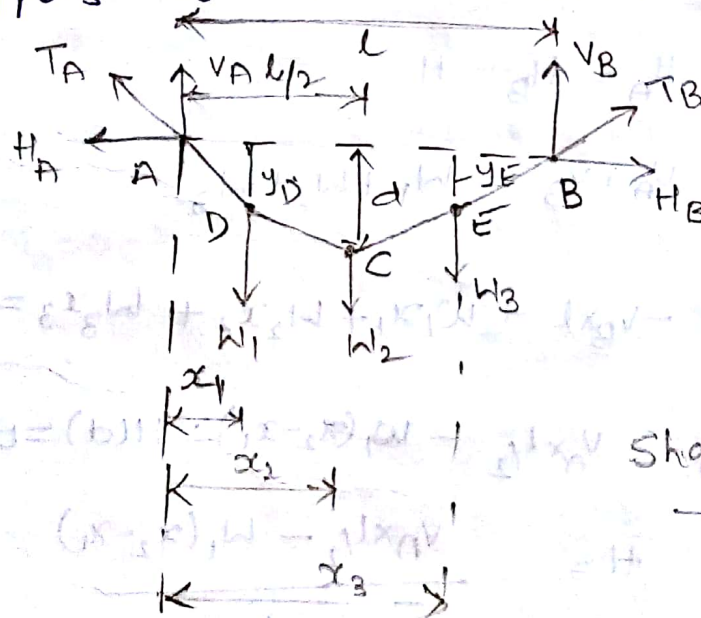
2) Cables are usually flexible and carries tension only.

3) Central dip of the cable varies from $1/10$ to $1/12$ greater the dip, less is the horizontal thrust.

4) Suspension cable bridge is very light, ^{cables} with stiffening girders are used for road ways.

Analysis of Cables with point loads:-

(a) Supports at the same Level.



Shape:-

- Funicular polygon

Let us consider that, the cable consisting (or) carrying 3 point loads they are W_1, W_2, W_3 and they are at a distances of x_1, x_2, x_3 from one end 'A' respectively.

The horizontal force at end A be " H_A " and the vertical force be " V_A " and the resultant force be " T_A " respectively.

The horizontal force at end other end B be " H_B " and vertical force be " V_B " and the resultant force be " T_B " respectively.

Let the central dip of the cable be ' d '.

Considering 3 equilibrium conditions, we have to calculate the unknown reactions in the cable.

$$\sum H = 0 \Rightarrow H_A = H_B = H$$

$$\sum V = 0 \Rightarrow V_A + V_B = W_1 + W_2 + W_3$$

$$\sum M_A = 0 \Rightarrow -V_B \times l + W_1 x_1 + W_2 x_2 + W_3 x_3 = 0$$

$$\sum M_C = 0 \Rightarrow V_A \times l_2 - W_1 (x_2 - x_1) - H(d) = 0$$

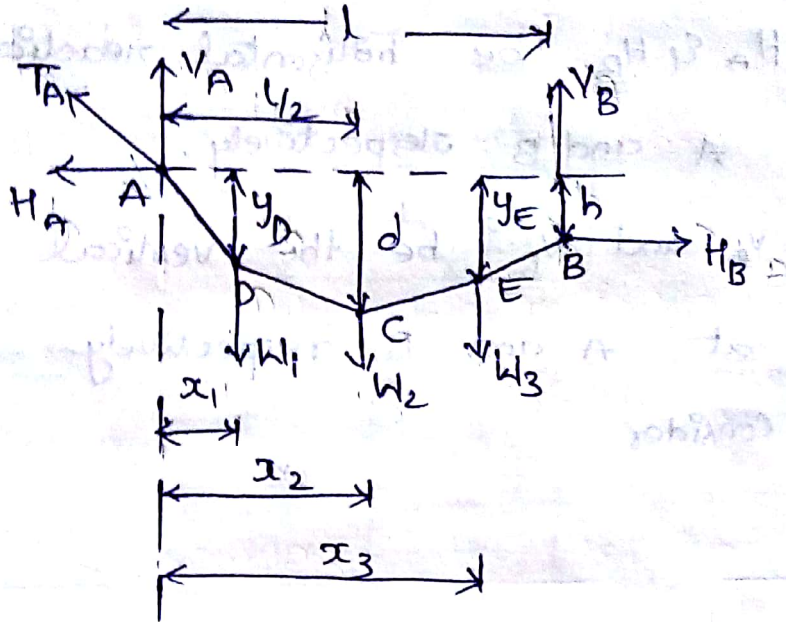
$$H = \frac{V_A \times l_2 - W_1 (x_2 - x_1)}{d}$$

$$\therefore T_A = \sqrt{H_A^2 + V_A^2}$$

$$T_B = \sqrt{H_B^2 + V_B^2}$$

$$S = \sqrt{x_1^2 + y_D^2} + \sqrt{(x_2 - x_1)^2 + (y_C - y_D)^2} + \sqrt{(x_3 - x_2)^2 + (y_E - y_C)^2} + \sqrt{(l - x_3)^2 + y_E^2}$$

(b) supports at different Level:-



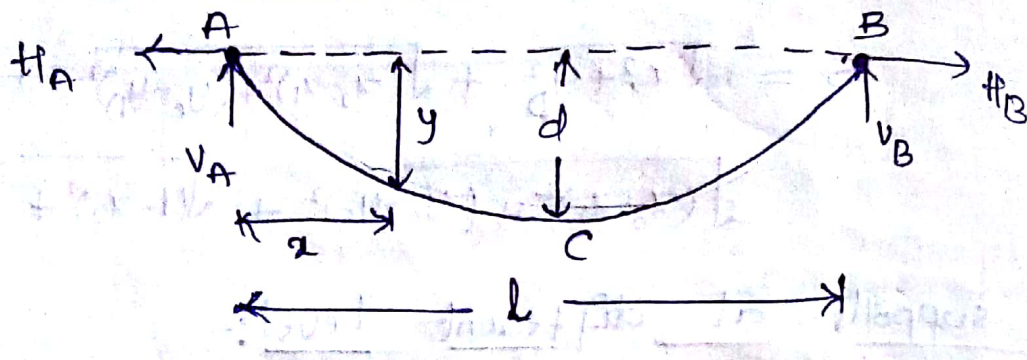
$$\sum H = 0 \Rightarrow H_A = H_B = H$$

$$\sum V = 0 \Rightarrow V_A + V_B = W_1 + W_2 + W_3$$

$$\sum M_A = 0 \Rightarrow -V_B \times l + W_1 x_1 + W_2 x_2 + W_3 x_3 - H_B h = 0$$

* Analysis of cable with UDL (Uniformly Distributed Load):-

Let 'd' be the dip of the cable at centre and a cable be loaded with UDL of intensity w/m length.



H_A & H_B be horizontal reactions at ends A and B respectively.

V_A and V_B be the vertical reactions at A and B respectively

Consider

$$\Sigma H = 0 \Rightarrow H_A = H_B = H$$

$$\Sigma V = 0 \Rightarrow V_A + V_B = wl$$

$$\Sigma M_A = 0 \Rightarrow -V_B(l) + wl \times l/2 = 0$$

$$V_B(l) = wl \times l/2$$

$$\boxed{V_B = wl/2}$$

$$V_A + V_B = wl$$

$$V_A = wl - V_B$$

$$= wl - wl/2$$

$$\boxed{V_A = wl/2}$$

$$Hd = \frac{wl}{2} \times l_2 - \frac{wl^2}{8}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$= \frac{2wl^2 - wl^2}{8}$$

$$\therefore Hd = \frac{wl^2}{8}$$

$$\boxed{H = \frac{wl^2}{8d}}$$

Maximum tension:

$$T_A = T_B = \sqrt{H_A^2 + V_A^2} = \sqrt{V_B^2 + H_B^2}$$

$$= \sqrt{\left(\frac{wl}{2}\right)^2 + \left(\frac{wl^2}{8d}\right)^2}$$

$$= \sqrt{\frac{w^2 l^2}{4} + \frac{w^2 l^4}{64d^2}}$$

$$= \sqrt{\left(\frac{wl}{2}\right)^2 \left(1 + \left(\frac{l^2}{2d}\right)^2\right)}$$

$$= \sqrt{\frac{w^2 l^2}{4} \left(1 + \frac{l^2}{16d^2}\right)}$$

$$= \frac{wl}{2} \sqrt{1 + \left(\frac{l^2}{2d}\right)^2}$$

$$= \frac{wl}{2} \sqrt{1 + \frac{l^2}{16d^2}}$$

$$= \frac{wl}{2} \sqrt{1 + \frac{l^2}{4d^2}}$$

$$\boxed{T = \frac{wl}{2} \sqrt{1 + \frac{l^2}{4d^2}}}$$

Shape of the cable:-

$$M_p = 0$$

$$wx - wx \times \frac{l}{2} - Hxy = 0$$

$$\frac{wl}{2} x - \frac{wx^2}{2} = \frac{wl^2}{8d} y$$

$$y = \frac{4d}{l^2} x(1-x)$$

∴ Cable with udl entire the span, the shape of the cable is "Parabolic" shape.

Length of the cable:-

$$ds = \sqrt{dx^2 + dy^2}$$

$$= dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$y = \frac{4d}{l^2} x(1-x)$$

$$\frac{dy}{dx} = \frac{4d}{l^2} (1-2x)$$

$$\therefore ds = dx \sqrt{1 + \left(\frac{4d}{l^2} (1-2x)\right)^2}$$

$$S = \int_0^l ds$$

$$= 2 \int_0^{l/2} \left(1 + \frac{16d^2}{l^4} (l-2x)^2\right)^{1/2} dx$$

$$= 2 \int_0^{l/2} \left(1 + \frac{16d^2}{l^4} x^4 \left(\frac{l}{2} - x\right)^2\right)^{1/2} dx$$

$$= 2 \int_0^{l/2} \left(1 + \frac{64d^2}{l^4} \left(\frac{l}{2} - x\right)^2\right)^{1/2} dx$$

$$= 2 \int_0^{l/2} \left(1 + \frac{1}{2} \times \frac{64d^2}{l^4} (l/2 - x)^2 \right) dx$$

$$= 2 \left[\left(x \right)_0^{l/2} + \left[\frac{32d^2}{l^4} \frac{(l/2 - x)^3}{3} (-1) \right]_0^{l/2} \right]$$

$$\left((1+x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 \right)$$

$$= 2 \left(\frac{l}{2} - \frac{32d^2}{3l^4} \left(\left(\frac{l}{2} - l/2 \right)^3 - \left(\frac{l}{2} - 0 \right)^3 \right) \right)$$

$$= 2 \left(\frac{l}{2} + \frac{32d^2}{3l^4} \left(\frac{l^3}{8} \right) \right)$$

$$= 2 \left(\frac{l}{2} + \frac{4d^2}{3l} \right)$$

$$= \frac{2l}{2} + \frac{8}{3} (d^2/l)$$

$$\therefore S = l + \frac{8}{3} (d^2/l)$$

(1) A suspension cable having supports at same level, as span of 30m. maximum dip of the cable 3m. The cable is loaded with UDL of 10 kN/m. throughout the length. Find the maximum tension in the cable and length of the cable.

Sol:- Given data:-

$$\text{span } l = 30 \text{ m}$$

$$\text{Central dip } (d) = 3 \text{ m}$$

$$\text{UDL } = 10 \text{ kN/m}$$

$$\sum H = 0 \Rightarrow$$

$$H_A = H_B = H$$

$$\sum V = 0 \Rightarrow$$

$$R_A + R_B = (10 \times 30)$$

$$R_A + R_B = 300$$

$$\sum M_A = 0 \Rightarrow$$

$$-R_B(30) + 10 \times 20 \times 30/2 = 0$$

$$R_B(30) = 5 \times 30 \times 30$$

$$R_B = \frac{5 \times 30 \times 30}{30}$$

$$\boxed{\therefore R_B = 150 \text{ kN}}$$

$$\therefore R_A = 300 - R_B$$

$$= 300 - 150$$

$$\boxed{\therefore R_A = 150 \text{ kN}}$$

$$\sum M_C = 0 \Rightarrow$$

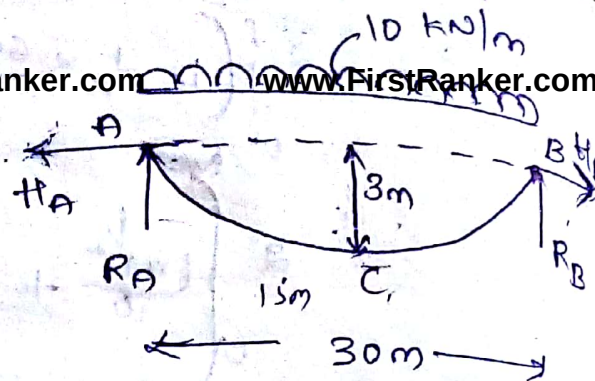
$$R_A \times 15 - 10 \times 15 \times 15/2 - H \times 3 = 0$$

$$H(3) = 150 \times 15 - 15 \times 15 \times 5$$

$$H = \frac{150 \times 15 - 15 \times 15 \times 5}{3}$$

$$\boxed{\therefore H = 375 \text{ kN}}$$

$$\therefore H_A = H_B = H = 375 \text{ kN}$$



$$\begin{aligned}
 T_A = T_B &= \sqrt{P_A^2 + H_A^2} \\
 &= \sqrt{150^2 + 375^2} \\
 &= \sqrt{163125}
 \end{aligned}$$

$$\therefore \text{maximum tension} = 403.88 \text{ KN}$$

Length of the cable:-

$$ds = \sqrt{dx^2 + dy^2}$$

$$= dx \sqrt{1 + (dy/dx)^2}$$

$$y = \frac{4d^2}{l^2} x(1-x)$$

$$y = \frac{4 \times 3^2}{30^2} (30x - x^2)$$

$$\frac{dy}{dx} = \frac{4 \times 3}{30^2} (30 - 2x)$$

$$\frac{dy}{dx} = \frac{12}{30^2} (30 - 2x)$$

$$s = \int_0^L ds$$

$$= 2 \int_0^{1/2} \left(1 + \left(\frac{12}{30^2} (30 - 2x) \right)^2 \right)^{1/2} dx$$

$$= 2 \int_0^{1/2} \left(1 + \left(\frac{16}{90000} (30 - 2x)^2 \right) \right)^{1/2} dx$$

$$= 2 \int_0^{1/2} \left(1 + \frac{1}{2} \times \frac{16}{90000} (30 - 2x)^2 \right) dx$$

$$= 2 \left(\frac{15}{1} + \frac{48}{3 \times 10000} \left(\frac{(30-2 \times 15)^3}{3} - \frac{(30-0)^3}{3} \right) \right)$$

$$= 2 \left(15 + \frac{48 \times 2}{3 \times 10000} \left(30 - (2 \times 15) \right)^3 - (30 - 0)^3 \right)$$

$$= 2 \left(15 - \left(\frac{48}{3 \times 10000} \times (0 - (30)^3) \right) \right)$$

$$= 2 \left(15 + \frac{48}{3 \times 10000} \times 27000 \right)$$

$$= 2(15 + 1200) = 2(15 + 0.8)$$

$$= 2430$$

$$= 31.6 \text{ m}$$

$$\therefore s = 2430 \text{ m}$$

$$\therefore s = 31.6 \text{ m}$$

check!—

$$s = 1 + \frac{8}{3} \left(\frac{d^2}{1} \right)$$

$$= 30 + \frac{8}{3} \left(\frac{3^2}{30} \right)$$

$$= 30 + 0.8$$

$$\therefore s = 30.80 \text{ m}$$

Consider a suspension of cable of span 'l' supported at same level. and having a maximum central dip 'd'. Let it be subjected to a change of temperature $t^{\circ}\text{C}$. When the cable is subjected to a change of temperature there will be a change in length of cable.

The supporting towers cannot change their span and hence there will be a change in dip of the cable.

$\therefore$ length of the cable

$$S = l + \frac{8}{3} \left(\frac{d^2}{l} \right)$$

$$\frac{\partial S}{\partial d} = \frac{\partial}{\partial d} (l) + \frac{\partial}{\partial d} \left(\frac{8}{3} \left(\frac{d^2}{l} \right) \right)$$

$$= 0 + \frac{8}{3l} (2d)$$

$$\frac{\partial S}{\partial d} = \frac{8}{3l} (2d)$$

$$\therefore \frac{\partial S}{\partial d} = \frac{16d}{3l}$$

$$\partial S = \left(\frac{16d}{3l} \right) \partial d$$

deformation due to change of temperature

$$e = l \alpha t$$

$$= S \alpha t$$

$$= \left(1 + \frac{8}{3} \frac{d^2}{l^2} \right) \alpha t$$

$$\therefore \Delta s \cong \Delta t$$

$$\Delta t = \left(\frac{16d}{3l} \right) \Delta s$$

$$\Delta s = \frac{3l^2 \Delta t}{16d}$$

Change of Horizontal thrust due to temperature
raise :

we k/r

$$H = \frac{wl^2}{8d}$$

$$\frac{\partial H}{\partial d} = \frac{\partial}{\partial d} \left(\frac{wl^2}{8d} \right)$$

$$= \frac{wl^2}{8} \cdot \frac{\partial}{\partial d} \left(\frac{1}{d} \right)$$

$$= \frac{wl^2}{8} \times \frac{-1}{d^2}$$

$$= \frac{-wl^2}{8d^2} \quad (\because \text{here negative sign indicates})$$

$$\frac{\partial H}{\partial d} = \frac{-wl^2}{8d^2}$$

decrease of horizontal thrust)

$$\Delta H = \frac{-wl^2}{8d^2} (\Delta d)$$

$$= \frac{-wl^2}{8d^2} \left(\frac{3l^2 \Delta t}{16d} \right)$$

$$\Delta H = -\frac{3wl^4 \Delta t}{128d^3}$$

$$\therefore \Delta H = -\frac{3\alpha l^2 t}{16d^2}$$

$$\Delta H = -H \left(\frac{3l^2 \alpha t}{16d^2} \right)$$

$$\therefore \Delta H = \frac{-3l^2 \alpha t}{16d^2} H$$

- (1) A suspension cable of horizontal span 210 m . is supported at the same level and as a central dip of 20 m . Find the increase in dip. of the cable, and change in horizontal thrust. If the cable is subjected to raise in temperature 28°C . Take $\alpha = 12 \times 10^{-6}/^\circ\text{C}$ and load udl 20 kN/m .

Sol:- Given data:-

$$\text{horizontal span } (l) = 210\text{ m.}$$

$$\text{central dip } (d) = 20\text{ m.}$$

$$\text{temperature } (t) = 28^\circ\text{C}$$

$$\text{Coefficient of linear expansion } (\alpha) = 12 \times 10^{-6}/^\circ\text{C}$$

$$\text{udl } (w) = 20\text{ kN/m.}$$

$$\Delta d = \frac{3l^2 \alpha t}{16d}$$

$$= \frac{3 \times 210^2 \times 12 \times 10^{-6} \times 28}{16 \times 20}$$

Horizontal thrust = HG

$$2H = \frac{3l^2 \alpha t \times H}{16d^2}$$

$$H = \frac{wl^2}{8d}$$

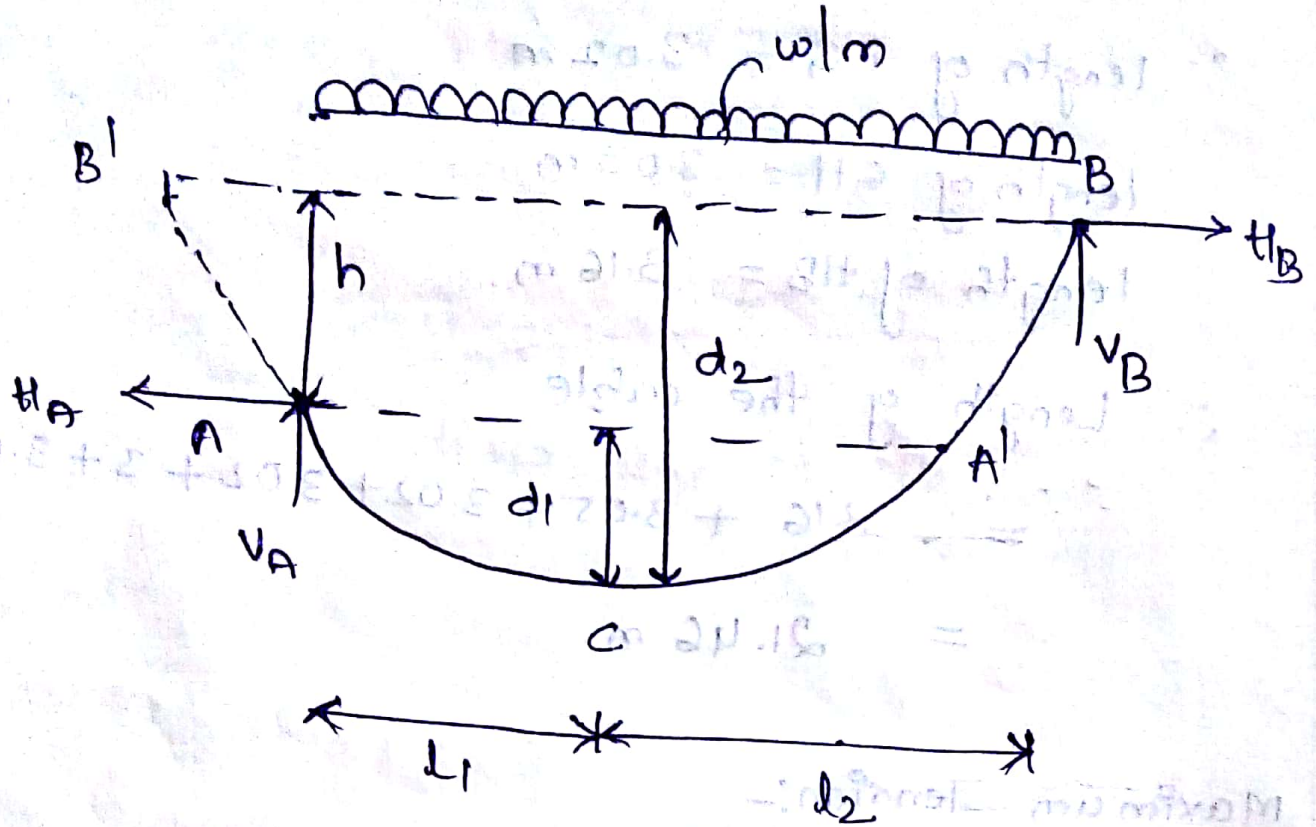
$$= \frac{20 \times 20^2}{8 \times 20}$$

$$\therefore H = 5512.5 \text{ kN}$$

$$\therefore 2H = \frac{3 \times 20^2 \times 12 \times 10^6 \times 28 \times 5512.5}{16 \times 20^2}$$

$$\therefore 2H = 38.28 \text{ kN}$$

* Analysis of cable with supports at different level:



Here the cable AB is supported at different levels at A and B . ' C ' is the lowest point of the cable. d_1 is the dip and L_1 is the length of the portion AA' d_2 is the dip and L_2 is the length of the extended part of the cable i.e., $B'B$.

Vertical reaction,

$$V_A = \frac{w(2l_1)}{2}$$

$$\therefore V_A = wl_1$$

Horizontal reaction,

$$H_A = \frac{wl_1^2}{8d_1}$$

$$\therefore H_A = \frac{w(l_1 \times d_1)^2}{8d_1}$$

$$\therefore H_A = \frac{wl_1^2}{2d_1}$$

$$S_{AC} = \frac{1}{2} \left(2l_1 + \frac{8}{3} \frac{d_1^2}{2l_1} \right)$$

$$\therefore S_{AC} = l_1 + \frac{2}{3} \frac{d_1^2}{l_1}$$

Tension at 'A'

$$T_A = \sqrt{V_A^2 + H_A^2}$$

$$= \sqrt{(wl_1)^2 + \left(\frac{wl_1^2}{2d_1} \right)^2}$$

$$\therefore T_A = \sqrt{w^2 l_1^2 + \frac{w^2 l_1^4}{4d_1^2}}$$

Consider the portion B'C B.

$$V_B = wl_2$$

$$H_B = \frac{wl_2^2}{2d_2}$$

$$S_{CB} = \frac{1}{2} \left(2l_2 + \frac{8}{3} \frac{d_2^2}{2l_2} \right)$$

∴ length of the cable:

$$S = S_{AC} + S_{CB}$$

$$= d_1 + \frac{2}{3} \frac{d_1^2}{l_1} + l_2 + \frac{2}{3} \frac{d_2^2}{l_2}$$

$$= (l_1 + l_2) + \frac{2}{3} \left(\frac{d_1^2}{l_1} + \frac{d_2^2}{l_2} \right)$$

$$\therefore S = l + \frac{2}{3} \left(\frac{d_1^2}{l_1} + \frac{d_2^2}{l_2} \right)$$

$$\Sigma H = 0 \Rightarrow$$

$$H_A = H_B$$

$$\frac{wl_1^2}{2d_1} = \frac{wl_2^2}{2d_2}$$

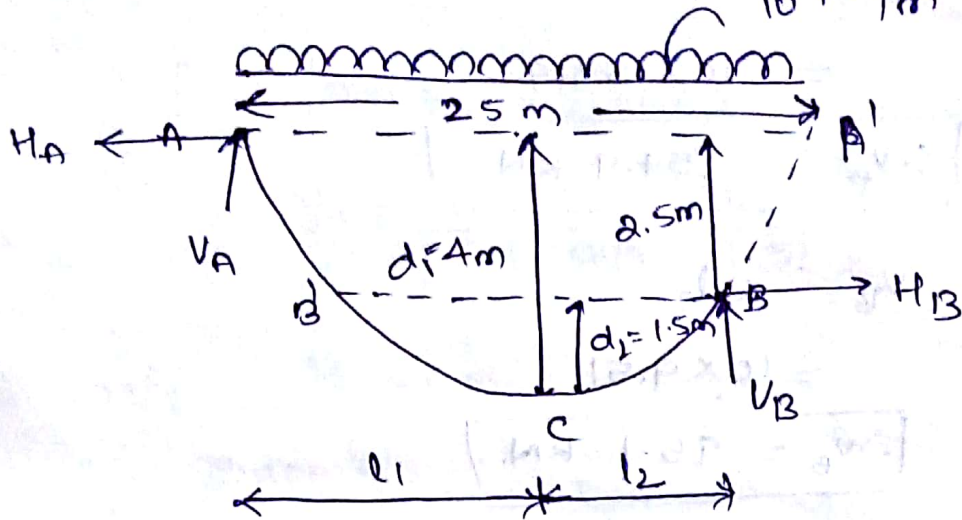
$$\frac{l_1^2}{d_1} = \frac{l_2^2}{d_2}$$

$$\frac{l_1^2}{l_2^2} = \frac{d_1}{d_2}$$

$$\therefore \frac{l_1}{l_2} = \sqrt{\frac{d_1}{d_2}}$$

- (1) A suspension cable is supported at two points 25m apart. The left support is 2.5m above the right support. The cable is loaded with a udl of 10kN/m throughout the span. The maximum dip of the cable from the left support is 4m. Find the maximum

sol:-



$$l_1 + l_2 = l = 25 \text{ m} \quad (1)$$

$$\frac{l_1}{l_2} = \sqrt{\frac{d_1}{d_2}}$$

$$= \sqrt{\frac{4}{1.5}}$$

$$\frac{l_1}{l_2} = 1.63$$

$$l_1 = 1.63(l_2) \quad (2)$$

sub. eqn (2) in eqn (1)

$$1.63(l_2) + l_2 = 25$$

$$2.63l_2 = 25$$

$$l_2 = 25 / 2.63$$

$$l_2 = 9.51 \text{ m}$$

$$l_1 = 1.63(l_2)$$

$$= 1.63 \times 9.51$$

$$l_1 = 15.49 \text{ m}$$

$$l_1 = 15.5 \text{ m}$$

$$V_A = w l_1$$
$$= 10 \times 15.49$$

$$\therefore V_A = 154.9 \text{ kN}$$

$$V_B = w l_2$$
$$= 10 \times 9.51$$

$$\therefore V_B = 95.1 \text{ kN}$$

$$H_A = H_B = H = \frac{w l_1^2}{2 d_1} \quad (\sigma) \quad \frac{w l_2^2}{2 d_2}$$
$$= \frac{10 \times (15.49)^2}{2 \times 4} = \frac{10 \times 9.51^2}{2 \times 1.5}$$
$$= 299.92 \quad = 301.4 \text{ kN}$$
$$\therefore H \approx 300 \text{ kN}$$

Tension:-

$$T_A = \sqrt{w^2 l_1^2 + \frac{w^2 l_1^4}{4 d_1}}$$
$$= \sqrt{10^2 \times 15^2 + \frac{10^2 \times 15^4}{4 \times 4}}$$
$$= 337.63 \text{ kN}$$

$$T_B = \sqrt{V_B^2 + H_B^2}$$
$$= \sqrt{(95.1)^2 + (300)^2}$$
$$= 314.71 \text{ kN}$$

$\therefore$ maximum Tension $T_A = 337.63 \text{ kN}$

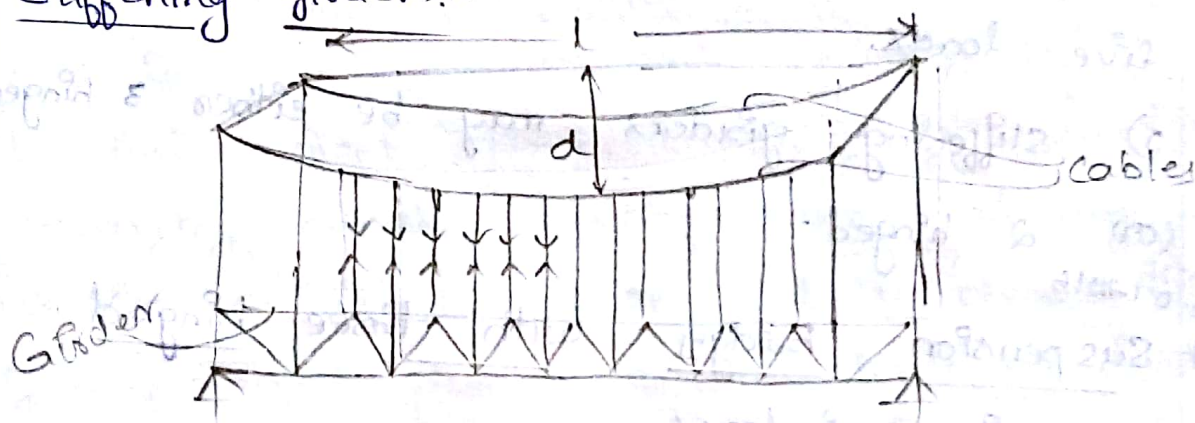
length of the cable

$$S = l + \frac{2}{3} \left(\frac{d_1^2}{l_1} + \frac{d_2^2}{l_2} \right)$$

$$= 25 + \frac{2}{3} \left(\frac{4^2}{15.49} + \frac{1.5^2}{9.51} \right)$$

$$S = 25.84 \text{ m}$$

* Stiffening girders:-



Stiffening girders are usually suspended from the cable through closely spaced suspenders one on each side of the width of the ~~Beam~~ Bridge.

* Functions:-

- 1) Stiffening girders enable the suspension bridge deck to remain fairly.
- 2) Whatever be the live load on the deck slab, the stiffening girders will convert and transmit on the deck slab as a u.d.l. & thereby help the cable remain the

during the passage of loads.

3) The dead load of the girder which is a udl is directly transmitted to the cable, and is taken up entirely by the tension in the cable.

4) The stiffening girders will have to resist the shear force and bending moment due to live loads.

5) stiffening girders may be either 3 hinged & 2 hinged.

* Suspension Bridges with three hinged stiffening girders:

The girder by itself will not be enough to span the enormous distances the cable by pulling the girder via by closely spaced suspenders relieves the girder of a huge part of the load by developing a uniform tension in suspension cable.

In other words, the suspenders apply on the girder an upward acting udl

Be

stiffening girders:-

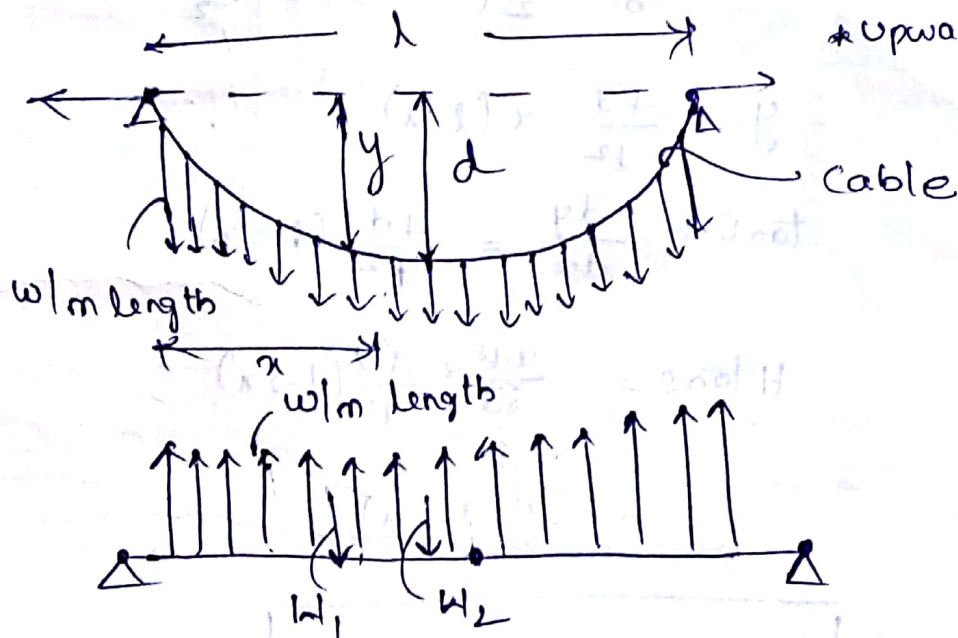
If the load on the bridge is made up of 2 point loads W_1 & W_2 then the system is going to develop an upward udl of w /m length on the girders. and a downward udl of w /m length on the cable. The girder load is W_1 & W_2 downwards, producing positive bending moment; w /m length acting upward, producing negative bending moment at any section x on the girder. Let us denote the bending moment due to external loads equal to

" M_x "

(loads:-

* Liveloads

* Upward udl)



∴ Bending moment at any distance on the girder

$$M_x = -M_x - \frac{wx}{2}(l-x)$$

$$y = \frac{4d}{l^2} x(l-x)$$

$$H = \frac{wl^2}{8d}$$

$$H_y = \frac{wl^2}{8d} \times \frac{4d}{l^2} x(l-x)$$

$$\therefore H_y = \frac{w}{2} x(l-x)$$

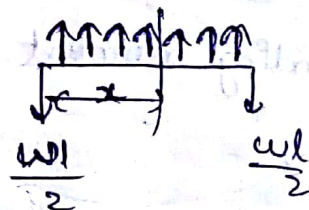
Bending moment at any distance.

$$\therefore M_x = M_2 - H_y$$

Shear Force:

$$S_x = S_b - \frac{wl}{2} + wx$$

$$= S_b - \frac{w}{2} (l - 2x)$$



$$y = \frac{4d}{l^2} x(l-x)$$

$$\tan \theta = \frac{dy}{dx} = \frac{4d}{l^2} (l - 2x)$$

$$H \tan \theta = \frac{wl^2}{8d} \times \frac{4d}{l^2} (l - 2x)$$

$$= \frac{w}{2} (l - 2x)$$

$$\therefore S_x = S_b - H \tan \theta$$

Shear force at 'x' distance

$$(x-1) \frac{wl}{2} S_x = S_b - H \tan \theta$$

Bending moment at any distance

$$M_x = M_c - Hy$$

At centre,

$$y = d$$

$$M_x = M_c$$

$$M_x = 0 \quad (\text{due to hinge at centre})$$

$$0 = M_c - Hd$$

$$Hd = M_c$$

$$H = \frac{M_c}{d}$$

$$H = \frac{wl^2}{8d}$$

$$w = \frac{H8d}{l^2}$$

$$= \frac{8d}{l^2} \times \frac{M_c}{d}$$

$$w = \frac{8M_c}{l^2}$$

MOMENT DISTRIBUTION METHOD

MDM - Analysis of single Bay single storey Portal Frames including side sway. Analysis of inclined frames.

MDM - The method was first introduced by Prof. Hardy Cross in 1930. The method could be used for the analysis of all types of statically indeterminate beams or rigid frames.

Stiffness: It is denoted by 'k'. The moment 'k' is also known as absolute stiffness or simply stiffness.

The stiffness of a member is a moment required to rotate the end under consideration through unit angle. It is the amount of force required to produce unit deflection or displacement is called stiffness.

Proposition - 1: The moment 'k' required to rotate the near end of a prismatic beam through a unit angle, without translation, the far end being freely supported is:

$$k = \frac{3EI}{L}$$



Proposition - 2: The moment 'k' required to rotate the near end of a prismatic beam through unit angle, without translation, the far end is fixed:

$$k = \frac{4EI}{L}$$



Proposition - 3: A moment which rotates the near end of beam without translation, the far end being fixed, induces at the far end a moment of one half its magnitude & in the same direction.

$$\therefore M_1 = \frac{3EI_1}{L_1} \theta = k_1 \theta$$

$$M_2 = \frac{4EI_2}{L_2} \theta = k_2 \theta$$

$$M_3 = \frac{3EI_3}{L_3} \theta = k_3 \theta$$

$$M_n = \frac{4EI_n}{L_n} \theta = k_n \theta$$



$$u = u_1 + u_2 + u_3 + u_4 = \dots$$

$$u_1 : u_2 : u_3 : u_4 :: k_1 : k_2 : k_3 : k_4 \quad \text{--- (i)}$$

$$\therefore u_1 = \frac{k_1}{k_1 + k_2 + k_3 + k_4} \cdot u = \frac{k_1}{\sum k} u$$

$$u_2 = \frac{k_2}{k_1 + k_2 + k_3 + k_4} \cdot u = \frac{k_2}{\sum k} \cdot u$$

$$u_3 = \frac{k_3}{\sum k} \cdot u$$

$$u_4 = \frac{k_4}{\sum k} \cdot u$$

Proposition-4 : A moment which tends to rotate a joint without translation, will be divided amongst the connecting members of the joint in proportion to their "stiffness".

The quantities $\frac{k_1}{\sum k}, \frac{k_2}{\sum k}, \frac{k_3}{\sum k}, \frac{k_4}{\sum k}$ are called distribution factors. The moments u_1, u_2, u_3, u_4 are called as distribution moments.

Fixed End Moments :

$$M_{FAB} = -\frac{Wab^2}{L^2}$$

$$M_{FBA} = \frac{Wa^2b}{L^2}$$

$$M_{FAB} = -\frac{WL^2}{8}$$

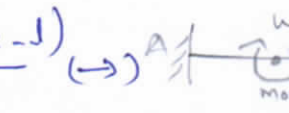
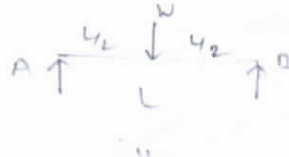
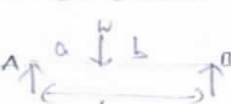
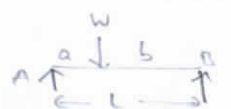
$$M_{FBA} = \frac{WL}{8}$$

$$M_{FAB} = -\frac{WL^2}{12}$$

$$M_{FBA} = \frac{WL^2}{12}$$

$$M_{FAB} = \frac{M \times b (3a - b)}{L^2} \quad (\rightarrow)$$

$$M_{FBA} = \frac{Ma (3b - a)}{L^2} \quad (\leftarrow)$$



Sinking of Supports :

$$M = \frac{6EI\delta}{L^2}$$

$$M = \frac{3EI\delta}{L^2}$$

Sinking of propped cantilever.

$$M_{FAB} = -\frac{WL^2}{20} (\rightarrow)$$

$$M_{FBA} = \frac{WL^2}{30} (\leftarrow)$$



Sway, one of the degrees of freedom.

In portal frames, the amount of 'sway' or joint movement is not known & the analysis is done by assuming some arbitrary fixed moments.

These assumed fixed moments due to side sway are then distributed & the reactions are found.

The algebraic sum of the horizontal reactions due to the assumed sway moments must be equal to the sway force.

Causes of Side sway :

- 1) Eccentric or unsymmetrical loading on the portal frame.
- 2) Unsymmetrical out-line of portal frame.
- 3) Different end conditions of the columns of the portal frame.
- 4) Non-uniform section of the members of the frame.
- 5) Horizontal loading on the columns of the frame.
- 6) Settlement of the supports of the frame.
- 7) Combination of all above.

Method of Analysis :

Analysis of ^{portal} frames with side sway is done as following

Step 81-a) Hold the joints against side sway by applying a force 'P'. calculate the fixed end moments due to external loads & distribute the moments.

b) Calculate the horizontal & vertical reactions. The algebraic sum of the two horizontal reactions at the column bases will give the value of the restraining or holding force 'P'.

The sway force S will be in the opposite direction & of the magnitude of 'P'.

Step-2: Remove the holden force P & permit the joints to sway. This will cause a set of fixed end moments. To start with, assume suitable sway moments at the four joints, A, B, C & D of the frame, in proportion.

b) Calculate the horizontal & vertical reactions due to the assumed sway moments. The algebraic sum of the horizontal reactions of two column bases must be equal to the sway force S . If not, reduce the sway moments assumed proportionately. The sway moments must be of such magnitude that the algebraic sum of the horizontal reactions due to sway is equal to the sway force S .

Let H_1 & H_2 be the horizontal reactions.

$$\text{Let } c(H_1 + H_2) = S$$

Then, actual sway moments = $c \times$ Assumed Sway moments.

Thus the actual sway moments are known.

Step-3: a) The final moments at each joint will be equal to the algebraic sum of the moments due to initial moments.

[a) (as obtained in step 1(a) & the moments due to actual sway (as obtained in step 2))]

b) The final reactions will be equal to the algebraic sum of those found in 1(b) & 2(b).

Ratio of Sway Moments at Column Heads:

When the joints sway, a set of moments are introduced at the two column heads (and bases) of a portal frame. The ratio of the sway moments at the two column heads (i.e. $M_{BA} : M_{CD}$) will depend upon the end conditions. Let us now take

different end conditions to derive the standard expressions for the ratio of the sway moments.

Dividing ① & ②

$$\frac{MBA}{MCD} = \frac{I_1/L_1^2}{I_2/L_2^2}$$

anticlockwise " , the " " " " +ve.

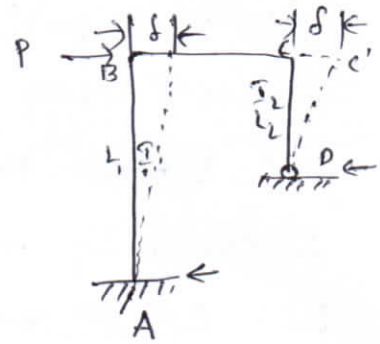
$$BB' = CC' = \delta$$

$$M_{BA} = M_{AB} = \frac{6EI_1\delta}{L_1^2} \quad \text{--- (1)}$$

$$M_{CD} = \frac{3EI_2\delta}{L_2^2} \quad \text{--- (2)}$$

$$\therefore \frac{M_{BA}}{M_{CD}} = \frac{2 \cdot \frac{6EI_1\delta}{L_1^2}}{\frac{3EI_2\delta}{L_2^2}} = 2 \cdot \frac{I_1}{I_2} \cdot \frac{L_2^2}{L_1^2}$$

$$\boxed{\frac{M_{BA}}{M_{CD}} = \frac{2 I_1 / L_1^2}{I_2 / L_2^2}}$$



$$\frac{M_{BA}}{M_{CD}} = \frac{I_1 / L_1^2}{I_2 / L_2^2} ; \quad \frac{M_{BA}}{M_{CD}} = \frac{I_1 / L_1^2}{I_2 L_2^2} ; \quad \frac{M_{BA}}{M_{CD}} = \frac{2 I_1 / L_1^2}{I_2 / L_2^2}$$

are the ratio of the moments induced at the column heads due to side sway, for various end conditions.

Note: Relative stiffness (k):

The absolute stiffness (k) of a member when far end is fixed = $\frac{4EI}{L}$ [when several members meeting at a joint]

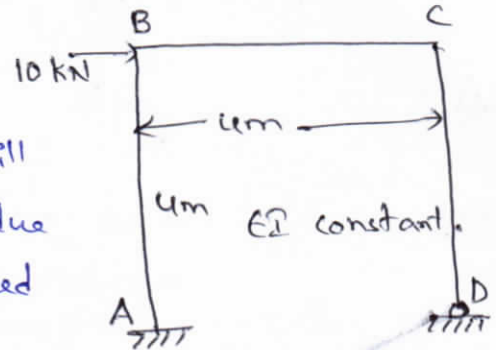
The members meeting at a joint are of same material & are fixed at the far end, the stiffness of each member is $\frac{I}{L}$.

The members meeting at a joint are freely supported at the other end, the stiffness is $\frac{3}{4} \cdot \frac{I}{L}$.

1. Analyse the portal frames shown below. The end A is fixed and D is hinged. The joints B & C are rigid. Draw the B.M.D & sketch the deflected shape of the frame.

Sol: Distribution factors:

∵ The load acting at the joint, there will be no fixed end moments. Therefore due to side sway, moments will be induced at joint A, B & C.



Joint	Member	Relative Stiffness K	Sum ΣK	D. F ($K/\Sigma K$)
B	BA	$\frac{I}{L} = \frac{I}{4}$	$\frac{2I}{4}$	0.5
	BC	$\frac{I}{L} = \frac{I}{4}$		0.5
C	CB	$\frac{I}{L} = \frac{I}{4}$	$\frac{7I}{16}$	0.57
	CD	$\frac{3}{4} \times \frac{I}{L} = \frac{3}{4} \times \frac{I}{4} = \frac{3I}{16}$		0.43

$$\frac{1}{4} + \frac{3}{16} = \frac{4+3}{16} = \frac{7}{16}$$

Side Sway :

The side sway will be clockwise in direction; at the columns AB & CD. Thus -ve moment will be induced at A, B & C. [One end fixed & other hinged]

$$\therefore \frac{M_{BA}}{M_{CD}} = \frac{2 \times \frac{I}{L^2}}{\frac{I}{L^2}} = \frac{\frac{2I}{4^2}}{\frac{I}{4^2}} = \frac{2}{1}$$

$$\therefore \frac{M_{BA}}{M_{CD}} = \frac{2}{1}$$

Let assume arbitrary moments :

$$M_{CD} = -5 \text{ kN}\cdot\text{m}$$

$$M_{AB} = M_{BA} = -10 \text{ kN}\cdot\text{m}$$

A	B		C		D	
	0.5	0.5	0.57	0.43		F.E.M
-10.0	-10.0	-	-	-5.0	0	Bal
-	+5	+5	+2.86	+2.14	-	
+2.50	-	+1.43	+2.50	-	-	C.O
-	-0.725	-0.715	-1.425	-1.075	-	Bal
-0.357	-	-0.712	-0.357	-	-	C.O
-	0.356	0.356	0.203	0.152	-	Bal
0.178	-	0.1015	0.178	-	-	C.O
-	-0.050	-0.050	-0.1014	-0.076	-	Bal
-0.025	-	-0.0507	-0.025	-	-0.056	C.O
-	0.025	0.025	0.0142	0.0107	-	Bal
-0.025	-	0.	-0.025	-	-0.056	
-7.70 kN·m	-5.384	5.384	3.845	-3.847	0	F.M

Final Moments : $M_{AB} = -7.70 \text{ kN}\cdot\text{m}$

$$M_{BA} = -5.384 \text{ kN}\cdot\text{m}$$

$$M_{BC} = 5.384 \text{ kN}\cdot\text{m}$$

$$M_{CB} = 3.845 \text{ kN}\cdot\text{m}$$

$$M_{CD} = -3.847 \text{ kN}\cdot\text{m}$$

$$\begin{aligned} \text{Horizontal Reaction at A} &= \frac{M_{AB} + M_{BA}}{4} \\ &= \frac{-7.70 + 5.38}{4} \\ &= \frac{-2.32}{4} = -0.58 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Reaction at A} &= 3.27 \text{ kN} (\leftarrow) \\ &= \frac{+13.08}{4} = 3.27 \text{ kN} (\leftarrow) \\ &= -3.27 (\leftarrow) \end{aligned}$$

$$\begin{aligned} \text{Horizontal Reaction at D} &= \frac{M_{DC} + M_{CD}}{4} \\ &= \frac{-3.85 + 0}{4} = -0.962 \\ &= 0.962 (\leftarrow) \end{aligned}$$

$$\text{Reaction D} = 0.962 \text{ kN} (\leftarrow)$$

∴ The sway force caused by the assumed moments

$$= 3.27 + 0.962$$

$$= 4.232 \text{ kN} (\rightarrow)$$

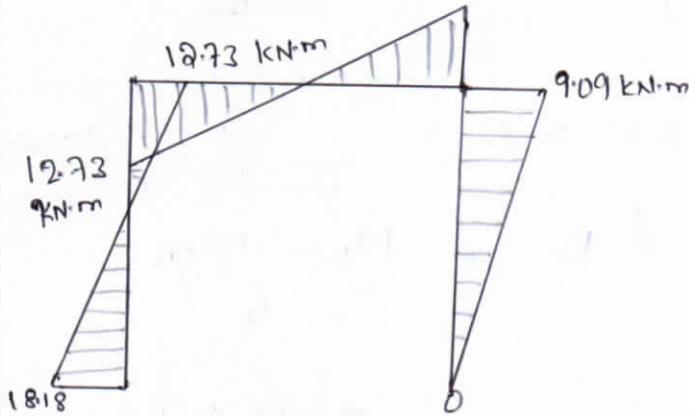
∴ The actual Sway force = 10 kN

∴ The increased moments proportionately in the ratio of $\frac{10}{4.232} = (2.36)$

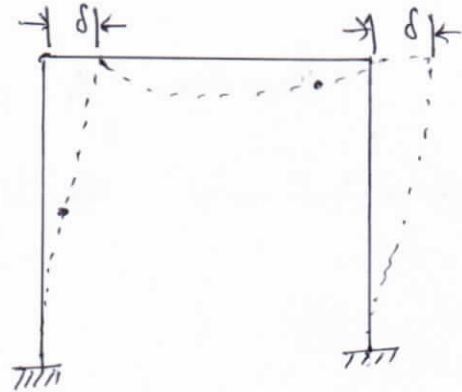
	A	B	C	D
Sway = 4.232 kN	-7.70	-5.38, +5.38	+3.85, -3.85	0
Sway = 10 kN	18.172	-12.69, 12.69	9.062, -9.062	0

The horizontal reaction @ A = $\frac{3.27}{4.233} \times 10 = 7.72 \text{ kN}$ ($\leftarrow$)

The horizontal reaction @ D = $\frac{0.963}{4.233} \times 10 = 2.28 \text{ kN}$ ($\leftarrow$)

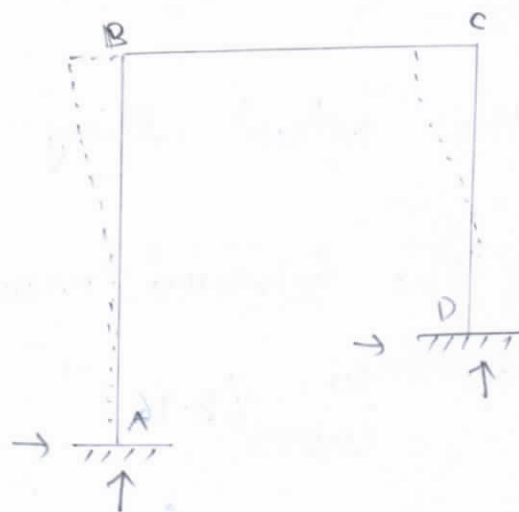
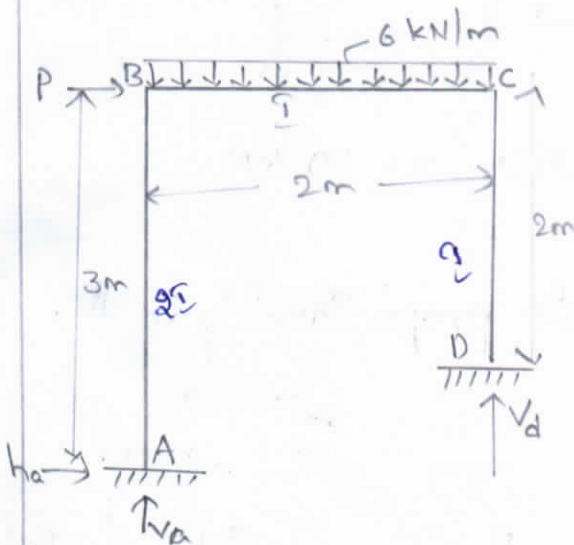


(a) B.M.D



(b) Deflected shape

Prb: Draw the B.M.D & sketch the deflected shape of the frame shown in fig. The ends A & B are fixed & BC is loaded with U.D.L of 6 kN/m.



Sol: Fixed End Moments :

$$M_{FBC} = M_{FCB} = \pm \frac{W \times l^2}{12} = \pm \frac{6 \times 2^2}{12}$$

$$M_{FBC} = -9.09 \text{ kN.m} \quad M_{FCB} = +2.0 \text{ kN.m}$$

Joint	Member	Relative (k) Stiffness	Sum ($k/\Sigma k$)	D.F ($k/\Sigma k$)
B	BA	$\frac{2I}{3} = \frac{2I}{3}$	$\frac{4I+3I}{6} = \frac{7I}{6}$	0.57
	BC	$\frac{I}{L} = \frac{I}{2}$		0.43
C	CB	$\frac{I}{L} = \frac{I}{4}$	$\frac{2I}{2} = I$	0.50
	CD	$\frac{I}{L} = \frac{I}{2}$		0.50

c) Moment Distribution Table :

A	B		C		D	
0	0.57	0.43	0.5	0.5	0	B.F.M
-	0	-2.0	+2.0	0	-	F.C.M
+0.57	+1.14	+0.86	-1.0	-1.0	-	Bal
-	-	-0.50	+0.43	-	-0.50	C.O
	+0.29	+0.21	-0.21	0.22	-	Bal
+0.15	-	-0.11	+0.10	-	-0.11	C.O
-	+0.06	+0.05	-0.05	-0.05	-	Bal
+0.03	-	-0.03	+0.03	-	-0.03	C.O
+0.01	+0.02	+0.01	-0.02	-0.01	-	B & C.O
+0.76	+1.51	-1.51	+1.28	-1.28	-0.64	Final moment

d) Reactions :

$$\text{Horizontal Reaction @ A} = h_a = \frac{M_{AB} + M_{BA}}{L} = \frac{0.76 + 1.51}{3}$$

$$h_a = \frac{2.27}{3} = 0.76 \text{ kN } (\rightarrow)$$

i horizontal reaction @ D, $h_d = -\left(\frac{1.28 + 0.64}{2}\right) = -\frac{1.92}{2}$

$$h_d = 0.96 \text{ kN } (\leftarrow) \quad \quad \quad = -0.96$$

The horizontal force $P = 0.96 - 0.76 = 0.20 \text{ kN}$
 $(\rightarrow)$

$$P = 0.20 \text{ kN } (\rightarrow)$$

The side Sway value of P

The value of P preventing side sway = $0.20 \text{ kN } (\rightarrow)$

e) Side Sway: Now let a side Sway $S = 0.20 \text{ kN } \leftarrow$ be applied at 'C'. $\therefore$ The columns AB & DE rotates in anticlockwise direction & thus clockwise moments will be induced at 'column heads'.

$$\frac{M_{BA}}{M_{CD}} = \frac{I_1/L_1^2}{I_2/L_2^2} = \frac{22/9}{214} = \frac{8}{9}$$

Let the arbitrary moments be:

$$M_{BA} = M_{AB} = +8 \text{ kNm}$$

$$M_{CD} = M_{DC} = +9 \text{ kNm}$$

g) Reactions:

$$\text{Horizontal Reaction at A} = \frac{6.26 + 4.53}{3} = 3.60 \text{ kN } (\rightarrow)$$

$$\text{Horizontal Reaction at D} = \frac{5.15 + 7.08}{2} = 6.12 \text{ kN } (\rightarrow)$$

	A	B	C	D	
	0.57	0.43	0.5	0.5	
+8.0	+8.0	-	-	+9.0	FEM
-	-4.57	-3.43	-4.50	-4.50	Bal
-2.29	-	-2.25	-1.72	-	CO
-	+1.29	+0.46	+0.86	+0.86	Bal
+0.64	-	+0.43	+0.48	-	CO
-	-0.25	-0.18	-0.24	-0.24	Bal
-0.13	-	-0.12	-0.09	-	CO
-	+0.07	+0.05	+0.05	+0.04	Bal
+0.04	-	+0.02	+0.03	-	CO
-	-0.01	-0.01	-0.02	-0.01	Bal
+6.26	+4.53	-4.53	-5.15	+5.15	Final Moments

Actual sway moments : 0.20 kN (←)

The sway force which induces the assumed moments

$$= 3.6 + 6.12 = 9.72 \text{ k (←)}$$

The corrected sway moment : Table.

	A	B	C	D	
1. Sway = 9.72 kN	+6.26	+4.53 -4.53	-5.15 +5.15	+7.08	(H)
2. Sway = 0.20 kN	+0.12	+0.09 -0.09	-0.10 +0.10	+0.15	
3. Non-sway	+0.76	+1.51 -1.51	+1.28 -1.28	+0.64	(C)
4. Final Moment	+0.88	+1.60 -1.60	+1.18 -1.18	-0.49	

Horizontal Reaction at A = $\frac{0.88 + 1.60}{3} = 0.83 \text{ kN} (\rightarrow)$

Horizontal Reaction at D = $\frac{-1.18 - 0.49}{2} = 0.83 \text{ kN} (\leftarrow)$

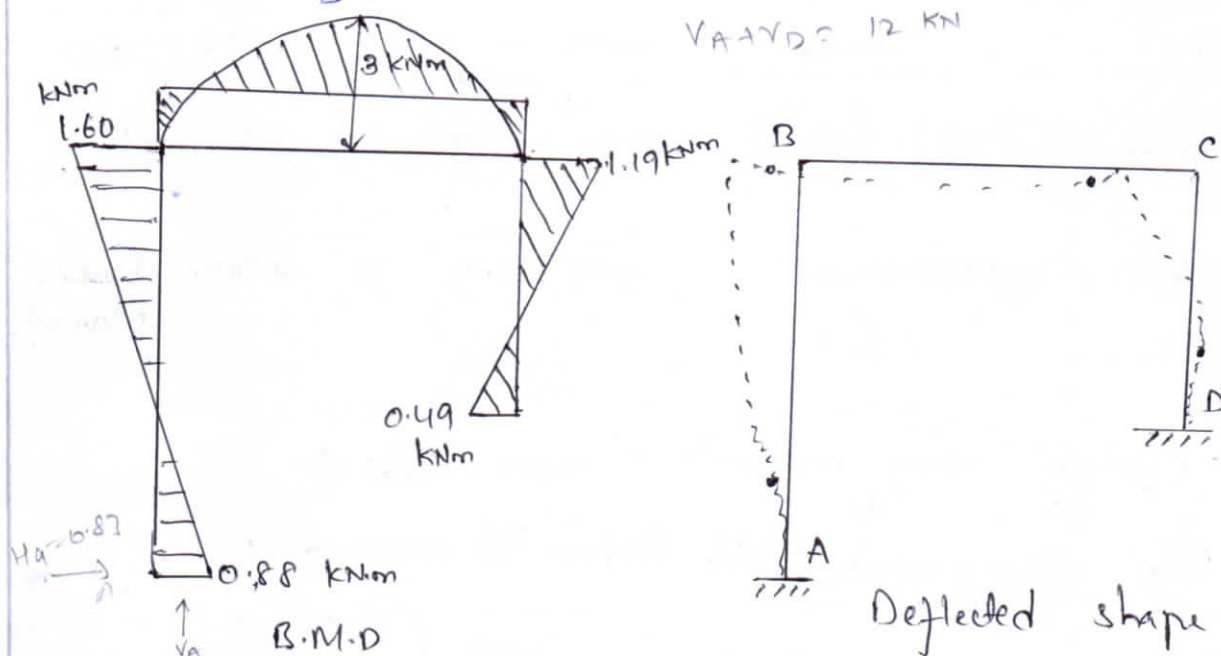
Vertical Reaction @ A = $\left. \begin{array}{l} \sum M_D = 0 \\ \sum M_A = 0 \end{array} \right\} \sum M_D = 0$

$2V_A - (0.83 \times 1) - (12 \times 1) + 0.88 - 0.49 = 0$

(or) $V_A = 6.22 \text{ kN}$ & $V_D = 12 - 6.22$

$V_D = 5.78 \text{ kN}$

$V_A + V_D = 12 \text{ kN}$



Prb: Use the method of Moment Distribution to analyse the portal frame shown: if the hinged support D sinks by an amount Δ , the members have the same uniform cross-section.

Sol: Let P be applied at C to prevent this sideways. When end D sinks, the end C will also settle. Due to settlement, there will be side sway in the right side.

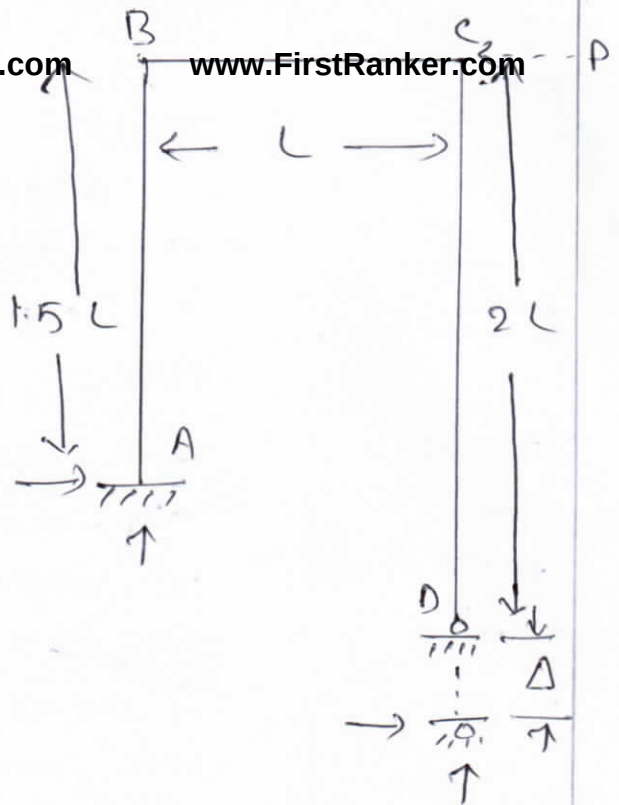
The settlement of end C will induce moments in BC in anticlockwise direction.

$$M_{FBC} = M_{FCB} = -\frac{6EI\Delta}{L^2}$$

$$= -6c \text{ [say]}$$

$$c = \frac{EI\Delta}{L^2}$$

b) Distribution factors?



Joint	Members	Relative Stiffness $\frac{I}{L}$	Sum $\sum \frac{I}{L}$	D.F $\frac{K}{\sum K}$
B	BA	$\frac{2I}{1.5L} = \frac{2I \cdot 2}{3 \cdot 0.5L}$	$\frac{5I}{3L}$	0.4
	BC	$\frac{I}{L} = \frac{3I}{3L}$		0.6
C	CB	$\frac{I}{L} = \frac{8I}{8L}$	$\frac{11I}{8L}$	0.73
	CD	$\frac{3}{4} \times \frac{I}{L} = \frac{3I \cdot 3}{4 \times 2 \cdot 8L}$		0.27

(c) Horizontal Reaction @ A = $\frac{0.85c + 1.71c}{1.5L} = \frac{1.713c}{L}$

$$= \frac{1.713 EI\Delta}{L^2} \quad (\rightarrow)$$

Horizontal reaction @ D = $\frac{1.28c}{2L} = \frac{0.64 EI\Delta}{L^2} \quad (\rightarrow)$

$$P = (1.173 + 0.64) \frac{EI\Delta}{L^2}$$

$$= 2.353 \frac{EI\Delta}{L^2} \quad (\rightarrow)$$

Marker's choice Distribution Table: [With no side Sway]

A	B	C	D	
	0.4	0.6	0.73	0.27
-	-	6.0 C	-6.0 C	-
-	+2.4 C	+3.6 C	+4.36 C	+1.64 C
+1.2 C	-	+2.18 C	+1.8 C	-
-	-0.87 C	-1.31 C	-1.31 C	-0.49 C
-0.44 C	-	-0.66 C	+0.66 C	-
-	+0.26 C	+0.40 C	+0.48 C	+0.18 C
+0.13 C	-	+0.24 C	+0.20 C	-
-	-0.10 C	-0.14 C	-0.14 C	-0.06 C
-0.05 C	-	-0.07 C	-0.07 C	-
-	+0.03 C	+0.04 C	+0.05 C	+0.02 C
+0.02 C	-	+0.03 C	+0.02 C	-
-0.01 C	← -0.01 C	-0.02 C	-0.01 C	-0.01 C
+0.85 C	+1.71 C	+1.71 C	-1.28 C	+1.28 C
			0	-

c) Distribution of correcting moments:

A	B	C	D	
	0.4	0.6	0.73	0.27
-8.0	-8.0	-	-2.25	-
-	+3.20	+4.80	+1.64	+0.61
+1.60	-	+0.82	+2.40	-
-	-0.33	-0.49	-1.75	-0.65
-0.16	-	-0.87	-0.24	-
-	+0.35	+0.52	+0.17	+0.07
+0.18	-	+0.09	+0.26	-
-	-0.04	-0.05	-0.19	-0.07
-0.02	-	-0.09	-0.03	-
+0.02 (←)	+0.04	+0.05	+0.02	+0.01
-6.38	-4.78	+4.78	+2.28	-2.28

Apply a force $S = \frac{2.353 \text{ EID}}{L^2}$ at B in opposite direction to that of P. Side sway will produce anticlockwise moment A, B & C,

$$\frac{M_{BA} \propto M_{AB}}{M_{CD}} = \frac{2I_1/L_1^2}{2I_2/L_2^2} = \frac{8/9}{1/4} = \frac{32}{9}$$

∴ arbitrary ^{sway} moments assumed are:

$$M_{BA} = M_{AB} = -8.0 \text{ kN.m}$$

$$M_{CD} = -2.25 \text{ kN.m}$$

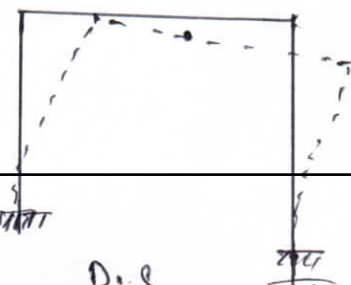
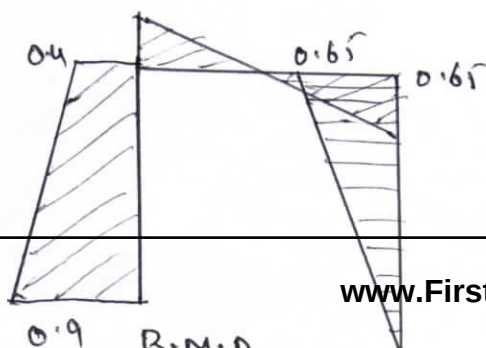
f) The horizontal reaction @ A = $-\frac{(6.38 + 4.78)}{1.5L} = \frac{7.44}{L} (\leftarrow)$

The horizontal reaction @ D = $\frac{2.28}{2L} = \frac{1.14}{L} (\leftarrow)$

The sway force = $\frac{7.44}{L} + \frac{1.14}{L} = \frac{8.58}{L} (\rightarrow)$

But the actual sway force is $\frac{2.353 \text{ EID}}{L^2}$ & hence the sway moments will have to be corrected according & added to the non-sway moments.

	A	B	C
1. Sway = $\frac{8.58}{L}$	-6.38	-4.78	+4.78
2. Sway = $\frac{2.353 \text{ EID}}{L^2}$	$-\frac{1.75 \text{ EID}}{L^2}$	$-\frac{1.31 \text{ EID}}{L^2}$	$+\frac{1.31 \text{ EID}}{L^2}$
3. Non sway moment	$+\frac{0.85 \text{ EID}}{L^2}$	$+\frac{1.71 \text{ EID}}{L^2}$	$+\frac{1.28 \text{ EID}}{L^2}$
4. Final moment	$-\frac{0.9 \text{ EID}}{L^2}$	$+\frac{0.4 \text{ EID}}{L^2}$	$+\frac{0.65 \text{ EID}}{L^2}$



Portal frame with inclined leg:

Prob: Inclined members are used, though less frequently, in pitched roofs, in high trestles, & in framed girders for bridges.

→ Mostly inclined members are used in buildings for elegance in appearance.

→ Rigid jointed structures involving inclined members in their construction are of two types:

→ First one is the single or multi bay rigid portal frame.

These types are used for construction of factories

& it not only presents a clean & elegant appearance of but also provides unrestricted internal space by bracing members.

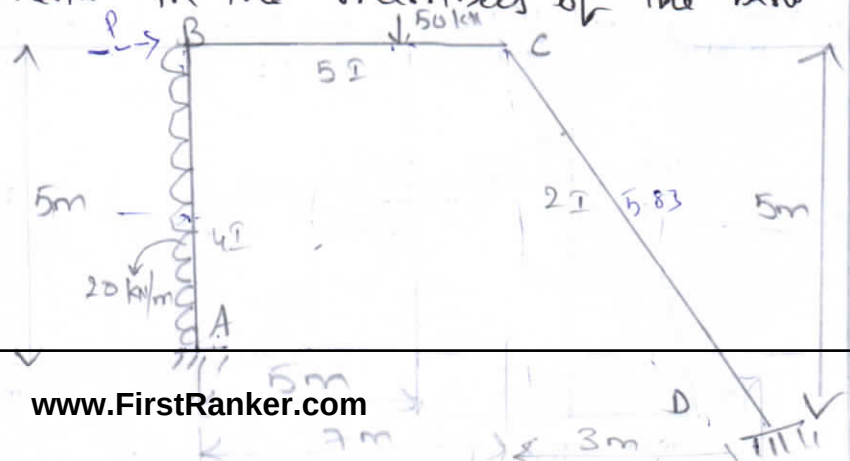
→ The second type is the open-frame cantilever or girder. This is used as trestles in vertical orientation or as a bridge in horizontal position.

→ In these types of applications, the inclined members also help to resist lateral forces better in open frame.

→ Supports the roof in portal frame construction.

→ In such applications, the inclined member is both functional & pleasing.

Prob: Find the end moments in the members of the bent as shown below:



Sol: The length of the member is $\sqrt{5^2 + 5^2} = 5.83m$.

The loading & geometrical configuration of the frame is unsymmetrical, it undergoes sway moment.

Moment Distribution under vertical loadings:

Relative stiffness:

Member	k	Σk	D.F (k/k)
AB	$4I/L$	$\frac{4I}{5} = 0.8I$	$0.8I / (0.8I + 0.714I) = 0.528 = D.F_{BA}$
BC	$5I/L$	$\frac{5I}{7} = 0.714I$	$0.714I / (0.714I + 0.343I) = 0.675$
CD	$2I/L$	$\frac{2I}{5.83} = 0.343I$	$0.343I / (0.714I + 0.343I) = 0.325$

$D.F_{AB} = 0, D.F_{BA} = 0.528, D.F_{BC} = 0.675, D.F_{CB} = 0.675$

$D.F_{CD} = 0.325$

Fixed-End Moments:

$M_{FBA} = \frac{20 \times 5^2}{12} = 41.67 \text{ kN.m}$

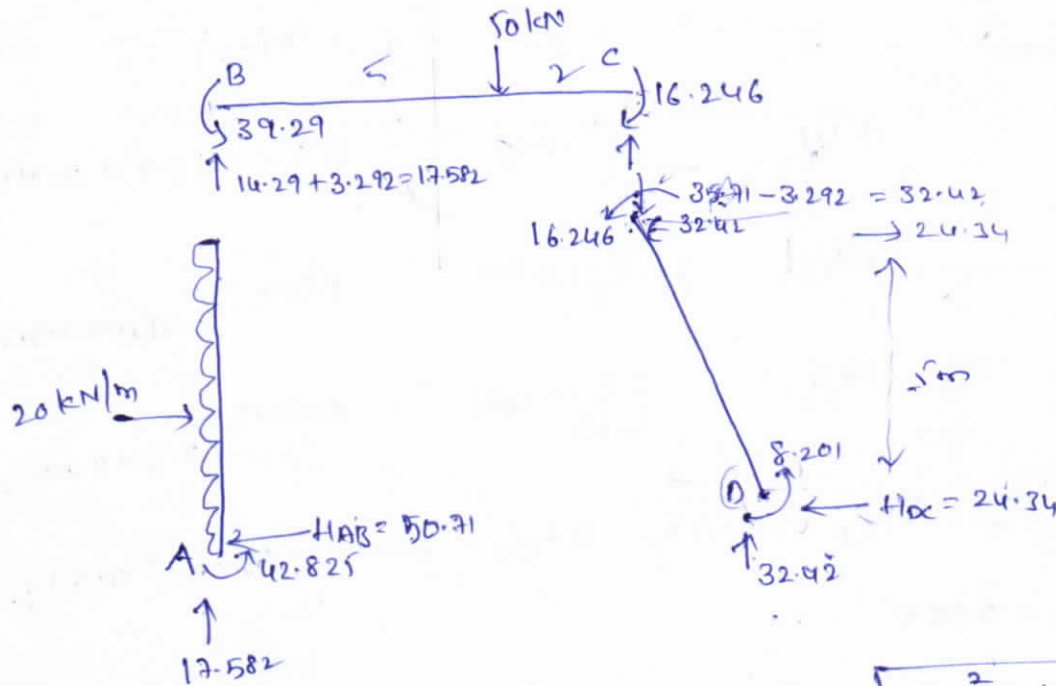
$M_{FBC} = \frac{50 \times 5 \times 2^2}{7^2} = 20.41 \text{ kN.m}$

$M_{FAB} = -41.67 \text{ kN.m}$

$M_{FCB} = \frac{-50 \times 1^2 \times 2}{7^2} = -5.02 \text{ kN.m}$

A	A	B	C	D	
0	0	0.528	0.675	0.325	0
41.67	41.67	-41.67	20.41	-5.02	0
	-	11.23	10.63	34.44	16.58
	5.615	-	17.22	5.015	-
	-	-9.09	-8.13	-3.385	-1.63
	-4.545	-	-1.693	-4.065	-
	-	0.894	0.8	2.744	1.321
	0.447	0	1.322	0.4	0
	-	-0.724	-0.648	-0.27	-0.12

A	B	C	D	
-0.362	-	-0.135	-0.324	-
-	0.0713	0.064	0.2187	0.1053
42.825	-39.29	39.29	-16.246	16.246
			8.201	F.M



$\sum M_B = 0$ for member AB

$$-H_{AB} \times 5 + 20 \times 5 \times \frac{5}{2} + 42.825 - 39.29 = 0$$

$$\therefore H_{AB} = 50.71 \text{ kN}$$

$\sum M_C = 0$ for member CD

$$-H_{DC} \times 5 + 8.201 + 32.42 \times 3 + 16.246 = 0$$

$$H_{DC} = 24.34 \text{ kN}$$

The force P acting at 'C',

$$P = 20 \times 5 - (50.71 + 24.34)$$

$$P = 24.95 \text{ kN}$$

$$= \frac{\sqrt{5.82^2 + 3.0^2}}{2}$$

$$= 3.27$$

$\sum M_B = 0$
 $R_B + R_C = 50$
 $\sum M_D = 0$

$$R_C \times 7 - 50 \times 5 = 0$$

$$R_C = 35.71$$

$$\therefore R_B = 14.29$$

$R_A + R_D = 20 \times 5$
 $R_D = 20 - R_A$
 $R_D \times 5 - 20 \times 2.5 = 0$

Moment Distribution for horizontal loading

A	B	C	D	
0	0.528	0.472	0.675	0.325
96	96	-73.47	-73.47	41.17
-	-11.90	-10.63	21.80	10.50
-5.95	-	10.9	-5.315	-
-	-5.755	-5.145	3.588	1.727
-2.878	-	1.794	-2.573	-
-	-0.947	-0.847	1.737	0.826
-0.4735	-	0.8685	-0.4235	-
-	-0.459	-0.41	0.286	0.138
86.7	76.94	-76.99	-54.37	54.37

The fixed end Moments due to translation is given by

$$\frac{6EI\Delta}{L^2} \text{ Assume } EI\Delta = 100$$

$$M_{FAB} = \frac{6EI(4I)\Delta}{L^2} = \frac{6 \times 100 \times 4}{5^2} = 0.96 EI\Delta = 96 \text{ kN.m}$$

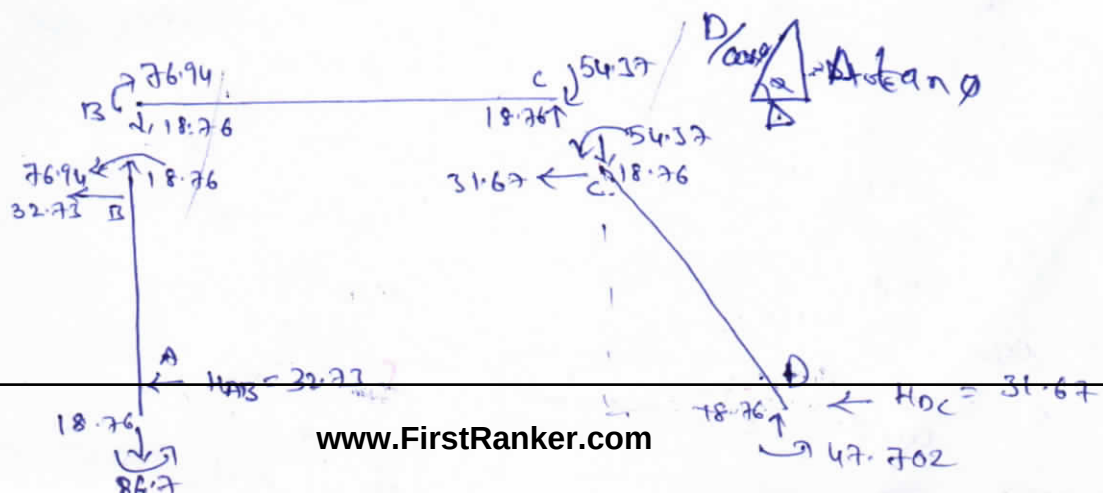
$$M_{FBA} = 96 \text{ kN.m}$$

$$M_{FCD} = \frac{6EI(2I)\Delta}{5.832^2} \times \frac{1}{\cos\theta} = \frac{12EI\Delta}{5.832 \times (5/5.38)} = 41.17 \text{ kN.m}$$

$$M_{FDC} = 41.17 \text{ kN.m}$$

$$M_{FBC} = M_{FCB} = -\frac{6EI(5I)}{7^2} (\Delta \tan\theta) = -\frac{6EI(5I)}{7^2} 2\Delta \left(\frac{3}{5}\right)$$

$$= -0.735 \times 100 = -73.47 \text{ kN.m}$$



$$\sum H_B = 0$$

$$-H_{AB} \times 5 + 47.702 + 54.37 + 18.76 \times 3 = 0$$

$$\therefore H_{AB} = 32.73 \text{ kN}$$

$$\sum M_C = 0$$

$$-H_{BC} \times 5 + 47.702 + 54.37 + 18.76 \times 3 = 0$$

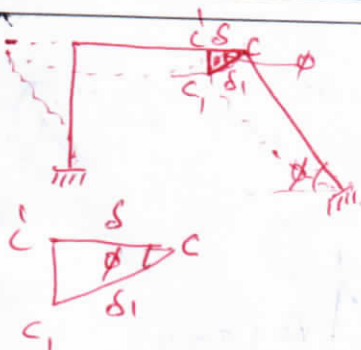
$$\therefore H_{BC} = 31.67 \text{ kN}$$

$$\therefore \text{The horizontal force } P' = 32.73 + 31.67 = 64.4 \text{ kN.}$$

$$\therefore \text{The correction factor } k = \frac{24.95}{64.4} = 0.387.$$

The End moments :

Members	AB	BA	BC	CB	CD	DC
Final moments under fixed loads	42.825	-39.29	39.29	-16.246	16.246	8.261
kx fixed moment under sway moment	33.55	29.78	-29.79	-21.04	21.04	18.46
final end moments kNm	76.38	-9.51	9.5	-37.29	37.29	26.66



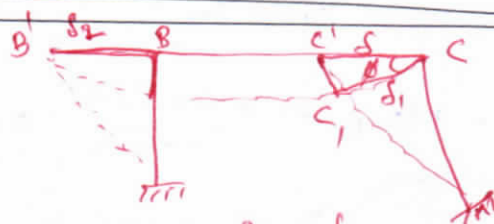
$$\cos \phi = \frac{\delta}{\delta_1}$$

$$\delta_1 = \frac{\delta}{\cos \phi}$$

(or)

$$\delta_1 = \frac{\Delta}{\cos \phi}$$

$$\delta_1 = \Delta \times \frac{1}{\cos \phi}$$



$$\delta_1 = \frac{\delta}{\cos \phi}$$

$$\sin \phi = \frac{C'C_1}{\delta_1}$$

$$B'B = C_1C' = \delta_2$$

$$C'C_1 = \delta_1 \sin \phi$$

$$C'C_1 = \frac{\delta}{\cos \phi} \cdot \sin \phi$$

$$\delta_2 = \delta \tan \phi$$

$$\delta_2 = \Delta \tan \phi$$

Kani's Method :
 ⇒ This method was developed by "Gasper Kani" of Germany.

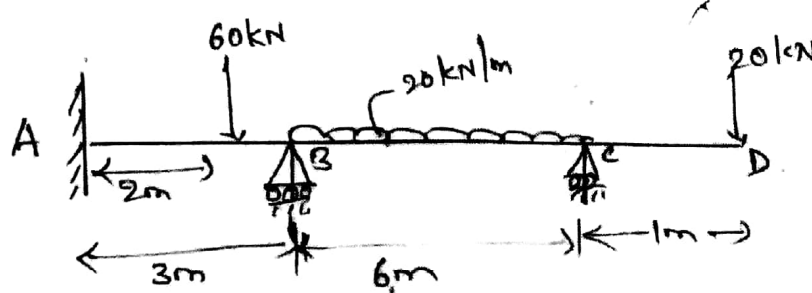
⇒ This method is an excellent method extension of the slope deflection method.

⇒ Kani's method is also known as "rotation contribution method".

⇒ Rotation contribution method is used to analysis of statically indeterminate structures, like continuous beams, portal frames etc.

Prb. Analysis of Continuous beam including settlement of support

1) Analyse the continuous beam shown below @ by Kani's method if the support B yields by 9mm. Take $EI = 1 \times 10^{12} \text{ Nmm}^2$ through out. Draw B.M.D



$$EI = 1 \times 10^{12} \text{ Nmm}^2 = 1 \times 10^3 \text{ kNm}^2$$

$$\Delta = 9 \text{ mm} = 0.009 \text{ m}$$

Sol: Fixed End Moments :

$$M_{FAB} = -\frac{Wab^2}{l^2} - \frac{6EI\delta}{l^2}$$

$$= -\frac{60 \times 2 \times 1^2}{3^2} - \frac{6 \times 1 \times 10^3 \times 0.009}{3^2}$$

$$= -12.22 - 12.22 = -24.44 \text{ kNm}$$

Note: The sum of rotation factors at a joint is equal to

$$\boxed{-0.5}$$

$$R_{BA} + R_{BC} = -(0.147 + 0.353) = -0.5$$

3. Resultant Restraint moments:-

At each joint.

$$M_{Fi} = \sum_j M_{Fij}$$

$$M_{FB} = M_{FBA} + M_{FBC} = +6 - 2.4 = +3.6 \text{ kN.m}$$

$$M_{FC} = M_{FCB} + M_{FCD} = +3.6 - 5.0 = -1.4 \text{ kN.m}$$

A	B	C	D
6.0	6.0	3.6	5.0
	$\boxed{+3.6}$	$\boxed{-1.4}$	
	-0.147	-0.353	
	-0.24	$+3.6$	
			-5.0
			$+5.0$
M_{AB}	M_{BA}	M_{CB}	M_{CD}
0	-0.598	+0.466	0
I	-0.668	+0.944	
II	-0.676	+1.080	
III	-0.677	+1.007	
IV	-0.677	+1.008	
V	-0.677	+1.008	

Step: 4 Kani's Iteration Cycle 8

$\theta_i = 0$; Note: M_{AB} & M_{DC} are zero, since ends A & D are fixed.

at joint C.

cycle-1 $M_{BC} = 0$

$$M_{CB} = R_{CB}(M_{FC}) = -0.373(-1.4) = +0.466$$

$$M_{CD} = R_{CD}(M_{FC}) = -0.167(-1.4) = +0.234$$

① joint B.

$$m_{BC} = R_{BC} (M_{FB} + m_{CB}) = -0.353 (+3.6 + 0.466) \\ = -1.435$$

$$m_{BA} = R_{BA} (M_{FB} + m_{CB}) = -0.147 (+3.6 + 0.466) \\ = -0.598$$

Cycle-2 :

$$m_{CB} = R_{CB} (M_{FC} + m_{BC}) = -0.333 (-1.4 - 1.435) = +0.944$$

$$m_{CD} = R_{CD} (M_{FC} + m_{BC}) = -0.167 (-1.4 - 1.435) = +0.473$$

② joint B

$$m_{BC} = R_{BC} (M_{FB} + m_{CB}) = -0.353 (+3.6 + 0.944) = -1.604$$

$$m_{BA} = R_{BA} (M_{FB} + m_{CB}) = -0.147 (+3.6 + 0.944) = -0.668$$

Cycle-3 :

$$m_{CB} = -0.333 (-1.4 - 1.604) = +1.000$$

$$m_{CD} = -0.167 (-1.4 - 1.604) = +0.502$$

② joint B

$$m_{BC} = -0.353 (+3.6 + 1.000) = -1.624$$

$$m_{BA} = -0.147 (+3.6 + 1.000) = -0.676$$

Cycle-4 :

① C, $m_{CB} = -0.333 (-1.4 - 1.624) = +1.007$

$$m_{CD} = -0.167 (-1.4 - 1.624) = +0.505$$

② B

$$m_{BC} = -0.353 (+3.6 + 1.007) = -1.626$$

$$m_{BA} = -0.147 (+3.6 + 1.007) = -0.677$$

Cycle-5 :

① C, $m_{CB} = -0.333 (-1.4 - 1.626) = +1.008$

$$m_{CD} = -0.167 (-1.4 - 1.626) = +0.505$$

② B, $m_{BC} = -0.353 (+3.6 + 1.008) = -1.627$

$$m_{BA} = -0.147 (+3.6 + 1.008) = -0.677$$

$$M_{BA} = -0.677; \quad m_{BC} = -1.627; \quad m_{CB} = +1.008; \quad m_{CD} = +0.505$$

displacement equations in iteration are:

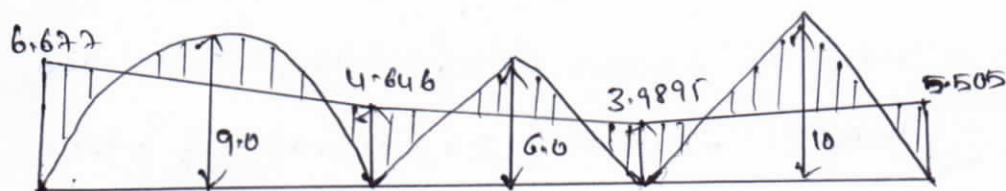
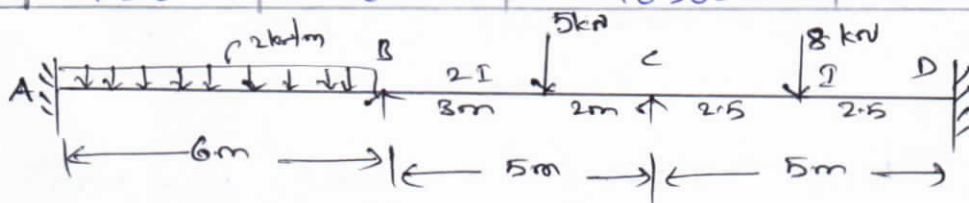
$$\Delta_B = \frac{m_{BA}}{2EI k_{BA}} = \frac{-0.677}{2EI (I/6)} = -\frac{2.031}{EI}$$

$$\Delta_C = \frac{m_{CB}}{2EI k_{CB}} = \frac{+1.008}{2EI (2I/5)} = \frac{1.26}{EI}$$

5. Computation of final moments at joint.

$$M_{AB} = M_{FAB} + 2M'_{AB} + M_{BA} = -6 + 2 \times 0 + (-0.677) = -6.677$$

M_{ij}	M_{Fij}	$2m'_{ij}$	m_{ji}	Sum (kN.m)
M_{AB}	-6.0	0	-0.677	-6.677
M_{BA}	+6.0	-1.354	0	+4.646
M_{BC}	-2.4	-3.254	+1.008	-4.646
M_{CB}	+3.6	+2.016	-1.627	+3.989
m_{CD}	-5.0	+1.010	0	-3.990
m_{DC}	+5.0	0	+0.505	+5.505



B.M.D & F.M.D



Analysis of Single bay single storey & single bay two storey frames by Kan's Method Including Side Sway.

Prob: Analyse the Portal frame shown below by Kan's Method. Draw the B.M.D & sketch the deflected shape of the frame. Take EI constant for all the members.

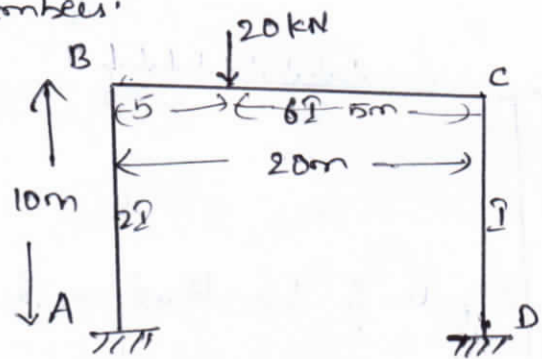
Sol: The frame unsymmetrical.

Case-1

Step-1: Fixed End moments.

$$M_{FBC} = - \frac{20 \times 5 (15)^2}{20^2} = -56.25 \text{ kNm}$$

$$M_{FCB} = + \frac{20 \times 15 \times 5^2}{20^2} = 18.75 \text{ kNm}$$



Step-2: Rotation Factors:

$$k_{ij}^o = \frac{I_{ij}^o}{L_{ij}^o} \quad \& \quad R_{ij}^o = -0.5 \frac{k_{ij}^o}{\sum k_{ij}^o}$$

$$k_{BA} = \frac{2I}{10}$$

$$k_{BC} = \frac{6I}{20} = \frac{3I}{10} = k_{CB}$$

$$k_{CD} = \frac{I}{10}$$

$$R_{BA} = -0.5 \frac{2I/10}{\frac{2I}{10} + \frac{3I}{10}} = -0.5 \times \frac{2}{5} = -0.2$$

$$R_{BC} = -0.5 \frac{3I/10}{\frac{2I}{10} + \frac{3I}{10}} = -0.5 \times \frac{3}{5} = -0.3$$

$$R_{CB} = -0.5 \frac{3I/10}{\frac{3I}{10} + \frac{I}{10}} = -0.5 \times \frac{3}{4} = -0.375$$

$$R_{CD} = -0.5 \frac{I/10}{\frac{3I}{10} + \frac{I}{10}} = -0.5 \times \frac{1}{4} = -0.125$$

Step-3 : Displacement factors:

$$D_{ij}^0 = -1.5 \frac{k_{ij}^0}{\sum k_{ij}^0} =$$

$$D_{BA} = -1.5 \left[\frac{k_{BA}}{k_{BA} + k_{CD}} \right] = -1.5 \left[\frac{2I/10}{\frac{2I}{10} + \frac{I}{10}} \right] = -1.5 \times \frac{2}{3}$$

$$D_{BA} = -1.0$$

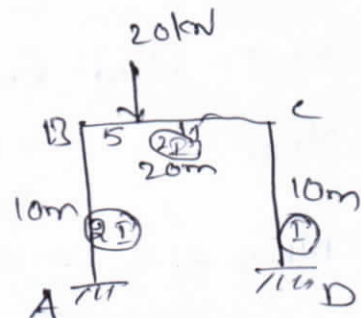
$$D_{CD} = -1.5 \left[\frac{k_{CD}}{k_{BA} + k_{CD}} \right] = -1.5 \left[\frac{I/10}{\frac{2I}{10} + \frac{I}{10}} \right] = -1.5 \times \frac{1}{3}$$

$$D_{CB} = -0.5$$

Step-4 : Resultant restaint moment

$$M_{PB} = -56.25 \text{ \& } M_{FC} = +18.75$$

Step-4 : Kani's Iteration Cycles :



B		C	
-56.25 0 1		+18.75 0 -0.125	
-56.25		+18.75	
m _{BC}		m _{CB}	
+16.88		-13.17	
+22.84		-14.17	
+24.28		-13.97	
m _{BA} +24.52		+13.88	
+11.25 +24.56		-13.86	
+15.26		m _{CD}	
+16.18		-4.39	
+16.35		-4.71	
+16.37		-4.66	
		-4.63	
		-4.62	
D _{BA} -1.0		D _{CB} -0.5	
m _{BA}		m _{CD}	
-6.86		-3.43	
-10.55		-5.28	
-11.52		-5.76	
-11.72		-5.88	
		-5.87	

$$m_{ij}^0 = R_{ij}^0 \left[M_{Fi} + \frac{1}{2} (m_{ji}^0 + m_{ij}^0) \right]$$

$$\textcircled{a} B = m_{BC} = -0.3 \left[-56.25 + 0 + 0 \right] = +16.88$$

$$m_{BA} = -0.2 \left[-56.25 + 0 + 0 \right] = +11.25$$

$$\textcircled{a} C = m_{CB} = R_{CB} \left[M_{Fc} + m_{BC} + m_{DC} + m_{CD}^0 \right]$$

$$= -0.375 \left[18.25 + 16.88 + 0 + 0 \right] = -13.17$$

$$m_{CD} = R_{CD} \left[M_{Fc} + m_{BC} + m_{DC} + m_{CD}^0 \right]$$

$$= -0.125 \left[18.25 + 16.88 + 0 + 0 \right] = -4.39$$

Displacement Contribution:

$$m'_{BA} = D_{BA} \left[m_{BA} + m_{AB} + m_{CD} + m_{DC} \right] = -1.0 \left[+11.25 + 0 - 4.39 + 0 \right] = -6.86$$

$$m'_{CD} = D_{CD} \left[m_{BA} + m_{AB} + m_{CD} + m_{DC} \right] = -0.5 \left[11.25 + 0 - 4.39 + 0 \right] = -3.43$$

Cycle-2:

Rotation factors

$$\begin{aligned} \textcircled{a} \text{ joint B} = m_{BC} &= R_{BC} \left[M_{FB} + m_{CB} + m_{AB} + m'_{BA} \right] \\ &= -0.3 \left[-56.25 - 13.17 + 0 - 6.86 \right] = 22.84 \end{aligned}$$

$$\begin{aligned} m_{BA} &= R_{BA} \left[M_{FB} + m_{CB} + m_{AB} + m'_{BA} \right] \\ &= -0.2 \left[-56.25 - 13.17 + 0 - 6.86 \right] = 15.26 \end{aligned}$$

joint C:

$$\begin{aligned} m_{CB} &= R_{CB} \left[M_{Fc} + m_{BC} + m_{DC} + m'_{CD} \right] \\ &= -0.375 \left[+18.25 + 22.84 + 0 - 3.43 \right] \end{aligned}$$

$$= -14.19$$

$$m_{CD} = R_{CD} \left[M_{Fc} + m_{BC} + m_{DC} + m'_{CD} \right]$$

$$= -0.125 [+18.25 + 22.84 + 0 - 3.43] = -4.71$$

b) Displacement factors:

$$m'_{BA} = -1.0 [+15.26 + 0 - 4.71 + 0] = -10.55$$

$$m'_{CD} = -0.5 [+15.26 + 0 - 4.71 + 0] = -5.28$$

Cycle 3:

a) Rotation factors:

$$\text{joint B: } m_{BC} = -0.3 [-56.25 - 14.12 + 0 - 10.55] \\ = +24.28$$

$$m_{BA} = -0.2 [-56.25 - 14.12 + 0 - 10.55] \\ = +16.18$$

$$\text{joint C: } m_{CB} = -0.375 [+18.25 + 24.28 + 0 - 5.28] \\ = -13.97$$

$$m_{CD} = -0.125 [18.25 + 24.28 + 0 - 5.28] \\ = -4.66$$

b) Displacement factors:

$$m'_{BA} = -1.0 [+16.18 + 0 - 4.66 + 0] = -11.52$$

$$m'_{CD} = -0.5 [+16.18 + 0 - 4.66 + 0] = -5.76$$

Cycle 4

a) Rotation Factors

$$\text{joint B: } m_{BC} = -0.3 [-56.25 - 13.97 + 0 - 11.52] = 24.52$$

$$m_{BA} = -0.2 [-56.25 - 13.97 + 0 - 11.52] = +16.35$$

$$\text{joint C: } m_{CB} = -0.375 [18.25 + 24.52 + 0 - 5.76] = -13.88$$

$$m_{CD} = -0.125 [18.25 + 24.52 + 0 - 5.76] = -4.63$$

b) Displacement factors:

$$m'_{BA} = -1.0 [+16.35 + 0 - 4.63 + 0] = -11.72$$

$$m'_{CD} = -0.5 [+16.35 + 0 - 4.63 + 0] = -5.86$$

Ⓐ Rotation factors:

$$\text{Joint B} : m_{BC} = -0.3 [-56.25 - 13.88 + 0 - 11.72] = +24.56$$

$$m_{BA} = -0.2 [-56.25 - 13.88 + 0 - 11.72] = +16.37$$

$$\text{Joint C} : m_{CB} = -0.375 [+18.25 + 24.56 + 0 - 5.86] = -13.86$$

$$m_{CD} = -0.125 [+18.25 + 24.56 + 0 - 5.86] = -4.62$$

Ⓑ Displacement factors:

$$m'_{BA} = -1.0 [+16.35 + 0 - 4.62 + 0] = -11.73$$

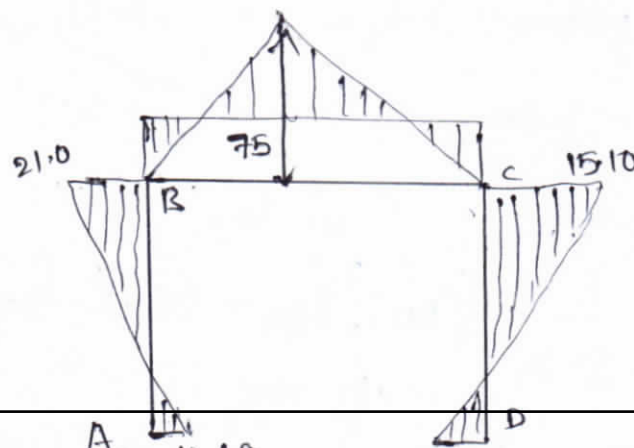
$$m'_{CD} = -0.5 [+16.35 + 0 - 4.62 + 0] = -5.87$$

Step-6: Final moments:

$$M_{ij} = M_{Fij} + 2m_{ij} + m_{ji} + m'_{ij}$$

$$M_{AB} = M_{FAB} + 2m_{ab} + m_{ba} + m'_{ab}$$

M_{ij}	M_{Fij}	$2m_{ij}$	m_{ji}	m'_{ij}	Total (kNm)
M_{BA}	0	+32.74	0	-11.73	+21.01
M_{BC}	-56.25	+49.12	-13.86	-	-20.99
M_{CB}	+18.25	-27.72	+24.56	-	+15.09
M_{CD}	0	-9.24	0	-5.87	-15.11
M_{DB}	0	0	+16.37	-11.73	+4.64
M_{DC}	0	0	-4.62	-5.87	-10.49



Prob. Draw the B.M.D & sketch the deflected shape of the frame:

Sol: 1) Fixed End Moments:

$$M_{FBC} = -\frac{6(2)^2}{12} = -2.0 \text{ kNm}$$

$$M_{FEB} = +2.0 \text{ kNm}$$

2) Rotation Factors:

$$k_{ij} = \frac{\Delta_{ij}}{L_{ij}} \quad \& \quad R_{ij} = -0.5 \frac{k_{ij}}{\sum_j k_{ij}}$$

$$k_{BA} = \frac{2I}{3}; \quad k_{BC} = \frac{I}{2} = k_{CB}; \quad k_{CD} = \frac{I}{2}$$

$$\therefore R_{BA} = -0.5 \frac{\frac{2I}{3}}{\frac{2I}{3} + \frac{I}{2}} = -0.5 \times \frac{2}{3} \times \frac{6}{7} = -0.2857$$

$$R_{BC} = -0.5 \frac{\frac{I}{2}}{\frac{2I}{3} + \frac{I}{2}} = -0.5 \times \frac{1}{2} \times \frac{6}{7} = -0.2143$$

$$R_{CB} = -0.5 \frac{\frac{I}{2}}{\frac{I}{2} + \frac{I}{2}} = -0.5 \times \frac{1}{2} \times 1 = -0.25$$

$$R_{CD} = -0.5 \frac{\frac{I}{2}}{\frac{I}{2} + \frac{I}{2}} = -0.5 \times \frac{1}{2} \times 1 = -0.25$$

Step: 3) Displacement factors:

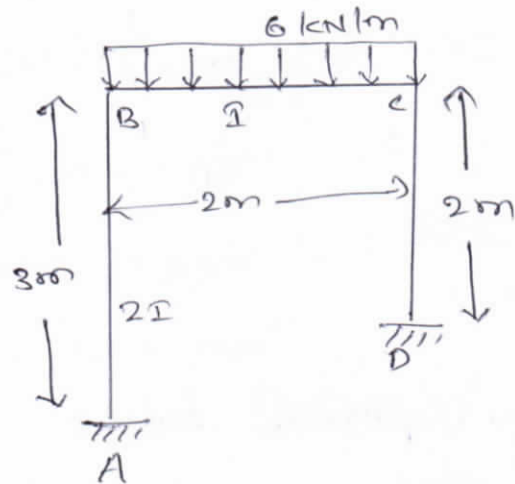
$$D_{ij} = -1.5 \frac{C_{ij} \cdot k_{ij}}{\sum_j C_{ij}^2 \cdot k_{ij}} \quad [\text{unequal legs}]$$

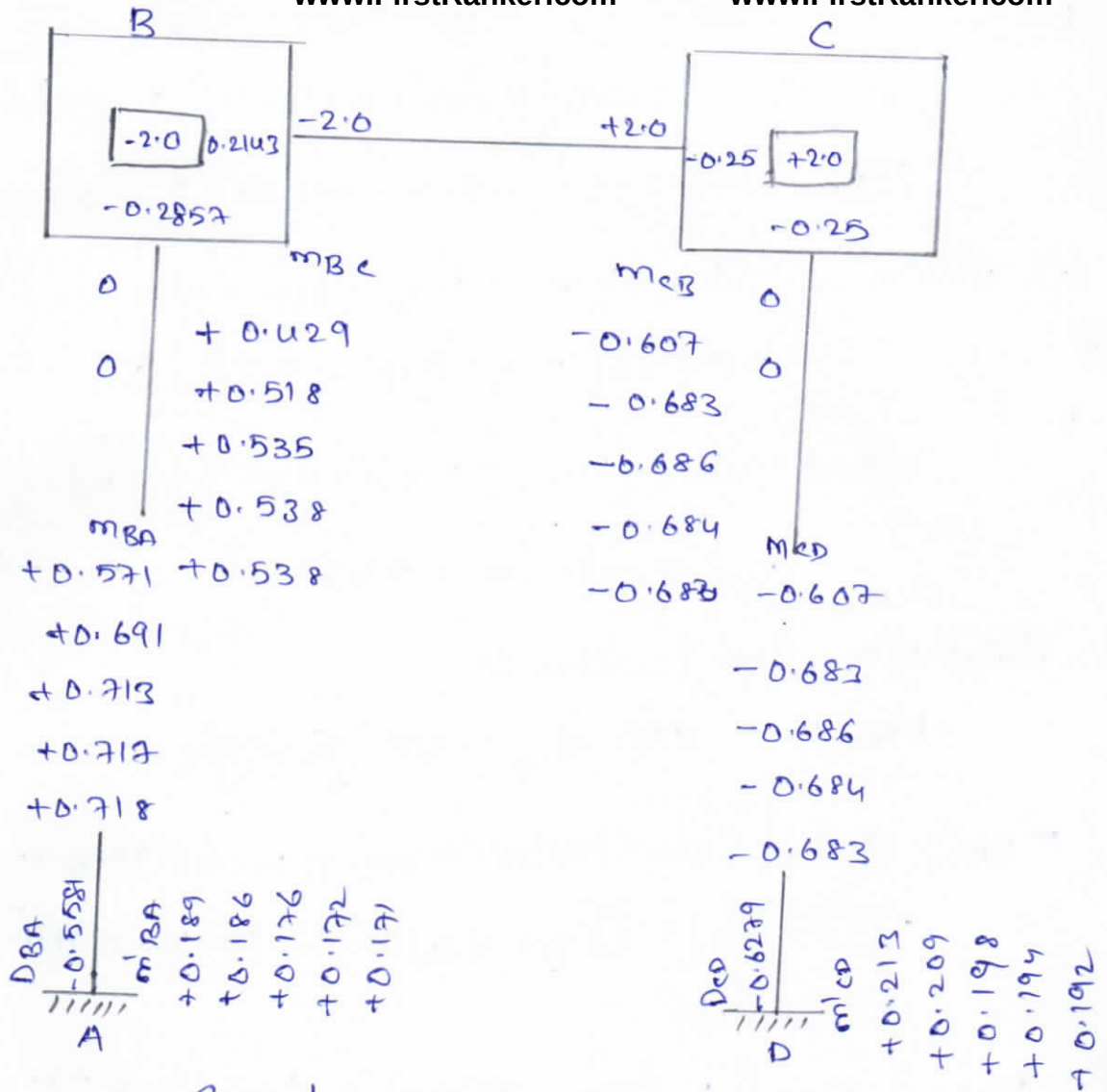
Let reference height is 3m

$$\therefore C_{BA} = \frac{3}{3} = 1 \quad \& \quad C_{CD} = \frac{3}{2} = 1.5$$

$$\therefore D_{BA} = -1.5 \frac{C_{BA} \cdot k_{BA}}{(C_{BA})^2 \cdot k_{BA} + (C_{CD})^2 \cdot k_{CD}}$$

$$= -1.5 \frac{1 \cdot \frac{2I}{3}}{(1)^2 \times \frac{2I}{3} + (1.5)^2 \frac{I}{2}} = -0.5581$$





$$D_{CD} = -1.5 \frac{C_{CD} \cdot K_{CD}}{(C_{BA})^2 \cdot K_{BA} + (C_{CD})^2 \cdot K_{CD}}$$

$$= -1.5 \frac{1.5 \times \frac{I}{2}}{(1)^2 \times \frac{2I}{3} + (1.5)^2 \cdot \frac{I}{2}}$$

$$D_{CD} = -0.6279$$

4) Resultant Restraint Moments:

$$M_{FB} = -2.0; M_{FC} = +2.0 \text{ kNm}$$

5) Kar's Iteration Cycle.

Cycle - 1: Rotation contribution m_{ij}^0 @ a joint i is

$$m_{ij}^0 = R_{ij} \left[M_{Fi} + \sum_j (m_{ji}^0 + m_{ij}^0) \right]$$

m_{ji}^0 & m_{ij}^0 are unknown ($\omega = 0$)

m_{ji}^0 & m_{ij}^0 are

$$\textcircled{a} B = m_{BC} = R_{BC} [M_{FB} + m_{CB} + m_{AB} + m'_{BA}]$$

$$= -0.2143 [-2.0 + 0 + 0 + 0] = +0.429$$

$$m_{BA} = -0.2857 [-2.0 + 0 + 0 + 0] = +0.571$$

$$\textcircled{a} C = m_{CB} = R_{CB} [M_{FC} + m_{BC} + m_{DC} + m'_{CD}]$$

$$= -0.25 [2.0 + 0.429 + 0 + 0] = -0.607$$

$$m_{CD} = R_{CD} [M_{FC} + m_{BC} + m_{DC} + m'_{CD}]$$

$$= -0.25 [+2.0 + 0.429 + 0 + 0] = -0.607$$

Distribution Contributions :

$$M_r = 0 ; \text{ ~~for~~ } m_{ij} = D_{ij} \left[\sum_r C_{ij} (m_{ij} + m_{ji}) \right]$$

$$m'_{BA} = D_{BA} [C_{BA} (m_{BA} + m_{AB}) + C_{CD} (m_{CD} + m_{DC})]$$

$$= -0.5581 [1 (0.571 + 0) + 1.5 (-0.607 + 0)]$$

$$= +0.189$$

$$m'_{CD} = D_{CD} [1 (m_{BA} + m_{AB}) + C_{CD} (m_{CD} + m_{DC})]$$

$$= -0.6279 [1 (0.571 + 0) + 1.5 (-0.607 + 0)]$$

$$= +0.213$$

Cycle - 2 : $\textcircled{a}$ Rotation factors.

$$\textcircled{a} B = m_{BC} = -0.2143 [-2.0 - 0.607 + 0 + 0.189] = +0.518$$

$$m_{BA} = -0.2857 [-2.0 - 0.607 + 0 + 0.189] = +0.691$$

$$\textcircled{a} C = m_{CB} = -0.25 [+2.0 + 0.518 + 0 + 0.213] = -0.682$$

$$m_{CD} = -0.25 [+2.0 + 0.518 + 0 + 0.213] = -0.682$$

$\textcircled{b}$ Displacement factors.

$$m_{BA} = -0.5581 [1 \times 0.691 - 1.5 \times 0.682] = +0.186$$

$$m'_{CD} = -0.6279 [1 \times 0.691 - 1.5 \times 0.682] = +0.209$$

① $B = m_{BC} = -0.2143 [-2.0 - 0.683 + 0 + 0.186] = +0.535$

$m_{BA} = -0.2857 [-2.0 - 0.683 + 0 + 0.186] = +0.713$

② $C = m_{CB} = -0.25 [+2.0 + 0.535 + 0 + 0.209] = -0.686$

$m_{CD} = -0.25 [+2.0 + 0.535 + 0 + 0.209] = -0.686$

③ Displacement factors:

$m'_{BA} = 0.5581 [+1 \times 0.713 + 0 - 1.5 \times 0.686 + 0] = +0.176$

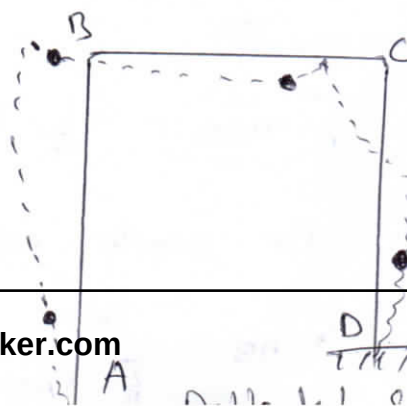
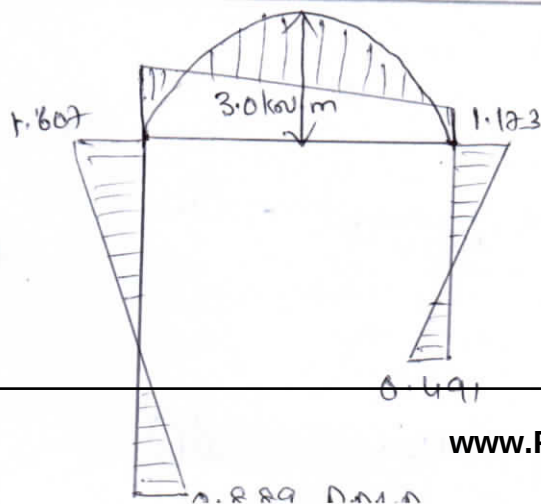
$m'_{CD} = -0.6229 [+1 \times 0.713 + 0 - 1.5 \times 0.686 + 0] = +0.192$

Similarly cycle -4 & cycle 5.

④ Final moments:

$M_{ij} = M_{Fij} + 2m'_{ij} + m_{ji} + m'_{ij} \Rightarrow M_{FAB} + 2m'_{ab} + m_{ba} + m'_{ab}$

M_{ij}	M_{Fij}	$2m'_{ij}$	m_{ji}	m'_{ij}	Total
M_{BA}	0	+1.436	0	+0.171	+1.607
M_{BC}	-2.0	+1.436	-0.683	-	-1.607
M_{CB}	+2.0	-1.366	0.538	-	+1.172
M_{CD}	+2.0	-1.366	+0.538	-	+1.174
M_{AD}	0	-1.366	0	+0.192	-1.174
M_{DC}	0	0	-0.687	+0.192	-0.491



The final deflected shape of a span AB of a continuous beam under imposed loads



Let M_{AB} & M_{BA} are moments developed

θ_A & θ_B are rotations at A & B due to imposed loads.

The slope deflection equation for span AB, at joint A is :

$$M_{AB} = M_{FAB} + 2EK_{AB} (2\theta_A + \theta_B) \quad \text{--- (1)}$$

where; $K_{AB} = \frac{I_{AB}}{L_{AB}}$

$$\therefore M_{AB} = M_{FAB} + 4EK_{AB}\theta_A + 2EK_{AB}\theta_B$$

$$M_{AB} = M_{FAB} + 2m_{AB} + m_{BA} \quad \text{--- (2)}$$

where by definition $m_{AB} = 2EK_{AB}\theta_A$ & $m_{BA} = 2EK_{AB}\theta_B$ --- (3)

m_{AB} = rotational contribution of end A to M_{AB} ,

m_{BA} = rotational contribution of end B to M_{AB} .

$$\therefore \sum_B M_{AB} = 0$$

from eq. (2)

$$\sum_B M_{FAB} + \sum_B (2m_{AB} + m_{BA}) = 0 \quad \text{--- (4)}$$

Let M_{FA} - resultant restraint moment at A.

$$M_{FA} = \sum_B M_{AB} \quad \text{--- (5)}$$

from eqn. (4)

$$\sum_B m_{AB} = -\frac{1}{2} (M_{FA} + \sum_B m_{BA}) \quad \text{--- (6)}$$

$\therefore$ For member AB,

$$m_{AB} = \frac{K_{AB}}{\sum_B K_{AB}} \sum_B m_{AB} \quad \text{--- (7)}$$

$$M_{AB} = \frac{\sum_B K_{AB}}{\sum_B K_{AB}} \quad \text{--- (7)}$$

$$(or) \quad M_{AB} = -\frac{1}{2} \left[\frac{K_{AB}}{\sum_B K_{AB}} \right] \left[M_{FA} + \sum_B M_{BA} \right]$$

$$M_{AB} = R_{AB} \left[M_{FA} + \sum_B M_{BA} \right] \quad \text{--- (8)}$$

where

R_{AB} = rotational factor for AB.

$$\therefore R_{AB} = -\frac{1}{2} \left[\frac{K_{AB}}{\sum_B K_{AB}} \right] \quad \text{--- (9)}$$

Note: Rotation factor $R = -\frac{1}{2}$ times D.F

i = Near end A & j = far free ends B.

$$M_{ij}^o = M_{Fij} + 2Ek_{ij}(\theta_i + \theta_j) \quad \text{where; } k_{ij} = \frac{I_{ij}}{L_{ij}}$$

$$k_{ij} = \frac{I_{ij}}{L_{ij}} \quad \text{--- (a)}$$

$$M_{ij}^o = M_{Fij} + 2m_{ij} + m_{ji} \quad \text{--- (b)}$$

where $m_{ij} = 2Ek_{ij}\theta_i$ & $m_{ji} = 2Ek_{ij}\theta_j$ --- (c)

$$\sum_j M_{Fij} + \sum_j (2m_{ij} + m_{ji}) = 0 \quad \text{--- (d)}$$

$$-M_{Fi} = \sum_j M_{Fij} \quad \text{--- (e)}$$

$$\sum_j m_{ij} = -\frac{1}{2} \left[M_{Fi} + \sum_j m_{ji} \right] \quad \text{--- (f)}$$

$$m_{ij} = R_{ij} (M_{Fi} + \sum_j m_{ji}) \quad \text{--- (g)}$$

where. $R_{ij} = -\frac{1}{2} \left[\frac{k_{ij}}{\sum_j k_{ij}} \right]$

Step-1: Calculate the fixed end moments (M_{Fij}) in all the members of the structure & enter them outside the outer square.

Step-2: Calculate the (k) values & rotation factors (R_{ij}) for all members meeting at each joint. These values are entered outside the first square but inside the second square towards each member.

Step-3: Find the resultant restraint moment at each joint using eq-⑤. Enter these values of resultant restraint moment within the inner square at each joint.

Step-4: Compute rotational contribution (m_{ij}) of the two ends of all the members by Gauss-Seidel iteration from eq-⑨, taking $m_{ij}' = 0$ at all joints starting with the approximation that $\theta_i = 0$. Continue the iteration through several cycles till practically the same values of m_{ij} are obtained in two successive cycles. Each cycle gives improved approximation for the rotational contribution. All these values of m_{ij} are entered.

Step-5: Using - eq-② determine member end moments M_{ij} .

⇒ This method was developed by "Gasper Kani" of Germany.

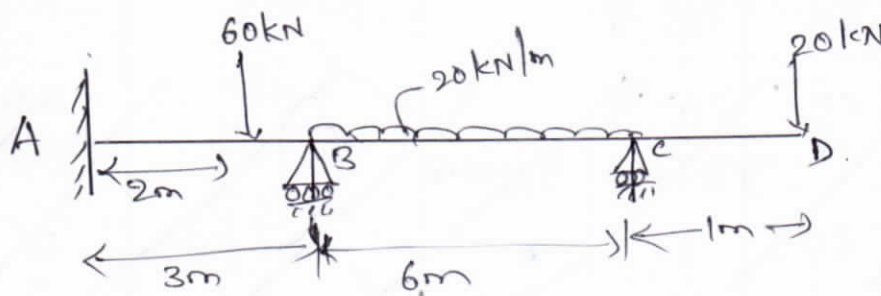
⇒ This method is an excellent method extension of the slope deflection method.

⇒ Kani's method is also known as "rotation contribution method".

⇒ Rotation contribution method is used to analysis of statically indeterminate structures, like continuous beams, Portal frames etc.

Prb. Analysis of Continuous beam including settlement of Support

1) Analyse the continuous beam shown below @ by Kani's method if the support B yields by 9mm. Take $EI = 1 \times 10^{12} \text{ Nmm}^2$ through out. Draw B.M.D



$$EI = 1 \times 10^{12} \text{ Nmm}^2$$

$$= 1 \times 10^3 \text{ kNm}^2$$

$$\Delta = 9 \text{ mm}$$

$$= 0.009 \text{ m}$$

Sol:- Fixed End Moments :

$$M_{FAB} = -\frac{Wab^2}{l^2} - \frac{6EId\delta}{l^2}$$

$$= -\frac{60 \times 2 \times 1^2}{3^2} - \frac{6 \times 1 \times 10^3 \times 0.009}{3^2}$$

$$= -13.33 - 6 = -19.33 \text{ kNm}$$

$$M_{FBA} = \left[\frac{W a^2 b}{L^2} \right] - \left[\frac{6 \Sigma I \delta}{L^2} \right] \Rightarrow \frac{60 \times 2^2 \times 1}{3^2} - \frac{6 \times 1000 \times 0.009}{2^2}$$

$$= 20.67 \text{ kN.m}$$

$$M_{FBC} = - \frac{WL^2}{12} + \frac{6 \Sigma I \delta}{L^2} = - \frac{20 \times 6^2}{6} + \frac{6 \times 1000 \times 0.009}{6^2} = -58.5 \text{ kN.m}$$

$$M_{FCB} = \frac{WL^2}{12} + \frac{6 \Sigma I \delta}{L^2} = \frac{20 \times 6^2}{6} + \frac{6 \times 1000 \times 0.009}{6^2}$$

$$= 61.5 \text{ kN.m}$$

$$M_{FCD} \text{ (cantilever moment)} = -W \times L \Rightarrow -20 \times 1 = -20 \text{ kN.m}$$

$$M_{FDC} = 0$$

2) Distribution Factor & Rotation Factors:

S.NO	Joint	Member	k	Σk	D.F = $k/\Sigma k$	R.F = -0.5 D.F
1.	B	BA	$I/l = \frac{I}{3}$	$0.5I$	0.667	-0.333
		BC	$I/l = \frac{I}{6}$		0.333	-0.166
2.	C	CB	$I/l = \frac{I}{6}$	$I/6 = 0.166$	1.0	-0.5
		CD	0		0	0

3) Rotation Contribution Moments:
Resultant Restraint

$$M_{FB} = M_{FBA} + M_{FBC} = 20.67 + (-58.5) = -37.82 \text{ kN.m}$$

$$M_{FC} = M_{FCB} + M_{FCD} = 61.5 - 20 = 41.5 \text{ kN.m}$$

$$M_{FB} = M_{FBA} + M_{FBC} = 20.67 + (-58.5) = -37.83 \text{ kNm}$$

4) Rotation Contribution Table:

A		B		C		D	
-19.33		20.67		61.5		-20	
M_{iab}		M_{ba}		M_{icb}		M_{idc}	
0		12.489		0		0	
0		6.279		0		0	

Rotation Contribution Moments:

$$M'_{ab} = 0 \quad M'_{ba} = 12.489 \quad M'_{bc} = 6.279 \quad M'_{cb} = 0 \quad M'_{dc} = 0 \quad M'_{cd} = 0$$

5) Final Moments:

$$M_{AB} = M_{FAB} + 2M'_{ab} + M'_{ba} = -19.33 + 0 + 12.489 = -6.841 \text{ kNm}$$

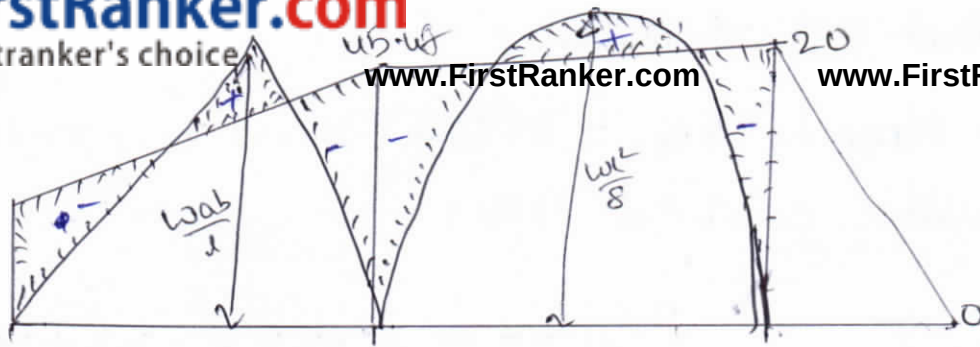
$$M_{BA} = M_{FBA} + 2M'_{ba} + M'_{ab} = 20.67 + 2 \times 12.489 + 0 = 45.658 \text{ kNm} \approx 45.47$$

$$M_{BC} = M_{FBC} + 2M'_{bc} + M'_{cb} = -58.5 + 2 \times 6.279 + 0 = -45.942 \text{ kNm} \approx -45.94$$

$$M_{CB} = M_{FCB} + 2M'_{cb} + M'_{bc} = 61.5 + (2 \times 0) + 6.279 = 67.779 \text{ kNm}$$

$$M_{CD} = M_{FCD} + 2M'_{cd} + M'_{dc} = -20 + 0 + 0 = -20 \text{ kNm}$$

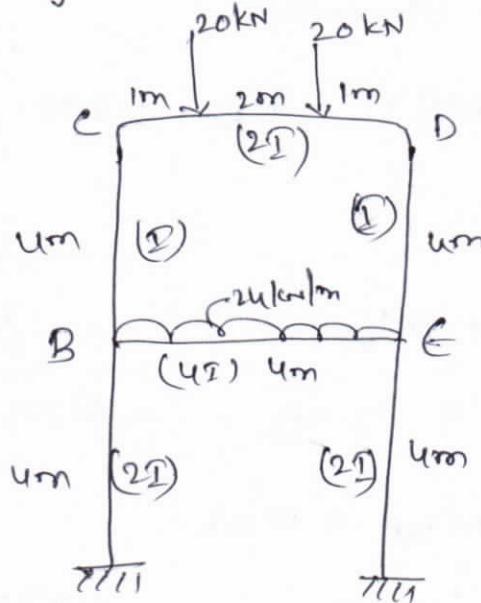
$$M_{DC} = 0$$



B.M.D

Analysis of single bay two-storey Portal Frame by Kani's Method :

Prb. Analyse the frame shown below by Kani's method.



Sol:- Analysis will be carried out taking the advantage of symmetry.

a) Fixed End Moments:

$$M_{FED} = - \left[\frac{wab^2}{l^2} + \frac{wab^2}{l^2} \right]$$

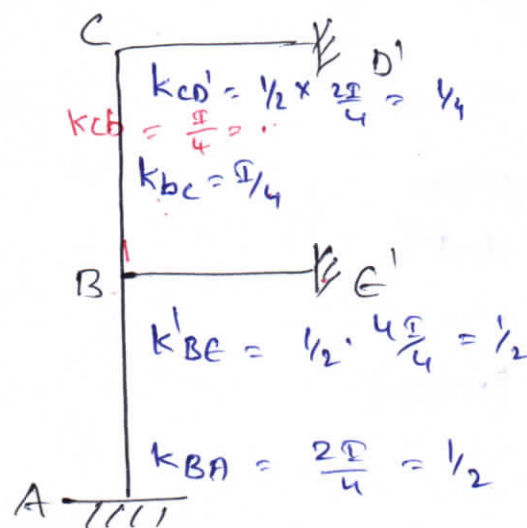
$$= - \left[\frac{20 \times 1 \times 3^2}{4^2} + \frac{20 \times 3 \times 1^2}{4^2} \right]$$

$$= - 15 \text{ kNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-24 \times 4^2}{12} = -32 \text{ kNm}$$

b) Rotation Factors :

Joint	Member	k	Σk	D.F $\frac{k}{\Sigma k}$	R.F $= -0.5 \times \text{D.F}$
B	BA	$2I/4$		0.4	$-1/5$
	BE'	$\frac{1}{2} \times \frac{4I}{4} = \frac{1}{2}$	$5I/4$	0.4	$-1/5$
	BC	$2I/4$		0.2	$-1/10$
C	CB	$2I/4$	$2I/4$	0.5	$-1/4$
	CD'	$\frac{1}{2} \times \frac{2I}{4} = \frac{1}{4}$		0.5	$-1/4$

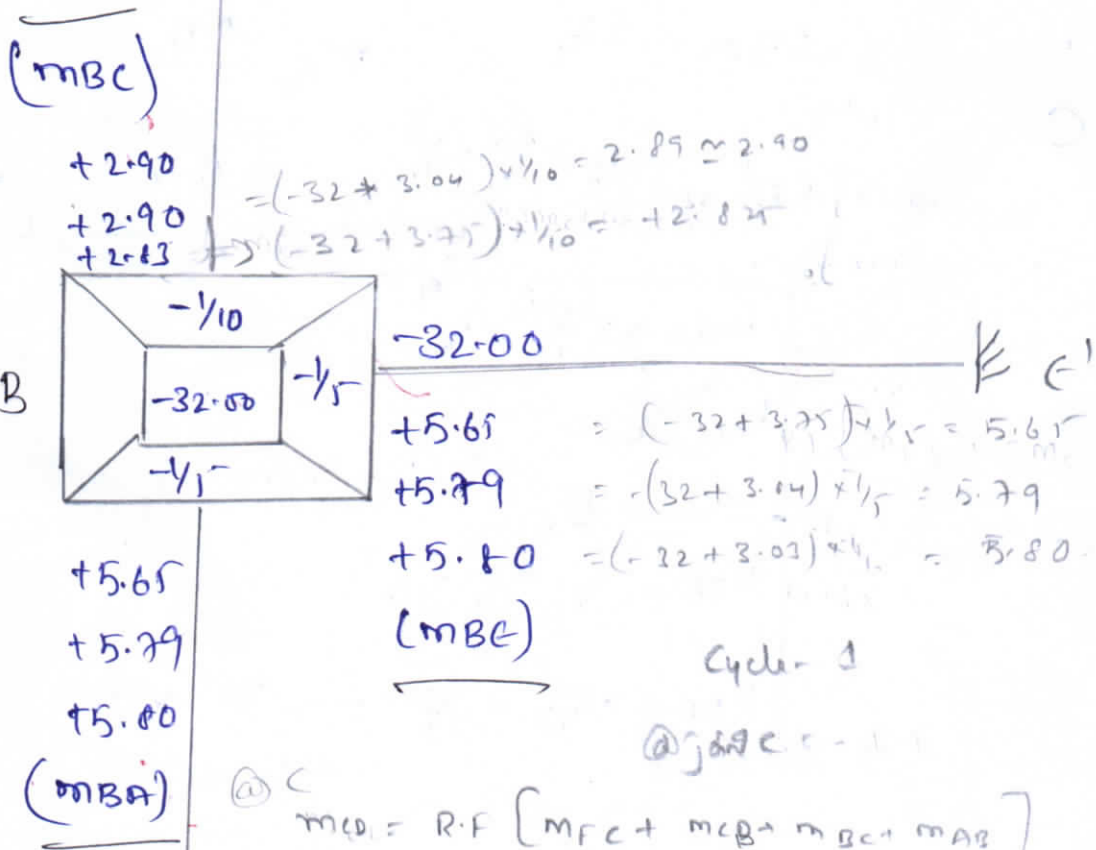
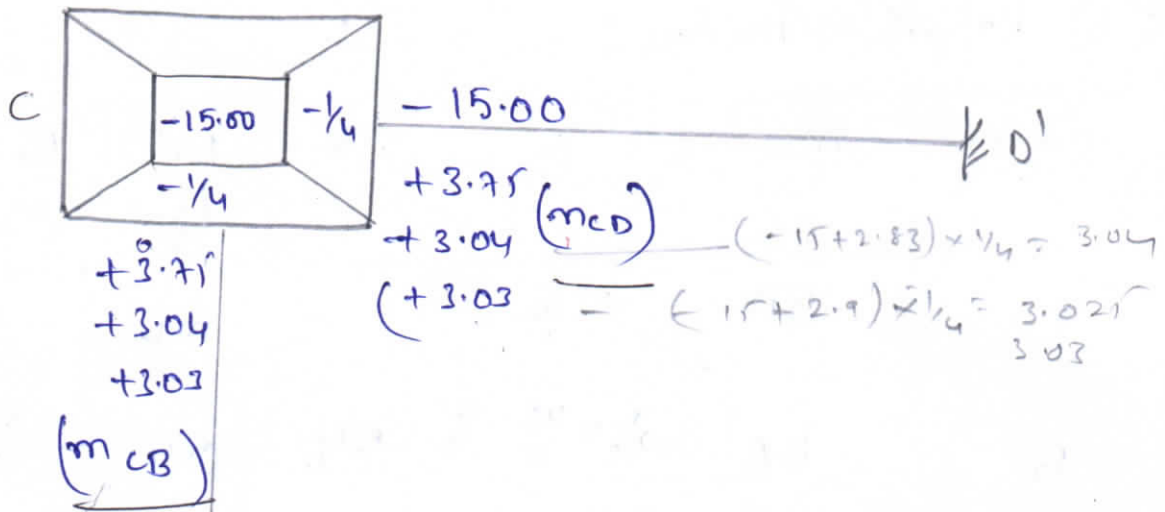


c) Resultant Restraint Moments :

$$M_{FC} = M_{FCD} + M_{FCB} = -15 + 0 = -15 \text{ kNm}$$

$$M_{FB} = M_{FBE} + M_{FBA} = -32 + 0 = -32 \text{ kNm}$$

Rotation Contribution calculation by Pradeep
cycles.



Cycle-1

@ joint C - 1

① C

$$m_{CD} = R.F [M_{FC} + m_{CB} + m_{BC} + m_{AB}]$$

$$= -1/4 [-15 + 0 + 0 + 0] = 3.75$$

$$m_{CB} = R.F [M_{FC} + m_{CD} + m_{BC} + m_{AB}]$$

$$= -1/4 [-15 + 0 + 0 + 0] = 3.75$$

② B

$$m_{BC} = R.F [M_{FB} + m_{CD} + m_{BA} + m_{AB}]$$

$$= -1/10 [-32 + 3.75 + 0 + 0]$$

$$m_{BA} = R.F [M_{FB} + m_{BC} + m_{CD} + m_{AB}]$$

$$= -1/5 [-32 + 0 + 3.75 + 0] = +5.65$$

$$m_{BE} = R \cdot F [M_{FB} + m_{BE} + m_{BA} + m_{CB} + 0] = -1/4 [-32 + 0 + 0 + 3.75 + 0] \\ = -1/4 [-32 + 3.75] = 5.65$$

Ly-81

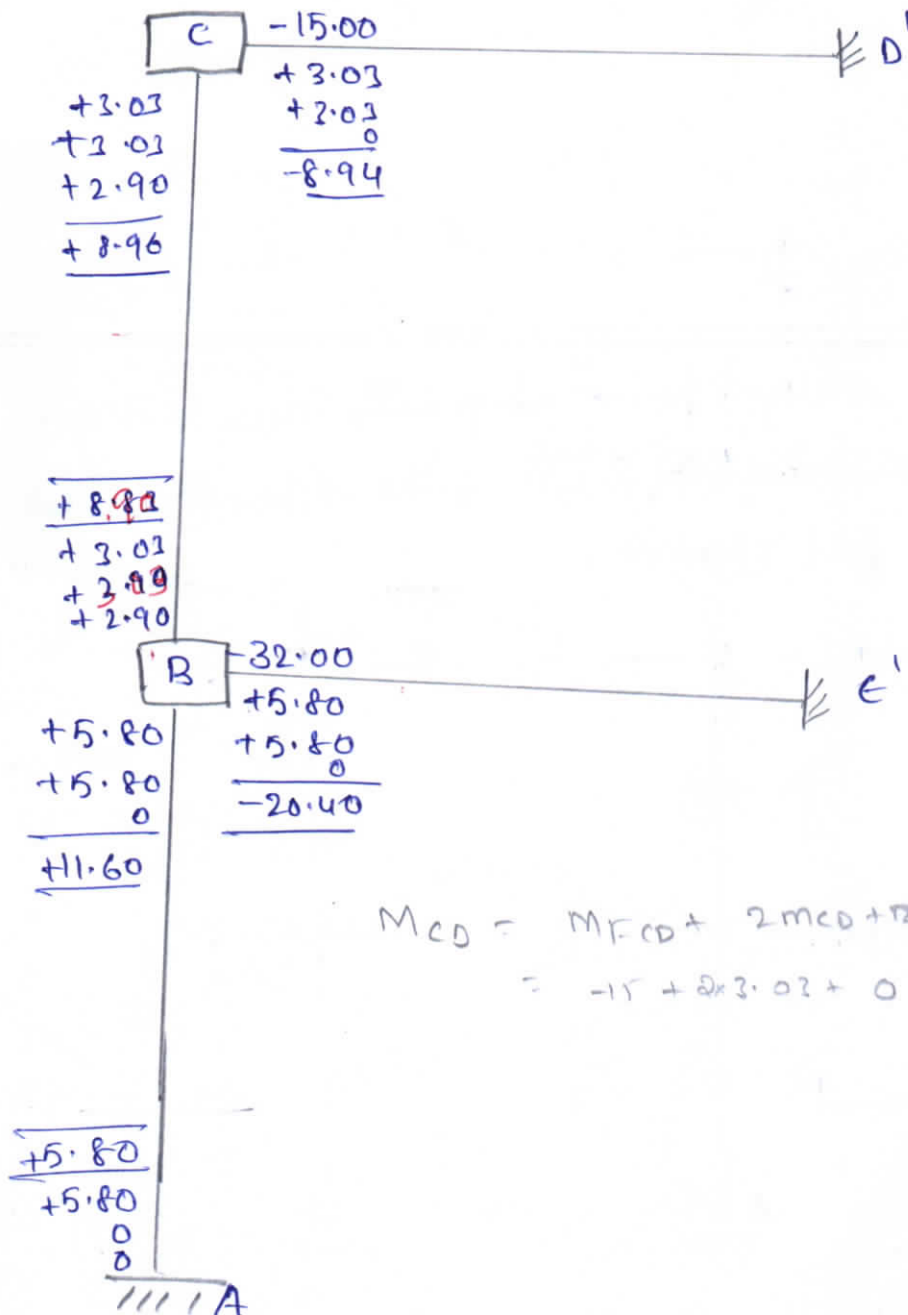
$$m_{CD} = R \cdot F [M_{FC} + m_{BC} + m_{CB}] = -1/4 [-15 + 2.03 + 0] = 3.0425$$

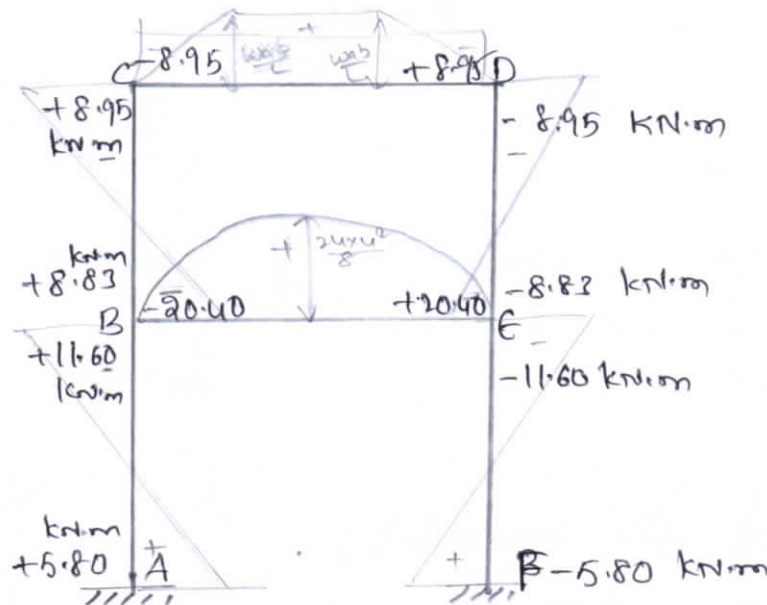
$$m_{CB} = R \cdot F [M_{FC} + m_{BC} + m_{CB}] = -1/4 [-15 + 2.83 + 0] = 3.04$$

$$m_{BC} = R \cdot F [M_{FB} + m_{CD} + m_{AB}] = -1/10 [-32 + 3.04 + 0] = 2.896$$

$$m_{BE} = R \cdot F [M_{FB} + 3.04 + 0] = -1/5 [-32$$

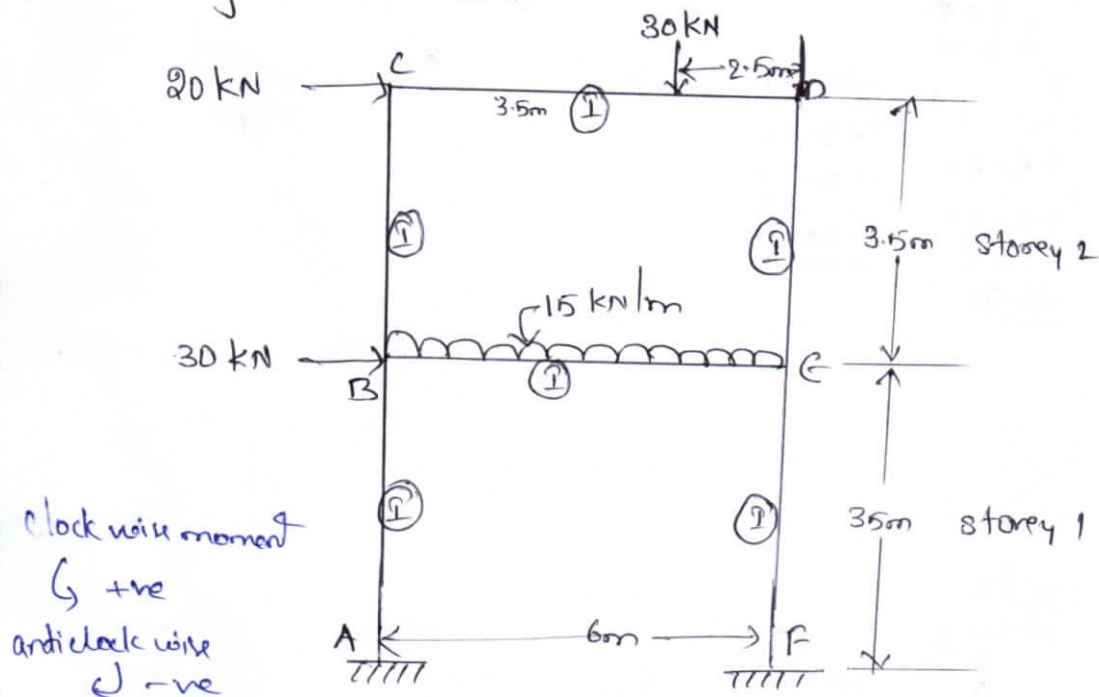
Calculation of final moments:





Two storey frames with lateral load: [Sway forces]

Prob: A two-storeyed frame along with lateral loading is shown below. The moments of fixed end of the different members are given. Find the joint moments.



Sol: Fixed-End Moments:

$$M_{FCD} = \frac{wab^2}{l^2} = + \frac{30 \times 3.5 \times 2.5^2}{6^2} = 18.125 \text{ kNm}$$

$$M_{FDC} = \frac{800 \times 3.5^2 \times 2.5}{6^2} = -25.52 \text{ kN}\cdot\text{m}$$

$$M_{FBE} = +\frac{wL^2}{12} = \frac{+15 \times 6^2}{12} = +45 \text{ kN}\cdot\text{m}; M_{FEB} = -45 \text{ kN}\cdot\text{m}$$

2. Rotation Factor Table:

Joint	Member	Relative stiffness 'K'	Sum ΣK	D.F = $1/\Sigma K$	R.F = $-0.5 \times \text{D.F}$
B	BA	$I/3.5$	$0.739 I$	0.386	-0.193
	BE	$I/6$		0.225	-0.112
	BC	$I/3.5$		0.386	-0.194
C	CD	$I/6$	$0.453 I$	0.363	-0.316
	DC	$I/6$		0.363	-0.184
D	DE	$I/3.5$	$0.453 I$	0.631	-0.316
	ED	$I/3.5$		0.631	-0.194
E	EB	$I/6$	$0.739 I$	0.225	-0.112
	ED	$I/3.5$		0.6386	-0.194
	EF	$I/3.5$		0.386	-0.194

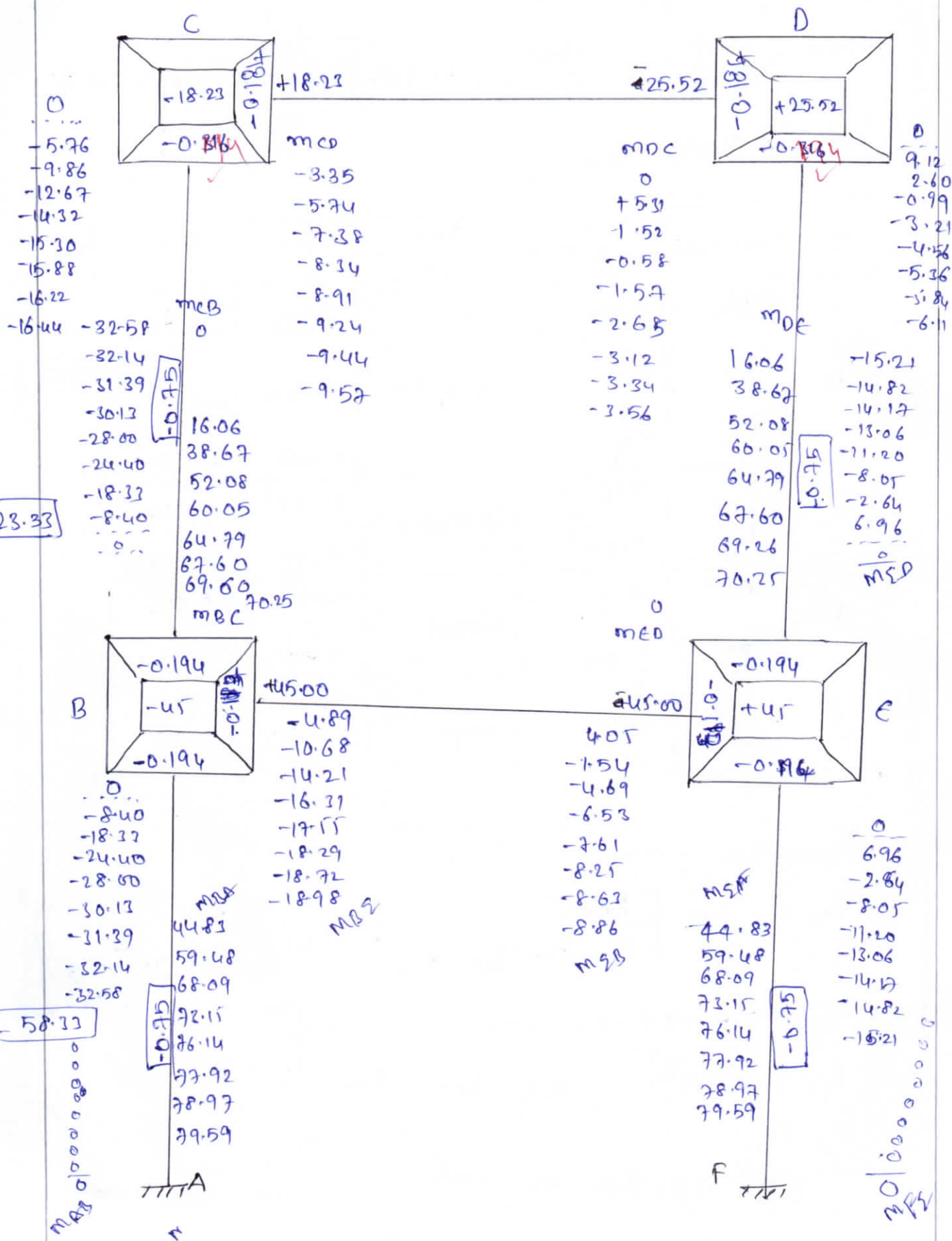
3. Displacement factors:

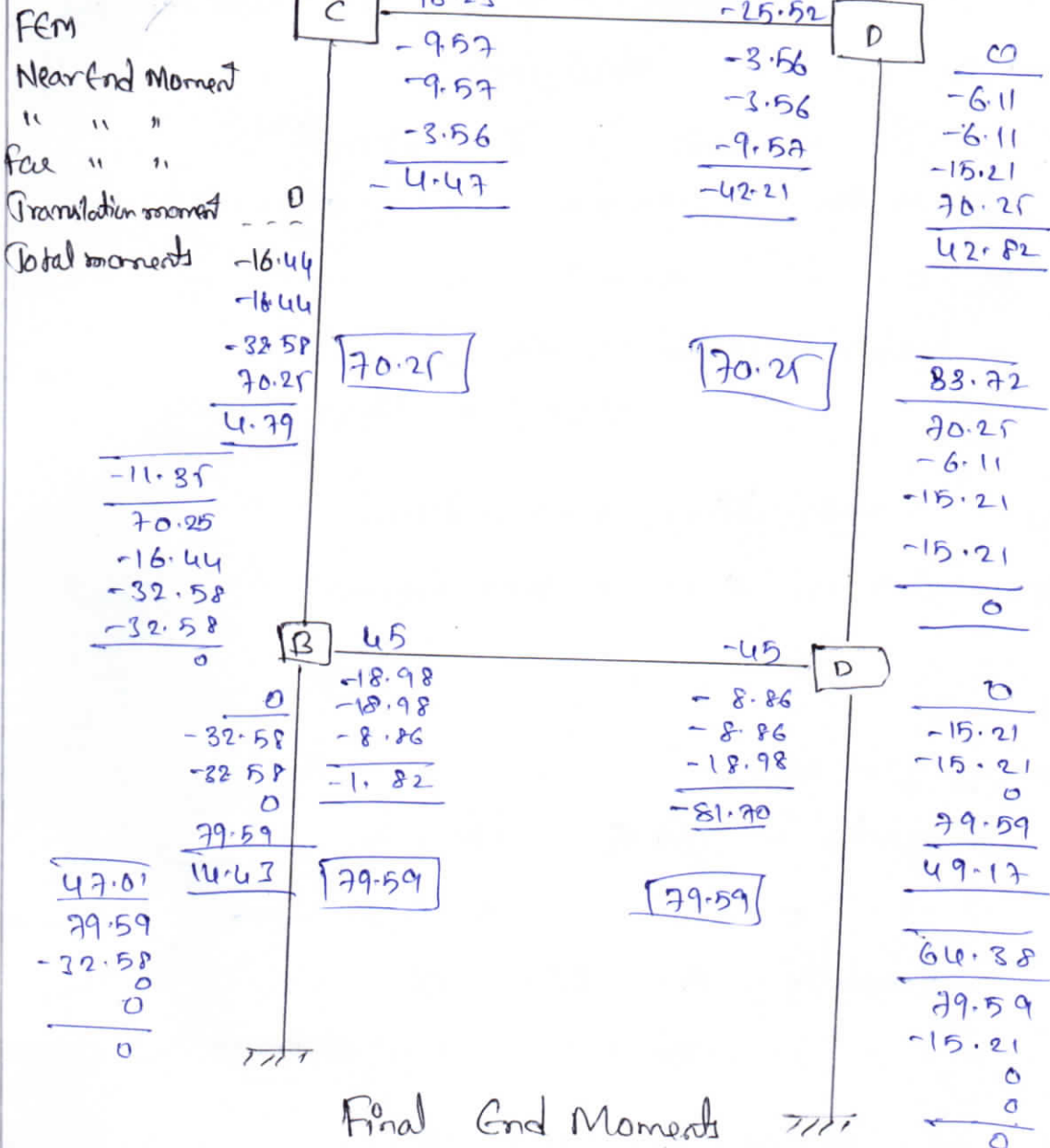
$$D_{BA} = -1.5 \left[\frac{k_{BA}}{k_{BA} + k_{CB} + k_{DE} + k_{EF}} \right] = -1.5 \times \left[\frac{1/3.5}{(1/3.5) + (1/3.5) + (1/3.5) + (1/3.5)} \right]$$

$$= -1.5 \times 0.25 = -0.375$$

$$D_{EF} = -0.375 \quad D_{BA} = D_{EF} = D_{DE} = D_{DE} = 0.375$$

$$D.F = 0.75 \times (-0.375 + 0.375)$$





Storey moments:

Storey 2: Due to lateral load of 20 kN @ C,

Storey shear $V_{r2} = -20 \text{ kN}$

Storey moment $M_{r2} = -20 \times \frac{3.1}{2} = -23.33 \text{ kN.m}$

Storey 1: Lateral loads @ B & C

Storey shear $V_{r1} = -20 - 30 = -50 \text{ kN}$

Storey moments $M_{r1} = \frac{-50 \times 3.1}{3} = -58.33 \text{ kN.m}$

Sum of joint moments = 18.23 kN.m

10 11 12 13 14 $\textcircled{a} \text{end } B = 0$ 15

1. 1. translation of column CB below = 0

Total = 18.23 km

$$M_{RCO} = (-0.184)(18.23) = -3.35 \text{ kN.m}$$

$$M_{RCB} = (-0.316)(18.27) = -5.761 \text{ kNm}$$

Sum of joint moments = -25.52 kNm

11. " radiation is (a) α $\rightarrow$ 3.35 cm, γ

2. $\lim_{x \rightarrow 0} \frac{1}{x} = \infty$ (assumed)

" " translation " Column DE = 0

Total = -28.87 kN.m

$$M_{RDC} = (-0.316) \times (-28.87) = 9.12 \text{ kN.m}$$

$$M_{PDE} = (-0.184) \times (-28.87) = 5.31 \text{ kNm}$$

Sum of joint moments = -45 kNm

11. n rotation 11. (a) $D = 5.31 \text{ n}$

11 11 11 11 (a) $B = 0$ (assume 2)

$$1A \quad B \quad 1A \quad B \quad \textcircled{e} \quad A = 0 \quad 1A$$

Sum of transition in all column
GF below = 0

 $\frac{1}{\lambda} \quad \frac{1}{\lambda} \quad \frac{1}{\lambda} \quad \frac{1}{\lambda} \quad \frac{1}{\lambda}$

ED above ε 0 4

Partial $\pi = -39.69 \text{ kJ/m}$

$$\therefore M_{RED} = (-0.194)(-37.69) = 7.20 \text{ kNm}$$

$$M_{REB} = (-0.113)(-37.69) = 4.48 \text{ kNm}$$

$$M_{REF} = (-0.194)(-37.69) = 7.20 \text{ kNm}$$

Joint B :

Sum of joint moments = 45.0 kNm

" " rotation " @ end E = 4.48 "

" " " " @ end C = -5.76 "

" " " " @ end A = 0 kNm (column)

" " translation " column BC below = 0 " "

" " " " " BA = 0 " "

Total = 43.72 kNm

$$M_{RBE} = (-0.113)(43.72) = -4.94 \text{ kNm}$$

$$M_{RBC} = M_{RBA} = (-0.194)(43.72) = -8.48 \text{ kNm}$$

$$M_{BC} = (-0.194)(43.72) = -8.48 = BA$$

Calculation of Storey Moments:

Storey - 2:

Storey moments: = -23.33 kNm

Rotation moments of columns @ end C of column BC = -5.76 kNm

" " " " " D " " DE = 5.31 "

" " " " " B " " BE = -8.48 "

" " " " " E " " EF = 7.20 "

Total = -24.56 kNm

$$M_{RBC} = M_{RED} = (-0.25) \times (-24.56) = 6.14 \text{ kNm}$$

Storey - 1:

Storey moments = -58.33 kNm

" " " " " B " " AB = -8.48 kNm

" " " " " E " " EF = 7.20 "

" " " " " A " " AB = 0 (column)

" " " " " F " " FF = 0 " "

Total = -59.11 kNm

$$M_{RAB} = M_{RFE} = (-0.25)(-59.11) = 14.78 \text{ kNm}$$

UNIT-6 FLEXIBILITY MATRIX METHOD

The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure.

This method is exactly opposite to stiffness matrix method.

The flexibility matrix equation is given by

$$[P] [F] = \{ \Delta \} - \{ \Delta_L \}$$

$$[P] = [F]^{-1} (\{ \Delta \} - \{ \Delta_L \})$$

Where,

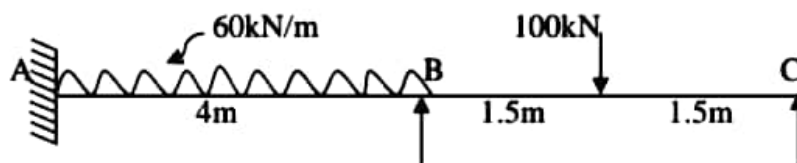
$[P]$ = Redundant in matrix form

$[F]$ = Flexibility matrix

$\{ \Delta \}$ = Displacement at supports

$\{ \Delta_L \}$ = Displacement due to load

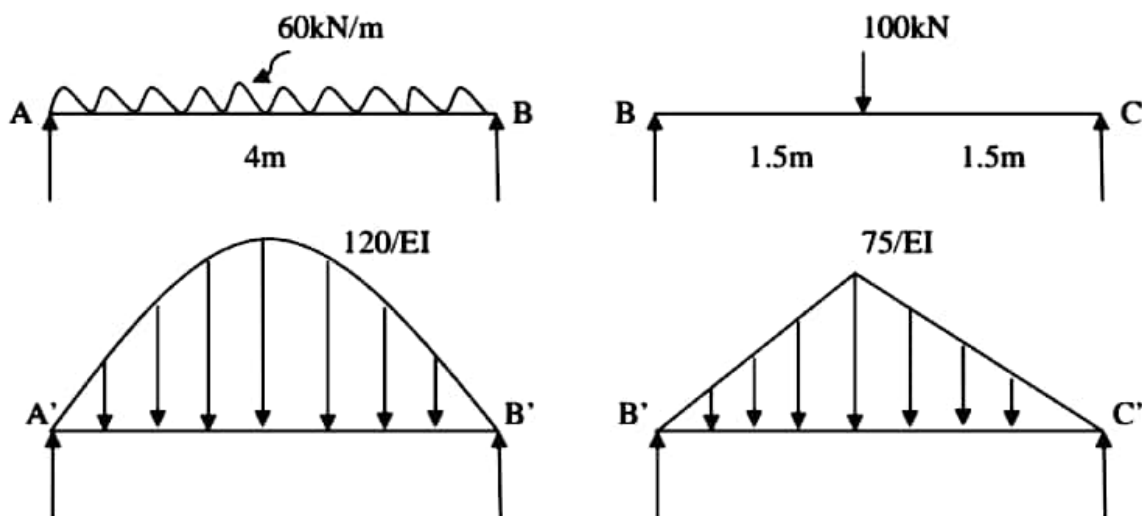
1. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD



Static Indeterminacy $SI = 2$ (M_A and M_B)

M_A and M_B are the redundant

Let us remove the redundant to get primary determinate structure



$$[\Delta] = \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix}$$

$$\Delta_1 = \text{Rotation at A} = \text{SF at A'}$$

$$\Delta_1 = \frac{1}{2} \left[\frac{2}{3} \times 4 \times \frac{120}{EI} \right]$$

$$\Delta_1 = \frac{160}{EI}$$

$$\Delta_2 = \text{Rotation at A} = \text{SF at B'}$$

$$= V_{B1'} + V_{B2'}$$

$$\Delta_2 = \frac{1}{2} \left[\frac{2}{3} \times 4 \times \frac{120}{EI} \right] + \frac{1}{2} \left[\frac{1}{2} \times 3 \times \frac{75}{EI} \right]$$

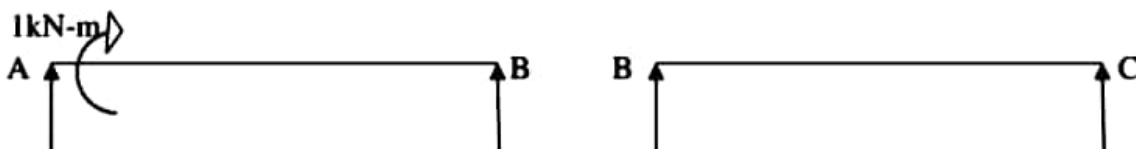
$$\Delta_2 = \frac{216.25}{EI}$$

$$[\Delta] = \frac{1}{EI} \begin{pmatrix} 160 \\ 216.25 \end{pmatrix}$$

Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

To get Flexibility Matrix

Apply unit moment to joint A

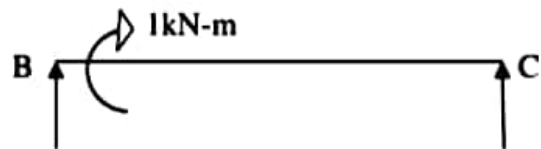
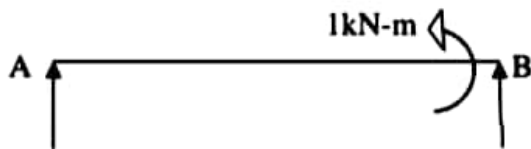


$$[F] = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}$$

$$11 = \frac{ml}{3EI} = \frac{1 \times 4}{3EI} = \frac{1.33}{EI}$$

$$21 = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI}$$

Apply unit moment to joint A



$$12 = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI}$$

$$22 = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 4}{3EI} + \frac{1 \times 3}{EI} = \frac{2.33}{EI}$$

$$[F] = \begin{bmatrix} 11 & 12 \\ 21 & 22 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}$$

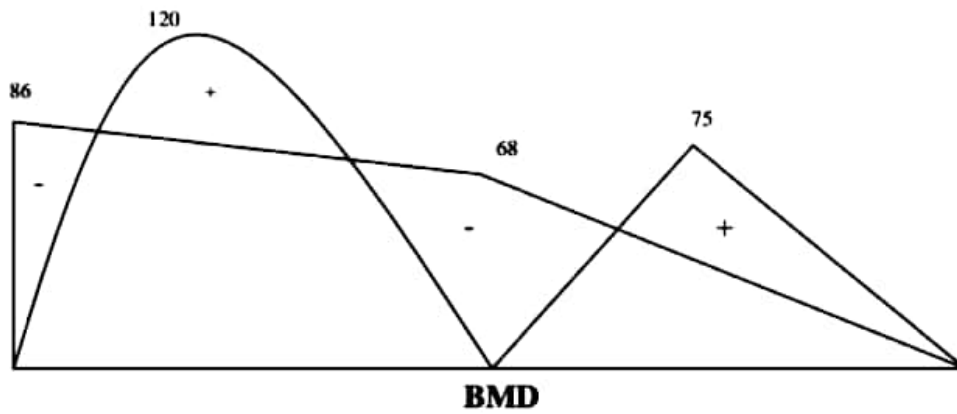
Apply the flexibility equation

$$[P] = [F]^{-1} ([\Delta] - [f_L])$$

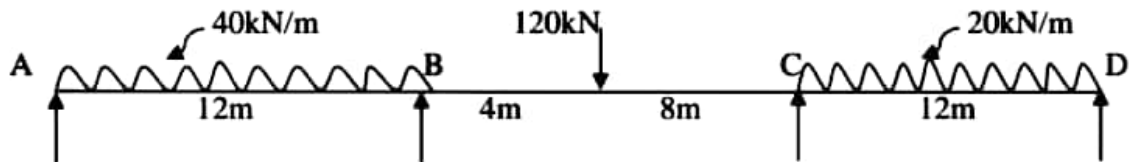
$$[f_L] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P] = EI \begin{bmatrix} 1.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 160 \\ 216.25 \end{bmatrix} \right\}$$

$$[P] = \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -86.00 \\ -68.08 \end{bmatrix} \text{ kN-m}$$



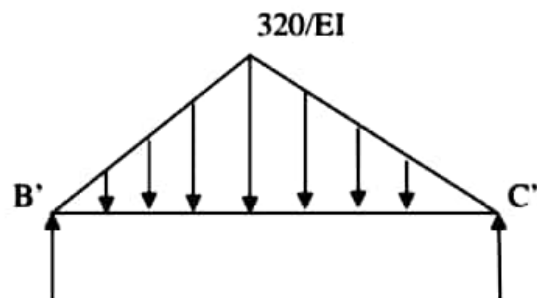
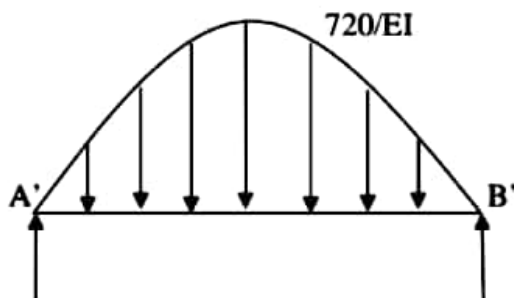
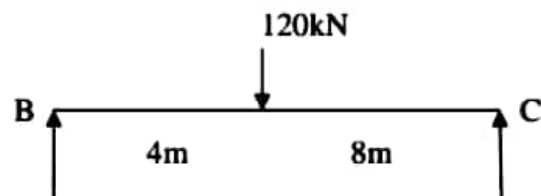
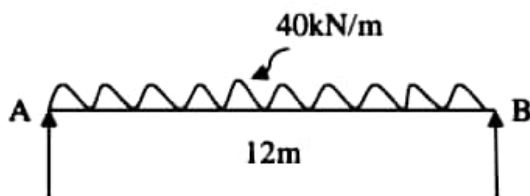
2. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD

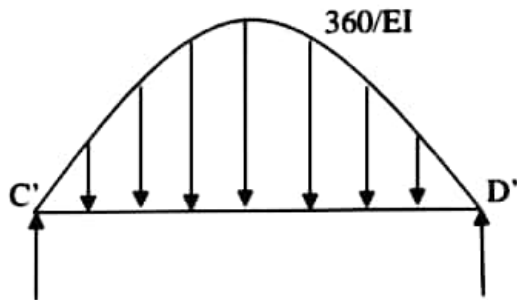
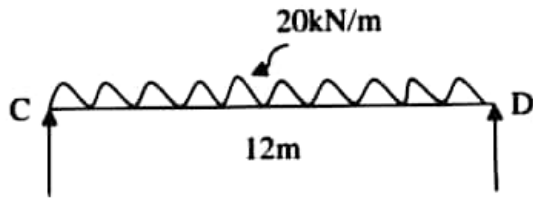


Static Indeterminacy $SI = 2$ (M_B and M_C)

M_B and M_C are the redundant

Let us remove the redundant to get primary determinate structure





$$[\Delta] = \begin{bmatrix} 1L \\ 2L \end{bmatrix}$$

$$1L = \text{Rotation at B} = \text{SF at B'}$$

$$= V_{B1}' + V_{B2}'$$

$$1L = \frac{3946.67}{EI}$$

$$2L = \text{Rotation at C} = \text{SF at C'}$$

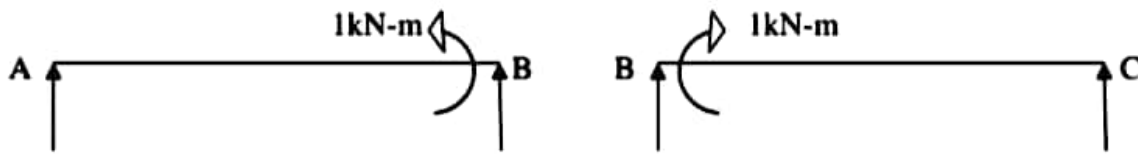
$$= V_{C1}' + V_{C2}'$$

$$2L = \frac{2293.33}{EI}$$

$$[\Delta] = \frac{1}{EI} \begin{bmatrix} 3946.67 \\ 2293.33 \end{bmatrix}$$

To get Flexibility Matrix

Apply unit moment to joint A

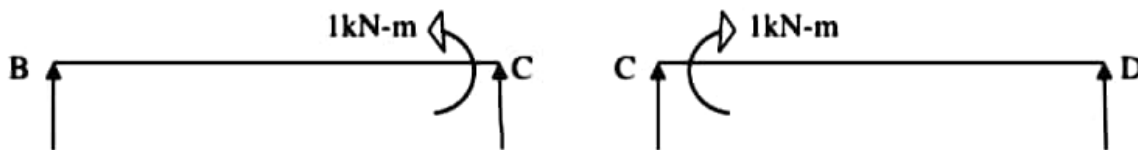


$$[F] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

$$f_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{3EI} = \frac{8}{EI}$$

$$f_{21} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}$$

Apply unit moment to joint B



$$f_{12} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}$$

$$f_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{3EI} = \frac{8}{EI}$$

$$[F] = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}$$

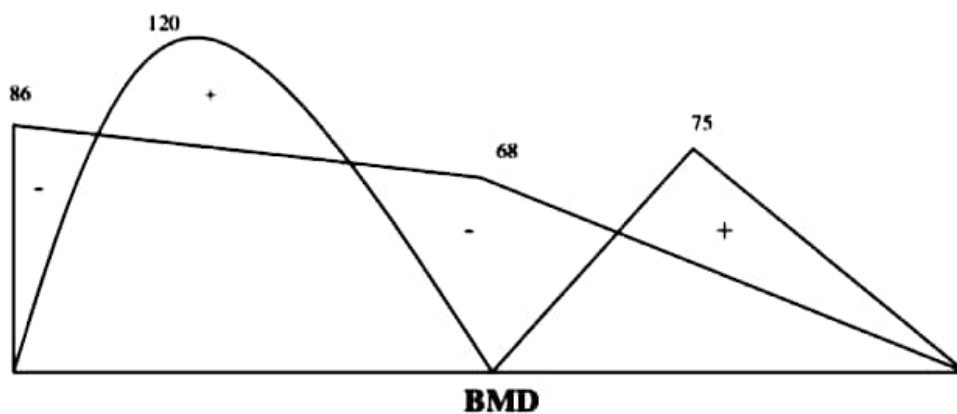
Apply the flexibility equation

$$[P] = [F]^{-1} ([\Delta] - [F]^{-1} [L])$$

$$[\Delta] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

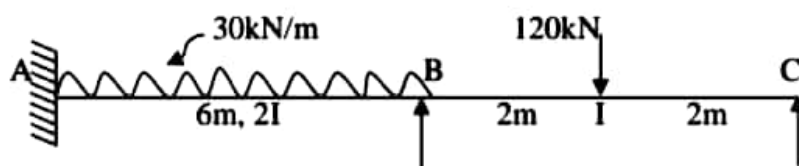
$$[P] = EI \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{EI} \begin{pmatrix} 3946 \\ 2293 \end{pmatrix} \right\}$$

$$[P] = \begin{pmatrix} M_{AB} \\ M_{BA} \end{pmatrix} = \begin{pmatrix} -449.97 \\ -174.22 \end{pmatrix} \text{ kN-m}$$



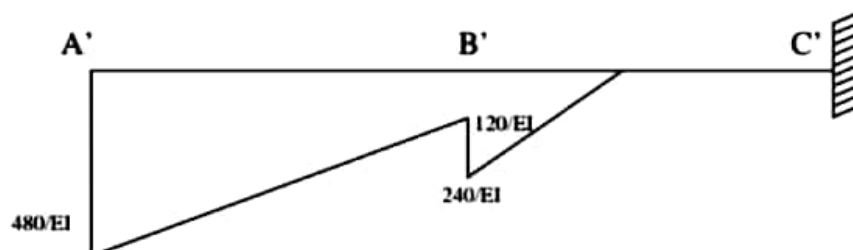
SINKING OF SUPPORT

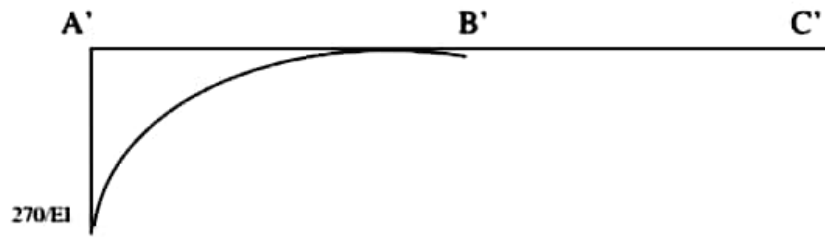
1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given $EI = 15 \times 10^3 \text{ kN-m}^2$



NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction.

Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.





$$[L] = \begin{bmatrix} 1L \\ 2L \end{bmatrix}$$

$1L$ = Displacement at B in primary determinate structure = BM at B' in conjugate beam

$$1L = \left[\frac{1}{2} \times 6 \times \frac{360}{EI} \times \left(\frac{2}{3} \times 6 \right) \right] + \left(6 \times \frac{120}{EI} \times \frac{6}{2} \right) + \left[\frac{1}{3} \times 6 \times \frac{270}{EI} \times \left(\frac{3}{4} \times 6 \right) \right]$$

$$1L = \frac{8910}{EI}$$

$2L$ = Displacement at C in primary determinate structure = BM at C' in conjugate beam

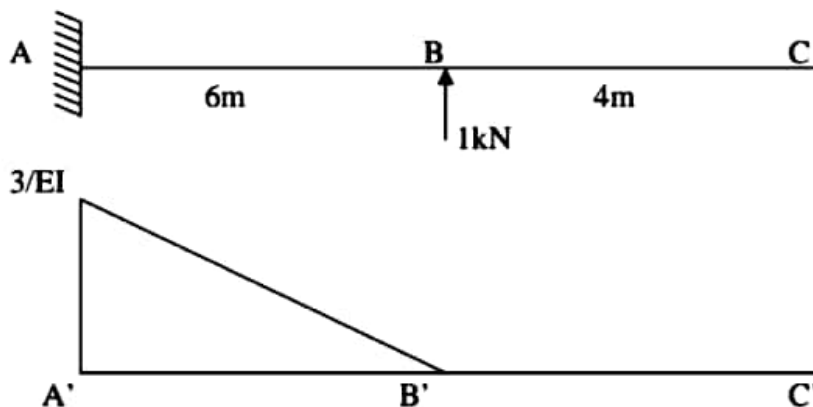
$$2L = \left[\frac{1}{2} \times 6 \times \frac{360}{EI} \times \left(\frac{2}{3} \times 6 + 4 \right) \right] + \left(6 \times \frac{120}{EI} \times \left(\frac{6}{2} + 4 \right) \right) - \left[\frac{1}{3} \times 6 \times \frac{270}{EI} \times \left(\frac{3}{4} \times 6 + 4 \right) \right]$$

$$2L = \frac{19070}{EI}$$

$$[L] = \frac{1}{EI} \begin{bmatrix} 8910 \\ 19070 \end{bmatrix}$$

To get Flexibility Matrix

Apply unit Load at B

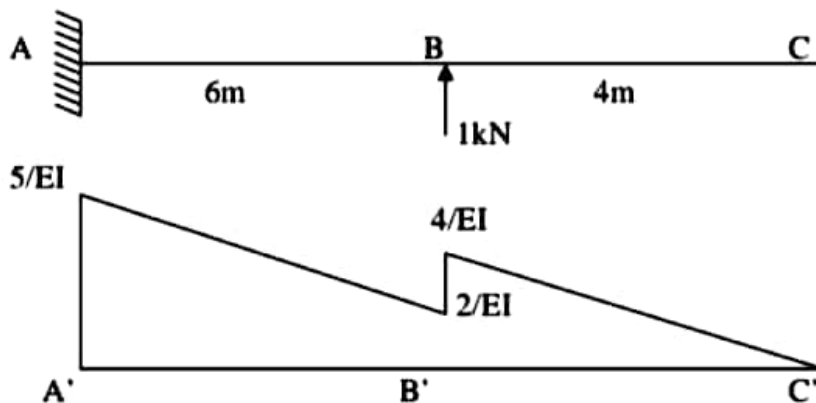


$$[F] = \begin{pmatrix} 11 & 12 \\ 21 & \delta_{22} \end{pmatrix}$$

$$11 = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6) = \frac{-36}{EI}$$

$$21 = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6 + 4) = \frac{-72}{EI}$$

Apply unit load at C



$$12 = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6) - [6 \times \frac{2}{EI} \times (6/2)] = \frac{-72}{EI}$$

$$22 = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6 + 4) - [6 \times \frac{2}{EI} \times (6/2 + 4)] - \frac{1}{2} \times 4 \times \frac{4}{EI} \times (2/3 \times 4) = \frac{-177.33}{EI}$$

$$[F] = \begin{pmatrix} 11 & 12 \\ 21 & 22 \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}$$

Apply the flexibility equation

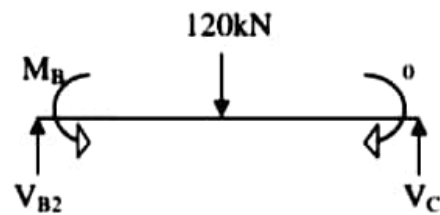
$$[P] = [F]^{-1} ([\Delta] - [f])$$

$$[\Delta] = \begin{pmatrix} 0.005 \\ 0 \end{pmatrix}$$

$$[P] = EI \begin{bmatrix} -36 & -72 \\ -72 & -177.33 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0.005 \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 8910 \\ 19070 \end{bmatrix} \right\}$$

$$[P] = \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 161.43 \\ 41.98 \end{bmatrix} \text{ kN-m}$$

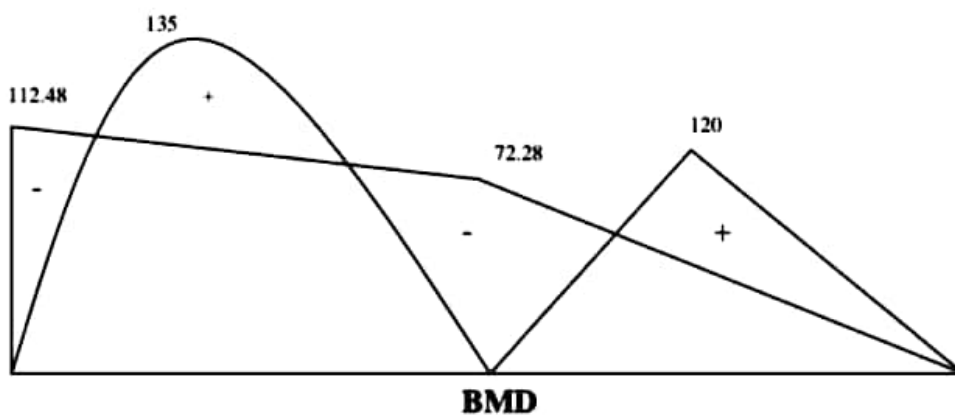
Support Reaction



$$V_A = 96.64 \text{ kN}, \quad V_{B1} = 83.36 \text{ kN}, \quad V_{B2} = 78.07 \text{ kN}, \quad V_C = 41.98 \text{ kN}$$

$$V_B = V_{B1} + V_{B2} = 161.43 \text{ kN}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 112.48 \\ 72.28 \end{bmatrix} \text{ kN-m}$$



STIFFNESS MATRIX METHOD

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

The stiffness matrix equation is given by

$$[F] [K] = ([P] - [P_L])$$

$$[F] = [K]^{-1} ([P] - [P_L])$$

Where,

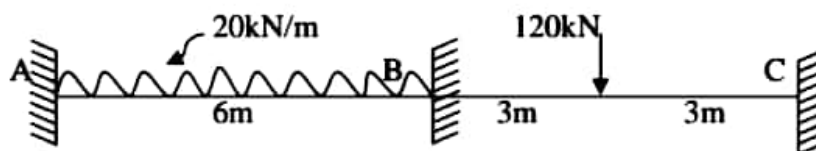
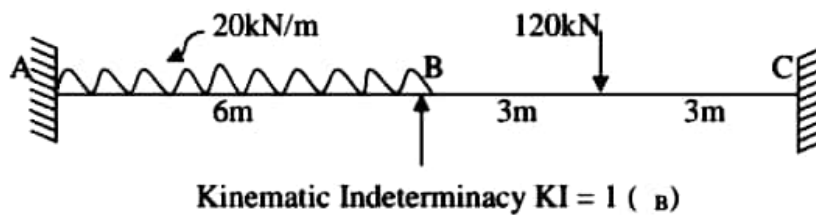
[P] = Redundant in matrix form

[F] = Stiffness matrix

[P] = Final force at the joints in matrix form

[P_L] = force at the joints due to applied load in matrix form

1. Analyse the continuous beam by Stiffness method Sketch the BMD



$$[P_L] = M_{FBA} + M_{FBC}$$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) = \frac{20 \times 6^2}{12} - \frac{120 \times 6}{8}$$

$$[P_L] = -30 \text{ kN-m}$$

Apply unit displacement at joint B.



$$[K] = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \quad (\theta = 1)$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[] = [K]^{-1} ([P] - [P_L])$$

$$= \frac{1}{K} ([P] - [P_L])$$

$$B = \frac{1}{1.33EI} \{ [0] - [-30] \} = \frac{22.56}{EI}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A - \theta_B)$$

$$= -60 + \frac{2EI}{6}(2\theta_A + \frac{22.5}{EI})$$

($\theta_A = 0$ due to fixity at support A)

$$M_{AB} = -52.5 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 60 + \frac{2EI}{6}(2 \times \frac{22.5}{EI} + \theta_A)$$

$$M_{BA} = 75.04 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

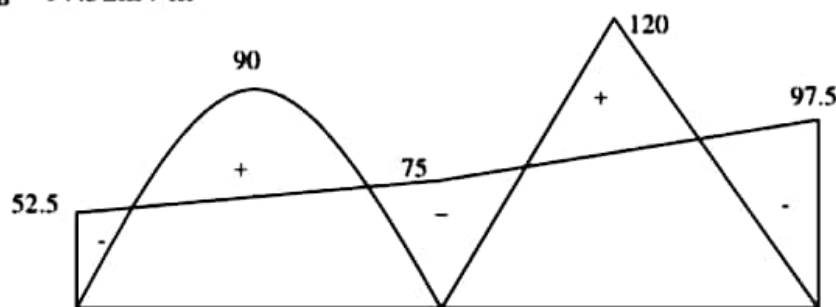
$$= -90 + \frac{2EI}{6}(2 \times \frac{22.5}{EI} + 0)$$

$$M_{BC} = -75 \text{ kN-m}$$

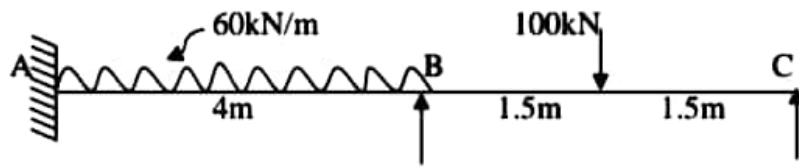
$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

$$= 90 + \frac{2EI}{6}(0 + \frac{22.5}{EI})$$

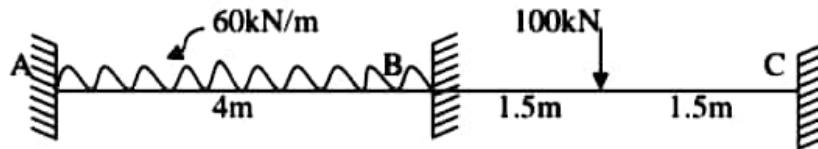
$$M_{CB} = 97.52 \text{ kN-m}$$



2. Analyse the continuous beam by Stiffness method Sketch the BMD



Kinematic Indeterminacy $KI = 2$ (B & C)



$$[P_{1L}] = M_{FBA} + M_{FBC}$$

$$= \frac{wl^2}{12} + \left(-\frac{wl}{8}\right) - \frac{60 \times 4^2}{12} - \frac{100 \times 3}{8} = 42.5 \text{ kN-m}$$

$$[P_{2L}] = M_{FCB} = \frac{wl}{8} = \frac{100 \times 3}{8} = 37.5 \text{ kN-m}$$

$$[P_L] = \begin{Bmatrix} P_{1L} \\ P_{2L} \end{Bmatrix} = \begin{Bmatrix} 42.5 \\ 37.5 \end{Bmatrix} \text{ kN-m}$$

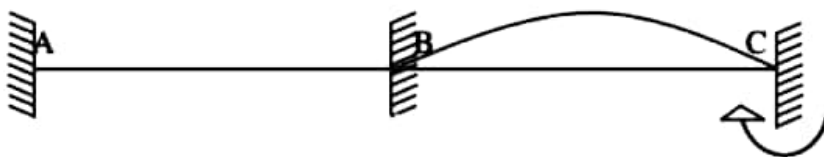
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} - \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI \quad (=1)$$

$$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

Apply unit displacement at joint B.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[] = [K]^{-1} ([P] - [P_L])$$

$$[] = \frac{1}{EI} \begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 42.5 \\ 37.5 \end{Bmatrix}$$

$$\begin{Bmatrix} B \\ C \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -11.88 \\ -22.19 \end{Bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -80 + \frac{2EI}{4} (2\theta_A + \frac{0}{EI})$$

($\theta_A = 0$ due to fixity at support A)

$$M_{AB} = -85.94 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 80 + \frac{2EI}{4} (2 \times \frac{-11.88}{EI} + \theta_A)$$

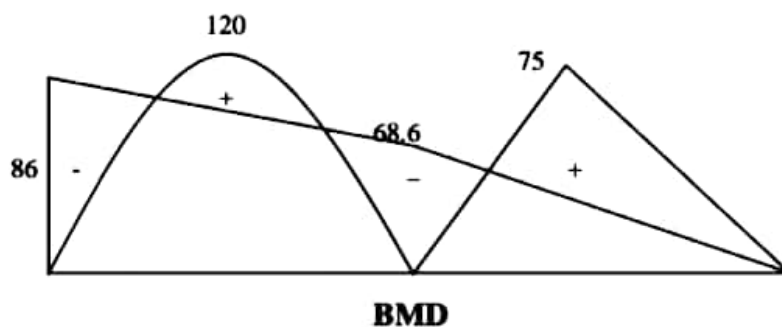
$$M_{BA} = 68.12 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -37.5 + \frac{2EI}{6} (2 \times \frac{-11.88}{EI} + \frac{-22.9}{EI})$$

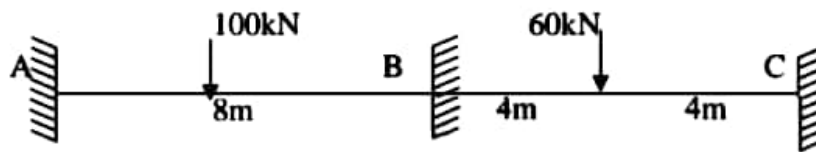
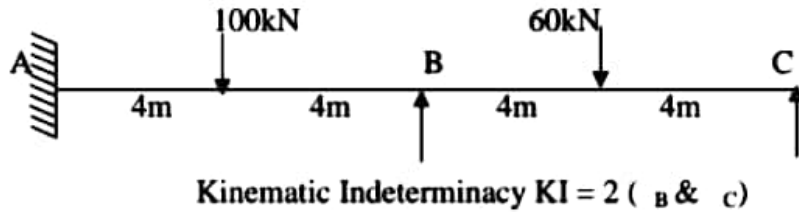
$$M_{BC} = -68.6 \text{ kN-m}$$

$$M_{CB} = 0$$



Sinking of support

1. Analyse the continuous beam shown in figure by stiffness method. Support B sinks by $300/EI$ units and support C sinks by $200/EI$ units



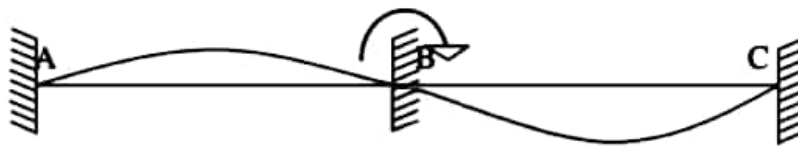
$$[P_{1L}] = M_{FBA} + M_{FBC} - \frac{6EI\delta}{l^2} - \frac{6EI\delta}{l^2}$$

$$= \frac{100 \times 8}{8} - \frac{60 \times 8}{8} - \frac{6 \times 300}{8^2} + \frac{6 \times 100}{8^2} = 21.25 \text{ kN-m}$$

$$[P_{2L}] = M_{FCB} - \frac{6EI\delta}{l^2} = \frac{60 \times 8}{8} + \frac{6 \times 100}{8^2} = 69.38 \text{ kN-m}$$

$$[P_L] = \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 21.25 \\ 69.38 \end{bmatrix} \text{ kN-m}$$

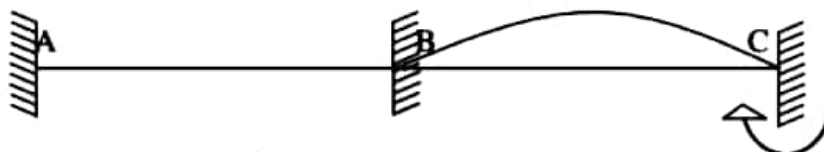
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} - \frac{4EI}{8} + \frac{4EI}{8} = EI \quad (\theta \approx 1)$$

$$K_{21} = \frac{2EI\theta}{8} = \frac{2EI}{3} = 0.25EI$$

Apply unit displacement at joint C.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{8} = 0.25EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{8} = 0.50EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[] = [K]^{-1} \{ [P] - [P_L] \}$$

$$[] = \frac{1}{EI} \begin{pmatrix} 1 & 0.25 \\ 0.25 & 0.50 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 21.25 \\ 69.38 \end{pmatrix} \right\}$$

$$\begin{pmatrix} B \\ C \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 15.36 \\ -146.44 \end{pmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$$

$$= -100 + \frac{2EI}{8} (2\theta_A + \frac{15.36}{EI} - \frac{3EI(300/EI)}{8}) \quad (\theta_A = 0 \text{ due to fixity at support A})$$

$$M_{AB} = -124.29 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$$

$$= 100 + \frac{2EI}{8} (0 + 2 \times \frac{15.36}{EI} - \frac{3EI(300/EI)}{8})$$

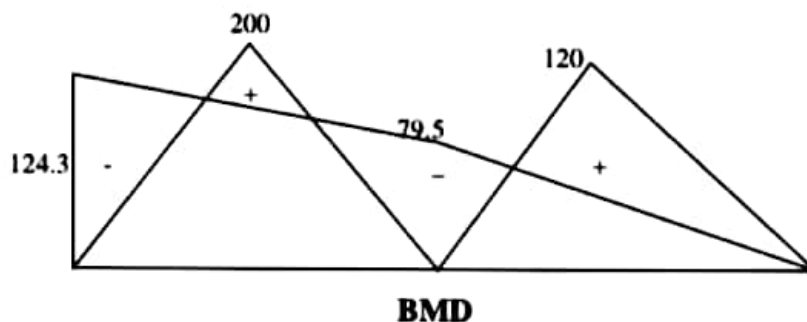
$$M_{BA} = 79.55 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$= -60 + \frac{2EI}{8} (2 \times \frac{15.36}{EI} + \frac{-146.44}{EI} - \frac{3EI(-100/EI)}{5})$$

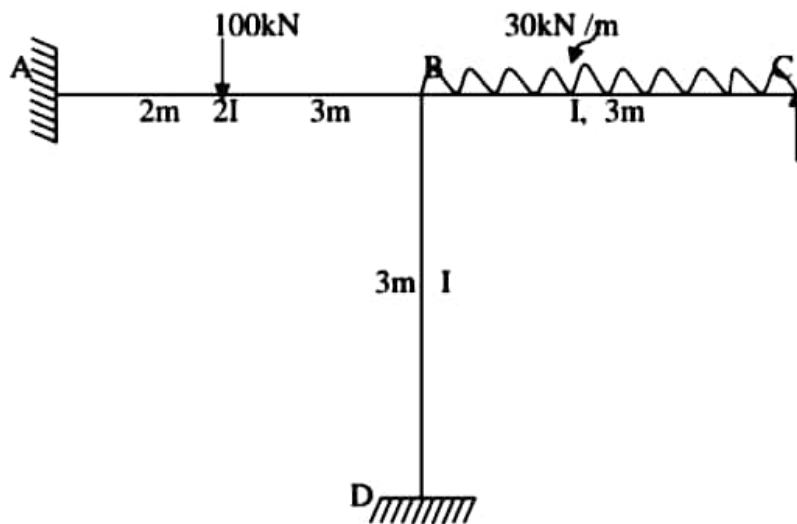
$$M_{BC} = -79.55 \text{ kN-m}$$

$$M_{CB} = 0$$

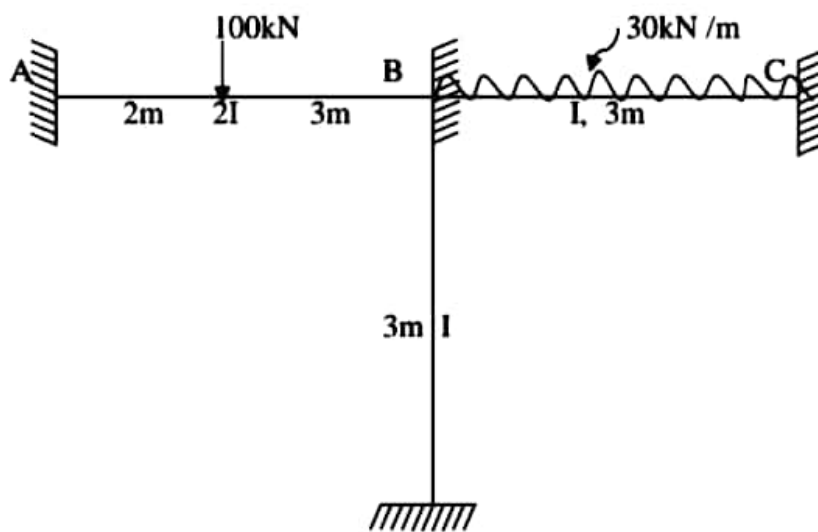


Analysis of frames

1. Analyse the frame by stiffness method



Kinematic Indeterminacy $KI = 2$ (B & C)



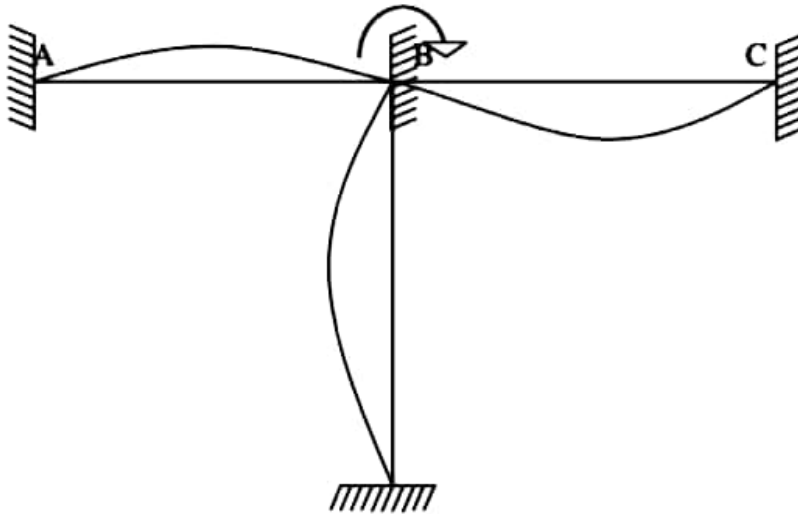
$$[P_{1L}] = M_{FBA} - M_{FBC} + M_{FCD}$$

$$= \frac{100 \times 2 \times 3^2}{5^2} - \frac{30 \times 3^2}{12} = 25.5 \text{ kN-m}$$

$$[P_{2L}] = M_{FCB} = \frac{30 \times 3^2}{12} = 22.5 \text{ kN-m}$$

$$[P_L] = \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} 25.5 \\ 22.5 \end{bmatrix} \text{ kN-m}$$

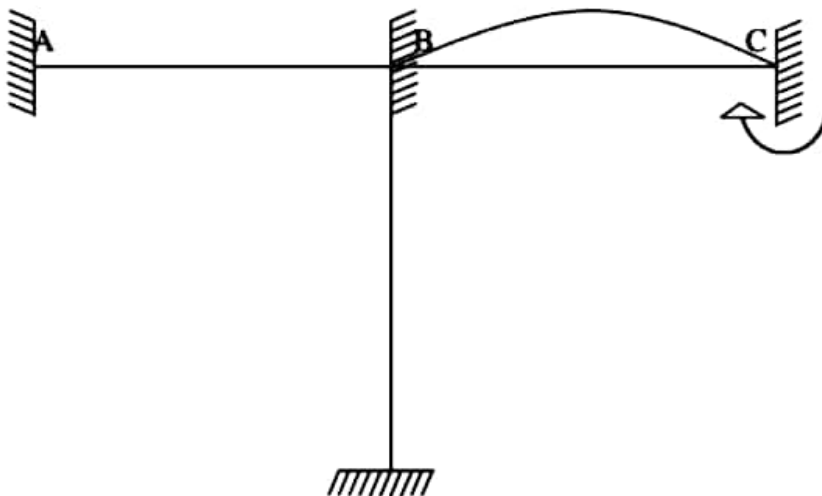
Apply unit displacement at joint B.



$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4 \times 2EI}{5} + \frac{4EI}{3} + \frac{4EI}{3} = 4.267EI \quad (=1)$$

$$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

Apply unit displacement at joint C.



$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[] = [K]^{-1} ([P] - [P_L])$$

$$[] = \frac{1}{EI} \begin{pmatrix} 4.267 & 0.67 \\ 0.67 & 1.33 \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 25.5 \\ 22.5 \end{Bmatrix}$$

$$\begin{Bmatrix} B \\ C \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} -3.604 \\ -15.01 \end{Bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -72 + \frac{2 \times 2EI}{5} \left(2\theta_A + \frac{-3.604}{EI} \right) \quad (\theta_A = 0 \text{ due to fixity at support A})$$

$$M_{AB} = -74.88 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= 72 + \frac{2 \times 2EI}{5} \left(2\theta_A + 2 \times \frac{-3.604}{EI} \right)$$

$$M_{BA} = 42.23 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -22.5 + \frac{2EI}{3} \left(2 \times \frac{-3.604}{EI} + \frac{-15.1}{EI} \right)$$

$$M_{BC} = -37.37 \text{ kN-m}$$

$$M_{BD} = M_{FBD} + \frac{2EI}{l}(2\theta_B + \theta_D)$$

$$= 0 + \frac{2EI}{3} \left(2 \times \frac{-3.604}{EI} + 0 \right)$$

$$M_{BD} = -4.81 \text{ kN-m}$$

$$M_{DB} = M_{FDB} + \frac{2EI}{l}(2\theta_D + \theta_B)$$

$$= 0 + \frac{2EI}{3} \left(2 \times 0 + \frac{-3.604}{EI} \right)$$

$$M_{DB} = -2.402 \text{ kN-m}$$

$$M_{CB} = 0$$