DATA TRANSMISSION

* Integrate and Dump Filter (Receiver):-
* The digital signal $x(t)$ is corrupted by white noise $n(t)$ during transmission over channel.
* Such noisy signal $[x(t) + n(t)]$ is given to the input integrate and Dump filter.
* The capacitor is discharged fully at the beginning of the bit interval
* This is acheived temporarily closing switch Sw_1 at the beginning of the bit interval. Integrator integrates noisy input signal over one bit period.
* At the $t=T$, the value $r(t)$ reaches the maximum amplitude.

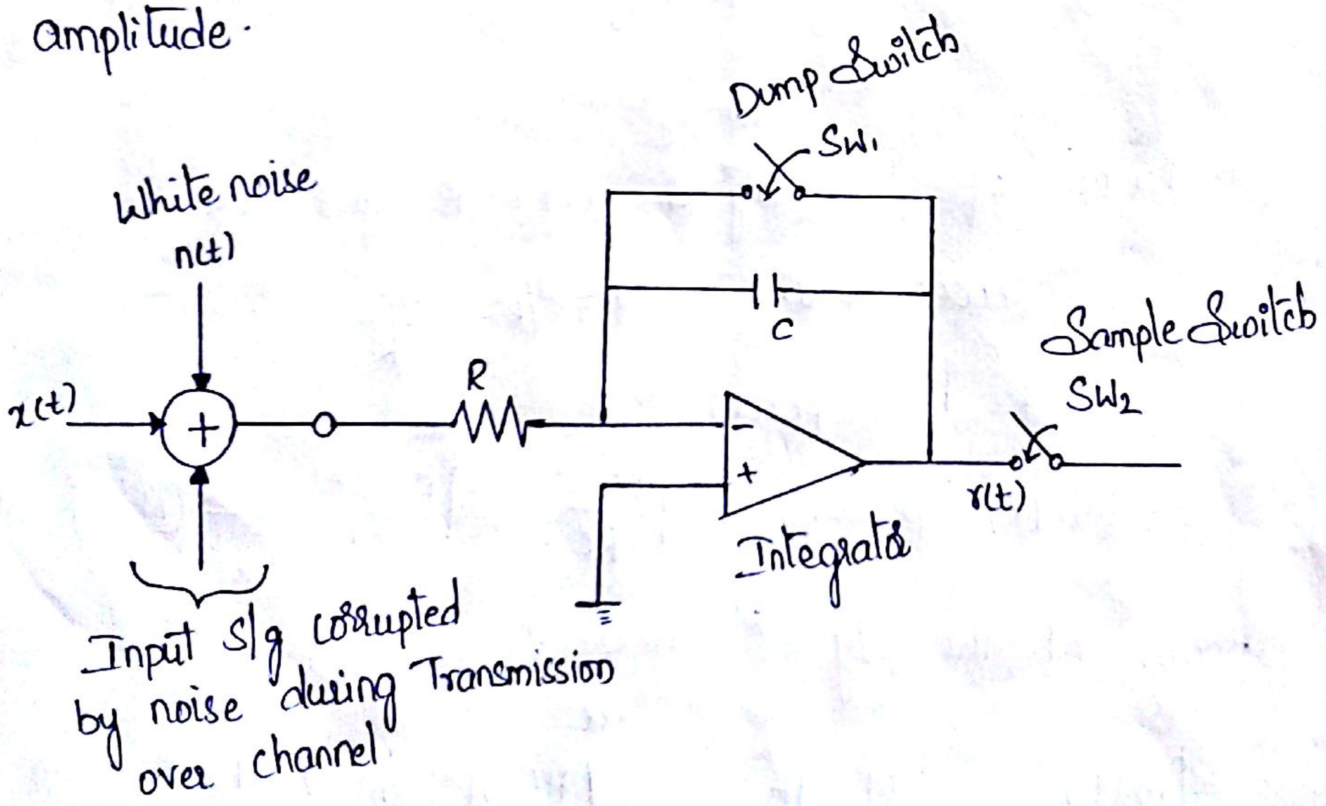

Fig:- Integrate and dump filter.

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* The dump switch Sw_2 is then closed momentarily to discharge the capacitor to receive next bit.
* The integrator integrates independent values of previous bit.
* (Thus integrator) This shows detection in integrate and dump filter is uneffected by values of previous bits.

* **Signal to Noise Ratio of the Integrator and Dump filter:-**

This ckt is a combination of integrator & low pass RC ckt to get base band s/g receiver

The o/p is calculated by

$$r(t) = \underbrace{\int \text{input data}}_{x_0(t)} + \underbrace{\int \text{Noise}}_{n_0(t)}$$

$$x_0(t) = 1/RC \int_0^T A\, dt$$

$$x_0(t) = \frac{AT}{RC}$$

Power of the signal $x(t)$ is given by

$$\text{power} = V \cdot I = V - V/R = \frac{V^2}{R}$$

$$R = 1\Omega;\ \text{power} = V^2$$

$$P_{x_0(t)}\ (\text{power}) = \left|\frac{AT}{RC}\right|^2 = \left|\frac{AT}{\tau}\right|^2 \longrightarrow (1)$$

power of the s/g of $n_0(t)$ is

$$P_{n_0(t)} = \int_{-\alpha}^{\alpha} S(F)\,dF = \int_{-\alpha}^{\alpha} |H(F)|^2 S_n(F)\,dF$$

PDF of noise

MND

$S_n:(F) = \frac{N_0}{2}$ [Gaussian] ③

$$H(F) = \frac{1-e^{-j\omega T}}{j\omega RC}$$

Substitute these values in $P_{no}(t)$

Now $H(F)$ is written as

$$H(F) = \frac{1-e^{j\omega T}}{j\omega RC}$$

$$= \frac{1-(\cos\omega t + j\sin\omega t)}{j\omega RC}$$

$$= \frac{\sin\omega T}{\omega RC} + \frac{i}{j}\left[\frac{(1-\cos\omega t)}{j\omega RC}\right]$$

$$= \frac{\sin\omega T}{\omega RC} - \frac{j(1-\cos\omega t)}{j\omega RC}$$

$$|H(F)|^2 = \left[\sqrt{\left[\frac{\sin\omega T}{\omega RC}\right]^2 + \left[\frac{(1-\cos\omega T)}{RC\omega}\right]^2}\right]^2$$

$$= \left[\frac{\sin\omega T}{\omega RC}\right]^2 + \frac{(1+\cos^2\omega T - 2\cos\omega T)}{\omega RC^2}$$

$$= \frac{\sin^2\omega T + \cos^2\omega T + 1 - 2\cos\omega T}{(\omega RC)^2}$$

$$= \frac{2(1-\cos\omega T)}{(\omega RC)^2}$$

$$|H(F)|^2 = \frac{2(1 \cdot \cos 2\pi FT)}{(2\pi FRC)^2}$$

$$= \frac{\cancel{2}(\cancel{2}\sin^2\pi FT)}{4(\pi FRC)^2}$$

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$$|H(F)|^2 = \frac{\sin^2 \pi FT}{(\pi FT)^2}$$

So, from (1)

$$P_{no}(t) = \int_{-\alpha}^{\alpha} \frac{\sin^2 \pi FT}{(\pi FT)^2} \frac{N_0}{2} dF$$

$$= \frac{N_0}{2} \int_{-\alpha}^{\alpha} \frac{\sin^2 \pi FT}{(\pi FT)^2} \frac{T^2}{\tau^2} dF$$

$$= \frac{N_0}{2} \frac{T^2}{\tau^2} \int_{-\alpha}^{\alpha} \frac{\sin^2 \pi FT}{(\pi FT)^2} dF$$

Let $\pi FT = x$.

$$dF = \frac{1}{\pi F} dx.$$

$$\text{So} \Rightarrow P_{no}(t) = \frac{N_0}{2} \frac{T^2}{\tau^2} \int_{-\alpha}^{\alpha} \left(\frac{\sin x}{x}\right)^2 \frac{1}{\pi T} dx.$$

$$P_{no}(t) = \frac{N_0}{2\pi} \frac{T}{\tau^2} \, 2\int_{0}^{\alpha} \left(\frac{\sin x}{x}\right)^2 dx.$$

$$= \frac{N_0}{\cancel{2}\cancel{\pi}} \frac{T}{\tau^2} \cdot 2 \cdot \cancel{\pi}/\cancel{2}\,]$$

$$P_{no}(t) = \frac{N_0 T}{2\tau^2}$$

$$r(t) = x_0(t) + n_0(t)$$

$$\boxed{r(t) = \left(\frac{AT}{\tau}\right)^2 + \left(\frac{N_0 T}{2\tau^2}\right)}$$

we know that power ration i.e signal to noise ratio

$$S/N = \left(AT/\tau\right)^2 \Big/ \frac{N_0 T}{2\tau^2}$$

$$S/N = \frac{A^2 T^2}{\tau^2} \cdot \frac{2\tau^2}{N_0 T}$$

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$$\boxed{S/N = \frac{2A^2T}{N_0}}$$

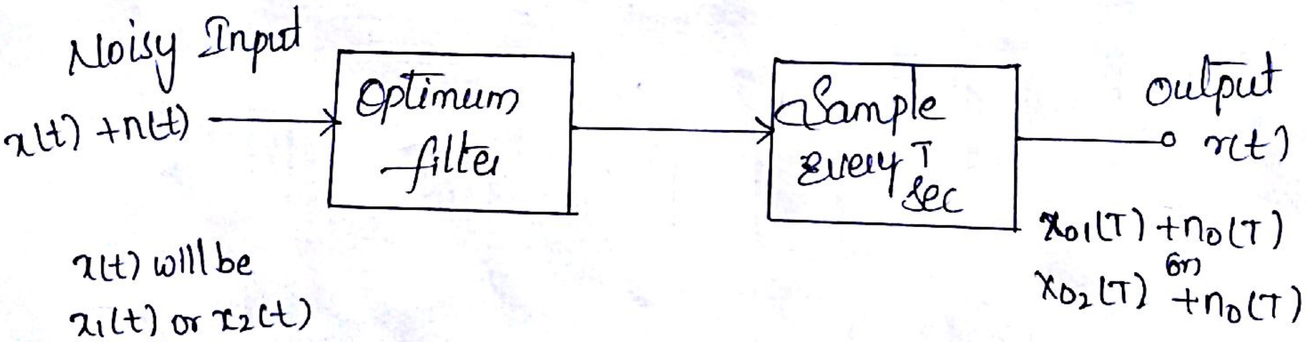
* Optimum filter :-

* Now we will consider the generalized gaussian noise of zero mean.
* Let us Assume that the Received signal is a binary waveform. Let's say that the polar NRZ signal is used to represent binary 1s and 0s.

for binary '1' : $x_1(t) = +A$ for one bit period T

for binary '0' ; $x_2(t) = -A$ for one bit period T

* Thus the input signal $x(t)$ will be either $x_1(t)$ or $x_2(t)$ depending upon the polarity of the NRZ signal.



$x(t)$ will be $x_1(t)$ or $x_2(t)$

Noise $n(t)$ added over the channel has psd of $S_n(f)$.

* Noise $n(t)$ is added to the signal $x(t)$ over the channel during transmission

$\therefore$ Input to the optimum filter is $x(t) + n(t)$ i.e.,

Input to the receiver $= x(t) + n(t)$

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and

output from the receiver $= x_{01}(T) + n_0(T)$ or $x_{02}(T) + n_0(T)$

* In the absence of noise, decisions are taken clearly. But if noise is present then select $x_1(t)$ if $r(T)$ is closer to $x_{01}(T)$ than $x_{02}(T)$ and select $x_2(t)$ if $r(T)$ is closer to $x_{02}(T)$ than $x_{01}(T)$.

* Therefore the decision boundary will be midway between $x_{01}(T)$ and $x_{02}(T)$. It is given as,

$$\text{Decision boundary} = \frac{x_{01}(T) + x_{02}(T)}{2}.$$

* Probability of Error:—

We know

power output $r(t) = \left(AT/\tau\right)^2 + \frac{N_0T}{2\tau^2}$

Signal o/p $r(t) = \frac{AT}{\tau} + \sqrt{\frac{N_0T}{2\tau^2}}$

$x(t) = A = 1 \Rightarrow r(t) > 0 \Rightarrow 1$
$x(t) = A = 0 \Rightarrow r(t) < 0 \Rightarrow 0$

error
if not $r(t) < 0 \Rightarrow 0$
if not $r(t) > 0 \Rightarrow 1$

error

$d(t) = 1 \Rightarrow 0$ if $AT/\tau < -n(t)$

$d(t) = 0 \Rightarrow 1$ if $AT/\tau > n(t)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

mean $= 0$

$$f(n(t)) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{[n(t)-0]^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi}\sigma} e^{-(n(t)^2)/2\sigma^2}$$

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σ = [mean square value − square of mean value]$^{1/2}$

$= [\overline{x^2} - m_x^2]$ $\quad m_x = 0$

$\sigma = \overline{x^2}$ [Noise power square]

$$\overline{x^2} = \overline{n_0^2(t)} = \frac{N_0 T}{2\tau^2}$$

$$\sigma = \left[\frac{N_0 T}{2\tau^2} - 0\right]^{1/2}$$

$$\sigma = \sqrt{\frac{N_0 T}{2\tau^2}}$$

$$f(n(t)) = \frac{1}{\sqrt{2\pi}}\sqrt{\frac{2\tau^2}{N_0 T}}\, e^{-\frac{n^2(t)}{\cancel{2}\left(\frac{N_0 T}{\cancel{2}\tau^2}\right)}}$$

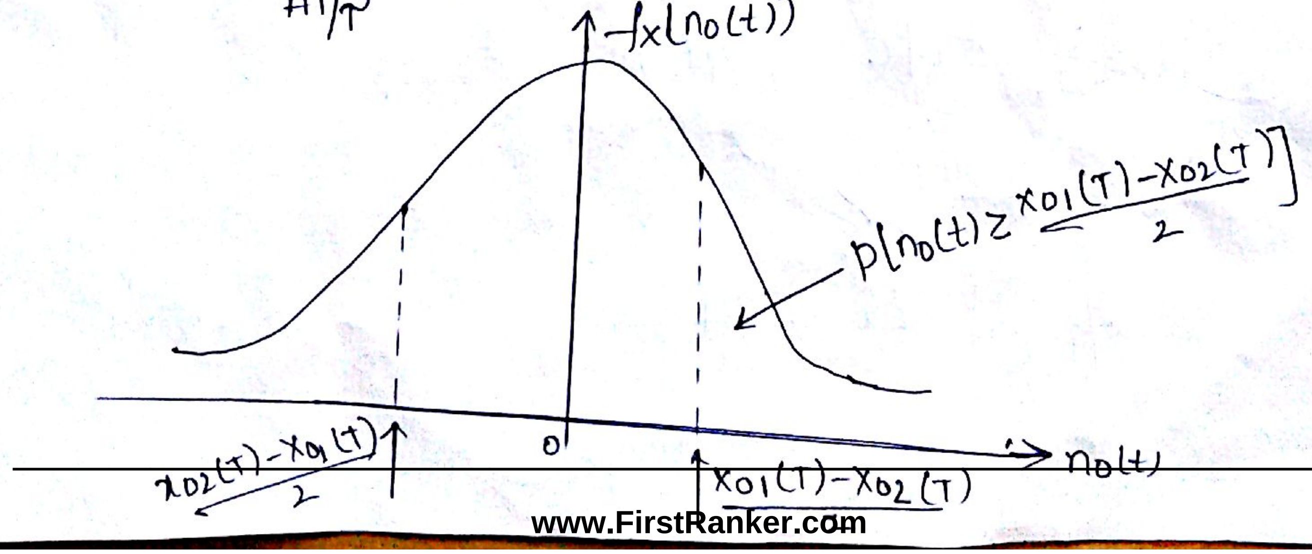
$$= \sqrt{\frac{\cancel{2}\tau^2}{\cancel{2}\pi N_0 T}}\, e^{-\frac{n^2(t)\tau^2}{N_0 T}}$$

$$f(n(t)) = \frac{\tau}{\sqrt{\pi N_0 T}}\, e^{-\frac{n^2(t)\tau^2}{N_0 T}}$$

Graphical Representation:-

$$P_e[f(n(t))] = \int_{\frac{AT}{\tau}}^{\alpha} F(n(t))\, d(n(t))$$

$$= \int_{AT/\tau}^{\alpha} \frac{\tau}{\sqrt{N_0 T \pi}}\, e^{-\frac{n^2(t)\tau^2}{N_0 T}}\, d(n(t))$$



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let

$$\frac{n^2(t)}{N_0T/\gamma^2} = y^2$$

$$\frac{n(t)\gamma}{\sqrt{N_0T}} = y$$

$$d(n(t)) = \frac{\sqrt{N_0T}}{\gamma}dy$$

limits:

$$n(t) \to \alpha \Rightarrow y \to \alpha$$

$$n(t) \to AT/\gamma \Rightarrow y = \frac{AT}{\sqrt{N_0T}} \frac{\cancel{\gamma}}{\cancel{\gamma}}$$

$$= A\sqrt{T/N_0}$$

$$Pe = \int_{A\sqrt{T/N_0}}^{\alpha} \frac{\cancel{\gamma}}{\sqrt{\cancel{N_0T}\pi}} e^{-y^2} \frac{\sqrt{\cancel{N_0T}}}{\cancel{\gamma}} dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{A\sqrt{T/N_0}}^{\alpha} e^{-y^2} dy$$

$$= \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{A\sqrt{T/N_0}}^{\alpha} e^{-y^2} dy\right]$$

$$= \frac{1}{2} Erfc\left[\sqrt{\frac{A^2T}{N_0}}\right]$$

$$= \frac{1}{2} Erfc\left(\sqrt{E/N_0}\right)$$

$$Erfc <<< 1$$

$$Pe = 1/2 \text{ if } Erfc \simeq 1$$

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* Optimum filter:— [Transfer function]

1) A —— $x_1(t)$ (or) 0 —— A → $x_2(t)$ → $\boxed{O/P}$ ← $n(t)$ —— $x_{01}(t) + n_0(t)$ (or) $x_{02}(t) + n_0(t)$; $\frac{x_1(t)+x_2(t)}{2}$ →

$$\begin{cases} x_1(t) \text{ if } x_2(t) \\ x_2(t) \text{ if } x_1(t) \end{cases}$$

data error

2 . 3 ⟹ $$\frac{x_1(t)+x_2(t)}{2} - x_2(t) < n(t)$$

$$\frac{x_1(t)-x_2(t)}{2} < n(t)$$

3. 2 ⟹ $$\frac{x_1(t)+x_2(t)}{2} - x_1(t) > n(t)$$

$$\frac{x_2(t)-x_1(t)}{2} > n(t)$$

$$Pe = \int_{\frac{x_1(t)-x_2(t)}{2}}^{\alpha} S(f)\,df$$

we know

$$\left[S(f) = f(n_0(t)) \cdot \frac{1}{\sqrt{2\pi}\sigma} e^{-(n_0(t)-0)^2/2\sigma^2} d(n_0(t)) \right.$$

$$\left[x\sigma^2 = \frac{N_0T}{2\tau^2} \Rightarrow \sqrt{\frac{N_0T}{2\tau^2}} = \sigma \right.$$

$$= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{2\tau^2}{N_0T}}\, e^{-\frac{n_0^2(t)}{2\frac{N_0T}{2\tau^2}}} x \Big]$$

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$$P_e = \int_{-\infty}^{\infty} f(n_0(t))\, dn_0(t)$$

$$P_e = \int_{\frac{x_{01}(1) - x_{02}(t)}{2}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(n_0(t))^2/2\sigma^2}\, d(n_0(t))$$

Let $y = \dfrac{n_0(t)}{\sqrt{2}\sigma} \Rightarrow d(n_0(t)) = \sqrt{2}\cdot\sigma\, dy$

$n_0(t) \to \infty \Rightarrow y \to \infty$

$n_0(t) \to \infty \Rightarrow y \to \infty$

$n_0(t) \to \dfrac{x_0(t) - x_{02}(t)}{2} \Rightarrow y \to \dfrac{x_0(T) - x_{02}(T)}{2\sqrt{2}\sigma}$

$$P_e = \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} \frac{1}{\sqrt{\cancel{2\pi}}\cancel{\sigma}}\, e^{-y^2}\, \sqrt{\cancel{2}}\cancel{\sigma}\, dy$$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} e^{-y^2}\, dy$$

$$= \frac{1}{2}\, \frac{2}{\sqrt{\pi}} \int_{\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}}^{\infty} e^{-y^2}\, dy$$

$$= \frac{1}{2}\, \mathrm{Erfc}\left(\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}\right)$$

$$P = \frac{x_0^2(T)}{\sigma^2} \left[(S/N) \text{ power} \right]$$

$$x_0(f) = H(f)\, x(f)$$

* $$x_0(T) = \int_{-\alpha}^{\alpha} H(f)\, x(f)\, e^{j\omega T}\, df$$

* $$\sigma^2 = \int_{-\alpha}^{\alpha} |H(f)|^2\, S_{ni}(f)\, df$$

$$\rho = \frac{|x_0(T)|^2}{|\sigma^2|}$$

$$= \frac{\left| \int_{-\alpha}^{\alpha} H(f)\, x(f)\, e^{j\omega T}\, df \right|^2}{\left| \int_{-\alpha}^{\alpha} (H(f))^2\, S_{ni}(f) \right|}$$

Let

$$\theta_1 = H(f) \sqrt{S_{ni}(f)}$$

$$\theta_2 = \frac{1}{\sqrt{S_{ni}(f)}}\, x(f)\, e^{j\omega T}$$

$$\rho = \frac{|\theta_1 \cdot \theta_2|^2}{|\theta_1|^2}$$

$$\boxed{\therefore \rho = \frac{|\theta_1 \cdot \theta_2|^2}{|\theta_1|^2}}$$

* Schwarz's Equations :-

$$\int_{-\alpha}^{\alpha} |O_1 \cdot O_2|^2 df \leq \int |O_1|^2 df \int |O_2|^2 df$$

$$\rho \leq \frac{|O_1|^2 df \; |O_2|^2 df}{|O_1|^2 df}$$

$$\rho \leq |O_2 (P)]^2 df$$

$$\leq \frac{1}{S_{ni}(f)} |x(f)\, e^{j\omega T}|^2$$

$$\leq \frac{|x(f)]^2}{S_{ni}(f)} |e^{j\omega T}|^2$$

$$\leq \frac{|x(f)|^2}{S_{ni}(f)} \qquad |e^{j\omega T}| = 1$$

According to schwarz's equation of the equation is to become equality. we should change the assomations of O_1 and O_2 when it should as following

$$O_1(f) = K\, O_2^*(f)$$

$$H(f)\sqrt{S_{ni}(f)} = K \cdot \frac{1}{\sqrt{S_{ni}(f)}} X^*(f)\, e^{-j\omega T}$$

$$\boxed{H(f) = K \frac{1}{S_{ni}(f)} X^*(f)\, e^{-j\omega T}}$$

$$\boxed{\rho = \frac{x_0(T)}{\sigma^2} = \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2 = \int_{-\alpha}^{\alpha} \frac{|x(f)|^2}{S_{ni}(f)} df}$$

* Matched filter :-

* for this filter we considered generalized gaussian noise.
* when the noise is white gaussian noise, then the optimum

Probability error for matched filter:-

$$\rho^2 = \left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2 = \frac{2}{N_0}\int_{-\alpha}^{\alpha} |X(f)|^2 df$$

from parasavals $\int_{-\alpha}^{\alpha} |X(f)|^2 df = \int_{-\alpha}^{\alpha} |x(t)|^2 dt$

$$\rho = 2/N_0 \int_{-\alpha}^{\alpha} (x_0(t))|^2 dt$$

$$= \frac{2}{N_0}\int_{-\alpha}^{\alpha} [x_{0_1} - x_{0_2}]^2 dt$$

$$\rho = \frac{2}{N_0}\left\{ \int_0^T x_{0_1}^2 dt + \int_0^T x_{0_2}^2 dt - 2\int_0^T 2x_{0_1} x_{0_2} dt \right.$$

$$\rho = 2/N_0 [\mathcal{E}_1 + \mathcal{E}_2 - 2(\mathcal{E}_{12})]$$

if $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}_{12} = \mathcal{E}$

$$\rho = 2/N_0 4\mathcal{E} = 8\mathcal{E}/N_0 \Rightarrow \sqrt{\rho} = \frac{2\sqrt{2}\sqrt{\mathcal{E}}}{\sqrt{N_0}}$$

$$P_e = 1/2\, Erfc\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}\right]$$

$$= \frac{1}{2} Erfc\left|\frac{1}{2\sqrt{2}}\sqrt{e}\right| \qquad \left[\because \frac{x_{01}(T)\cdot x_{02}(T)}{\sigma} = \sqrt{e}\right]$$

$$= \frac{1}{2} Erfc\left[\frac{1}{2\sqrt{2}}\,\frac{2\sqrt{2}}{1}\sqrt{\mathcal{E}/N_0}\right]$$

for matched filter

$$P_e = 1/2\, Erfc\left[\sqrt{\epsilon/N_0}\right]$$

$$\boxed{\therefore P_e = 1/2\, Erfc\left[\sqrt{\epsilon/N_0}\right]}$$

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filter is called matched filter.

* for the white gaussian noise the power spectral density is given as.

$$S_{ni}(f) = \frac{N_0}{2}$$

General gaussian noise filter

T.F for matched filter

$$H(f) = k\, {}^{2}/_{N_0}\, X^*(f)\, e^{-j\omega T}$$

Impulse response of matched filter h(t)

$$h(t) = IFT\,(H(f))$$

$$H(t) = \int_{-\alpha}^{\alpha} k\, \frac{2k}{N_0}\, X^*(f)\, e^{-j\omega T} e^{j\omega T}\, df$$

$$= IFT\left[{}^{2k}/_{N_0}\, X^*(f) e^{-j\omega T}\right]$$

$$= IFT\left[{}^{2k}/_{N_0}\, X(-f)\, e^{-j\omega T}\right]$$

we know that

$$FT[x(-t)] = X(-f)$$

$$\& \quad FT[x(T-t)] = X(-f)\, e^{-j\omega T}$$

$$So = IFT\left[{}^{2k}/_{N_0}\, FT[x(-f)]\, e^{-j\omega T}\right]$$

$$= IFT\left[\frac{2k}{N_0}\, FT\; x(T-t)\right]$$

$$h(t) = \frac{2k}{N_0}\, x_0(T-t)$$

$$h(t) = \frac{2k}{N_0}\left[x_1(T-t) - x_2(T-t)\right]$$

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Detection of ASK — Pe of ASK

Consider pe for optimum $= \frac{1}{2} \operatorname{Erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\sigma}\right]$

Output function $H(F)$

Let

$$\rho = \frac{x_0^2(T)}{\sigma^2} - \left[\frac{(x_{01}(T) - x_{02}(T)}{\sigma}\right]^2$$

$$= \int_{-\alpha}^{\alpha} \frac{|X(F)|^2}{S_{ni}(F)} dF$$

$$P = 2/N_0 \int_{-\alpha}^{\alpha} |X(F)|^2 dF$$

parseval's power theorem

$$\int_{-\alpha}^{\alpha} |X(F)|^2 dF = \int_{-\alpha}^{\alpha} |x(t)|^2 dt = \int_0^T |x(t)|^2 dt$$

$$P = 2/N_0 \int_0^T |x(t)|^2 dt$$

$$= \frac{2}{N_0} \int_0^T (x_{01}(T) - x_{02}(T)]^2 dt$$

for ASK $x_{01}(T) = A\cos\omega t$

$x_{02}(T) = 0$

$$\rho = 2/N_0 \int_0^T (A\cos\omega t)^2 dt$$

$$= \frac{2A^2}{N_0} \int_0^T \cos^2\omega t \, dt$$

$$= \frac{A^2}{N_0} \int_0^T 2\cos^2\omega t \, dt = \frac{A^2}{N_0} \int_0^T 1 - \cos 2\omega t \, dt$$

$$= \frac{A^2 \cdot}{N_0}\left[[1]_0^T - \int_0^T \cos 2\omega t \, dt\right]$$

→ it must be zero.

$$\boxed{\therefore \rho = \frac{A^2}{N_0}[T]}$$

$$\therefore \sqrt{e} = A\sqrt{T/N_0}$$

$$P_e = \frac{1}{2}\, \mathrm{Erfc}\left[\frac{1}{2\sqrt{2}} A\sqrt{T/N_0}\right]$$

$$\therefore A = \sqrt{2P_s}$$

$$P_e = \frac{1}{2}\, \mathrm{Erfc}\left[\frac{1}{2\sqrt{2}}\sqrt{2P_s}\sqrt{T/N_0}\right]$$

$$= \frac{1}{4}\, \mathrm{Erfc}\sqrt{\frac{P_sT}{N_0}}$$

$$\boxed{P_e = \frac{1}{2}\, \mathrm{Erfc}\sqrt{E/4N_0}}$$

*Detection of PSK:— Pe of PSK

we know that $x_{01}(T) = A\cos\omega t$

$x_{02}(T) = -A\cos\omega t$

for PSK

$$\rho = 2/N_0 \int_0^T (2A\cos\omega T)^2 dt$$

$$= 8/N_0 \int_0^T (A\cos\omega T)^2 dt$$

$$= \frac{4A^2}{N_0}\int_0^T 2\cos^2\omega T \, dt$$

$$= \frac{4A^2}{N_0}\int_0^T [1 + \cos 2\omega t) dt$$

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$$\rho = \frac{4A^2}{N_0}[T]$$

$$\sqrt{e} = 2A\sqrt{T/N_0}$$

$$P_e = \frac{1}{2}\,\mathrm{Erfc}\left[\frac{1}{2\sqrt{2}}\,2A\sqrt{T/N_0}\right]$$

$$= \frac{1}{2}\,\mathrm{Erfc}\left[\frac{1}{\cancel{2}\cancel{\sqrt{2}}}\,\cancel{2}\cancel{\sqrt{2}}\sqrt{P_s}\sqrt{T/N_0}\right]$$

$$= \frac{1}{2}\,\mathrm{Erfc}\left[\sqrt{\frac{P_sT}{N_0}}\right] \qquad \left[\because \sqrt{P_sT} = \sqrt{\epsilon}\right]$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{Erfc}\sqrt{\epsilon/N_0}}$$

* Detection of FSK:– Pe of FSK

We know that

$$x_{01}(T) = A\cos(\omega+\phi)t$$
$$x_{02}(t) = A\cos(\omega-\phi)t$$

$$\rho = \frac{2}{N_0}\int_0^T [A\cos(\omega+\phi)t - A\cos(\omega-\phi)t]^2\,dt$$

$$= \frac{2A^2}{N_0}\int_0^T[-2\sin\omega t\ \sin\phi t]^2\,dt = 2/N_0\int_0^T 2\sin^2\omega t\ 2\sin^2\phi t\,dt$$

$$= \frac{2A^2}{N_0}\left[\int_0^T(1-\cos 2\omega t)\,(1-\cos 2\phi t)\,dt\right]$$

$$= \frac{2A^2}{N_0}\left[\int_0^T(1-\cos 2\phi t - \cos 2\omega t + \cos 2\phi t\ \cos 2\omega t)\,dt\right.$$

$$= \frac{2A^2}{N_0}\int_0^T \cancel{1-\cos 2\phi t - \cos 2\omega t + 1/2\cos(\omega+\phi)t + 1/2\cos(\omega-\phi)t}\,]\,dt$$

$$= \frac{2A^2}{N_0}\left[\int_0^T 1\,dt \cdot \int_0^T \cos 2\phi t\,dt \cdot \int_0^T \cos 2\omega t\,dt + \frac{1}{2}\int_0^T \cos(\omega+\phi)t\,dt + \frac{1}{2}\int_0^T \cos(\omega-\phi)\,dt\right]$$

$$= \frac{2A^2}{N_0}\left[T - \left[\frac{\sin 2\phi T}{2\phi}\right]\right] - 0 - 0 - 0\Bigg]$$

$$\rho = \frac{2A^2T}{N_0}\left[1 - \frac{\sin 2\phi T}{2\phi T}\right]$$

P_{max} of $2\phi T = 3\pi/2$

$$P_{max} = \frac{2A^2T}{N_0}\left[1 - \frac{\sin 3\pi/2}{3\pi/2}\right]$$

$$= \frac{2A^2T}{N_0}\left[1 - \frac{\sin 225°}{225}\right]$$

$$= \frac{2(\sqrt{2p})^2T}{N_0}[1.21] = \frac{4pT}{N_0}(1.21)$$

$$P_{max} = \frac{4.84(E)}{N_0}$$

$$\sqrt{\rho_{max}} = \sqrt{\frac{4.84(E)}{N_0}}$$

$$P_e = \frac{1}{2}\,\mathrm{Erfc}\left[\frac{1}{2\sqrt{2}}\sqrt{e}\right]$$

$$= \frac{1}{2}\,\mathrm{Erfc}\left[\frac{1}{2\sqrt{2}}\sqrt{\frac{4.84E}{N_0}}\right]$$

$$= \frac{1}{2}\,\mathrm{Erfc}\left[\sqrt{\frac{0.6E}{N_0}}\right]$$ for FSK.

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Digital Communications

Analog Communication:-

In this communication, the message signal is continuous in nature i.e; the modulating signal is a continuous signal.

Digital Signal:-

In this communication, the message signal is in the form of binary digits. i.e; 1's and 0's.

Classification of Modulation:-

- Modulation
  - Continuous wave modulation
    - Amplitude modulation
    - Angle modulation
      - Frequency modulation
      - Phase modulation.
  - Pulse modulation.
    - Pulse analog modulation
      - Pulse amplitude modulation. (PAM)
      - pulse width modulation (PWM)
      - pulse position modulation (PPM)
    - Pulse digital modulation
      - pulse code modulation (PCM)
      - Delta modulation (DM)
      - adaptive delta modulation (ADM)
      - differential pulse code modulation (DPCM)

In general modulation can be classified into two types:-

i) Continuous wave modulation.
ii) Pulse modulation.

i) Continuous Wave Modulation:-

If the carrier signal is continuous, then that modulation is called continuous modulation.

Amplitude modulation and Angle modulation are the types of continuous wave modulation.

Phase modulation and frequency modulation comes under angle modulation.

ii) Pulse Modulation:-

If the carrier signal is a pulse signal, then that modulation is called pulse modulation. Pulse modulation is again classified as two types:-

i. Pulse analog modulation.
ii. Pulse digital modulation.

i. Pulse Analog Modulation:-

If the message signal is continuous in pulse modulation, then it is called pulse analog modulation.

Pulse amplitude modulation (PAM), Pulse width modulation (PWM) and Pulse position modulation (PPM) comes under pulse analog modulation.

ii) Pulse digital modulation:-

In pulse digital modulation, message signal (or) modulating signal is in the form of binary digits i.e, 0's and 1's.

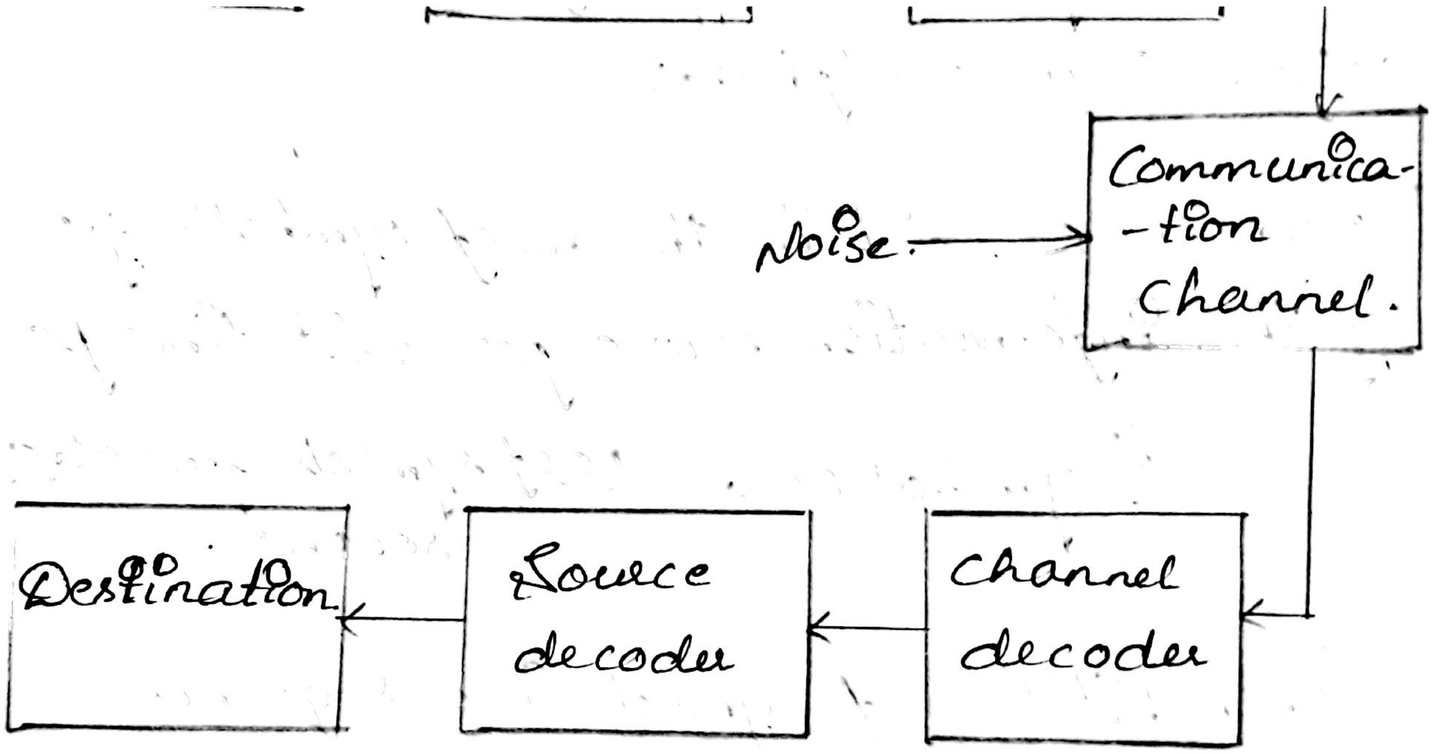
Pulse code modulation (PCM), Delta modulation (DM), Adaptive delta modulation (ADM) and Differential Pulse Code modulation (DPCM) comes under Pulse digital modulation

Applications of digital communications:-

i) Cellular Communications.

ii) Computer Communications.

iii) RADARs



Information Source:- Information source produce information in the form of symbols. The symbols may be either letters (or) special characters (or) digits.

There are two types of information sources.

If it is analog communication, analog information source is used.

In digital communication, digital information source is used which generates binary digits like 0's and 1's.

Important parameters of information source:

i) Source alphabet :- The letters, digits special characters generated by information source is called Source alphabet.

ii) Symbol rate:- It is the no. of symbols generated by the information source per unit time period.

$$\text{Symbol rate} = \frac{\text{no. of symbols generated}}{\text{second}}.$$

iii) Probabilities of symbols in a sequence:-

This represents occurance of symbol in a sequence. If we consider a sequence, the symbols will present number of times in a sequence with various probabilities. This will be one of the important parameter of the information source.

$$\text{Information rate} = \text{Symbol rate} \times \text{Source Entropy}.$$

= Symbol/sec × bits/symbol

= bits/sec.

Occurrance of source alphabet in a sequence:-

This parameter represents chance of occurrance of each source alphabet in a sequence.

Source Encoder:-

Source encoder converts the symbols generated by the information source into bits. Bit means binary digit i.e; binary 1's and 0's.

Code word:-

Group of bits is called code word.

Code word length:-

The no.of bits in a code word represents code word length. It may be 2, 4, 8, 16, 32. The no.of symbols represented by the each code word depends upon code word length.

Ex:- i) If code word length is 4, then no.of symbols represented by that code word is $2^4 = 16$.

ii) If code word length is 8, then no.of symbols represented by code word is $2^8 = 256$.

Important parameters of source encoder

i) Block size:- It is the maximum no. of code words represented by source encoder.

ii) Data rate:- It will represent the information content from the output of the source encoder.

Data rate = symbol rate × code word length.

= symbol/sec × no. of bits/symbol.

= bits/sec.

iii) Efficiency of the encoder:- It is the ratio of output of the encoder to the maximum input of the source encoder.

Source Decoder:-

At the receiver side source decoder converts bits into the symbols.

Channel Encoder:-

By using source encoder the message signal can be converted into binary digits. If these bits are transmitted through the communication channel, noise gets interfered with the information and wrong information can be received at the receiver.

To reduce these errors, channel encoder adds some redundant bits to the output of the source encoder.

Output of source encoder.			bit to be added by channel encoder to get even parity.
b_3	b_2	b_1	b_0
0	0	1	1
1	0	0	1
1	0	1	0
1	1	0	0

Important parameters:-

i) Type of the coding used.

ii) Coding rate:- It depends upon the various words assigned by the channel encoder

iii) Coding efficiency:- It is the ratio of input of channel encoder to the output of channel encoder

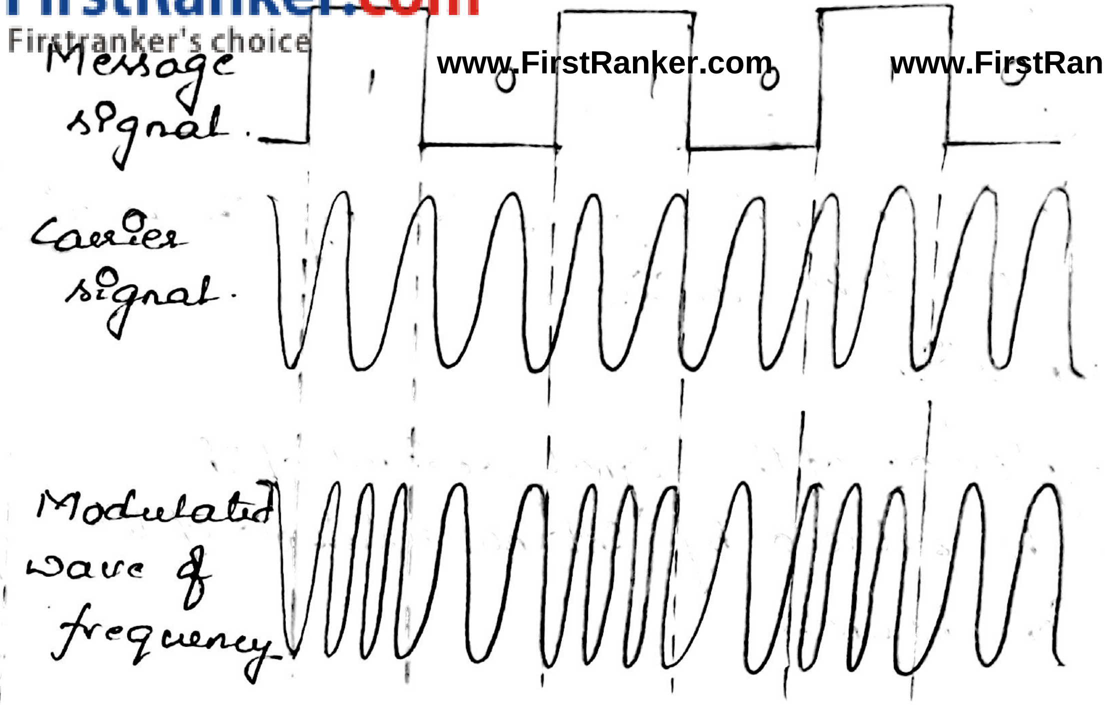
Digital modulators and demodulators:-

Since, the message signal is of digital, digital modulators and demodulators are used. In digital modulation, carrier signal is a continuous signal. ie; continuous sinusoidal signal. Hence these modulators are called continuous-wave digital modulators.

Amplitude shift keying (ASK)
Phase shift keying (PSK).
Frequency shift keying (FSK)
Differential phase shift keying (DPSK).
Minimum shift keying (MSK).

These are the examples of digital modulators.

The example of digital modulation is,



Digital demodulators:-

These will demodulate the received signal at the receiver.

Important parameters:-

i) Probability of error in the bit or symbol.

ii) Bandwidth needed to transmit the signal.

iii) Synchronous (or) asynchronous method of detection.

Communication Channels:-

Various communication channels transmit data through them. Wire lines, wireless, optical fibers, etc can be considered as communication channels and also magnetic disks, magnetic tapes and optical disks, can also be considered as communication channel. Because, they can transmit data through them.

Important parameters:-

i) Additive noise interference

This is the noise generated by the solid state components like resistors.

ii) Signal attenuation:-

The amplitude of the signal received at the receiver can be reduced by internal resistance of the communication channel.

iii) Amplitude and phase distortion:-

The amplitude and phase of the received signal can be distorted by the non linear characteristics of communication channel.

iv) Multi path distortion:-

This distortion obtained in wireless channels. The signals from various transmitters will overlap each other.

② Advantages and Disadvantages of digital communication:-

Advantages:-

1) Digital communication systems are simpler and cheaper compared to analog communication systems.

2) In digital communication, multiplexing of speech,

video data can be done.

3) Since the data is converted into digital form, noise interference can be tolerated.

4) Wide dynamic range is possible since, the data is converted in to digital form.

5) Since, the channel encoding is used, errors can be detected and corrected easily.

6) Digital communications is adaptive to other branches like, digital signal processing and image processing.

Disadvantages:-

1) Since the data is converted into digital form, data rate becomes high. Hence, transmission band width will be increased.

2) It requires synchronization in case of synchronous modulation.

Element of Pulse Code Modulation

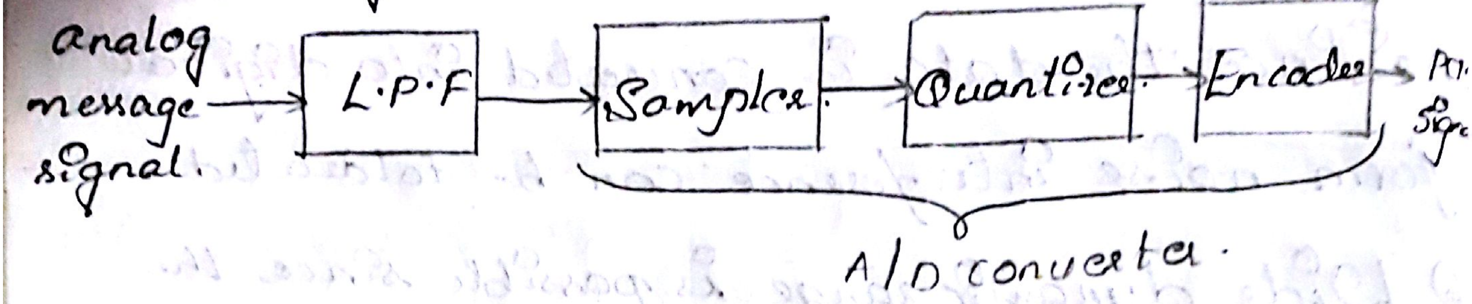
There are three basic sections in the Pulse code modulation and it is a digital pulse modulation technique.

i) transmitting section.

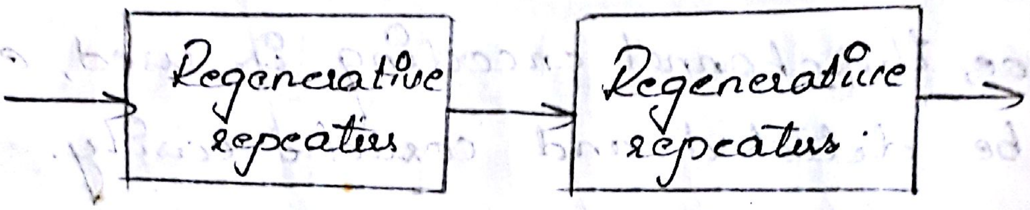
ii) transmitting path

iii) Receiving section.

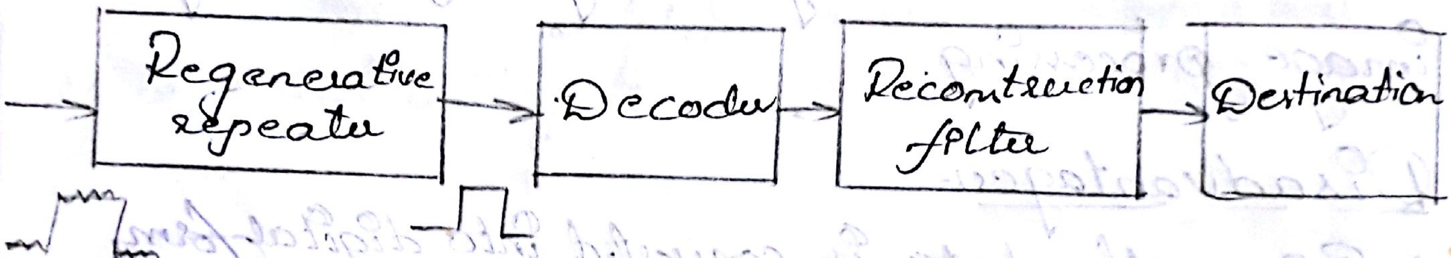
Transmitting section:-



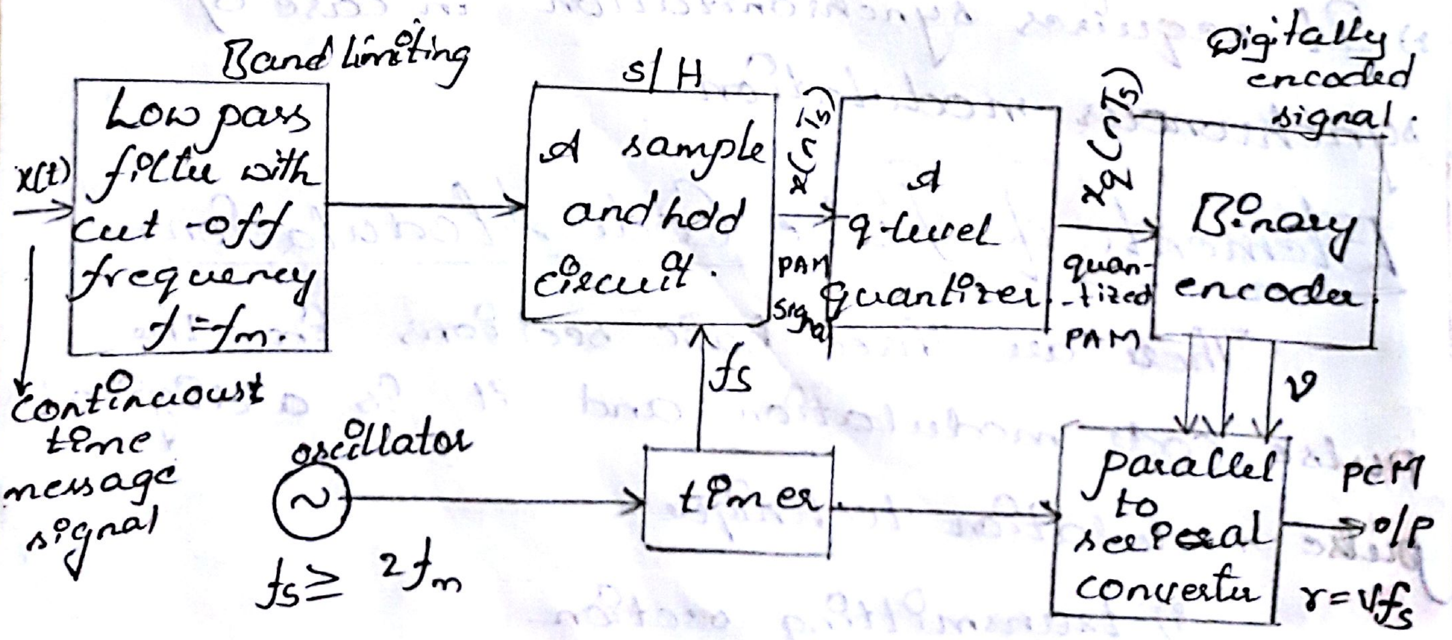
transmitting path:-



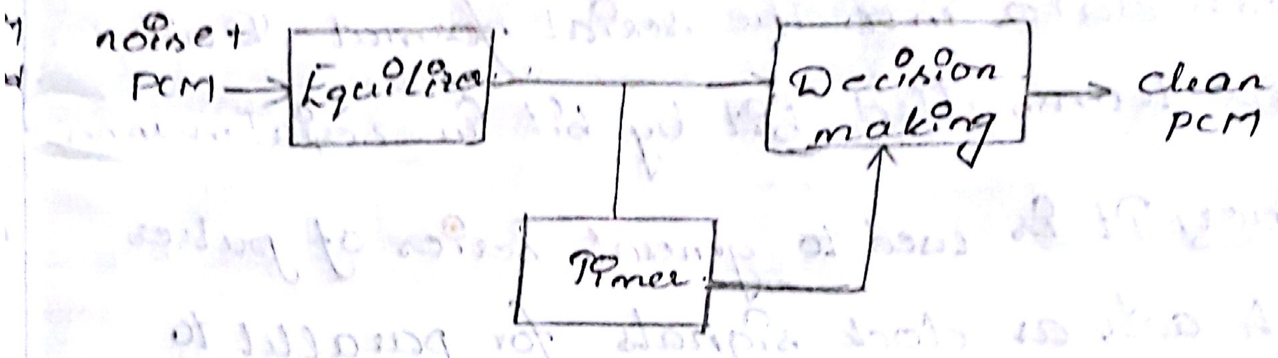
Receiving Section:-



i) PCM Generator (or) Transmitter:-



ii) Receiver



1) Transmitter

i) Low pass filter:- Low pass filter is used to band limit the message signal upto a frequency fm and all the other frequencies gets eliminated which are greater than frequency fm.

ii) Sample and hold circuit:- It is used to convert continuous time signal into descrete time signal by sampling.

iii) Q-level quantizer:- Quantizer compares the amplitudes of samples and it will assigns fixed digital levels to the amplitudes of im-pulses which are called as quantization levels. and this quantization can be done with minimum error called quantization error.

iv) Encoder:- It will converts the signal into the form of bits.

v) Parallel to Serial converter:- It is difficult to transmit all bits parallely at a time. Hence, Parallel to serial converter is used which converts data into the serial format. Data can be transmitted bit by bit in serial manner.

vi) Timer:- It is used to generate series of pulses which acts as clock signals for parallel to serial converter and sample and hold circuit based on the output of oscillator.

Repeaters:- Repeaters consists of three basic processes

i) Equalizing ii) Timing iii) Decision Making.

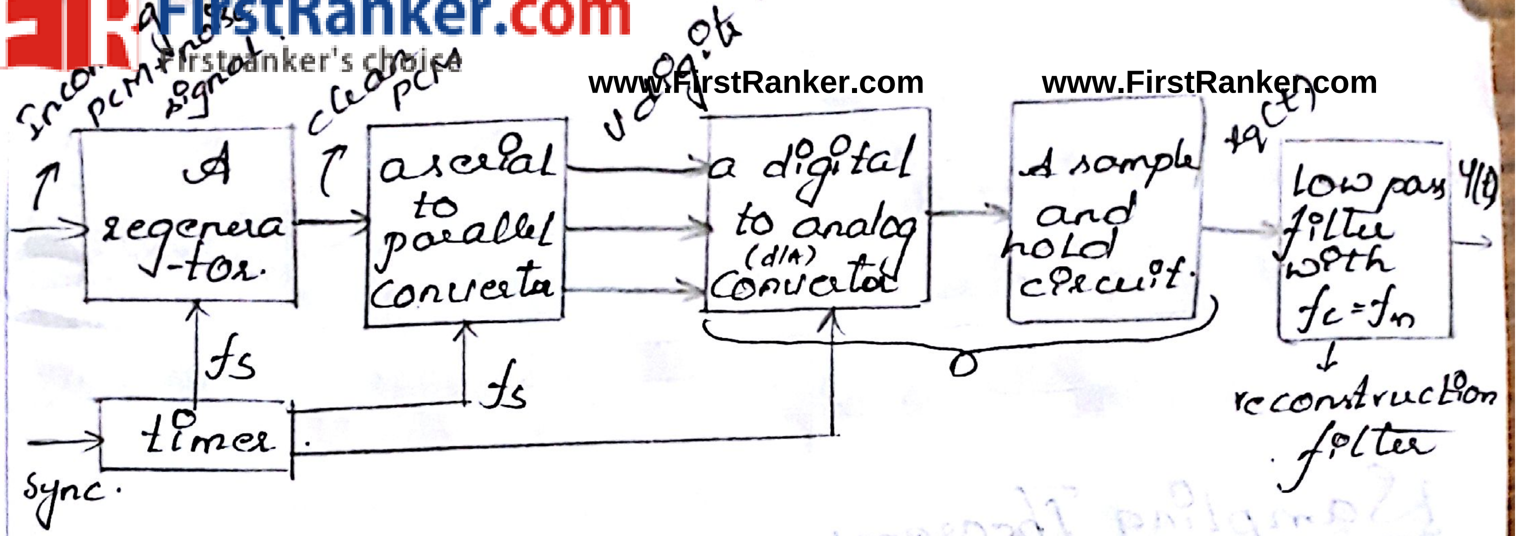
i) Equalizer:- It will reduce phase and amplitude distortion in the pulse code modulated wave.

ii) Timer:- It is used to generate series of pulses based on the input provided by the equalizer.

iii) Decision Making Device:- It compares the two signals from the equalizer and the timer and will performs sampling to the signal where the signal to noise ratio is maximum and gives clean PCM.

iii) PCM Receiver:-

The block diagram of PCM receiver is as follows.

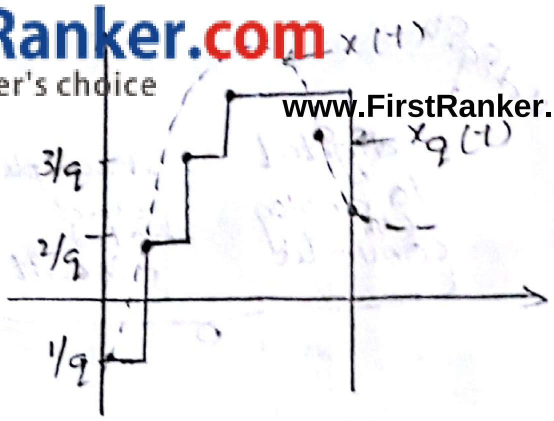


i) <u>Regenerator</u> :- It is used to reduce the distortion present in the PCM signal and gives clean PCM

ii) <u>Serial to parallel Convertor</u> :- It is used to convert serial data into the parallel format of v bits.

iii) <u>Digital to analog converter</u> and <u>Sample and hold circuits</u> :- These two circuits combinely converts digital signal into the analog signal.

iv) <u>Low pass filter</u> :- It is also called as reconstruction filter which is used to reconstruct original signal from its samples. Some times it would be difficult to reconstruct original signal from its samples because of quantization error is introduced permanently in the signal at the transmitter side. At the transmitter side to reduce this quantization error, data rate should ~~be increased which increases the transmission band~~



Sampling Theorem:-

This theorem is used to convert continuous time signal to descrete time signal.

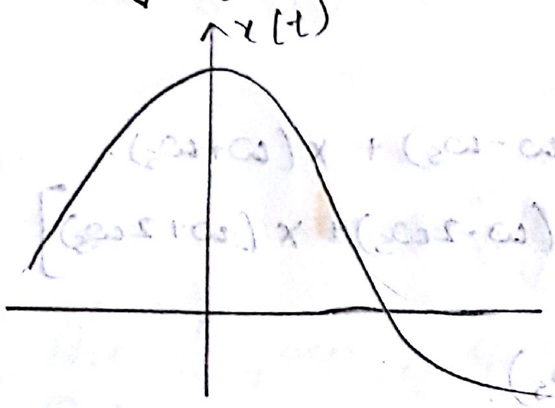
Statement1:- A band limited signal of finite energy which has no frequency components greater than f_m Hz can be converted to its samples with time intervals less than $\frac{1}{2f_m}$ seconds (sampling statement)

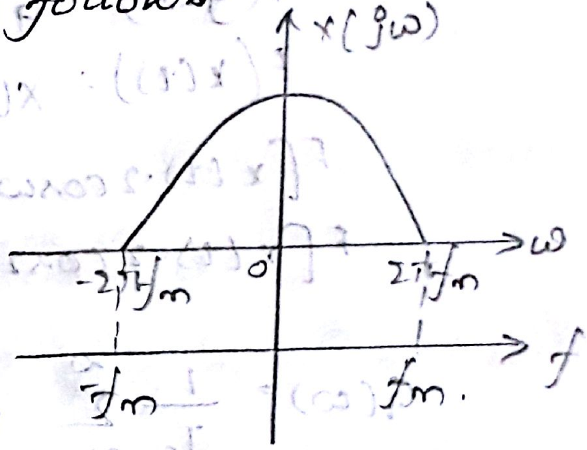
Statement 2:- A band limited signal of finite energy which has no frequency components greater than f_m Hz can be completely reconstructed from its samples under the condition $f_s \geq 2f_m$ (reconstruction statement)

Combined Statement:- A band limited signal of finite energy which has no frequency components greater than f_m Hz can be converted into its samples and reconstructed from its samples under the condition $f_s \geq 2f_m$.

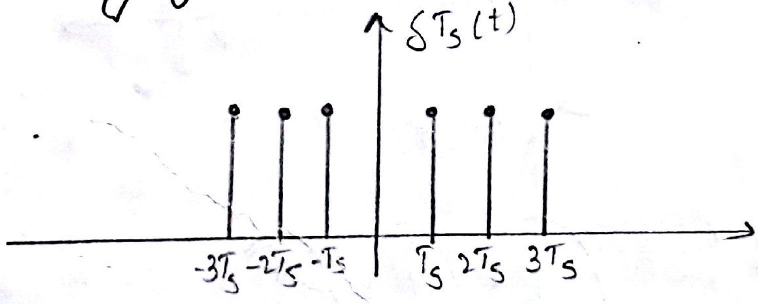
Proof:-

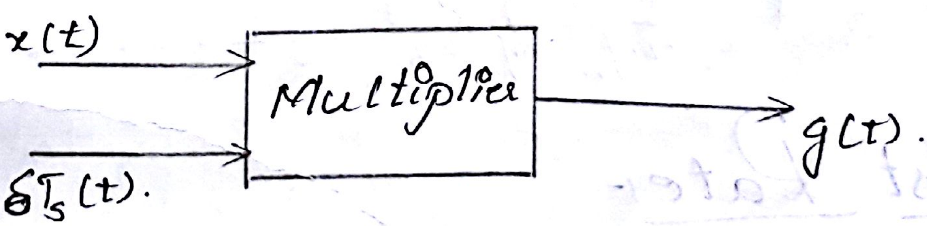
Consider a continuous time signal, $x(t)$ which has band limited to frequency f_m. The signal and its frequency spectrum will be as follows:





To perform sampling to $x(t)$, it can be multiplied with an impulse series of impulses with regularly spaced intervals represented by $\delta T_s(t)$





from multiplier, $g(t) = x(t) \cdot \delta T_s(t)$.

The trigonometric fourier series of $\delta T_s(t)$ is,

$$\delta T_s(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \cdots\right]$$

$$g(t) = x(t) \cdot \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \cdots\right]$$

Apply fourier transforms on both sides,

$$G(\omega) = \frac{1}{T_s}\left[X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + X(\omega+2\omega_s) + X(\omega-3\omega_s) + X(\omega+3\omega_s)\right] + X(\omega)\right]$$

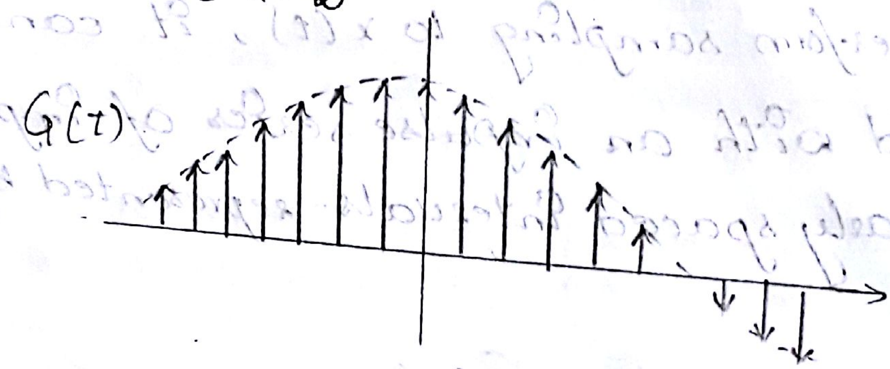
[∵ Since $F(g(t)) = G(\omega)$

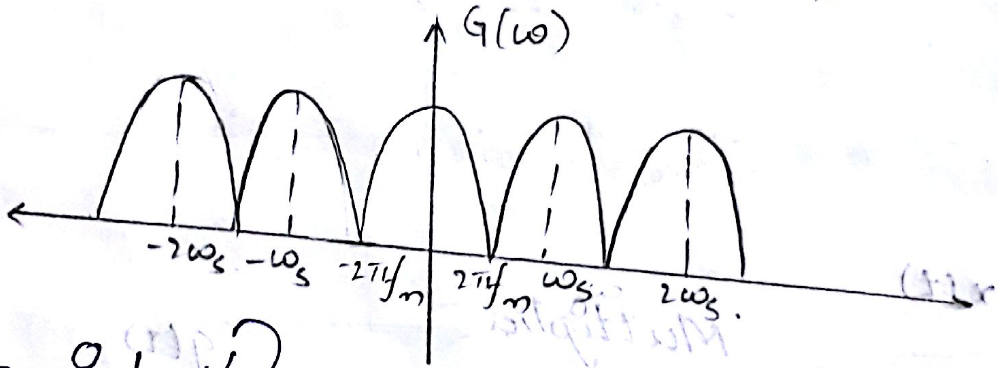
$F(x(t)) = X(\omega)$.

$F[x(t)\cdot 2\cos\omega_s t] = X(\omega-\omega_s) + X(\omega+\omega_s)$.

$F[x(t)\cdot 2\cos 2\omega_s t] = X(\omega-2\omega_s) + X(\omega+2\omega_s)$]

$$G(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s).$$





Nyquist Rate:-

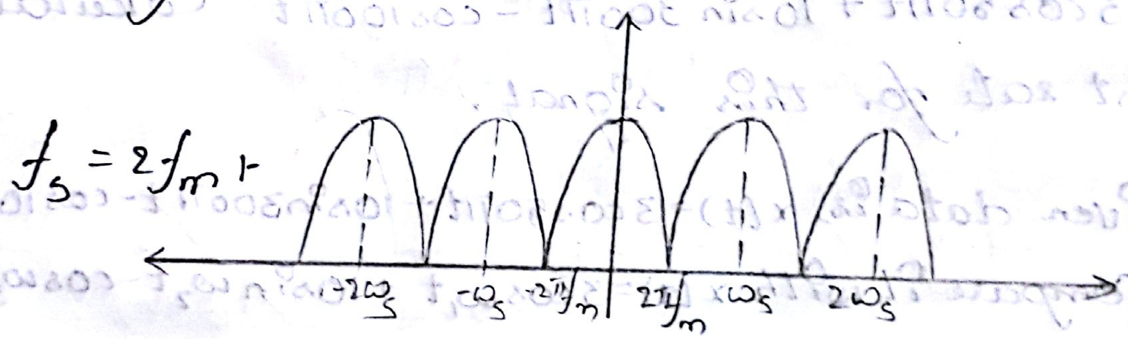
For the process of sampling the sampling frequency f_s should be greater than twice of the maximum frequency of the message signal.

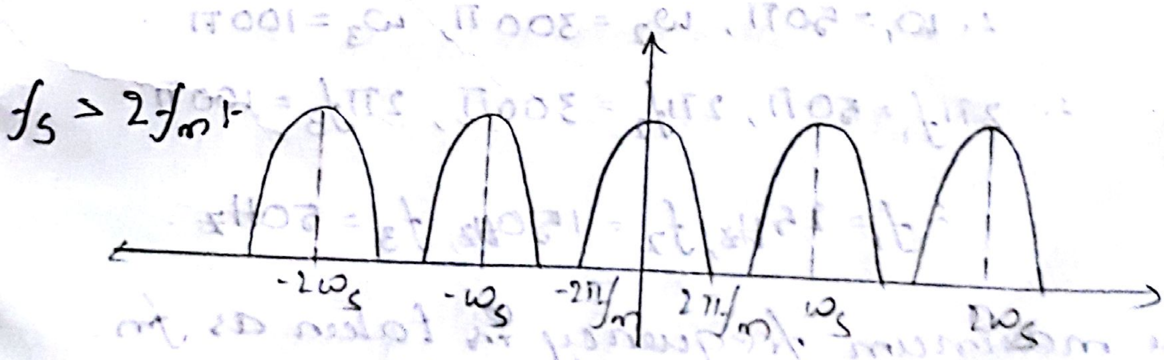
If the sampling frequency f_s is exactly ~~equal to the twice of the~~ maximum signal frequency of the message signal, then it is called "Nyquist Rate". It is also called as minimum sampling rate.

$$\text{Nyquist Rate} = f_s = 2f_m$$

$\therefore T_s = 1/2f_m$ which is called as "Nyquist interval."

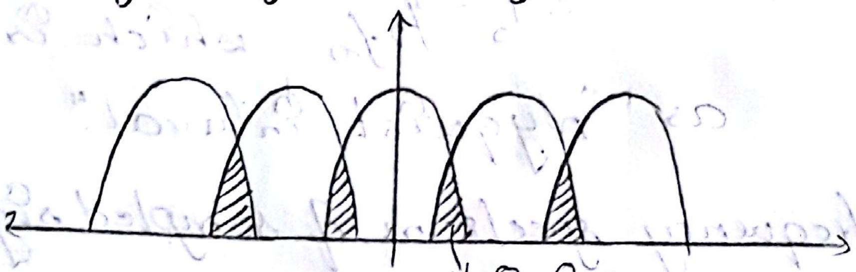
The frequency spectrum of sampled signal will consists of frequency bands under the condition of Nyquist rate will touch each other and will be as follows,





Aliasing Effect:

Under the condition $f_s < 2f_m$, the frequency bands of spectrum of sampling signal will be over-lap with each other because of this overlaping signal reconstruction will not be possible. The overlapping of frequency bands of spectrum is called as "Aliasing Effect". To reduce aliasing effect, the signal can be band limited by using a low pass filter before sampling.



Problem:-

An analog signal is expressed as $x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$. Calculate Nyquist rate for this signal.

Given data is, $x(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$

Compare it with $x(t) = 3\cos \omega_1 t + 10\sin \omega_2 t - \cos \omega_3 t$.

$\therefore \omega_1 = 50\pi, \ \omega_2 = 300\pi, \ \omega_3 = 100\pi$.

$\therefore 2\pi f_1 = 50\pi, \ 2\pi f_2 = 300\pi, \ 2\pi f_3 = 100\pi$.

$\therefore f_1 = 25\,Hz, \ f_2 = 150\,Hz, \ f_3 = 50\,Hz$.

The maximum frequency is taken as f_m.

$\therefore f_m = f_2 = 150\,Hz$.

Nyquist rate: $f_s = 2f_m$

$= 2 \times 150$

$\therefore f_s = 300\,Hz$.

Problem 2:-

Find the nyquist rate and nyquist interval for the signal, $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cos(1000\pi t)$

Given $x(t) = \frac{1}{2\pi}\cos(4000\pi t)\cdot\cos(1000\pi t)$.

$$= \frac{1}{4\pi}\cdot 2\cos(4000\pi t)\cdot\cos(1000\pi t)$$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$.

$$\therefore x(t) = \frac{1}{4\pi}\left[\cos(5000\pi t) + \cos(3000\pi t)\right]$$

Compare with $x(t) = \frac{1}{4\pi}\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right]$.

$\therefore \omega_1 = 5000\pi$, $\omega_2 = 3000\pi$

$2\pi f_1 = 5000\pi$, ω_2 $2\pi f_2 = 3000\pi$.

$\therefore f_1 = 2{,}500\ Hz$, $f_2 = 1500\ Hz$.

Hence, $f_m = f_1 = 2{,}500\ Hz$.

$\therefore$ Nyquist rate, $f_s = 2f_m$.

$= 2\times 2500$.

$= 5000\ Hz$.

$= 5\ kHz$.

Nyquist interval $T_s = \frac{1}{2f_m}$.

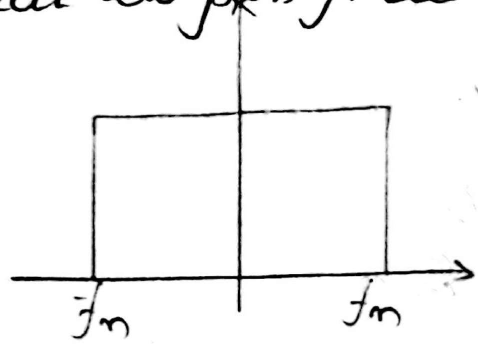
$= \frac{1}{5}$

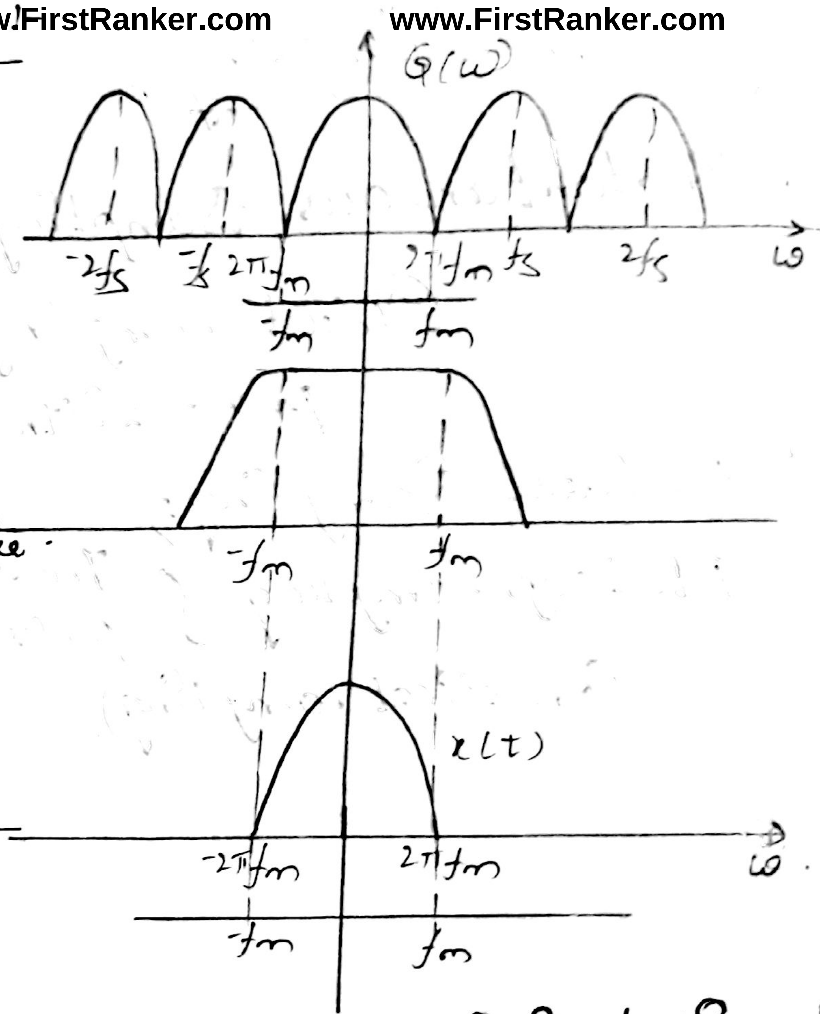
$= 0.2$ ~~kHz~~ msec.

$= 200\mu sec$.

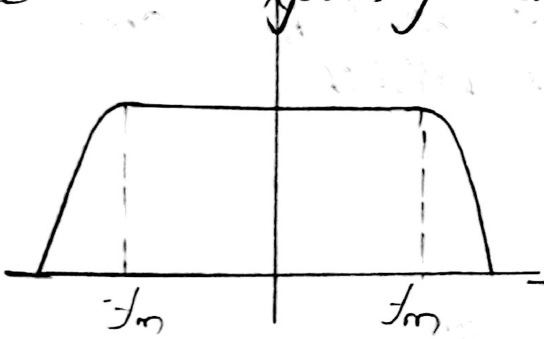
Reconstruction filter:

Ideal low pass filter





practical low pass filter.



In general to recover the original signal from its samples reconstruction filter is used, It is an ideal lowpass filter. which allows the frequency upto some cut off frequency.

Ideal LPF cannot be exist but practical LPF having transition bands nearly cutoff frequency will be as shown.

When we pass the sampling signal through the reconstruction filter we will get the original signal x(t).

Problem 3:-

If sampling frequency f_s =

A continuous time signal is,

$x(t) = 8\cos 200\pi t$ calculate minimum sampling rate. i)

ii) If the sampling frequency $f_s = 400\,Hz$. what is the discrete time signal $x(n)$ after sampling.

i) Given $x(t) = 8\cos 200\pi t$.

compare with $x(t) = 8\cos\omega_f t$

$$\therefore \omega_f = 200\pi$$
$$2\pi f_f = 200\pi$$
$$f = 100\,Hz \quad \therefore f_m = f = 100\,Hz$$

Sampling rate $f_s = 2f_m$.

$$= 2\times 100 = 200\,Hz$$

ii) $F = \frac{f_m}{f_s}$; given $f_s = 400\,Hz$.

$$= \frac{100}{400} = \cancel{4\,Hz}\ 0.25\,Hz$$

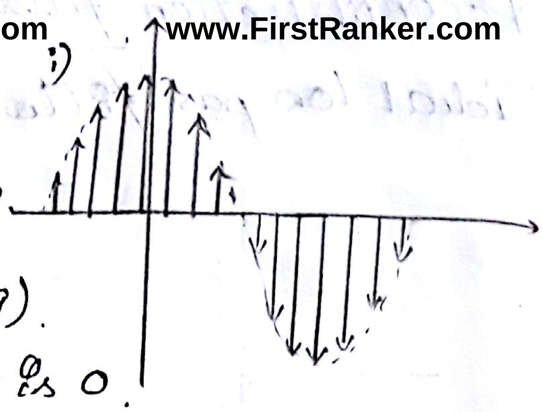
$$\therefore x(n) = 8\cos 2\pi F n$$
$$= 8\cos 2\pi(0.25)n$$
$$= 8\cos(0.5\pi n)$$
$$= 8\cos\frac{\pi n}{2}$$

Types of Sampling :-

i) Instantaneous Sampling

(Ideal sampling)

pulse width is 0.

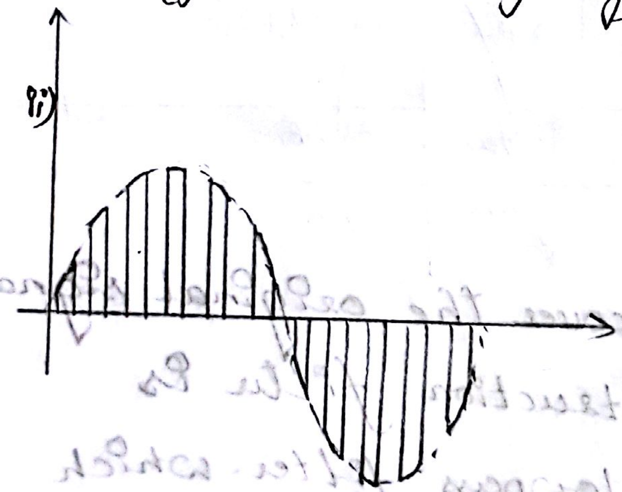


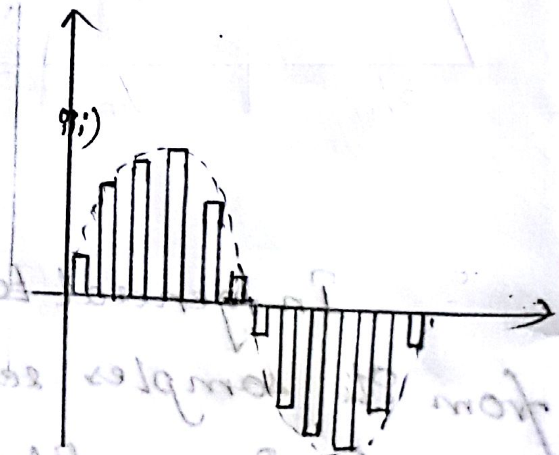
ii) Natural Sampling

iii) Flat top sampling

There will be some pulse width.

(practical sampling)





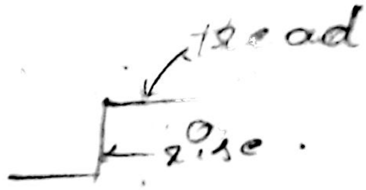
Quantization:-

Quantizer assigns some fixed digital levels to the amplitudes of the sampled signal with minimum distortion and error, which is called as quantization error.

There are many types of quantizers. They are:-

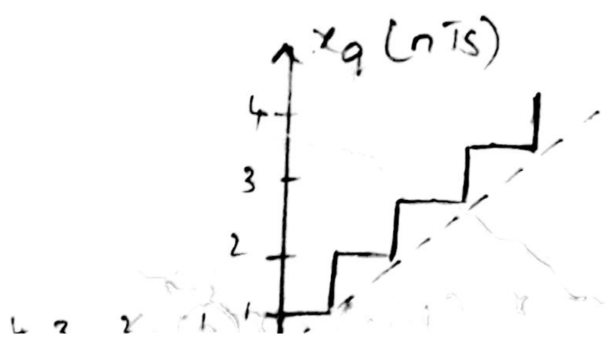
Quantizer

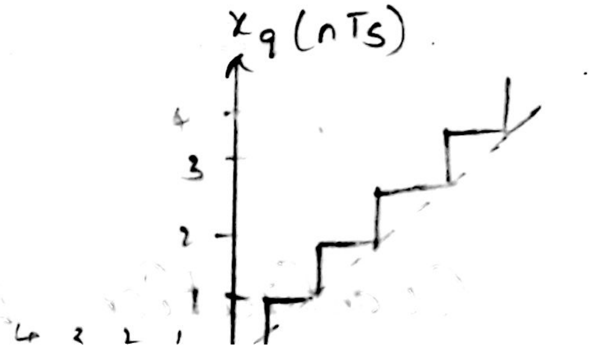
- Uniform
  - Midtread
  - Midrise.
- Non-uniform.



i) Uniform Quantizer:- In uniform quantizer the step signal size is remains same through out the input range. (This is a symmetrical type of quantizer).

ii) Non-uniform Quantizer:- In non uniform quantizer the step size varies based on the instantaneous values of the input signal.



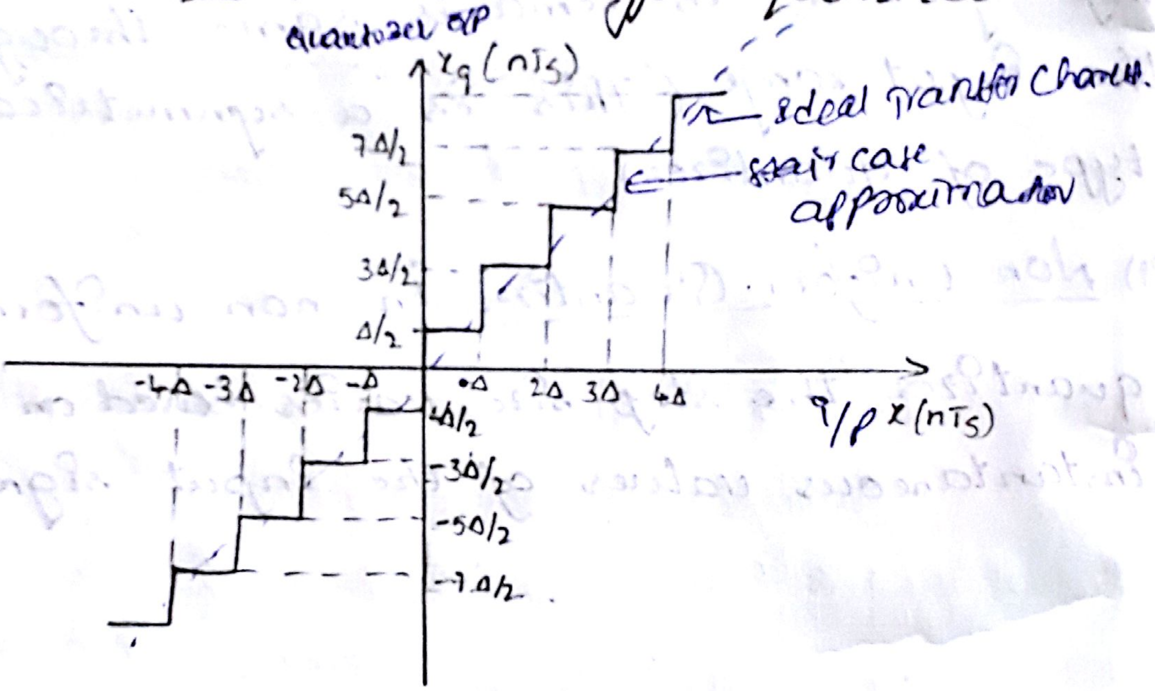


Symmetrical midrise quantizer

In this quantizer transfer characteristics origin will passes through the rising part of the stair case signal.

Working principle of quantizer

To know the working principle of quantizer we will take mid rise type quantizer.



Quantization Error $E = x_q(nT_s) - x(nT_s)$.

$\Delta/2 - 0 = \Delta/2$.

at $x(nT_s) = 0$, $x_q(nT_s) = \Delta/2, -\Delta/2$.

$\therefore E = \frac{\Delta}{2} - 0, \; -\frac{\Delta}{2} - 0$.

$E = \Delta/2 \quad E = -\Delta/2 \; . \; \therefore E = \pm \Delta/2$.

at every point i.e, $x(nT_s) = \pm\Delta, 2\Delta, 3\Delta$, $x_q(nT_s) = \pm\frac{\Delta}{2}$

$E = \frac{\pm 3\Delta}{2} - \Delta = \frac{\pm\Delta}{2}$.

Hence, at every point, i.e.

$$e = \pm \Delta/2$$

∴ The maximum limits of error are $-\Delta/2$ & $\Delta/2$

Transmission Band width in PCM

If 'v' digits represented by the encoder of the PCM, then the no. of quantization levels represented by the quantizer of PCM is 2^v.

Information rate or signaling rate

r = no. of samples per second × no. of bits per samples

$$r = f_s \times v$$

$$\therefore \boxed{r = v \cdot f_s}$$

Transmission band width should be greater than half of the signaling rate.

$$B.W \geq \frac{1}{2} r.$$

$$\therefore B.W \geq \frac{1}{2}(v f_s).$$

Nyquist rate $f_s = 2 f_m$

$$\therefore B.W \geq \frac{1}{2}(v \cdot 2 f_m).$$

$$\therefore \boxed{B.W \geq v f_m}$$

Quantization error in pulse code modulation:

The quantization error in pulse code modulation can be expressed as.

$$E = x_q(nT_s) - x(nT_s)$$

Consider a symmetrical midrise quantizer whose maximum and minimum amplitudes are $+7\Delta/2$ and $-7\Delta/2$ at maximum and minimum sampled values ie; $+4\Delta$ and -4Δ.

Let the amplitudes be ~~x~~ maximum and $-x_{maximum}$ and the total amplitude range becomes

as $(x_{max}) - (-x_{max}) = 2x_{max}$.

If q-quantization levels used to represent the signal, then step size $= \dfrac{2x_{max}}{q} = \dfrac{\text{Total amplitude level}}{\text{quantization levels}}$.

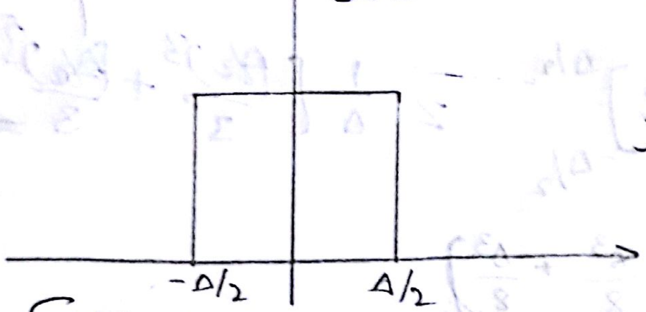
Let $x_{max} = 1$ and $-x_{max} = -1$ (Normalized signal)

then step size (δ) $= \dfrac{2(x_{max})}{q} = \dfrac{2}{q}$.

The quantization error can be considered as the uniformly distributed random variable.

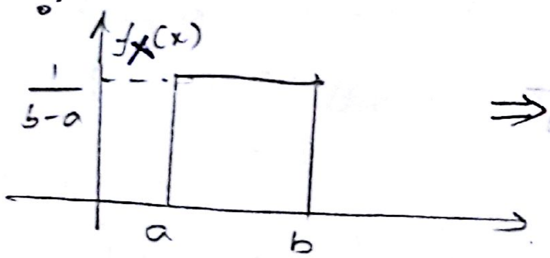
The uniformly distributed random variable can be represented as shown in the below diagram

$f_\epsilon(\epsilon)$ → quantization error



$$f_\epsilon(\epsilon) = \begin{cases} 0 & \text{for } \epsilon \le -\Delta/2 \\ 1/\Delta & \text{for } -\Delta/2 < \epsilon < \frac{\Delta}{2} \\ 0 & \text{for } \epsilon > \Delta/2. \end{cases}$$

$\because$



$$\Rightarrow \quad f_X(x) = \begin{cases} 0 & \text{for } x \le a. \\ \frac{1}{b-a} & \text{for } a < x \le b. \\ 0 & \text{for } x > b. \end{cases}$$

Signal to Noise Ratio $\frac{S}{N}$ = $\dfrac{\text{Signal power (Normalized)}}{\text{Noise power (Normalized)}}$.

Quantization Error = Noise power (Normalized).

$$\text{Noise power} = \frac{V^2_{\text{Noise}}}{R}.$$

V^2_{Noise} = mean square value of error.
$= E(\epsilon^2)$.

$$\therefore E(\epsilon^2) = \overline{\epsilon}^2 = \int_{-\infty}^{\infty} \epsilon^2 \cdot f_\epsilon(\epsilon) \cdot d\epsilon .$$

$$\left[\because E(X^2) = \overline{X}^2 = \int_{-\infty}^{\infty} x^2 \cdot f_X(x) \cdot dx \right].$$

$$\therefore E(\epsilon^2) = \int_{-\Delta/2}^{\Delta/2} \epsilon^2 \cdot \frac{1}{\Delta}\, d\epsilon .$$

$$E(\epsilon^2) = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} \epsilon^2 \cdot d\epsilon$$

$$= \frac{1}{\Delta}\left[\frac{\epsilon^3}{3}\right]_{-\Delta/2}^{\Delta/2} = \frac{1}{\Delta}\left[\frac{(\Delta/2)^3}{3} + \frac{(\Delta/2)^3}{3}\right]$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right].$$

$$= \frac{2\Delta^3}{8} \times \frac{1}{3\Delta}.$$

$$= \Delta^2/12.$$

enc

Now, normalized power, $= \frac{V^2_{Noise}}{R} = \frac{\Delta^2/12}{R}$.

assume that $R = 1$.

$\therefore$ Noise power $= \frac{\Delta^2/12}{1} = \frac{\Delta^2}{12}$.

$\therefore$ Quantization error $= \frac{\Delta^2}{12}$.

Signal to noise ratio

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}}.$$

Noise power (normalized) $= \Delta^2/12$.

Let signal power $= P$.

$$\therefore \frac{S}{N} = \frac{P}{\Delta^2/12} \longrightarrow (1)$$

Δ = Step size $= \frac{2x_{max}}{q}$

---

for normalized wave $x_{max} = 1$.

$\Delta = 2/q$

$\left(\frac{\Delta/2)^3}{3} + \frac{(\Delta/2)^3}{3}\right]$

By substituting in equ. 1

$$\frac{S}{N} = \frac{P}{\left(\frac{2}{q}\right)^2 \cdot \frac{1}{12}}.$$

But, no. of quantization levels $q = 2^v$.

if v is the no. of bits generated by the encoder.

By substituting the q value,

$\frac{\Delta^2/12}{R}$

$$S/N = \frac{P}{\left(\frac{2}{2^v}\right)^2 \cdot \frac{1}{12}} = \frac{P}{\frac{4}{2^{2v}} \cdot \frac{1}{12}} = \frac{P}{\frac{1}{2^{2v}} \cdot \frac{1}{3}}$$

$$\therefore \boxed{\frac{S}{N} = P \cdot 3 \cdot 2^{2v}}.$$

assume $p=1$, then, $\boxed{\frac{S}{N} = 3 \cdot 2^{2v}}$.

S/N in decimals is, $\boxed{(S/N)_{db} = 10 \log_{10}\left(\frac{S}{N}\right)}$

$$\therefore \left(\frac{S}{N}\right)_{db} = 10 \log_{10}(3 \cdot 2^{2v})$$

$$= 10\left[\log_{10} 3 + \log_{10} 2^{2v}\right].$$

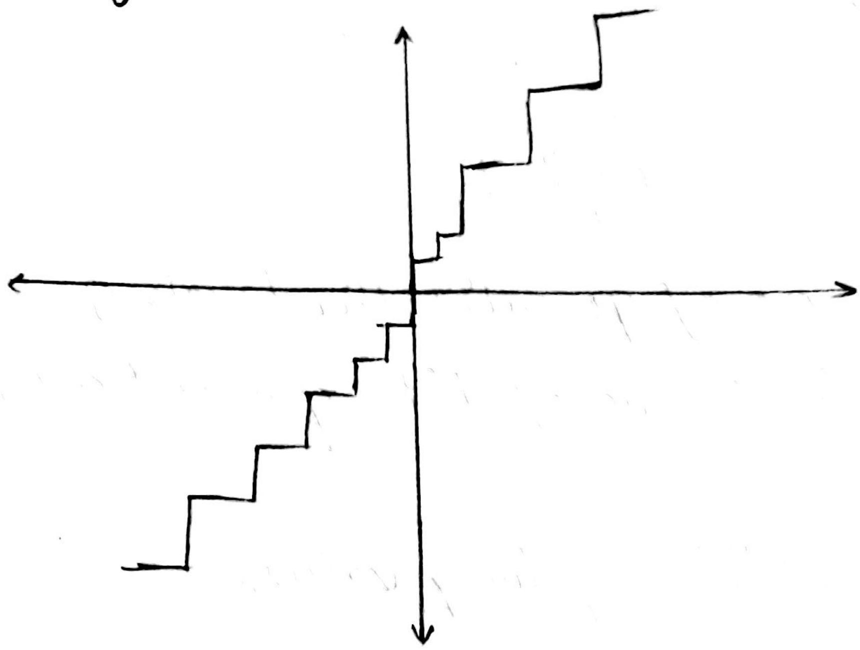
$$= 10\left[\log_{10} 3 + 2v \cdot \log_{10} 2\right].$$

$$= 10\left[0.48 + 2v(0.3)\right]$$

$$\therefore \boxed{\left(\frac{S}{N}\right)_{db} = 4.8 + 6v}$$

=1.

Non-Uniform Quantization:-



If the step size is varied, according to the input signal range, then the quantization is called as non-uniform quantization.

Necessity of Non-uniform quantization:-

In uniform quantization,

quantization error $\epsilon = \Delta/2$.

$$\Delta = \frac{2x_{max}}{q}$$

If $x_{max} = 1$, $\Delta = 2/q$.

$$q = 2^{v}.$$

If $v=4$, $q = 2^4 = 16$.

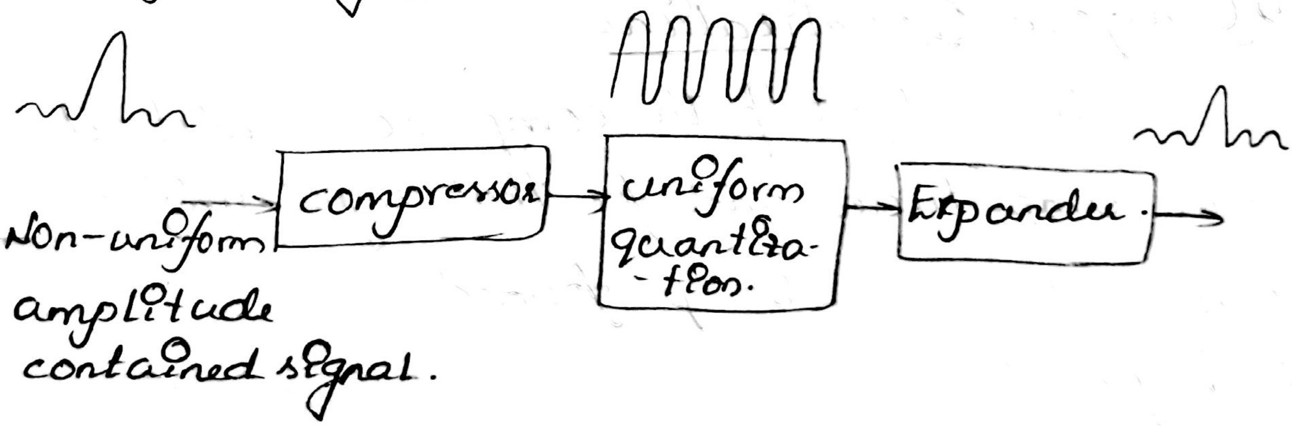
Substitute q value in Δ,

$$\therefore \Delta = \frac{2}{q} = \frac{2}{16} = \frac{1}{8}.$$

$$\therefore \epsilon = \frac{\Delta}{2} = \frac{1}{8 \times 2} = \frac{1}{16}. \quad \therefore \boxed{\epsilon = 1/16}.$$

from the above calculation, 1/16th part of the signal consists of quantization error. If the signal having 16V amplitude, 1V will be the quantization error. If the signal have 2V, 3V amplitudes, then quantization error is nearly 50% to 30% of the signal. In uniform quantization, error can be introduced to the signal equally at all amplitudes which gives ~~wrong~~ maximum error. Hence, non-uniform quantization is used which adds errors based on amplitude levels. Hence, non-uniform quantization is called as "Robust quantization".

Companding:-

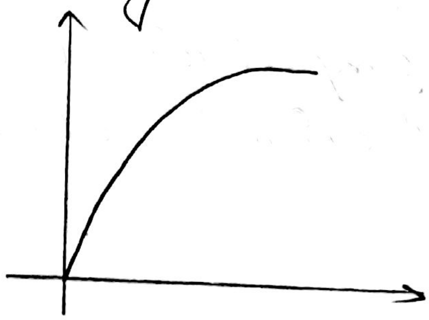


Basically, companding consist of two processes, i) Compressing.
ii) Expanding.

Compressing can be done at the transmitter side, expanding can be done at the receiver side.

i) <u>Compressor:-</u>

Compressor can amplify the low amplitude signals and gives output as high amplitude signals. It will performs attenuation for the high amplitude signals which can becomes as low amplitude signals. Thus, overall amplitude of all signals becomes equal.

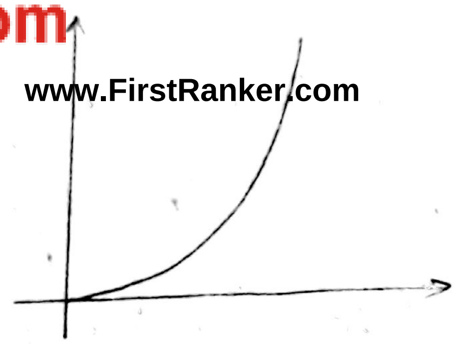


ii) <u>Uniform-Quantization:-</u>

The output of the compressor consists of equal amplitudes and for that signal uniform quantization can be performed.

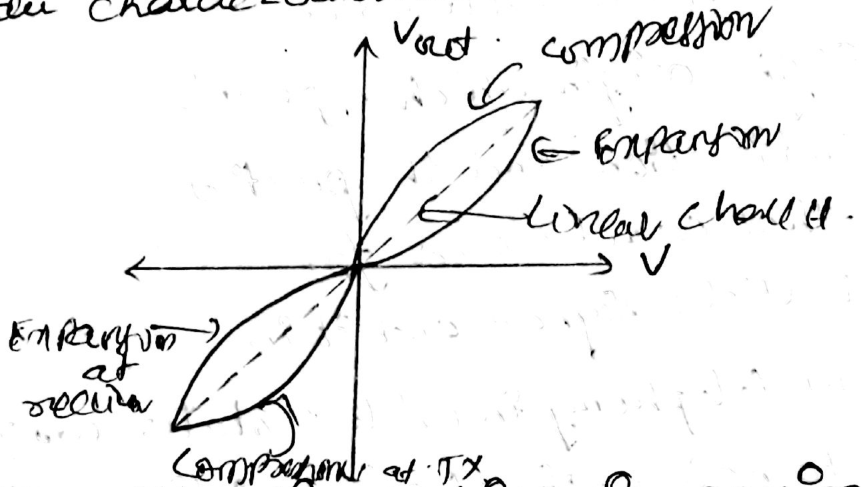
iii) <u>Expander:-</u>

Expander can perform reverse process of the compressor i.e; it will attenuate low frequency signals and amplifies the high frequency signals. Thus the original signal can be build by using Expander.



Companding Characteristics:-

It is the combination of compressor and expander characteristics.



The dotted line which is passing through the origin indicates uniform quantization.

There are two types of compandings:-

i) μ-law companding ii) A-law companding.

i) μ-Law companding:-

μ-Law compander characteristics are linear and continuous and it will acts as linear amplifier for small signals and logarithmic amplifier for high amplitude containing signals in ~~this~~ its compressor. The compressor characteristics of μ-law compander can be expressed by the formula.

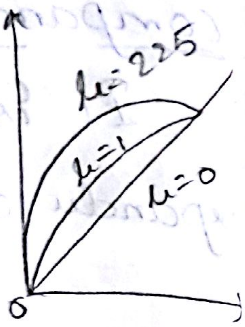
$$z(x) = \frac{(\mathrm{sgn})(x)\ln\left(1+\mu|x|/x_{max}\right)}{\ln(1+\mu)}$$

where $0 \le |x|/x_{max} \le 1$, $z(x)$ = o/p of compressor

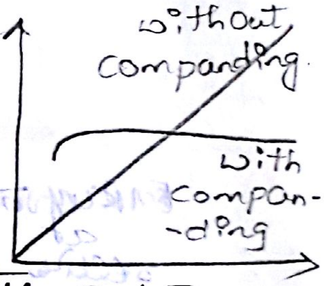
$sgn(x)$ = It will represents the sign of x i.e; ±1.

$\frac{|x|}{x_{max}}$ = normalized value of the i/p w.r.t max value

The μ-law compressor characteristics will be as shown.



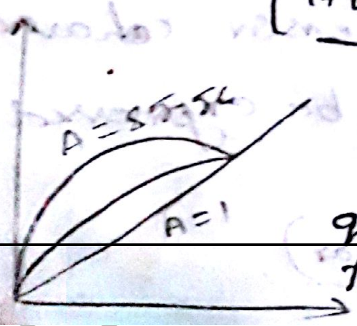
If μ=0 then characteristics passes through origin which represents uniform quantization and the practical value of μ that can be used for speech and video signals and it is used in PCM telephony in united states and Japan.



The signal to noise ratio characteristics with and without companding are as shown

A-Law Companding:- This companding uses piece wise-linear segment for low signals and Logarthemic amplifier for high signals. This companding is used in PCM telephony in germany. The characteristics of A law compressor is represented by the equation,

$$z(x) = \begin{cases} \dfrac{A \cdot |x|/x_{max}}{1+\log_e A} & \text{for } 0 \le |x|/x_{max} \le A \\ \dfrac{1+\log_e A|x|/x_{max}}{1+\log_e A} & \text{for } A \le \dfrac{|x|}{x_{max}} \le 1. \end{cases}$$



If A=1 that represents uniform quantization i.e; the characteristics passes through origin. The practical value of A

1) It is used in telephony.

2) It is used in space communication where the signal power is minimum and huge distance is present.

Advantages of PCM:-

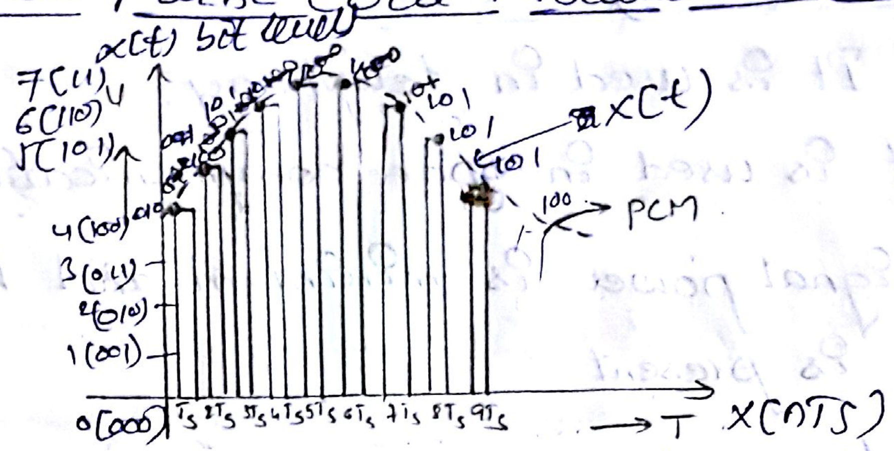
1) Noise immunity is more.
2) In PCM, between transmitter and receiver regenerative repeaters are used. These type of repeaters will not be used in analog communication. These repeaters are used to reduce the noise.
3) Because of encoding is used only one person can detect the original data.
4) Data can be stored easily because of data is in digital form.

Disadvantages of PCM:-

1) Because of the data is in digital, transmission band width will be increased.
2) System becomes complex because of sampling, quantization and encoding.

[P.T.O]

Differential Pulse Code Modulation Ahead

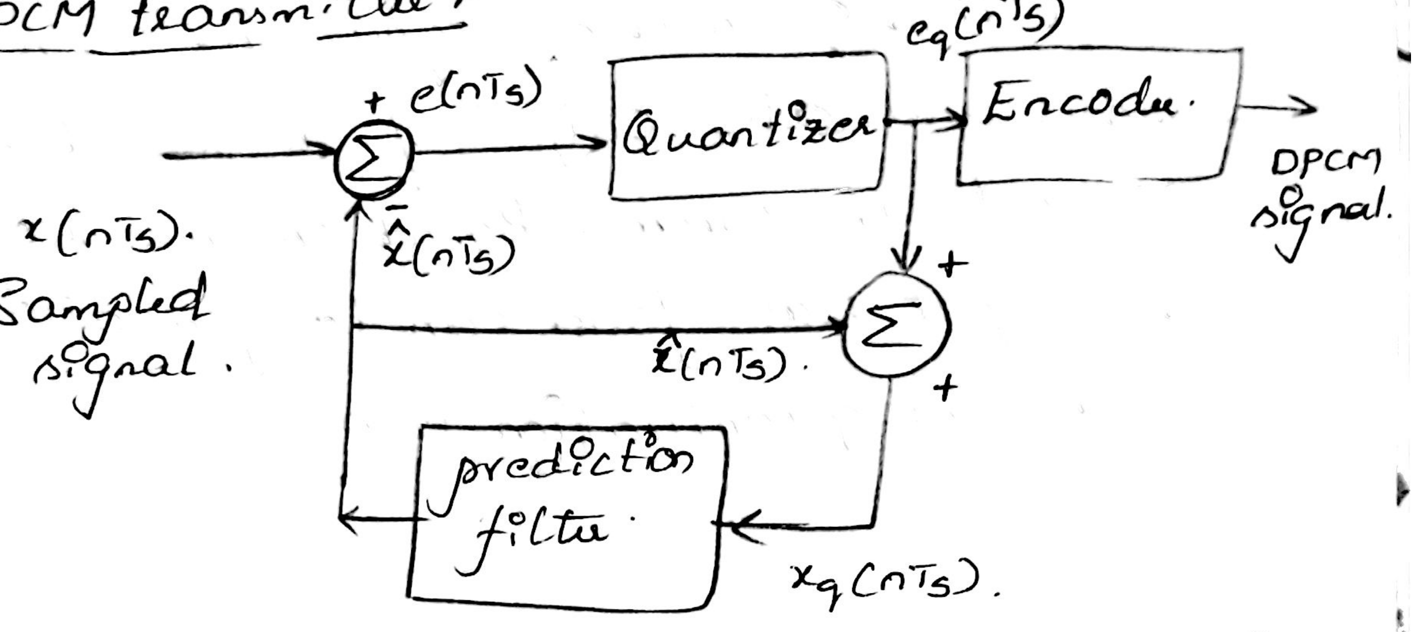


In PCM, Let an analog signal is converted in to discrete signal by flat top sampling with in time intervals T_s, $2T_s$, $3T_s$ ...... After the sampling quantization is performed and the amplitudes of samplings are rounded off and then encoding is done. If we observe the above signal, the samplings at T_s and $2T_s$ carry same information and those samplings are said to be redundant and also the samples at time intervals $4T_s$, $5T_s$ and $6T_s$ carries the same information 100. These samples are also said to be redundant. And also, the samples present at $6T_s$ and $7T_s$ carry information with a difference of one bit only and the remaining two bits are same and those two bits are said to be ~~de~~ redendent.

Differential pulse code modulation

DPCM reduces the redundancy present in the PCM signal. By reducing this redundancy, DPCM decreases the data rate and also decreases the transmission band width.

i) DPCM transmitter :-



DPCM works on the principle of prediction i.e; it DPCM works based on the previous sample value.

$x(nT_s)$ = sample signal
$\hat{x}(nT_s)$ predicted signal

From figure,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s). \longrightarrow (1)$$

$$e_q(nT_s) = e(nT_s) + q(nT_s). \longrightarrow (2)$$

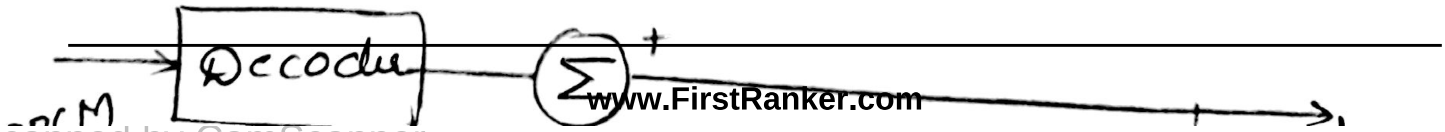
$$x_q(nT_s) = e_q(nT_s) + \hat{x}(nT_s). \longrightarrow (3)$$

from (2), $x_q(nT_s) = e(nT_s) + q(nT_s) + \hat{x}(nT_s)$.

from (1), $x_q(nT_s) = x(nT_s) - \cancel{\hat{x}(nT_s)} + q(nT_s) + \cancel{\hat{x}(nT_s)}$.

$$\boxed{x_q(nT_s) = x(nT_s) + q(nT_s).}$$

ii) DPCM Receiver :-



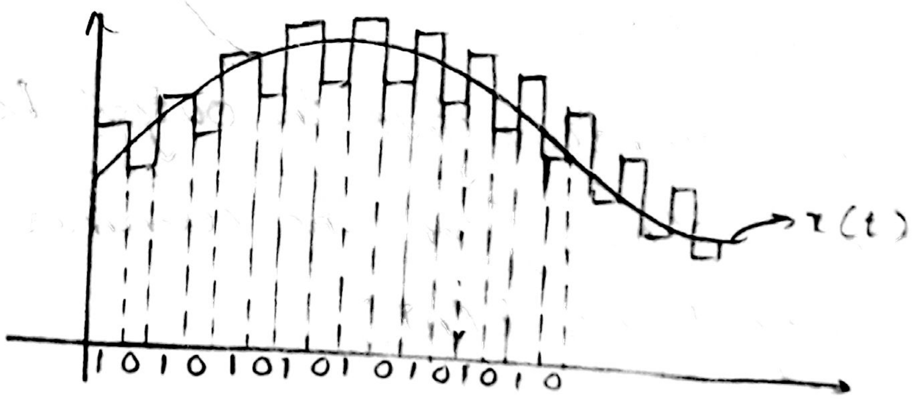
Decoder converts binary digits of the DPCM signal into the form of quantized signal and this quantized signal is given to the summer and also output of the prediction filter given to the summer. Based on those two values output wave form can be generated with permanent quantization error.

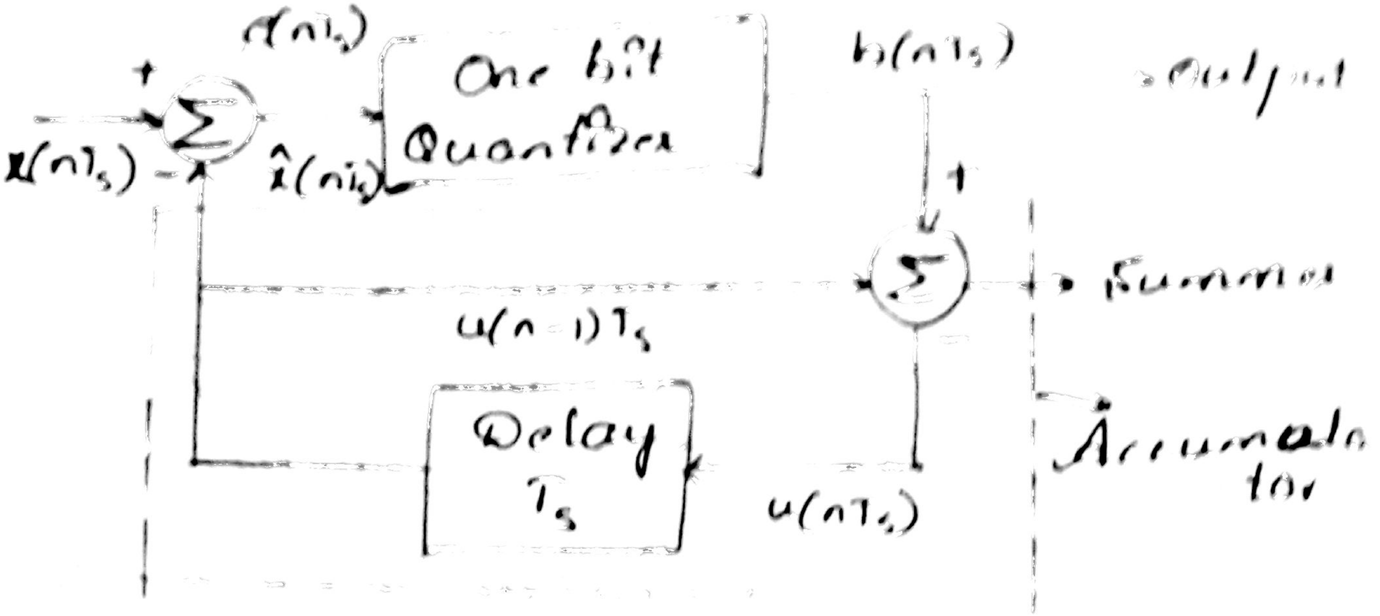
DELTA MODULATION

Delta Modulation:

In pulse code modulation, number of bits per one sample can be transmitted which increases the transmission bandwidth, hence to decrease the transmission bandwidth in delta modulation, only one bit per sample can be transmitted.

In delta modulation, an analog signal can be approximated with a staircase waveform and comparison between these two analog and staircase waveform can be done. The comparison result & can be expressed in form of $\pm\Delta$. Based on this $\pm\Delta$, bit 1 (or) 0 can be transmitted.





Initially, summer compares the present sample value with the previous sample value and produces an error signal.

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

where $x(nT_s)$ = Present sample value.

$\hat{x}(nT_s)$ = Previous sample value.

$e(nT_s)$ = Error signal.

One bit Quantizer:-

One bit quantizer gives the output based on error signal $e(nT_s)$ from the summer and produces output as $b(nT_s)$.

$$b(nT_s) = \Delta \; \text{sign}[e(nT_s)]$$

$$= \begin{cases} +\Delta & \text{for } x(nT_s) \geq \hat{x}(nT_s) \\ -\Delta & \text{for } x(nT_s) \leq \hat{x}(nT_s) \end{cases}$$

If $b(nT_s) = +\Delta$ then bit '1' is transmitted

If $b(nT_s) = -\Delta$ then bit '0' is transmitted.

Accumulator:- Let us assume that $u(nT_s)$ as the present sample value. i.e;

$$u(nT_s) = x(nT_s) = \text{present sample value}$$

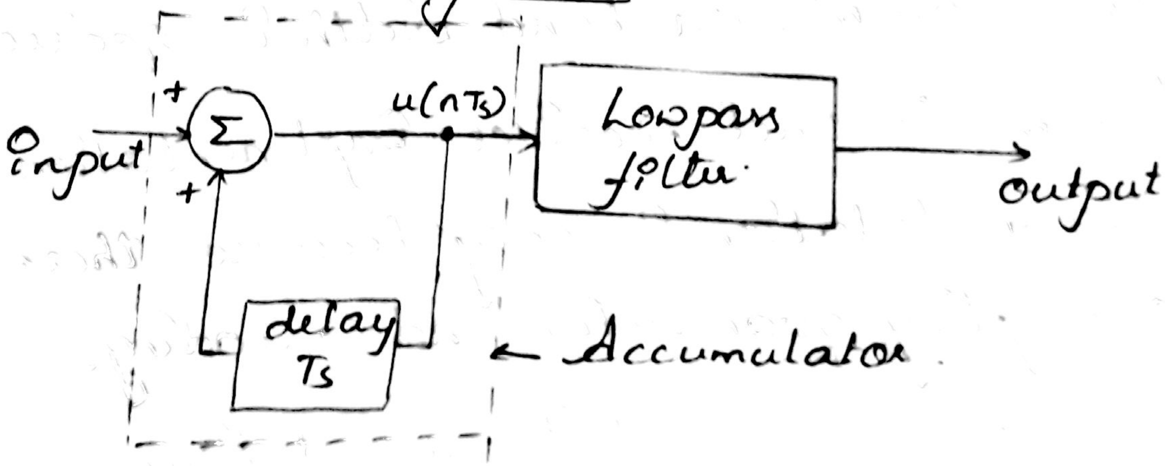
$$u[(n-1)T_s] = \hat{x}(nT_s) = \text{previous sample value.}$$

The summer in the accumulator adds, output from the quantizer and previous sample value, which gives present sample value.

$$\therefore u(nT_s) = u(n-1)T_s \pm b(nT_s)$$

$$\therefore u(nT_s) = u(n-1)T_s \pm \Delta$$

Receiver Block diagram:-



At the receiver side, the received information is in the form of bit.

Accumulator:- Accumulator used to convert incoming bits in the form of staircase signal. Accumulator generates the signal

(nT_s) and a delay is provided to the $u(nT_s)$ which becomes $u(n-1)T_s$. Again the summer in the accumulator compares the present sample with the previous sample and based on this comparison step is increased (or) decreased by the amount of Δ.

Low pass filter:- The stair case signal can be smoothened by using low pass filter to give the original signal.

Advantages of delta modulation:-

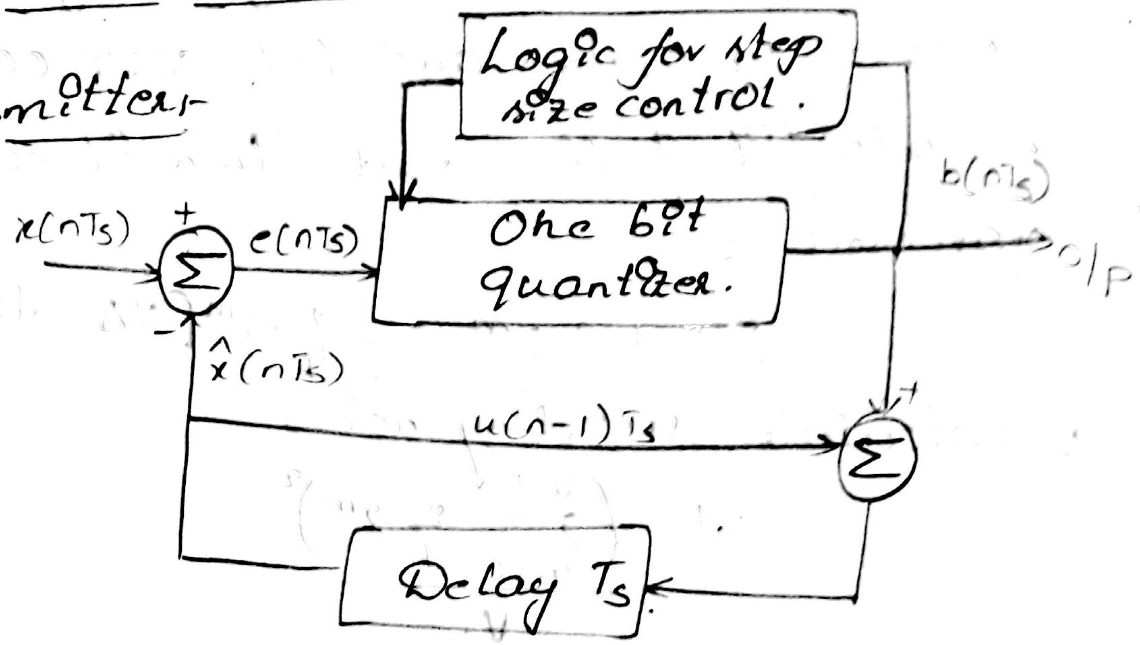
1) Only one bit for sample is transmitted. Hence transmission band width is reduced.
2) Transmitter and receiver implmentation of delta modulation is easy because there is no sampling quantization and encoding

Disadvantages of delta modulation:-

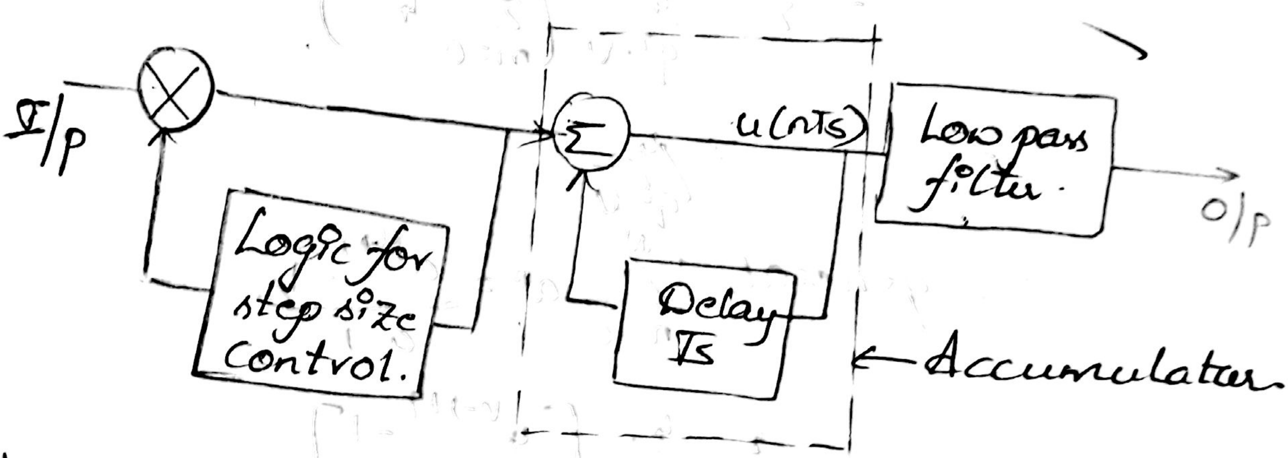
1) Slope over load distortion.
2) Granular noise.

Adaptive Delta Modulation:

Transmitter:-



Receiver:-



Noise in PCM:-

The noise in PCM presents at the decoder. While decoding is performed, some errors are introduced in the decoded information which is called as decoding noise.

Let, an information word containing 'v' no.of bits. Then probability of error in word is represented as,

P(error in word) = $P_e \cdot v$.

Let the bits of the word of the form $b_{v-1}, b_{v-2} \ldots b_2, b_1, b_0$ and let error occurs in the 'mth' bit of the word. Then it can be expressed as,

$$\varepsilon = \pm \Delta \cdot 2^m \Rightarrow \pm \frac{2}{q} \cdot 2^m \left[\because \Delta = \text{step size} = 2/q \right]$$

To calculate decoding noise,

$$\overline{E}^2 = \frac{\left(\sum_{m=0}^{v-1} \frac{2}{q} \cdot 2^m \right)^2}{v}$$

$$\therefore \overline{\varepsilon}^2 = \frac{4}{q^2 v} \left(\sum_{m=0}^{v-1} 4^m \right).$$

$$= \frac{4}{q^2 v}$$

In general, $\sum_{n=0}^{N} a^n = \frac{a^{N+1} - 1}{a-1}$.

$$\therefore \overline{E}^2 = \frac{4}{q^2 v} \left[\frac{4^{v-1+1} - 1}{4-1} \right]$$

$$\overline{E}^2 = \frac{4}{3q^2 v} \left[(2^2)^v - 1 \right]$$

$$= \frac{4}{3q^2 v} \left[(2^v)^2 - 1 \right]$$

In PCM, $2^v = q$ (no. of quantization levels)

$$\overline{E}^2 = \frac{4}{3q^2 v} \left[q^2 - 1 \right]$$

Let $q^2 >> 1$.

$$\overline{E}^2 = \frac{4}{3q^2 v} q^2 \therefore \boxed{\overline{E}^2 = \frac{4}{\ }}$$

Decoding noise = P(error in word) × $\overline{\epsilon}^2$

$$= P_e \nu \times \frac{4}{3\nu}$$

$$= \frac{4P_e}{3}$$

Total noise in PCM = decoding noise + quantization noise.

$$= \frac{4P_e}{3} + \frac{\Delta^2}{12}$$

but, $\Delta = \frac{2}{q}$ = step size.

$$= \frac{4P_e}{3} + \frac{4/q^2}{12}$$

$$= \frac{4P_e}{3} + \frac{1}{3q^2}$$

Total noise (or) noise power $= \frac{4P_e q^2 + 1}{3q^2}$

Signal to Noise Ratio $S/N = \frac{\text{signal power}}{\text{Noise power}}$

Let signal power = S_x.

$$S/N = \frac{S_x}{\frac{4P_e q^2+1}{3q^2}} = \frac{S_x\, 3q^2}{1+4P_e q^2}$$

If $4P_e q^2 \ll 1$, $S/N = S_x \cdot 3q^2$.

If $4P_e q^2 \gg 1$, $S/N = \frac{S_x \cdot 3q^2}{4P_e \cdot q^2}$

$$\therefore \boxed{S/N = \frac{3 \cdot S_x}{4P_e}}$$

Noise in Delta Modulation

Let $x(t) = A_m \sin(2\pi f_m t)$ is a continuous time signal in delta modulation.

Slope of delta modulator $= \frac{\text{step size}}{\text{sampling period}}$

$= \Delta / T_s$

$=$ Slope of stair case signal.

Slope over load condition occurs when,

slope of $x(t) >$ slope of stair case.

$$\therefore \text{Max} \left| \frac{d}{dt} (x(t)) \right| > \frac{\Delta}{T_s}$$

$$\text{Max} \left| \frac{d}{dt} (A_m \sin 2\pi f_m t) \right| > \Delta / T_s$$

$$\text{Max} \left| A_m \cos 2\pi f_m t \cdot 2\pi f_m \right| > \frac{\Delta}{T_s}$$

Since, for maximum value, $\cos 2\pi f_m = 1$

$$\therefore A_m 2\pi f_m > \frac{\Delta}{T_s}$$

$$\boxed{A_m > \frac{\Delta}{2\pi f_m T_s}}$$

The slope over load will not occur when

$$A_m \leq \frac{\Delta}{2\pi f_m T_s}$$

$$\therefore A_m = \frac{\Delta}{2\pi f_m T_s}$$

Signal to Noise Ratio S/N = $\frac{\text{signal power (Normalized)}}{\text{Noise power (Normalized)}}$.

Signal power (Normalized):-

$$\text{signal power} = \frac{V^2}{R}$$

V = peak voltage of $x(t) = A_m$.

$$\therefore \text{signal power} = \frac{A_m^2}{R}.$$

Here we will take RMS value of A_m

$$A_m(RMS) = \frac{A_m}{\sqrt{2}}.$$

$$\therefore \text{signal power} = \frac{(A_m/\sqrt{2})^2}{R} = \frac{A_m^2}{2R}.$$

Normalized signal power $= \frac{A_m^2}{2}$ $[\because R = 1]$

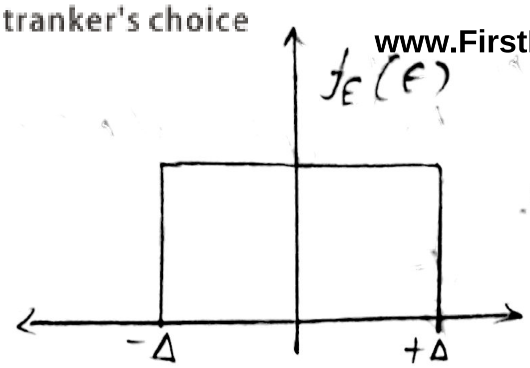
But, $A_m = \Delta / 2\pi f_m T_s$

$$\therefore \text{Normalized signal power} = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2}$$

Noise power (Normalized):-

The quantization error in delta modulation varies with in the range $\pm\Delta$.

Let the quantization error is a uniformly distributed random variable and the probability density function of this random variable within the interval $\pm\Delta$ will be as follows.



$$f_E(\epsilon) = \frac{1}{\Delta - (-\Delta)}$$

$$= \frac{1}{2\Delta}$$

$$f_E(\varepsilon) = \begin{cases} 0 & \text{for } \epsilon < -\Delta \\ 1/2\Delta & \text{for } -\Delta < \epsilon < \Delta \\ 0 & \text{for } \epsilon \geq \Delta \end{cases}$$

Let quantization error represented by ϵ, then

$$\text{Noise power} = \frac{V^2_{Noise}}{R}$$

$$V^2_{Noise} = E(\varepsilon^2) = \bar{\varepsilon}^2$$

In general, $E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f_x(x) \cdot dx$.

$$E(\varepsilon^2) = \int_{-\Delta}^{+\Delta} \varepsilon^2 \cdot f_\varepsilon(\varepsilon) \cdot d\varepsilon$$

$$= \cancel{\varepsilon^2} \int_{-\Delta}^{\Delta} \varepsilon^2 \frac{1}{2\Delta} \cdot d\varepsilon$$

$$= \cancel{\varepsilon^2} \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} \varepsilon^2 \, d\varepsilon$$

$$= \frac{1}{2\Delta} \left[\frac{\varepsilon^3}{3}\right]_{-\Delta}^{\Delta}$$

$$= \frac{1}{2\Delta} \left[\frac{\Delta^3}{3} + \frac{\Delta^3}{3}\right]$$

$$= \frac{1}{2\Delta} \left[\frac{2\Delta^3}{3}\right]$$

$$\bar{\varepsilon}^2 = \frac{\Delta^2}{3}$$

$\therefore$ Noise power $= \frac{(P_{Noise})}{R}$

$$= \frac{\Delta^2}{3R}$$

Normalized noise power $= \frac{\Delta^2}{3}$ $[\because R = 1]$

This is the output noise power. But in delta modulator, at the receiver side, a low pass filter is used. Let the band width of low pass filter is "W" and also assume that noise is a distributed random variable with in the interval of T_s and $-T_s$ with sampling frequencies as f_s and $-f_s$.

By the consideration of low pass filter,

Total noise power $= \frac{W}{f_s} \times$ output noise power.

$$= \frac{W}{f_s} \times \frac{\Delta^2}{3}$$

$$= \frac{W \cdot \Delta^2}{f_s \cdot 3}$$

$$\therefore S/N = \frac{\text{signal power (Normalized)}}{\text{Noise power (Normalized)}}$$

$$= \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2} \times \frac{3 f_s}{W \cdot \Delta^2}$$

$$= \frac{3 f_s}{8\pi^2 f_m^2 T_s^2 W} \quad \left[\because \frac{1}{f_s} = T_s\right]$$

$$\boxed{S/N = \frac{3}{8\pi^2 f_m^2 T_s^3 W}}$$

Basically transmission can be divided into two types

i) Base band transmission.

ii) Pass band transmission.

i) Base band transmission

In this transmission no carrier presents and the message signal is directly transmitted from source to destination and this is used for short distances.

ii) Pass band transmission

In this transmission, message signal can be modulated with a carrier signal. This is used for long distances.

Types of Pass Band Transmission:-

i) Amplitude shift keying (ASK):-

In this the amplitude of the carrier is varied according to the instantaneous value of the message signal.

ii) Frequency shift keying (FSK):-

In this the frequency of the carrier is varied according to the instantaneous values of the message signal.

iii) Phase shift keying (PSK):-

In this the phase of the carrier is varied according to the instantaneous values of the message signal.

Types of detection in pass band detection:-

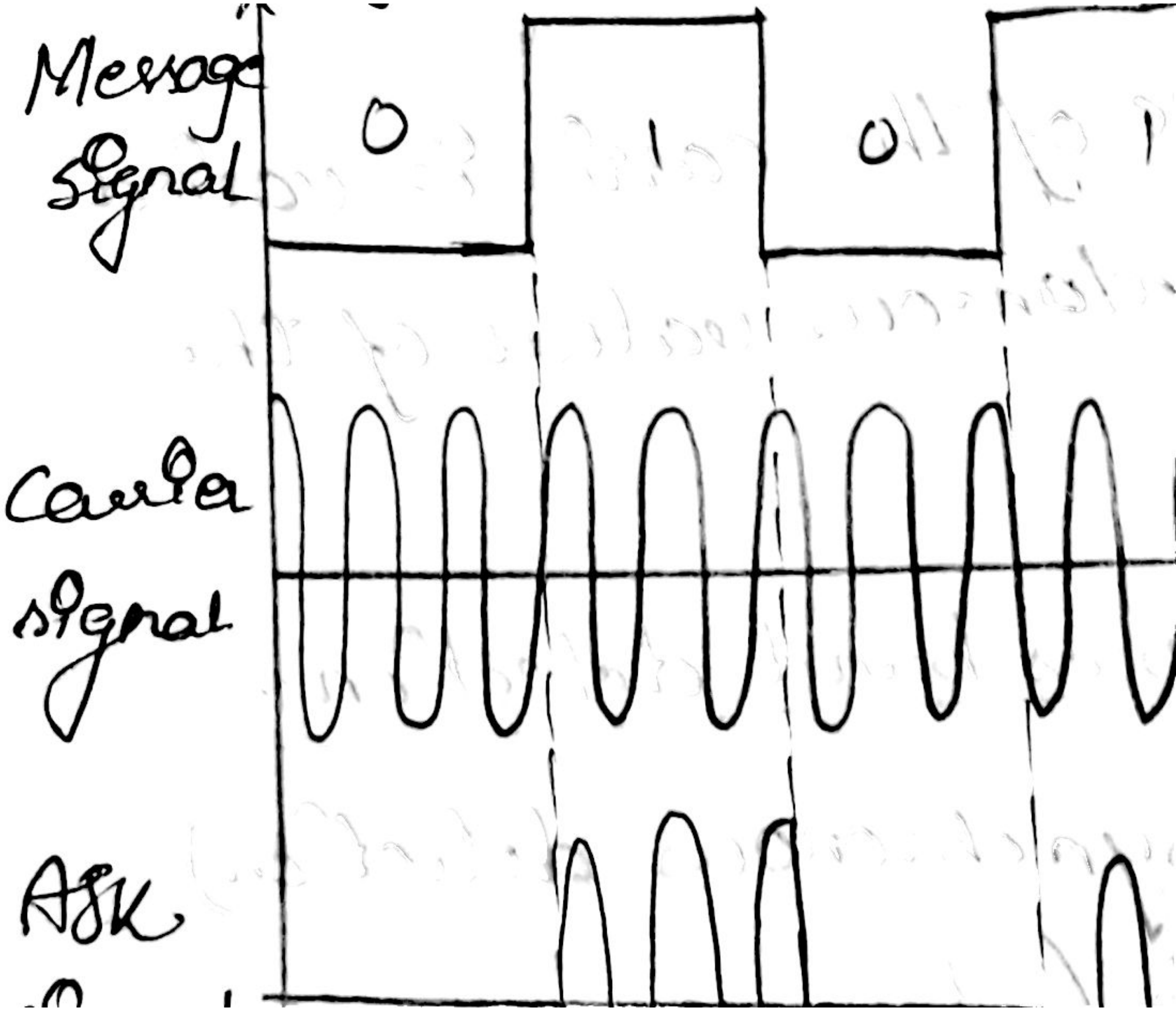
i) Coherent detection (synchronous detection)

ii) Non-Coherent detection (Asynchronous (detection).

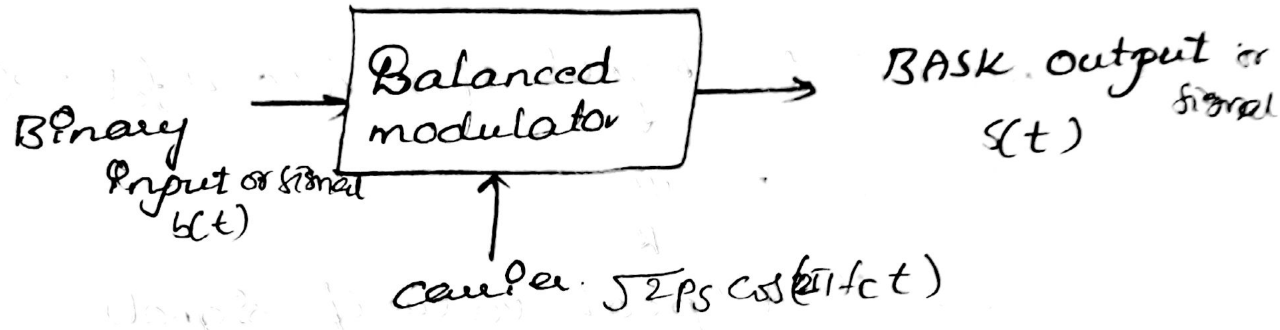
i) Coherent detection:- In this detection, synchronization presents between the carrier at the receiver and the carrier at the transmitter

ii) Non-coherent detection:- In this no synchronization presents between the carrier at the transmitter and the carrier at the receiver.

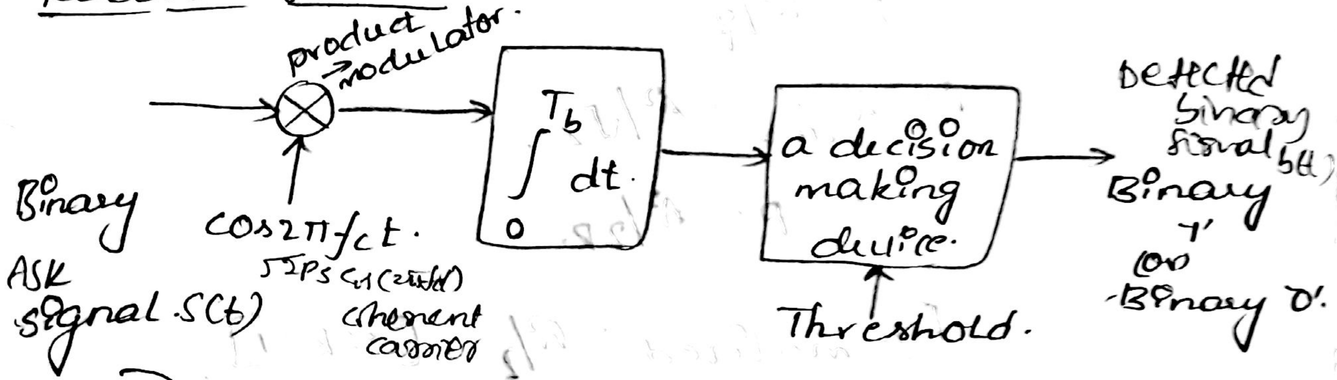
Block diagram of pass band transmission:-


Message signal
0
1
0
1
Carrier signal
ASK

Transmitter (BASK):-



Receiver (BASK):-



Product modulator modulates the incoming BASK signal with the locally generated carrier. Then this product is given to the integrator and integrator performs integration and it also acts as low pass filter. Output of the integrator is given to the decision making device. Decision making device compares output of the integrator to a threshold value.

If threshold value > output of integrator → Binary '1'.

If threshold value < output of integrator → Binary '0'.

Binary PSK (BPSK):-

Let modulated signal $s(t) = A\cos 2\pi f_c t \rightarrow (1)$

power $P = \frac{V^2}{R}$

here, $V = A$ (peak value of signal).

$P = A^2/R$

$A = A_{rms} = A^2/\sqrt{2}$.

$\therefore P = A^2/2R$

$P_{Normalized} = A^2/2 \quad [\because R=1]$.

$P = A^2/2 \Rightarrow A = \sqrt{2P}$.

By substituting in equ (1)

$s(t) = \sqrt{2P}\cos 2\pi f_c t$.

$\therefore s(t) = \sqrt{2P}\cos 2\pi f_c t$ when bit '1' is transmitted

$= \sqrt{2P}\cos(2\pi f_c t + \pi)$ when bit '0' is transmitted

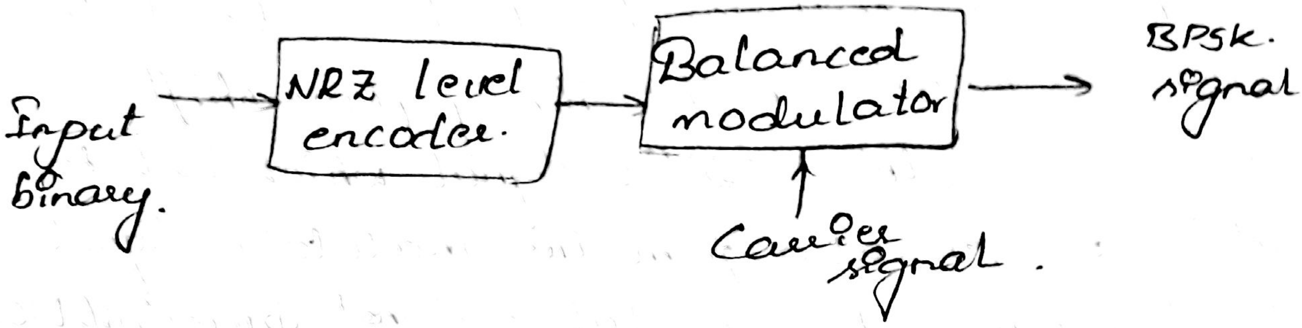
$= -\sqrt{2P}\cos(2\pi f_c t)$ when bit '0' is transmitted

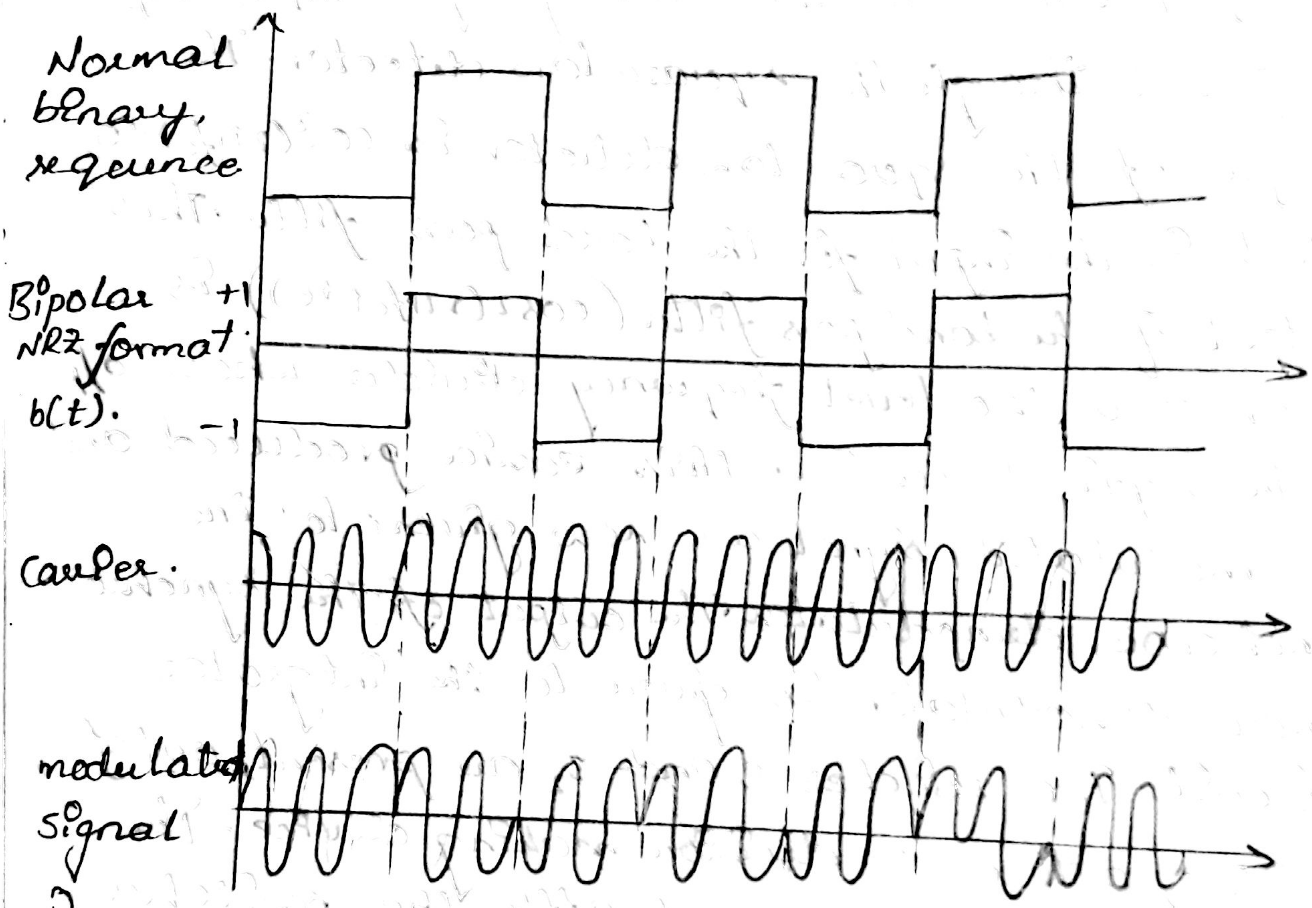
$\therefore s(t) = b(t)\sqrt{2P}\cos 2\pi f_c t$.

where, $b(t) = +1$ when bit '1' is transmitted

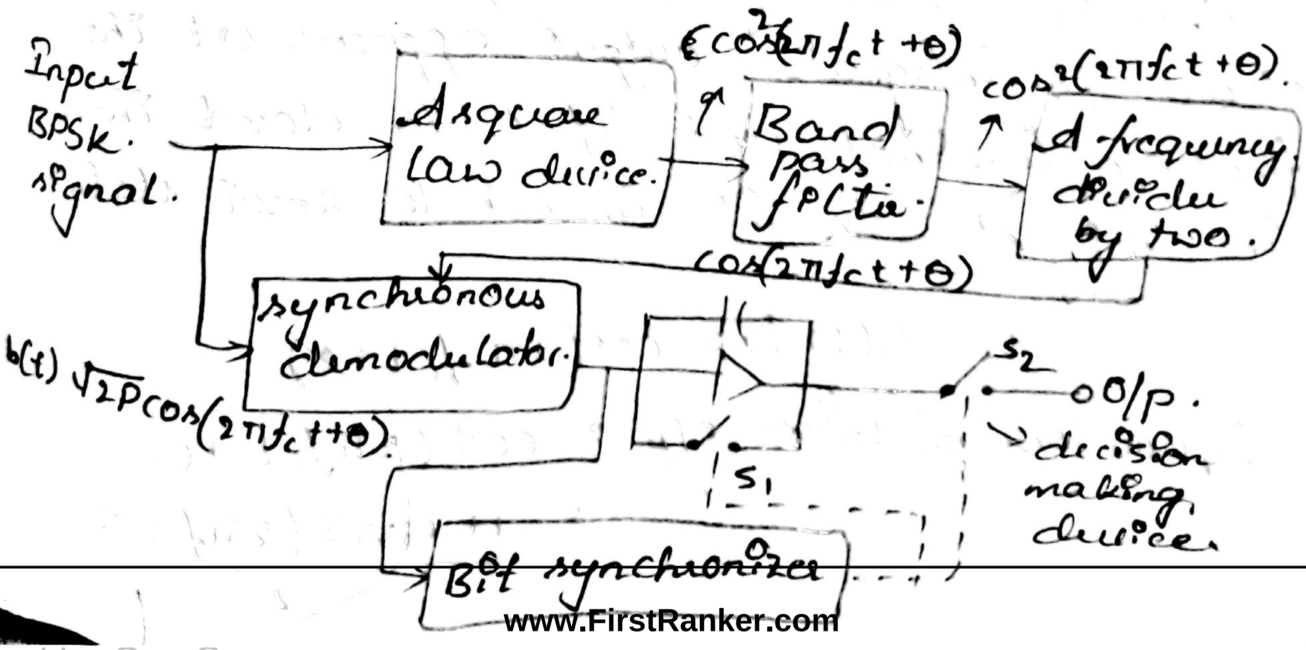
$b(t) = -1$ when bit '0' is transmitted.

Transmitter (BPSK):-





Receiver (BPSK):-



In the band pass detection, a carrier signal from the local oscillator has to be added to the modulated signal. In the binary phase shift keying receiver, this carrier signal which is to be added is generated from the modulated signal itself. For this the modulating signal $b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)$ is passed through the square law detector. The output of the square law detector is $\cos^2(2\pi f_c t+\theta)$ which is the input for the band pass filter. This output of the band pass filter $(\cos 2(2\pi f_c t+\theta))$ is given to a two level frequency divider whose o/p is the required carrier. This carrier produced and the modulated signal are now given to the synchronous demodulator. The output of the synchronous demodulator is given to the integrator circuit. Two switches S_1 and S_2 are present, out of which S_2 acts as a decision making device. The bit synchronizer ons and off's the switches S_1 and S_2 alternately. If the switch two is closed the integrator output appears at the final o/p terminal and if S_1 is closed then the integrator circuit is set to reset state.

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_c t+\theta)$$

Square law ~~modulator~~ device o/p $= \cos^2(2\pi f_c t+\theta)$

$$= \underbrace{\frac{1}{2}}_{\text{dc signal.}} + \frac{\cos 2(2\pi f_c t + \theta)}{2}$$

Band pass filter o/p $= \cos(2\pi f_c t + \theta)$

frequency divider by two o/p $= \cos(2\pi f_c t + \theta)$.

Synchronous demodulator i/p $= s(t) \cdot \cos(2\pi f_c t + \theta)$.

$$= b(t)\sqrt{2P}\cos^2(2\pi f_c t + \theta).$$

$$= b(t)\sqrt{2P}\left[\frac{1 + \cos 2(2\pi f_c t + \theta)}{2}\right]$$

$$= b(t)\sqrt{\frac{P}{2}}\left[1 + \cos 2(2\pi f_c t + \theta)\right]$$

<u>To show the output depends upon $v(t)$</u>:-

Let kth bit in the information is applied to the integrator.

$$S_o(kT_b) = b(t)\cdot\sqrt{\frac{P}{2}}\int_{(k-1)T_b}^{kT_b}\left[1 + \cos 2(2\pi f_c t + \theta)\right]dt$$

$$= b(t)\cdot\sqrt{P/2}\left[\int_{(k-1)T_b}^{kT_b} 1\cdot dt + \int_{(k-1)T_b}^{kT_b}\cos 2(2\pi f_c t + \theta)\,dt\right].$$

$$\therefore S_o(kT_b) = b(t)\cdot\sqrt{\frac{P}{2}}\left[\int_{(k-1)T_b}^{kT_b} 1\cdot dt\right]$$

$$= b(t)\sqrt{\frac{P}{2}}\left[t\right]_{(k-1)T_b}^{kT_b}.$$

$$= b(t)\sqrt{\frac{P}{2}}\left[kT_b - (k-1)T_b\right]$$

$$S_o(kT_b) = b(t)\cdot\sqrt{\frac{P}{2}}\,T_b.$$

$s(t) = \sqrt{2P_s}\cos 2\pi f_c t$ if '1' is transmitted

$\quad\quad 0$ if '0' is transmitted.

$$s(t) = \sqrt{2P_s}\cos 2\pi f_c t .$$

Multiply and divide with $\sqrt{T_b}$.

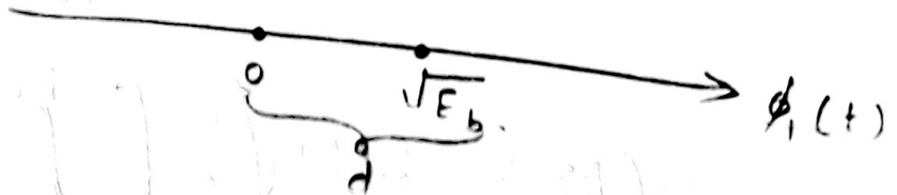
$$s(t) = \sqrt{P_s T_b}\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t .$$

put $\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t = \phi_1$

$$\therefore s(t) = \sqrt{P_s T_b}\cdot\phi_1$$

Let $P_s T_b = E_b$.

$$\therefore s(t) = \sqrt{E_b}\cdot\phi_1 .$$



$$\therefore \boxed{d = \sqrt{E_b}.}$$

Geometrical Representation of BPSK.

for BPSK,

$$s(t) = b(t)\cdot\sqrt{2P}\cos(2\pi f_c t).$$

$b(t) = +1$ when message signal ...

$\quad = -1$ when ...

$$s(t) = b(t) \cdot \sqrt{PT_b}\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$$

Put $\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t = \phi_1$.

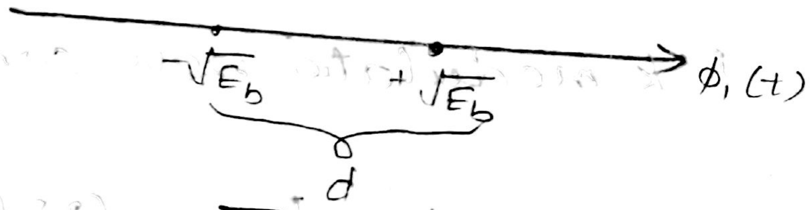
$$\therefore s(t) = b(t) \cdot \sqrt{PT_b} \cdot \phi_1$$

Let $P \cdot T_b = E_b$.

$$s(t) = b(t)\sqrt{E_b} \cdot \phi_1$$

if $b(t) = +1$, $s(t) = \sqrt{E_b} \cdot \phi_1$

if $b(t) = -1$, $s(t) = -\sqrt{E_b} \cdot \phi_1$.



$$d = \sqrt{E_b} - (-\sqrt{E_b})$$

$$\boxed{d = 2\sqrt{E_b}}$$

Band width of BASK:-

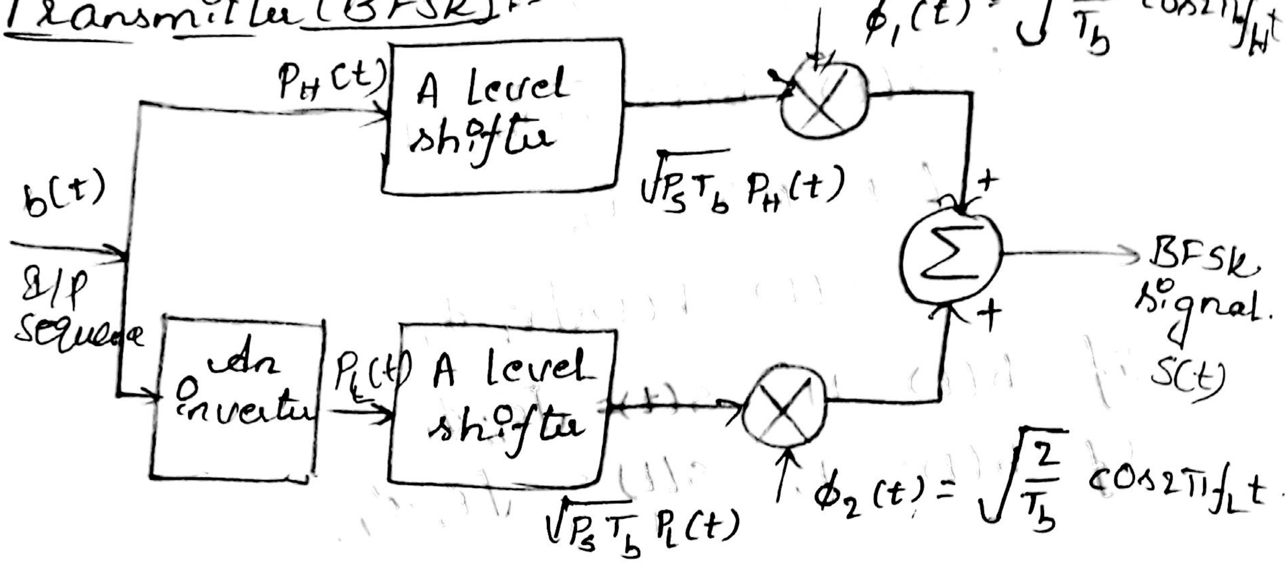
Band width of BASK $= 3/2\, f_b$.

Band width of BPSK:-

Band width of BPSK $= 2 f_b$

Binary frequency shift keying (BFSK):-

Transmitter (BFSK):-



The FSK modulated wave can be written as,

$$b(t) = 1 \Rightarrow S_H(t) = \sqrt{2P_s} \cos(2\pi f_c t + \Omega)t$$

$$b(t) = 0 \Rightarrow S_L(t) = \sqrt{2P_s} \cos(2\pi f_c t - \Omega)t$$

By combining the above two equations,

$$s(t) = \sqrt{2P_s} \cos(2\pi f_c t + d(t)\Omega)t$$

$d(t) = \pm 1V$ based on the message signal i/p

In fsk, frequency of the carrier is varied according to the instantaneous values of the message signal. Here we will use two orthogonal carriers $\phi_1(t)$ and $\phi_2(t)$.

Message signal is applied to a level shifter and this level shifter generates unipolar signals based on the input.

If the input is '0', o/p of the level shifter

is zero. If the input is '1', Level shifter generates $P_H(t)$ (or) $P_L(t)$

<u>b(t)=1:-</u>

If $b(t)=1$, level shifter generates, $\sqrt{P_S T_b}\cdot P_H(t)$. beacuse of the input of the level shifter is the message signal in the upper part. In the lower part, message signal is inverted ie; $b(t)=0$, no signal can be generated by the lower level shifter. The o/p from the upper level shifter multiplied with carrier $\phi_1(t)$ by the product modulator. At the end two outputs from the two product modulators can be added.

$b(t)$	$d(t)$	$P_H(t)$	$P_L(t)$
1	+1V	1V	0V
0	-1V	0V	1V

$$\text{o/p BFSK signal} = \sqrt{P_S T_b}\; P_H(t)\cdot\sqrt{\frac{2}{T_b}}\cos 2\pi f_H t$$

But $P_H(t)=1$.

$$\therefore \text{o/p BFSK signal} = \sqrt{P_S T_b}\sqrt{\frac{2}{T_b}}\cos 2\pi f_H t$$

<u>b(t)=0:-</u>

If $b(t)=0$, the upper level shifter can not generate any signal. This $b(t)=0$ is given to the inverter in the lower part and hence it becomes $b(t)=1$. Therefore this

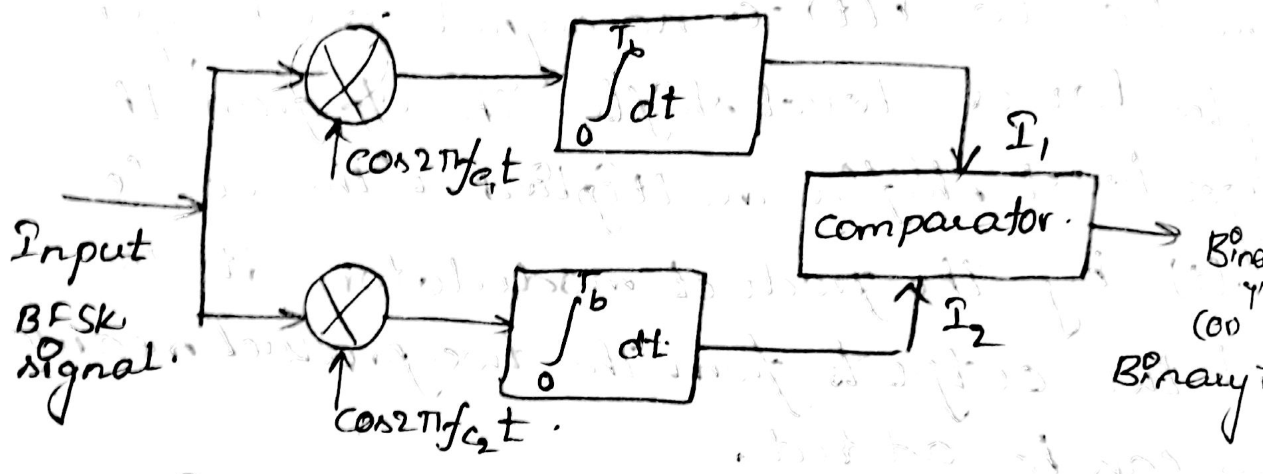
~~$b(t)=1$ is given to the level shifter in the~~

generated $\sqrt{P_s T_b}\, P(t) \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t$.

But $P(t) = 1$

$\therefore$ o/p of BFSK signal $= \sqrt{P_s T_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi$

Receiver (BFSK):



Received BFSK signal can be multiplied with two carriers $\cos 2\pi f_{c_1} t$ and $\cos 2\pi f_{c_2} t$. The two product can be applied to two integrators and by performing integration, the two output from the two integrators are I_1 and I_2. I_1 and I_2 are applied to the comparator. Comparator compares the two values of I_1 and I_2

If $I_1 > I_2$ then comparator gives binary as the output.

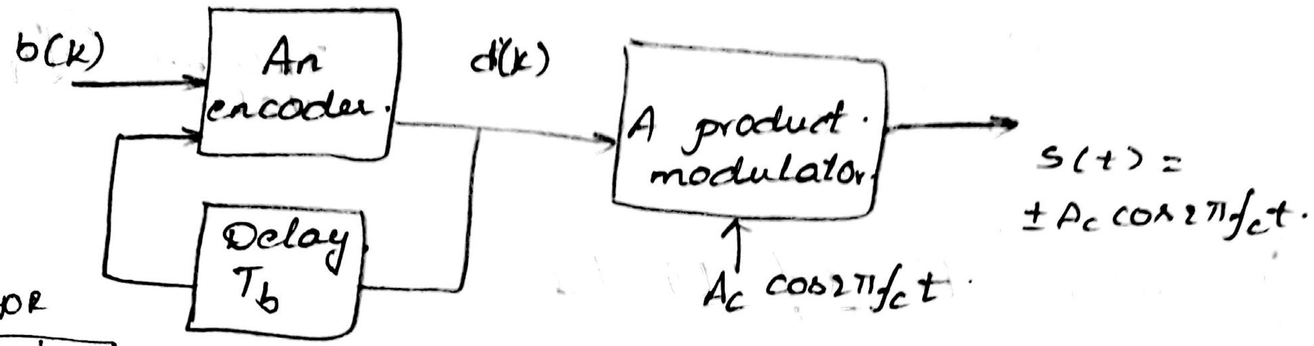
If $I_1 < I_2$ then comparator o/p becomes binary '0'. //

Bandwidth of BFSK:-

Band width of BFSK $= 4f_b$.

Differential phase shift keying:- (DPSK):-

Modulator:-

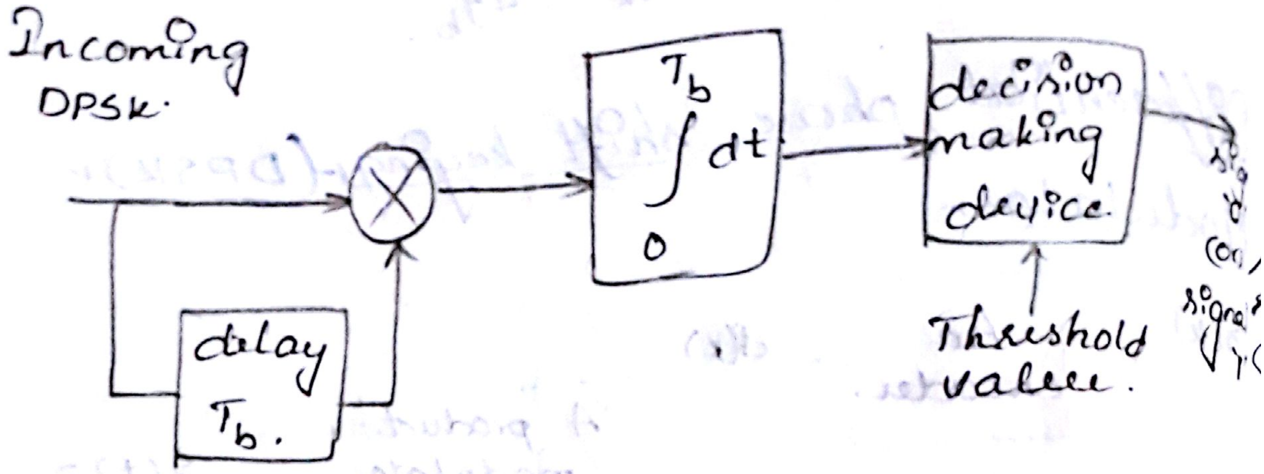


XNOR

0	0	1
0	1	0
1	0	0
1	1	1

Binary data b(t)		0	0	1	0	0	1	0	0	1	1
Encoded data	1	0	1	1	0	1	1	0	1	1	1
Phase of DPSK	0	π	0	0	π	0	0	π	0	0	0
shifted encoded data.		1	0	1	1	0	1	1	0	1	1
phase of DPSK		0	π	0	0	π	0	0	π	0	0
Comparator		−	−	+	−	−	+	−	−	+	+
Detector sequence.		0	0	1	0	0	1	0	0	1	1

Demodulation:-



Band width of DPSK:-

Band width of DPSK is $= f_b$.

Advantages of DPSK:-

1) Band width is reduced compared to PSK.
2) There is no carrier at the receiver. Hence circuit complexity decreases.

Disadvantages of DPSK:-

In demodulation present bit can be compared with the previous bit value. Hence error propagation is maximum in DPSK.

Quadrature Phase Shift Keying (QPSK)

In transmission of any signal, two parameters have to be considered basically

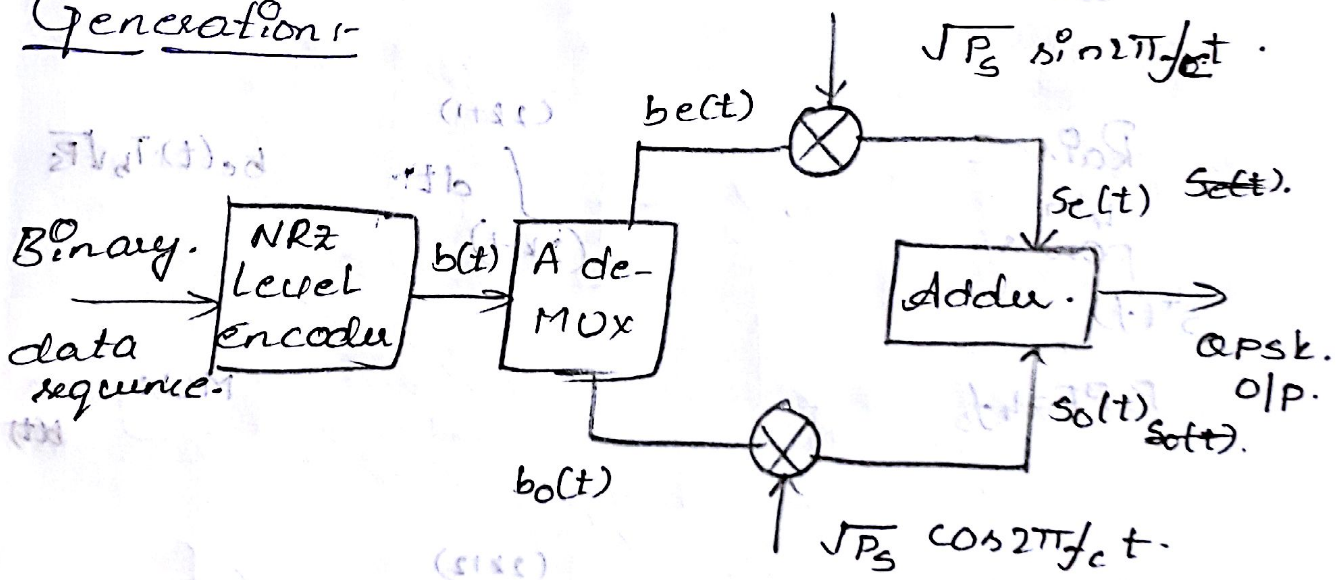
i) Signal Power.

ii) Transmission band...

To reduce the transmission band width two bits are combined in the message signal and will be transmitted. These two bits forms four combinations which will give four symbols and phase shift in QPSK is $\pi/4$.

Input bits		symbols.	phase shift.
0(-1)	0(-1)	S_1	$\pi/4$
0(-1)	1(+1)	S_2	$3\pi/4$
1(+1)	0(-1)	S_3	$5\pi/4$
1(+1)	1(+1)	S_4	$7\pi/4$.

Generation:-



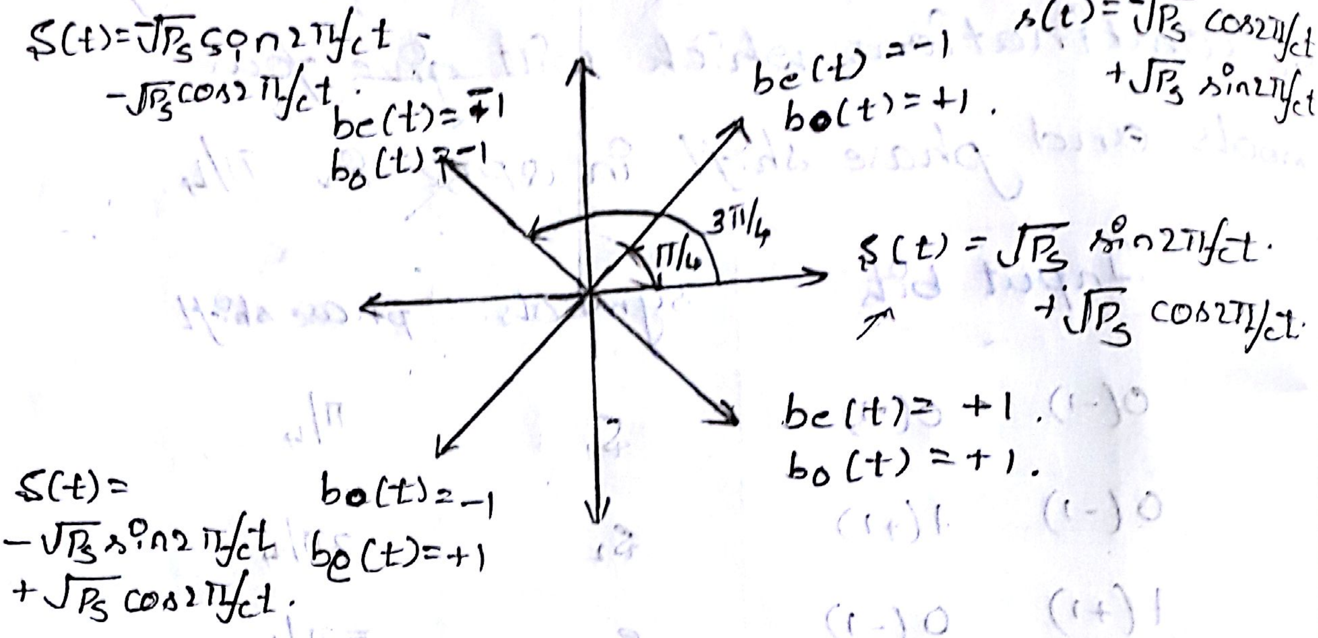
$S_e(t) = b_e(t) \cdot \sqrt{P_s} \sin 2\pi f_c t$.

$S_o(t) = b_o(t) \cdot \sqrt{P_s} \cos 2\pi f_c t$.

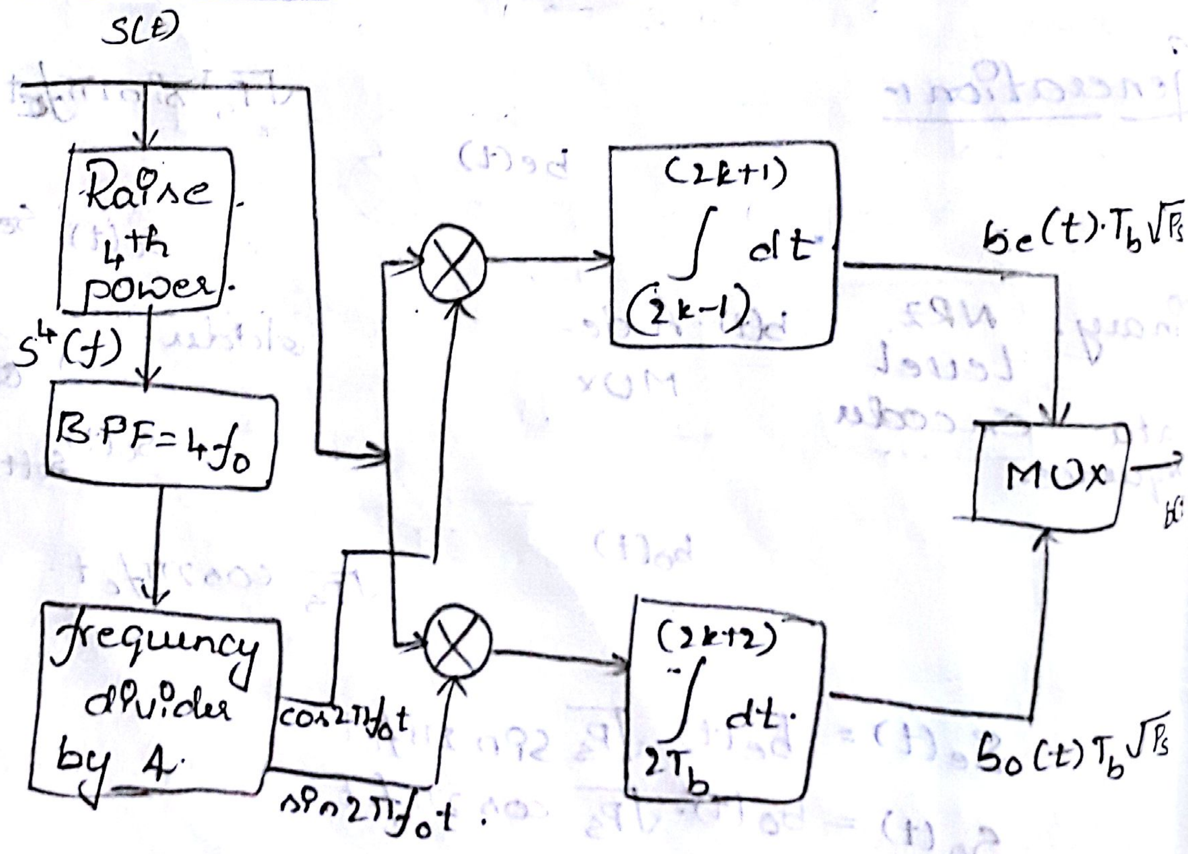
$S(t) = S_e(t) + S_o(t)$.

$= b_e(t) \cdot \sqrt{P_s} \sin 2\pi f_c t + b_o(t) \cdot \sqrt{P_s} \cos 2\pi f_c t$.

Representation of QPSK:-

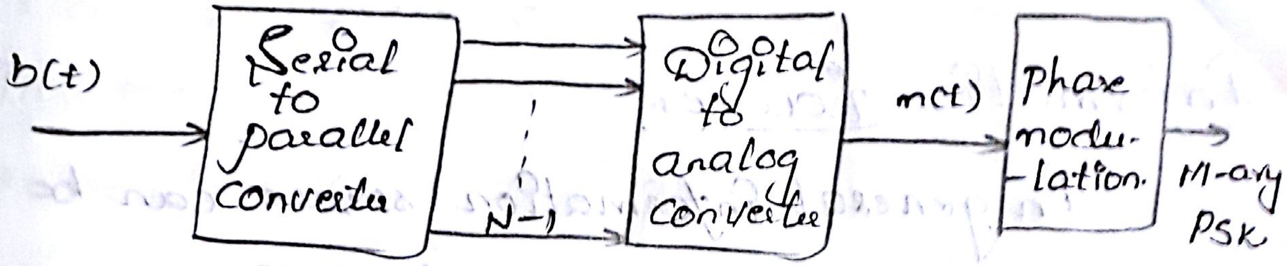


Demodulation:-

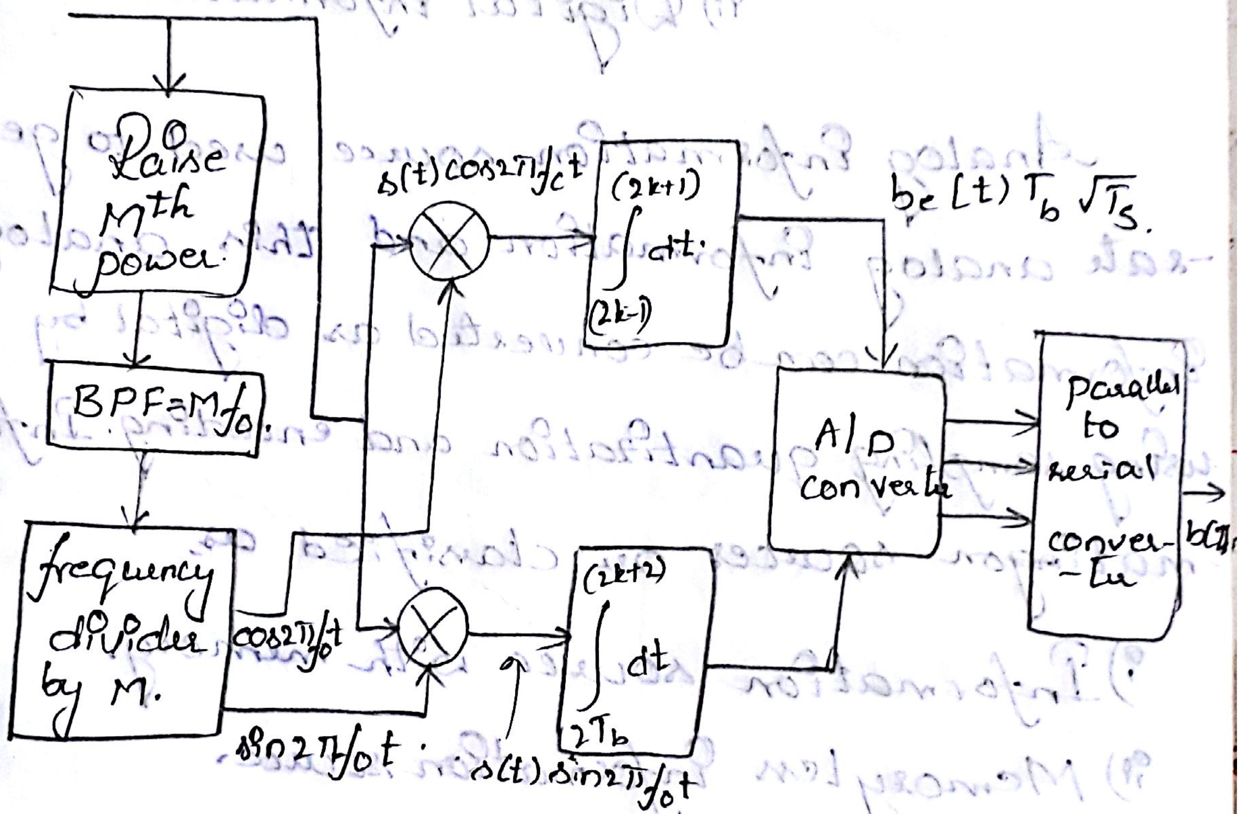


M-ary PSK modulation and demodulation

Modulator:-



Demodulator:-



UNIT - IV

Information Theory

Information theory deals with representation and the transfer of information.

There are two fundamentally different ways to transmit **messages**: via **discrete** signals and via continuous signals. ... For example, the letters of the English alphabet are commonly thought of as **discrete** signals.

Information sources

Definition:

The set of source symbols is called the **source alphabet**, and the elements of the set are called the **symbols or letters.**

The number of possible answers ‘ r ’ should be linked to "information." "Information" should be additive in some sense. We define the following measure of information:

$$\tilde{I}(U) \triangleq \log_b r,$$

Where ‘ r ’ is the number of all possible outcome so far an do m message U.

Using this definition we can confirm that it has the wanted property of additivity:

$$\tilde{I}(U_1, U_2, \ldots, U_n) = \log_b r^n = n \cdot \log_b r = n\tilde{I}(U).$$

The basis ‘b’ of the logarithm b is only a change of units without actually changing the amount of information it describes.

Classification of information sources

1. Discrete memory less.
2. Memory.

Discrete memory less source (DMS) can be characterized by "the list of the symbols, the probability assignment to these symbols, and the specification of the rate of generating these symbols by the source".

1. Information should be proportion to the uncertainty of an outcome.
2. Information contained in independent outcome should add.

Scope of Information Theory

1. Determine the irreducible limit below which a signal cannot be compressed.

2. Deduce the ultimate transmission rate for reliable communication over a noisy channel.

3. Define Channel Capacity - the intrinsic ability of a channel to convey information.

The basic setup in Information Theory has:

– a source,

– a channel and

– destination.

The output from source is conveyed through the channel and received at the destination. The source is a random variable S
which takes symbols from a finite alphabet i.e.,

$$S = \{s_0, s_1, s_2, \cdots, s_{k-1}\}$$

With probabilities

$P(S = s_k) = p_k$ where $k = 0, 1, 2, \cdots, k - 1$
and
$\sum_{k=0}^{k-1} p_k = 1$

The following assumptions are made about the source

1. Source generates symbols that are statistically independent.
2. Source is memory less i.e., the choice of present symbol does not depend on the previous choices.

Properties of Information

1. Information conveyed by a deterministic event is nothing
2. Information is always positive.
3. Information is never lost.
4. More information is conveyed by a less probable event than a more probable event

Entropy:

The Entropy (H(s)) of a source is defined as the average information generated by a discrete memory less source.

Information content of a symbol:

Let us consider a discrete memory less source (DMS) denoted by X and having the alphabet $\{U_1, U_2, U_3, \ldots\ldots U_m\}$. The information content of the symbol xi, denoted by $I(x_i)$ is defined as

$$I(U) = \log_b \frac{1}{P(u)} = -\log_b P(U)$$

Where P (U) is the probability of occurrence of symbol U

Units of $I(x_i)$:

For two important and one unimportant special cases of b it has been agreed to use the following names for these units:

b =2(log2): bit,

b = e (ln): nat (natural logarithm),

b =10(log10): Hartley.

The conversation of these units to other units is given as

$$\log_2 a = \frac{\ln a}{\ln 2} = \frac{\log a}{\log 2}$$

Uncertainty or Entropy (i.e Average information)

Definition:

In order to get the information content of the symbol, the flow information on the symbol can fluctuate widely because of randomness involved into the section of symbols.

The uncertainty or entropy of a discrete random variable (RV) ‘U’ is defined as

$$H(U) = E[I(u)] = \sum_{i=1}^{m} P(u) I(u)$$

$$H(U) \triangleq -\sum_{u \in \mathrm{supp}(P_U)} P_U(u) \log_b P_U(u),$$

Where PU (·) denotes the probability mass function (PMF) 2 of the RV U, and where the support of P U is defined as

$$\text{supp}(P_U) \triangleq \{u \in \mathcal{U}: P_U(u) \neq 0\}.$$

We will usually neglect to mention "support" when we sum over PU (u) · $\log_b$ PU (u), i.e., we implicitly assume that we exclude all u

With zero probability PU (u) =0.

Entropy for binary source

It may be noted that for a binary source U which genets independent symbols 0 and 1 with equal probability, the source entropy H (u) is

$$H(u) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2} = 1 \text{ b/symbol}$$

Bounds on H (U)

If U has r possible values, then 0 ≤ H(U) ≤ log r,

Where

H(U)=0 if, and only if, PU(u)=1 for some u,

H(U)=log r if, and only if, PU(u)= 1/r ∀ u.

Hence, H(U) ≥ 0.Equalitycanonlybeachievedif −PU(u)log2 PU(u)=0

Proof. Since $0 \le P_U(u) \le 1$, we have

$$-P_U(u)\log_2 P_U(u) \begin{cases} = 0 & \text{if } P_U(u) = 1, \\ > 0 & \text{if } 0 < P_U(u) < 1. \end{cases}$$

For all u ∈ supp (PU), i.e., PU (u) =1forall u ∈ supp (PU).

To derive the upper bound we use at rick that is quite common in.

Formation theory: We take the deference and try to show that it must be non positive.

$$
\begin{aligned}
H(U) - \log r &= - \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \log P_U(u) - \log r \\
&= - \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \log P_U(u) - \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \log r \\
&= - \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \log \left(P_U(u) \cdot r \right) \\
&= \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \log \Big(\underbrace{\frac{1}{r \cdot P_U(u)}}_{\triangleq \xi} \Big) \\
&\le \sum_{u \in \operatorname{supp}(P_U)} P_U(u) \left(\frac{1}{r \cdot P_U} - 1 \right) \cdot \log e \\
&= \Big(\sum_{u \in \operatorname{supp}(P_U)} \frac{1}{r} - \underbrace{\sum_{u \in \operatorname{supp}(P_U)} P_U(u)}_{=1} \Big) \cdot \log e \\
&= \left(\frac{1}{r} \sum_{u \in \operatorname{supp}(P_U)} 1 - 1 \right) \log e \\
&\le \left(\frac{1}{r} \sum_{u \in \mathcal{U}} 1 - 1 \right) \log e \\
&= \left(\frac{1}{r} \cdot r - 1 \right) \log e \\
&= (1 - 1) \log e = 0.
\end{aligned}
$$

Equality can only be achieved if

1. In the IT Inequality $\xi = 1$, i.e., if $\frac{1}{r \cdot P_U(u)} = 1 \Longrightarrow P_U(u) = \frac{1}{r}$, for all u;
2. |supp (PU)| = r.

Note that if Condition1 is satisfied, Condition 2 is also satisfied.

Conditional Entropy

Similar to probability of random vectors, there is nothing really new about conditional probabilities given that a particular event Y = y has occurred.

The conditional entropy or conditional uncertainty of the RV X given the event Y = y is defined as

$$H(X|Y=y) \triangleq -\sum_{x \in \mathrm{supp}(P_{X|Y}(\cdot|y))} P_{X|Y}(x|y) \log P_{X|Y}(x|y)$$
$$= \mathsf{E}\left[-\log P_{X|Y}(X|Y) \,\middle|\, Y=y\right].$$

Note that the definition is identical to before apart from that everything is conditioned on the event Y = y

$$0 \le H(X|Y=y) \le \log r;$$
$$H(X|Y=y) = 0 \quad \textit{if, and only if,} \quad P(x|y) = 1 \quad \textit{for some } x;$$
$$H(X|Y=y) = \log r \quad \textit{if, and only if,} \quad P(x|y) = \frac{1}{r} \quad \forall x.$$

Note that the conditional entropy given the event Y = y is a function of y. Since Y is also a RV, we can now average over all possible events Y = y according to the probabilities of each event. This will lead to the averaged.

Mutual Information

Although conditional entropy can tell us when two variables are completely independent, it is not an adequate measure of dependence. A small value for **H(Y| X)** may implies that **X** tells us a great deal about **Y** or that **H(Y)** is small to begin with. Thus, we measure dependence using *mutual information*:

I(X,Y) =H(Y)–H(Y|X)

Mutual information is a measure of the reduction of randomness of a variable given knowledge of another variable. Using properties of logarithms, we can derive several equivalent definitions

$$I(X,Y)=H(X)-H(X|Y)$$

$$I(X,Y) = H(X)+H(Y)-H(X,Y) = I(Y,X)$$

In addition to the definitions above, it is useful to realize that mutual information is a particular case of the Kullback-Leibler divergence. The KL divergence is defined as:

$$D(p||q) = \int p(x)\log\frac{p(x)}{q(x)}.$$

KL divergence measures the difference between two distributions. It is sometimes called the relative entropy. It is always non-negative and zero only when **p**=**q**; however, it is not a distance because it is not symmetric.

In terms of KL divergence, mutual information is:

$$D(P(X,Y)||P(X)P(Y)) = \int P(X,Y)\log\frac{P(X,Y)}{P(X)P(Y)}$$

In other words, mutual information is a measure of the difference between the joint probability and product of the individual probabilities. These two distributions are equivalent only when **X** and **Y** are independent, and diverge as **X** and **Y** become more dependent.

UNIT – V

Source coding

Coding theory is the study of the properties of codes and their respective fitness for specific applications. Codes are used for data compression, cryptography, error-correction, and networking. Codes are studied by various scientific disciplines—such as information theory, electrical engineering, mathematics, linguistics, and computer science—for the purpose of designing efficient and reliable data transmission methods. This typically involves the removal of redundancy and the correction or detection of errors in the transmitted data.

The aim of source coding is to take the source data and make it smaller.

All source models in information theory may be viewed as random process or random sequence models. Let us consider the example of a discrete memory less source (DMS), which is a simple random sequence model.

A DMS is a source whose output is a sequence of letters such that each letter is independently selected from a fixed alphabet consisting of letters; say a_1, a_2 , ……….a_k. The letters in the source output sequence are assumed to be random and statistically

Independent of each other. A fixed probability assignment for the occurrence of each letter is also assumed. Let us, consider a small example to appreciate the importance of probability assignment of the source letters.

Let us consider a source with four letters a_1, a_2, a_3 and a_4 with $P(a_1)=0.5$, $P(a_2)=0.25$, $P(a_3)= 0.13$, $P(a_4)=0.12$. Let us decide to go for binary coding of these four

Source letters While this can be done in multiple ways, two encoded representations are shown below:

Code Representation#1:

a_1: 00, a_2:01, a_3:10, a_4:11

Code Representation#2:

a_1: 0, a_2:10, a_3:001, a_4:110

It is easy to see that in method #1 the probability assignment of a source letter has not been considered and all letters have been represented by two bits each. However in

The second method only a_1 has been encoded in one bit, a_2 in two bits and the remaining two in three bits. It is easy to see that the average number of bits to be used per source letter for the two methods is not the same. (~~a~~ for method #1=2 bits per letter and ~~a~~ for method #2 < 2 bits per letter). So, if we consider the issue of encoding a long sequence of

Letters we have to transmit less number of bits following the second method. This is an important aspect of source coding operation in general. At this point, let us note

a) We observe that assignment of small number of bits to more probable letters and assignment of larger number of bits to less probable letters (or symbols) may lead to efficient source encoding scheme.

b) However, one has to take additional care while transmitting the encoded letters. A careful inspection of the binary representation of the symbols in method #2 reveals that it may lead to confusion (at the decoder end) in deciding the end of binary representation of a letter and beginning of the subsequent letter.

1) The average number of coded bits (or letters in general) required per source letter is as small as possible and

2) The source letters can be fully retrieved from a received encoded sequence.

Shannon-Fano Code

Shannon–Fano coding, named after Claude Elwood Shannon and Robert Fano, is a technique for constructing a prefix code based on a set of symbols and their probabilities. It is suboptimal in the sense that it does not achieve the lowest possible expected codeword length like Huffman coding; however unlike Huffman coding, it does guarantee that all codeword lengths are within one bit of their theoretical ideal $I(x) = -\log P(x)$.

In Shannon–Fano coding, the symbols are arranged in order from most probable to least probable, and then divided into two sets whose total probabilities are as close as possible to being equal. All symbols then have the first digits of their codes assigned; symbols in the first set receive "0" and symbols in the second set receive "1". As long as any sets with more than one member remain, the same process is repeated on those sets, to determine successive digits of their codes. When a set has been reduced to one symbol, of course, this means the symbol's code is complete and will not form the prefix of any other symbol's code.

The algorithm works, and it produces fairly efficient variable-length encodings; when the two smaller sets produced by a partitioning are in fact of equal probability, the one bit of information used to distinguish them is used most efficiently. Unfortunately, Shannon–Fano does not always produce optimal prefix codes.

For this reason, Shannon–Fano is almost never used; Huffman coding is almost as computationally simple and produces prefix codes that always achieve the lowest expected code word length. Shannon–Fano coding is used in the IMPLODE compression method, which is part of the ZIP file format, where it is desired to apply a simple algorithm with high performance and minimum requirements for programming.

Shannon-Fano Algorithm:

A Shannon–Fano tree is built according to a specification designed to define an effective code table. The actual algorithm is simple:

For a given list of symbols, develop a corresponding list of probabilities or frequency counts so that each symbol's relative frequency of occurrence is known.

- Sort the lists of symbols according to frequency, with the most frequently occurring

  Symbols at the left and the least common at the right.
- Divide the list into two parts, with the total frequency counts of the left part being as

  Close to the total of the right as possible.
- The left part of the list is assigned the binary digit 0, and the right part is assigned the digit 1. This means that the codes for the symbols in the first part will all start with 0, and the codes in the second part will all start with 1.

Recursively apply the steps 3 and 4 to each of the two halves, subdividing groups and adding bits to the codes until each symbol has become a corresponding code leaf on the tree.

Example:

The source of information A generates the symbols {A0, A1, A2, A3 and A4} with the corresponding probabilities {0.4, 0.3, 0.15, 0.1 and 0.05}. Encoding the source symbols using binary encoder and Shannon-Fano encoder gives

Source Symbol	**P_i**	**Binary Code**	**Shannon-Fano**
A0	0.4	000	0
A1	0.3	001	10
A2	0.15	010	110
A3	0.1	011	1110
A4	0.05	100	1111
L_{avg}	**H = 2.0087**	**3**	**2.05**

The average length of the Shannon-Fano code is

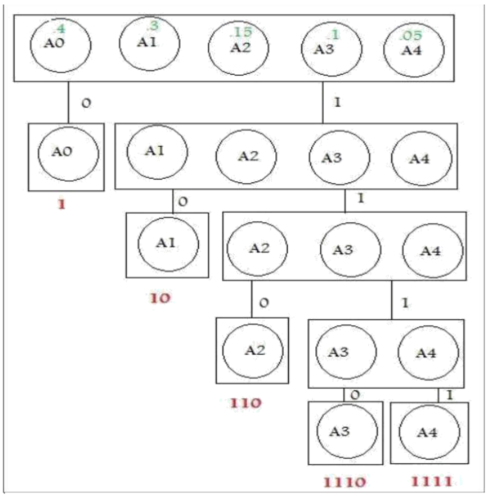
$$\text{Lavg} = \sum_{i=0}^{4} \text{Pi li} = 0.4 * 1 + 0.3 * 2 + 0.15 * 3 + 0.1 * 4 + 0.05 * 4 = 2.05 \text{ bit/symbol}$$

Thus the efficiency of the Shannon-Fano code is

$$\eta = \frac{H}{\text{Lavg}} = \frac{2.0087}{2.05} = 98\%$$

This example demonstrates that the efficiency of the Shannon-Fano encoder is much higher than that of the binary encoder.

Shanon-Fano code is a top-down approach. Constructing the code tree, we get



The Entropy of the source is

$$H = -\sum_{i=0}^{4} \text{Pi} \log_2 \text{Pi} = 2.0087 \text{ bit/symbol}$$

Since we have 5 symbols ($5<8=2^3$), we need 3 bits at least to represent each symbol in binary (fixed-length code). Hence the average length of the binary code is

$$\text{Lavg} = \sum_{i=0}^{4} \text{Pi li} = 3\,(0.4 + 0.3 + 0.15 + 0.1 + 0.05) = 3 \text{ bit/symbol}$$

Thus the efficiency of the binary code is

$$\eta = \frac{H}{\text{Lavg}} = \frac{2.0087}{3} = 67\%$$

Binary Huffman Coding (an optimum variable-length source coding scheme)

In Binary Huffman Coding each source letter is converted into a binary code word. It is a prefix condition code ensuring minimum average length per source letter in bits.

Let the source letters a_1, a_2, ……….a_K have probabilities $P(a_1)$, $P(a_2)$,………… $P(a_K)$ and let us assume that $P(a_1) \geq P(a_2) \geq P(a_3) \geq \ldots \geq P(a_K)$.

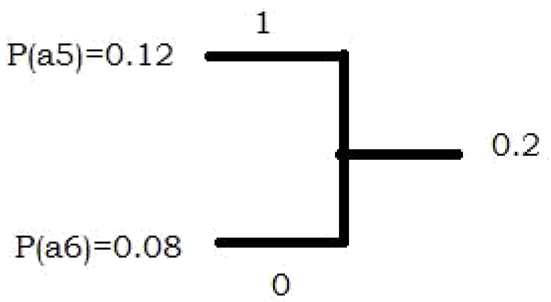
We now consider a simple example to illustrate the steps for Huffman coding.

Steps to calculate Huffman Coding

Example Let us consider a discrete memory less source with six letters having

$P(a_1)=0.3$,$P(a_2)=0.2$, $P(a_3)=0.15$, $P(a_4)=0.15$, $P(a_5)=0.12$ and $P(a_6)=0.08$.

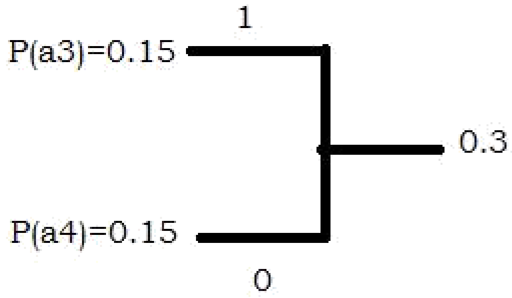
- Arrange the letters in descending order of their probability (here they are arranged).
- Consider the last two probabilities. Tie up the last two probabilities. Assign, say, 0 to the last digit of representation for the least probable letter (a_6) and 1 to the last digit of representation for the second least probable letter (a_5). That is, assign '1' to the upper arm of the tree and '0' to the lower arm.



(3) Now, add the two probabilities and imagine a new letter, say b_1, substituting for a_6 and a_5. So $P(b_1)$ =0.2. Check whether a_4 and $b_1$are the least likely letters. If not, reorder the letters as per Step#1 and add the probabilities of two least likely letters. For our example, it leads to:

$P(a_1)=0.3$, $P(a_2)=0.2$, $P(b_1)=0.2$, $P(a_3)=0.15$ and $P(a_4)=0.15$

(4) Now go to Step#2 and start with the reduced ensemble consisting of a_1 , a_2 , a_3 ,

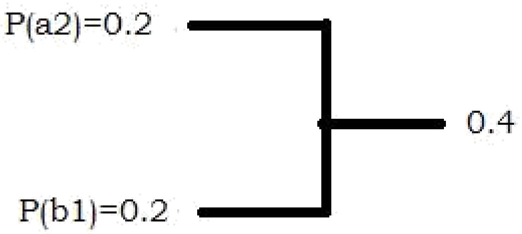


a_4 and b_1. Our example results in:

Here we imagine another letter b_1, with $P(b_2)=0.3$.

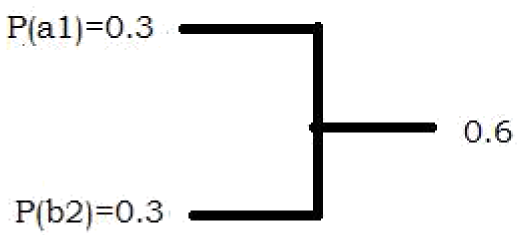
Continue till the first digits of the most reduced ensemble of two letters are assigned a '1' and a '0'.

Again go back to the step (2): $P(a_1)=0.3$, $P(b_2)=0.3$, $P(a_2)=0.2$ and $P(b_1)=0.2$. Now we consider the last two probabilities:

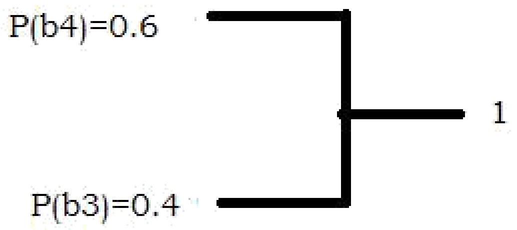


So, $P(b_3)=0.4$. Following Step#2 again, we get, $P(b_3)=0.4$, $P(a_1)=0.3$ and $P(b_2)=0.3$.

Next two probabilities lead to:



With $P(b_4) = 0.6$. Finally we get only two probabilities



6. Now, read the code tree inward, starting from the root, and construct the code words. The first digit of a codeword appears first while reading the code tree inward.

Hence, the final representation is: a_1=11, a_2=01, a_3=101, a_4=100, a_5=001, a_6=000.

A few observations on the preceding example

1. The event with maximum probability has least number of bits

2. Prefix condition is satisfied. No representation of one letter is prefix for other. Prefix condition says that representation of any letter should not be a part of any other letter.

3. Average length/letter (in bits) after coding is

$$= \sum P(a_i) n_i = 2.5 \text{ bits/letter.}$$

4. Note that the entropy of the source is: H(X)=2.465 bits/symbol. Average length per source letter after Huffman coding is a little bit more but close to the source entropy. In fact, the following celebrated theorem due to C. E. Shannon sets the limiting value of average length of code words from a DMS.

Shannon–Hartley theorem

In information theory, the Shannon–Hartley theorem tells the maximum rate at which information can be transmitted over a communications channel of a specified bandwidth in the presence of noise. It is an application of the noisy-channel coding theorem to the archetypal case of a continuous-time analog communications channel subject to Gaussian noise. The theorem establishes Shannon's channel capacity for such a communication link, a bound on the maximum amount of error-free information per time unit that can be transmitted with a specified bandwidth in the presence of the noise interference, assuming that the signal power is bounded, and that the Gaussian noise process is characterized by a known power or power spectral density.

The law is named after Claude Shannon and Ralph Hartley.

Hartley Shannon Law

The theory behind designing and analyzing channel codes is called Shannon's noisy channel coding theorem. It puts an upper limit on the amount of information you can send in a noisy channel using a perfect channel code. This is given by the following equation:

$$C = B \times \log_2(1 + SNR)$$

where C is the upper bound on the capacity of the channel (bit/s), B is the bandwidth of the channel (Hz) and SNR is the Signal-to-Noise ratio (unit less).

Bandwidth-S/N Tradeoff

The expression of the channel capacity of the Gaussian channel makes intuitive sense:

1. As the bandwidth of the channel increases, it is possible to make faster changes in the information signal, thereby increasing the information rate.

2 As S/N increases, one can increase the information rate while still preventing errors due to noise.

3. For no noise, S/N tends to infinity and an infinite information rate is possible irrespective of bandwidth.

Thus we may trade off bandwidth for SNR. For example, if S/N = 7 and B = 4kHz, then the channel capacity is C = 12 $\times 10^3$ bits/s. If the SNR increases to S/N = 15 and B is decreased to 3kHz, the channel capacity remains the same. However, as B tends to 1, the channel capacity does not become infinite since, with an increase in bandwidth, the noise power also increases. If the noise power spectral density is $\eta/2$, then the total noise power is N = ηB, so the Shannon-Hartley law becomes

$$
\begin{aligned}
C &= B \log_2 \left(1 + \frac{S}{\eta B}\right) = \frac{S}{\eta}\left(\frac{\eta B}{S}\right) \log_2 \left(1 + \frac{S}{\eta B}\right) \\
&= \frac{S}{\eta} \log_2 \left(1 + \frac{S}{\eta B}\right)^{\eta B / S}.
\end{aligned}
$$

Noting that

$$\lim_{x \to 0} (1 + x)^{1/x} = e$$

and identifying x as $x = S/\eta B$, the channel capacity as B increases without bound becomes

$$C_\infty = \lim_{B \to \infty} C = \frac{S}{\eta} \log_2 e = 1.44 \frac{S}{\eta}.$$

UNIT - VI
Forward Error Correction (FEC)

- The key idea of FEC is to transmit enough redundant data to allow receiver to recover from errors all by itself. No sender retransmission required.
- The major categories of FEC codes are
  - Block codes,
  - Cyclic codes,
  - Convolutional codes,
  - and Turbo codes.

Linear Block Codes

- Information is divided into blocks of length *k*
- *r* parity bits or check bits are added to each block (total length $n = k + r$).
- Code rate $R = k/n$
- Decoder looks for codeword closest to received vector (received vector = code vector + error vector)
- Tradeoffs between
  - Efficiency
  - Reliability
  - Encoding/Decoding complexity

- In Maximum-likelihood decoding, we compare the received vector with all possible transmitted codes and choose that which is closest in Hamming distance (i.e., which is differs in the fewest bits). This results in a minimum probability of a code word error.

Linear Block Codes

- The uncoded k data bits be represented by the **m** vector:

  $\mathbf{m}=(m_1, m_2, \ldots, m_k)$

  The corresponding codeword be represented by the n-bit **c** vector:

  $\mathbf{c}=(c_1, c_2, \ldots c_k, c_{k+1}, \ldots, c_{n-1}, c_n)$

- Each parity bit consists of weighted modulo 2 sum of the data bits represented by ⊕symbol.

$$\begin{cases} c_1 = m_1 \\ c_2 = m_2 \\ \ldots \\ c_k = m_k \\ c_{k+1} = m_1 p_{1(k+1)} \oplus m_2 p_{2(k+1)} \oplus \ldots \oplus m_k p_{k(k+1)} \\ \ldots \\ c_n = m_1 p_{1n} \oplus m_2 p_{2n} \oplus \ldots \oplus m_k p_{kn} \end{cases}$$

Block Codes: Linear Block Codes

- Linear Block Code

The codeword block $\boldsymbol{C}$ of the Linear Block Code is

$$\boldsymbol{C} = m\,\boldsymbol{G}$$

where m is the information block, $\boldsymbol{G}$ is the generator matrix.

$$\boldsymbol{G} = [\mathbf{I}_k \mid \mathbf{P}]_{k \times n}$$

where p_i = Remainder of $[x^{n-k+i-1}/g(x)]$ for $i=1, 2, .., k$, and $\mathbf{I}$ is unit matrix.

- The parity check matrix

$\boldsymbol{H} = [\mathbf{P}^{\mathbf{T}} \mid \mathbf{I}_{n-k}]$, where $\mathbf{P}^{\mathbf{T}}$ is the transpose of the matrix $\mathbf{p}$.

Block Codes: Example

Example : Find linear block code encoder **G** if code generator polynomial $g(x)=1+x+x^3$ for a (7, 4) code.

We have n = Total number of bits = 7, k = Number of information bits = 4, r = Number of parity bits = n - k = 3.

- $$G = [I \mid P] = \begin{bmatrix} 1 & 0 & \cdots & 0 & p_1 \\ 0 & 1 & \cdots & 0 & p_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & p_k \end{bmatrix},$$

where

$$p_i = \text{Remainder of } \left[\frac{x^{n-k+i-1}}{g(x)} \right], \quad i = 1, 2, \cdots, k$$

Block Codes: Example (Continued)

$$p_1 = \mathrm{Re}\left[\frac{x^3}{1+x+x^3}\right] = 1 + x \rightarrow [110]$$

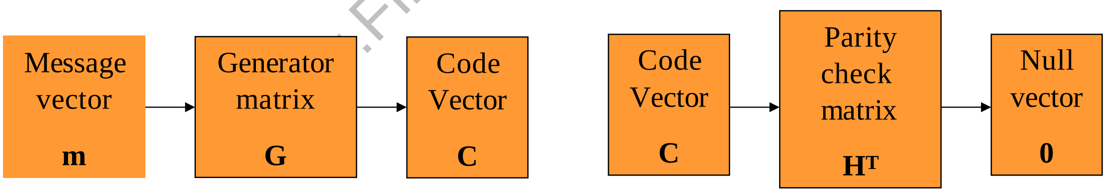
$$p_2 = \mathrm{Re}\left[\frac{x^4}{1+x+x^3}\right] = x + x^2 \rightarrow [011]$$

$$p_3 = \mathrm{Re}\left[\frac{x^5}{1+x+x^3}\right] = 1 + x + x^2 \rightarrow [111]$$

$$p_4 = \mathrm{Re}\left[\frac{x^6}{1+x+x^3}\right] = 1 + x^2 \rightarrow [101]$$

$$G = \begin{bmatrix} 1000110 \\ 0100011 \\ 0010111 \\ 0001101 \end{bmatrix}$$

Block Codes: Linear Block Codes



Operations of the generator matrix and the parity check matrix

The parity check matrix H is used to detect errors in the received code by using the fact that $c * H^T = \mathbf{0}$ (null vector)

Let $x = c \oplus e$ be the received message where c is the correct code and e is the error

Compute $S = x * H^T = (c \oplus e) * H^T = c H^T \oplus e H^T = e H^T$ (s is know as syndrome matrix)

If S is 0 then message is correct else there are errors in it, from common known error patterns the correct message can be decoded.

Linear Block Codes

- Consider a (7,4) linear block code, given by **G** as

$$G = \begin{bmatrix} 1000111 \\ 0100110 \\ 0010101 \\ 0001011 \end{bmatrix} \qquad \text{Then,} \qquad H = \begin{bmatrix} 1110100 \\ 1101010 \\ 1011001 \end{bmatrix}$$

For **m** = [1 0 1 1] and **c** = **mG** = [1 0 1 1 0 0 1].

If there is no error, the received vector **x**=**c**, and **s**=**cH**^{T}=[0, 0,0]

Linear Block Codes

Let c suffer an error such that the received vector

x=**c**⊕ **e**

=[1 0 1 1 0 0 1] ⊕[0 0 1 0 0 0 0]

=[1 0 0 1 0 0 1].

Then,

$s = xH^T$

$$= \begin{bmatrix} 1001001 \end{bmatrix} \begin{bmatrix} 111 \\ 110 \\ 101 \\ 011 \\ 100 \\ 010 \\ 001 \end{bmatrix} = [101]$$

$= (eH^T)$

Cyclic Codes

It is a block code which uses a shift register to perform encoding and Decoding (all code words are shifts of each other)
The code word with n bits is expressed as

$$c(x)=c_1x^{n-1}+c_2x^{n-2}\ldots\ldots+c_n$$

where each c_i is either a 1 or 0.

$$c(x) = m(x)\,x^{n-k} + c_p(x)$$

where $c_p(x)$ = remainder from dividing $m(x)\,x^{n-k}$ by generator g(x)
if the received signal is c(x) + e(x) where e(x) is the error.

To check if received signal is error free, the remainder from dividing c(x) + e(x) by g(x) is obtained(syndrome). If this is 0 then the received signal is considered error free else error pattern is detected from known error syndromes.

Cyclic Code: Example

Example : Find the codewords c(x) if $m(x)=1+x+x^2$ and $g(x)=1+x+x^3$ for (7,4) cyclic code.

We have n = Total number of bits = 7, k = Number of information bits = 4,
r = Number of parity bits = n - k = 3.

□ $$c_p(x) = rem\left[\frac{m(x)x^{n-k}}{g(x)}\right] = rem\left[\frac{x^5+x^4+x^3}{x^3+x+1}\right] = x$$

Then,

$$c(x) = m(x)x^{n-k} + c_p(x) = x + x^3 + x^4 + x^5$$

Cyclic Redundancy Check (CRC)

- Cyclic redundancy Code (CRC) is an error-checking code.
- The transmitter appends an extra n-bit sequence to every frame called Frame Check Sequence (FCS). The FCS holds redundant information about the frame that helps the receivers detect errors in the frame.
- CRC is based on polynomial manipulation using modulo arithmetic. Blocks of input bit as coefficient-sets for polynomials is called message polynomial. Polynomial with constant coefficients is called the generator polynomial.

Cyclic Redundancy Check (CRC)

- Generator polynomial is divided into the message polynomial, giving quotient and remainder, the coefficients of the remainder form the bits of final CRC.
- Define:

M – The original frame (k bits) to be transmitted before adding the Frame Check Sequence (FCS).

F – The resulting FCS of n bits to be added to M (usually n=8, 16, 32).

T – The cascading of M and F.

P – The predefined CRC generating polynomial with pattern of n+1 bits.

The main idea in CRC algorithm is that the FCS is generated so that the remainder of T/P is zero.

Cyclic Redundancy Check (CRC)

- The CRC creation process is defined as follows:
  - Get the block of raw message
  - Left shift the raw message by n bits and then divide it by p
  - Get the remainder R as FCS
  - Append the R to the raw message . The result is the frame to be transmitted.
- CRC is checked using the following process:
  - Receive the frame
  - Divide it by P
  - Check the remainder. If the remainder is not zero, then there is an error in the frame.

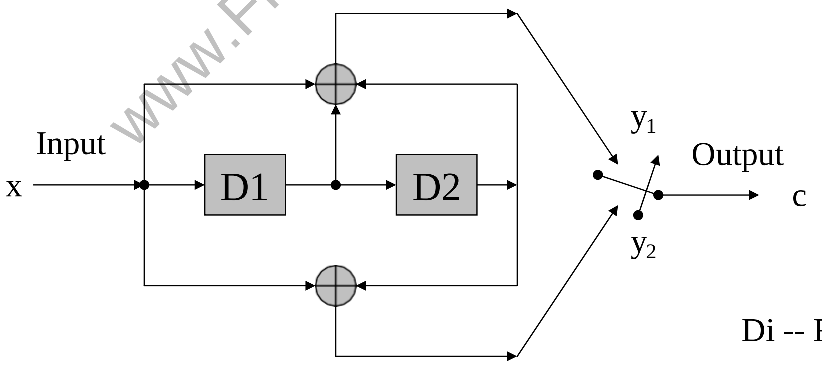
Common CRC Codes

Code	Generator polynomial g(x)	Parity check bits
CRC-12	$1+x+x^2+x^3+x^{11}+x^{12}$	12
CRC-16	$1+x^2+x^{15}+x^{16}$	16
CRC-CCITT	$1+x^5+x^{15}+x^{16}$	16

Convolutional Codes

- Encoding of information stream rather than information blocks
- Value of certain information symbol also affects the encoding of next *M* information symbols, i.e., memory *M*
- Easy implementation using shift register
  - Assuming *k* inputs and *n* outputs
- Decoding is mostly performed by the Viterbi Algorithm (not covered here)

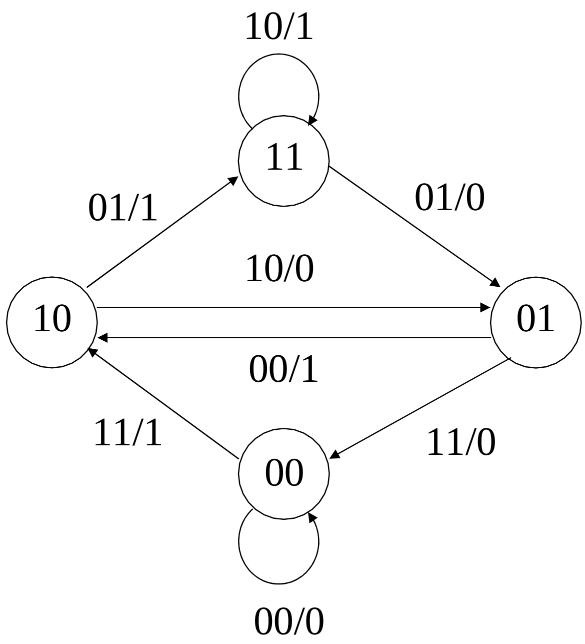
Convolutional Codes: (n=2, k=1, M=2) Encoder



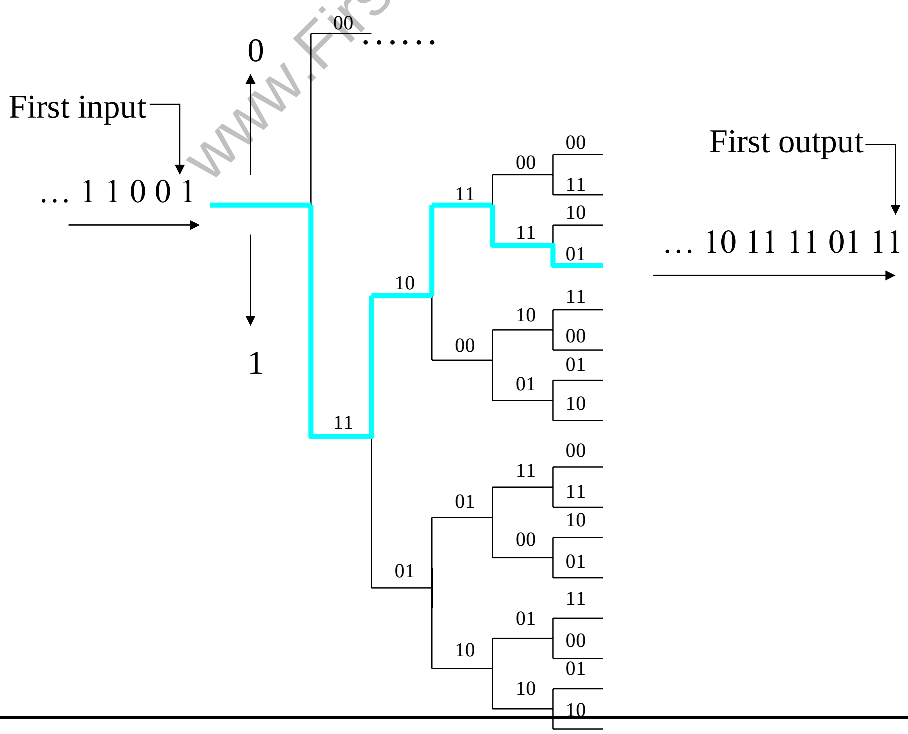
Di -- Register

Input:	1	1	1	0	0	0	…
Output:	11	01	10	01	11	00	…
Input:	1	0	1	0	0	0	…
Output:	11	10	00	10	11	00	…

State Diagram



Tree Diagram



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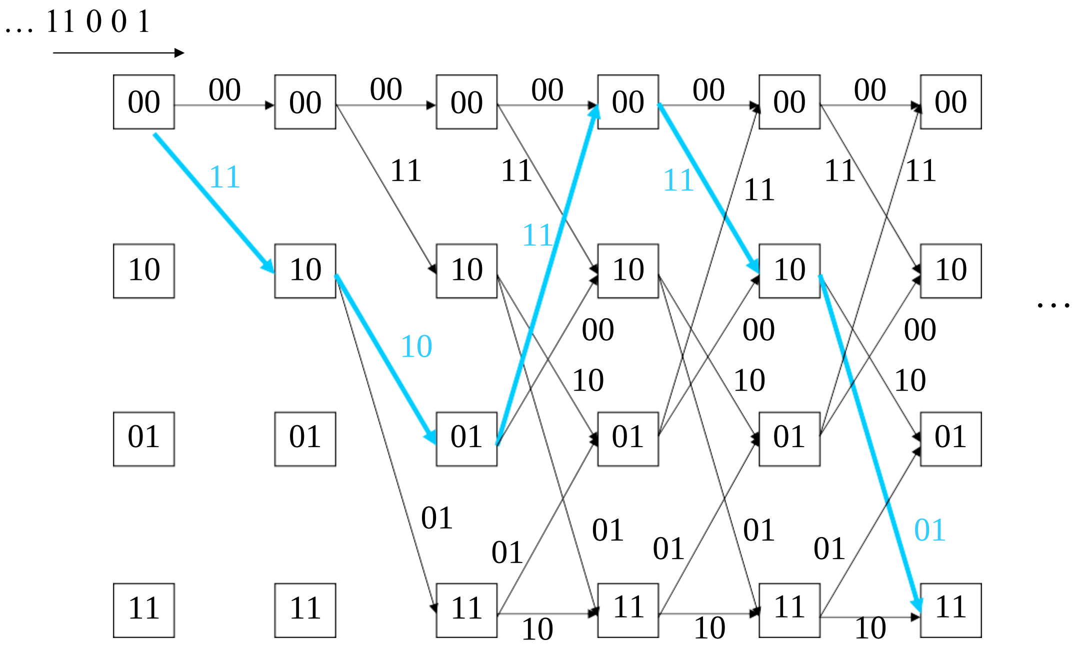


Interleaving

Input Data: a1, a2, a3, a4, a5, a6, a7, a8, a9, …

Interleaving (Write →, Read ↓):

a1,	a2,	a3,	a4
a5,	a6,	a7,	a8
a9,	a10,	a11,	a12
a13,	a14,	a15,	a16

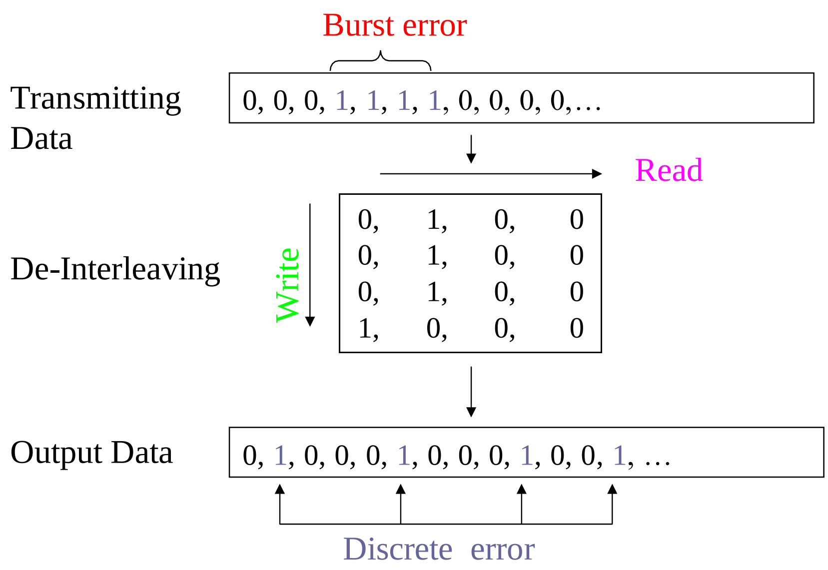
Transmitting Data: a1, a5, a9, a13, a2, a6, a10, a14, a3, …

De-Interleaving (Read →, Write ↓):

a1,	a2,	a3,	a4
a5,	a6,	a7,	a8
a9,	a10,	a11,	a12
a13,	a14,	a15,	a16

Output Data: a1, a2, a3, a4, a5, a6, a7, a8, a9, …

Interleaving (Example)

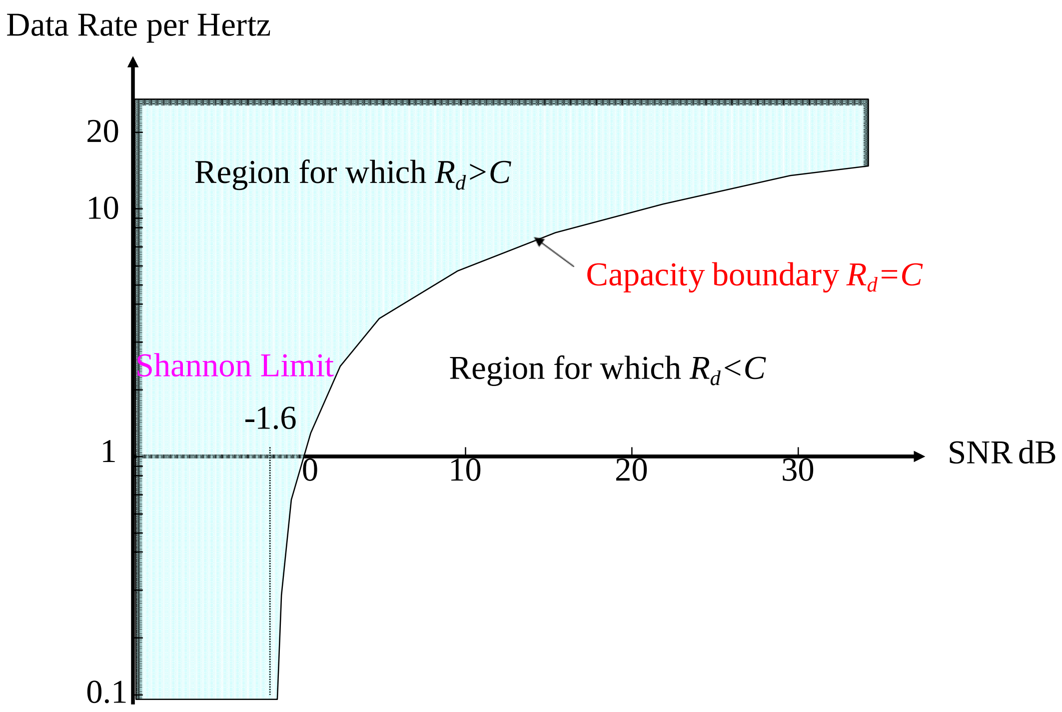


Information Capacity Theorem (Shannon Limit)

- The information capacity (or channel capacity) *C* of a continuous channel with bandwidth *B* Hertz satisfies

$$C = B \log_2 \left(1 + \frac{S}{N}\right) \quad bits / second$$

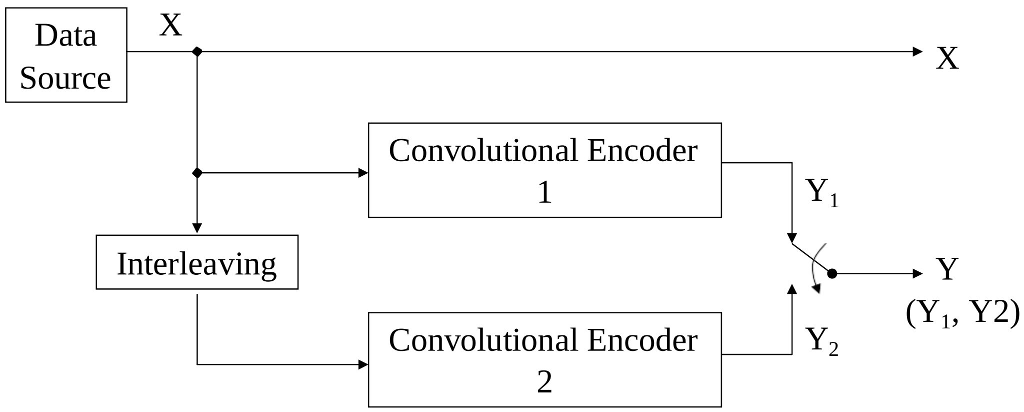
Shannon Limit



Turbo Codes

- A brief historic of turbo codes:
  The turbo code concept was first introduced by C. Berrou in 1993. Today, Turbo Codes are considered as the most efficient coding schemes for FEC.
- Scheme with known components (simple convolutional or block codes, interleaver, and soft-decision decoder)
- Performance close to the Shannon Limit at modest complexity!
- Turbo codes have been proposed for low-power applications such as deep-space and satellite communications, as well as for interference limited applications such as third generation cellular, personal communication services, ad hoc and sensor networks.

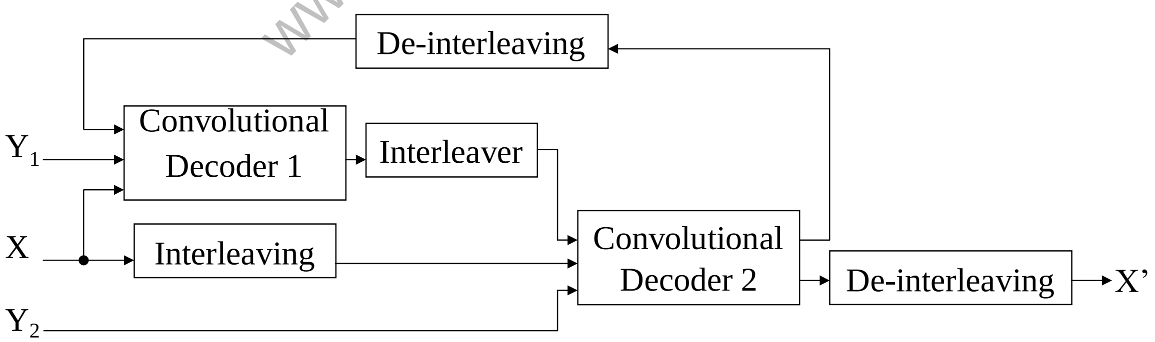
Turbo Codes: Encoder



X: Information

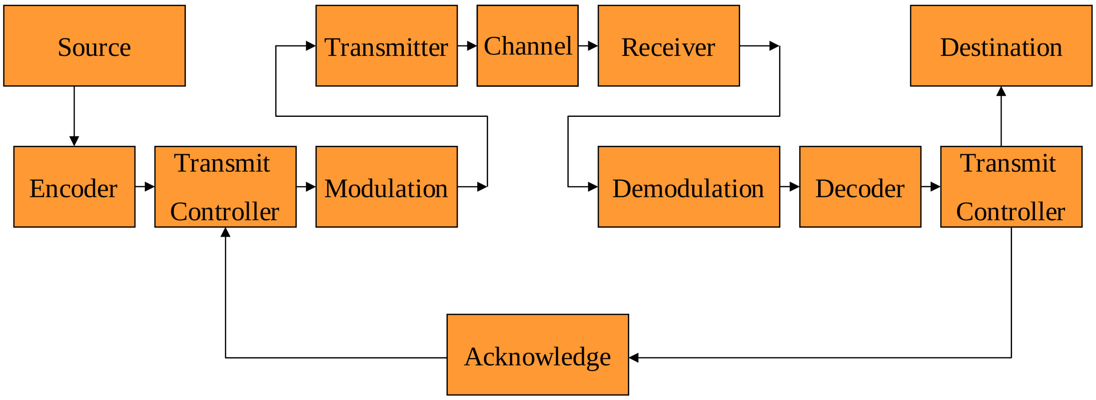
Y_i: Redundancy Information

Turbo Codes: Decoder

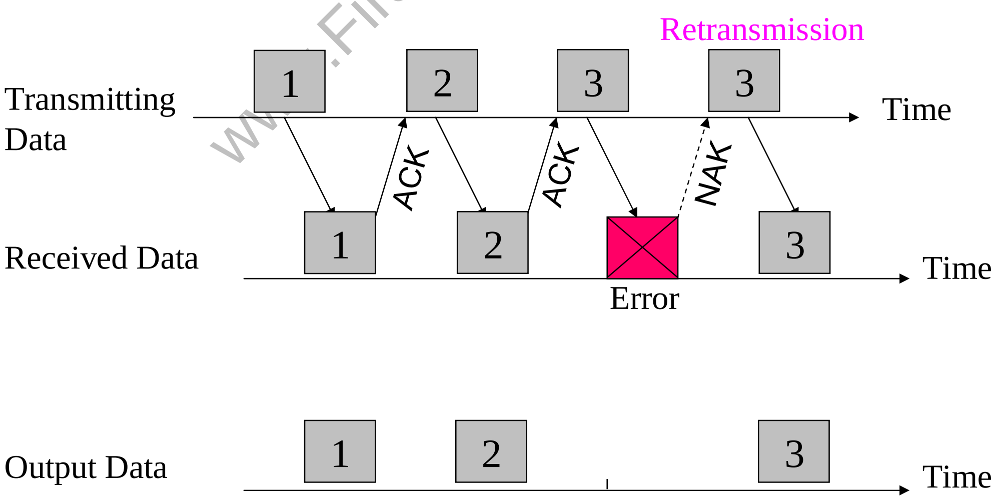


X': Decoded Information

Automatic Repeat Request (ARQ)



Stop-And-Wait ARQ (SAW ARQ)



ACK: Acknowledge

NAK: Negative ACK

Stop-And-Wait ARQ (SAW ARQ)

Throughput:

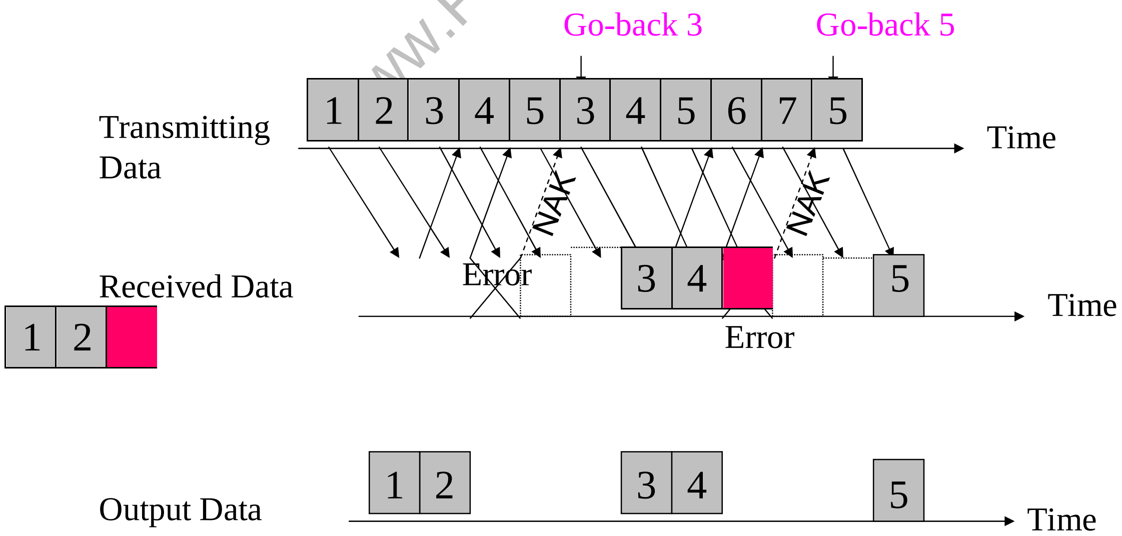
$$S = (1/T) * (k/n) = [(1- P_{\mathbf{b}})^n / (1 + D * R_b/ n)] * (k/n)$$

where T is the average transmission time in terms of a block duration

$$T = (1 + D * R_b/ n) * P_{ACK} + 2 * (1 + D * R_b/ n) * P_{ACK} * (1- P_{ACK})$$
$$+ 3 * (1 + D * R_b/ n) * P_{ACK} * (1- P_{ACK})^2 + \ldots..$$
$$= (1+ D * R_b/n) * P_{ACK} \sum_{i=1}^{\infty} i * (1-P_{ACK})^{i-1}$$
$$= (1+ D * Rb/n) * P_{ACK}/[1-(1-P_{ACK})]^2$$
$$= (1 + D * R_b/ n) / P_{ACK}$$

where n = number of bits in a block, k = number of information bits in a block, D = round trip delay, R_b= bit rate, $P_{\mathbf{b}}$ = BER of the channel, and $P_{ACK} = (1- P_{\mathbf{b}})^n$

Go-Back-N ARQ (GBN ARQ)



Go-Back-N ARQ (GBN ARQ)

Throughput

$S = (1/T) * (k/n)$

$= [(1- P_b)^n / ((1- P_b)^n + N * (1-(1- P_b)^n))]* (k/n)$

where

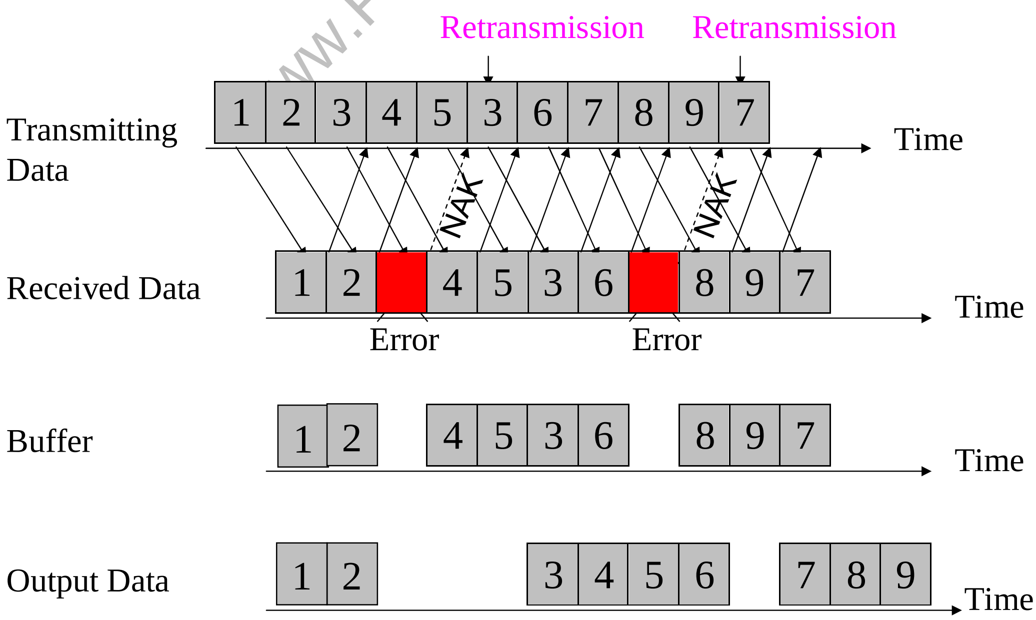
$T = 1 * P_{ACK} + (N+1) * P_{ACK} * (1- P_{ACK}) +2 * (N+1) * P_{ACK} * (1- P_{ACK})^2 + \ldots$

$= P_{ACK}+P_{ACK} * [(1-P_{ACK})+(1-P_{ACK})^2 +(1-P_{ACK})^3+\ldots]+ P_{ACK}[N * (1-P_{ACK})+2 * N * (1-P_{ACK})^2 +3 * N * (1-P_{ACK})^3+\ldots]$

$= P_{ACK}+P_{ACK} *[(1-P_{ACK})/P_{ACK} + N * (1-P_{ACK})/P_{ACK}^2$

$= 1 + (N * [1 - (1- P_b)^n])/ (1- P_b)^n$

Selective-Repeat ARQ (SR ARQ)



Selective-Repeat ARQ (SR ARQ)

Throughput

$S = (1/T) * (k/n)$

$= (1- P_b)^n * (k/n)$

where

$T = 1 * P_{ACK} + 2 * P_{ACK} * (1- P_{ACK}) + 3 * P_{ACK} * (1- P_{ACK})^2 + \ldots.$

$= P_{ACK} \sum_{i=1}^{\infty} i * (1-P_{ACK})^{i-1}$

$= P_{ACK}/[1-(1-P_{ACK})]^2$

$= 1/(1- P_b)^n$