UNIT-I

BEARINGS

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

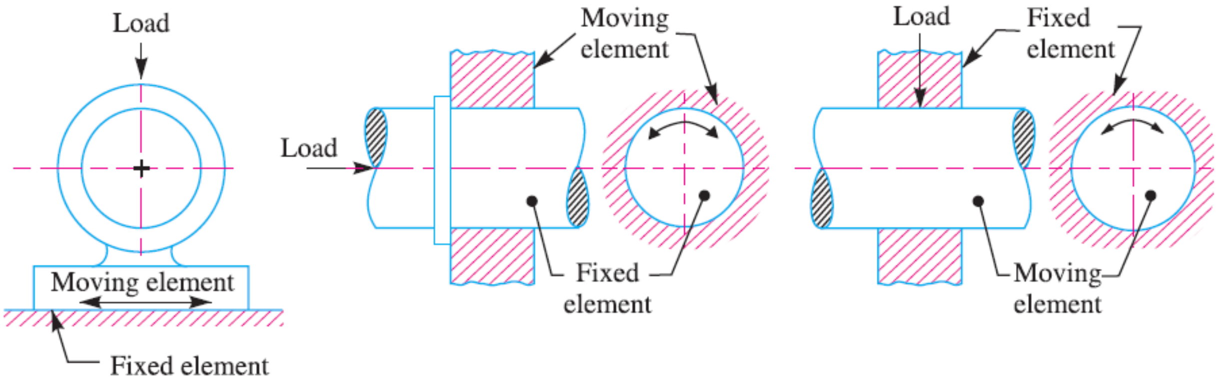
Classification of Bearings

1. Depending upon the direction of load to be supported. The bearings under this group are
classified as:

(a) Radial bearings, and (b) Thrust bearings.

In radial bearings, the load acts perpendicular to the direction of motion of the moving element as shown in Fig (a) and (b).

In thrust bearings, the load acts along the axis of rotation as shown in Fig (c).



(*a*) Radial bearing. (*b*) Radial bearing. (*c*) Thrust bearing.

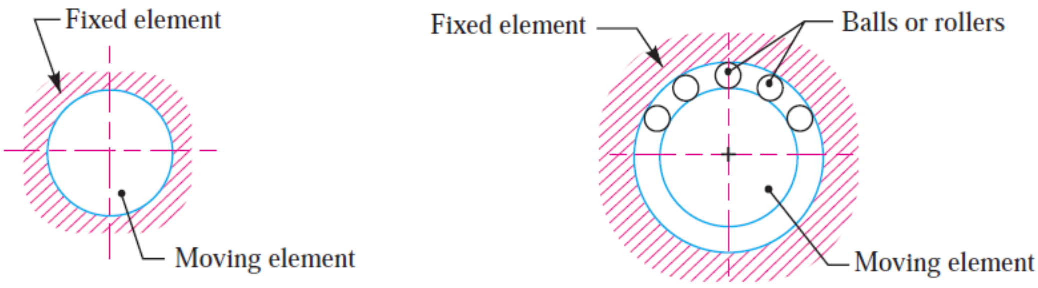
Fig: Sliding and rolling contact bearings

2. Depending upon the nature of contact. The bearings under this group are classified as:

(a) Sliding contact bearings, and (b) Rolling contact bearings.

In sliding contact bearings, as shown in Fig (a), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

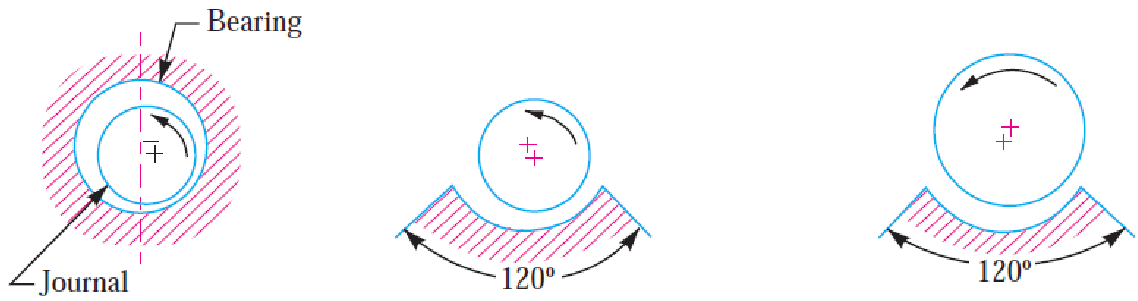
In rolling contact bearings, as shown in Fig (b), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.



(a) Sliding contact bearing. (b) Rolling contact bearings.

Types of Sliding Contact Bearings

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig (a), may be called slipper or guide bearings. Such type of bearings are usually found in cross-head of steam engines.



(a) Full journal bearing. (b) Partial journal bearing. (c) Fitted journal bearing.

Fig: Journal or sleeve bearings

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearings.

When the angle of contact of the bearing with the journal is 360° as shown in Fig (a), then the bearing is called a full journal bearing. This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig (b), then the bearing is said to be partial journal bearing. This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as clearance bearings because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance i.e. the diameters of the journal and bearing are equal, then the bearing is called a fitted bearing, as shown in Fig (c).

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows:

1. Thick film bearings. The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.

2. Thin film bearings. The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called boundary lubricated bearings.

3. Zero film bearings. The zero film bearings are those which operate without any lubricant present.

4. Hydrostatic or externally pressurized lubricated bearings. The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

Hydrodynamic Lubricated Bearings

In hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load.

The load supporting pressure in hydrodynamic bearings arises from either

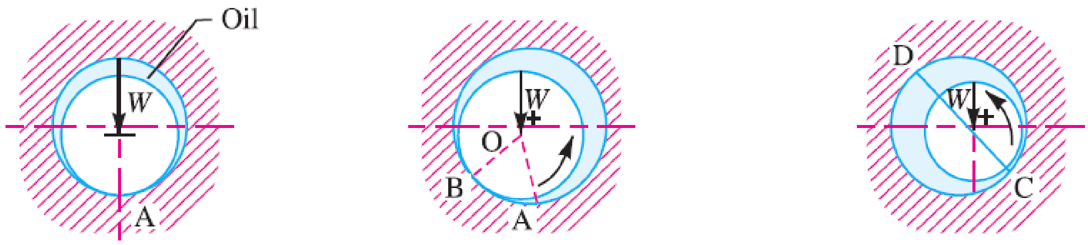
1. the flow of a viscous fluid in a converging channel (known as wedge film lubrication), or

2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces

(known as squeeze film lubrication).

Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (nonrotating) bearing and a rotating journal. Fig (a) shows a journal at rest with metal to metal contact at A on the line of action of the supported load. When the journal rotates slowly in the anticlockwise direction, as shown in Fig (b), the point of contact will move to B, so that the angle AOB is the angle of sliding friction of the surfaces in contact at B. In the absence of a lubricant, there will be dry metal to metal friction. If a lubricant is present in the clearance space of the bearing and journal, then a thin absorbed film of the lubricant may partly separate the surface, but a continuous fluid film completely separating the surfaces will not exist because of slow speed.



(*a*) At rest. (*b*) Slow speed. (*c*) High speed.

Fig: Wedge film journal bearing.

When the speed of the journal is increased, a continuous fluid film is established as in Fig(c). The center of the journal has moved so that the minimum film thickness is at C. It may be noted that from D to C in the direction of motion, the film is continually narrowing and hence is a converging film. The curved converging film may be considered as a wedge-shaped film of a slipper bearing wrapped around the journal. A little consideration will show that from C to D in the direction of rotation, as shown in Fig (c), the film is diverging and cannot give rise to a positive pressure or a supporting action.

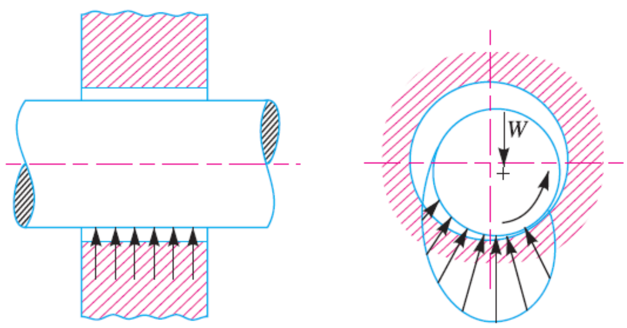


Fig: Variation of pressure in the converging film

Figure shows the two views of the bearing shown in Fig (c), with the variation of pressure in the converging film. Actually, because of side leakage, the angle of contact on which pressure acts is less than 180°.

Squeeze Film Journal Bearing

In a wedge film journal bearing, the bearing carries a steady load and the journal rotates relative to the bearing. But in certain cases, the bearings oscillate or rotate so slowly that the wedge film cannot provide a satisfactory film thickness. If the load is uniform or varying in magnitude while acting in a constant direction, this becomes a thin film or possibly a zero-film problem. But if the load reverses its direction, the squeeze film may develop sufficient capacity to carry the dynamic loads without contact between the journal and the bearing. Such bearings are known as squeeze film journal bearing.

Properties of Sliding Contact Bearing Materials

When the journal and the bearings are having proper lubrication i.e. there is a film of clean, non-corrosive lubricant in between, separating the two surfaces in contact, the only requirement of the bearing material is that they should have sufficient strength and rigidity. However, the conditions under which bearings must operate in service are generally far from ideal and thus the other properties as discussed below must be considered in selecting the best material.

1. Compressive strength: The maximum bearing pressure is considerably greater than the average pressure obtained by dividing the load to the projected area. Therefore, the bearing material should have high compressive strength to withstand this maximum pressure so as to prevent extrusion or other permanent deformation of the bearing.

2. Fatigue strength: The bearing material should have sufficient fatigue strength so that it can withstand repeated loads without developing surface fatigue cracks. It is of major importance in aircraft and automotive engines.

3. Comformability: It is the ability of the bearing material to accommodate shaft deflections and bearing inaccuracies by plastic deformation (or creep) without excessive wear and heating.

4. Embeddability: It is the ability of bearing material to accommodate (or embed) small particles of dust, grit etc., without scoring the material of the journal.

5. Bondability: Many high capacity bearings are made by bonding one or more thin layers of a bearing material to a high strength steel shell. Thus, the strength of the bond i.e. bondability is an important consideration in selecting bearing material.

6. Corrosion resistance: The bearing material should not corrode away under the action of lubricating oil. This property is of particular importance in internal combustion engines where the same oil is used to lubricate the cylinder walls and bearings. In the cylinder, the lubricating oil comes into contact with hot cylinder walls and may oxidize and collect carbon deposits from the walls.

7. Thermal conductivity: The bearing material should be of high thermal conductivity so as to permit the rapid removal of the heat generated by friction.

8. Thermal expansion: The bearing material should be of low coefficient of thermal expansion, so that when the bearing operates over a wide range of temperature, there is no undue change in the clearance. All these properties as discussed above are, however, difficult to find in any particular bearing material. The various materials are used in practice, depending upon the requirement of the actual service conditions. The choice of material for any application must represent a compromise.

Materials used for Sliding Contact Bearings

The materials commonly used for sliding contact bearings are discussed below:

1. Babbit metal: The tin base and lead base babbits are widely used as a bearing material, because they satisfy most requirements for general applications. The babbits are recommended where the maximum bearing pressure (on projected area) is not over 7 to 14 N/mm^2. When applied in automobiles, the babbit is generally used as a thin layer, 0.05 mm to 0.15 mm thick, bonded to an insert or steel shell. The composition of the babbit metals is as follows:

Tin base babbits : Tin 90% ; Copper 4.5% ; Antimony 5% ; Lead 0.5%.

Lead base babbits : Lead 84% ; Tin 6% ; Anitmony 9.5% ; Copper 0.5%.

2. Bronzes: The bronzes (alloys of copper, tin and zinc) are generally used in the form of machined bushes pressed into the shell. The bush may be in one or two pieces. The bronzes commonly used for bearing material are gun metal and phosphor bronzes.

The gun metal (Copper 88% ; Tin 10% ; Zinc 2%) is used for high grade bearings subjected to high pressures (not more than 10 N/mm2 of projected area) and high speeds.

The phosphor bronze (Copper 80% ; Tin 10% ; Lead 9% ; Phosphorus 1%) is used for bearings subjected to very high pressures (not more than 14 N/mm2 of projected area) and speeds.

3. Cast iron: The cast iron bearings are usually used with steel journals. Such type of bearings are fairly successful where lubrication is adequate and the pressure is limited to 3.5 N/mm^2 and speed to 40 metres per minute.

4. Silver: The silver and silver lead bearings are mostly used in aircraft engines where the fatigue strength is the most important consideration.

5. Non-metallic bearings: The various non-metallic bearings are made of carbon-graphite, rubber, wood and plastics. The carbon-graphite bearings are self-lubricating, dimensionally stable over a wide range of operating conditions, chemically inert and can operate at higher temperatures than other bearings. Such type of bearings are used in food processing and other equipment where contamination by oil or grease must be prohibited. These bearings are also used in applications where the shaft speed is too low to maintain a hydrodynamic oil film.

The soft rubber bearings are used with water or other low viscosity lubricants, particularly where sand or other large particles are present. In addition to the high degree of embeddability and comformability, the rubber bearings are excellent for absorbing shock loads and vibrations. The rubber bearings are used mainly on marine propeller shafts, hydraulic turbines and pumps.

The wood bearings are used in many applications where low cost, cleanliness, inattention to lubrication and anti-seizing are important.

The commonly used plastic material for bearings is Nylon and Teflon. These materials have many characteristics desirable in bearing materials and both can be used dry i.e. as a zero-film bearing.

The Nylon is stronger, harder and more resistant to abrasive wear. It is used for applications in which these properties are important e.g. elevator bearings, cams in telephone dials etc. The Teflon is rapidly replacing Nylon as a wear surface or liner for journal and other sliding bearings because of the following properties:

1. It has lower coefficient of friction, about 0.04 (dry) as compared to 0.15 for Nylon.

2. It can be used at higher temperatures up to about 315°C as compared to 120°C for Nylon.

3. It is dimensionally stable because it does not absorb moisture, and

4. It is practically chemically inert.

Lubricants

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups:

1. Liquid, 2. Semi-liquid, and 3. Solid.

The liquid lubricants usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained. A grease is a semi-liquid lubricant having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable. The solid lubricants are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease.

Properties of Lubricants

1. Viscosity: It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure. The greater the heat and pressure, the greater viscosity is required of a lubricant to prevent thinning and squeezing out of the film. The fundamental meaning of viscosity may be understood by considering a flat plate moving under a force P parallel to a stationary plate, the two plates being separated by a thin film of a fluid lubricant of thickness h, as shown in Fig. The particles of the lubricant adhere strongly to the moving and stationary plates. The motion is accompanied by a linear slip or shear between the particles throughout the entire height (h) of the film thickness. If A is the area of the plate in contact with the lubricant, then the unit shear stress is given by

$$\tau = \frac{P}{A}$$

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient (dV/dy) .It is assumed that

(a) the lubricant completely fills the space between the two surfaces,

(b) the velocity of the lubricant at each surface is same as that of the surface, and

(c) any flow of the lubricant perpendicular to the velocity of the plate is negligible.

$$\therefore \tau = \frac{P}{A} \alpha \frac{dV}{dy} \text{ or } \tau = Z \times \frac{dV}{dy}$$

where Z is a constant of proportionality and is known as absolute viscosity (or simply viscosity) of the lubricant.

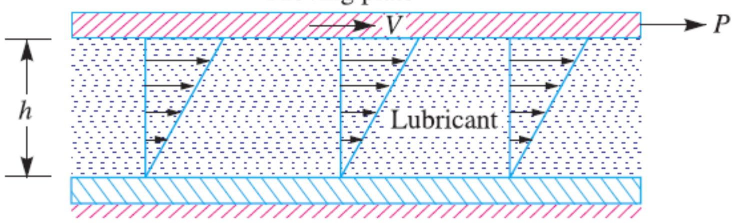


Fig: Viscosity

When the thickness of the fluid lubricant is, small which is the case for bearings, then the velocity gradient is very nearly constant as shown in Fig, so that

$$\frac{dV}{dy} = \frac{V}{y} = \frac{V}{h}$$

$$\tau = Z \times \frac{V}{h} \quad \text{or} \quad Z = \tau \times \frac{h}{V}$$

When τ is in N/m^2, h is in metres and V is in m/s, then the unit of absolute viscosity is given by

$$Z = \tau \times \frac{h}{V} = \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}}{\text{m/s}} = \text{N-s/m}^2$$

However, the common practice is to express the absolute viscosity in mass units, such that

$$1 \text{ N-s} / \text{m}^2 = \frac{1\text{kg-m}}{s^2} \times \frac{\text{s}}{\text{m}^2} = 1 \text{ kg} / \text{m-s} \qquad \ldots (\because \ 1 \text{ N} = 1 \text{ kg-m} / \text{s}^2)$$

Thus, the unit of absolute viscosity in S.I. units is kg / m-s.

The viscocity of the lubricant is measured by Saybolt universal viscometer. It determines the time required for a standard volume of oil at a certain temperature to flow under a certain head through a tube of standard diameter and length. The time so determined in seconds is the Saybolt universal viscosity. In order to convert Saybolt universal viscosity in seconds to absolute viscosity (in kg / m-s), the following formula may be used:

$$Z = \text{Sp. gr. of oil} \left(0.000\ 22\, S - \frac{0.18}{S}\right) \text{kg} / \text{m-s} \qquad \ldots (i)$$

where Z = Absolute viscosity at temperature t in kg / m-s, and

S = Saybolt universal viscosity in seconds.

2. Oiliness: It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness.

3. Density: This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically

Absolute viscosity = ρ × Kinematic viscosity (in m^2/s)

where ρ = Density of the lubricating oil.

The density of most of the oils at 15.5°C varies from 860 to 950 kg / m^3 (the average value may

be taken as 900 kg / m^3). The density at any other temperature (t) may be obtained from the following relation, i.e.

$$\rho_t = \rho_{15.5} - 0.000\ 657\ t$$

where $\rho_{15.5}$ = Density of oil at 15.5° C.

4. Viscosity index: The term viscosity index is used to denote the degree of variation of viscosity with temperature.

5. Flash point: It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.

6. Fire point: It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.

7. Pour point or freezing point: It is the temperature at which an oil will cease to flow when cooled.

Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig, in which O is the center of the journal and O′ is the center of the bearing.

Let D = Diameter of the bearing,

d = Diameter of the journal, and

l = Length of the bearing.

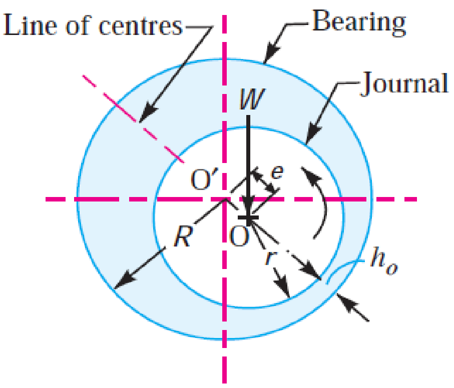


Fig: Hydrodynamic journal bearing.

The following terms used in hydrodynamic journal bearing are

1.Diametral clearance: It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note: The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also, the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. Radial clearance: It is the difference between the radii of the bearing and the journal.

Mathematically, radial clearance,

$$c_1 = R - r = \frac{D-d}{2} = \frac{c}{2}$$

3. Diametral clearance ratio: It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= \frac{c}{d} = \frac{D-d}{d}$$

4.Eccentricity: It is the radial distance between the centre (O) of the bearing and the displaced centre (O′) of the bearing under load. It is denoted by e.

5. Minimum oil film thickness: It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h0 and occurs at the line of centres as shown in Fig. Its value may be assumed as c / 4.

6. Attitude or eccentricity ratio: It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio,

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_0}{c_1} = 1 - \frac{h_0}{c_1} = 1 - \frac{2h_0}{c} \qquad \ldots (\because\ c_1 = c/2)$$

7. Short and long bearing: If the ratio of the length to the diameter of the journal (i.e. l / d) is less than 1, then the bearing is said to be short bearing. On the other hand, if l / d is greater than 1, then the bearing is known as long bearing.

Notes: 1. When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called square bearing.

2. Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large l/d ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of l / d may be taken as 1 to 2 for general industrial machinery. In crank shaft bearings, the l/ d ratio is frequently less than 1.

Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, i.e.

$$(i)\ \frac{ZN}{p}; \qquad (ii)\ \frac{d}{c}; \qquad \text{and} \qquad (iii)\ \frac{l}{d}$$

Therefore, the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

where μ = Coefficient of friction,

φ = A functional relationship,

Z = Absolute viscosity of the lubricant, in kg / m-s,

N = Speed of the journal in r.p.m.,

p = Bearing pressure on the projected bearing area in N/mm^2,

= Load on the journal ÷ l × d

d = Diameter of the journal,

l = Length of the bearing, and

c = Diametral clearance.

The factor ZN / p is termed as bearing characteristic number and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number (ZN / p) as obtained by McKee brothers (S.A. McKee and T.R. McKee) in an actual test of friction is shown in Fig. The factor ZN/p helps to predict the performance of a bearing.

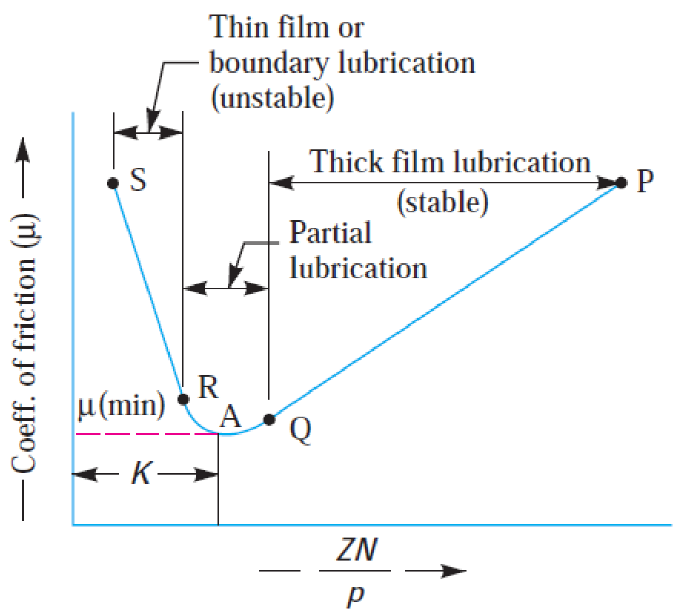


Fig: Variation of coefficient of friction with ZN/p.

The part of the curve PQ represents the region of thick film lubrication. Between Q and R, the viscosity (Z) or the speed (N) are so low, or the pressure (p) is so great that their combination ZN / p will reduce the film thickness so that partial metal to metal contact will result. The thin film or boundary lubrication or imperfect lubrication exists between R and S on the curve. This is the region where the viscosity of the lubricant ceases to be a measure of friction characteristics but the oiliness of the lubricant is effective in preventing complete metal to metal contact and seizure of the parts. It may be noted that the part PQ of the curve represents stable operating conditions, since from any point of stability, a decrease in viscosity (Z) will reduce ZN / p. This will result in a decrease in coefficient of friction (μ) followed by a lowering of bearing temperature that will raise the viscosity (Z). From Fig, we see that the minimum amount of friction occurs at A and at this point the value of ZN / p is known as bearing modulus which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. **This will result in high friction, wear and heating. In order to prevent such conditions, the**

bearing should be designed for a value of ZN / p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of ZN / p = 15 K may be used. From above, it is concluded that when the value of ZN / p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions. On the other hand, when the value of ZN / p is less than K, then the oil film will rupture and there is a metal to metal contact.

Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \qquad \text{... (when Z is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (i.e. l / d).

k= 0.002 for l / d ratios of 0.75 to 2.8.

Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known

as critical pressure or the minimum operating pressure of the bearing. It may be obtained by the following empirical relation, i.e.

Critical pressure or minimum operating pressure,

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) \text{N/mm}^2 \qquad \text{...(when Z is in kg / m-s)}$$

Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

For design purposes, its value is taken as follows :

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6 \qquad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu \times W \times V \text{ N-m/s or J/s or watts ...(i)}$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

W = Pressure on the bearing in N/mm^2 × Projected area of the bearing in mm^2 = $p(l \times d)$

V = Rubbing velocity in m/s = $\frac{\pi dN}{60}$, d is in meters, and

N = Speed of the journal in r.p.m.

After the thermal equilibrium, has been reached, heat will be dissipated at the outer surface of the bearing at the same rate at which it is generated in the oil film. The amount of heat dissipated will depend upon the temperature difference, size and mass of the radiating surface and on the amount of air flowing around the bearing. However, for the convenience in bearing design, the actual heat dissipating area may be expressed in terms of the projected area of the journal.

Heat dissipated by the bearing,

$Q_d = C \times A(t_b - t_a)$ J/s or W ... (Q 1 J/s = 1 W) ...(ii)

where C = Heat dissipation coefficient in W/m^2/°C,

A = Projected area of the bearing in m^2 = $l \times d$

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

The value of C depends upon the type of bearing, its ventilation and the temperature difference. The average values of C (in W/m^2/°C), for journal bearings, may be taken as follows:

For unventilated bearings (Still air)

C= 140 to 420 W/m^2/°C

For well ventilated bearings

C= 490 to 1400 W/m^2/°C

It has been shown by experiments that the temperature of the bearing (t_b) is approximately

mid-way between the temperature of the oil film (t_0) and the temperature of the outside air (t_a). In other words,

$$(t_b - t_a) = \frac{1}{2}(t_o - t_a)$$

Notes:

1. For well-designed bearing, the temperature of the oil film should not be more than 60°C, otherwise the viscosity of the oil decreases rapidly and the operation of the bearing is found to suffer. The temperature of the oil film is often called as the operating temperature of the bearing.

2. In case the temperature of the oil film is higher, then the bearing is cooled by circulating water through coils built in the bearing.

3. The mass of the oil to remove the heat generated at the bearing may be obtained by equating the heat generated to the heat taken away by the oil. We know that the heat taken away by the oil,

$$Q_t = m.S.t \text{ J/s or watts}$$

where m = Mass of the oil in kg / s,

S = Specific heat of the oil. Its value may be taken as 1840 to 2100 J / kg / °C,

t = Difference between outlet and inlet temperature of the oil in °C.

Design Procedure for Journal Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load,

the diameter and the speed of the shaft are known.

1. Determine the bearing length by choosing a ratio of $\frac{l}{d}$ from Table.

2. Check the bearing pressure, $p = \frac{W}{l \times d}$ from Table for probable satisfactory value.

3. Assume a lubricant from Table and its operating temperature (t_0). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.

4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table, to determine the possibility of maintaining fluid film operation.

5. Assume a clearance ratio c / d from Table.

6. Determine the coefficient of friction (μ) by using the relation.

7. Determine the heat generated by using the relation

8. Determine the heat dissipated by using the relation

9. Determine the thermal equilibrium to see that the heat dissipated becomes at least equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

Rolling Contact Bearings

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction. It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called antifriction bearings.

Advantages and Disadvantages of Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.

2. Ability to withstand momentary shock loads.

3. Accuracy of shaft alignment.

4. Low cost of maintenance, as no lubrication is required while in service.

5. Small overall dimensions.

6. Reliability of service.

7. Easy to mount and erect.

8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.

2. Low resistance to shock loading.

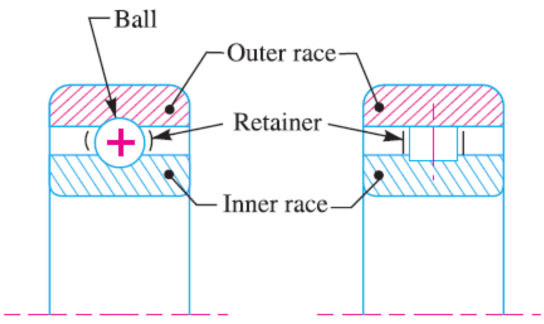
3. More initial cost.

4. Design of bearing housing complicated.

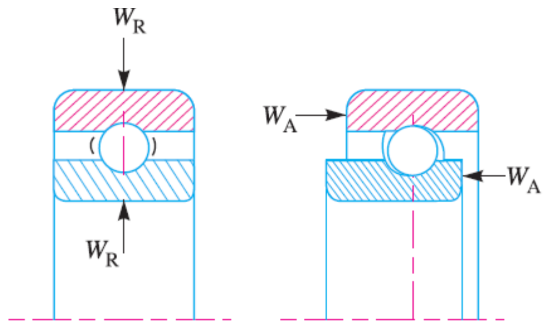
Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:

1. Ball bearings; and 2. Roller bearings.



(*a*) Ball bearing. (*b*) Roller bearing.



(*a*) Radial ball bearing. (*b*) Thrust ball bearing.

Fig. 1. Ball and roller bearings. **Fig.2.** Radial and thrust ball bearings.

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig 1. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as:

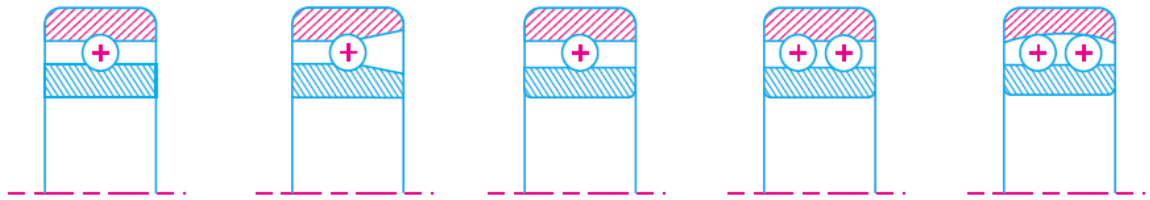
(a) Radial bearings, and (b) Thrust bearings.

The radial and thrust ball bearings are shown in Fig2 (a) and (b) respectively. When a ball bearing supports only a radial load (W_R), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig.2 (a). The action of thrust load (W_A) is to shift the plane of rotation of the balls, as shown in Fig2 (b). The radial and thrust loads both may be carried simultaneously.

Types of Radial Ball Bearings

Following are the various types of radial ball bearings:

1. Single row deep groove bearing. A single row deep groove bearing is shown in Fig(a).



(*a*) Single row deep groove. (*b*) Filling notch. (*c*) Angular contact. (*d*) Double row. (*e*) Self-aligning.

Fig: Types of radial ball bearings.

During assembly of this bearing, the races are offset and the maximum number of balls are placed between the races. The races are then centred and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds. The load carrying capacity of a ball bearing is related to the size and number of the balls.

2. Filling notch bearing. A filling notch bearing is shown in Fig(b). These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearings. The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position. Since this type of bearing contains larger number of balls than a corresponding unnotched one, therefore it has a larger bearing load capacity.

3. Angular contact bearing. An angular contact bearing is shown in Fig (c). These bearings have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load. The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. Double row bearing. A double row bearing is shown in Fig (d). These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings. The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. Self-aligning bearing. A self-aligning bearing is shown in Fig (e). These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing. If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of self-aligning bearings:

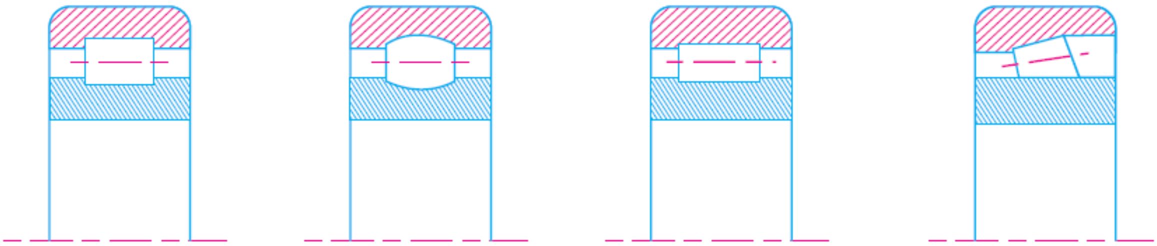
(a) Externally self-aligning bearing, and **(b)** Internally self-aligning bearing.

In an **externally self-aligning bearing,** the outside diameter of the outer race is ground to a spherical surface which fits in a mating spherical surface in a housing, as shown in Fig (e). In case of **internally self-aligning bearing,** the inner surface of the outer race is ground to a spherical surface. Consequently, the outer race may be displaced through a small angle without interfering with the normal operation of the bearing. The internally self-aligning ball bearing is interchangeable with other ball bearings.

Types of Roller Bearings

Following are the principal types of roller bearings:

1. Cylindrical roller bearings. A cylindrical roller bearing is shown in Fig. 27.6 (a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such type of bearings are used in high speed service.



(*a*) Cylindrical roller. (*b*) Spherical roller. (*c*) Needle roller. (*d*) Tapered roller.

2. Spherical roller bearings. A spherical roller bearing is shown in Fig (b). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order of ± $1½^0$ and when used with a double row of rollers, these can carry thrust loads in either direction.

3. Needle roller bearings. A needle roller bearing is shown in Fig(c). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

4. Tapered roller bearings. A tapered roller bearing is shown in Fig(d). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.

Life of a Bearing

The **life** of an individual ball (or roller) bearing may be defined as the number of revolutions (or hours at some given constant speed) which the bearing runs before the first evidence of fatigue develops in the material of one of the rings or any of the rolling elements. The **rating life** of a group of apparently identical ball or roller bearings is defined as the number of revolutions (or hours at some given constant speed) that 90 per cent of a group of bearings will

complete or exceed before the first evidence of fatigue develops (i.e. only 10 per cent of a group of bearings fail due to fatigue). The term **minimum life** is also used to denote the rating life. It has been found that the life which 50 per cent of a group of bearings will complete or exceed is approximately 5 times the life which 90 per cent of the bearings will complete or exceed. In other words, we may say that the average life of a bearing is 5 times the rating life (or minimum life). It may be noted that the longest life of a single bearing is seldom longer than the 4 times the average life and the maximum life of a single bearing is about 30 to 50 times the minimum life.

Dynamic Load Rating for Rolling Contact Bearings under Variable Loads

The approximate rating (or service) life of ball or roller bearings is based on the fundamental equation,

$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ revolutions}$$

or

$$C = W\left(\frac{L}{10^6}\right)^{1/k}$$

where L = Rating life,
C = Basic dynamic load rating,
W = Equivalent dynamic load, and
k = 3, for ball bearings,
= 10/3, for roller bearings.

The relationship between the life in revolutions (L) and the life in working hours (L_H) is given by

$$L = 60 \times N \times L_H \text{ revolutions}$$

where N is the speed in r.p.m.

Now consider a rolling contact bearing subjected to variable loads. Let W_1, W_2, W_3 etc., be the loads on the bearing for successive n_1, n_2, n_3 etc., number of revolutions respectively. If the bearing is operated exclusively at the constant load W_1, then its life is given by

$$L_1 = \left(\frac{C}{W_1}\right)^k \times 10^6 \text{ revolutions}$$

Fraction of life consumed with load W_1 acting for n_1 number of revolutions is

$$\frac{n_1}{L_1} = n_1 \left(\frac{W_1}{C}\right)^k \times \frac{1}{10^6}$$

Similarly, fraction of life consumed with load W_2 acting for n_2 number of revolutions is

$$\frac{n_2}{L_2} = n_2 \left(\frac{W_2}{C}\right)^k \times \frac{1}{10^6}$$

and fraction of life consumed with load W_3 acting for n_3 number of revolutions is

$$\frac{n_3}{L_3} = n_3 \left(\frac{W_3}{C}\right)^k \times \frac{1}{10^6}$$

But $\frac{n_1}{L_1} + \frac{n_2}{L_2} + \frac{n_3}{L_3} + ... = 1$

or $n_1\left(\frac{W_1}{C}\right)^k \times \frac{1}{10^6} + n_2\left(\frac{W_2}{C}\right)^k \times \frac{1}{10^6} + n_3\left(\frac{W_3}{C}\right)^k \times \frac{1}{10^6} + ... = 1$

$$\therefore n_1(W_1)^k + n_2(W_2)^k + n_3(W_3)^k + ... = C^k \times 10^6 \quad ...(i)$$

If an equivalent constant load (W) is acting for n number of revolutions, then

$$n = \left(\frac{C}{W}\right)^k \times 10^6$$

$$\therefore \qquad n\,(W)^k = C^k \times 10^6 \qquad \ldots(ii)$$

where $\qquad n = n_1 + n_2 + n_3 + \ldots..$

From equations (i) and (ii), we have

$$n_1\,(W_1)^k + n_2\,(W_2)^k + n_3\,(W_3)^k + \ldots.. = n\,(W)^k$$

$$\therefore \qquad W = \left[\frac{n_1\,(W_1)^k + n_2\,(W_2)^k + n_3\,(W_3)^k + \ldots.}{n}\right]^{1/k}$$

Substituting $n = n_1 + n_2 + n_3 + \ldots\ldots.$, and $k = 3$ for ball bearings, we have

$$W = \left[\frac{n_1\,(W_1)^3 + n_2\,(W_2)^3 + n_3\,(W_3)^3 + \ldots.}{n_1 + n_2 + n_3 + \ldots.}\right]^{1/3}$$

The above expression may also be written as

$$W = \left[\frac{L_1\,(W_1)^3 + L_2\,(W_2)^3 + L_3\,(W_3)^3 + \ldots}{L_1 + L_2 + L_3 + \ldots.}\right]^{1/3}$$

Reliability of a Bearing

The rating life is the life that 90 per cent of a group of identical bearings will complete or exceed before the first evidence of fatigue develops. The reliability (R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. The relation between the bearing life and the reliability is given as

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \qquad \text{or} \qquad \frac{L}{a} = \left[\log_e\left(\frac{1}{R}\right)\right]^{1/b} \qquad \ldots(i)$$

where L is the life of the bearing corresponding to the desired reliability R and a and b are constants whose values are

a = 6.84, and b = 1.17

If L_{90} is the life of a bearing corresponding to a reliability of 90% (i.e. R_{90}), then

$$\frac{L_{90}}{a} = \left[\log_e\left(\frac{1}{R_{90}}\right)\right]^{1/b} \quad \text{...}(ii)$$

Dividing equation (i) by equation (ii), we have

$$\frac{L}{L_{90}} = \left[\frac{\log_e (1/R)}{\log_e (1/R_{90})}\right]^{1/b} = 6.85\,[\log_e (1/R)]^{1/1.17} \quad \text{... } (\because\ b = 1.17)$$

This expression is used for selecting the bearing when the reliability is other than 90%.

Materials and Manufacture of Ball and Roller Bearings

Since the rolling elements and the races are subjected to high local stresses of varying magnitude with each revolution of the bearing, therefore the material of the rolling element (i.e. steel) should be of high quality. The balls are generally made of high carbon chromium steel. The material of both the balls and races are heat treated to give extra hardness and toughness.

The balls are manufactured by hot forging on hammers from steel rods. They are then heat-treated, ground and polished. The races are also formed by forging and then heat-treated, ground and polished.

Lubrication of Ball and Roller Bearings

The ball and roller bearings are lubricated for the following purposes:

1. To reduce friction and wear between the sliding parts of the bearing,

2. To prevent rusting or corrosion of the bearing surfaces,

3. To protect the bearing surfaces from water, dirt etc., and

4. To dissipate the heat.

In general, oil or light grease is used for lubricating ball and roller bearings. Only pure mineral oil or a calcium-base grease should be used. If there is a possibility of moisture contact, then potassium or sodium-base greases may be used. Another additional advantage of the grease is that it forms a seal to keep out dirt or any other foreign substance. It may be noted that too much oil or grease cause the temperature of the bearing to rise due to churning. The temperature should be kept below 90ºC and in no case a bearing should operate above 150ºC.

Unit – II

Piston & Cylinder

* Design of a piston :-

1. piston head (or) Crown
2. Radial ribs
3. Piston Rings
4. piston barral
5. piston skirt
6. piston Pin

* 1. Piston Head :-

Treating the piston head has a flat circular plate of uniform thickness.

Pg. no 361 Equation no. 18.18a —— ①

Thickness of piston head

t or $t_1 = 0.43\, D\sqrt{P/\sigma_t}$ —— ①

Treating the piston has flat circular plate, its thickness given by

Pg. no 361 Equation no. 18.19 —— ②

Thickness of Crown $t_1 = \dfrac{D^2 q}{1600k\,(T_c - T_e)}$

The heat flowing through the piston head may be determined

$$q = C \times HCV \times m \times B.P$$

$$C = 0.05$$

HCV → disel → 45×10^3 kJ/kg

→ petrol → 47×10^3 kJ/kg

The thickness of the piston head is calculated by using equation (1) & (2) and taking large value of thickness of piston head.

2. Radial Ribs :-

1. The thickness of Ribs may be taken as

$$\frac{t_H}{3} \text{ to } \frac{t_H}{2}$$

2. $t_H < 6mm$ (no Ribs are required to strengthen the piston head against gas loads)

3. $t_H > 6mm$ (Suitable no. of Ribs at the centre line of the boss extending around the skirt should be provided to distribute the side thrust from the

Connecting rod and thus to prevent distortion of the Skirt.

3. Piston Rings :-

Pg no :- 363 Eq no :- 18.27 upto 18.31

4. Piston Barrel :-

Pg no 362 Equ. no :- 18.20

$t_3 = 0.03D + b + 4.5$ mm

Pg no 362 Equ. no :- 18.21

$t_4 = 0.25\, t_3$ to $0.35\, t_3$

5. Piston Skirt :-

The Side thrust on the Cylinder Liner is usually taken as $\frac{1}{10}$ of the Maximum gas load on the piston

$$R = \frac{P}{10} \quad \text{—} (1)$$

P = Max. gas load on the piston

$$P = p \times \frac{\pi}{4} D^2$$

The Side thrust also given

R = bearing pressure × projected area of the piston Skirt

$$= P_b \times D \times l \quad \text{—— (2)}$$

where,

l : length of the piston skirt

from (1) & (2) the length of the piston skirt is determined.

Total length of the piston

$$L = l + (5h + 4t_{band}) + t_g$$

The length of the piston varies between D and 1.5D L = D and 1.5 D

6. **Piston Pin:-**

Pg. no:- 362 Equ. no 18.24

The diameter of the piston pin

$$d = \frac{\frac{\pi}{4} D^2 \, p_{max}}{4 P_b}$$

pg. no :- 363 Equ. no :- 18.26

$$\sigma_b = \frac{F_p D}{8Z}$$

$$Z = \frac{\pi}{32} d^3$$

$$F_p = p \times \frac{\pi}{4} D^2$$

Problem:

1. Design a Cast Iron piston for a single acting four stroke engine for the following data.
Cylinder bore : 100mm, Stroke 125mm,
Maximum gas pressure $5\,N/mm^2$, Indicated Mean effective pressure $0.75\,N/mm^2$, Mechanical efficiency 80%, fuel consumption 0.15 kg per brake power per hour. HCV = 42×10^3 kJ/kg
Speed = 2000 RPM any other data required for the design may be assumed.

A:- given data,

$D = 100mm$

$L = 125\,mm$

$P = 5\,N/mm^2$

$P_m = 0.75\,N/mm^2$

$M = 0.15$ kg per Bp per hour

$H.C.V = 42\times10^3\,kJ/kg$

$N = 2000$ RPM

$\eta_m = 0.80$

Piston Head :-

(i) The thickness of the piston head

$$t_H \text{ or } t_4 = 0.43D\sqrt{P/\sigma_t}$$

$$\sigma_t = 38\ MN/m^2$$

$$= 38\ N/mm^2$$

$$t_H = 0.43 \times 100\sqrt{5/38}$$

$$= 15.59\ mm$$

(ii) The thickness of the Crown

$$t_H \text{ or } t_1 = \frac{D^2 q}{1600k(T_c - T_e)}$$

$q = 32000$ to 128000 for C.I

$= 128000$

$k = 460$

$T_c - T_e = 222°C$

$$t_H = \frac{100^2 \times 128000}{1600 \times 460 (222)}$$

$$= \frac{128 \times 10^7}{}$$

$$= 7.83\ mm$$

Larger value of t_H (or) $t_1 = 15.59\ mm$

2. Radial ribs :-

$\frac{t_H}{3}$ or $\frac{t_H}{2}$

Thickness of Radial ribs $\Rightarrow \frac{t_H}{2} = \frac{15.59}{2}$

$= 7.795 mm$

3. Piston Rings :- Po = pg no 366 Table no 18.6

$t_r = D\sqrt{3p_r/\sigma} = 100\sqrt{3\times0.03090/110}$

$= 2.90 mm$

p_r = diesel engine = $0.03090 N/m^2$

$\sigma = 110 N/m^2$

depth

(a) $h = 0.7 \times t_r$ to $t_r = 0.7 \times 2.90$ to 2.90

$= 2.03$ to 2.90

$= 2.90 mm$

Minimum depth

$h = \frac{D}{10i} = \frac{100}{10\times5} = 2 mm$

The total depth piston Rings

$h_{total} = \frac{D}{7} + 6 = \frac{100}{7} + 6 = 20.28 mm$

The distance from the top to the first groove

$t_g = t_1$ to $1.2t = 15.59$ to 18.708

$= 18.70mm$

The land between the Ring grooves

t_{land} = hor Slightly less than h

$= 2.90mm$

4. piston Barrel :-

The thickness of the Barrel

$t_3 = 0.03D + b + 4.5mm$

$b = t_r + 0.4 = 2.9 + 0.4 = 3.3mm$

$t_3 = 0.03 \times 100 + 3.3 + 4.5$

$= 10.8 MM$

The wall thickness towards the open end of the Piston

$t_4 = 0.25 t_3$ to $0.35 t_3$

$= 2.7$ to 3.78 mm

$t_4 = 3.7mm$

5) Piston Skirt :-

$$R_1 = \frac{P}{10}$$

$$\frac{R \cdot P \times \frac{\pi}{4} \times D}{10} = \frac{5 \cdot 7 \times \frac{\pi}{4} \times 100^2}{10}$$

$$= 3926 \cdot 9 \, N \quad -(1)$$

$$R = P_b \times D \times L$$

$P_b = 0.25 \; N/mm^2$ for low Speed engines

$= 0.5 \; N/mm^2 \longrightarrow$ high speed engines.

$$R = 0.5 \times 100 \times L \quad -(2)$$

(1) & (2)

$$3926 \cdot 9 = 0.5 \times 100 \times L$$

$$L = 78.52$$

$$L = L + (5h + 4t \text{ end}) + t_g$$

$$= 78.52 + [(5 \times 2.90) + (4 \times 2.90)] + 18.70$$

$$= 123.3 \text{ mm}$$

The length of the piston varies between D and 1.5

$$L = 100 \text{ to } 1.5 \times 100 = 100 \text{ to } 150 \text{ mm}$$

6. piston pin:

The diameter of the piston pin

$$\frac{\pi}{4} d^2 = \frac{\pi D^2 P_{max}}{4\, l_1 P_b} = \frac{\pi \times 100^2 \times 5}{4 \times 2d \times 150}$$

$$t_1 = k_1 d = 2d$$

$$120 d^2 = \pi \times 100^2 \times 15$$

$$d = \sqrt{\frac{\pi \times 100^2 \times 15}{120}}$$

$$= \cancel{198.16}\ mm \approx 36.18\ mm$$

$$\sigma_b = \frac{F_p D}{8 Z}$$

$$= \frac{p \times \frac{\pi}{4} D^2 \times D}{8 \times \frac{\pi}{32} d^3}$$

$$= \frac{5 \times \frac{\pi}{4} \times 100^2 \times 100}{8 \times \frac{\pi}{32} \times 36.18^3}$$

$$= 105\ N/mm^2$$

2. Design a trunk type cast iron piston for an i.c engine for the following data diameter of cylinder 100mm stroke 150mm

Maximum Ex closion pressure, 3.5 mpa. Maximum permissible tension for Cast iron for the design of head thickness is 30 mpa and flex tural stresses for the pin may be taken from 90 to 120 mpa. Piston pin must be hardend and Ground and should turn in prosphor bronze burhing bearing pressure should not Exceed 20Mpa

✱ given data

$D = 100$ MM

$L = 150$ MM

$P = 3.5$ Mpa

$\sigma_t = 30$ Mpa

90 to 120 MpA

20 Mpa

1. piston head :-

$$t_1 \text{ (or) } t_H = 0.43 D \sqrt{\frac{P}{\sigma_t}}$$

$$= 0.43(100) \sqrt{\frac{3.5}{30}}$$

$$= 14.68 \text{ mm}$$

t_H or t_1 $= \sqrt{\dfrac{D^2 q}{1600\,k\,(T_c - T_e)}}$

$q = 128000$

$k = 460$

$T_c - T_e = 222$

$t_H = \sqrt{\dfrac{100^2 \times 128000}{1600 \times 460\,(222)}}$

$= 7.83\ mm$

larger value t_H or t_1 $= 14.68$ mm

2. Radial Ribs:-

thickness of Ribs

$\dfrac{t_H}{3}$ to $\dfrac{t_H}{2}$

$\dfrac{14.68}{3}$ to $\dfrac{14.68}{2}$

4.89 to 7.34

$t = 7.34$ mm

3. piston Rings :-

$t_r = D\sqrt{\dfrac{3 P_r}{\sigma}}$

$P_r = 0.02746$

$\sigma = 110\ MN/m^2$

D_{oo} ($0.0548\ ...$) www.FirstRanker.com

$$= 2.73\ mm$$

depth of (h) piston Ring

$$h = 0.7 t_r \text{ to } t_r$$

$$= 1.911 \text{ to } 2.73$$

$$h = 2.73\ mm$$

The maximum depth of the piston Rings

$$h = \frac{D}{10 i}$$

$$= \frac{100}{10 \times 4}$$

$$= 2.5\ mm$$

$$h_{total} = \frac{D}{5.5} \text{ for petrol}$$

$$= \frac{100}{5.5} = 18.18$$

The distance from the top the first groove $t_g = t_1$ to $1.2 t_1$

$$= 14.68 \text{ to } 17.616$$

$$t_g = 17.61\ mm$$

The lands between the Ring grooves

t land = h or slightly less than h

$= 2.73$ or 2.5

$= 2.73$ MM.

4. piston barrel :-

The maximum thickness of the piston barrel

$t_3 = 0.03D + b + 4.5$ mm

$b = 2.73 + 0.4$ mm

$= 3.13$ mm

$t_3 = (0.03 \times 100) + 3.13 + 4.5$

$= 10.63$ mm

$t_4 = 0.25 t_3$ to $0.35 t_3$

$= 2.6575$ to 3.7205 mm

$= 3.72$ mm

5. piston Skirt :-

$R = \frac{P}{10}$

$$R = \frac{P \times \frac{\pi}{4} D^2}{10} = \frac{3.5 \times \frac{\pi}{4} \times 100^2}{10}$$

$= 2748.89\,N$ ——— (1)

$R = P_D \times l \times D =$

$P_b = 0.5$ for high Speed for petrol

$R = P_b \times D \times l$

$= 0.5 \times 100 \times l \longrightarrow$ (2)

$2748.89 = 0.5 \times 100 \times l$

$$l = \frac{2748.89}{0.5 \times 100} = 54.97\,mm$$

$l = l + 5h + 4t$ bnd $+ t_g$

$= 54.9 + (5 \times 2.73) + 4\,(2.73)$

$+ 17.61$

$= 97.08\,mm$

6. piston pin :-

$$d = \frac{\pi D^2 p_{max}}{4 l_1 P_b}$$

$l = k_1 d = 1.5 \times d$

$P_b = 15.7\,MPa$

$$d = \frac{\pi (100)^2 \times 3.5}{4 \times 1.5 d \times 15.7}$$ $d = 34.16\,mm$

$$\sigma_b = \frac{\frac{P D}{8}}{Z}$$

$$= \frac{p \times \frac{\pi}{4} D^2 \times D}{8 \times \frac{\pi}{32} d^3}$$

$$= \frac{3.5 \times \frac{\pi}{4} \times (100)^2 \times 100}{8 \times \frac{\pi}{32} \times (39.16)^3}$$

$$= 87.80 \text{ Mpa}.$$

Design of the piston pin is with in the limit.

* Design of Cylinder :-

1. Thickness of Cylinder Wall

$$t = \frac{P \times D}{2\sigma_c} + C$$

P - Maximum pressure inside the cylinder

D - inside diameter of the cylinder bore

σ_c - circumferential or hoop stress for the cylinder Material 35 to 100 mpa.

C - allowance for Reboring. it is depending up on the cylinder bore for IC engine thickness of cylinder wall may also may be obtain

$t = 0.045D + 1.6mm$

2. Bore & length of cylinder

Power produced inside the engine cylinder that is Indicated Power.

$$I.P = \frac{P_m \times l \times A \times n}{60} \text{ watts}$$

from this Expression (the bore (D) and length of the stroke (l) is determined. the length of the stroke generally taken as 1.25D to 2D

length of cylinder $L = 1.15 \times$ stroke length
$= 1.15\, l$.

Mechanical efficiency $\eta_m = \frac{B.P}{I.P}$

The Maximum gas pressure P may be taken as 9 to 10 times Mean effective pressure (P_m)

3. Cylinder flanges & studs

The diameter of studs are bolt may be obtained equating the gas load due to

The Maximum pressure in the cylinder to the Resisting force offered by all the studs are bolts. n_s = no. of studs

$n_s = (0.01D+4)$ to $(0.02D+4)$

σ_t = Allowable tensile stress for Materials of studs (or) bolts

= 35 to 70 mpa

d_c = core diameter

The nominal or Major diameter of the stud or bolt Usually lies between $0.75 t_f$ to t_f

t_f → Thickness of flange $d = 0.75 t_f$ to t_f

The distance of the flange from the centre of hold for the stud or bolt should not less then d+6mm are not more than 1.5d.

In Order to Make a leak proof Joint the pitch of the studs are bolts should lie between $19\sqrt{d}$ to $28.5\sqrt{d}$.

Cylinder head :—

thickness of cylinder head

$$t_h = D\sqrt{\frac{C.P}{\sigma_c}}$$

σ_c = allowable circumferential stress
= 30 to 50 mpa.

C = constant = 0.1

Pitch circle diameter $D_p = D + 3d$.

* Problems

*1 A 4 stroke diesel engine has the following specifications. brake power 5kW integrated mean effect pressure 0.357 N/mm² speed = 1200 Rpm. Mechanical efficiency = 80%. determine 1. bore and length of cylinder 2. thickness of cylinder head 3. size of studs for the cylinder head.

Ans :- Given data :—

BP = 5kW

$P_m = 0.35 N/mm^2$

N = 1200 RPM.

$\Rightarrow n = 600$.

$\eta_m = 80\%$

1. Bore and length of the cylinder

$$I.P = \frac{P_m . L . A N}{60}$$

$$\eta_m = \frac{Bp}{IP}$$

$$Ip = \frac{P_m \times L \times A \times N}{60}$$

$$\eta_m \; Ip = \frac{B.P}{\eta_m}$$

$$= \frac{5 \times 10^3}{0.8}$$

$$= 6250W$$

$$6250 = \frac{0.35 \times 2D \times \frac{\pi}{4} D^2 \times 600}{60 \times 1000}$$

$375 \times 10^6 = 210 \times 2 x$

$$D = 104mm$$

length of cylinder, $L = 1.15 l$

$$= 1.15 \times 2D$$

$$L = 1.15 \times 2 \times 104$$

$$L = 240mm.$$

2. Thickness of cylinder head

$t_H = D\sqrt{\frac{C \cdot P}{\sigma_c}}$

$P_{\pi} = 10 \times 0.35 = 3.5\, N/mm^2$

$\sigma_c = 30 \times 50\, mpa$

$\sigma_c = 40\, mpa$

($C = 0.1 \rightarrow$ Constant)

$$t_H = 104\sqrt{\frac{0.1 \times 3.5}{40}}$$

$= 9.7\, mm$

3. Size of studs for the cylinder head.

$$\frac{\pi}{4} D^2 p = n_s \frac{\pi}{4} d_c^2 \sigma_t$$

$n_s = (0.01D + 4) + 0(0.02D + 4)$

$= 0.01 \times 104 + 4 = 5.04 \approx 5$

$$\frac{\pi}{4} D^2 p = n_s \, \pi/4 \, d_c^2 \sigma_t$$

$$\frac{\pi}{4} (104)^2 \times 3.5 = 5 \times \frac{\pi}{4} d_c^2 \times 50$$

$d_c = 12.3\, mm$

Assume $d_c = 0.84 d$.

$12.3 = 0.84 d$

$$d = \frac{12.3}{0.84} = 14.6\, mm$$

Pitch diameter $D_p = D + 3d$

$= 104 + 3(14.6)$

$= 147.8$

Pitch of studs $= \frac{\pi D_p}{n_s}$

$= \frac{\pi \times 147.8}{5}$

$= 92.86 \text{ mm}$

for leak proof joint pitch of studs lie between $19\sqrt{d}$ to $28.5\sqrt{d}$

$19\sqrt{14.6}$ to $28.5\sqrt{14.6}$

72.5 to 108.8

The design cylinder is safe.

2. A four stroke IC engine has the following specification brake power 7.5 kW. Speed 1000 rpm. Indicated mean effective pressure 0.35 Mechanical efficiency = 80% determine

1. Dimensions of the cylinder. if the length of stroke 1.4 times the bore of the cylinder

2. Wall thickness of the cylinder if hoop stress is 35 MPa

3. Thickness of cylinder head and size of studs when permissible stress for cylinder head and stud materials are 45mpa & 65 mpa.

Sol:- Given data,

$B.P = 7.5 \, KW$

$l = 1.4D$

$\sigma_c = 35 \, mpa$

$P_m = 0.35 \, N/mm^2$

$N = 1000 \, Rpm$

$\eta_m = 80\%$

$n = \frac{N}{2}$

$\eta_m = \frac{B.P}{I.P}$

$I.P = \frac{7.5 \times 10^3}{0.8} = 9375$

$I.P = 9375$

$I.P = \frac{P_m \times l \times A \times n}{60}$

$9375 = \frac{0.35 \times 1.4D \times \frac{\pi}{4} D^2 \times 500}{60 \times 1000}$

$D = 142.8 \, mm.$

Length of cylinder $L = 1.5l$

$= 1.15 \times 1.4D$

$= 1.15 \times 1.4 \times 142.8$

$L = 230.6mm.$

2. Wall thickness

$$t = \frac{P_D}{2\sigma_c} + C$$

C = constant = 6.3

$$t = \frac{3.5 \times 142.8}{2 \times 35} + 6.3$$

$= 13.4mm.$

3. Thickness of cylinder head.

$$t_h = D\sqrt{\frac{C.P}{\sigma_c}}$$

$\sigma_c = 45mpa$ $C = 0.1$

$$t_h = 142.8\sqrt{\frac{0.1 \times 3.5}{45}}$$

$t_h = 12.6mm$

Studs for cylinder head.

$$\frac{\pi}{4} D^2 p = n_s \frac{\pi}{4} d_c^2 \sigma_t$$

$n_s = 0.01 \times 142.8 + 4$

$n_s = 5.42 \simeq 6$

$\sigma_t = 65 mpa$

$\frac{\pi}{4} (142.8)^2 \times 35 = 16 \times \frac{\pi}{4} d_c^2 \times 65$

$d_c = 13.53 mm.$

Assume $d_c = 0.84d.$

$13.53 = 0.84d.$

$d = 16.1 mm$

$D_p = D + 3d$

$= 142.8 + 3(16.1)$

$= 191.2 mm.$

Pitch of stud $= \frac{\pi D_p}{n_s}$

$= \frac{\pi \times 191.2}{6}$

$= 100.1 mm$

for leak proof Joint Pitch of studs lie between $19\sqrt{d}$ to $28.5\sqrt{d}$.

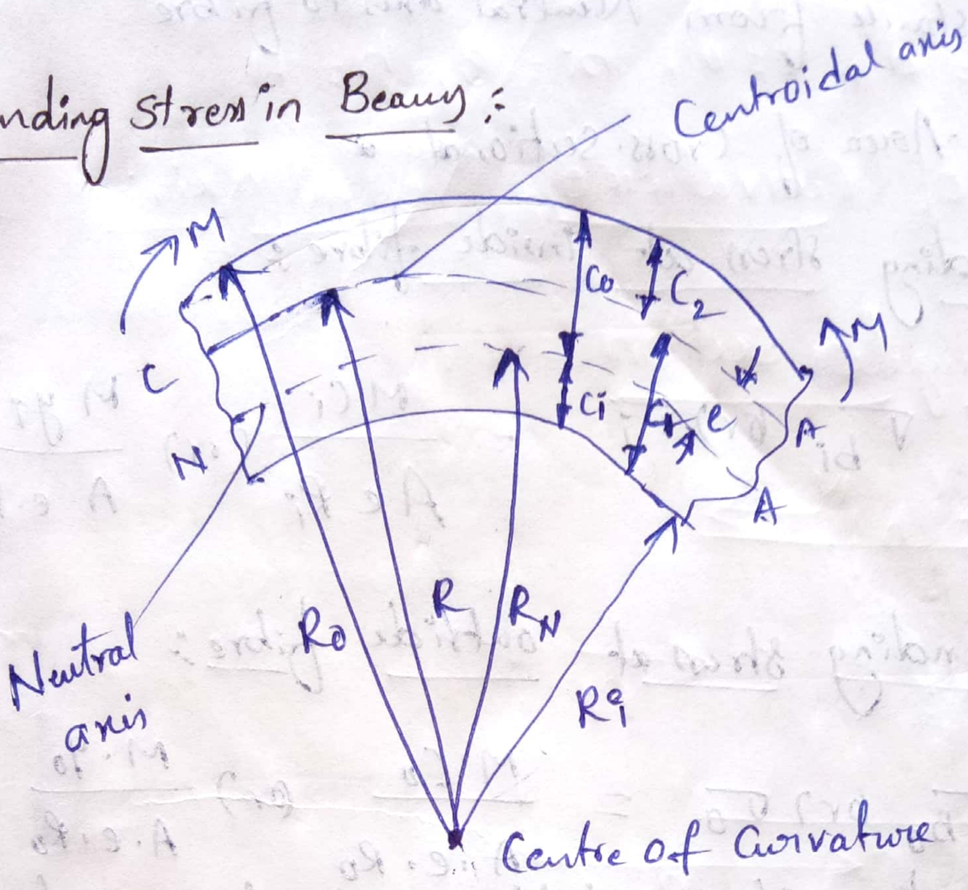
$19\sqrt{16.1}$ to $28.5\sqrt{16.1}$

76.2 to 114.35

the design of cylinder is safe.

Design of Curved beam

Bending stress in Beams :-



R_i = Radius of Curvature of inside fibre

R_o = Radius of Curvature of outside fibre

R_N = Radius of Curvature of Neutral axis

R = Radius of Curvature of Centroidal axis

$C_i = R_N - R_i$ = distance from Neutral axis to inside fibre

$C_o = R_o - R_N$ = distance from Neutral axis to outside fibre.

$C_1 = R - R_i$ = Distance from Central axis to inside fibre

$C_2 = R_o - R$ = Distance from Central axis to outside

e = Distance b/w centroidal axis & Neutral axis

M = bending Moment

y = Distance from Neutral axis to fibre

A = Area of Cross-sectional

Max bending stress at inside fibre :-

$$\sigma_{bi} \text{ (or) } \sigma_i = \frac{M C_i}{A e R_i} \text{ (or) } \frac{M y_i}{A \cdot e \cdot R_i}$$

Max bending stress at outside fibre :-

$$\sigma_{bo} \text{ (or) } \sigma_o = \frac{M C_o}{A \cdot e \cdot R_o} \text{ (or) } \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

the bending stress at the inside fibre is tensile while the bending stress & the outside fibre is compressive. In symmetrical sections the max bending stress will always occur at the inside fibre in symmetrical section. Resultant stresses on the section

$$\sigma = \sigma_d \pm \sigma_b$$

σ_d = axial (or) direct stress

Resultant stresses

Inner surface $= \sigma_t + \sigma_i \rightarrow$ Tensile

Outer surface $= \sigma_t - \sigma_o$ — Compressive

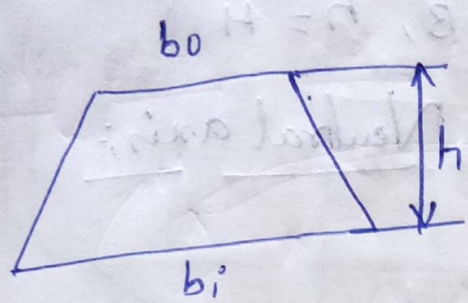
Bending stress :

Inner fibre $\sigma_i = \dfrac{M C_i}{A e R_i} \rightarrow$ Tensile

outer fibre $\sigma_o = -\dfrac{M C_o}{A e R_o}$ — Compressive

Radius of curvature of the Centroidal axis:

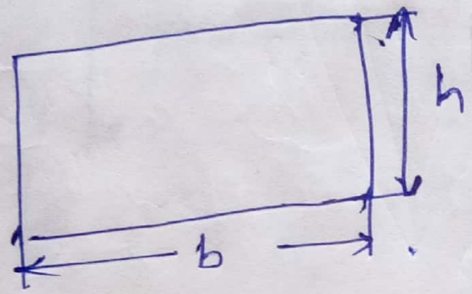
1. Trapezoidal section:



Area $A = \left(\frac{b_i + b_o}{2}\right) h$

$R = R_i + \frac{h(b_i + 2b_o)}{3(b_i + b_o)}$

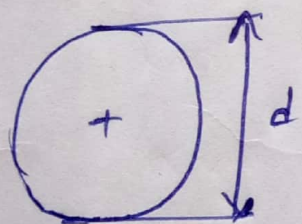
2. Rectangular Section:



Area $A = bh$

$R = R_i + h/2$

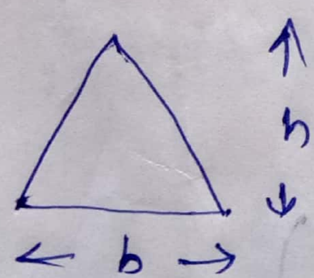
3. Circular Section:



$A = \pi/4\, d^2$

$R = R_i + d/2$

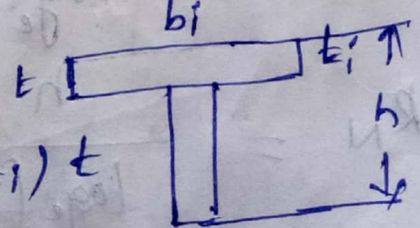
4. Triangular Section:



$A = 1/2\, bh$

$R = R_i + h/3$

5. T-section:



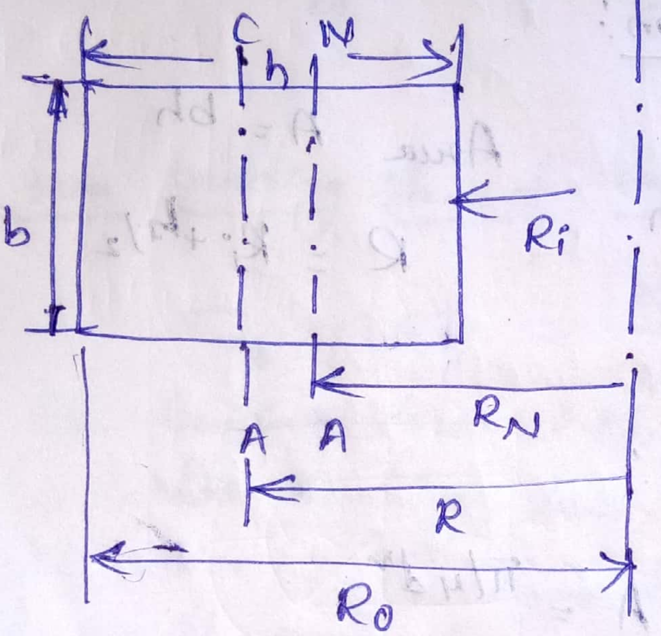
$A = b_i t_i + (h - t_1) t$

$$R = R_i + \frac{\frac{1}{2}h^2 t + \frac{1}{2}t_i^2 (b_i - t)}{ht + t_i (b_i - t)}$$

$t_i = d,\ t = a,\ b_i = B,\ h = H$

Radius of Curvature of Neutral axis:-

1) Rectangular section:



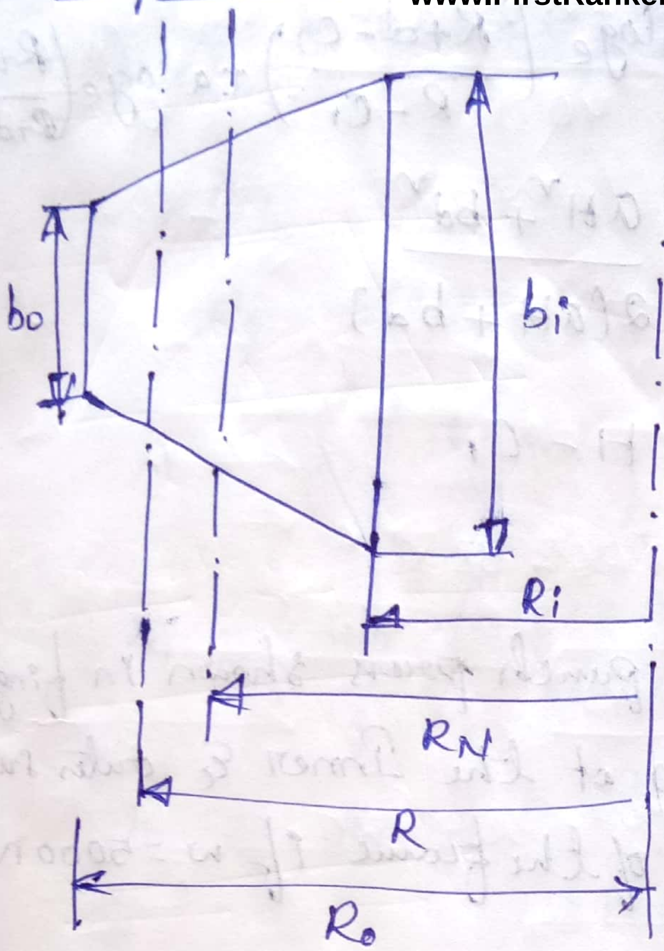
$$R_N = \frac{h}{\log_e\left(\frac{R+c}{R-c}\right)}$$

$$c = h/2$$

$$R_N = \frac{h}{\log_e\left(\frac{h/2 + R}{R - h/2}\right)}$$

$$R_N = \frac{h}{\log_e\left(\frac{2R+h}{2R-h}\right)}$$

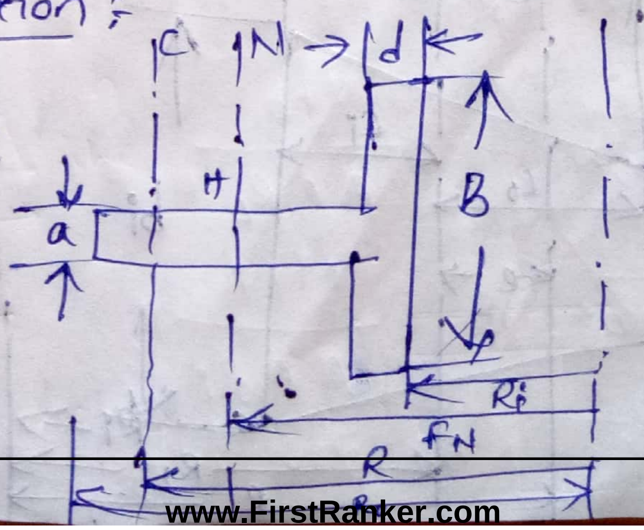
2. Trapezoidal section :-



$$R_N = \frac{A}{\left[\frac{b_i(R+C_2) - b_o(R-C_1)}{h}\log_e\left(\frac{R+C_2}{R-C_1}\right) - (b_i - b_o)\right]}$$

$$C_1 = \frac{h\,(3b_i + 2b_o)}{3\,(2b_i + b_o)}$$

3. T- section :-



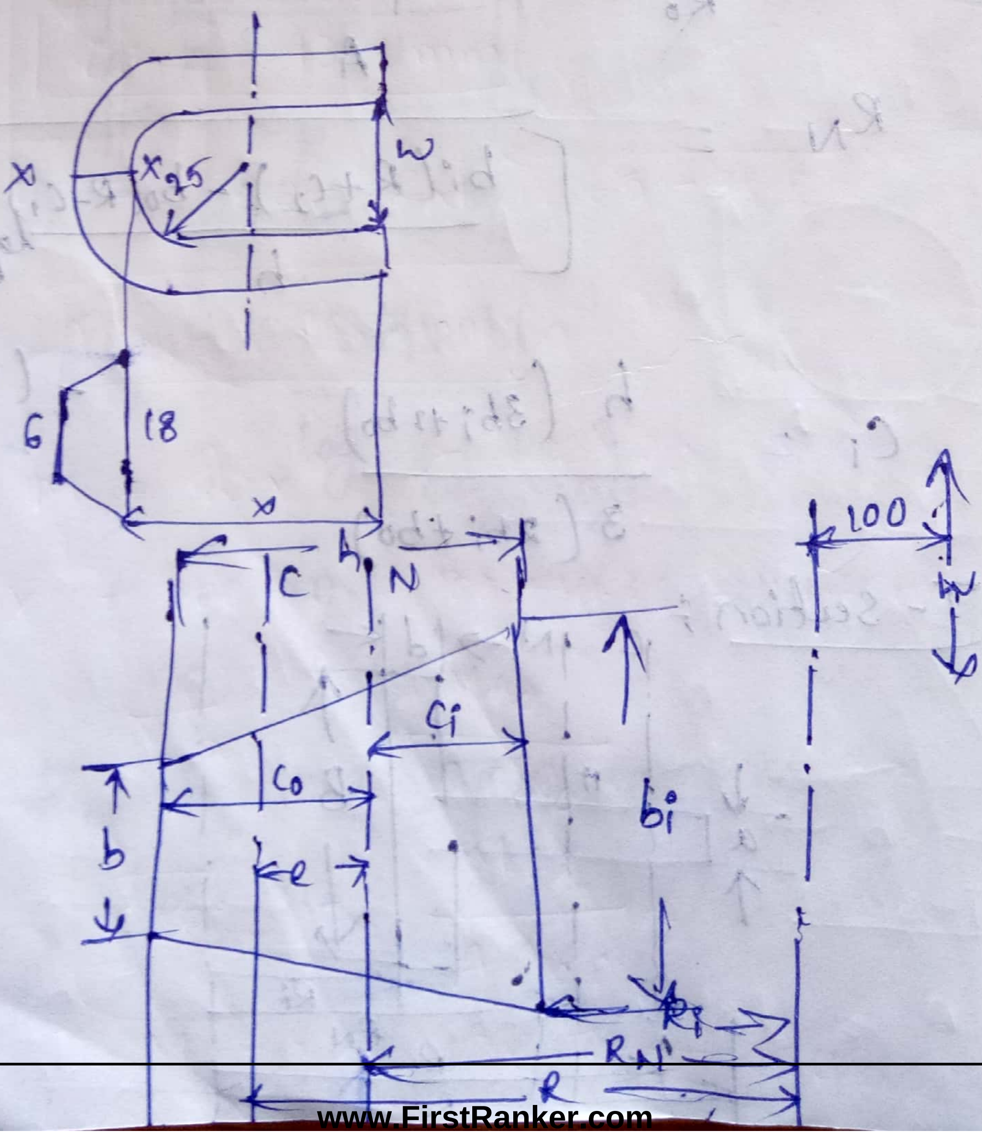
$$R_N = \frac{A}{b \log_e\left(\frac{R+d-c_1}{R-c_1}\right) + a \log_e\left(\frac{R+c_2}{R+d-c_2}\right)}$$

$$c_1 = \frac{aH^2 + bd^2}{2(aH + bd)}$$

$$c_2 = H - c_1$$

Problems:

1) The frame of a punch press shown in figure. find the stresses at the inner & outer surface at section x - x of the frame if w = 5000 N



$b_i = 18\,mm$

$b_o = 6\,mm$

$h = 40\,mm$

$R_i = 25\,mm$

$R_o = R_i + 40 = 65\,mm$

$$A = h\left(\frac{b_i + b_o}{2}\right)$$

$$A = 40\left(\frac{18+6}{2}\right) = 480\,mm^2$$

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)}$$

$$= 25 + \frac{40\,(18 + 2(6))}{3(18+6)}$$

$$\boxed{R = 41.66\,mm}$$

$$R_N = \frac{A}{\left[\frac{b_i(R+C_2) - b_o(R-C_1)}{h}\log\frac{R+C_2}{R-C_1}\right] - (b_i - b_o)}$$

$C_1 = R - R_i = 41.66 - 25$

$= 16.66\,mm$

$C_2 = R_o - R = 65 - 41.66 = 23.34\,mm$

480

$$R_N = \frac{480}{\left[\frac{18(41.66+23.4)-6(41.66-16.66)}{40} \cdot \log\left(\frac{41.66+23.4}{41.66-16.66}\right) - (18-6)\right]}$$

$$\boxed{R_N = 38.8\,mm}$$

$$e = R - R_N = 41.66 - 38.8$$

$$\boxed{e = 2.86\,mm}$$

$$C_i = R_N - R_i = 38.8 - 25$$

$$\boxed{C_i = 13.8\,mm}$$

$$C_o = R_o - R_N = 65 - 38.8$$

$$\boxed{C_o = 26.2\,mm}$$

$\Rightarrow M = W x$

$x = 100 + R = 100 + 41.66 = 141.66\,mm$

$$\sigma_{bo} = \frac{M C_o}{A e R_o}$$

$$\sigma_{bi} = \frac{M C_i}{A e R_i}$$

$M = 5000 \times 141.66 = 708{,}300$

$$\sigma_{bo} = \frac{708300 \times 26.2}{480 \times 2.286 \times 65}$$

$$\boxed{\sigma_{bo} = 207.96 \ N/mm^2}$$

$$\sigma_{bi} = \frac{708300 \times 13.8}{480 \times 2.86 \times 25}$$

$$\boxed{\sigma_{bi} = 284.8 \ N/mm^2}$$

Resultant stresses on Inner surface

$$\sigma = \sigma_t + \sigma_{bi}$$

$$\sigma_t = W/A = \frac{5000}{480} = 10.416 \ N/mm^2$$

Resultant stress on $\sigma_i = 10.416 + 284.8$

$$\boxed{\sigma_i = 295.216 \ N/mm^2 \ (\text{Tensile})}$$

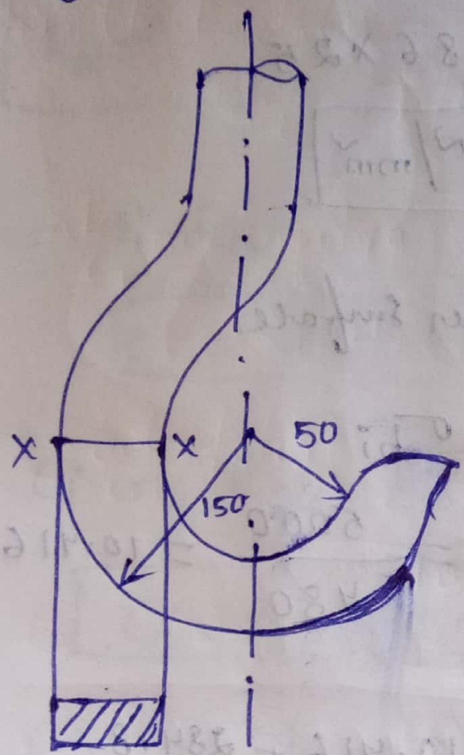
Resultant stresses on outer surface

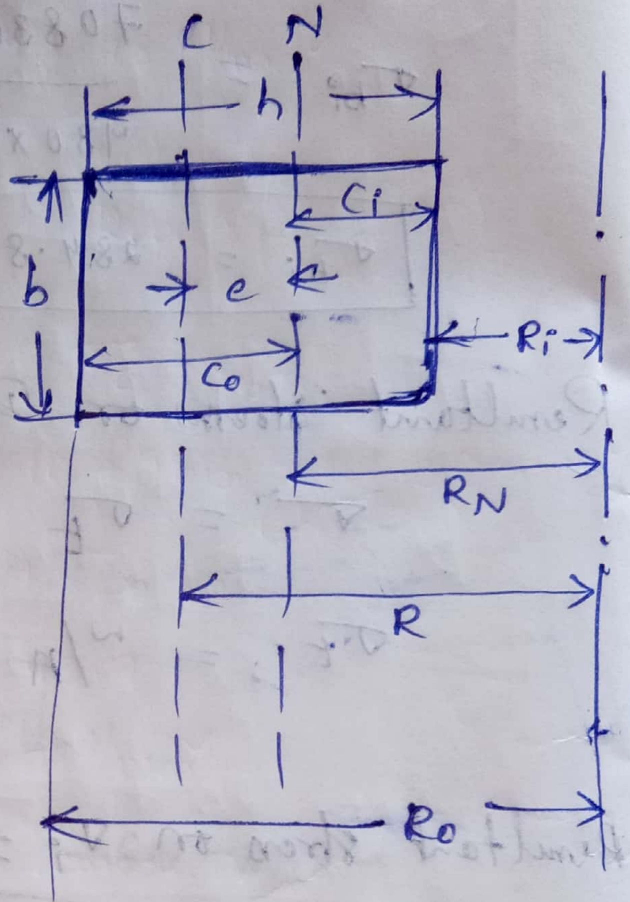
$$\sigma_o = \sigma_t - \sigma_{bo} = 10.416 - 207.96$$

$$\boxed{\sigma_o = -197.544 \ N/mm^2}$$

(compressive)

2. A crane hook carries a load of 20kN as shown in figure the section at x-x is Rectangular grooves horizontal side is 100 mm find the stresses in the inner & outer fibres at the given section





Area of Cross section

$$A = bh = 100 \times 20 = 2000 \, mm^2$$

$$R_i = 50 \, mm$$

$$R_o = 150 \, mm$$

Radius of curvature of ~~Cond~~ Centroidal axis

$$R = R_i + h/2$$

$$R = 50 + \frac{100}{2} = 100 \, mm$$

Radius of curvature of Neutral axis

$$R_N = \frac{h}{\log_e\left(\frac{R+C}{R-C}\right)} \quad \left(\because C = h/2\right)$$

$$C = h/2 = 100/2 = 50\,mm$$

$$R_N = \frac{100}{\log_e\left(\frac{100+50}{100-50}\right)}$$

$$\boxed{R_N = 91.02\,mm}$$

distance b/w the C.A & N.A

$$e = 100 - 91.02 = 8.98\,mm$$

$$C_o = R_o - R_N = 150 - 91.02 = 58.98\,mm$$

$$C_i = R_N - R_i = 91.02 - 50 = 41.02$$

$$x = R = 100\,mm$$

$$M = W \times x = (20 \times 10^3) \times 100 = 2000\,kN\text{-}mm$$

Bending Stress : inner surface $\sigma_{bi} = \frac{M C_i}{A e R_i}$

$$\sigma_{bi} = \frac{2000 \times 41.02}{2000 \times 8.98 \times 50}$$

$$= 0.09 \times 10^3 = 91.3\,N/mm^2$$

outer fibres $\sigma_{bo} = \dfrac{M C_o}{A e R_o}$

$$= \frac{2000 \times 58.98 \times 10^3}{2000 \times 8.98 \times 150}$$

$$\sigma_{bo} = 43.78\ N/mm^2$$

Resultant stress

$$\sigma_t = W/A = \frac{20 \times 10^3}{2000} = 10\ N/mm^2$$

at inner fibre $= \sigma_t + \sigma_{bi}$

$= 10 + 91.3 = 101.3\ N/mm^2$ (tensile)

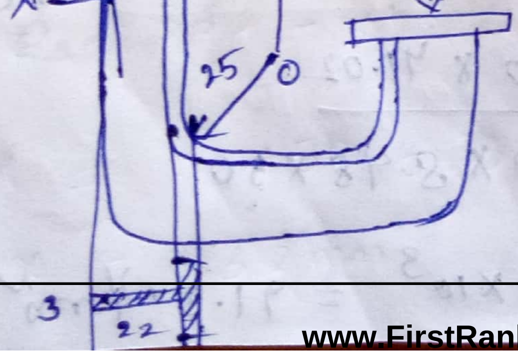
outer fiber $= \sigma_t + \sigma_{bo}$

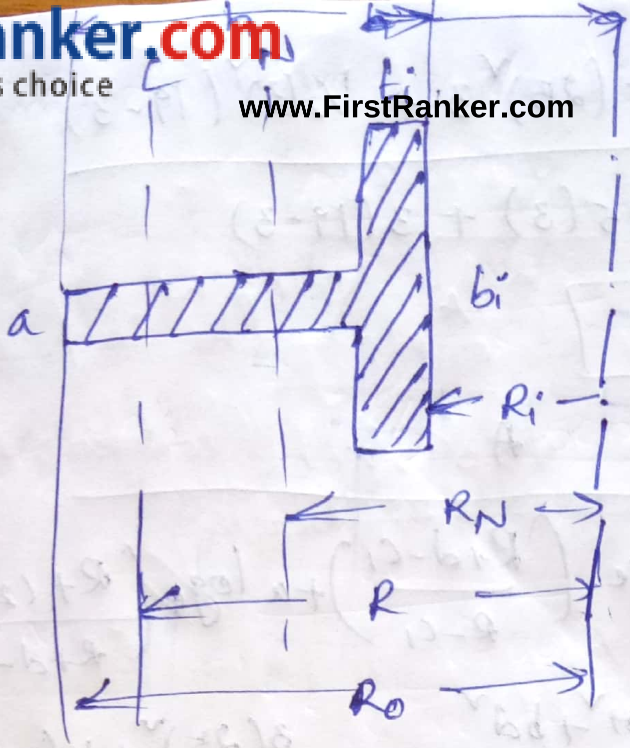
$= 10 - 43.7$

$= -33.7\ N/mm^2$

(compressive stress)

3. A C-clamp is subjected to a maximum load of w as shown in figure if the maximum tensile stress in the clamp is limited to 140 MPa find the value of the w.





$a = t = 3\,mm$

$B = b_i = 19\,mm$

$d = t_i = 3\,mm$

$H = h = 22+3 = 25\,mm$

$R_i = 25\,mm$

$R_o = 25+25 = 50\,mm$

$b = B - a = 19-3 = 16\,mm$

$A = Bd + (H-d)a$

$= 19\times 3 + (25-3)\,3$

$A = 123\,mm^2$

$$R = \frac{R_i + \frac{1}{2}h^2 t + \frac{1}{2}t_i^2(b_i - t)}{ht + t_i(b_i - t)}$$

$$= 25 + \frac{\frac{1}{2}(25)^2 3 + \frac{1}{2}(3)^2 (19-3)}{25(3) + 3(19-3)}$$

$$\boxed{R = 33.2\,mm}$$

$$R_N = \frac{A}{B \log_e\left(\frac{R+d-c_1}{R-c_1}\right) + a \log_e\left(\frac{R+c_2}{R+d-c_1}\right)}$$

$$c_1 = \frac{aH^2 + bd^2}{2(aH+bd)} = \frac{3(25)^2 + 16(3)^2}{2(3\times 25 + 16\times 3)}$$

$$\boxed{c_1 = 8.20\,mm}$$

$$c_2 = 25 - 8.2 = 16.8\,mm$$

$$R_N = \frac{123}{19 \log_e\left(\frac{33.2+3-8.2}{33.2-8.2}\right) + 3 \log_e\left(\frac{33.2+16.8}{33.2+3-8.2}\right)}$$

$$\boxed{R_N = 31.6\,mm}$$

$$e = R - R_N$$

$$e = 33.2 - 31.6 = 1.6\,mm$$

$$x = R + 50 = 33.2 + 50$$

$$x = 83.2\,mm$$

$= 83.2\,W$

Bending stress

$$\sigma_{bi} = \frac{M C_i}{A e R_i}$$

$$C_i = R_N - R_i = 31.6 - 25$$

$$C_i = 6.6$$

$$C_o = R_o - R_N = 50 - 31.6$$

$$C_o = 18.4\,mm$$

$$\sigma_{bi} = \frac{83.2W \times 6.6}{123 \times 1.6 \times 25} = 0.112W$$

$$\sigma_{bo} = \frac{M C_o}{A e R_o} = \frac{83.2W \times 18.4}{123 \times 1.6 \times 50}$$

$$\sigma_{bo} = 0.155W$$

Resultant stresses

$$\sigma = \sigma_t + \sigma_i$$

$$140 = W/A + 0.112\,W$$

$$140 = W/123 + 0.112\,W$$

$$\boxed{W = 1185.13\,N} \text{ (Tensile)}$$

Outer fibre

$$\sigma = \sigma_E - \sigma_b$$

$$140 = W/A - 0.155W$$

$$\boxed{W = -953.22N}$$ (compressive)

INTRODUCTION

Power is transmitted from the prime mover to a machine by means of intermediate mechanism called drives. This intermediate mechanism known as drives may be belt or chain or gears. Belt is used to transmit motion from one shaft to another shaft with the help of pulleys, preferably if the centre distance is long. It is not positive drive since there is slip in belt drive.

Three types of belt drives are commonly used. They are:

- Flat belt drive
- V-belt drive
- Rope or circular belt drive

•

FLAT BELT DRIVE

When the distance between two pulleys is around 10 meters and moderate power is required then flat belt drive is preferred. This may be arranged in two ways

- Open belt drive
- Cross belt drive

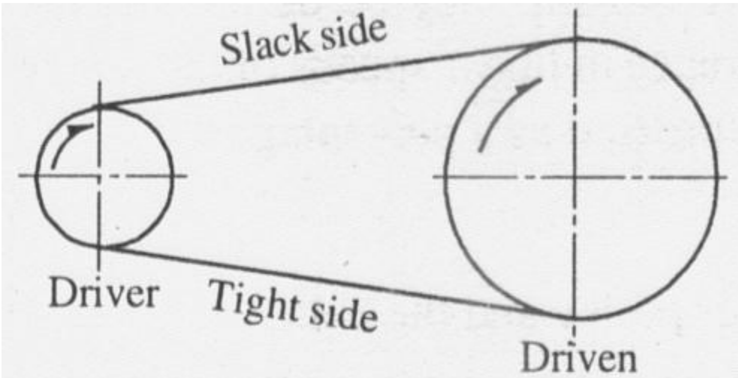
When the direction of rotation of both the pulleys are required in the same direction , then we can use open belt drive; if direction of rotation of pulleys are required in opposite direction then cross belt is used. The pulleys which drives the belt is known as driver and the pulley which follows driver is known as driven or follower.

MERITS AND DEMERITS OF FEAT BELT DRIVE

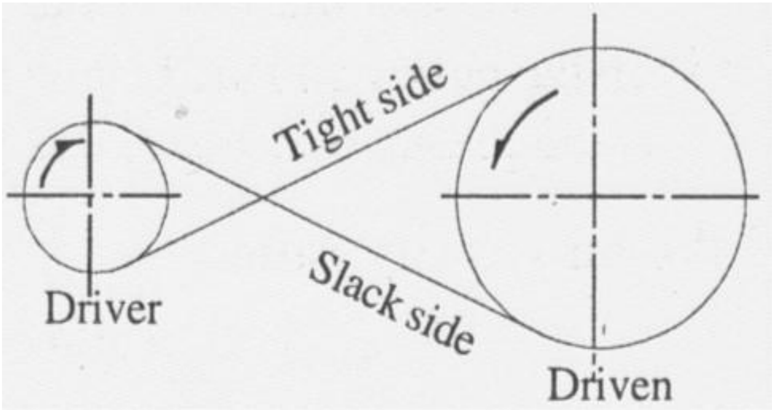
Merits:

- Simplicity, low cost, smoothness of operation, ability to absorb shocks, flexibility and efficiency at high speeds.
- Protect the driven mechanism against breakage in case of sudden overloads owing to belt slipping.
- Simplicity of care, low maintenance and service.
- Possibility to transmit power over a moderately long distance.

Open belt drive



Cross belt drive



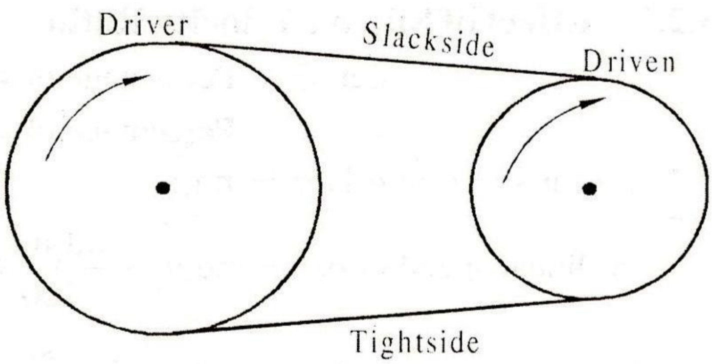
Demerits:

- It is not a positive drive.
- Comparatively large size.
- Stretching of belt calling for resewing when the centre distance is constant.
- Not suitable for short centre distance.
- Belt joints reduce the life of the belt.
- High bearing loads and belt stresses.
- Less efficiency due to slip and creep.

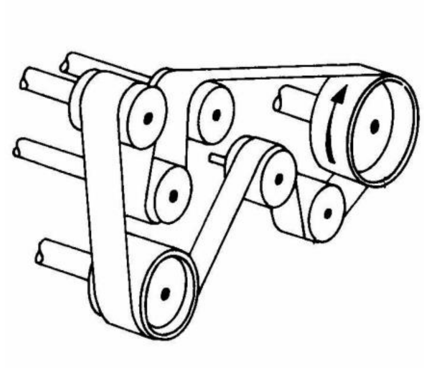
Creep in Belts

Consider an open belt drive rotating in clockwise direction as shown in figure. The portion of the belt leaving the driven and entering the driver is known as tight side and portion of belt leaving the driver and entering the driven is known as slack side.

During rotation there is an expansion of belt on tight side and contraction of belt on the slack side. Due to this uneven expansion and contraction of the belt over the pulleys, there will be a relative movement of the belt over the pulleys, this phenomenon is known as creep in belts.

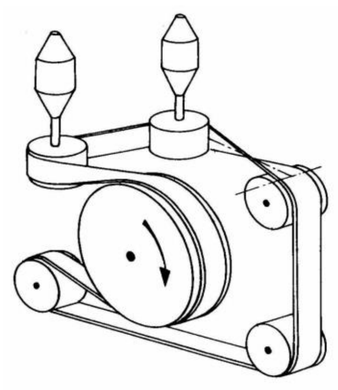


Flat belt drive system



Serpentine belt system

Flat belt drive system



3D arrangement of belt drive

Velocity Ratio

The ratio of angular velocity of the driver pulley to the angular velocity of the driven pulley is known as velocity ratio or speed ratio or transmission ratio.

Let d1 = Speed of driver pulley
d2 = Speed of driver pulley
n1 = Speed of driver pulley
n2 = Speed of driver pulley

Neglecting slip and thickness of belt,

Linear speed of belt on driver = Linear speed of belt on driven

Velocity Ratio

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n1 = Speed of driver pulley
n2 = Speed of driver pulley
Neglecting slip and thickness of belt,
Linear speed of belt on driver = Linear speed of belt on driven

i.e., $\pi d_1 n_1 = \pi d_2 n_2$

$$\frac{n_1}{n_2} = \frac{d_2}{d_1}$$

$$\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{Diameter of the driven pulley}}{\text{Diameter of the driver pulley}}$$

Considering the thickness of belt

$$\frac{n_1}{n_2} = \frac{d_2 + t}{d_1 + t}$$

Slip in Belts

Consider an open belt drive rotating in clockwise direction, this rotation of belt over the pulleys is assumed to be due to firm frictional grip between the belt and pulleys.

When this frictional grip becomes in sufficient, there is a possibility of forward motion of driver without carrying belt with it and there is also possibility of belt rotating without carrying the driver pulley with it, this is known as slip in belt.

Therefore slip may be defined as the relative motion between the pulley and the belt in it. This reduces velocity ratio and usually expressed as a percentage.

Effect of Slip on Velocity Ratio

Let s1 = Percentage of slip between driver pulley rim and the belt.
s2 = Percentage of slip between the belt and the driven pulley rim.
Linear speed of driver = $\pi d_1 n_1$

$$\therefore \text{ linear speed of belt} = \pi d_1 n_1 - \frac{\pi d_1 n_1 s_1}{100} = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)$$

$$\text{Hence, speed of driven} = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\text{i.e., } \pi d_2 n_2 = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\therefore \text{ Velocity ratio } \frac{n_1}{n_2} = \frac{d_2}{d_1\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)}$$

$$= \frac{d_2}{d_1\left(1 - \frac{s_1 + s_2}{100}\right)} = \frac{d_2}{d_1\left(1 - \frac{s}{100}\right)} \text{ Neglect } \frac{s_1 . s_2}{10000} \text{ since very small}$$

Where s = Total percentage slip = $s_1 + s_2$

$$\text{Considering thickness velocity ratio } \frac{n_1}{n_2} = \frac{d_2 + t}{(d_1 + t)\left(1 - \frac{s}{100}\right)}$$

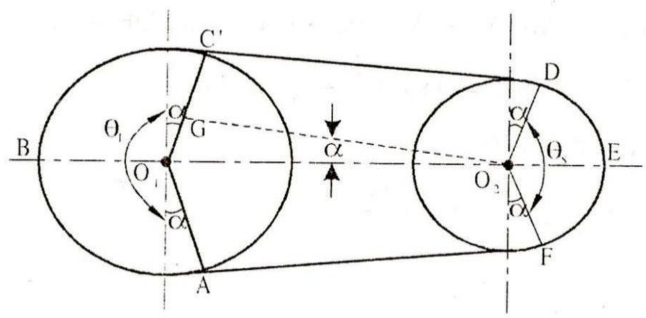
Material Used for Belt

Belts used for power transmission must be strong, flexible, and durable and must have a coefficient of friction. The most common belt materials are leather, fabric, rubber, balata, Camel's hair and woven cotton.

Length of Open Belt

Consider an open belt drive as shown in Figure.

Let, D = diameter of larger pulley
d = diameter of smaller pulley
C = distance between centers of pulley
L = length of belt



Length of open belt = Arc AB + Arc BC' + C'D + Arc DE + Arc EF + FA

$$\therefore\ L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\,\theta_L + d\theta_s)$$

Angle of contact on the larger pulley

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

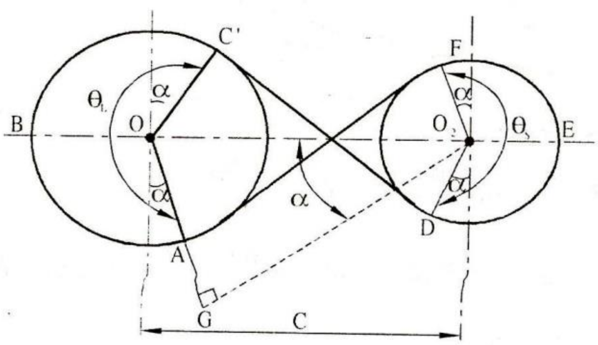
Angle of contact on the smaller pulley

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

Where L and s are in radians.
For equal diameter pulleys L = s = radians.
For unequal diameters pulleys, since slip will occur first on the smaller diameter pulley, it is necessary to consider s while designing the belt.

Length of Cross Belt

Consider a cross-belt drive as shown in Figure
Let, D = diameter of larger pulley
d = diameter of smaller pulley
L = Length of belt



$$\text{Length of cross belt} = \text{Arc AB} + \text{Arc BC}' + \text{C}'\text{D} + \text{Arc DE} + \text{Arc EF} + \text{FA}$$

$$\therefore\ L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D+d)$$

$$\text{In cross belt } \theta_L = \theta_s = \theta$$

$$\therefore\ \text{Angle of contact } \theta = \pi + \left\{2\sin^{-1}\left(\frac{D+d}{2C}\right)\right\}\frac{\pi}{180}$$

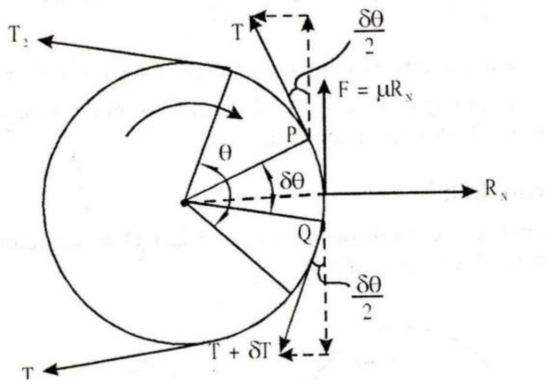
Where θ in radians.

Ratio of Belt Tensions

Consider a driven pulley rotating in clockwise direction as shown in Figure.

Let, T1 = Tension on tight side
T2 = Tension on slack side
= Angle of lap RN
= Normal Reaction
F = Frictional force = RN

Now consider a small elemental portion of the belt PQ subtending an angle at the centre. The portion of the belt PQ is in equilibrium under the action of the following forces, (i) Tension T at P (ii) Tension T + T at Q (iii) Normal reaction RN (iv) Frictional force F = RN



Resolving the forces horizontally

$$R_N = T\sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2}$$

$$= T\delta\theta \left[\text{Since small, } \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2} \text{ and neglect } \delta_T.\frac{\delta\theta}{2}\right]$$

Resolving the forces vertically

$$\mu R_N + T\cos\frac{\delta\theta}{2} = (T+\delta_T)\cos\frac{\delta\theta}{2}$$

$$\therefore\ R_N = \frac{\delta_T}{\mu} \left[\text{Since } \frac{\delta\theta}{2} \text{ is small, } \cos\frac{\delta\theta}{2} \approx 1\right]$$

Equating (i) and (ii)

$$\frac{\delta_T}{\mu} = T\delta\theta$$

$$\text{i.e., } \frac{\delta_T}{T} = \mu\delta\theta$$

Integrating between their respective limits

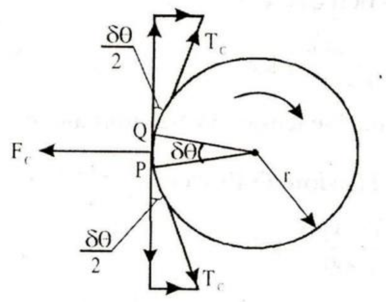
$$\text{i.e., } \int_{T_2}^{T_1} \frac{\delta_T}{T} = \int_{o}^{\theta} \mu\delta\theta$$

$$\therefore\ \frac{T_1}{T_2} = e^{\mu\theta} \text{ where } \theta \text{ in radians.}$$

Centrifugal Tension

Consider a driver pulley rotating in clockwise direction, because of rotation of pulley there will be centrifugal force which acts away from the pulley. The tensions created because of this centrifugal force both on tight and slack side are known as centrifugal tension.

Let, m = Mass of belt per meter length
v = Velocity in m/sec
TC = Centrifugal tension in N
r = Radius of pulley
FC = Centrifugal force



Consider a small elemental portion of the belt PQ subtending an angle shown in Figure.
Now the mass of belt PQ = M = Mass per unit length x Arc length PQ = mrd

Centrifugal force at the elemental portion PQ = $F_c = \frac{Mv^2}{r} = \frac{mr\delta\theta . v^2}{r} = mv^2\delta\theta$

Also resolving the forces horizontally,

$$F_c = T_c \sin\frac{\delta\theta}{2} + T_c \sin\frac{\delta\theta}{2}$$

$$= 2T_c\frac{\delta\theta}{2}\left[\text{Since } \frac{\delta\theta}{2} \text{ is small, } \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}\right]$$

Equating equations (i) and (ii)

$$2T_c\frac{\delta\theta}{2} = mv^2\delta\theta$$

$$\therefore T_c = mv^2$$

Considering centrifugal tension,

Tension on tight side = $T_1 + T_c$; Tension on slack side = $T_2 + T_c$.

Effect of Centrifugal Tension on Ratio ofTensions

Ratio of belt tension considering the effect of centrifugal tension is

$$\frac{T_1 - T_C}{T_2 - T_C}$$

Power transmitted by belt drive

$$\text{Power } N = \frac{(T_1 - T_2).v}{1000} \text{ kW}$$

where $T_1 - T_2$ = Effective tension in Newtons and v = Velocity in m/sec.

Effect of Centrifugal Tension on Power

$$\text{We have } N = \frac{(T_1 - T_2)v}{1000} \text{ kW}$$

Ratio of tension considering centrifugal tension $\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$

$$\therefore T_2 = \frac{T_1 - T_c}{e^{\mu\theta}} + T_c = \frac{T_1}{e^{\mu\theta}} + T_c\left(1 - \frac{1}{e^{\mu\theta}}\right)$$

$$= \frac{T_1}{e^{\mu\theta}} + T_c.k \text{ where } k = 1 - \frac{1}{e^{\mu\theta}}$$

$$\therefore N = \frac{\left\{T_1 - \left(\frac{T_1}{e^{\mu\theta}} + T_c.k\right)\right\}v}{1000} = \frac{\left\{T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) - T_c k\right\}}{1000}.v$$

$$= \frac{\{T_1 k - T_c k\}v}{1000} = \frac{\{T_1 - T_c\}kv}{1000} \text{ kW}$$

Neglecting the effect of centrifugal tension

$$N = \frac{T_1 kv}{1000} \text{ kW}$$

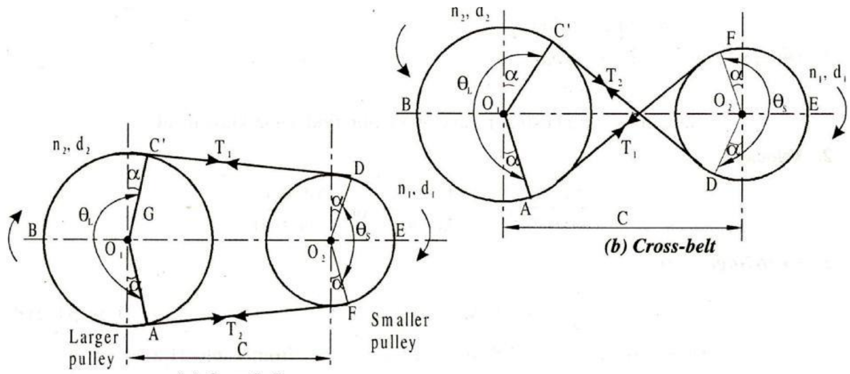
Initial Tension

The motion of the belt with the pulleys is assumed to be due to firm frictional grip between the belt and pulleys surface. To increase this grip the belt is mounted on the pulleys with some tension when the pulleys are stationary.

The tension provided in the belt while mounting on the pulley is "Initial tension'' and is represented by T0. Since in actual practice the belt is not perfectly elastic, C.G.Barth has given the relation as

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2}$$

Design Procedure for Flat Belt Drive



(a) Open belt

d_1 = Diameter of smaller pulley = d

n_1 = Speed of smaller pulley

d_2 = Diameter of larger pulley = D

n_2 = Speed of larger pulley

C = Centre distance

b = Width of belt

t = Thickness of belt

L = Length of belt

g = Acceleration due to gravity = 9810 mm/sec^2

μ = Coefficient of friction

w = Specific weight of belt material = 10×10^{-6} N/mm^2 for leather

σ_1 = Allowable or safe stress on the tight side

σ_2 = Slack side stress

σ_c = Centrifugal stress

v = Velocity of belt = $\dfrac{\pi dn}{60{,}000}$ m/sec, where d in mm

θ_s = Angle of contact for smaller pulley
θ_L = Angle of contact for larger pulley } For open belt
θ = Angle of contact for cross-belt
T_1 = Tension on tight side
T_2 = Tension on slack side
T_c = Centrifugal tension
T_0 = Initial tension

1. Unknown diameter or speed

$$n_1 d_1 = n_2 d_2$$

i.e., $n_1 d = n_2 D$ using the above relation find the unknown value.

2. Velocity

$$v = \frac{\pi d_1 n_1}{60,000} \text{ or } \frac{\pi(d_1+t)n_1}{60,000} \text{ or } \frac{\pi(d_2+t)n_2}{60,000}$$

3. Centrifugal stress

$$\sigma_c = \frac{w}{g}.v^2 \times 10^6 \text{ N/mm}^2$$

where $w = 10 \times 10^{-6}$ N/mm³ for leather

4. Capacity

Calculate e L L and e s s and take the smaller value as the capacity. If the coefficient of friction is same for both pulleys then find only e s since it is smaller than e L

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

For cross-belt

$$\theta = \pi + \left\{2\sin^{-1}\left(\frac{D+d}{2C}\right)\right\}\frac{\pi}{180}$$

5. ***Find 'k'***

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}}$$

6. ***Width of belt***

$$\text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_c)kv}{1000} \frac{kW}{mm^2}$$

where σ_1 in N/mm²; σ_c in N/mm² and v in m/sec

$$\text{Area of cross section of belt} \quad A = \frac{\text{Total given power}}{\text{Power transmitted / mm}^2}$$

$$\text{Also } A = b \times t \therefore \; b = \text{width of belt}$$

7. ***Length of belt***

For open belt

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_L + d\theta_s)$$

For cross belt

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D+d)$$

8. ***Initial tension in the belt***

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2}$$

$$T_1 = \sigma_1 A$$

$$T_2 = \sigma_2 A$$

σ_2 is obtained using the following relation.

$$\frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = e^{u\theta}$$

Example

A belt is required to transmit 18.5 kW from a pulley of 1.2 m diameter running at 250 rpm to another pulley which runs at 500 rpm.The distance between the centers of pulleys is 2.7 m. The following data refer to an open belt drive, = 0.25. Safe working stress for leather is 1.75 N/mm2. Thickness of belt = 10mm. Determine the width and length of belt taking centrifugal tension into account. Also find the initial tension in the belt and absolute power that can be transmitted by this belt and the speed at which this can be transmitted.

Data :

Open belt drive; N = 18.5 kW; n_1 = 500 rpm = Speed of smaller pulley;
d_2 = 1.2 m = 1200 mm = D = Diameter of larger pulley; n_2 = 250 rpm = Speed of larger pulley;
C = 2.7 m = 2700 mm; μ = 0.25; σ_1 = 1.75 N/mm²; t = 10 mm

(i) Diameter of smaller pulley

$$n_1 d_1 = n_2 d_2$$

$$500 \times d_1 = 250 \times 1200$$

$$\therefore \text{Diameter of smaller pulley } \mathbf{d_1} = \mathbf{600\ mm = d}$$

(ii) Velocity

$$v = \frac{\pi(D+t)n_2}{60{,}000} = \frac{\pi(1200+10)250}{60{,}000} = 15.839 \text{ m/sec.}$$

(iii) Centrifugal stress

$$\sigma_C = \frac{wv^2}{g} \times 10^6$$

Assume specific weight of leather as 10×10^{-6} N/mm³

$$\therefore \sigma_C = \frac{10 \times 10^{-6}}{9810} \times 15.839^2 \times 10^6 = 0.25573 \text{ N/mm}^2$$

(iv) Capacity

Since coefficient of friction is same for both smaller and larger pulleys, capacity = $e^{\mu\theta_s}$

$$\text{i.e., } e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1200-600}{2\times 2700}\right)\right\}\frac{\pi}{180} = 2.92 \text{ radians}$$

$$\therefore e^{\mu\theta} = e^{0.25\times 2.92} = 2.075$$

(v) Constant

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{2.075 - 1}{2.075} = 0.52$$

(vi) Width of belt

$$\text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_c)kv}{1000}$$

$$= \frac{(1.75 - 0.25573)0.52 \times 15.839}{1000} = 0.01231 \text{ kW}$$

Area of cross section of belt

$$= \frac{\text{Total given power}}{\text{Power transmitted per mm}^2 \text{ area}} = \frac{18.5}{0.01231} = 1503.18 \text{ mm}^2$$

Also A = b x t

1503.18

= b x 10

Therefore, b = 150.318 mm

Standard width b = 152 mm

(vii) Length of belt

$$\text{Length of open belt } L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_L + \theta_s d)$$

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180}$$

$$= \pi + \left\{2\sin^{-1}\left(\frac{1200-600}{2 \times 2700}\right)\right\}\frac{\pi}{180} = 3.364 \text{ radians}$$

$$L = \sqrt{4 \times 2700^2 - (1200-600)^2} + \frac{1}{2}(3.364 \times 1200 + 2.92 \times 600)$$

$$\therefore \mathbf{L = 8260.96 \text{ mm}}$$

(viii) Initial tension

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2}$$

$$T_1 = \sigma_1 A = 1.75 \times 1503.18 = 2630.566 \text{ N}$$

$$\frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = e^{\mu\theta}; \quad \frac{1.75 - 0.25573}{\sigma_2 - 0.25573} = 2.075; \quad \therefore \sigma_2 = 0.97586 \text{ N/mm}^2$$

$$T_2 = \sigma_2 A = 0.97586 \times 1503.18 = 1466.894 \text{ N}$$

$$2\sqrt{T_o} = \sqrt{2630.566} + \sqrt{1466.894}$$

$$\therefore \mathbf{T_o = 2006.552 \text{ N}}$$

(ix) Absolute power

For maximum power transmission

$$\sigma_C = \frac{\sigma_1}{3} = \frac{1.75}{3} = 0.5833 \text{ N/mm}^2$$

$$\text{Also } \sigma_C = \frac{w}{g} v^2 \times 10^6$$

$$\therefore 0.5833 = \frac{10 \times 10^{-6}}{9810} \times v^2 \times 10^6$$

$$\therefore \mathbf{v = 23.92 \text{ m/sec}}$$

$$\therefore \text{Power transmitted} \setminus \text{mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000}$$

$$= \frac{(1.75 - 0.5833)0.52 \times 23.92}{1000}$$

$$= 0.0145 \text{ kW}$$

$$\therefore \text{Total absolute power} = \text{Area of c/s of belt} \times \text{power per mm}^2$$

$$= 1503.18 \times 0.0145 = 21.7961 \text{ kW}$$

$$\therefore \text{Absolute power} = 21.8 \text{ kW.}$$

V- BELT DRIVE
Introduction

When the distance between the shafts is less, then V-belts are preferred. These are endless and of trapezoidal cross section as shown in Figure. It consists of central layer of fabric and moulded in rubber or rubber like compound. This assembly is enclosed in an elastic wearing cover. The belt will have contact at the two sides of the groove in the pulley. The wedging action between the belt and groove will increase the coefficient of friction making the drive a positive one.

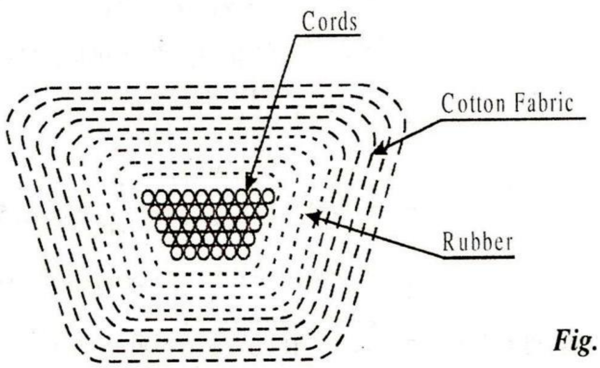
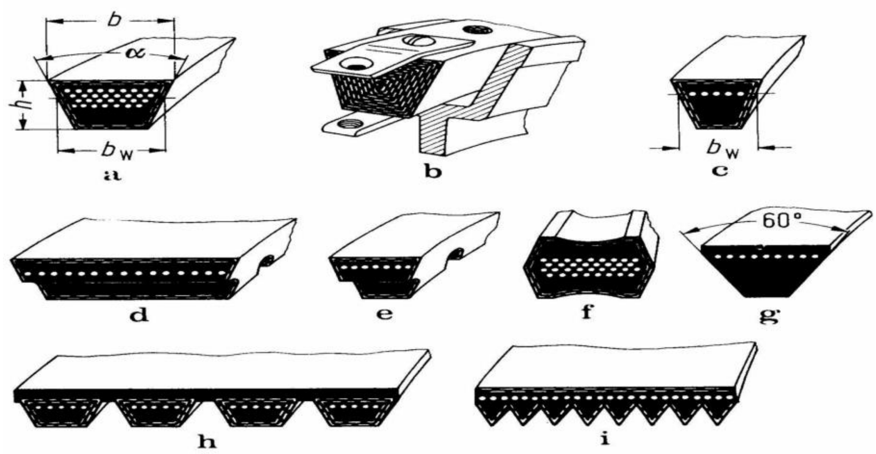


Fig.

Types of V - belts



a) Endless V-belt
b) Assembling of V-belt
c) Narrow V-belt
d) Wide V-belt with cogs
e) Narrow V-belt with cogs
f) Double angle V-belt
g) Great angle V-belt
h) Vee-band
i) Pol-rib belt

Advantages of V-belt over Flat belt
Advantages:

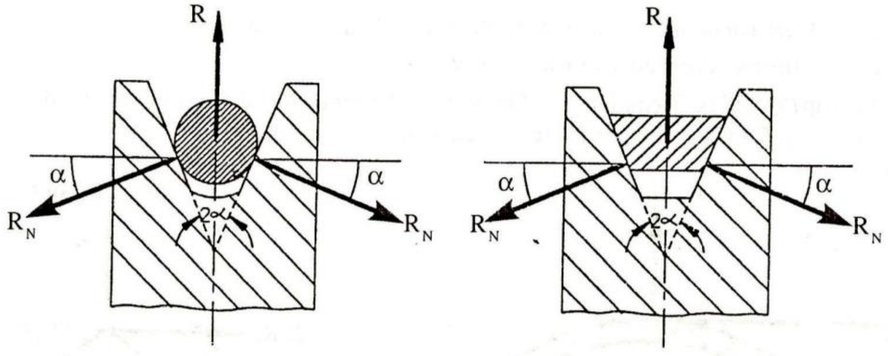
- Compact and give high velocity ratio.
- Provides shock absorption between driver and driven shafts.
- Positive and reliable drive.
- Because of wedging action in the grooves, loss of power due to slip is less.
- There is no joints problem as the drive is of endless type.

Disadvantages:

- Initial cost is more as the fabrication of pulleys with V-grooves are complicated.
- Cannot be used when the center distance is large.
- Improper belt tensioning and mismatching of belt results in reduction in service life.

Ratio of belt tensions for V-belt or rope drive

V-Belt drive or rope drive runs in a V-grooved pulley as discussed earlier. The cross-section of V-belt is shown in Figure.



(a) Rope *(b) V-belt*

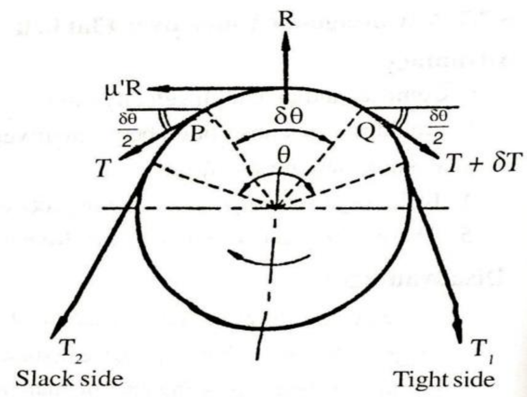
Let, 2 = angle of groove
RN = normal reaction between each side of groove
and the corresponding side of the belt strip PQ

From Figure Resolving Forces Vertically,

R = RN sin + RN sin = 2 RN sin

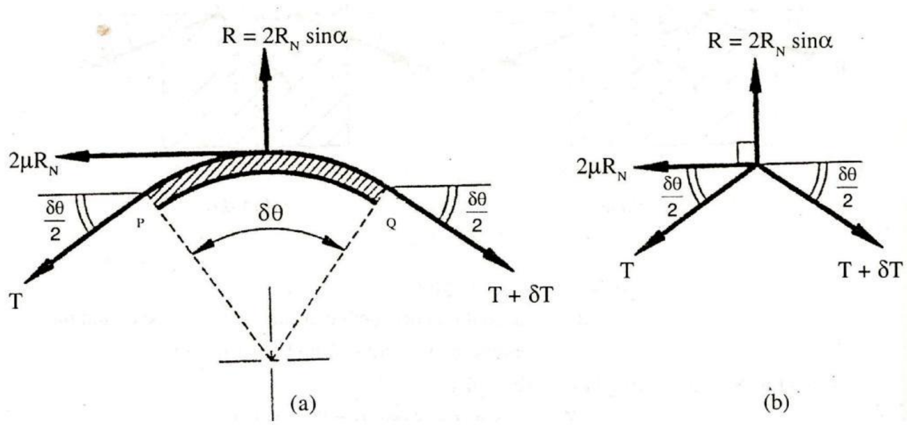
Total frictional force = RN + RN = 2 RN

In case of V-belt or rope, there are two normal reactions as shown in Figure, so that the radial reaction R is 2 RN *sin* and the total frictional force = 2(μ RN) = 2μ RN
Consider a short length *PQ* of belt subtending angle at the center of the pulley as shown in Figure.



Smaller Pulley (Driven)

Let, *R* = radial reaction between the belt length
PQ and the pulley rim = 2RN sin
RN = Normal reaction between the belt length *PQ* and
the pulley rim.
T = Tension on slack side of the shot strip *PQ* *T*+
T = Tension on tight side of short strip *PQ*
T = Difference in tension due to friction between
the length *PQ* and the surface pulley rim
μ' = Coefficient of friction between the belt and
pulley surface
μ = effective coefficient of friction = μ/sin
The strip *PQ* will be in equilibrium (figure) under the action of four forces *T*, *T*+ *T*, 2 μ*RN* and R where 2 μRN is the frictional force which is opposing the motion



Resolving the Forces Vertically

$$2\,R_N \sin\alpha = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

$$= T\sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

$$= 2T \times \sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2}$$

$$= 2T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} \qquad \text{... [As } \delta\theta \text{ is small, } \sin\frac{\delta\theta}{2} \sim \frac{\delta\theta}{2}]$$

$$= 2T\frac{\delta\theta}{2} \qquad \text{... [neglecting } \frac{\delta T.\delta\theta}{2}]$$

$$\therefore \quad 2R_N \sin\alpha = T\delta\theta$$

$$\therefore \quad R_N = \frac{1}{2\sin\alpha} T\delta\theta \qquad \text{..... (iii)}$$

Resolving the Forces Horizontally

$$2\mu R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$= T\cos\frac{\delta\theta}{2} + \delta T\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$= \delta T\cos\frac{\delta\theta}{2}$$

$$= \delta T \text{ [since } \delta\theta \text{ is small, } \cos\frac{\delta\theta}{2} \to 1]$$

$$\therefore 2\mu R_N = \delta T$$

Substituting value of R_N from (iii) in (iv), we get

$$2\mu\left[\frac{1}{2\sin\alpha} T\delta\theta\right] = \delta T$$

$$\text{i.e.} \quad \frac{\mu}{\sin\alpha} T\delta\theta = \delta T$$

$$\therefore \quad \frac{\delta T}{T} = \frac{\mu}{\sin\alpha}\delta\theta$$

Integrating both sides of equation between their limits

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \frac{\mu\delta\theta}{\sin\alpha}$$

$$\text{i.e.} \quad (\log_e T)_{T_2}^{T_1} = \frac{\mu}{\sin\alpha} (\theta)_0^{\theta}$$

$$\log_e T_1 - \log_e T_2 = \frac{\mu}{\sin\alpha}\theta$$

$$\log_e \frac{T_1}{T_2} = \frac{\mu}{\sin\alpha}\theta$$

$$\therefore \frac{T_1}{T_2} = e^{\frac{\mu\theta}{\sin\alpha}}$$ where θ in radians and α in degrees.

The above equation is called the 'limiting tension ratio' of the *V-belt or rope* and is valid only when the belt is on the point of slipping on the pulleys.

Considering centrifugal tension, ratio of belt tension $= \frac{T_1 - T_C}{T_2 - T_C} = e^{\frac{\mu\theta}{\sin\alpha}}$

DESIGN PROCEDURE FOR V-BELT

1. Selection of belt *c/s*

Equivalent pitch diameter of smaller pulley

de = dp.Fb

Where , dp = d1

Fb = smaller diameter factor

Based on 'de' select the c/s of belt

If 'd' is not given them based on power select the c/s of beltand diameter d1

2. Velocity

$$v = \frac{\pi d_1 n_1}{60000} \quad \text{m/sec}$$

3. Power capacity

Based on the cross-section selected, calculated the power capacity N* from the formulas.

4. Number of 'V' belts

$$i = \frac{N\,F_a}{N^* F_C \cdot F_d}$$

N= Total power transmitted in kW

N*= power capacity

Fa= Service factor

If the condition is not given then assume medium duty and 10-16 hours duty per day.

$$\text{Pitch length} \quad L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

If centre distance 'C' is not given, then calculate

$$C_{max} = 2(D+d)$$

$$C_{min} = 0.55(D+d) + T$$

For 'T' from the table for the selected cross section

Top width = W

Thickness = T

From the calculated value of C_{max} and C_{min}

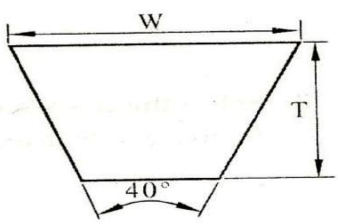
Select 'C' and find 'L' Then from table select the nearest standard value of pitch length 'L' and Nominal inside length.

Now from table for the nominal inside length and the selected cross section, correction factor for length = F_C

$$\theta = 2\cos^{-1}\frac{D-d}{2C}$$

or

$$\theta = 180 - 2\sin^{-1}\frac{D-d}{2C}$$



Correction factor for angle of contact = Fd

(If the type is not given select V - V - Belts)

Therefore, Number of belts = i

5. Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left\{\frac{L}{4} - \frac{\pi(D+d)}{8}\right\}^2 - \frac{(D-d)^2}{8}}$$

D = larger pulley diameter

d = smaller pulley diameter

L = standard pitch length

6. Specify the V-belt by the cross section letter followed by the inside length of belt.

Select a V-belt drive to transmit 10 kW of power froma pulley of 200 mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The service is heavy duty varying from 10 hours to 14 hours per day and centre distance between centre of pulleys is 600 mm.

Data:

$N = 10\,kW;\ d_1 = 200\ mm = d;\ n_1 = 720\ rpm;\ n_2 = 200\ rpm;\ C = 600\ mm$

Heavy duty 10 hours to 14 hours per day.

Solution :

i. Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$

$$720 \times 200 = 200 \times d_2$$

$$\therefore\ d_2 = 720\ mm = D = \text{diameter of larger pulley}$$

ii. Select the cross-section of belt

Equivalent Pitch diameter of smaller pulley $d_e = d_p F_b$ where $d_p = d_1 = 200$ mm

$$\frac{n_1}{n_2} = \frac{720}{200} = 3.6$$

From Table when $\frac{n_1}{n_2} = 3.6$

Smaller diameter factor $F_b = 1.14$

$$\therefore\ d_e = 200 \times 1.14 = 228\ mm.$$

iii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 200 \times 720}{60000} = 7.54 \text{ m/sec}$$

iv. Power capacity

For 'C' cross-section belt

$$N^* = v\left[\frac{1.47}{V^{0.09}} - \frac{143.27}{d_e} - \frac{2.34v^2}{10^4}\right]$$

$$= 7.54\left[\frac{1.47}{7.54^{0.09}} - \frac{143.27}{228} - \frac{2.34 \times 7.54^2}{10^4}\right]$$

$$N^* = 4.4 \text{ kW}$$

v. Number of Belts

$$i = \frac{NF_a}{N^*F_c.F_d}$$

for heavy duty 10 – 14 hours/day correction factor for service $F_a = 1.3$

$$L = 2C + \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4C}$$

$$= 2 \times 600 + \frac{\pi}{2}(720+200) + \frac{(720-200)^2}{4 \times 600} = 2757.8 \text{ mm}$$

The nearest standard value of nominal pitch length for the selected C-cross section belt L = 2723 mm
Nominal inside length = 2667 mm

For nominal inside length = 2667 mm, and C-cross section belt, correction factor for length Fe = 0.94

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right)$$

$$= 2\cos^{-1}\left(\frac{720-200}{2 \times 600}\right) = 128.64°$$

From Table when $\theta = 128.64°$

Correction factor for angle of contact $F_d = 0.86$ (Assume V-V belt)

$$\therefore \ i = \frac{10 \times 1.3}{4.4 \times 0.94 \times 0.86} = 3.655$$

$\therefore$ Number of V belts **i = 4**

vi. Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left\{\frac{L}{4} - \frac{\pi(D+d)}{8}\right\}^2 - \frac{(D-d)^2}{8}}$$

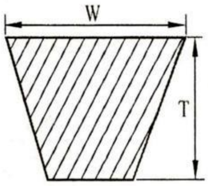
$$= \frac{2723}{4} - \frac{\pi(720+200)}{8} + \sqrt{\left\{\frac{2723}{4} - \frac{\pi(720+200)}{8}\right\}^2 - \frac{(720-200)^2}{8}}$$

$\therefore$ Correct centre distance $C = 580.73\,\text{mm}$

vii. Specification of V-belt

The V-belt selected is , C 2667

W = 22 m
T = 14 mm



ROPE DRIVES
Introduction

When power is to be transmitted over long distances then belts cannot be used due to the heavy losses in power. In such cases ropes can be used. Ropes are used in elevators, mine hoists, cranes, oil well drilling, aerial conveyors, tramways, haulage devices, lifts and suspension bridges etc. two types of ropes are commonly used. They are fiber ropes and metallic ropes. Fiber ropes are made of Manila, hemp, cotton, jute, nylon, coir etc., and are normally used for transmitting power. Metallic ropes are made of steel, aluminium. alloys, copper, bronze or stainless steel and are mainly used in elevator, mine hoists, cranes, oil well drilling, aerial conveyors, haulage devices and suspension bridges.

Hoisting tackle (Block and Tackle Mechanism)

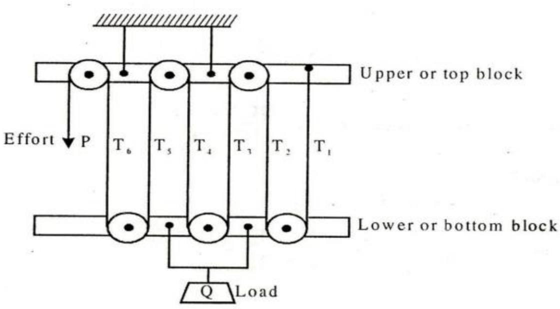
It consists of two pulley blocks one above the other. Each block has a series of sheaves mounted side by side on the same axle. The ropes used in hoisting tackle are

- Cotton ropes
- Hemp ropes and
- Manila ropes

The pulleys are manufactured in two designs i.e. fixed pulley and movable pulley.

Pulley system

A pulley system is a combination of several movable and fixed pulleys or sheaves. The system can be used for a gain in force or for a gain in speed. Hoisting devices employ pulleys for a gain in force predominantly. Pulley systems for a gain in forces are designed with the rope running off a fixed pulley and with the rope running off a movable pulley. Consider a hoisting tackle (block and tackle mechanism) as shown in fig.



Let Q = Load to be lifted
P = Effort required to raise the load
n = Total number of pulleys or sheaves

T_1, T_2, T_3, ---- etc. = Tensions in the rope
C = Pulley coefficient or Ratio of adjacent tensions.

i) Raise the load

For raising the load the block is pulled downward at P

$$C = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{T_5}{T_4} = \frac{T_6}{T_5} = \frac{P}{T_6}$$

$$\therefore \quad T_2 = CT_1$$
$$T_3 = CT_2 = C^2 T_1$$
$$T_4 = CT_3 = C^3 T_1$$
$$T_5 = CT_4 = C^4 T_1$$
$$T_6 = CT_5 = C^5 T_1$$
$$P = CT_6 = C^6 T_1$$

For equilibrium at the lower block

$$Q = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$$
$$= T_1 + CT_1 + C^2 T_1 + C^3 T_1 + C^4 T_1 + C^5 T_1$$
$$= T_1 [1 + C + C^2 + C^3 + C^4 + C^5]$$

$$\therefore \quad Q = T_1 \left[\frac{C^6 - 1}{C - 1}\right] \quad (\because \ 1 + C + C^2 + \text{-----} \text{ is the geometric progression})$$

$$= \frac{P}{C^6}\left(\frac{C^6 - 1}{C - 1}\right) \quad (\because \ P = C^6 T_1]$$

$\therefore$ Effort required to raise the load $\mathbf{P = \frac{QC^6 (C - 1)}{(C^6 - 1)}}$

For 'n' number of pulleys or sheaves $\mathbf{P = \frac{QC^n (C - 1)}{(C^n - 1)}}$ ---- 21.60 (DDHB)

ii) Lowering the load

For lowering the load the block is pulled up at 'P'

$$\text{Hence } C = \frac{T_1}{T_2} = \frac{T_2}{T_3} = \frac{T_3}{T_4} = \frac{T_4}{T_5} = \frac{T_5}{T_6} = \frac{T_6}{P}$$

$$T_6 = CP$$
$$T_5 = CT_6 = C^2 P$$
$$T_4 = CT_5 = C^3 P$$
$$T_3 = CT_4 = C^4 P$$
$$T_2 = CT_3 = C^5 P$$
$$T_1 = CT_2 = C^6 P$$

STEEL WIRE ROPES

A wire rope is made up of stands and a strand is made up of one or more layers of wires as shown in fig. the number of strands in a rope denotes the number of groups of wires that are laid over the central core. For example a 6× 19 construction means that the rope has 6 strands and each strand is composed of 19(12/6/1) wires. The central part of the wire rope is called the core and may be of fiber, wire, plastic, paper or asbestos. The fiber core is very flexible and very suitable for all conditions.

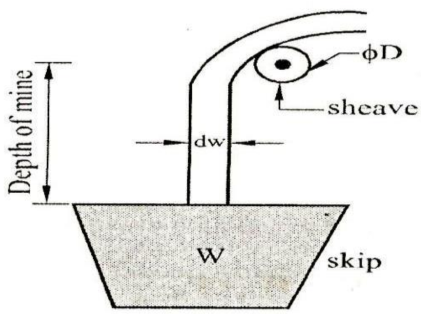
The points to be considered while selecting a wire rope are

- Strength
- Abrasion resistance
- Flexibility
- Resistance of crushing
- Fatigue strength
- Corrosion resistance.

Ropes having wire core are stronger than those having fiber core. Flexibility in rope is more desirable when the number of bends in the rope is too many.

DESIGN PROCEDURE FOR WIRE ROPE

Let d=Diameter of rope
D=Diameter of sheave
H= Depth of mine or height of building
W= total load
WR= Weight of rope
dw= Diameter of wire
A= Area of c/s of rope
Pb= Bending load in the rope
Fa= allowable pull in the rope
Fu= Ultimate of breaking load of rope
n= Factor of safety
Ws= Starting load



1. *Total load*

$$W = \text{Load to be lifted} + \text{Weight of skip}$$

2. *Total weight of rope*

$$W_R = \text{Weight per meter length} \times \text{Length of rope}$$
$$= \text{Weight per meter length} \times H$$

3. *Inertia load due to Acceleration*

$$W_I = Ma = \left(\frac{W + W_R}{g}\right).a$$

4. *Bending load*

$$P_b = \frac{k.A\,d_w}{D} \qquad \text{---- 21.63 b (}$$

Where
k = 82728.5 MPa = Modulus of elasticity of rope in N/mm²
A = Area of c/s of rope in mm²
d_w = Diameter of wire in mm
D = Diameter sheave in mm

5. *Starting load*

$$W_s = 2\,[W + W_R]$$

6. ***Maximum load***

The maximum load on the rope can be determined into the following ways.

(i) Load on the rope during uniform velocity = $W + W_R + P_b$

(ii) Load on the rope during acceleration = $W + W_R + W_I + P_b$

(iii) Load on the rope during starting = $W_s + P_b$

F_{max}, is the maximum among the above three values.

Neglecting impact $F_{max} = W + W_R + W_I + P_b$

7. *Diameter of rope*

Allowable pull $F_a \geq F_{max}$

Since $F_a = \dfrac{F_u}{n}$

$$\frac{F_u}{n} \geq F_{max}$$

Neglecting impact,

$$\mathbf{\frac{F_u}{n} \geq W + W_R + W_I + P_b}$$

From Table 21.47, it is found that the most commonly used type of rope is 6 × 19. From Tal 21.46 for 6 × 19 rope

$$F_u = 500.8\ d^2\ \text{MN where d in meters} = 500.8\ d^2\ \text{N where d in mm}$$

Weight per unit length of rope = $36.3\ d^2$ kN/m = $36.3\ d^2 \times 10^{-3}$ N/m where d in mm

Wire diameter $d_w = 0.063$ d mm

Area of c/s of rope $A = 0.38\ d^2\ mm^2$, d in mm

Average sheave diameter $D = 45$ d in mm

To find the acceleration any one of the following equations may be used

$$v = u + at \quad \text{---- (i)}$$

$$s = ut + \frac{1}{2}at^2 \quad \text{---- (ii)}$$

$$v^2 = u^2 + 2\,as \quad \text{---- (iii)}$$

problem

Select a wire rope to lift a load of 10kN through a height of 600m from a mine. The weight of bucket is 2.5kN. the load should attain a maximum speed of 50m/min in 2 seconds.

Solution:

From table select the most commonly used type of rope i.e. 6×19

From table for 6×19 rope Fu= 500.8 d2 N where d in mm

Weight per meter length = 36.3×10-3 d2 N/m where d in mm

Wire diameter dw = 0.063 d, mm

Area of c/s A = 0.38 d2, mm2

Sheave diameter D= 45 d, mm

From table for 600 m depth

F.O.S = n = 7

1. Total load

W= Load to be lifted + weight of skip = 10000+ 2500= 12500N

2. Total weight of rope

$$W_R = \text{Weight per meter length} \times \text{length of rope}$$

$$= 36.3 \times 10^{-3}\, d^2 \times 600\,\frac{N}{m} \text{ where d in mm } = 21.78\, d^2$$

3. Inertia load due to Acceleration

$$W_I = Ma = \left(\frac{W + W_R}{g}\right).a$$

$$v = u + at \quad \text{Since } u = 0$$

$$v = at$$

$$\therefore\ a = \frac{v}{t} = \frac{50}{60 \times 2} = 0.417\ \text{m/sec}^2$$

$$\therefore\ W_I = \left(\frac{12500 + 21.78d^2}{9.81}\right) \times 0.417 = \mathbf{531.345 + 0.9258\,d^2}$$

4. Bending load

$$P_b = k.A\frac{d_w}{D} \quad \text{---- 21.63}$$

where $k = 82728.5$ MPa

$$\therefore P_b = 82728.5 \times 0.38\,d^2 \times \frac{0.063d}{45d} = \mathbf{44.01\,d^2}$$

5. Starting load

$$W_s = 2[W + W_R] = 2[12500 + 21.78\,d^2] = 25000 + 43.56\,d^2$$

6. Maximum load

Assume impact load is neglected.

$$\therefore \text{ Max. load on the rope } F_{max} = W + W_R + W_1 + P_b$$

$$\therefore F_{max} = 12500 + 21.78\,d^2 + 531.345 + 0.9258\,d^2 + 44.01\,d^2$$

$$= 13031.345 + 66.7158\,d^2$$

7. Diameter of rope

$$F_a \geq F_{max}$$

$$\text{i.e., } \frac{F_u}{n} \geq F_{max}$$

$$\therefore \frac{500.8d^2}{7} \geq 13031.345 + 66.7158\,d^2$$

$$d \geq 51.96\,\text{mm}$$

From Table 21.40 for 6×19 rope std diameter $d = 54$ mm

CHAIN DRIVE

Introduction

Chain is used to transmit motion from one shaft to another shaft with the help of sprockets. Chain drives maintain a positive speed ratio between driving and driven components, so tension on the slack side is considered is as zero. They are generally used for the transmission of power in cycles, motor vehicles, agricultural machinery, road rollers etc.

Merits and demerits of chain drives Merits

1. Chain drives are positive drives and can have high efficiency when operating under ideal conditions.
2. It can de used for both relatively long centre distances.
3. Less load on shafts and compact in size as compared to belt drive.

Demerits

1. Relatively high production cost and noisy operation.
2. Chain drives require more amounts of servicing and maintenance as compared to belt drives.

Velocity ratio in chain drive

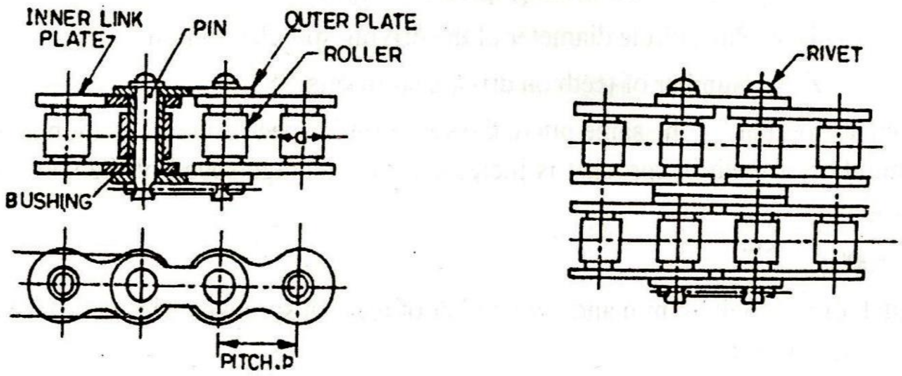
Let n1= speed of driver sprocket in rpm
n2 = speed of driven sprocket in rpm
z1= number of teeth on drivers sprocket
z2 = number of teeth on driven sprocket
Therefore Velocity ratio n1/n2= z1/z2

Chains for power transmission

The different types of chain used for power transmission are:
i. Block chain ii. Roller chain iii. Inverted-tooth chain or silent chain.

ROLLER CHAIN

It consists of two rows of outer and inner plates. The outer row of plates in known as pin link or coupling link whereas the inner row of plates is called roller link. A Pin passes through the bush which is secured in the holes of the inner pair of links and is riveted to the outer pair of links as shown in Fig. Each bush is surrounded by a roller. The rollers run freely on the bushes and the bushes turn freely on the pins.

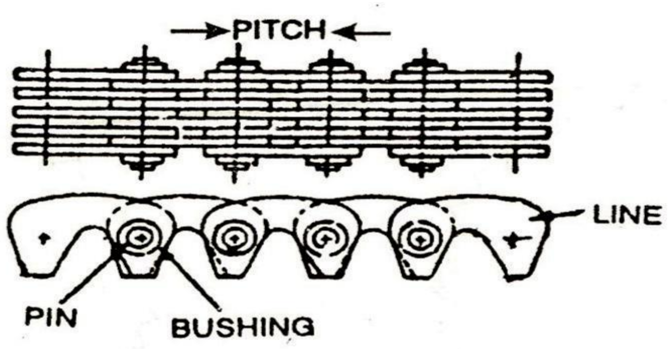


(a) (b)

A roller chain is extremely strong and simple in construction. It gives good service under severe conditions. To avoid longer sprocket diameter, multi-row-roller chains or chains with multiple strand width are used. Theoretically, the power capacity multistrand chain is equal to the capacity of the single chain multiplied by the number of strand, but actually it is reduced by 10 percent.

Inverted tooth chain or silent chain

It is as shown Fig. these chains are not exactly silent but these are much smoother and quieter in action than a roller chain. These chains are made up of flat steel stamping, which make it easy to built up any width desired. The links are so shaped that they engage directly with sprocket teeth. In design, the silent chains are more complex than brush roller types, more expensive and require more careful maintenance.



Chordal action

When a chain passes over a sprocket, it moves as a series of chords instead of a continuous arc as in the case of a belt drive. Thus the center line of a chain is not a uniform radius. When the driving sprocket moves at a constant speed, the driven sprocket rotates at a varying speed due to the continually varying radius of chain line. This variation in sped ranges from

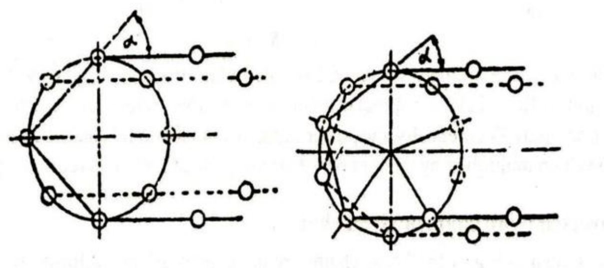
$$v_{min} = \frac{\pi d_1 n_1}{60 \times 1000} \times \frac{180}{z_1} \text{ to } v_{max} = \frac{\pi d_1 n_1}{60 \times 1000} \text{ m/sec}$$

Where
n_1= Speed of the driving sprocket in rpm
d_1 Pitch circle diameter of the driving sprocket in mm
z1 = number of teeth on driving sprockets.

It is clear from above that for the same pitch, the variation in speed and articulation angle decreases, if the number of teeth in sprocket is increased. The average speed of the sprocket as given by

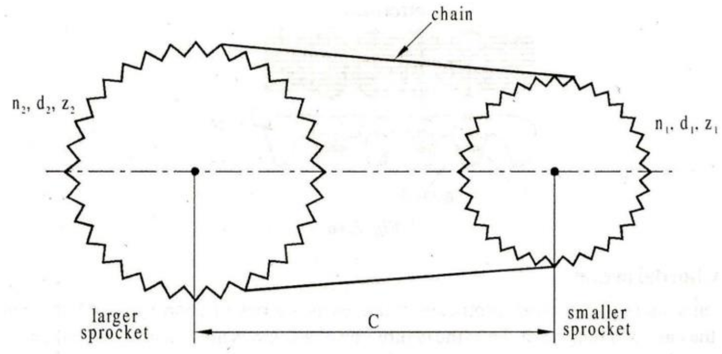
$$v = \frac{pzn}{60 \times 1000} \text{ m/sec.}$$

Where p = pitch of the chain in mm and z = number of teeth in sprocket. This chordal action of the chain is shown in Fig.



a. Four teeth sprocket *b. Five teeth sprocket*

DESIGN PROCEDURE FOR ROLLER CHAIN



Let p = Pitch
d1 = diameter of smaller sprocket
d2 = diameter of larger sprocket
n1 = speed of smaller sprocket
n2 = speed of larger sprocket
z1 = number of teeth on smaller sprocket
z2 = Number of teeth on larger sprocket
L = Length of chain in pitches
C = Center diameter
Cp = Center distance in pitches

1. Pitch of chain

$$p \leq 25\left(\frac{900}{n_1}\right)^{\frac{2}{3}}$$

Where p in mm, and n1= speed of smaller sprocket
Select standard nearest value of pitch from Table
Chain number
Breaking load Fu
Measuring load w

2. Number of teeth on the sprockets

From Table for the given ratio select the number of teeth on the smaller sprocket (z1)
Since

$$\frac{n_1}{n_2} = \frac{z_2}{z_1}$$

Number of teeth on larger sprocket = z2

3. Pitch diameters

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)}$$

$$\therefore\ d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)}$$

= pitch diameter of smaller sprocket

$$d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)}$$

= pitch of larger sprocket

4. Velocity

$$v = \frac{p z_1 n_1}{60000} \text{ m/sec}$$

5. Required pull

$$\text{Power required} \quad N = \frac{F_\theta . v}{1000 k_l k_s} \quad \text{---- 21.115a}$$

k_l = Load factor

= 1.1 to 1.5 or from Table 14.4 (Old DDHB) or Table 14.7 (New DDHB)

k_s = Service factor

= 1 for 10 hours per day service

= 1.2 for 24 hours operation or from Table 21.61 (Old DDHB) or Table 21.54 F (New DDHB)

6. Allowable pull

$$\text{Allowable pull} \quad F_a = \frac{F_u}{n_o}$$

where n_o = working factor of safety from Table 21.75

7. Number of strands in a chain

$$\text{Number of strands} \quad i = \frac{F_\theta}{F_a}$$

8. Check for actual factor of safety

$$\text{Actual factor of safety} \quad n_a = \left(\frac{F_u}{F_\theta + F_{cs} + F_s}\right) i \quad \text{---}$$

$$\text{where} \quad F_\theta = \frac{1000N}{v} \quad \text{----}$$

$$F_{cs} = \frac{wv^2}{g} \quad \text{----}$$

$$F_s = k_{sg} wC$$

C = centre distance. If not given for medium centre distance C_p = 30 to 50

k_{sg} = coefficient of sag from Table
i = number of strands

If $n_a > n_o$, then safe

6. Allowable pull

$$L_p = 2\,C_p \cos\alpha + \frac{z_2 + z_1}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

Where $\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right)$

10. length of chain

$$L = p.L_p$$

11. Correct centre distance

$$L_p = 2\frac{C}{p}\cos\alpha + \frac{z_2 + z_1}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

$\therefore$ Correct centre distance C

Select a roller chain drive to transmit power of 10 kw from a shaft rotating at 750 rpm to another shaft to run at 450 rpm. The distance between the shaft centers could be taken as 35 pitches.

Data:

N= 10 kw; n1 = 750 rpm; n2 = 450 rpm; C = 35 pitches

1. Pitch of chain

$$p \le 25\left(\frac{900}{n_1}\right)^{2/3}$$

$$\le 25\left(\frac{900}{750}\right)^{2/3}$$

$$\le 28.23 \text{ mm}$$

From table 21.64, the nearest standard value of pitch **p = 25.4 mm**

Select chain number 208 B

Breaking load F_u = 17.9 kN = 17900 N

Measuring load w = 127.5 N

2. Number of teeth on the sprockets

$$\frac{n_1}{n_2} = \frac{750}{450} = 1.667$$

From Table 21.60 for $\frac{n_1}{n_2} = 1.667$, select number of teeth on the sumaller sprocket $z_1 = 27$

$$\text{Now} \quad \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{750}{450} = \frac{z_2}{27}$$

Number of teeth on larger sprocket $z_2 = 45$

3. Pitch diameter

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)}$$

$$\text{Pitch diameter of smaller sprocket} \quad d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{25.4}{\sin\left(\frac{180}{27}\right)} = 218.79 \text{ mm}$$

$$\text{Pitch diameter of larger sprocket} \quad d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{25.4}{\sin\left(\frac{180}{45}\right)} = 364.124 \text{ mm}$$

4. Velocity

$$v = \frac{p z_1 n_1}{60000} = \frac{25.4 \times 27 \times 750}{60000} = 8.57 \text{ m/sec}$$

5. Required pull

$$\text{Power} \quad N = \frac{F_\theta . v}{1000 k_l k_s} \qquad \text{---- 21.115a (DDHB)}$$

k_l = Load factor = 1.1 – 1.5

k_s = Service factor

= 1.2 for 24 hours operation (Assume 24 hours operation)

Take $k_l = 1.3$

$$\therefore \quad 10 = \frac{F_\theta \times 8.57}{1000 \times 1.3 \times 1.2}$$

$$\therefore \quad \mathbf{F_\theta = 1820.3\ N}$$

6. Allowable pull

$$F_a = \frac{F_u}{n_o} \quad \text{where} \quad n_o = \text{Working factor of safety}$$

From Table 21.75 for n_1 = 750 rpm and p = 25.4 mm

Select the working factor of safety $n_o = 11.7$ [n_o is not equal to 10.7, printing error in DDHB]

$$\therefore \quad F_a = \frac{17900}{11.7} = 1529.914$$

7. Number of strands

$$i = \frac{F_\theta}{F_a} = \frac{1820.3}{1529.914} = 1.189$$

$\therefore$ Number of strands $i = 2$

8. Check for actual factor of safety

$$\text{Actual factor of safety } n_a = \left(\frac{F_u}{F_\theta + F_{cs} + F_s}\right) i$$

$$F_\theta = \frac{1000\,N}{v} = \frac{1000 \times 10}{8.57} = 1166.86\ N$$

$$F_{cs} = \frac{wv^2}{g} = \frac{127.5 \times 8.57^2}{9.81} = 954.56\ N$$

$$F_s = k_{sg}\, w\, C$$

From Table 21.58 for horizontal drive, $k_{sg} = 6$

$$\therefore\ F_s = 6 \times 127.5 \times \frac{35 \times 25.4}{1000} = 680.085\ N$$

$$\therefore\ n_a = \left(\frac{17900}{1166.86 + 954.56 + 680.085}\right) \times 2 = 12.778$$

Since $n_a > n_o$, the selection of the chain is safe.

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \frac{z_1 + z_2}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right) \quad \text{---- 21.122 (DDHB)}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) \quad \text{---- 21.122 (DDHB)}$$

$$= \sin^{-1}\left(\frac{364.124 - 218.79}{2 \times 35 \times 25.4}\right) = 4.6886°$$

$$\therefore\ L_p = 2 \times 35 \cos 4.6886 + \left(\frac{27 + 45}{2}\right) + 4.6886\left(\frac{45 - 27}{180}\right)$$

$$= 106.2346 \text{ pitches}$$

The nearest even number of pitches is 106

$$\therefore\ \mathbf{L_p = 106 \text{ pitches}}$$

11. Correct centre distance

$$L_p = 2\frac{C}{p}\cos\alpha + \frac{(z_2 + z_1)}{2} + \alpha\frac{(z_2 - z_1)}{180}$$

$$106 = 2 \times \frac{C}{25.4}\cos 4.6886 + \left(\frac{27 + 45}{2}\right) + 4.6886\left(\frac{45 - 27}{180}\right)$$

$$\therefore\ C = 886 \text{ mm}$$

1. Drives that transmit power by means of friction: eg: belt drives and rope drives.
2. Drives that transmit power by means of engagement: eg: chain drives and gear drives.

However, the selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, velocity ratio, shifting arrangement, Maintenance and cost.

GEAR DRIVES

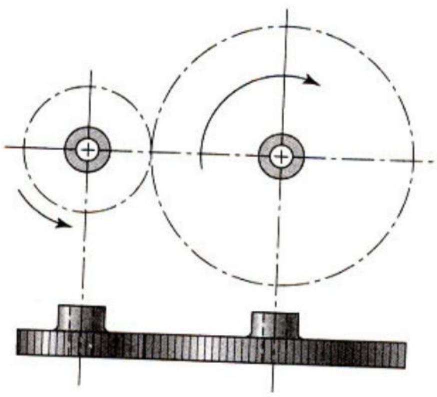
Gears are defined as toothed wheels, which transmit power and motion from one shaft to another by means of successive engagement of teeth

1. The centre distance between the shafts is relatively small.
2. It can transmit very large power
3. It is a positive, and the velocity ratio remains constant.
4. It can transmit motion at a very low velocity.

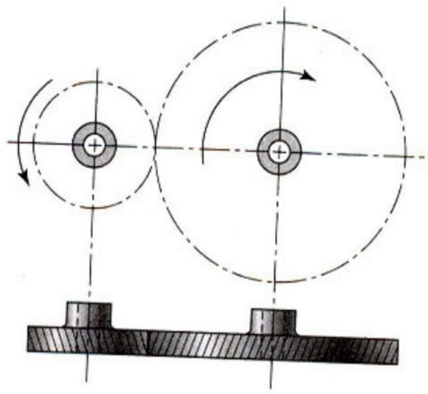
CLASSIFICATION OF GEARS:

Four groups:

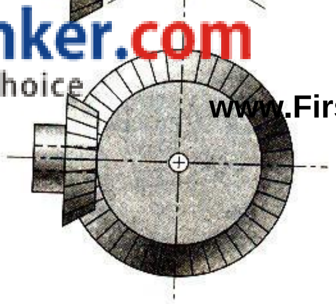
1) Spur Gears
2) Helical gears
3) Bevel gears and
4) Worm Gears



Spur Gear



Helical Gear



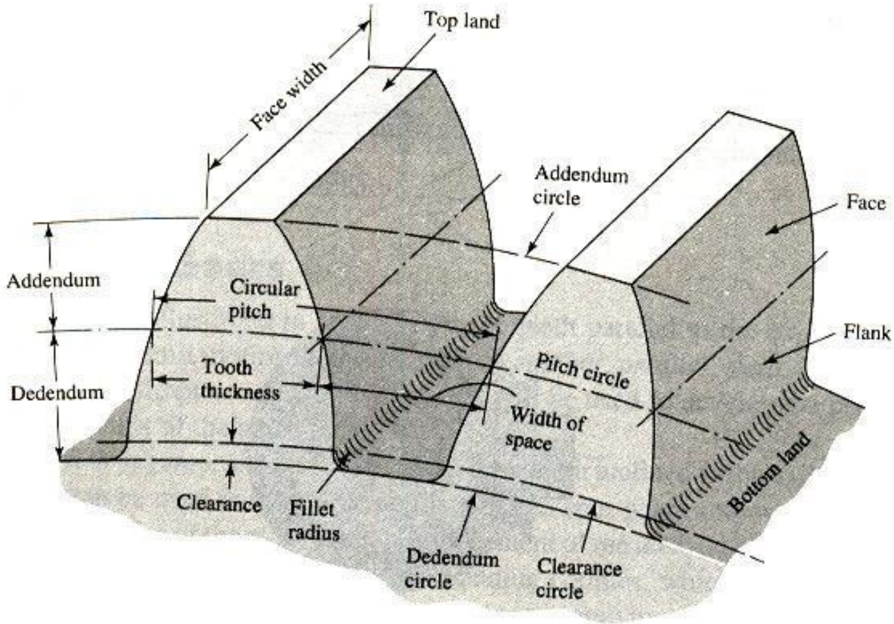
Bevel Gear

Worm Gear Set

NOMEN CLATURE

Spur gears are used to transmit rotary motion between parallel shafts. They are usually cylindrical is shape and the teeth are straight and parallel to the axis of rotation.

In a pair of gears, the larger is often called the GEAR and, the smaller one is called the PINION



Nomenclature of Spur Gear

1. **Pitch Surface:** The pitch surfaces of the gears are imaginary planes, cylinders or cones that roll together with out slipping.
2. **Pitch circle:** It is a theoretical circle upon which all calculations are usually based. It is an imaginary circle that rolls with out slipping with the pitch circle of a mating gear. Further, pitch circles of a mating gears are tangent to each other.

6. **Addendum**: The Addendum is the radial distance between the pitch and addendum circles. Addendum indicates the height of tooth above the pitch circle.

7. **Dedendum**: The dedendum is the radial distance between pitch and the dedendum circles. Dedendum indicates the depth of the tooth below the pitch circle.
8. **Whole Depth**: The whole depth is the total depth of the tooth space that is the sum of addendum and Dedendum.
9. **Working depth:** The working depth is the depth of engagement of two gear teeth that is the sum of their addendums
10. **Clearance:** The clearance is the amount by which the Dedendum of a given gear exceeds the addendum of it's mating tooth.
11. **Face:** The surface of the gear tooth between the pitch cylinder and the addendum cylinder is called face of the tooth.
12. **Flank:** The surface of the gear tooth between the pitch cylinder and the root cylinder is called flank of the tooth.
13. **Face Width:** is the width of the tooth measured parallel to the axis.
14. **Fillet radius:** The radius that connects the root circle to the profile of the tooth is called fillet radius.
15. **Circular pitch:** is the distance measured on the pitch circle, from a point on one tooth to a corresponding point on an adjacent tooth.
16. **Circular tooth thickness:** The length of the arc on pitch circle subtending a single gear tooth is called circular tooth thickness. Theoretically circular tooth thickness is half of circular pitch.
17. **Width of space:** (tooth space) The width of the space between two adjacent teeth measured along the pitch circle. Theoretically, tooth space is equal to circular tooth thickness or half of circular pitch
18. **Working depth:** The working depth is the depth of engagement of two gear teeth, that is the sum of their addendums
19. **Whole depth:** The whole depth is the total depth of the tooth space, that is the sum of addendum and dedendum and (this is also equal to whole depth + clearance)
20. **Centre distance:** it is the distance between centres of pitch circles of mating gears. (it is also equal to the distance between centres of base circles of mating gears)
21. **Line of action:** The line of action is the common tangent to the base circles of mating gears. The contact between the involute surfaces of mating teeth must be on this line to give smooth operation. The force is transmitted from the driving gear to the driven gear on this line.
22. **Pressure angle:** It is the angle that the line of action makes with the common tangent to the pitch circles.

26. **Contact Ratio?**

Velocity ratio: if the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

27. **Module:** It is the ratio of pitch circle diameter in mith meters to the number of teeth. it is usually denoted by 'm' Mathematically

$$m = \frac{D}{Z}$$

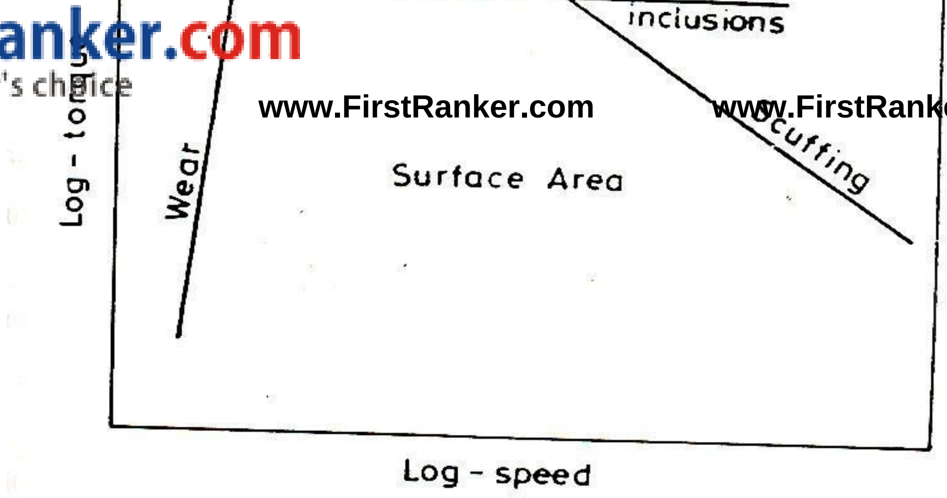
28. **Back lash**: It is the difference between the tooth space and the tooth thickness as measured on the pitch circle.

29. **Velocity Ratio**: Is the ratio of angular velocity of the driving gear to the angular velocity of driven gear. It is also called the speed ratio.

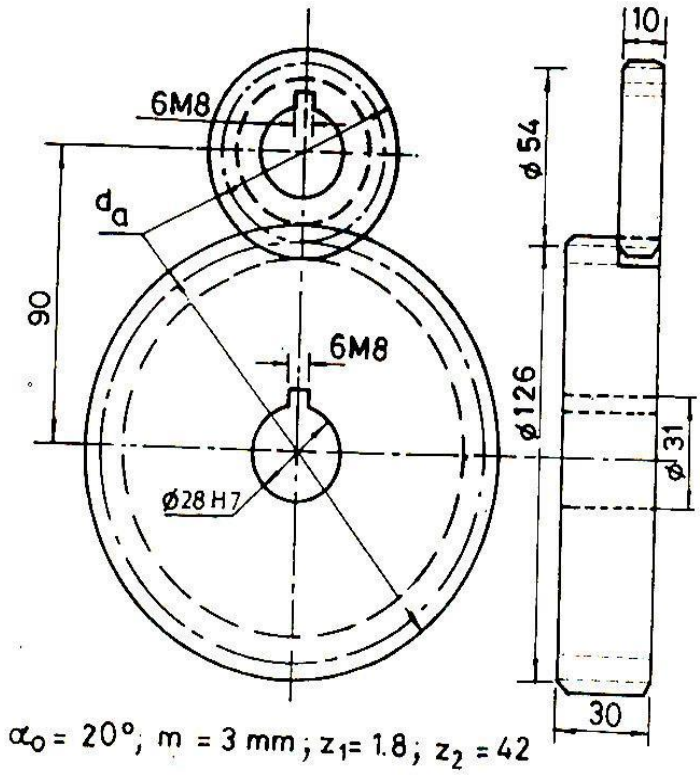
d_a	Addendem circle diameter
d_o	Pitch circke diameter
d_r	Root circle diameter
m	Module
r_a	Addendem circle radius
r_b	Base circle radius
r_o	Pitch circle radius
R	Radius of curvature of tooth profile
Z	Number of teeth
α	Preasure angle
σ	Stress value
σ_b	Bending stress
σ_H	Hertz contact stress
σ_{HB}	Contact stress at the beginning of the engagement
σ_{HE}	Contact stress at the end of the engagement
σ_{HL}	Pitting limit stress
τ	Shear stress
ω	Angle velocity
Suffix 1	Pinion
Suffix 1	Gear

Nomenclature of Spur Gear

Failure Map of Involute Gears

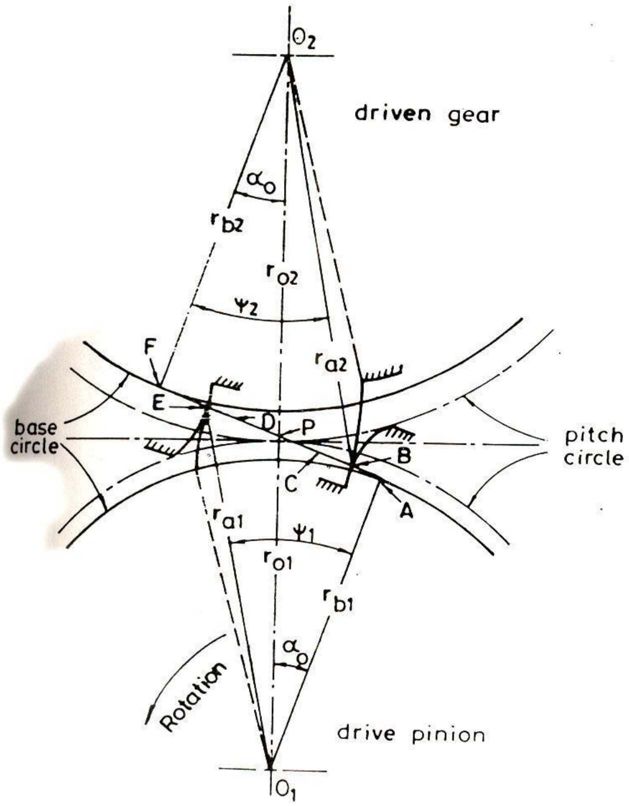


Failure Map of Involute Gears



Gear Set

			Pinion		Gears
Module	m	(mm)		3.0	
Pressure angle	α_0	(deg)		20	
Number of teeth	z	(--)	18		42
Pitch circle diameter	d	(mm)	54.0		126.0
Centre distance	a_o	(mm)		90.0	
Addendum circle diameter	d_a	(mm)	60.0		132.0
Root circle diameter	d_r	(mm)	46.5		118.5
Face width	B	(mm)	10.0		30.0



Different Phases of Gear Tooth Contact

Transition phase	C	2 to 1	$C_1 - C_6$	$C_3 - C_1 - C_6$
Pitch point	P	1	C_4	C_5
Transition phase	D	1 to 2	$C_3 - C_2 + C_6$	$C_2 - C_6$
End of engagement	E	2	C_1	$C_3 - C_1$

Expressions for the Calculation of Equivalent Radii of Curvature at Various Phases of Contact

Design consideration for a Gear drive

In the design of gear drive, the following data is usually given

i. The power to be transmitted
ii. The speed of the driving gear
iii. The speed of the driven gear or velocity ratio
iv. The centre distance

The following requirements must be met in the design of a gear drive

(a) The gear teeth should have sufficient strength so that they will not fail under static loading or dynamic loading during normal running conditions
(b) The gear teeth should have wear characteristics so that their life is satisfactory.
(c) The use of space and material should be recommended
(d) The alignment of the gears and deflections of the shaft must be considered because they effect on the performance of the gears
(e) The lubrication of the gears must be satisfactory

1. **Spur & Helical Gears** – When the shaft are parallel

2. **Bevel Gears** – When the shafts intersect at right angles, and,

3. **Worm & Worm Gears** – When the axes of the shaft are perpendicular and not intersecting. As a special case, when the axes of the two shafts are neither intersecting nor perpendicular crossed helical gears are employed.

The speed reduction or velocity ratio for a single pair of spur or helical gears is normally taken as 6: 1. On rare occasions this can be raised to 10: 1. When the velocity ratio increases, the size of the gear wheel increases. This results in an increase in the size of the gear box and the material cost increases. For high speed reduction two stage or three stage construction are used.

The normal velocity ratio for a pair of bend gears is 1: 1 which can be increased to 3: 1 under certain circumstances.

For high-speed reduction worm gears offers the best choice. The velocity ratio in their case is 60: 1, which can be increased to 100: 1. They are widely used in materials handling equipment due to this advantage.

Further, spur gears generate noise in high-speed applications due to sudden contact over the entire face with between two meeting teeth. Where as, in helical gears the contact between the two meshing teeth begans with a point and gradually extends along the tooth, resulting in guite operations.

From considerations spurgears are the cheapest. They are not only easy to manufacture but there exists a number of methods to manufacture them. The manufacturing of helical, bevel and worm gears is a specialized and costly operation.

Low of Gearing:

The fundamental law of gearing states “The common normal to the both profile at the point of contact should always pass through a fixed point called the pitch point, in order to obtain a constant velocity ratio.

MODULE:

The module specifies the size of gear tooth. Figure shows the actual sizes of gear tooth with four different modules. It is observed that as the modules increases, the size of the gear tooth also increases. It can be said that module is the index of the size of gear tooth.

Standard values of module are as shown

Recommended Series of Modules (mm)

Preferred (1)	Choice 2 (2)	Choice 3 (3)	Preferred (1)	Choice 2 (2)	Choice 3 (3)
1			8	7	(6.5)
1.25	1.125		10	9	
1.5	1.375		12	11	
2	1.75		16	14	
2.5	2.25		20	18	
3	2.75	(3.25)	25	22	
4	3.5		32	28	
5	4.5	(3.75)	40	36	
6	5.5		50	45	

Note: The modules given in the above table apply to spur and helical gears. In case of helical gears and double helical gears, the modules represent normal modules

The module given under choice 1, is always preferred. If that is not possible under certain circumstances module under choice 2, can be selected.

Standard proportions of gear tooth in terms of module m, for 20º full depth system.

Addendum = m
Dedendum = 1.25 m
Cl earance (c) = 0.25 m
Working depth = 2 m
Whole depth = 2.25 m
Tooth thickness = 1.5708 m $\frac{\pi d}{2z} = \frac{\pi mz}{2z} = 1.5708m$
Tooth space = 1.5708 m
Fillet radius = 0.4 m

Addendum	0.3183 p	1/P	m
Dedendum	0.3977 p	1.25/P	1.25 m
Tooth thickness	0.5 p	1.5708/P	1.5708 m
Tooth space	0.5 p	1.5708/P	1.5708 m
Working depth	0.6366 p	2/P	2 m
Whole depth	0.7160 p	2.25/P	2.25 m
Clearance	0.0794 p	0.25/P	0.25 m
Pitch diameter	zp/π	z/P	zm
Outside diameter	$(z+2)p/\pi$	(z+2)/P	$(z+2)\ m$
Root diameter	$(z-2.5)p/\pi$	(z − 2.5)/P	$(z-2.5)\ m$
Fillet radius	0.1273p	0.4/P	0.4 m

Selection of Material :

- The load carrying capacity of the gear tooth depends upon the ultimate tensile strength or yield strength of the material.
- When the gear tooth is subjected to fluctuating forces, the endurance strength of the tooth is the deciding factor.
- The gear material should have sufficient strength to resist failure due to breakage of tte tooth.
- In many cases, it is wear rating rather than strength rating which decides the dimensions of gear tooth.
- The resistance to wear depends upon alloying elements, grawn size, percentage of carbon and surface hardness.
- The gear material should have sufficient surface endurance strength to avoid failure due to destructive pitting.
- For high-speed power transmission, the sliding velocities are very high and the material should have a low co-efficient of friction to avoid failure due to scoring.
- The amount of thermal distortion or wrapping during the heat treatment process is a major problem on gear application.
- Due to wraping the load gets concentrated at one corner of the gear tooth.
- Alloy steels are superior to plain carbon steel in this respect (Thermal distortion)

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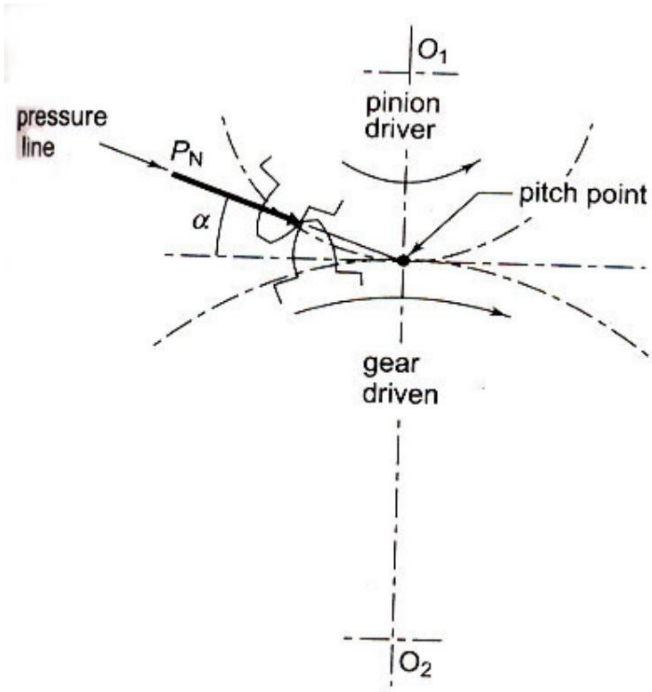
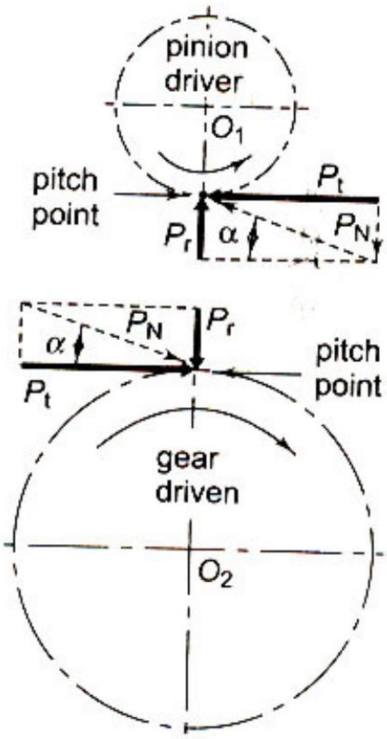
Material	Allowable static stress σ_0, MN/m^2 (kgf/mm^2)	BHN
Cast Iron Grade 20 ..	47.1 (4.80)	200
Cast Iron Grade 25 ..	56.4 (5.75)	220
Cast Iron Grade 35 ..	56.4 (5.75)	225
Cast Iron Grade 35 (Heat treated) ..	78.5 (8.00)	300
Cast steel, 0.20%C, untreated ..	138.3 (14.10)	180
Cast steel, 0.20%C, heat treated ..	193.2 (19.70)	250
Bronze ..	68.7 (7.00)	80
Phosphor gear bronze ..	82.4 (8 .40)	100
Manganese bronze ..	138.3 (14.10)	100
Aluminium bronze ..	152.0 (15.50)	180
Forged steel, about 0.30%C (untreated) ..	172.6 (17.60)	150
Forged steel, about 0.30%C (heat treated) ..	220.0 (22.40)	200
Steel, C30 (heat treated) ..	220.6 (22.50)	300
Steel, C40, untreated ..	207.0 (21.10)	150
Steel, C45, untreated ..	233.4 (23.80)	200
Alloy steel, case hardened ..	345.2 (35.20)	650
Cr-Ni Steel, about 0.45%C heat treated ..	462.0 (47.10)	400
Cr-Va steel, about 0.45%C, heat treated ..	516.8 (52.70)	450
Rawhide, Fabroil, etc. ..	41.2 (4.20)	—
Plastic ..	58.8 (6.00)	—
Laminated phenolic materials (Bakelite, Micarta, Celoron) ..	41.2 (4.20)	—
Laminated steel (silent material) ..	82.4 (8.40)	—

According to fundamental law of gear this resultant force P_N always acts along the pressure line.

This resultant force P_N, can be resolved into two components – tangential component P_t and radial components P_r at the pitch point.





The torque transmitted by the gear is given by

$$M_t = \frac{P \times 60}{2\,\pi\, N_1}$$

Where,

M_t = Torque transmitted gears (N- m)
PkW = Power transmitted by gears
N_1 = Speed of rotation (rev / mn)

The tangential component F_t acts at the pitch circle radius.

$$\therefore M_t = F_t \frac{d}{2}$$

OR

$$F_t = \frac{2M_t}{d}$$

Where,

M_t = Torque transmitted gears N- mm
d = Pitch Circle diameter, mm

Further, we know,

Power transmitted by gears = $\dfrac{2\pi\, N\, M_t}{60}$ (kW)

Where

$$F_r = F_t \tan\alpha$$

and

resultant force,

$$F_N = \frac{F_t}{Cos\alpha}$$

two pairs that are simultaneously in contact and share the load. This aspects is also neglected

iii) This analysis is valid under static conditions for example, when the gears are running at very low velocities. In practice there are dynamic forces in addition to force due to power transmission.

For gear tooth forces, It is always required to find out the magnitude and direction of two components. The magnitudes are determined by using equations

$$M_t = \frac{P \times 60}{2\pi N_1}$$

$$F_t = \frac{2M_t}{d_1}$$

Further, the direction of two components F_t and F_r are decided by constructing the free body diagram.

?

How

Minimum Number of Teeth:

The minimum number of teeth on pinion to avoid interference is given by

$$Z_{min} = \frac{2}{\sin^2 \alpha}$$

For 20˚full depth involute system, it is always safe to assume the number of teeth as 18 or 20

Once the number of teeth on the pinion is decided, the number of teeth on the gear is calculated by the velocity ratio $i = \frac{Z_2}{Z_1}$

Face Width:

In designing gears, it is required to express the face width in terms of module.

$\therefore \boxed{b = 12.5m}$

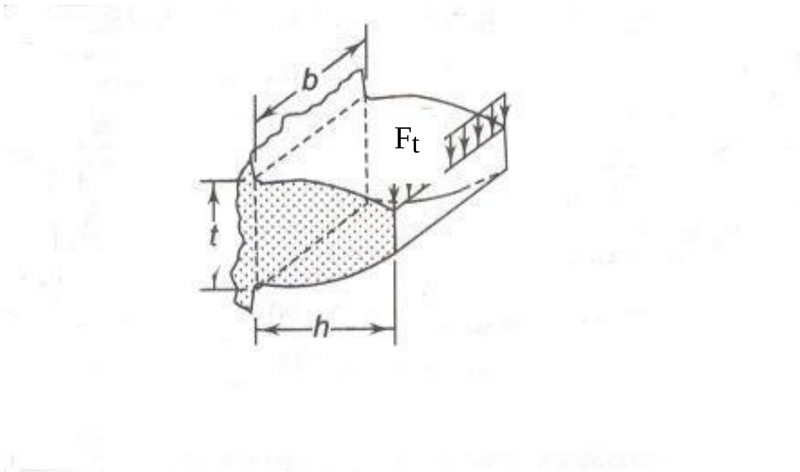
The LEWIS Bending Equation:

Wilfred Lewis introduced an equation for estimating the bending stress in gear teeth.

This equation announced in 1892 still remains the basis for most gear design today.

In the lewis analysis, the gear tooth is treated as a cantilever beam and the tangential component (F_t) causes the bending moment about the base of the tooth.

GEAR TOOTH AS CANTILEVER



The Lewis equation is based on the following assumption.

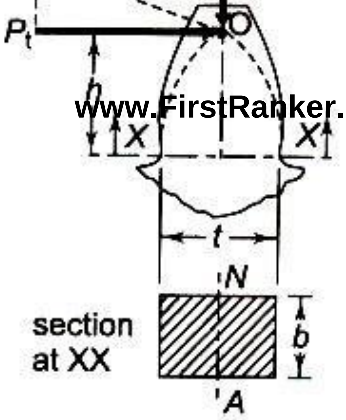
(i) The effect of radial component (F_r) which induces compressive stresses is neglected.

(ii) It is assumed that the tangential component (F_t) is uniformly distributed over the face width of the gear (this is possible when the gears are rigid and accurately machined.)

(iii) The effect of stress concentration is neglected.

(iv) It is assumed that at any time only one pair of teeth is in contact and Takes total load

The advantage of parabolic out lines is that it is a beam of uniform strength and the stress at any cross section is uniform.



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We know

$$M_b = F_t \times h$$

$$I = \frac{bt^3}{12} \text{ and } y = \frac{t}{2}$$

$$\therefore \sigma_b = \frac{M_b y}{I} = \frac{M_b}{(I/y)} = \frac{M_b}{Z}$$ (Z= Section modulus)

$$= \frac{bt^3}{12} / \frac{t}{2} = \frac{bt^2}{6}$$

σ_b = Permissible bending stress (N/mm^2) $\quad \frac{6F_b \times h}{bt^2}$

$$\therefore F_t = \frac{bt^2 \sigma_b}{6h}$$

Multiplying the numerator and denominator of the right hand side by m, (m=Module)

$$F_t = mb\sigma_b \frac{t^2}{6hm}$$

The bracketed quantity depends on the from of the tooth and is termed as **lewis from stress factor Y**

Let $\quad y = \frac{t^2}{6hm}$

Then the equation can be rewritten as $\quad F_t = mb\sigma_b y$

This y is called as <u>lewis form factor</u>

S_b = beam strength of gear teeth (N)

σ_b = Permissible bending stress (N/mm^2)

The above equation is known as LEWIS EQUATION

The values of the lewis from factor y is given in table below,

Values of Tooth-Form Factor (Lewis), Load at Tip of Tooth

z	y					
	$14\frac{1}{2}$-deg form	$14\frac{1}{2}$-deg variable centre distance	20-deg full depth form	20-deg stub-tooth form	Internal Gears: Spur pinion	Internal Gears: Internal gear
12	0.067	0.125	0.078	0.099	0.104	
13	0.071	0.123	0.083	0.103	0.104	
14	0.075	0.121	0.088	0.108	0.105	
15	0.078	0.120	0.092	0.111	0.105	
16	0.081	0.120	0.094	0.115	0.106	
17	0.084	0.120	0.096	0.117	0.109	
18	0.086	0.120	0.098	0.120	0.111	*
19	0.088	0.119	0.100	0.123	0.114	
20	0.090	0.119	0.102	0.125	0.116	
21	0.092	0.119	0.104	0.127	0.118	
22	0.093	0.119	0.105	0.129	0.119	
24	0.095	0.118	0.107	0.132	0.122	
26	0.098	0.117	0.110	0.135	0.125	
28	0.100	0.115	0.112	0.137	0.127	0.220
30	0.101	0.114	0.114	0.139	0.129	0.216
34	0.104	0.112	0.118	0.142	0.132	0.210
38	0.106	0.110	0.122	0.145	0.135	0.205
43	0.108	0.108	0.126	0.147	0.137	0.200
50	0.110	0.110	0.130	0.151	0.139	0.195
60	0.113	0.113	0.134	0.154	0.142	0.190
75	0.115	0.115	0.138	0.158	0.144	0.185
100	0.117	0.117	0.142	0.161	0.147	0.180
150	0.119	0.119	0.146	0.165	0.149	0.175
300	0.122	0.122	0.150	0.170	0.152	0.170
Rack	0.124	0.124	0.154	0.175		

*Internal gears with less than 28 teeth must be designed specially for the particular application, and their values of y must be determined for each one individually.

- We know that $S_b = mb\sigma_b y$

- It can be observed that 'm' and 'b' are same for pinion and as well as for gear in a gear pair,
- When different materials are used, the product $\sigma_b . y$ decides the weaker between the pinion and gear
- The lewis form factor y is always less for pinion compared to gear
- Therefore, when the same material is used for pinion and gear, the pinion is always weaker than the gear.

Effective load-Calculation

Earlier we have seen how to determine the tangential component of the resultant force between two meshing teeth.

This component can be calculated by using

I. $$M_t = \frac{P \times 60}{\quad}$$

And

II. $$F_t = \frac{2M_t}{d_1}$$

The value of the tangential component, depends upon rated power and rated speed.

In gear design, the maximum force (due to maximum torque) is the criterion. This is accounted by means of a factor called service factor – (C_S)

This service factor (C_S) is defined as

$$C_S = \frac{Maximum\ Torque}{Rated\ Torque}$$

$$(F_t)_{max} = C_s F_t$$

The values of service factors are given in table…

Service factor C_s for gears

Type of load	Type of service		
	Intermittent or 3 h per day	8 to 10 h per day	Continuous 24 h/day
Steady	0.80	1.0	1.25
Ligh shocks	1.00	1.25	1.50
Medium shocks	1.25	1.50	1.80
Heavy shocks	1.50	1.80	2.00

We know, that

σ_b is permissible static bending stress which is modified to $C_v \sigma_b$ where, C_V is the velocity factor used for taking into account the fatigue loading

This velocity factor C_V developed by. **Carl. G. Barth,** expressed in terms of pitch line velocity.

The values of velocity factor are as below

(i) $C_V = \dfrac{3}{3+V}$, for ordinary and commercially cut gears

(made with form cutters) and V<10 m / Sec

(ii) $C_v = \dfrac{6}{6+V}$,For accurately hobbed and generated gears and V < 20 m/Sec.

60 × 10³ d, mm

n, rev/min

(The velocity factor is an empirical relationship developed by past experience).

Dynamic effects (Dynamic Tooth Load)

When gears rotate at very low speed, the transmitted load P_t can be considered to be the actual force present between two meshing teeth

However in most of the cases the gears rotate at appreciable speed and it becomes necessary to consider the dynamic force resulting from impact between mating teeth.

The **Dynamic force** is induced due to the following factors

1. Inaccuracies of the tooth profile
2. Errors in tooth spacings
3. Misalignment between bearings
4. Elasticity of parts, and
5. Inertia of rotating masses.

There are two methods to account for Dynamics load.

I. Approxinate estimation by the velocity factor in the preliminary stages of gear design

II. Precise estimation by **Bucking Hams** equation in the final stages of gear design.

Note: Approximate estimation, Using velocity factor (C_v) developed by Barth discussed earlier.

In the final stages of gear desing when gear dimensions are known errors specified and quality of gears determined, the Dynamic load is calculated by equation derived by

Earle Buckingham

Where, F_d = Dynamic load

$= F_t + F_i$

Where, F_t = Tangential tooth load

F_i = Inevement load due to dynamic action

V = Pitch line Velocity (m/Sec)
C = Dynamic factor (N/mm^2) depending upon machining errors e = measured error in action between gears in mm
b = face width of tooth (mm)
F_t = tangential force due to rated torque (N) K_3 = 20.67 in SI units

The Dynamic factor C, depends upon modulus of elasticity of materials for pinion and gear and the form tooth or pressure angle and it is given by

$$C = \frac{e}{K\left(\frac{1}{E_1} + \frac{1}{E_2}\right)}$$

Where

K = Constant depending upon the form of tooth – (take from DDH)

E_2 = Modulus of elasticity of gear material (N/mm^2)

The Values of K, for various tooth forms are given as.

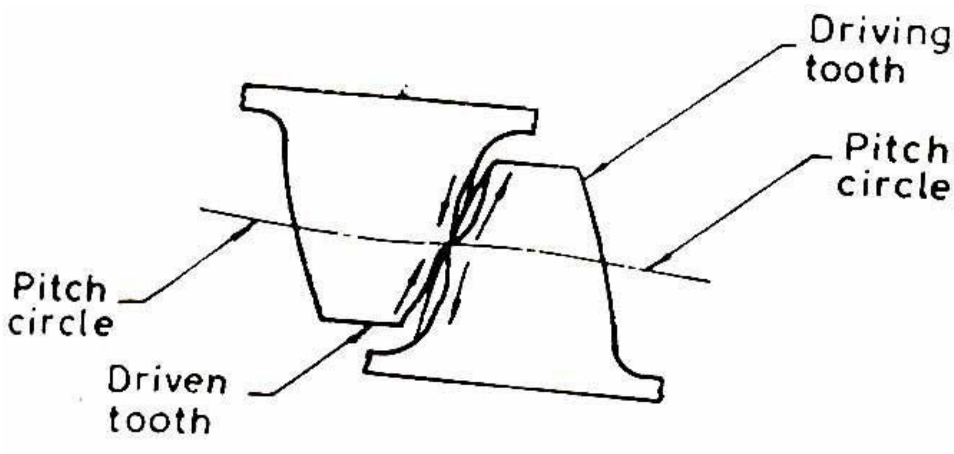
The error, e, in the dynamic load equation is measured error in action between gears in mm

This error depends upon the quality of gear and manufacturing methods.

- Normal wear
- Moderate wear
- Destructive wear
- Abrasive wear
- Scratching and etc.

Generally, normal wear (Polishinging in) does not constitute failure because it involves loss of metal at a rate too slow to affect performance

- Moderate wear refers to loss of metal more rapid than normal wear.
- This need not necessarily be destructive and may develop on heavily loaded gear teeth.
- Destructive wear usually results from loading that is excessive for the lubricant employed.
- The effect of destructive wear on the tooth profile of an involute gear is depicted in the figure.



PITTING

Pitting is the principal mode of failure of rolling surfaces. The details of the process vary with the material and operating conditions, but in all cases it manifests itself by the initiation and propagation of cracks in the near surface layer until microscopic pieces detach themselves to form a pit or a spall.

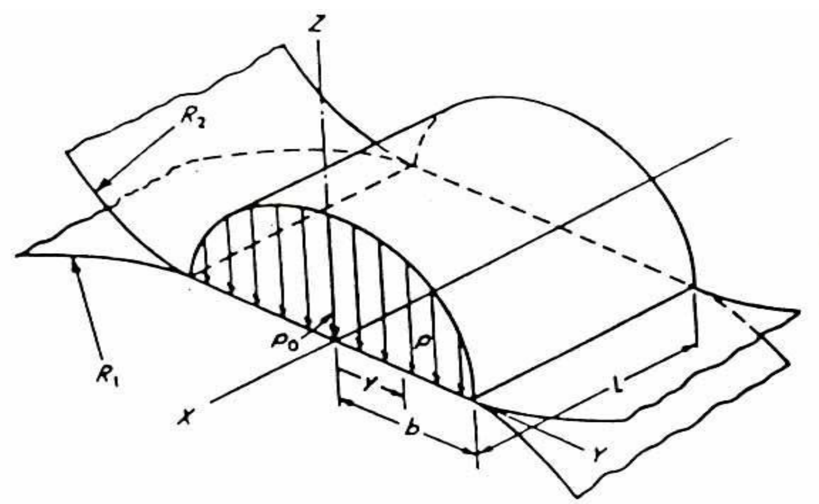
In spur gears surface pitting has long been recognised as one of the failure modes. This is often refereed to as “Pitch line Pitting”

The main factors affecting pitting type of failure,

- Contact stress.
- Material pouring and hardness.
- Surface finish and lubrication

In order to avoid this type of failure, the proportions of the gear tooth and surface properties such as surface hardness should be selected in such a way that the wear strength of the gear tooth is more than the effective load between the meshing teeth.

The Hertz stress is based on the assumptions of electric and isometric material behaviours, load is compressive and normal to the contacting surfaces which are stationary and the size of contacting area whose dimensions are relatively smaller compared with the curvature radius of the contacting bodies



The above figure,
Illustrates the contact are and corresponding stress distribution between two cylinders.

Here the area of contact stress which is theoretically rectangular with one dimension being the cylinder length L. (i.e. corresponding to face width of the gear)

The distribution of pressure is represented by a semi elliptical prism and the maximum contact pressure Po exists on the load axis,

The current gear design practice is to estimate the contact stress at the pitch point of the teeth by assuming line contact between two cylinders whose radil of contact depends on the gear geometry at the pitch point.

The analysis of wear strength was done by Earle Buckingham and was accepted by AGMA (American Gear Manufacturing Association) in 1926. This Buckingham equation gives the wear strength of the gear tooth based on Hertz theory of contact stress.

Hence, the maximum tooth load from wear consideration as evaluated from Hertz contact stress equation applied for pitch point contact is given by

$$\boxed{F_t = d_1 b Q K}$$

$$= \frac{2Z_2}{Z_2 + Z_1}$$

and

K = Load stress factor (also known as material combination factor in N / mm^2)

This load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle, and the modulus of elasticity of the materials of the gear.

According to Buckingham, this load stress factor is given by

$$K = \frac{(\sigma_{es})^2 \ Sin\,\alpha}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2} \right)$$

Where, σ_{es} = Surface endurance limit of a gear pair. (N / mm^2)

= [2.75 (BHN) -70]

Where BHN = the average Brinall hardness number of gear and pinion for the steels

Design procedure for spur gears:

(i) Find design tangential tooth load, from power transmitted and pitch line velocity

$$= F_T = \frac{1000\,p}{V} C_s \,.$$

(ii) Apply lewis relationship i.e. $F_t = \sigma_d \,.\, P_C \,.b \,.y = \sigma_d \ C_v \ b\pi \ m \ y$

a) This lewis equation is applied only to the weaker of the two wheels
b) When both the pinion and gear are made of the same material, then pinion is weaker.
c) When the pinion and gear are made of different materials then the product of (σ_o x y) is the deciding factor. The lewis equation is used to that wheel for which (σ_o x y) is less.
d) The product of [(σ_o . C_v) .y] is called as strength factor of the gear
e) The face width may be taken as 9.5m to 12.5m for cut teeth and 6.5m to 9.5 m for cast teeth.

(iii) Calculate the dynamic load (F_d) on the tooth by using Backingham equation, i.e.,

$F_s = \sigma_e \; b \; p_c \; y$

$= \sigma_e \; b \; \pi \; m \; y$

for safety against breakage $F_S > F_d$

(v) Finally find the wear tooth load by using the relation

$$F_W = d_1 b \, Q \, K$$

The wear load (F_W) should not be less than the dynamic load (F_D)

Design a pair of spur gears to transmit 20kW of power while operating for 8 – 10 hrs/day sustaining medium shock, from shaft rotating at 1000 rev/min to a parallel shaft which is to rotate at 310 rev/min. Assume the number of teeth on pinion to be 31 and 20° full depth involute tooth profile. if load factor C = 522.464 N/mm and also for wear load taking load stress factor, K = 0.279 N/mm^2. Suggest suitable hardness. Boththe pinio gears are made of cast steel 0.2% corbon (untreated) whose σ_d = 137.34 N/mm
check the design for dynamic load if

Given: P = 20kW, =20x10^{3}W, Z_1 = 31, Z_2 = 100, V. R = 1:3.225, ∝ = 20°Full depth. N_1 = 1000 rev/min, N_2 = 310 rev/min

Material: Cast steel 0.2% C, (untreated) σ_d = 137.34 N/mm^2

Type of load: Medium skock, with 8-10hrs/day.

C = dynamic factors depending up on machining errors 522.464 N/mm

K = load stress factor (wear) = 0.279 N/mm^2

Solution:

σ_{d1} = Allowable static stress = 207.0 N/mm2(Pinion)
σ_{d2}= 138.3N/mm2(Gear)

Let $C_v = \dfrac{3.05}{3.05 + V}$ (assume)

V = pitch lin velocity $= V = \dfrac{\pi \; d_1 \; N_1}{60}$

$V = 1.623m$ mm/Sec

For, Medium shock, with 08- 10 hrs/day the service factor C_S, for gear, C_S =1.5

The tangential tooth load $= F_t = \frac{1000\,p}{V} . C_s$ P, in kW,
V, m/ Sec

$$= \frac{20 \times 10^3}{1.623m} . \times 1.5$$

$$= \frac{18484}{m} . \text{ N}$$

Now $C_v = \frac{3.05}{3.05 + 1.623\,m.}$

W.K.T, Tooth form factor for the pinion,

$$= 0.154 - \frac{0.912}{Z_1} \quad \text{(for 20° full depth)}$$

$$= 0.154 - \frac{0.912}{31}$$

$$= 0.154 - 0.0294$$

$$= 0.1246$$

and Tooth from factor, for the gear

$$= 0.154 - \frac{0.912}{Z_2}$$

$$= 0.154 - \frac{0.912}{100} \qquad (\because Z_2 = 100)$$

$$= 0.154 - 0.00912$$

$$= 0.1448$$

Now, by using the lewis equation to the pinion, We have,

$$F_t = \sigma_d C_v b \pi m y_1$$

$$= \frac{18484}{m} \quad \frac{3.05}{3.05 + 1.623\, m} \times 10m \times \pi\, m \times 0.1246$$

$$= \frac{18484}{m} = \frac{2471.37\, m^2}{3.05 + 1.623\, m}$$

By hit and trial methd,

m =	LHS	RHS
01	18484	528.86
02	9242	1570.12
03	6161	2808.75
04	4621	4143.98
05	3696	5533.74
Let m =	4107	4833.65

M 04.5 Hence, m = module = 4.5 is OK.

But the standard module is 5.0 mm

∴Let us take $\boxed{m = 5.0 \text{ mm}}$

Face width = b = 10m (assumed)
= 10 x 5 =50mm

Pitch circle diameter of

i) Pinion , $d_1 = mz_1$
= 5 x 31 = 155mm

ii)Gear, $d_2 = mz_2$
= 5 x 100 = 500mm

$$= F_t = \frac{18484}{m5} = \frac{18484}{.} = 3696.8\, N_s$$

$F_d = F_t + F_i$

$$= F_i = \frac{K_3 v (Cb + F_t)}{K_3 v + \sqrt{Cb + F_t}}$$

$$\therefore F_d = 3696.8 + \frac{20.67 \times 8.115 (522.464 \times 50 + 3696.9)}{20.67 \times 8.115 + \sqrt{522.464 \times 50 + 3696.8}}$$

$$= 3696.8 + \frac{167.73 (29820)}{167.73 + 129820} = 3696.8 + \frac{5001708.6}{167.73 + 172.68}$$

$$= 3696.8 + \frac{5001708.6}{340.41}$$

$$= 3696.8 + 14693.18$$

$$\boxed{\therefore F_d = 18389.98 \text{ N}}$$

Assuming:

$\sigma_{en} = 259.0 \text{ N/mm}^2$

State tooth load or endurance strength of the tooth $= F_{en} = \sigma_{en} . b\pi$

$F_{en} = 259 \times 50 \times \pi \times 5 \times 0.1246$

$= 25345.89$ N

For medium shock taking $F_{en} = 1.35 F_d$

$= 1.35 \times 18389.98$

$= 24826.473$

i.e., $\frac{F_{en}}{F_d} = \frac{25345.89}{18389.98} = 1.378$

Design is safe

$F_w = d, b\ Q\ K$

$= 155 \text{ x } 50 \text{ x } 1.138 \text{ x } 0.279$

$= 2460.6$ N

Is the design is safe from the point of wear?

∴ find new k

$$= \frac{F_d}{155 \times 50 \times 0.138}$$

$$= \frac{18389}{8819.5}$$

$$\boxed{\therefore k = 2.08}$$

Heat treaded for 350 BHN

∴ $F_w > F_d$ design is safe

A pair of carefully cut spur gears with 20° stub involute profile is used to transmit a maximum power 22.5 kW at 200 rev/min. The velocity ratio is 1:2. The material used for both pinion and gear is medium cast iron, whose allowable, static stress may be taken as 60 Mpa. The approximate center distance may be taken as 600 mm, determine module and face width of the spur pinion and gear. Check the gear pair for dynamic and wear loads

The dynamic factor or deformations factor in Bucking hams dynamic load equation may be taken as 80, and material combination/load stress factor for the wear may be taken as 1.4

Given: VR = 2, N_1 = 200 rev/min, N_2 = 100 rev/ min, P = Power transmitted, 22.5 kW

Center distance = L = 600mm $\sigma_{d1} = \sigma_{d2}$ = 60Mpa, C= 80, K= 1.4

Assumption:

i) b = face width = 10m

ii) Steady load condition and 8 – 10 hrs/day

∴ $C_s = 1.0$

$= \frac{d_1 + d_2}{2}$.

and $= \frac{d_1 + 2d}{2}$. = 600 *mm*

∴ d_1 = 400mm = 0.4 m

d_2 = 800mm = 0.8m

V_1 = Pitch line velocity of pinion $= \frac{\pi d_1 N_1}{60}$.

$$V = \frac{\pi \times 0.4 \times 200}{60} = 4.2 m/\sec$$

Since V_i = pitch line velocity is less than 12 *m/sec* the velocity factor = C_v, may be taken as

$$= \frac{3.05}{3.05 + v}.$$

$$= \frac{3.05}{3.05 + v} = \frac{3.05}{3.05 + 4.2} = \frac{3.05}{7.25}.$$

= 0.421

Now, $Z_1 = \frac{d_1}{m} = \frac{400}{m}$.

∴ y_1 = tooth form factor $= 0.175 - \frac{0.910}{Z_1}$ (for 20° stub systems)

$$0.175 - \frac{0.910}{400} m$$

= 0.175 – 0.002275 m

W.K.T,

Design tangential tooth load $= F_t = \frac{P \times 10^3}{v} \times C_s$

$$= \frac{22.5 \times 10^3}{4.2} \times 1.0$$

= 5357 N

∴ m = 8.0 (standard)

Face width = b = 10m = 10 x 8 = 80mm

$$Z_1 = \frac{d_1}{m} = \frac{400}{m} = 50$$

$$Z_2 = \frac{d_2}{m8} = \frac{800}{} = 100$$

Checking two gears for dynamic and wear load

W.K.T

(i) Dynamic load = $F_d = F_T + F_i$

$$g = F_T + \frac{20.67 \times 4.2\,(80 \times 80 + 53.57)}{20.67 \times 4.2 + \sqrt{(80 \times 80 + 53.57)}}$$

= 5357 + 5273

= 10630 N

W.K.T,

y_1 = Tooth form factor for pinion = 0.175 – 0.002275m

= [0.175 – 0.002275 x 8]

= 0.175 – 0.018200

= 0.1568

Let flexural endurance limit (σe) for cast iron may be taken as 85 Mpa = (85 N/mm)

$$\therefore Fen = \sigma_{en} \,.\,.\, b\, \pi\, my$$

$$= 85 \times 80 \times \pi \times 8 \times 0.1568$$

= 26720 N

For steady loads $Fen = 1.25\, fd$.

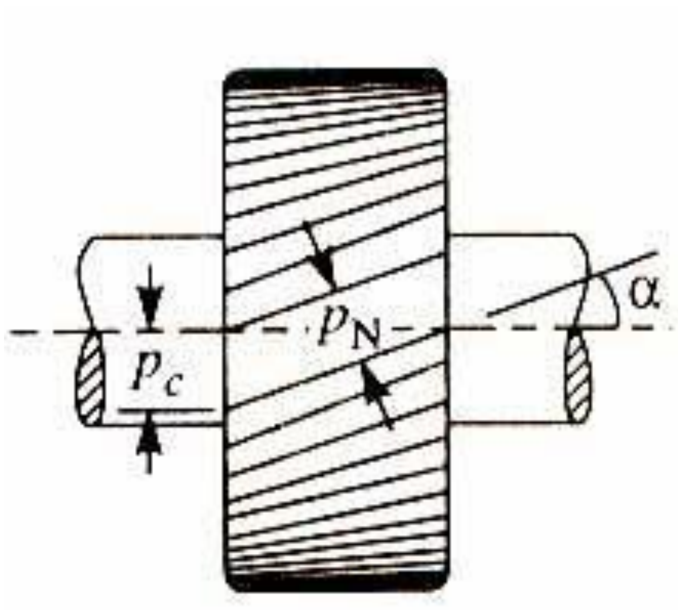
$VR + 1 \quad 2 + 1$

$= 1.33$

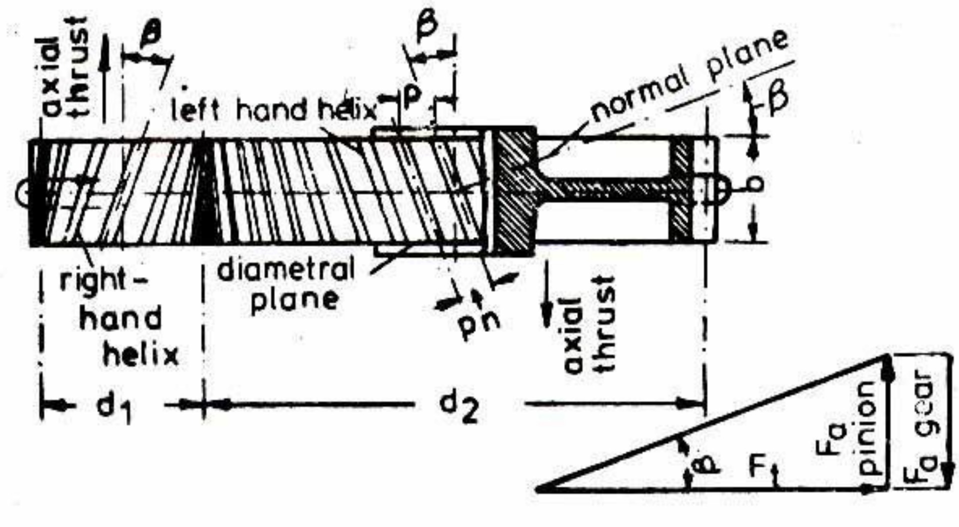
$F_w = d_1$ b Q K

$= 400 \times 80 \times 1.33 \times 1.4 = 59584$ N

Since both F_{en} and F_w are greater than F_d, the design is safe provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with high efficiency of transmission.



The helical gears may be single helical type or double helical type. In case of single helical type there is some axial thrust between the teeth which is a disadvantage. In order to eliminate this axial trust double helical gears (i.e., herning bone gears) are used. It is equivalent to two single helical gears, In which equal and opposite thrusts are provided on each gear and the resulting axial thrust is zero.



Normal pitch: It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by P_N.

$$P_N = P_C \cos\beta$$

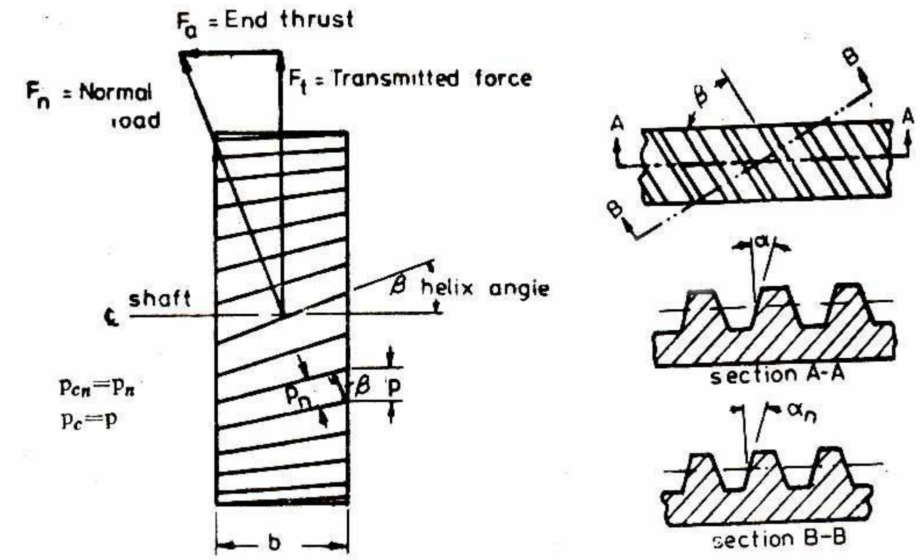
again α_N = Normal pressure angle

α = Pr. angle

$$\boxed{\tan\alpha_N = \tan\alpha \cos\beta}$$

Face width: In order to have more than one pair of teeth in contact, the tooth displacement (i.e., the advancement of one end of tooth over the other end) or over lap should be atleast equal to the axial pitch such that, over lap $P_C = b\tan\beta$--------- (i)

The normal tooth load (F_N) has two components, one is tangential component (F_t) and the other axial component (F_A) as shown in fig



The axial or end thrust is given by

$$F_A = F_N \sin\beta = F_t \tan\beta \text{ --------(ii)}$$

From the above equation (i), we see that as the helix angle increases then the tooth over lop increases. But at the same time the end thrust as given by the equation (ii) also increases which is not desirable. It is usually recommended that the over lop should be 15% of the circular pitch.

Over lop = $b\tan\beta = 1.11\ P_C$

$b = \frac{2.3 P_c}{\tan \beta}$ $= b = \frac{2.3 \times \pi\ m}{\tan \beta}$ $\geq = \frac{2.3 \times \pi\ m}{\sin \beta}$

3. In a single helical gears, the helix angle ranges from 20° to 35°, while for double helical gears it may be made up to 45°

$$b = 12.5\ m_n \text{ To } 20.m_n.$$

Formative or equivalent number of teeth for helical gear:

The formative or equivalent number of teeth for a helical gear may be defined as the number of teeth that can be generated on the surface of a cylinder having a radius equal to the radius of curvature at a point at the tip of the minor axis of an ellipse obtained by taking a section of the gear in the normal plane. Mathematically, formative or equivalent number of teeth an a helical gear

$$Z_E = Z / \cos^3. \beta$$

Z = Actual number of teeth on a helical gear and
β = helix angle.

Proportion of Helical Gears:

AGMA Recommendations.

Pressure angle in the plane of rotation	$\propto$ = 15° to 25°
Helix angle,	β = 20 - 45°
Addendum	= 0.8 m (maximum)
Dedendum	= 1.0 m
Minimum total depth	= 1.8 m (maximum)
Minimum clearance	= 0.2 m
Thickness of tooth	= 1.5708 m

STRENGTH OF HELICAL GEARS: (P962 K/G)

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore, in order to find the strength of helical gear, a modified lewis equation is used.

It is given by, $F_T = \sigma_o . C_V\ b\ \pi\ m\ y'$.

Where

(i) F_T, σ_o, C_V, b, π, m, as usual , with same meanings,

Item	Equation
(a) For low-angle helical gears when v is less than 5 m/s	$C_v \dfrac{4.58}{4.58+v}$
(b) For all helical and herringbone gears when v is 5 to 10 m/t	$C_v \dfrac{6.1}{6.1+v}$
(c) For gears when v is 10 to 20 m/s (Barth's formula)	$C_v \dfrac{15.25}{15.25+v}$
(d) For precision gear with v greater than 20 m/s	$C_v \dfrac{5.55}{5.55+\sqrt{v}}$
(e) For non metallic gears	$C_v \dfrac{0.7625}{1.0167+v} + 0.25$

(ii) The dynamic tooth load, $F_d = F_t + F_i$

$$\text{Where} \quad F_i = \frac{K_3 v \left(cb \cos^2 \beta + F_t\right) \cos \beta}{K_3 v + \left(cb \cos^2 \beta + F_t\right)^{1/2}}$$

K_S = 20.67 in SI units
= 6.60 in metric units,

(iii) The static tooth load or endurance strength of the tooth is given by

$$F_S = \sigma_e \, b \, \pi \, m y' \geq F_d$$

The maximum or limiting wear tooth load for helical gears is given by,

$$F_w = \frac{d_1 \, b \, Q \, K}{\cos^2 \beta} \geq F_d$$

Where d_1. b, Q and K have usual meanings as discussed in spur gears

In this case,

Where K = The load stress factor

$$K = \frac{(\sigma_{es})^2 \sin \alpha_N}{1.4} \left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

Given: $P = 15kW = 15 \times 10^3$ W, $\alpha = 20°$, $\beta = 45°$, $N_1 = 10{,}000$ rev/min,

$d_1 = 80mm = 0.08$ m, $d_2 = 320$ mm $= 0.32$ m, $\sigma_{d1} = \sigma_{d2} = 100$ MPa $= 100$ N/mm^2,

$\sigma_{es} = 618$ MPa $= 618$ N/mm^2

Since, both the pinion and gear are made of the same material (i.e., cast steel) the pinion is weaker. Thus the design is based on the pinion.

W K T,

Torque transmitted by the pinion

$$T = \frac{P \times 60}{2\pi N_1} = \frac{15 \times 10^3 \times 60}{2\pi \times 10{,}000} = 14.32 \text{ N-m}$$

∴ Tangential tooth load on the pinion $F_t = \frac{T}{d_1/2} = \frac{14.32}{0.08/2} = 358$ N

W.K.T

Number of teeth on the pinion $= Z_1 = \frac{d_1}{m} = \frac{80}{m}$

And formative or equivalent number of teeth for pinion $= Z_{E1} = \frac{Z_1}{\cos^3 \beta}$

$$= \frac{80/m}{\cos^3 45°} = \frac{80/m}{(0.707)^3} = \frac{226.4}{m}.$$

∴ Tooth from factor for pinion for 20°stub teeth

$$y'_1 = 0.175 - \frac{0.841}{Z_{E1}}$$

$$= 0.175 - \frac{0.841}{226.4/m} = 0.175 - 0.0037\,m$$

W.K.T

$$V = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.08 \times 10{,}000}{60} = 42\ m/Sec$$

∴ $C_v = \frac{5.55}{5.55 + \sqrt{V}} = \frac{5.55}{5.55 + \sqrt{42}}$ $\quad$ ∵ V is greater than 20 m/sec

W.K.T

tangential tooth load (F_t)

$= 358 = (\sigma_{d1}.\ C_V)\ b\ \pi\ m\ y_1$

$= (100 \times 0.461) \times 12.5m \times \pi \times m \times (0.175-0.003)$

$= 72m^2 - 1.5m^3$

By Trial and hit method,
Solution for m, =

m = 2.1 say 2.5 mm (standard)

and face width = b = 12.5 m = 12.5 x 2.5 = 31.25 mm say 32.0 mm

cheeking the gear for wear:

WKT. $V.R = \dfrac{d_2}{d_1} = \dfrac{320}{80} = 4$

$$Q = \frac{2 \times VR}{VR + 1} = \frac{2 \times 4}{4 + 1} = \frac{8}{5} = 1.6$$

WKT. $\tan \alpha_N = \tan \propto \cos \beta$

$= \tan 20° \cos 45°$

$= 0.2573$

$\therefore \alpha_N = 14.4°$

Since, both the gears are made of same material (i.e., cast steel).

Therefore, let

$E_1 = E_2 = 200 \times 10^3\ N/mm^2$

Load stress factor = $K = \dfrac{\sigma_{es}^2 . Sin\, \alpha_N}{1.4}\left(\dfrac{1}{E_1} + \dfrac{1}{E_2}\right)$

$$= \frac{618^2 \times Sin\, 14.4}{1.4}\left(\frac{1}{200 \times 10^3} + \frac{1}{200 \times 10^3}\right)$$

$= 0.678\ N/mm^2$

satisfactory for wear consideration.

Similarly try for F_d = dynamic load

$$F_d = F_t + F_i$$

$$= F_t + \frac{k_3 v (C_b \cos^2 \beta) \cos\beta}{k_3 v + \sqrt{C_b Cos^2 \beta + F_t}}$$

$$\left.\begin{array}{c}\text{C= dynamic factor depending} \\ \text{upon machine error} \\ \text{(for an error of 0.04)}\end{array}\right\} = 712.0$$

$$= 358 + \frac{20.67 \times 42\ (712 \times 32\ Cos^2\ 45 + 358) \cos 45}{(20.67 \times 42) + \sqrt{(712 \times 32\ Cos^2\ 45 + 358) \cos 45}}$$

$$= F_D = ?$$

Unit-VI
MACHINE TOOL ELEMENTS

A lever is a rigid rod or bar capable of turning about a fixed point called ***fulcrum***. It is used as a machine to lift a load by the application of a small effort. The ratio of load lifted to the effort applied is called ***mechanical advantage***. Sometimes, a lever is merely used to facilitate the application of force in a desired direction. A lever may be ***straight*** or ***curved*** and the forces applied on the lever (or by the lever) may be parallel or inclined to one another. The principle on which the lever works is same as that of moments.

effort point and fulcrum (l_2) is called ***effort arm***. According to the principle of moments,

$$W \times l_1 = P \times l_2 \quad \text{or} \quad \frac{W}{P} = \frac{l_2}{l_1}$$

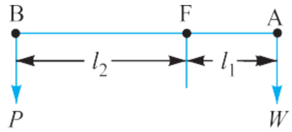


Fig. 15.1. Straight lever.

i.e. Mechanical advantage,

$$M.A. = \frac{W}{P} = \frac{l_2}{l_1}$$

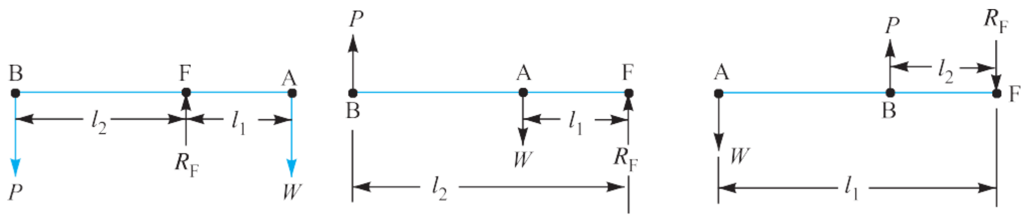
The ratio of the effort arm to the load arm *i.e.* l_2 / l_1 is called ***leverage***.

A little consideration will show that if a large load is to be lifted by a small effort, then the effort arm should be much greater than the load arm. In some cases, it may not be possible to provide a lever with large effort arm due to space limitations. Therefore in order to obtain a great leverage, ***compound levers*** may be used. The compound levers may be made of straight pieces, which may be attached to one another with pin joints. The bell cranked levers may be used instead of a number of jointed levers. In a compound lever, the leverage is the product of leverages of various levers.

Application of Levers in Engineering Practice

The load *W* and the effort *P* may be applied to the lever in three different ways as shown in Fig. 15.2. The levers shown at (*a*), (*b*) and (*c*) in Fig. 15.2 are called ***first type***, ***second type*** and ***third type*** of levers respectively.

In the ***first type*** of levers, the fulcrum is in between the load and effort. In this case, the effort arm is greater than load arm, therefore mechanical advantage obtained is more than one. Such type of levers are commonly found in bell cranked levers used in railway signalling arrangement, rocker arm in internal combustion engines, handle of a hand



(*a*) First type of lever. (*b*) Second type of lever. (*c*) Third type of lever.

pump, hand wheel of a punching press, beam of a balance, foot lever etc.

Fig. 15.2. Type of levers.

In the ***second type*** of levers, the load is in between the fulcrum and effort. In this case, the effort arm is more than load arm, therefore the mechanical advantage is more than one. The application of such type of levers is found in levers of loaded safety valves.

In the ***third type*** of levers, the effort is in between the fulcrum and load. Since the effort arm, in this case, is less than the load arm, therefore the mechanical advantage is less that one. The use of such type of levers is not recommended in engineering practice. However a pair of tongs, the treadle of a sewing machine etc. are examples of this type of lever.

Design of a Lever

The design of a lever consists in determining the physical dimensions of a lever when forces acting on the lever are given. The forces acting on the lever are

1. Load (W), **2.** Effort (P), and **3.** Reaction at the fulcrum F (R_F).

The load and effort cause moments in opposite directions about the fulcrum. The following procedure is usually adopted in the design of a lever :

1. Generally the load W is given. Find the value of the effort (P) required to resist this load by taking moments about the fulcrum. When the load arm is equal to the effort arm, the effort required will be equal to the load provided the friction at bearings is neglected.

2. Find the reaction at the fulcrum (R_F), as discussed below:

(i) When W and P are parallel and their direction is same as shown in Fig. 15.2 (*a*), then

$$R_F = W + P$$

The direction of R_F will be opposite to that of W and P.

(ii) When W and P are parallel and acts in opposite directions as shown in Fig. 15.2 (*b*) and (*c*), then R_F will be the difference of W and P. For load positions as shown in Fig. 15.2 (*b*),

$$R_F = W - P$$

and for load positions as shown in Fig. 15.2 (*c*),

$$R_F = P - W$$

The direction of R_F will be opposite to that of W or P whichever is greater.

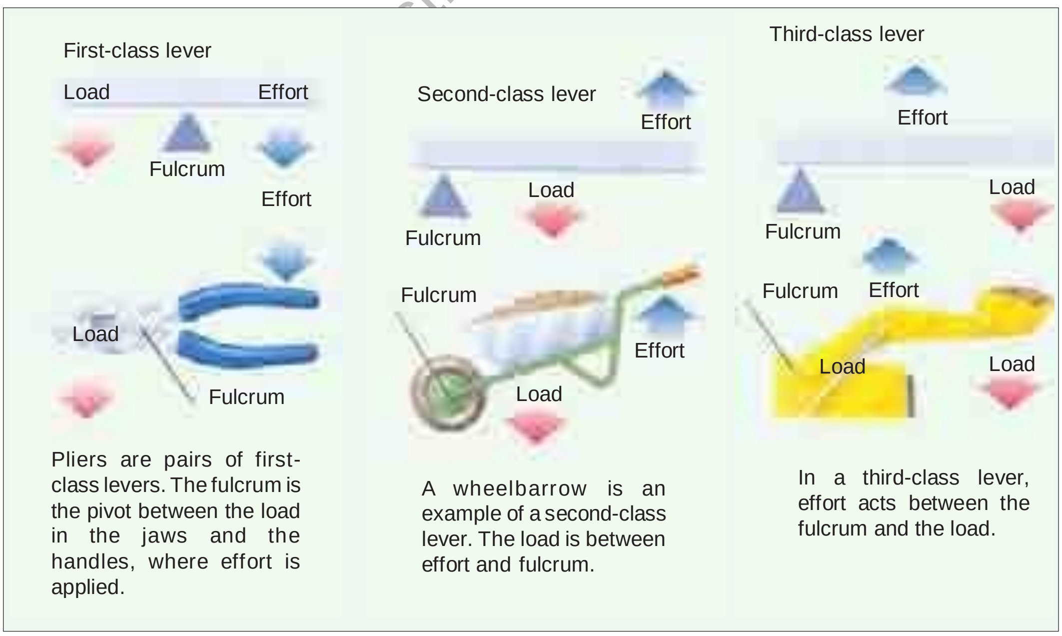
(iii) When W and P are inclined to each other as shown in Fig. 15.3 (*a*), then R_F, which is equal to the resultant of W and P, is determined by parallelogram law of forces. The line of action of R_F passes through the intersection of W and P and also through F. The direction of R_F depends upon the direction of W and P.

(iv) When W and P acts at right angles and the arms are inclined at an angle θ as shown in Fig. 15.3 (*b*), then R_F is determined by using the following relation:

$$R_F = \sqrt{W^2 + P^2 - 2W \times P \cos \theta}$$

In case the arms are at right angles as shown in Fig. 15.3 (*c*), then

$$R_F = \sqrt{W^2 + P^2}$$



Pliers are pairs of first-class levers. The fulcrum is the pivot between the load in the jaws and the handles, where effort is applied.

A wheelbarrow is an example of a second-class lever. The load is between effort and fulcrum.

In a third-class lever, effort acts between the fulcrum and the load.

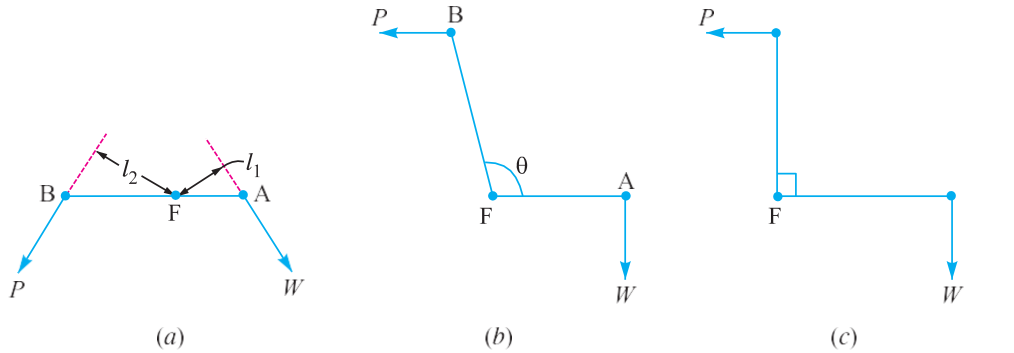


Fig. 15.3

3. Knowing the forces acting on the lever, the cross-section of the arm may be determined by considering the section of the lever at which the maximum bending moment occurs. In case of levers having two arms as shown in Fig. 15.4 (*a*) and cranked levers, the maximum bending moment occurs at the boss. The cross-section of the arm may be rectangular, elliptical or *I*-section as shown in Fig. 15.4 (*b*). We know that section modulus for rectangular section,

$$Z = \frac{1}{6} \times t \times h^2$$

where t = Breadth or thickness of the lever, and

h = Depth or height of the lever.

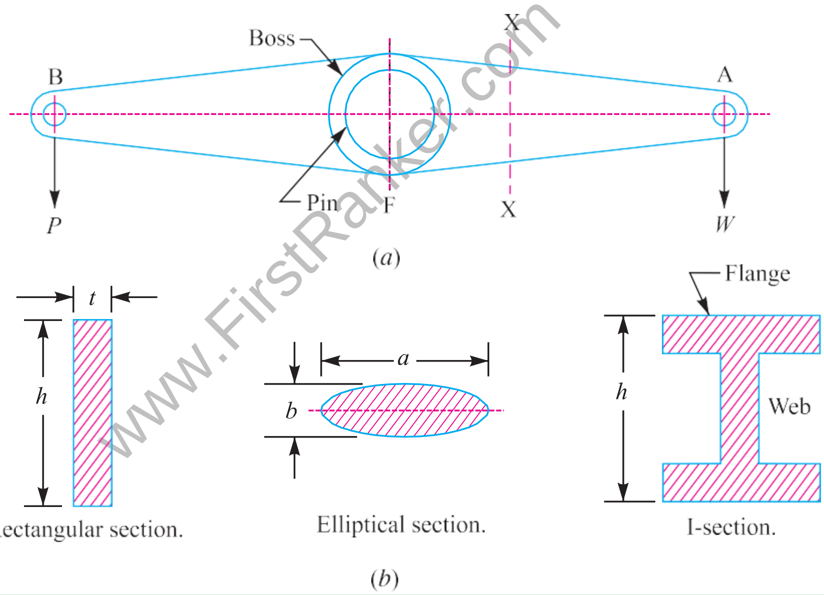


Fig. 15.4. Cross-sections of lever arm (Section at X-X).

The height of the lever is usually taken as 2 to 5 times the thickness of the lever. For elliptical section, section modulus,

$$Z = \frac{\pi}{32} \times b \times a^2$$

where a = Major axis, and b = Minor axis.

The major axis is usually taken as 2 to 2.5 times the minor axis.

For *I*-section, it is assumed that the bending moment is taken by flanges only. With this assumption, the section modulus is given by

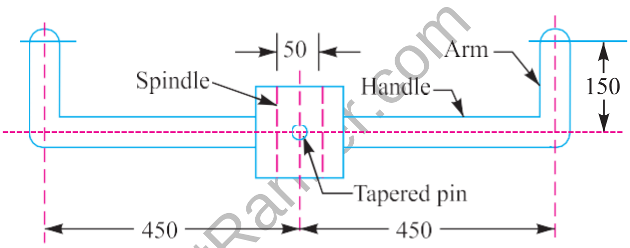
$$Z = \text{Flange area} \times \text{depth of section}$$

The section of the arm is usually tapered from the fulcrum to the ends. The dimensions of the arm at the ends depends upon the manner in which the load is applied. If the load at the end is applied by forked connections, then the dimensions of the lever at the end can be proportioned as a knuckle joint.

4. The dimensions of the fulcrum pin are obtained from bearing considerations and then checked for shear. The allowable bearing pressure depends upon the amount of relative motion between the pin and the lever. The length of pin is usually taken from 1 to 1.25 times the diameter of pin. If the forces on the lever do not differ much, the diameter of the pins at load and effort point shall be taken equal to the diameter of the fulcrum pin so that the spares are reduced. Instead of choosing a thick lever, the pins are provided with a boss in order to provide sufficient bearing length.

5. The diameter of the boss is taken twice the diameter of pin and length of the boss equal to the length of pin. The boss is usually provided with a 3 mm thick phosphor bronze bush with a dust proof lubricating arrangement in order to reduce wear and to increase the life of lever.

Example 15.1. *A handle for turning the spindle of a large valve is shown in Fig. 15.5. The length of the handle from the centre of the spindle is 450 mm. The handle is attached to the spindle by means of a round tapered pin.*



All dimensions in mm.

Fig. 15.5

If an effort of 400 N is applied at the end of the handle, find: 1. mean diameter of the tapered pin, and 2. diameter of the handle.

The allowable stresses for the handle and pin are 100 MPa in tension and 55 MPa in shear.

Solution. Given : L = 450 mm ; P = 400 N ; σ_t = 100 MPa = 100 N/mm^2 ; τ = 55 MPa= 55 N/mm^2

1. Mean diameter of the tapered pin

Let d_1 = Mean diameter of the tapered pin, and

d = Diameter of the spindle = 50 mm ...(Given)

We know that the torque acting on the spindle,

$$T = P \times 2L = 400 \times 2 \times 450 = 360 \times 10^3 \text{ N-mm} \quad ...(i)$$

Since the pin is in double shear and resists the same torque as that on the spindle, therefore resisting torque,

$$T = 2 \times \frac{\pi}{4} (d_1)^2 \tau \times \frac{d}{2} = 2 \times \frac{\pi}{4} (d_1)^2\, 55 \times \frac{50}{2} \text{ N-mm}$$

$$= 2160\, (d_1)^2 \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$(d_1)^2 = 360 \times 10^3 / 2160 = 166.7 \text{ or } d_1 = 12.9 \text{ say } 13 \text{ mm}$$ **Ans.**

2. Diameter of the handle

Let D = Diameter of the handle.

Since the handle is subjected to both bending moment and twisting moment, therefore the design will be based on either equivalent twisting moment or equivalent bending moment. We know that bending moment,

$$M = P \times L = 400 \times 450 = 180 \times 10^3 \text{ N-mm}$$

The twisting moment depends upon the point of application of the effort. Assuming that the effort acts at a distance 100 mm from the end of the handle, we have twisting moment,

$$T = 400 \times 100 = 40 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(180 \times 10^3)^2 + (40 \times 10^3)^2} = 184.4 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$184.4 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 55 \times D^3 = 10.8\, D^3$$

$\therefore$ $D^3 = 184.4 \times 10^3 / 10.8 = 17.1 \times 10^3$ or $D = 25.7$ mm

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(180 \times 10^3 + 184.4 \times 10^3) = 182.2 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$182.2 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82\, D^3 \qquad \text{...}(\because \sigma_b = \sigma_t)$$

$\therefore$ $D^3 = 182.2 \times 10^3 / 9.82 = 18.6 \times 10^3$ or $D = 26.5$ mm

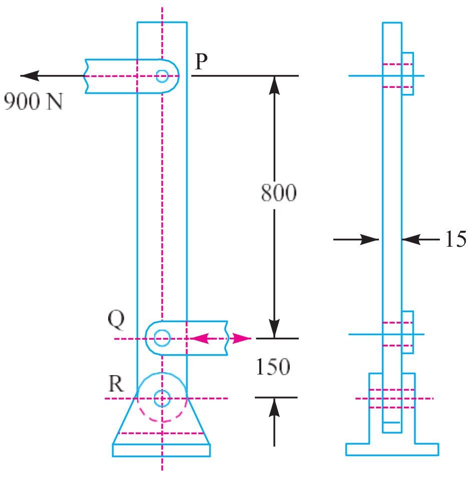
Taking larger of the two values, we have

$D = 26.5$ mm **Ans.**

Example 15.2. *A vertical lever PQR, 15 mm thick is attached by a fulcrum pin at R and to a horizontal rod at Q, as shown in Fig. 15.6.*

An operating force of 900 N is applied horizontally at P. Find :

1. *Reactions at Q and R,*
2. *Tensile stress in 12 mm diameter tie rod at Q*
3. *Shear stress in 12 mm diameter pins at P, Q and R, and*
4. *Bearing stress on the lever at Q.*



All dimensions in mm.

Fig. 15.6

Solution. Given : $t = 15$ mm ; $F_P = 900$ N

1. Reactions at Q and R

Let R_Q = Reaction at Q, and

R_R = Reaction at R,

Taking moments about R, we have

$$R_Q \times 150 = 900 \times 950 = 855\,000$$

$\therefore$ R_Q = 855 000 / 150 = 5700 N **Ans.**

These levers are used to change railway tracks.

Since the forces at P and Q are parallel and opposite as shown in Fig. 15.7, therefore

reaction at R, $R_R = R_Q - 900 = 5700 - 900 = 4800$ N **Ans.**

2. Tensile stress in the tie rod at Q

Let d_t = Diameter of tie rod = 12 mm ...(Given)

$\therefore$ Area, $A_t = \dfrac{\pi}{4}(12)^2 = 113 \text{ mm}^2$

We know that tensile stress in the tie rod,

$$\sigma_t = \frac{\text{Force at } Q\ (R_Q)}{\text{Cross-sectional area } (A_t)} = \frac{5700}{113}$$

$= 50.4 \text{ N/mm}^2 = 50.4$ MPa **Ans.**

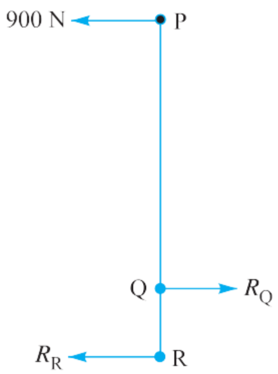


Fig. 15.7

3. Shear stress in pins at P, Q and R

Given : Diameter of pins at P, Q and R,

$$d_P = d_Q = d_R = 12 \text{ mm}$$

$\therefore$ Cross-sectional area of pins at P, Q and R,

$$A_P = A_Q = A_R = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$$

Since the pin at P is in single shear and pins at Q and R are in double shear, therefore shear stress in pin at P,

$$\tau_P = \frac{F_P}{A_P} = \frac{900}{113} = 7.96 \text{ N/mm}^2 = 7.96 \text{ MPa}$$ **Ans.**

Shear stress in pin at Q,

$$\tau_Q = \frac{R_Q}{2A_Q} = \frac{5700}{2 \times 113} = 25.2 \text{ N/mm}^2 = 25.2 \text{ MPa}$$ **Ans.**

and shear stress in pin at R,

$$\tau_R = \frac{R_R}{2A_R} = \frac{4800}{2 \times 113} = 21.2 \text{ N/mm}^2 = 21.2 \text{ MPa}$$ **Ans.**

4. Bearing stress on the lever at Q

Bearing area of the lever at the pin Q,

A_b = Thickness of lever × Diameter of pin = 15 × 12 = 180 mm^2

∴ Bearing stress on the lever at Q,

$$\sigma_b = \frac{R_Q}{A_b} = \frac{5700}{180} = 31.7 \text{ N/mm}^2 = 31.7 \text{ MPa} \quad \textbf{Ans.}$$

Hand Levers

A hand lever with suitable dimensions and proportions is shown in Fig. 15.8. Let

P = Force applied at the handle,

L = Effective length of the lever,

σ_t = Permissible tensile stress, and

τ = Permissible shear stress.

For wrought iron, σ_t may be taken as 70 MPa and τ as 60 MPa.

In designing hand levers, the following procedure may be followed :

1. The diameter of the shaft (d) is obtained by considering the shaft under pure torsion. We know that twisting moment on the shaft,

$$T = P \times L$$

and resisting torque,

$$T = \frac{\pi}{16} \times \tau \times d^3$$

From this relation, the diameter of the shaft (d) may be obtained.

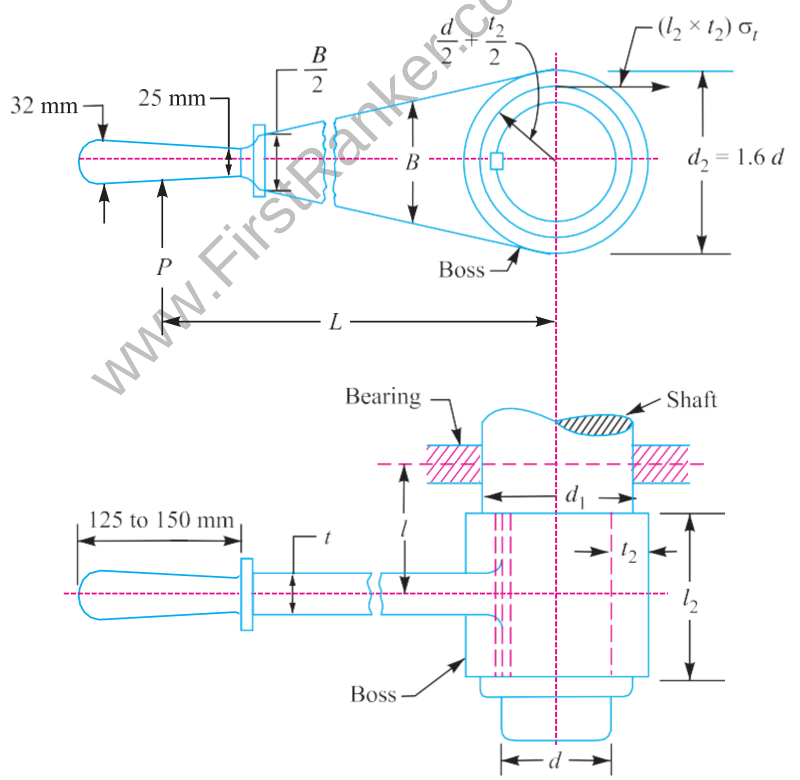


Fig. 15.8. Hand lever.

2. The diameter of the boss (d_2) is taken as 1.6 d and thickness of the boss (t_2) as 0.3d.

3. The length of the boss (l_2) may be taken from d to 1.25 d. It may be checked for a trial thickness t_2 by taking moments about the axis. Equating the twisting moment ($P \times L$) to the moment of resistance to tearing parallel to the axis, we get

$$P \times L = l_2 t_2 \sigma_t \left(\frac{d + t_2}{2}\right) \quad \text{or} \quad l_2 = \frac{2P \times L}{t_2 \sigma_t (d + t_2)}$$

4. The diameter of the shaft at the centre of the bearing (d_1) is obtained by considering the shaft in combined bending and twisting.

We know that bending moment on the shaft,

$$M = P \times l$$

and twisting moment, $T = P \times L$

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(P \times l)^2 + (P \times L)^2} = P\sqrt{l^2 + L^2}$$

We also know that equivalent twisting moment,

$$T_e = \frac{\pi}{16} \times \tau (d_1)^3 \text{ or } P\sqrt{l^2 + L^2} = \frac{\pi}{16} \times \tau (d_1)^3$$

The length l may be taken as 2 l_2.

From the above expression, the value of d_1 may be determined.

5. The key for the shaft is designed as usual for transmitting a torque of $P \times L$.

6. The cross-section of the lever near the boss may be determined by considering the lever in bending. It is assumed that the lever extends to the centre of the shaft which results in a stronger section of the lever.

Let t = Thickness of lever near the boss, and

B = Width or height of lever near the boss.

We know that the bending moment on the lever,

$$M = P \times L$$

Section modulus, $Z = \frac{1}{6} \times t \times B^2$

We know that the bending stress,

$$\sigma_b = \frac{L}{Z} = \frac{P \times L}{\frac{1}{6} \times t \times B^2} = \frac{6\,P \times L}{t \times B^2}$$

The width of the lever near the boss may be taken from 4 to 5 times the thickness of lever, *i.e.* $B = 4\,t$ to $5\,t$. The width of the lever is tapered but the thickness (t) is kept constant. The width of the lever near the handle is $B/2$.

Note: For hand levers, about 400 N is considered as full force which a man is capable of exerting. About 100 N is the mean force which a man can exert on the working handle of a machine, off and on for a full working day.

Foot Lever

A foot lever, as shown in Fig. 15.9, is similar to hand lever but in this case a foot plate is provided instead of handle. The foot lever may be designed in a similar way as discussed for hand lever. For foot levers, about 800 N is considered as full force which a man can exert in pushing a foot lever. The proportions of the foot plate are shown in Fig.

Example 15.3. *A foot lever is 1 m from the centre of shaft to the point of application of 800 N load. Find :*

1. Diameter of the shaft, 2. Dimensions of the key, and 3. Dimensions of rectangular arm of the foot lever at 60 mm from the centre of shaft assuming width of the arm as 3 times thickness.

The allowable tensile stress may be taken as 73 MPa and allowable shear stress as 70 MPa.

15.9.

Solution. Given : L = 1 m = 1000 mm ; P = 800 N ; σ_t = 73 MPa = 73 N/mm^2 ; τ = 70 MPa = 70 N/mm^2

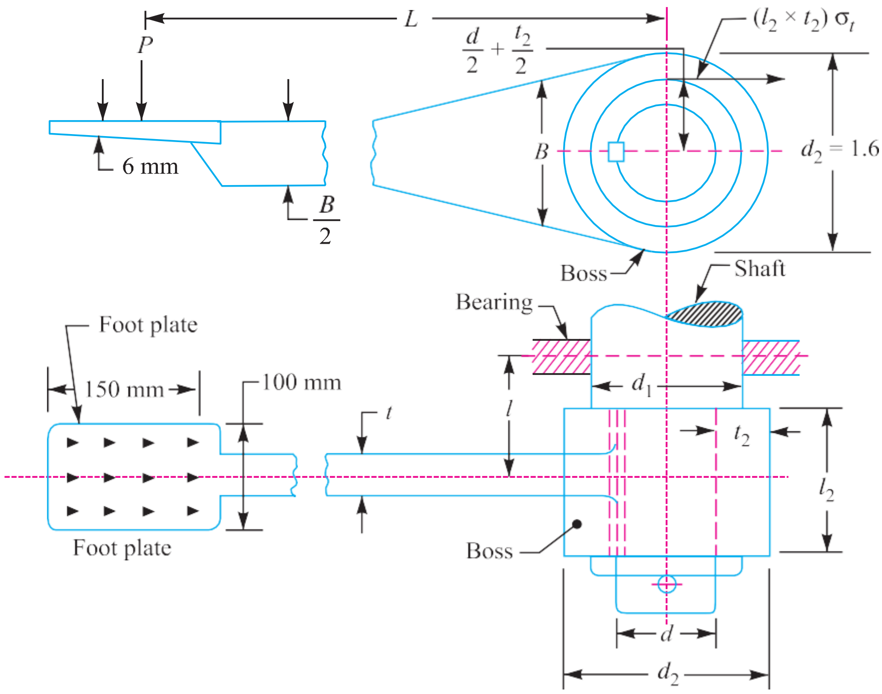


Fig. 15.9. Foot lever.

1. Diameter of the shaft

Let d = Diameter of the shaft. We know that the twisting moment on the shaft,

$$T = P \times L = 800 \times 1000 = 800 \times 10^3 \text{ N-mm}$$

We also know that the twisting moment on the shaft (T),

$$800 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$\therefore$ $d^3 = 800 \times 10^3 / 13.75 = 58.2 \times 10^3$

or d = 38.8 say 40 mm **Ans.**

We know that diameter of the boss,

$$d_2 = 1.6\, d = 1.6 \times 40 = 64 \text{ mm}$$

Thickness of the boss,

$$t_2 = 0.3\, d = 0.3 \times 40 = 12 \text{ mm}$$

and length of the boss, $l_2 = 1.25\, d = 1.25 \times 40 = 50$ mm

Now considering the shaft under combined bending and twisting, the diameter of the shaft at the centre of the bearing (d_1) is given by the relation

$$\frac{\pi}{16} \times \tau\, (d_1)^3 = P\sqrt{l^2 + L^2}$$

$$\frac{\pi}{16} \times 70 \times (d_1)^3 = 800\sqrt{(100)^2 + (1000)^2} \quad \text{...(Taking } l = 2\, l_2)$$

or $13.75\,(d_1)^3 = 804 \times 10^3$

$\therefore$ $(d_1)^3 = 804 \times 10^3 / 13.75 = 58.5 \times 10^3$ or d_1 = 38.8 say 40 mm **Ans.**

2. Dimensions of the key

The standard dimensions of the key for a 40 mm diameter shaft are :

Width of key, w = 12 mm **Ans.**

and thickness of key = 8 mm **Ans.**

The length of the key (l_1) is obtained by considering shearing of the key.

We know that twisting moment (T),

$$800 \times 10^3 = l_1 \times w \times \tau \times \frac{d}{2}$$

$$= l_1 \times 12 \times 70 \times \frac{40}{2} = 16\,800\, l_1$$

$\therefore$ $l_1 = 800 \times 10^3 / 16\,800 = 47.6$ mm

It may be taken as equal to the length of boss (l_2).

$\therefore$ $l_1 = l_2 = 50$ mm **Ans.**

Accelerator and brake levers inside an automobile.

3. Dimensions of the rectangular arm at 60 mm from the centre of shaft

Let t = Thickness of arm in mm,

and B = Width of arm in mm = $3\,t$ ...(Given)

$\therefore$ Bending moment at 60 mm from the centre of shaft,

$$M = 800\,(1000 - 60) = 752 \times 10^3 \text{ N-mm}$$

and section modulus, $Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\,(3t)^2 = 1.5\, t^3$ mm^3

We know that the tensile bending stress (σ_t),

$$73 = \frac{M}{Z} = \frac{752 \times 10^3}{1.5\, t^3} = \frac{501.3 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 501.3 \times 10^3 / 73 = 6.87 \times 10^3$

or t = 19 say 20 mm **Ans.**

and $B = 3\,t = 3 \times 20 = 60$ mm **Ans.**

The width of the arm is tapered while the thickness is kept constant throughout. The width of the arm on the foot plate side,

$$B_1 = B / 2 = 30 \text{ mm } \textbf{Ans.}$$

Cranked Lever

A cranked lever, as shown in Fig. 15.10, is a hand lever commonly used for operating hoisting winches.

The lever can be operated either by a single person or by two persons. The maximum force in order to operate the lever may be taken as 400 N and the length of handle as 300 mm. In case the lever is operated by two persons, the maximum force of operation will be doubled and length of handle may be taken as 500 mm. The handle is covered in a pipe to prevent hand scoring. The end of the shaft is usually squared so that the lever may be easily fixed and removed. The length (L) is usually from 400 to 450 mm and the height of the shaft centre line from the ground is usually one metre. In order to design such levers, the following procedure may be adopted :

1. The diameter of the handle (d) is obtained from bending considerations. It is assumed that the effort (P) applied on the handle acts at $\frac{2}{3}$ rd of its length (l).

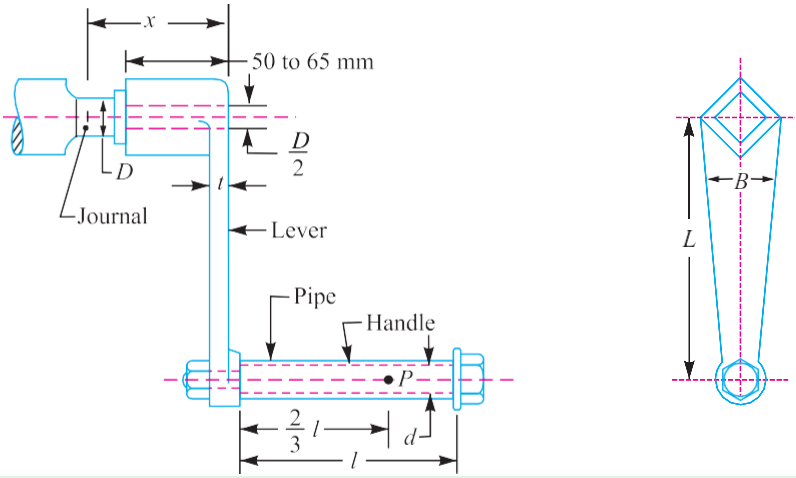


Fig. 15.10. Cranked lever.

$\therefore$ Maximum bending moment,

$$M = P \times \frac{2l}{3} = \frac{2}{3} \times P \times l$$

and section modulus,
$$Z = \frac{\pi}{32} \times d^3$$

$$\therefore \text{ Resisting moment } = \sigma_b \times Z = \sigma_b \times \frac{\pi}{32} \times d^3$$

where σ_b = Permissible bending stress for the material of the handle.

Equating resisting moment to the maximum bending moment, we have

$$\sigma_b \times \frac{\pi}{32} \times d^3 = \frac{2}{3} \times P \times l$$

From this expression, the diameter of the handle (d) may be evaluated. The diameter of the handle is usually proportioned as 25 mm for single person and 40 mm for two persons.

2. The cross-section of the lever arm is usually rectangular having uniform thickness through- out. The width of the lever arm is tapered from the boss to the handle. The arm is subjected to constant twisting moment, $T = \frac{2}{3} \times P \times l$ and a varying bending moment which is maximum near the boss. It is assumed that the arm of the lever extends upto the centre of shaft, which results in a slightly stronger lever.

$\therefore$ Maximum bending moment = $P \times L$

Since, at present time, there is insufficient information on the subject of combined bending and twisting of rectangular sections to enable us to find equivalent bending or twisting, with sufficient accuracy, therefore the indirect procedure is adopted.

We shall design the lever arm for 25% more bending moment.

$\therefore$ Maximum bending moment

$$M = 1.25\, P \times L$$

Let t = Thickness of the lever arm, and

B = Width of the lever arm near the boss.

∴ Section modulus for the lever arm,

$$Z = \frac{1}{6} \times t \times B^2$$

Now by using the relation, $\sigma_b = M / Z$, we can find t and B. The width of the lever arm near the boss is taken as twice the thickness *i.e.* $B = 2t$.

After finding the value of t and B, the induced bending stress may be checked which should not exceed the permissible value.

3. The induced shear stress in the section of the lever arm near the boss, caused by the twisting moment, $T = \frac{2}{3} \times P \times l$ may be checked by using the following relations :

$$T = \frac{2}{9} \times B \times t^2 \times \tau \qquad \text{...(For rectangular section)}$$

$$= \frac{2}{9} \times t^3 \times \tau \qquad \text{...(For square section of side } t\text{)}$$

$$= \frac{\pi}{16} \times B \times t^2 \times \tau \qquad \text{...(For elliptical section having major axis } B \text{ and minor axis } t\text{)}$$

4. Knowing the values of σ_b and τ, the maximum principal or shear stress induced may be checked by using the following relations:

Maximum principal stress,

$$\sigma_{b(max)} = \frac{1}{2}\left[\sigma_b + \sqrt{(\sigma_b)^2 + 4\tau^2}\right]$$

Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

5. Since the journal of the shaft is subjected to twisting moment and bending moment, there- fore its diameter is obtained from equivalent twisting moment.

We know that twisting moment on the journal of the shaft,

$$T = P \times L$$

and bending moment on the journal of the shaft,

$$M = P\left(\frac{2l}{3} + x\right)$$

where x = Distance from the end of boss to the centre of journal.

∴ Equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = P\sqrt{\left(\frac{2l}{3} + x\right)^2 + L^2}$$

We know that equivalent twisting moment,

$$T_e = \frac{\pi}{16} \times \tau \times D^3$$

From this expression, we can find the diameter (D) of the journal. The diameter of the journal is usually taken as

D = 30 to 40 mm, for single person

= 40 to 45 mm, for two persons.

Note: The above procedure may be used in the design of overhung cranks of engines.

Example 15.4. *A cranked lever, as shown in 15.10, has the following dimensions :*

Length of the handle = 300 mm

Length of the lever arm = 400 mm

Overhang of the journal = 100 mm

If the lever is operated by a single person exerting a maximum force of 400 N at a distance of $\frac{1}{3}$ rd length of the handle from its free end, find : 1. Diameter of the handle, 2. Cross-section of the lever arm, and 3. Diameter of the journal.

The permissible bending stress for the lever material may be taken as 50 MPa and shear stress for shaft material as 40 MPa.

Solution. Given : l = 300 mm ; L = 400 mm ; x = 100 mm ; P = 400 N ; σ_b = 50 MPa = 50 N/mm^2 ; τ = 40 MPa = 40 N/mm^2

1. Diameter of the handle

Let d = Diameter of the handle in mm.

Since the force applied acts at a distance of 1/3 rd length of the handle from its free end, therefore maximum bending moment,

$$M = \left(1 - \frac{1}{3}\right) P \times l = \frac{2}{3} \times P \times l = \frac{2}{3} \times 400 \times 300 \text{ N-mm}$$

$$= 80 \times 10^3 \text{ N-mm} \qquad ...(i)$$

Section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$

$\therefore$ Resisting bending moment,

$$M = \sigma_b \times Z = 50 \times 0.0982\, d^3 = 4.91\, d^3 \text{ N-mm} \qquad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$d^3 = 80 \times 10^3 / 4.91 = 16.3 \times 10^3 \text{ or } d = 25.4 \text{ mm } \textbf{Ans.}$$

2. Cross-section of the lever arm

Let t = Thickness of the lever arm in mm, and

B = Width of the lever arm near the boss, in mm.

Since the lever arm is designed for 25% more bending moment, therefore maximum bending moment,

$$M = 1.25\, P \times L = 1.25 \times 400 \times 400 = 200 \times 10^3 \text{ N-mm}$$

Section modulus, $Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\,(2t)^2 = 0.667\, t^3$ ...(Assuming $B = 2t$)

We know that bending stress (σ_b),

$$50 = \frac{M}{Z} = \frac{200 \times 10^3}{0.667\, t^3} = \frac{300 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 300 \times 10^3/50 = 6 \times 10^3$ or t = 18.2 say 20 mm **Ans.**

and $B = 2\,t = 2 \times 20 = 40$ mm **Ans.**

Let us now check the lever arm for induced bending and shear stresses.

Bending moment on the lever arm near the boss (assuming that the length of the arm extends upto the centre of shaft) is given by

$$M = P \times L = 400 \times 400 = 160 \times 10^3 \text{ N-mm}$$

and section modulus, Z =

$$\frac{1}{6} \times t \times B^2 = \frac{1}{6} \times 20\,(40)^2 = 5333 \text{ mm}^3 \quad \_ \qquad \_$$

∴ Induced bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{160 \times 10^3}{5333} = 30 \text{ N/mm}^2 = 30 \text{ MPa}$$

The induced bending stress is within safe limits. We know that the twisting moment,

$$T = \frac{2}{3} \times P \times l = \frac{2}{3} \times 400 \times 300 = 80 \times 10^3 \text{ N-mm}$$

We also know that the twisting moment (T),

$$80 \times 10^3 = \frac{2}{9} \times B \times t^2 \times \tau = \frac{2}{9} \times 40\,(20)^2\,\tau = 3556\,\tau$$

$$\therefore \quad \tau = 80 \times 10^3 / 3556 = 22.5 \text{ N/mm}^2 = 22.5 \text{ MPa}$$

The induced shear stress is also within safe limits.

Let us now check the cross-section of lever arm for maximum principal or shear stress. We know that maximum principal stress,

$$\sigma_{b\,(max)} = \frac{1}{2}\left[\sigma_b + \sqrt{(\sigma_b)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[30 + \sqrt{(30)^2 + 4\,(22.5)^2}\right]$$

$$= \frac{1}{2}(30 + 54) = 42 \text{ N/mm}^2 = 42 \text{ MPa}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{\sqrt{(30)^2 + 4\,(22.5)^2}}{2} = 27 \text{ N/mm}^2 = 27 \text{ MPa}$$

The maximum principal and shear stresses are also within safe limits.

3. Diameter of the journal

Let D = Diameter of the journal.

Since the journal of the shaft is subjected to twisting moment and bending moment, therefore its diameter is obtained from equivalent twisting moment.

We know that equivalent twisting moment,

$$T_e = P\sqrt{\left(\frac{2l}{3} + x\right)^2 + L^2} = 400\sqrt{\left(\frac{2 \times 300}{3} + 100\right)^2 + (400)^2}$$

$$= 200 \times 10^3 \text{ N-mm}$$

We know that equivalent twisting moment (T_e),

$$200 \times 10^3 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times D^3 = 7.86\,D^3$$

$$\therefore \quad D^3 = 200 \times 10^3 / 7.86 = 25.4 \times 10^3 \text{ or } D = 29.4 \text{ say } 30 \text{ mm}$$ **Ans.**

15.7 Lever for a Lever Safety Valve

A lever safety valve is shown in Fig. 15.11. It is used to maintain a constant safe pressure inside the boiler. When the pressure inside the boiler increases the safe value, the excess steam blows off through the valve automatically. The valve rests over the gunmetal seat which is secured to a casing fixed upon the boiler. One end of the lever is pivoted at the fulcrum F by a pin to the toggle, while the other end carries the weights. The valve is held on its seat against the upward steam pressure by the force P provided by the weights at B. The weights and its distance from the fulcrum are so adjusted that when the steam pressure acting upward on the valve exceeds the normal limit, it lifts the valve and the lever with its weights. The excess steam thus escapes until the pressure falls to the required limit.

The lever may be designed in the similar way as discussed earlier. The maximum steam load (W), at which the valve blows off, is given by

$$W = \frac{\pi}{4} \times D^2 \times p$$

where D = Diameter of the valve, and

p = Steam pressure.

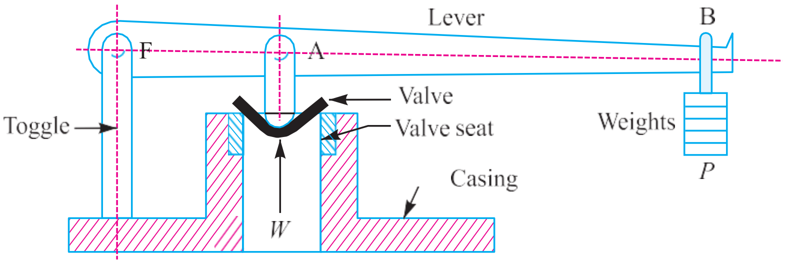


Fig. 15.11. Lever safety valve.

Example 15.5. *A lever loaded safety valve is 70 mm in diameter and is to be designed for a boiler to blow-off at pressure of 1 N/mm² gauge. Design a suitable mild steel lever of rectangular cross-section using the following permissible stresses :*

Tensile stress = 70 MPa; Shear stress = 50 MPa; Bearing pressure intensity = 25 N/mm².

The pin is also made of mild steel. The distance from the fulcrum to the weight of the lever is 880 mm and the distance between the fulcrum and pin connecting the valve spindle links to the lever is 80 mm.

Solution. Given : $D = 70$ mm ; $p = 1$ N/mm² ; $\sigma_t = 70$ MPa $= 70$ N/mm² ; $\tau = 50$ MPa $= 50$ N/mm² ; $p_b = 25$ N/mm² ; $FB = 880$ mm ; $FA = 80$ mm

We know that the maximum steam load at which the valve blows off,

$$W = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4}(70)^2 \times 1 = 3850 \text{ N} \quad \text{...(i)}$$

Taking moments about the fulcrum F, we have

$$P \times 880 = 3850 \times 80 = 308 \times 10^3 \text{ or } P = 308 \times 10^3 / 880 = 350 \text{ N}$$

Since the load (W) and the effort (P) in the form of dead weight are parallel and opposite, therefore reaction at F,

$$R_F = W - P = 3850 - 350 = 3500 \text{ N}$$

This rection will act vertically downward as shown in Fig. 15.12.

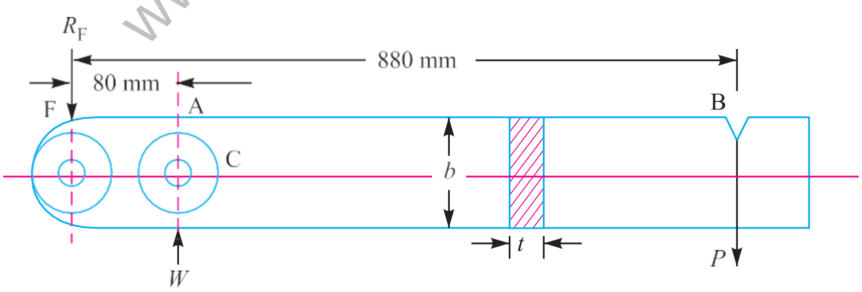


Fig. 15.12

First of all, let us find the diameter of the pin at A from bearing considerations.

Let d_p = Diameter of the pin at A, and

l_p = Length of the pin at A.

∴ Bearing area of the pin at A

$$= d_p \times l_p = 1.25\,(d_p)^2 \qquad \text{...(Assuming } l_p = 1.25\,d_p\text{)}$$

and load on the pin at A = Bearing area × Bearing pressure

$$= 1.25\,(d_p)^2\,p_b = 1.25\,(d_p)^2\;25 = 31.25\,(d_p)^2 \qquad \text{...}(ii)$$

Since the load acting on the pin at A is $W = 3850$ N, therefore from equations **(*i*)** and **(*ii*)**, we get $(d_p)^2 = 3850 / 31.25 = 123.2$ or $d_p = 11.1$ say 12 mm **Ans.**

and $l_p = 1.25\,d_p = 1.25 \times 12 = 15$ mm **Ans.**

Let us now check the pin for shearing. Since the pin is in double shear, therefore load on the pin at A (W),

$$3850 = 2 \times \frac{\pi}{4}(d_p)^2\,\tau = 2 \times \frac{\pi}{4}(12)^2\,\tau = 226.2\,\tau$$

∴ $\tau = 3850 / 226.2 = 17.02$ N/mm^2 = 17.02 MPa

This value of shear stress is less than the permissible value of 50 MPa, therefore the design for pin at A is safe. Since the load at F does not very much differ with the load at A, therefore the same diameter of pin may be used at F, in order to facilitate the interchangeability of parts.

∴ Diameter of the fulcrum pin at F

= 12 mm

A gun metal bush of 2 mm thickness is provided in the pin holes at A and F in order to reduce wear and to increase the life of lever.

∴ Diameter of hole at A and F

= 12 + 2 × 2 = 16 mm

and outside diameter of the boss

= 2 × Dia. of hole = 2 × 16 = 32 mm

Power clamp of an excavator.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Now let us find out the cross-section of the lever considering the bending moment near the boss at A.

Let t = Thickness of the lever, and

b = Width of the lever.

Bending moment near the boss at A *i.e.* at point C,

$$M = P \times BC = P\,(BF - AF - AC) = 350\left(880 - 80 - \frac{16}{2}\right) \text{ N-mm}$$

$$= 277\,200 \text{ N-mm}$$

and section modulus,

$$Z = \frac{1}{6} \times t.b^2 = \frac{1}{6} \times t\,(4\,t)^2 = 2.67\,t^3 \qquad \text{...(Assuming } b = 4\,t\text{)}$$

We know that the bending stress (σ_b)

$$70 = \frac{M}{Z} = \frac{277\,200}{2.67\,t^3} = \frac{104 \times 10^3}{t^3} \qquad \text{...}(\because \sigma_b = \sigma_t)$$

$\therefore$ $t^3 = 104 \times 10^3 / 70 = 1.5 \times 10^3$ or $t = 11.4$ say 12 mm **Ans.**

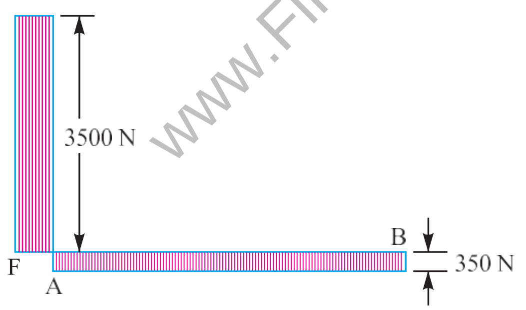
and $b = 4\,t = 4 \times 12 = 48$ mm **Ans.**

Now let us check for the maximum shear stress induced in the lever. From the shear force diagram as shown in Fig. 15.13 (*a*), we see that the maximum shear force on the lever is ($W - P$) *i.e.* 3500 N.

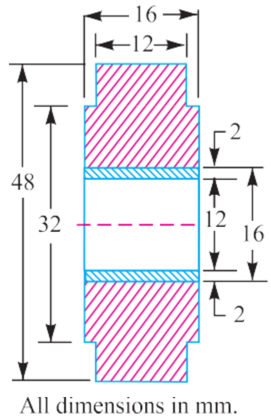
$\therefore$ Maximum shear stress induced,

$$\tau_{max} = \frac{\text{Maximum shear force}}{\text{Cross-sectional area of the lever}} = \frac{3500}{12 \times 48}$$

$$= 6.07 \text{ N/mm}^2 = 6.07 \text{ MPa}$$



(*a*) Shear force diagram.



(*b*) Section at A through the centre of hole.

Fig. 15.13

Since this value of maximum shear stress is much below the permissible shear stress of 50 MPa therefore the design for lever is safe.

Again checking for the bending stress induced at the section passing through the centre of hole at A. The section at A through the centre of the hole is shown in Fig. 15.13 (*b*).

$\therefore$ Maximum bending moment at the centre of hole at A,

$$M = 350\ (880 - 80) = 280 \times 10^3 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 12 \left[(48)^3 - (16)^3\right] + 2 \times \frac{1}{12} \times 2 \left[(32)^3 - (16)^3\right]}{48/2}$$

$$= \frac{106\,496 + 9557}{24} = 4836 \text{ mm}^3$$

$\therefore$ Maximum bending stress induced,

$$\sigma_t = \frac{M}{Z} = \frac{280 \times 10^3}{4836} = 58 \text{ N/mm}^2 = 58 \text{ MPa}$$

Since this maximum stress is below the permissible value of 70 MPa, therefore the design in safe.

15.8 Bell Crank Lever

In a bell crank lever, the two arms of the lever are at right angles. Such type of levers are used in railway signalling, governors of Hartnell type, the drive for the air pump of condensors etc. The bell crank lever is designed in a similar way as discussed earlier. The arms of the bell crank lever may be assumed of rectangular, elliptical or I-section. The complete design procedure for the bell crank lever is given in the following example.

Example 15.6. *Design a right angled bell crank lever. The horizontal arm is 500 mm long and a load of 4.5 kN acts vertically downward through a pin in the forked end of this arm. At the end of the 150 mm long arm which is perpendicular to the 500 mm long arm, a force P act at right angles to the axis of 150 mm arm through a pin into a forked end. The lever consists of forged steel material and a pin at the fulcrum. Take the following data for both the pins and lever material:*

Safe stress in tension = 75 MPa

Safe stress in shear = 60 MPa

Safe bearing pressure on pins = 10 N/mm²

Solution. Given : FB = 500 mm ; W = 4.5 kN = 4500 N ; FA = 150 mm ; σ_t = 75 MPa = 75 N/mm² ; τ = 60 MPa = 60 N/mm² ; p_b = 10 N/mm²

The bell crank lever is shown in Fig. 15.14.

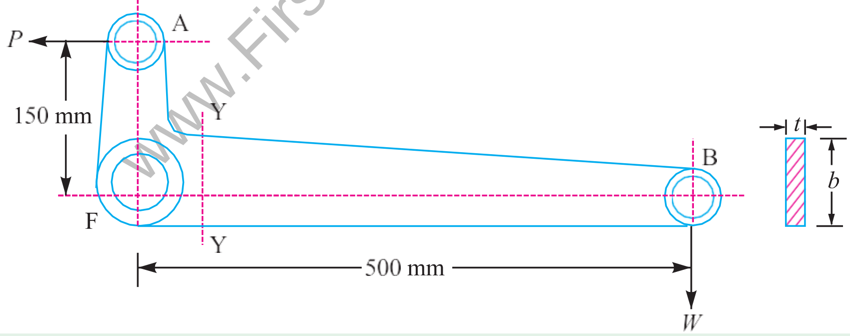


Fig. 15.14

First of all, let us find the effort (P) required to raise the load (W). Taking moments about the fulcrum F, we have

$$W \times 500 = P \times 150$$

$$\therefore \quad P = \frac{W \times 500}{150} = \frac{4500 \times 500}{150} = 15\,000 \text{ N}$$

and reaction at the fulcrum pin at F,

$$R_F = \sqrt{W^2 + P^2} = \sqrt{(4500)^2 + (15\,000)^2} = 15\,660 \text{ N}$$

1. Design for fulcrum pin

Let d = Diameter of the fulcrum pin, and

l = Length of the fulcrum pin.

Considering the fulcrum pin in bearing. We know that load on the fulcrum pin (R_F),

$$15\,660 = d \times l \times p_b = d \times 1.25\,d \times 10 = 12.5\,d^2 \quad \text{...(Assuming } l = 1.25\,d\text{)}$$

$\therefore$ $d^2 = 15\,660 / 12.5 = 1253$ or $d = 35.4$ say 36 mm **Ans.**

and $l = 1.25\,d = 1.25 \times 36 = 45$ mm **Ans.**

Let us now check for the shear stress induced in the fulcrum pin. Since the pin is in double shear, therefore load on the fulcrum pin (R_F),

$$15\,660 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (36)^2\,\tau = 2036\,\tau$$

$\therefore$ $\tau = 15\,660/2036 = 7.7 \text{ N/mm}^2 = 7.7$ MPa

Since the shear stress induced in the fulcrum pin is less than the given value of 60 MPa, therefore design for the fulcrum pin is safe.

A brass bush of 3 mm thickness is pressed into the boss of fulcrum as a bearing so that the renewal become simple when wear occurs.

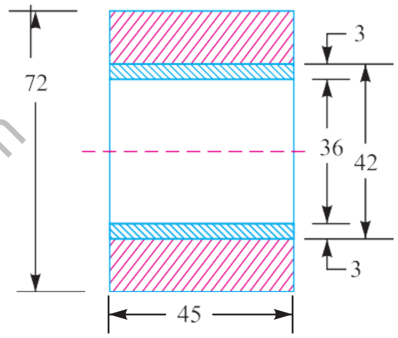
$\therefore$ Diameter of hole in the lever

$= d + 2 \times 3$

$= 36 + 6 = 42$ mm

and diameter of boss at fulcrum

$= 2\,d = 2 \times 36 = 72$ mm



All dimensions in mm.

Fig. 15.15

Now let us check the bending stress induced in the lever arm at the fulcrum. The section of the fulcrum is shown in Fig. 15.15.

Bending moment at the fulcrum

$$M = W \times FB = 4500 \times 500 = 2250 \times 10^3 \text{ N-mm}$$

Section modulus,

$$Z = \frac{\frac{1}{12} \times 45 \left[(72)^3 - (42)^3\right]}{72/2} = 311\,625 \text{ mm}^3$$

$\therefore$ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2250 \times 10^3}{311\,625} = 7.22 \text{ N/mm}^2 = 7.22 \text{ MPa}$$

Since the bending stress induced in the lever arm at the fulcrum is less than the given value of 85 MPa, therefore it is safe.

2. Design for pin at A

Since the effort at A (which is 15 000 N), is not very much different from the reaction at fulcrum (which is 15 660 N), therefore the same dimensions for the pin and boss may be used as for fulcrum pin to reduce spares.

$\therefore$ Diameter of pin at A = 36 mm **Ans.**

Length of pin at A = 45 mm **Ans.**

and diameter of boss at A = 72 mm **Ans.**

3. Design for pin at B

Let d_1 = Diameter of the pin at *B*, and

l_1 = Length of the pin at *B*.

Considering the bearing of the pin at *B*. We know that load on the pin at

$B (W)$, $4500 = d_1 \times l_1 \times p_b = d_1 \times 1.25\, d_1 \times 10 = 12.5\, (d_1)^2$

... (Assuming $l_1 = 1.25\, d_1$)

$\therefore$ $(d_1)^2 = 4500 / 12.5 = 360$ or $d_1 = 18.97$ say 20 mm **Ans.**

and $l_1 = 1.25\, d_1 = 1.25 \times 20 = 25$ mm **Ans.**

Let us now check for the shear stress induced in the pin at *B*. Since the pin is in double shear, therefore load on the pin at *B* (*W*),

$$4500 = 2 \times \frac{\pi}{4} (d_1)^2\, \tau = 2 \times \frac{\pi}{4} (20)^2\, \tau = 628.4\, \tau$$

$\therefore$ $\tau = 4500 / 628.4 = 7.16\ \text{N/mm}^2 = 7.16$ MPa

Since the shear stress induced in the pin at *B* is within permissible limits, therefore the design is safe.

Since the end *B* is a forked end, therefore thickness of each eye,

$$t_1 = \frac{l_1}{2} = \frac{25}{2} = 12.5 \text{ mm}$$

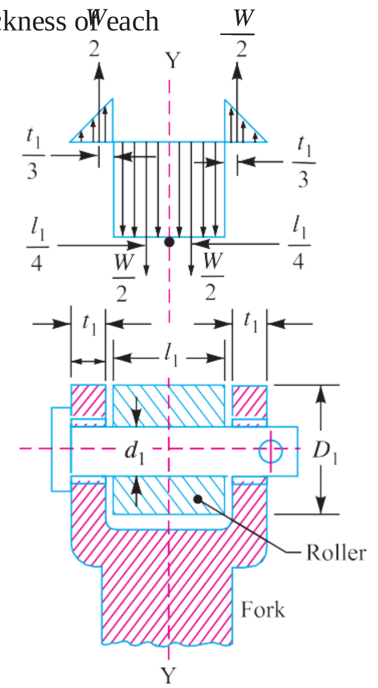


Fig. 15.16

In order to reduce wear, chilled phosphor bronze bushes of 3 mm thickness are provided in the eyes.

$\therefore$ Inner diameter of each eye

$= d_1 + 2 \times 3 = 20 + 6 = 26$ mm

and outer diameter of eye,

$D = 2\, d_1 = 2 \times 20 = 40$ mm

Let us now check the induced bending stress in the pin. The pin is neither simply supported nor rigidly fixed at its ends. Therefore the common practice is to assume the load distribution as shown in Fig. 15.16. The maximum bending moment will occur at *Y-Y*.

$\therefore$ Maximum bending moment at *Y-Y*,

$$M = \frac{W}{2}\left(\frac{l_1}{2} + \frac{t_1}{3}\right) - \frac{W}{2} \times \frac{l_1}{4}$$

$$= \frac{5}{24} W \times l_1 \qquad \text{...}(\because t_1 = l_1/2)$$

$$= \frac{5}{24} \times 4500 \times 25 = 23\,438 \text{ N-mm}$$

and section modulus,

$$Z = \frac{\pi}{32} (d_1)^3 = \frac{\pi}{32} (20)^3 = 786 \text{ mm}^3$$

$\therefore$ Bending stress induced,

$$\sigma_b = \frac{M}{Z} = \frac{23\,438}{786} = 29.8\ \text{N/mm}^2 = 29.8 \text{ MPa}$$

This induced bending stress is within safe limits.

4. Design of lever

It is assumed that the lever extends upto the centre of the fulcrum from the point of application of the load. This assumption is commonly made and results in a slightly stronger section. Considering the weakest section of failure at *Y-Y*.

Let t = Thickness of the lever at *Y-Y*, and

b = Width or depth of the lever at *Y-Y*.

Taking distance from the centre of the fulcrum to *Y-Y* as 50 mm, therefore maximum bending moment at *Y-Y*,

$$= 4500\,(500 - 50) = 2025 \times 10^3 \text{ N-mm}$$

and section modulus,

$$Z = \frac{1}{6} \times t \times b^2 = \frac{1}{6} \times t\ (3t)^2 = 1.5\ t^3 \qquad \text{...(Assuming } b = 3\,t)$$

We know that the bending stress (σ_b),

$$75 = \frac{M}{Z} = \frac{2025 \times 10^3}{1.5 t^3} = \frac{1350 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 1350 \times 10^3 / 75 = 18 \times 10^3$ or t = 26 mm **Ans.**

and $b = 3\,t = 3 \times 26 = 78$ mm **Ans.**

Bucket of a bulldozer.

Note : This picture is given as additional information and is not a direct example of the current chapter.

Example 15.7. *In a Hartnell governor, the length of the ball arm is 190 mm, that of the sleeve arm is 140 mm, and the mass of each ball is 2.7 kg. The distance of the pivot of each bell crank lever from the axis of rotation is 170 mm and the speed when the ball arm is vertical, is 300 r.p.m. The speed is to increase 0.6 per cent for a lift of 12 mm of the sleeve.*

(a) Find the necessary stiffness of the spring.

(b) Design the bell crank lever. The permissible tensile stress for the material of the lever may be taken as 80 MPa and the allowable bearing pressure at the pins is 8 N/mm².

Solution. Given : x = 190 mm ; y = 140 mm ; m = 2.7 kg ; r_2 = 170 mm = 0.17 m ; N_2 = 300 r.p.m. ; h = 12 mm ; σ_t = 80 MPa = 80 N/mm^2 ; p_b = 8 N/mm^2

A Hartnell governor is shown in Fig. 15.17.

(a) Stiffness of the spring

Let s_1 = Stiffness of the spring.

We know that minimum angular speed of the ball arm (*i.e.* when the ball arm is vertical),

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 300}{60} = 31.42 \text{ rad/s}$$

Since the increase in speed is 0.6 per cent, therefore maximum angular speed of the ball arm,

$$\omega_1 = \omega_2 + \frac{0.6}{100} \times \omega_2 = 1.006\,\omega_2 = 1.006 \times 31.42 = 31.6 \text{ rad/s}$$

We know that radius of rotation at the maximum speed,

$$r_1 = r_2 + h \times \frac{x}{y} = 170 + 12 \times \frac{190}{140} = 186.3 \text{ mm} = 0.1863 \text{ m}$$

$$\ldots \left(\because h = (r_1 - r_2)\frac{y}{x} \right)$$

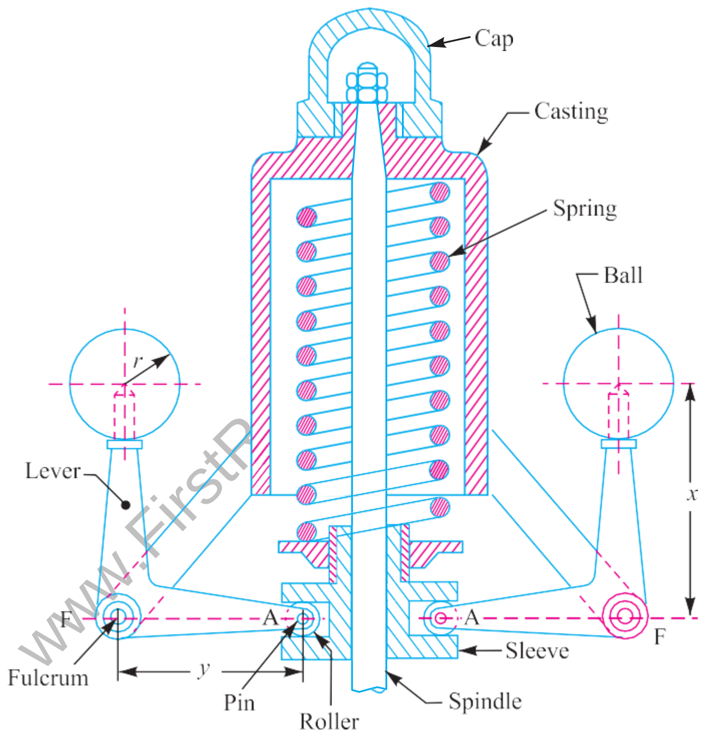
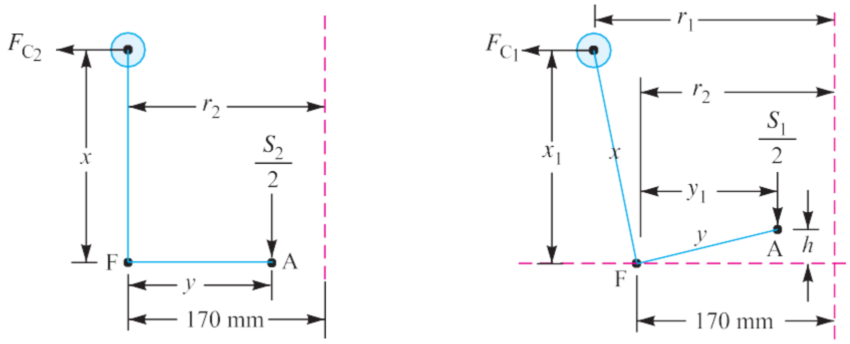


Fig. 15.17

The minimum and maximum position of the ball arm and sleeve arm is shown in Fig. 15.18 (*a*) and (*b*) respectively.

Let F_{C1} = Centrifugal force at the maximum speed = $m\,(\omega_1)^2\, r_1$, F_{C2} = Centrifugal force at the minimum speed = $m\,(\omega_2)^2\, r_2$, S_1 = Spring force at the maximum speed (ω_1), and

S_2 = Spring force at the minimum speed (ω_2).



(*a*) Minimum position. (*b*) Maximum position.

Fig. 15.18

Taking moments about the fulcrum *F* of the bell crank lever, neglecting the obliquity effect of the arms (*i.e.* taking $x_1 = x$ and $y_1 = y$) and the moment due to mass of the balls, we have for

*maximum position,

$$S_1 = 2 F_{C1} \times \frac{x}{y} = 2\, m\, (\omega_1)^2\, r_1 \times \frac{x}{y} \qquad \ldots \left[\because \frac{S_1}{2} \times y = F_{C1} \times x\right]$$

$$= 2 \times 2.7\, (31.6)^2\, 0.1863 \times \frac{190}{140} = 1364 \text{ N}$$

Similarly

$$S_2 = 2 F_{C2} \times \frac{x}{y} = 2\, m\, (\omega_2)^2\, r_2 \times \frac{x}{y}$$

$$= 2 \times 2.7\, (31.42)^2\, 0.17 \times \frac{190}{140} = 1230 \text{ N}$$

We know that

$$S_1 - S_2 = h \times s_1$$

$$\therefore \quad s_1 = \frac{S_1 - S_2}{h} = \frac{1364 - 1230}{12} = 11.16 \text{ N/mm } \textbf{Ans.}$$

(b) *Design of bell crank lever*

The bell crank lever is shown in Fig. 15.19. First of all, let us find the centrifugal force (or the effort *P*) required at the ball end to resist the load at *A*.

We know that the maximum load on the roller arm at *A*,

$$W = \frac{S_1}{2} = \frac{1364}{2} = 682 \text{ N}$$

Taking moments about *F*, we have

$$P \times x = W \times y$$

$$\therefore \quad P = \frac{W \times y}{x} = \frac{682 \times 140}{190} = 502 \text{ N}$$

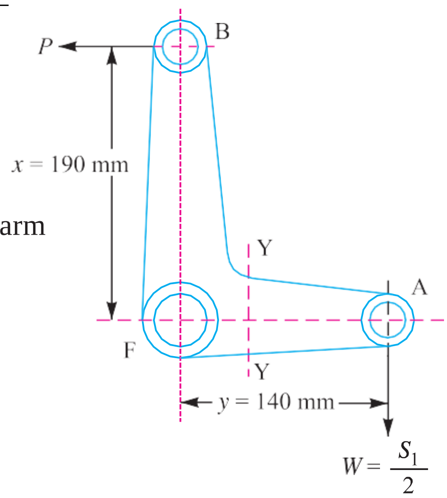


Fig. 15.19

* For further details, please refer chapter on 'Governors' of authors' popular book on **'Theory of Machines'.**

We know that reaction at the fulcrum F,

$$R_F = \sqrt{W^2 + P^2} = \sqrt{(682)^2 + (502)^2} = 847 \text{ N}$$

1. Design for fulcrum pin

Let d = Diameter of the fulcrum pin, and

l = Length of the fulcrum pin = 1.25 d ...(Assume)

The fulcrum pin is supported in the eye which is integral with the frame for the spring. Considering the fulcrum pin in bearing. We know that load on the fulcrum pin (R_F),

$$847 = d \times l \times p_b = d \times 1.25\, d \times 8 = 10\, d^2$$

$\therefore$ $d^2 = 847 / 10 = 84.7$ or $d = 9.2$ say 10 mm **Ans.**

and $l = 1.25\, d = 1.25 \times 10 = 12.5\, d = 12.5$ mm **Ans.**

Let us now check for the induced shear stress in the pin. Since the pin is in double shear, therefore load on the fulcrum pin (R_F),

$$847 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (10)^2 \tau = 157.1\, \tau$$

$\therefore$ $\tau = 847 / 157.1 = 5.4 \text{ N/mm}^2 = 5.4$ MPa

This induced shear stress is very much within safe limits.

A brass bush of 3 mm thick may be pressed into the boss. Therefore diameter of hole in the lever or inner diameter of boss

$= 10 + 2 \times 3 = 16$ mm

and outer diameter of boss $= 2\, d = 2 \times 10 = 20$ mm

2. Design for lever

The cross-section of the lever is obtained by considering the lever in bending. It is assumed that the lever arm extends upto the centre of the fulcrum from the point of application of load. This assumption results in a slightly stronger lever. Considering the weakest section of failure at *Y-Y* (40 mm from the centre of the fulcrum).

Lapping is a surface finishing process for finishing gears, etc.

Note : This picture is given as additional information and is not a direct example of the current chapter.

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∴ Maximum bending moment at *Y-Y*,

$= 682\ (140 - 40) = 68\ 200$ N-mm

Let t = Thickness of the lever, and

B = Depth or width of the lever.

∴ Section modulus,

$$Z = \frac{1}{6} \times t \times B^2 = \frac{1}{6} \times t\ (3t)^2 = 1.5\ t^3 \quad \text{...(Assuming } B = 3\ t\text{)}$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{68\ 200}{1.5t^3} = \frac{45\ 467}{t^3}$$

∴ $t^3 = 45\ 467 / 80 = 568$ or $t = 8.28$ say 10 mm **Ans.**

and $B = 3\ t = 3 \times 10 = 30$ mm **Ans.**

3. Design for ball

Let r = Radius of the ball.

The balls are made of cast iron, whose density is 7200 kg/m^3. We know that mass of the ball (m),

$$2.7 = \text{Volume} \times \text{density} = \frac{4}{3} \pi r^3 \times 7200 = 30\ 163\ r^3$$

∴ $r^3 = 2.7 / 30\ 163 = 0.089/10^3$

or $r = 0.0447$ m $= 44.7$ say 45 mm **Ans.**

The ball is screwed to the end of the lever. The screwed length of lever will be equal to the radius of ball.

∴ Maximum bending moment on the screwed end of the lever,

$$M = P \times r = 502 \times 45 = 22\ 590 \text{ N-mm}$$

Let d_c = Core diameter of the screwed length of the lever.

∴ Section modulus,

$$Z = \frac{\pi}{32} (d_c)^3 = 0.0982\ (d_c)^3$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{22\ 590}{0.0982\ (d_c)^3} = \frac{230 \times 10^3}{(d_c)^3}$$

∴ $(d_c)^3 = 230 \times 10^3 / 80 = 2876$ or $d_c = 14.2$ mm

We shall take nominal diameter of the screwed length of lever as 16 mm. **Ans.**

4. Design for roller end A

Let d_1 = Diameter of the pin at *A*, and

l_1 = Length of the pin at $A = 1.25\ d_1$ ...(Assume)

We know that the maximum load on the roller at *A*,

$$W = S_1 / 2 = 1364 / 2 = 682 \text{ N}$$

Considering the pin in bearing. We know that load on the pin at

$$A\ (W),\ 682 = d_1.l_1.p_b = d_1 \times 1.25\ d_1 \times 8 = 10$$

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$(d_1)^2$

$\therefore \quad (d_1)^2 = 682 / 10 = 68.2$ or $d_1 = 8.26$ say 10 mm **Ans.**

and $\quad l_1 = 1.25\, d_1 = 1.25 \times 10 = 12.5$ mm **Ans.**

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Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin at A (W),

$$682 = 2 \times \frac{\pi}{4} (d_1)^2 \tau = 2 \times \frac{\pi}{4} (10)^2 \tau = 157.1 \tau$$

$$\therefore \quad \tau = 682 / 157.1 = 4.35 \text{ N/mm}^2 = 4.35 \text{ MPa}$$

This induced stress is very much within safe limits.

The roller pin is fixed in the forked end of the bell crank lever and the roller moves freely on the pin. Let us now check the pin for induced bending stress. We know that maximum bending moment,

$$M = \frac{5}{24} \times W \times l_1 = \frac{5}{24} \times 682 \times 12.5 = 1776 \text{ N-mm}$$

and section modulus of the pin,

$$Z = \frac{\pi}{32} (d_1)^3 = \frac{\pi}{32} (10)^3 = 98.2 \text{ mm}^3$$

$\therefore$ Bending stress induced

$$= \frac{M}{Z} = \frac{1776}{98.2} = 18.1 \text{ N/mm}^2 = 18.1 \text{ MPa}$$

This induced bending stress is within safe limits. We know that the thickness of each eye of the fork,

$$t_1 = \frac{l_1}{2} = \frac{12.5}{2} = 6.25 \text{ mm}$$

and outer diameter of the eye,

$$D = 2\, d_1 = 2 \times 10 = 20 \text{ mm}$$

The outer diameter of the roller is taken slightly larger (at least 3 mm more) than the outer diameter of the eye. In the present case, 23 mm outer diameter of the roller will be sufficient. The roller is not provided with bush because after sufficient service, the roller has to be replaced due to wear on the profile. A clearance of 1.5 mm is provided between the roller and fork on either side of roller.

$\therefore$ Total length of the pin,

$$l_2 = l_1 + 2\, t_1 + 2 \times 1.5 = 12.5 + 2 \times 6.25 + 3 = 28 \text{ mm}$$ **Ans.**