

Importance and Scope of Operations Research

Introduction

Operation is simply defined as the research of operations. Operations may be called a set of acts required for the achievement of a desired outcome. Operation research is used to maximize the utility of limited resources. The objective is to select the best alternative that is the one leading to the best result. O.R is scientific approach to problem solving for executive management

Scope of operations research in Management

The technique used in operations research has very wide application in various fields of business/industrial/government/social sector. Few areas of applications are mentioned below:

Marketing and sales:

1. Product selection and competitive strategies
2. Utilization of salesmen, their time and territory control, frequency of visits in sales force analysis.
3. Marketing advertising decision for cost and time effectiveness.
4. Forecasting and decision trends

Production Management

1. Product mix and product proportioning
2. Facility and production planning and scheduling sequencing
3. Physical distribution, warehousing and retail outlets planning-nature and localities
4. Project planning, scheduling , allocation of resources, monitoring and control systems
5. Queuing system design.
6. Quality control decision

Purchasing, procurement and inventory controls

1. Buying policies and prices
2. Negotiation and bidding policies
3. Time and quantity of procurement

Finance, Investments and Budgeting

1. Profit Planning
2. Cash flow analysis
3. Investment policy for maximum return
4. Dividend policies
5. Portfolio analysis

Personnel Management

1. Determination of optimum organizational level
2. Job evaluation and assignment analysis
3. Mixes of age and skills
4. Recruitment policies and job description

Research and Development

1. Determination of areas of thrust for research and development
2. Selection criteria for specific project
3. 'what if' analysis for alternative design and reliability
4. Trade –off analysis for time-cost relationship and control of development project.

Importance of Operations Research

Business Operations

OR could be very effective in handling issues of inventory planning and scheduling, production planning, transportation, financial and revenue management and risk management. Basically, OR could be used in any situation where improvements in the productivity of the business are of paramount importance.

Control

With OR, organizations are greatly relieved from the burden of supervision of all the routine and mundane tasks. The problem areas are identified analytically and quantitatively. Tasks such as scheduling and replenishment of inventories benefit immensely from OR.

Decision Making

OR is used for analyzing problems of decision making in a superior fashion. The organization can decide on factors such as sequencing of jobs, production scheduling and replacements. Also the organization can take a call on whether or not to introduce new products or open new factories on the basis of a good OR plan.

Coordination

Various departments in the organization can be coordinated well with suitable OR. This facilitates smooth functioning for the entire organization.

Systems

With OR, any organization follows a systematic approach for the conduct of its business. OR essentially emphasizes the use of computers in decision making; hence the chances of errors are minimum.

Classification of OR Models

The first thing one has to do to use O.R. technique after formulating a practical problem is to construct a suitable model to represent the a practical problem. A model is a reasonably simplified representation of a real-world situation. It is an abstraction of reality. The model can broadly be classified as

Iconic(Physical) Models

Analog Models

Mathematical Models

Also as

Static Models

Dynamic Models

Deterministic Models

Stochastic Models

Models can further be subdivided as

Descriptive Models

Prescriptive Models

Predictive Models

Analytic Models

Simulation Models

Iconic Model:

This is physical or pictorial representation of various aspects of a system.

Example: Toy, Miniature Model of a building, scaled up model of a cell in biology etc.

Analogue or Schematic Model:

This uses one set of properties to represent another set of properties which a system under study has

Example: A network of water pipes to represent the flow of current in an electrical network or graph, organizational charts etc.

Mathematical model or Symbolic model:

This uses a set of mathematical symbols (letters, numbers etc) to represent the decision variables of a system consideration. These variables related by mathematical equations or in equations which describes the properties of the system.

Example: A linear programming model, A system of equations representing an electrical network or differential equations representing dynamic systems etc.

Static Model:

This is a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.

Example: A linear programming problem, an assignment problem, transportation problem etc.

Dynamic Model:

This is a model which considers time as one of the important variables.

Example: A dynamic programming problem. A replacement problem.

Deterministic model:

This is a model which does not take uncertainty into account.

Example: A linear programming problem, an assignment problem etc.

Stochastic model:

This is a model which considers uncertainty as an important aspect of the problem

Example: Any stochastic programming problem, stochastic inventory models etc,

Descriptive model:

This is one which just describes a situation or system.

Example: An opinion poll, any survey.

Predictive Model is one which predicts something based on some data. Predicting election results before actually the counting is completed.

Prescriptive model is one which prescribes or suggests a course of action for a problem

Example: Any programming (linear, non linear, dynamic, geometric etc.) problem

Analytic model:

This is a model in which exact solution is obtained by mathematical methods in closed form.

Simulation model: This is a representation of reality through the use of a model or device which will react in the same manner as reality under a given set of conditions. Once a simulation model is designed, it takes only a little time, in general, to run a simulation on a computer.

Phases of O.R

- 1) Formulation of the problems: Identifying the objective, the decision variables involved and the constraints that arise involving the decision variables.
- 2) Construction of a Mathematical Model: Expressing the measure of effectiveness which may be total profit, total cost, utility etc., to be optimized by a Mathematical function called objective function. Representing the constraints like budget constraints, raw materials constraints, resource mathematical equations or inequalities.
- 3) Solving the Model constructed: Determining the solution by analytic or iterative or Monte-Carlo method depending upon the structure of the mathematical model.
- 4) Controlling and Updating
- 5) Testing the model and its solution i.e., validating the model: Checking as far as possible either from the past available data or by expertise and experience whether the model gives a solution which can be used in practice.
- 6) Implementation: Implement using the solution to achieve the desired goal.

Linear Programming

Linear programming is a mathematical programming technique to optimize the performance (profit or cost), under a set of resource constraints (Machine hours, money, material, time men hours etc). As a special by an organization the usefulness of this technique is maximize even though the competitions in a linear programming model are two elaborate a sample list of applications of the linear programming problem is presented below.

1. Product mix problem
2. Diet planning problem
3. Manpower planning problem
4. Capital budgeting problem

Concept of Linear programming problem

The model of any linear programming problem will contain objective fuction, set of constraints and non – ve restrictions. Each of the company may consist of one of more of the following.

1. Decision variables
2. Objective function coefficients
3. Technological co efficient
4. Resource availabilities

Formulation of Linear programming problem (LPP)

A company manufactures two different types of product P_1 and P_2 . Each type of product requires processing of milling machine and drilling machine. Each type of machine as limited hours available for week the net profit per unit of the product resource requirement of the products of the products and availability of the sources are summarizing in table.

Machine type	Processing time (in hours)		Machine hours Available per week
	P_1	P_2	
Milling machine	2	5	200
Drilling machine	4	2	240
Profit (Per unit in Rs.)	250	400	

Solution

Let 'n' be the no. of products to be manufacture and 'm' be the different machine types

Decision variables

The decision variable is used represent the levels of achievement of a particular course of action. The solution of LPP will provide the optimal value for each and every decision variable of the model a generalize definition of a decision variable is presented below

From the example 'x₁' is the production volume of product 'P₁'. 'x₂' is the production volume of the 'P₂' and 'x₁' and 'x₂' are called decision variables.

Objective function coefficients

It is a constant representing the profit or cost per unit of the caring out an activity.

Let C₁, C₂, C_n be the profit per unit of the products P₁, P₂, P₃ ... P_n respectively then from the example C₁ is the profit per unit of the product 'P₁' which is equal to Rs. 250 and 'C₂' be the profit per unit of the product 'P₂'. Which is equal to Rs.400 In this example 'C₁' and 'C₂' are called objective function coefficients.

Objective function

It is an expression representing the total profit or cost per caring our. A set of objectives at some levels the objective function will be maximization or minimization. The benefit related objective function will come under maximization. Whereas the cost related objective function will come under minimization type a generalized format of the objective function is presented below.

$$\text{Min or Max } Z = C_1x_1 + C_2x_2 + C_3x_3 + \dots C_nx_n$$

From the example

$$\text{Max } Z = 250x_1 + 400x_2$$

Technological coefficients

The technological coefficients are the amount of resource 'i' required for activity 'j'. Where 'i' varies from 1, 2, 3 ...n and j varies from 1, 2, 3, m. A generalized formal of the technological coefficients matrix is

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

From the example $\begin{bmatrix} 2 & 5 \\ 4 & 2 \end{bmatrix}$

Where $a_{11} = 2$, $a_{12} = 5$, $a_{21} = 4$ and $a_{22} = 2$.

Resource Availability

The constraints 'b' is the amount of resource 'i' available during the planning period.

A generalized format of the resource availability matrix is

$$\begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$$

From the example $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 240 \end{bmatrix}$

Set of constraints or subject to constraints

A constraint is kind of restriction and the total amount of a particular resource required to carry out at various levels in a model there will be many such constraints so the constraints will reveal the levels of achievement different decision variables. A generalized form of the constraints can be given as

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq \text{or} \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq \text{or} \geq b_2 \\ a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n &\leq \text{or} \geq b_3 \\ \cdot & \\ \cdot & \\ \cdot & \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq \text{or} \geq b_n \end{aligned}$$

From the example

$$a_{11}x_1 + a_{12}x_2 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 \leq b_2$$

substitute the value of $a_{11}, a_{12}, a_{21}, a_{22}, b_1$ and b_2

$$2x_1 + 5x_2 \leq 200$$

$$4x_1 + 2x_2 \leq 240$$

Non – negative Restrictions

Each and every decision variable in the linear programming model is a non negative variable. The condition is represents as complete model this example

$$\text{Max } z = 250x_1 + 400x_2$$

Set of constraints

$$2x_1 + 5x_2 \leq 200$$

$$4x_1 + 2x_2 \leq 240$$

$$x_1, x_2 \geq 0$$

Properties of Linear Programming model**Optimum solution**

If there is no other superior solution to the solution obtained for a given linear programming model then the solution is obtained a treated as the optimum solution.

Feasible solution

If all the constraints of the linear programming model are satisfied by the solution of the model then the solution is called feasible solution.

Infeasible solution

When the constraints are not satisfied simultaneously, the LPP has no feasible solution. That solution is called infeasible solution. This means that there is no solution for the given model which can be implements.

Unbounded solution

When the values of the decision variable may be increased indefinitely or infinitely without violating any of constraints, the solution space (feasible region) is unbounded. The value of objective function in such cases, may increase (for maximization) or decrease (for minimization) indefinitely. Thus both solution space and objective function value are unbounded.

Bounded solution

When the values of the decision variable may be definite or finite without violating any of the constraints, the solution space (feasible region) is bounded.

Linear Programming Problem Methods

Algorithm for Graphical Method:

The major steps in the solution of a linear programming problem by graphical method are summarized as follows:

Step 1: identify the problem – the decision variables, the objective and restrictions.

Step 2: Set up the mathematical formulation of the problem.

Step 3: Plot a graph representing all the constraints of the problem and identify the feasible region (solution space). The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

Step 4: The feasible region obtained in “step – 3” may be bounded or unbounded. Compute the co-ordinates of all the corner points of the feasible region.

Step 5: Find out the value of the objective function at each corner (solution) point determined in “step -4”.

Step 6: Select the corner point that optimizes (maximizes or minimizes) the value of the objective function. It gives the “optimum feasible solution”.

Slack Variables:

If the constraints have $\leq$ type in order to make it an equation we have to add something +ve quantity to the left hand side. +ve quantities are called slack variables

Ex:

$$2x_1 + 4x_2 \leq 3$$

$$3x_1 + 5x_2 \leq 4$$

convert them into equations

$$2x_1 + 4x_2 + s_1 = 3$$

$$3x_1 + 5x_2 + s_2 = 4$$

where s_1 and s_2 are slack variables

Surplus variables

If the constraints have $\geq$ type in order to make it in equation we have to add something -ve quantity to left hand side. -ve quantities are called surplus variables.

Ex:

$$2x_1 + 4x_2 \geq 3$$

$$3x_1 + 5x_2 \geq 4$$

convert them into equations

$$2x_1 + 4x_2 - s_1 = 3$$

$$3x_1 + 5x_2 - s_2 = 4$$

where s_1 and s_2 are surplus variables

Algorithm for Simplex Method:

Step 1: Check whether the given objective function is maximization type

Case a: If it is maximization type then go to next step.

Case b: If is minimization problem then we converting them into maximization type by using relation.

$$\text{Min } Z = -\text{Max } Z^*$$

Step 2: check whether the right hand side constants are non – negative (positive)

Case a: if is non – negative then go to next step.

Case b: if at least one of the constant is negative (-ve) then we converting into +ve value by multiplying the corresponding constraint by “-1”.

Step 3: Introduce slack and surplus variables to the constraints and the cost of objective function becomes zero.

Step 4: Express the given objective functions and constraints in matrix form.

Step 5: Check whether there exist initial basic feasible solution (IBFS).

Step 6: prepare initial simplex table by using the following columns.

C_j			C_1	C_2	C_3 C_n				
C_B	S_B	X_B	X_1	X_2	X_3 X_n	S_1	S_2	S_3 S_n	
$Z_j - C_j$									

Step 7: Find z-value i.e., $z = C_B X_B$

Step 8: Find net evaluations i.e., $Z_j - C_j$ where $j = 1, 2, 3, \dots, n$

Step 9: Check whether all the net evaluations are non -ve.

Case a: if it is so then there exist optimum solution.

Case b: If at least one net evaluation is -ve then go to next step

Step 10: Find entering variable which is most -ve net evaluation i.e., $\text{Min} \{-(Z_j - C_j)\}$

Step 11: Find leaving variable which is

$$\text{Min} \left\{ \frac{X_B}{\text{Entering column elements}} \right\}, \text{Entering column elements} > 0.$$

Step 12: Find pivotal or key element which is intersection of both entering and leaving variables.

Step 13: Do the simplex operations, new values of

$$\text{Key row} = \frac{\text{Old values of the key row}}{\text{Key element}}$$

$$\text{New values of the Non key row} = \text{Old values of the Non key row} - (\text{Non key element}) \times (\text{New values of the key row})$$

Step 14: Repeat the procedure. Step 5 to step 12. Until optimum solution are unbounded solution has been obtained.

Big – M or Mixed or Penalty Method or Alternate Method:

Step 1: Check whether the given objective function is maximization type.

Case a: If it is maximization then go to next step

Case b: If it is minimization type then we convert them into maximization type by using relation.

$$\text{Min } Z = -\text{Max } Z^*$$

Step 2: Check whether the right hand side constants are non –ve

Case a: If it is non –ve or positive then go to next step.

Case b: If it is –ve or negative then we convert them into positive (+ve) by multiplying the corresponding constraint by “-1” Automatically the signs will also change.

Step 3: Express the given objective function and constraints in matrix form.

Step 4: Check whether there exist IBFS

Case a: If it is exit so then go to next step

Case b: If it is not so, then introduce artificial variable to the constraints $\geq$ type and the cost co-efficient objective function becomes “-M”.

Step 5: Prepare Initial Simplex Table

C_j			C_1	C_2	C_3		C_n						
C_B	S_B	X_B	X_1	X_2	X_3		X_n	S_1	S_2	S_3		S_n	$A_1, A_2, \dots, A_n$
$Z_j - C_j$													

Step 7: Find Z-value $Z = \sum C_B X_B$.

Step 8: Find Net evaluations i.e., $Z_j - C_j$ ($j = 1, 2, 3 \dots n$)

Step 9: Check whether the net evaluations are non –ve.

Case a: If it is so and verifies the following conditions.

- i) If all net evaluations are non –ve. If at least one artificial variable. Still in the basis not at zero level then there exist infeasible solution.
- ii) If all net evaluations are non –ve and if at least one artificial variable still in the basis at zero level then there exist optimum solution.
- iii) If all net evaluations are non –ve there is no artificial variable in the basis then there exist optimum solution.

Case b: If it is so then go to next step.

Step 10: Find entering variable which is most –ve net evaluation i.e., $\text{Min } \{-(Z_j - C_j)\}$

Step 11: Find leaving variable which is

$$\text{Min } \left\{ \frac{X_B}{\text{Entering column elements}} \right\}, \text{ Entering column elements} > 0.$$

Step 12: Find pivotal or key element which is intersection of both entering and leaving variables.

Step 13: Do the simplex operations, new values of

$$\text{Key row} = \frac{\text{Old values of the key row}}{\text{Key element}}$$

$$\text{New values of the Non key row} = \text{Old values of the Non key row} - (\text{Non key element}) \times (\text{New values of the key row})$$

Step 14: Repeat the procedure. Step 7 to step 13. Until optimum solution are unbounded solution has been obtained.

Duality in Linear programming problem

Every LPP there always exist another LPP which is basing on the same data and having same solution. The original problem is called “Primal problem” the associated one is called “Dual Problem”.

Rules for Converting Primal problem Into Dual:

If the system of constraints in a given LPP consist of mixed equations (or) in equations non –ve variables or unrestricted variables then the dual of the given problem can be obtained by reducing to standard primal problem.

Step 1: first convert the objective function to maximization form, if not.

Step 2: If the constraints as $\geq$ type then we converts them into $\leq$ type by multiplying -1.

Step 3: If the constraints as an equal sign then it is replaced by two constraints involving the in equalities is in opposite direction simultaneously.

Step 4: Every unrestricted variables is replaced by difference of “2” Non –ve variable.

Step 5: We get the standard primal form of the given LPP in which

Case a: All the constraints have $\leq$ type the objective function is maximization.

Case b: All the constraints have $\geq$ type when the objective function is minimization.

Step 6: Identify the variables to be used in the dual problem. The no. of these variables equals to the no. of constraints in the primal problem.

Step7: Write the objective function. Objective function coefficient is the Right hand side constraints of primal constraints. If the primal problem is of maximization type the dual will be minimization problem and vice versa.

Step 8: Making use of “Step 6” write the constraints for the dual problem

Case a: If the primal is a maximization problem the dual constraints must be all are $\geq$ type, If the primal is minimization problem the dual constraints must be all are $\leq$ type.

Case b: The column co-efficient of the primal constraints become the row coefficients of dual constraints.

Case c: The co-efficient the primal objective function become the right hand side constants of the dual constraints.

Applications of Linear Programming

Linear programming is the most widely used techniques of decision –making in business and industry and in various other fields. In this section, we will discuss a few of the broad application areas of linear programming.

Agricultural Application:

This application fall into categories of farm economics and farm management. The farm deals with agricultural economy of a nation or region.

Linear programming study of farm economics deals with interregional competition and optimum allocation of crop production.

Linear programming can be applied in agricultural planning e.g. allocation of limited resources such as acreage, labor, water supply and working capital, etc. in a way so as to maximize net revenue.

Production Management:

Product Mix: A company can produce several different products each of which requires the use of limited production resources. In such cases, it is essential to determine the quantity of each product to be produced knowing its marginal contribution and amount of available resources used by it. The objective is to maximize the total contribution, subject to all constraints.

Production planning: This deals with determination of minimum cost⁶ production plan over planning period of an item with a fluctuating demand, considering the initial number of units in inventory, production, man power and all relevant cost factors.

Financial Management

Portfolio Selection: This deals with the selection of specific investment activity among several other activities. The objective is to find the allocation which maximizes the total expected return or minimize risk. Under certain limitations.

Profit planning: This deals with the maximization of the profit margin form investment in plant facilities and equipment, cash in hand and inventory.

Marketing management:

Medial selection: Linear programming techniques help in determining the advertising media mix. So as to maximize the effective exposure, subject to limitation of budget, specified

exposure rates. To different market segments, specified minimum and maximum no. of advertisements in various media.

Transportation Problem

Introduction:

The transportation problem is one of the subclasses of LPPs in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively, such that the total cost of transportation is minimized.

Examples of Transportation Problem

Source	Destination	Commodity	Objective
Plants	Markets	Finished goods	Minimizing total cost of shipping
Plants	Finished goods warehouses	Finished goods	Minimizing total cost of shipping
Suppliers	Raw material warehouses	Raw material	Minimizing total cost of shipping

The solution of transportation problem can be obtained in two stages, namely initial solution or initial basic feasible solution and optimum solution.

Initial solution

Initial solution can be obtained by using any one of the three methods viz.,

- I. North West corner rule (NWCR).
- II. Row minimum rule
- III. Column minimum rule.
- IV. Least cost method or Matrix minima method.

V. Vogel's approximation method (VAM).

VAM is preferred over the other two methods, since the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution.

Optimum solution

The improved solution of the initial basic feasible solution is called optimal solution which is the second stage of solution that can be obtained by MODI (modified distribution method) or U-V method.

Algorithm for Northwest-Corner rule

Step 1 : Find the minimum of the supply and demand values with respect to the current northwest corner cell of the cost matrix.

Step 2 : Allocate this minimum value to the current northwest-corner cell and subtract this minimum from the supply and demand values with respect to the current northwest-corner cell.

Step 3 : Check whether exactly one of the row/column corresponding to the northwest-corner cell has zero supply/demand, respectively. If so, go to step 4.

Step 4 : Delete that row/column with respect to the current northwest-corner cell which has the zero supply/demand and go to next step.

Step 5 : Repeat above steps moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

Algorithm for Least Cost or Matrix Minima Method

Step 1 : Find the minimum of the values in the cost matrix (i.e. find the matrix minimum).

Step 2 : Find the minimum of the supply and demand values(X) with respect to the cell corresponding to the matrix minimum.

Step 3 : Allocate X units to the cell with the matrix minimum. Also, subtract X units from the supply and the demand values corresponding to the cell with the matrix minimum.

Step 4 : Check whether exactly one of the row/column corresponding to the cell with the matrix minimum has zero supply/demand, respectively. If yes, go to step 5.

Step 5 : Delete that row/column with respect to the cell with the matrix minimum which has the zero supply/zero demand.

Step 6 : Repeat above steps for the resulting reduced transportation table until all the rim requirements are satisfied.

Algorithm for Vogel's Approximation Method (VAM)

Step 1 : Find row penalties, i.e. the difference between the first minimum and the second minimum in each row . If the two minimum values are equal, then the row penalty is zero.

Step 2 : Find column penalties, i.e. the difference between the first minimum and the second minimum in each column. If the two minimum values are equal, then the column penalty is zero.

Step 3 : Find the maximum amongst the row penalties and column penalties and identify whether it occurs in a row or in a column. If the maximum penalty is in a row, go to step 4 otherwise go to step 7.

Step 4 : Identify the cell for allocation which has the least cost in that row.

Step 5 : Find the minimum of the supply and demand values with respect to the selected cell.

Step 6 : Allocate this minimum value to that cell and subtract this minimum from the supply and demand values with respect to the selected cell and go to step 10.

Step 7 : Identify the cell for allocation which has the least cost in that column.

Step 8 : Find the minimum of the supply and demand values with respect to the selected cell.

Step 9 : Allocate this minimum value to the selected cell and subtract this minimum from the supply and demand values with respect to the selected cell.

Step 10 : Check whether exactly one of the rows and the column corresponding to the selected cell has zero supply/zero demand, respectively. If yes go to step 11.

Step 11 : Delete the row/column which has the zero supply/zero demand and revise the corresponding row/column penalties.

Step 12 : Repeat the procedure until all the rim requirements are satisfied.

Algorithm for MODI (modified distribution method) or U-V method.

Step 1 : Find the initial basic feasible solution of a Transportation Problem by using any one of the above methods.

Step 2 : Check whether the number of basic cells (occupied cells) in the set of initial basic feasible solution is equal to $m+n-1$. If yes go to next step.

Step 3 : Compute the values for $U_1, U_2, U_3, \dots, U_m$ and $V_1, V_2, V_3, V_4, \dots, V_n$ by applying the following formula to all the basic cells only.

$$U_i + V_j = c_{ij}$$

Step 5 : Compute penalties P_{ij} for the non-basic cells by using the formula.

$$P_{ij} = U_i + V_j - c_{ij}$$

Step 6 : Check whether all P_{ij} values are less than or equal to zero . If yes, go to step 11, otherwise, go to step 7.

Step 7 : Identify the non-basic cell which has the maximum positive penalty, and term that cell as the new basic cell.

Step 8 : Starting from the new cell, draw a closed loop consisting of only horizontal and vertical lines passing through some basic cells.(Note : Change of direction of the loop should be with 90 degrees only at some basic cell.)

Step 9 : Starting from the new basic cell, alternatively assign positive(+) and negative (-) signs at the corners of the closed loop.

Step 10: Find the minimum of the allocations made amongst the negatively signed cells.

Step 11 : The optimality is reached. Treat the present allocations to the set of basic cells as the optimum allocations.

Assignment Problem

Introduction

The objective of assignment problem is to assign a number of origins (jobs) to the equal number of destinations (persons) at a minimum time or minimum cost or maximize the profit.

Algorithm for Assignment problem

Step 1: Determine the cost table from given problem

- I. If the number of source is equal to the number of destinations, go to step 3
- II. If the number of sources is not equal to the number of destinations, go to step 2

Step 2: Add a dummy source or dummy destination, so that the cost table becomes a square matrix. The cost entries of dummy source/destinations are always zero.

Step 3: locate smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step 4: in the reduced matrix obtained in step3, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have at least one zero.

Step 5: in the modified matrix obtained in step4, search for an optimal assignment as follows:

- a. Examine the rows successively until a row with a single zero is found. Enrectangle this zero ($\square$) and cross off (x) all other zeros in its column. Continue in its manner until all the rows have been taken care of.
- b. Repeat the procedure of each column of the reduced matrix.
- c. If a row and/or column has two or more zeros and one cannot be chosen by inspection, then assign arbitrary any one of these zeros and cross off all other zero of that row/column.
- d. Repeat (I) through (III) above successively until the chain of assigning ($\square$) or cross (x) ends. Solution is reached.

Step 6: if the number of assignments ($\square$) is equal to n (the order of matrix) , an optimum solution is reached

If the number of assignment is less than n (the order of matrix), go to next step.

Step 7: Draw the minimum number of horizontal and/or vertical lines to cover all the zeros of the reduced matrix. This can be conveniently done by using a simple procedure:

- a. Mark ($\vee$) rows that do not have any assigned zero.
- b. Mark ($\vee$) columns that have zeros in the marked rows.
- c. Mark ($\vee$) rows that have assigned zeros in the marked columns.
- d. Repeat (b) and (c) above until the chain of making is completed.
- e. Draw lines through all the unmarked rows and marked columns. This gives us the desired minimum number of lines.

Step 8: Develop the new revised cost matrix as follows:

- a. Find the smallest element of the reduced matrix not covered by any of the lines.
- b. Subtract this element from all the unmarked elements and add the same to all the Elements lying at the intersection of any two lines.

Step 9: Go to step 6 and repeat the procedure until an optimum solution is attained.

Sequencing problems

Introduction:

The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing.

A practical situation may corresponds to an industry producing a number of products, each of which is to be processed through different machines, of course, finite in number

The general sequencing problem may be defined as: Let there be n jobs to be performed one at a time on each of m machines. The sequence (order) of the machines in which each job should be performed is given. The actual or expected time required by the jobs on each of the machines is also given. The general sequencing problem, therefore, is to find the sequence out of $(n!)^m$ possible sequences which minimize the total elapsed time between the start of the job in the first machine and the completion of the job on the last machine.

Assumptions:

- Each job once started on a machine, is to be performed up to completion on that machine.
- The processing time on each machine is known. Such a time is independent of the order of the jobs in which they are to be processed.
- The time taken by each job in changing over from one machine to another is negligible.
- A job starts on the machine as soon as the job and the machine both are idle and job is next to the machine and the machine is also next to the job.
- No machine may process more than one job simultaneously.
- The order of completion of job has no significance, i.e. no job is to be given priority. The order of completion of jobs is independent of sequence of jobs.

Basic Terms:

Number of Machines: It refers to the no. of service facilities through which a job must pass before it is assumed to be completed.

Processing Order: It refers to the order (sequence) in which given machines are required for completing the job.

Processing Time: It indicates the time required by a job on each machine.

Total Elapsed Time: It is the time interval between starting the 1st job and completing the last job including the idle time in a particular order by the given set of machines.

Idle time on a machine: It is the time for which a machine does not have a job to process, i.e. idle time from the end of job (i-1) to the start of jobs i.

Processing Time: It indicates the time required by a job on each machine.

No passing rule: It refers to the rule of maintaining the order in which jobs are to be processed on given machines.

Process 'n' jobs through two Machines:

Optimum Sequence of Algorithm:

1. List the jobs along with their processing times in a table.

Job Number	:	J_1	J_2	J_3	J_n
ProcessingTime on machine	M_1 :	t_{11}	t_{12}	t_{13}	t_{1n}
	M_2 :	t_{21}	t_{22}	t_{23}	t_{2n}

2. Examine the rows for processing times on machines M_1 and M_2 , and find the smallest processing time in each row, i.e. find out $\min.(t_{1j}, t_{2j})$ for all j.
3. If the smallest processing time is for the first machine M_1 , then place the corresponding job in the 1st available position in the sequence, otherwise place 2nd machine M_2 .
4. If there is a tie in selecting the minimum of all the processing times, then there may be 3 situations
 - a) Minimum among all processing times is same for the machines, i.e., $\min.(t_{1j}, t_{2j}) = t_{1k} = t_{2r}$, then process the kth job first and the rth job last.
 - b) If the tie for minimum occurs among processing times t_{1j} on machine M_1 only, then select arbitrarily the job to process first.
 - c) If the tie for minimum occurs among processing times t_{2j} on machine M_2 , then select arbitrarily the job to process last.

d) Cross off the jobs already assigned and repeat steps 1 through 4, placing the remaining jobs next to first or next to last, until all the jobs have been assigned

➤ Calculate idle time for machines M_1 and M_2 :

Idle time for M_1 = Total elapsed time – (time when the last job in a sequence finishes on M_1).

Idle time for M_2 = Time at which the 1st job in a sequence finishes on M_1 + (time when the j th job in a sequence starts on M_2) – {(time when the $(j-1)$ th job in a sequence finishes on M_2)}

➤ The total elapsed time to process all jobs through 2 machines as under:

Total elapsed time = Time when the n th job in a sequence finishes on machine M_2

$$= \sum_{j=2}^n t_{2j} + \sum_{j=2}^n I_{2j}$$

Where, t_{2j} = Time required for processing j th job on machine M_2

I_{2j} = Time for which machine M_2 remains idle after processing $(j-1)$ th job and before starting work on j th job.

Process 'n' jobs through 'K' Machines:

Let there be 'n' jobs, each of which is to be processed through 'm' machines, say $M_1, M_2, M_3, \dots, M_k$, in order $M_1, M_2, M_3, \dots, M_k$. the list of jobs with their processing times is:

Job number		1	2	3		n
Machines	M_1 :	t_{11}	t_{12}	t_{13}		t_{1n}
	M_2 :	t_{21}	t_{22}	t_{23}		t_{2n}
	.	.	.	.	.	t_{3n}
	.	.	.	.	.	t_{3n}
	.	.	.	.	.	t_{3n}
	M_k :	t_{k1}	t_{k2}	t_{k3}		t_{kn}

Optimum Sequence of Algorithm:

The iterative procedure for determining the optimal sequence for 'n' jobs on 'k' machines can be summarized as follows:

Step 1: Find Min t_{1j} , Min t_{kj} and Maximum of each of $t_{2j}, t_{3j}, \dots, t_{k-1j}$ for all $j = 1, 2, \dots, n$

Step 2: Check the following

$$\text{Min}(t_{1j}) \geq \text{Max}(t_{ij}) \quad \text{for } i = 2, 3, 4, \dots, k-1$$

or

$$\text{Min}(t_{kj}) \geq \text{Max}(t_{ij}) \quad \text{for } i = 2, 3, 4, \dots, k-1$$

Step 3: If the inequalities of step 2 are not satisfied, method fails. Otherwise go to next step.

Step 4: Convert the k machine problem into two-machine problem by introducing two fictitious machine G and H, such that

$$G = t_{1j} + t_{2j} + t_{3j} \dots t_{k-1j}$$

$$H = t_{2j} + t_{3j} + t_{4j} \dots t_{kj}$$

Step 5: Determine the optimal sequence for the 'n' jobs and '2' machines equivalent sequencing problems with the prescribed order GH in the same way as discussed earlier the resulting sequence shall be optimum for the given problem.

Processing Two Jobs through m machines

Example 1. Use graphical method to minimize the time needed to process the following jobs on the machines shown below also calculate the total time needed to complete both the jobs.

Job1	A	B	C	D	E
Job 2	C	A	D	E	B

Solution:

Step -1: First draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2.

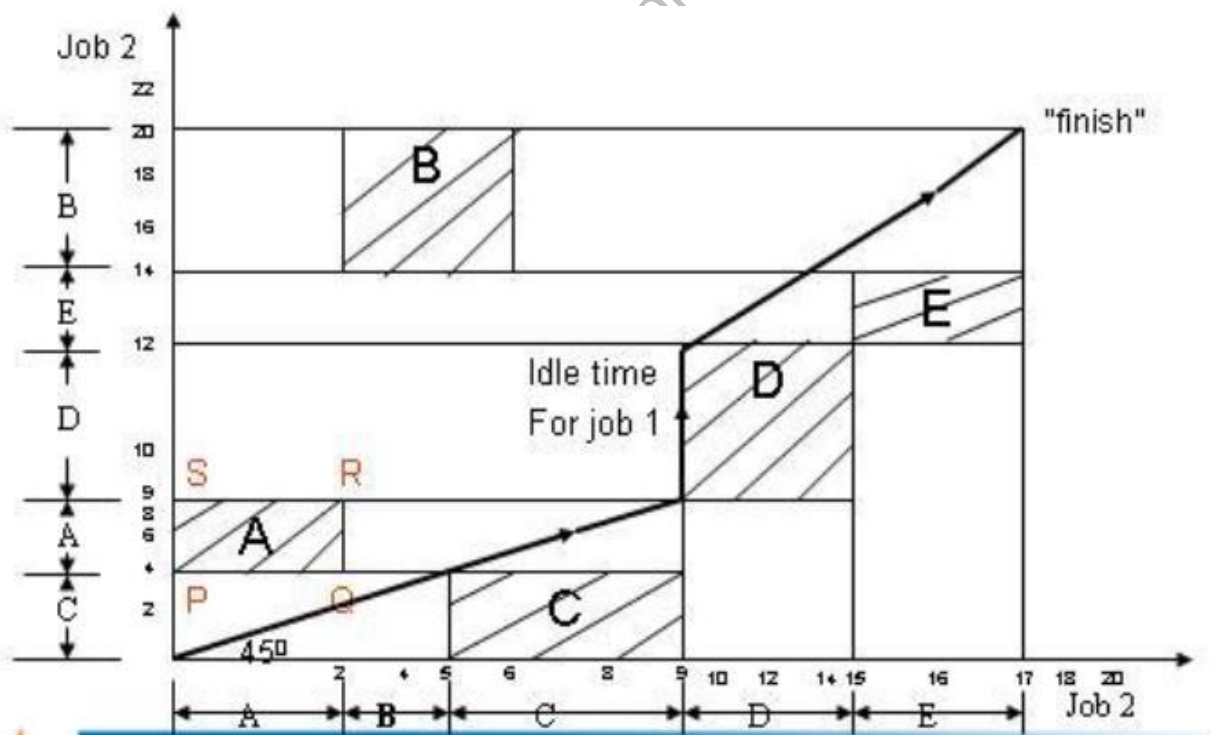
Step 2: layout the machine time for two jobs on the corresponding axes in the technological order machine A takes 2 hours for job 1 and 5 hours for job 2.

Construct the rectangular PQRS for the machine A. Similarly, other rectangular for machine B,C,D and E are constructed as shown in fig.

Step 3: make a program by starting from the origin O and moving through various stages of Completion (points) until the point marked finish is obtained. Physical interpretation of the path thus chosen involves the series of segments which are horizontal or vertical or diagonal making an angle of 45° with the horizontal. Moving to the right means that job1 is proceeding while job 2 is idle and moving upward means that job 2 is proceeding while job 1 is idle and moving diagonally means that both the jobs are proceeding simultaneously. Further both the jobs cannot be processed simultaneously on the same machine. Graphically diagonal movement through the blocked out (shaded) area is not allowed and similarly for other machines too.

Step 4: (To find an optimal path) An optimal path is one that minimizes idle for job1 (horizontal) and job 2 (vertical movement). Choose such a path on which diagonal movement is as much as possible.

Step 5: (to find the elapsed time). The elapsed time is obtained by adding the idle time either of the job to the processing time for that job. In this problem the idle time for the chosen path is seen to be 3hrs for the 1 and 0 for the job 2. Thus the total elapsed time, $17 + 3 = 20$ hrs



Replacement problems

Introduction: Replacement problems are concerned with the situation that arise when some items such as men, machines, electric-light bulbs, etc. Need replacement due to their decreased efficiency, failure or break down. The deteriorating efficiency or complete breakdown may be either gradual or sudden,

Broadly speaking, the requirement of a replacement may be in any of the following situations:

- (i) An item fails and does not work at all or the item is expected to fail shortly.
- (ii) An item deteriorates and need expensive maintenance.
- (iii) A better design of the equipment is available.
- (iv) It is economical to replace equipment in anticipation of costly failure.

Failure Mechanism of Items

The term 'failure' has a wider meaning in business than what it has in our daily life. There are two kinds of failure.

- 1) **Gradual Failure:** The mechanism under this category is progressive. That is, as life of an item increases, its efficiency deteriorates causing.
 - a) Increased expenditure for operating costs,
 - b) Decreased productivity of the equipments,
 - c) Decrease in the value of equipment, i.e., the resale or saving value decreaseFor example: Mechanical items like bearings, rings etc. Another example is 'Automobile tires,
- 2) **Sudden Failure:** This type of failure is applicable to those items that do not deteriorate markedly with service but which ultimately fail after some period of using. The period between installation and failure is not constant for any particular type of equipment but will follow some frequency distribution which may be progressive, retrogressive or random in nature.

Replacement Policy when money value is not counted with time:

The aim here is to determine the optimum replacement age of an equipment/item whose running/maintenance cost increases with time and the value of money remains static during that period. Let

C: Capital Cost of equipment

S: Scrap value of equipment

n: Number of years that equipment would be in use.

f(t) : Maintenance cost functions,

A(n): Average total annual cost.

Case I: When time (t) is a continuous variable: If the equipment is used for 'n' years, then the total cost incurred during this period is given by

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$$TC = \text{Capital cost} - \text{Scrap value} + \text{Maintenance Cost}$$

$$= C - S + \int_0^n f(t) dt$$

Average annual total cost, therefore is

$$A(n) = \frac{TC}{n}$$

$$= \frac{1}{n} \left(C - S + \int_0^n f(t) dt \right)$$

$$= \frac{C - S}{n} + \frac{1}{n} \left(\int_0^n f(t) dt \right)$$

For minimum cost, we must have $\frac{d}{dn} [A(n)] = 0$

$$\frac{d}{dn} \left[\frac{C - S}{n} + \frac{1}{n} \left(\int_0^n f(t) dt \right) \right] = 0$$

$$\frac{d}{dn} \left(\frac{C - S}{n} \right) + \frac{d}{dn} \left(\frac{1}{n} \left(\int_0^n f(t) dt \right) \right) = 0 \quad \left(\begin{array}{l} \text{Q Using } \frac{d}{dn} (UV) = V \frac{d}{dn} (U) + U \frac{d}{dn} (V) \\ U = \frac{1}{n} \text{ and } V = \int_0^n f(t) dt \end{array} \right)$$

$$= -\frac{C - S}{n^2} + \left(-\frac{1}{n^2} \int_0^n f(t) dt + \frac{1}{n} f(n) \right) = 0 \quad \left(\text{Q } \frac{d}{dn} \left(\int_0^n f(t) dt \right) = f(n) \right)$$

$$= \frac{C - S}{n^2} + \frac{1}{n^2} \int_0^n f(t) dt - \frac{1}{n} f(n) = 0$$

$$\frac{1}{n} f(n) = \frac{C-S}{n^2} + \frac{1}{n^2} \int_0^n f(t) dt$$

$$f(n) = \frac{C-S}{n} + \frac{1}{n} \int_0^n f(t) dt$$

$$f(n) = A(n)$$

$$\frac{d^2}{dn^2} [A(n)] \geq 0 \text{ at } f(n) = A(n)$$

i.e., when the maintenance cost becomes equal to the average annual cost, the decision should be to replace the equipment.

Case 2: when time(t) is a discrete variable:

Here, the period of time is considered as fixed and n, time(t) take the values 1,2,3... Then

$$A(n) = \frac{C-S}{n} + \frac{1}{n} \sum_{i=0}^n f(t)$$

Now, A(n) will be a minimum for that value of n, for which

$$A(n+1) \geq A(n) \text{ and } A(n-1) \geq A(n) \text{ or}$$

$$A(n+1) - A(n) \geq 0 \text{ and } A(n-1) - A(n) \geq 0$$

For this, we write

$$\begin{aligned} A(n+1) &= \frac{C-S}{n+1} + \frac{1}{n+1} \sum_{i=0}^{n+1} f(t) \\ &= \frac{C-S}{n+1} + \frac{1}{n+1} \left(\sum_{i=0}^n f(t) + f(n+1) \right) \\ &= \frac{C-S}{n+1} + \frac{1}{n+1} \left(\sum_{i=0}^n f(t) \right) + \frac{1}{n+1} f(n+1) \\ &= \frac{1}{n+1} \left(C-S + \sum_{i=0}^n f(t) \right) + \frac{1}{n+1} f(n+1) \end{aligned}$$

$$= \frac{1}{n+1}(nA(n)) + \frac{1}{n+1} f(n+1) \quad \left(Q nA(n) = C - S + \sum_{i=0}^n f(t) \right)$$

We subtract $A(n)$ on both sides

$$\begin{aligned} A(n+1) - A(n) &= \frac{1}{n+1}(nA(n)) + \frac{1}{n+1} f(n+1) - A(n) \\ &= \frac{1}{n+1}(f(n+1) - A(n)) \end{aligned}$$

$$A(n+1) - A(n) \geq 0$$

$$\frac{1}{n+1}(f(n+1) - A(n)) \geq 0$$

$$f(n+1) - A(n) \geq 0$$

$$f(n+1) \geq A(n)$$

Similarly it can shown that

$$A(n) - A(n-1) \leq 0 \Rightarrow f(n) \leq A(n-1)$$

This suggest the optimal replacement policy:

Replace the equipment at the end of n years, if the maintenance cost in the $(n+1)^{th}$ year is more than the average cost in the n^{th} year and the n^{th} year's maintenance cost is less than the previous year's average total cost.

Replacement Policy when Value of Money is counted with time:

In this case, the investor is paying interest on the money that has been invested. We assume that

- (i) The maintenance costs are incurred at the beginning of time periods; and
- (ii) The maintenance costs increase with time.

Let the money carry an interest rate of $r\%$ per year, i.e., one rupee in n years time is equivalent to Rs. $(1+r)^{-n}$ today.

$(1+r)^{-n}$ is called the present worth factor of rupee one spent n years after now.

$(1+r)^n$ is the payment compound amount factor of rupee one spent in n years time.

Let

C : Initial cost of equipment

R : Running cost of the equipment in year .

V : Rate of interest or Present worth factor

$$V = (1 + r)^{-1}$$

V_n : Future discounted costs associated with the policy of replacing the equipment at the end of each n years.

$$V_n = \frac{C + \sum_{k=0}^{n-1} V^k R_k}{1 - V^n}$$

Now V_n is minimum

$$V_{n+1} > V_n \text{ and } V_{n-1} > V_n$$

$$V_{n+1} - V_n = \frac{C + \sum_{k=0}^n V^k R_k}{1 - V^{n+1}} - V_n$$

$$= \frac{C + \sum_{k=0}^n V^k R_k - V_n (1 - V^{n+1})}{1 - V^{n+1}}$$

$$= \frac{\left(C + \sum_{k=0}^{n-1} V^k R_k \right) + V^n R_n - V_n (1 - V^{n+1})}{1 - V^{n+1}}$$

$$\left(\begin{array}{l} QV_n = \frac{C + \sum_{k=0}^{n-1} V^k R_k}{1 - V^n} \\ (1 - V^n)V_n = C + \sum_{k=0}^{n-1} V^k R_k \end{array} \right)$$

$$\begin{aligned}
 &= \frac{(1-V^n)V_n + V^n R_n - V_n(1-V^n \times V)}{1-V^{n+1}} \\
 &= \frac{V_n - V^n V_n + V^n R_n - V_n + V_n \times V^n \times V}{1-V^{n+1}} \\
 &= \frac{V^n (R_n - V_n + V_n \times V)}{1-V^{n+1}} \\
 &= \frac{V^n (R_n - (1-V)V_n)}{1-V^{n+1}}
 \end{aligned}$$

$$V_{n+1} - V_n > 0$$

$$\frac{V^n (R_n - (1-V)V_n)}{1-V^{n+1}} > 0$$

$$V^n (R_n - (1-V)V_n) > 0$$

$$(R_n - (1-V)V_n) > 0$$

$$R_n > (1-V)V_n \text{------(1)}$$

and similarly

$$V_n - V_{n-1} = \frac{V^{n-1} (R_{n-1} - (1-V)V_n)}{1-V^{n-1}}$$

$$V_n - V_{n-1} < 0$$

$$\frac{V^{n-1} (R_{n-1} - (1-V)V_n)}{1-V^{n-1}} < 0$$

$$R_{n-1} < (1-V)V_n \text{------(2)}$$

From (1) and (2)

$$R_{n-1} < (1-V)V_n < R_n$$

Replacement of items that fail completely:

Some situations, failure of a certain item occurs all of a sudden, instead of gradual deterioration (e.g., failure of light bulbs, tubes, etc.).

The failure of the item may result in complete breakdown of the system. The breakdown implies loss of production, idle inventory, idle labour, etc. Therefore, an organization must prepare itself against these failures.

Thus, to avoid the possibility of a complete breakdown, it is desirable to formulate a suitable replacement policy. The following two courses can be followed in such situations.

- **Individual replacement policy.** Under this policy, an item may be replaced immediately after its failure.
- **Group replacement policy.** Under this policy, the items are replaced in group after a certain period, say t , irrespective of the fact that items have failed or not. If any item fails before its preventive replacement is due, then individual replacement policy is used.

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GAME THEORY

Introduction:

A competitive situation will be called a “Game”. Many practical problems require decision-making in a competitive situation where there are two or more opposing parties where the action of one depends up on the one taken by the opponent.

For example: candidates for an election advertising and marketing campaigns by competing business firms, countries involved in military bottles etc.

If Game has the following properties

- Each player has all relevant information available
- Each Game has finite number of players
- Each player can use finite number of strategies.

Some Basic Terms:

Player: the participants or competitors in the game are known as players. A player may be individual or group of individuals.

Strategy: A strategy for a player is defined as a set of rules of alternative courses of action available to him in advance, by which player decides the course of action that he should adopt strategy may be of two types.

- Pure strategy
- Mixed strategy

Pure strategy: If the player select the game strategy each time , then it is referred to as pure-strategy. The objective of the players is to maximize gains or to minimize losses.

Mixed strategy: When the player use a combination of strategies and each player always kept guessing as to which course of action is to be selected by the other player at a particular occasion then this is known as mixed strategy. There is a probabilistic situation and objective of the player is to maximize expected gains or to minimize losses.

Optimum strategy: A course of action or play which puts the player in the most preferred position, irrespective of the strategy of his competitors is called an optimum strategy.

Pay-off matrix:

When the players select their particular strategies. The pay-off (gains or losses) can be represented in the form of a matrix called the payoff matrix.

$$\begin{array}{c}
 \text{Player B} \\
 \begin{array}{c}
 \text{Player A} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}
 \end{array}
 \end{array}$$

Let player 'A' have 'm' strategies A and player 'B' have 'n' strategies. It is assumed that each player has his choices from amongst the pure strategies. Also it is assumed that player 'A' is always the gainer and player 'B' is always loser.

Value of the Game: It is the expected pay off of play when all the players of the game follow their optimum strategies. The game is called value of the game

Optimum strategy:

A course of action or play which puts the player in the most preferred position irrespective of the strategy of his competitors is called an optimum strategy.

Value of the game:

It is the expected pay off of play when all the players of the game follow their optimum strategies. The game is called fair if the value of the game is zero and unfair if it is non-zero.

Saddle Point:

The definition of pure strategy states that both the players use the same strategy every time and that is the best strategy from the points of both players. In such case, the strategy indicates the saddle point, which is the point maximum gain for A and minimum loss for B.

Rules for determine a saddle point:

We may now summarize the procedure of locating the saddle point of a pay off matrix as follows:

Step 1: select the minimum element of each row of the pay off matrix and mark them (*)

Step 2: select the maximum element of each column of the pay off matrix and mark them (+)

Step 3: if there appears an element in the pay off matrix in the pay off matrix marked (*) and (+) both, the position of that element is a saddle point of the pay off matrix

Methods form solving game without saddle points

When there are no saddle points in the game, pure strategies as optimal strategies are not available. Such games can be resolved by using mixed strategy and following methods can be adopted

1. Algebraic method.
2. Graphic method
3. Dominance method.
4. Linear programming method

Algebraic method:

Algebraic formula for matrix method:

The general formulae can be derived algebraically from the matrix in the following manner. The matrix so obtained from rule of dominance can be written as:

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cc} q_1 & q_2 \end{array} \\ \text{Player A} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{array}{c} p_1 \\ p_2 \end{array} \end{array}$$

Strategy for Player A:

$$S_A = \begin{bmatrix} A_1 & A_2 \\ p_1 & p_2 \end{bmatrix}$$

$$p_1 = \frac{a_{22} - a_{21}}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$

$$p_2 = 1 - p_1$$

Strategy for Player B:

$$S_B = \begin{bmatrix} A_1 & A_2 \\ q_1 & q_2 \end{bmatrix}$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$

$$q_2 = 1 - q_1$$

Value of the game

$$V = \frac{(a_{22} \times a_{11}) - (a_{12} \times a_{21})}{(a_{22} + a_{11}) - (a_{12} + a_{21})}$$

Where P = probability or chance of player A using strategy and q of player B using strategy.

Graphical Method:

Some games will be specialized nature like $2 \times n$ or $m \times 2$ $m \times 2$ games. The pay off matrix for the $2 \times n$ games will contain 2 rows and n columns, where as the pay off matrix of the $m \times 2$ game will contain 'm' rows and 2 columns. If there is no saddle point for these games, one can solve them using graphical method.

Algorithm for $2 \times n$ games:

If the given pay off matrix is $2 \times n$ (no. of rows are 2 and no. of columns are more than 2). Then we follow the steps

Step 1: Plot the gain function on a graph assuming a suitable scale. Keep x on X-axis and gain on y-axis.

Step 2: Since the player A is a maximum player, find the highest intersection point in lower boundary of the graph. Let it be maximum point

Step 3: if the number of lines passing through the maximum point is only two, from 2×2 pay off matrix form the original problem by retaining and go to step 8 otherwise go to step 7.

Step 4: identify any two lines with opposite slopes passing through that point. Then from 2×2 pay off matrix form the original problem by retaining only column corresponding to those two lines which is having opposite slopes.

Step 5: Solve 2×2 game using oddments and find strategies for player 'A' and 'B' and also value of game.

Algorithm for $m \times 2$ game:

If the given pay off matrix is $m \times 2$ (no. of rows are more than 2 and no. of columns are 2). Then we follow the steps

Step 1: plot the gain function on a graph function on a graph by assuming a suitable scale, keep 'y' on 'x-axis' and gain on 'y – axis'.

Step 2: since B is minimax player, find lowest intersection point in the upper boundary of the graph. Let it be the minimax point.

Step 3: if number of lines passing through minimax point is only two, form a $m \times 2$ pay off matrix form original problem by retaining only the row corresponding to those two lines and go to step 8, otherwise go to step 7.

Step 4: identify any two lines with opposite slopes passing through that point the form 2×2 pay off matrix form the original by retaining only the rows corresponding to two lines which are

Step 5: solve the 2×2 game using oddments and find strategies for player 'A' and player 'B' and also the value of the game.

Dominance Method:

Some times it is observed that one of pure strategies of either player is always inferior to at least one of remaining one. The superior strategies are said to dominate the inferior one. Clearly a player would have to incentive to use inferior strategies which are dominated by superior one. In such cases of dominance, we can reduce the size of pay off matrix by deleting those strategies which are dominated by others. Thus if each element in one row say k^{th} of pay off matrix (a_{ij}) is less than or equal to the corresponding elements in some other row say r^{th} then player A will never choose k^{th} strategy in other words probability $P_k = 0$ (choosing the k^{th} strategy) is zero, if $a_{kj} < a_{rj}$ for all $j = 1, 2, 3, \dots, n$

The value of the game and the non – zero choice of probabilities remain unchanged even after the deletion of k^{th} row from the pay off matrix. In such a case the k^{th} strategy is said to be dominated by r^{th} one.

General rules of dominance are:

- 1) If all the elements of a row say r^{th} less than or equal to the corresponding elements of any other row say k^{th} then k^{th} is dominated by r^{th} the row.

- 2) If all elements of a column, say k^{th} are greater than or equal to the corresponding elements of any other column say r^{th} then k^{th} column is dominated by r^{th} column.
- 3) Dominated rows and columns can be deleted to reduce the size of pay off matrix is optimal strategies will remain unaffected.

The modified Dominance Property:

The dominance property is not always based on superiority of pure strategies only. A given strategy can also be said to be dominated if it is inferior to an average of two or more other pure strategies more generally. If some convex linear combination of some rows dominates the i^{th} row, then i^{th} row will be deleted similar arguments follow for columns.

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Queuing Theory(Waiting Lines)

Queue called the waiting line in order to maintain a proper discipline. Here the arriving people are called customers and the persons issuing the ticket are called a Server.

Queuing System:

A Queuing system can be completely described by

- I. The input (or arrival pattern)
- II. The service mechanism (or service pattern)
- III. The Queue discipline
- IV. Customer's behavior.

The input pattern (or arrival pattern)

The input describes the way in which the customers arrive and join the system. Generally the customers arrive in more or less random fashion which is not worth making the prediction. Thus the arrival pattern can be described in terms of probability distribution for inter arrival times (the time between two successive arrivals) must be defined. We deal with those queuing system in which the customers arrive in 'Poisson' fashion. Mean arrival rate is denoted by λ .

The service pattern (Out pattern)

It described how many customer can be served at a time, what the statistical distribution of the service time is, and when the service is available. Service time may be a constant or a random variable. Distributions of service time which are important in practice are the exponential distribution. The mean service rate is denoted by μ .

The Queue Discipline:

The Queue discipline is the rule determining the formation of the queue, the manner of the customer's behaviors while waiting, and the manner in which they are chosen for service. The simplest discipline is "First come, first served". Such type of Queue discipline is observed at a ration shop.

Some of the Queue service disciplines are

FIFO – "First in, First out"

LIFO – "Last in, First out"

SIRO – “Service in Random order”

Customer’s Behavior:

The customer generally behaves in 4 ways

1. Balking: A customer may leave the Queue, if there is no waiting space
2. Reneging: This occurs when the waiting customer leaves the Queue due to Impatience.
3. Priorities: In certain applications some customers are served before others regardless of their order of arrival.
4. Jockeying: Customers may jockey from one waiting line to another.

Queuing Models:

Generally Queuing Model may be completely specified in the following symbol form: (a/b/c: d/e)

Where ‘a’ = inter arrival time

‘b’ = inter service time

‘c’ = Number of service channels

‘d’ = system capacity

‘e’ = Queue discipline

Model I $\{M / M / 1: \infty / FIFO\}$

This model deals with a queuing system having single service channel. Poisson input, Exponential service and there is no limit on the system capacity while the customers are served on a “First in, First out” basis.

Characteristics of Model I

1. Average number of customers in system is given by

$$L_s = \frac{\lambda}{\mu - \lambda}$$

2. Average number of customer in Queue

$$L_s = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

3. Average waiting time in system

$$w_s = \frac{1}{(\mu - \lambda)}$$

4. Average waiting time in queue

$$w_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

5. Probability of Queue size being greater than or equal to N

$$P(\text{Queue size} \geq N) = \left(\frac{\lambda}{\mu}\right)^N$$

6. Average length of non – empty queue is given by

$$E\left[\frac{m}{m > 0}\right] = \frac{\mu}{\mu - \lambda}$$

7. Probability of 'n' customer in the system

$$P_n = \left[\frac{\lambda}{\mu}\right]^n \left[1 - \frac{\lambda}{\mu}\right]$$

8. Probability of no customers in the system

$$P_0 = \left[1 - \frac{\lambda}{\mu}\right]$$

9. Expected waiting time of a customer who has to wait

$$E(W / W > 0) = \frac{1}{\mu - \lambda}$$

10. Probability (waiting time in the system $\geq t$)

$$= \int_t^{\infty} (\mu - \lambda) e^{-(\mu - \lambda)w} dw$$

11. Probability (waiting time in the Queue $\geq t$)

$$= \int_t^{\infty} \left(\frac{\lambda}{\mu} \right) (\mu - \lambda) e^{-(\mu - \lambda)w} dw$$

Model II $\{M / M / 1: N / FIFO\}$

This model deals with a queuing system having single service channel. Poisson input, Exponential service and there is 'N' customers in the system capacity while the customers are served on a "First in, First out" basis.

Characteristics of Model II

1. Probability of 'n' customer in the system

$$P_n = \frac{[1 - (\rho)]}{[1 - (\rho)^{N+1}]} (\rho)^n$$

$$\text{Where } \rho = \frac{\lambda}{\mu}$$

2. Probability of no customers in the system

$$P_0 = \frac{[1 - \rho]}{[1 - \rho^{N+1}]}$$

3. Average no. of customers in the system is

$$L_s = P_0 \sum_{n=0}^N n \rho^n$$

$$\text{where } P_0 = \frac{1 - \rho}{1 - \rho^{N+1}}$$

4. Average no. of customer in the Queue

$$L_q = L_s - \frac{\lambda}{\mu}$$

5. Average waiting time in the system

$$W_s = \frac{L_s}{\lambda}$$

6. Average waiting in the Queue

$$W_q = \frac{L_q}{\lambda}$$

Model III $\{M / M / C : \infty / FIFO\}$

This model deals with a queuing system having more than one service channels. Poisson input, Exponential service and there is no limit on the system capacity while the customers are served on a “First in, First out” basis.

Model IV $\{M / M / C : N / FIFO\}$

This model deals with a queuing system having more than one service channels. Poisson input, Exponential service and there is ‘N’ customers in the system capacity while the customers are served on a “First in, First out” basis.

Simulation

Simulation is a imitation of reality. It representation of reality through the use of a model or other device which will react in the same manner as reality under a set of condition.

Examples:

- a. Testing of medicines on animals in laboratories. In this case, the animal responses simulate the human responses.

- b. Driving automobiles on test tracks. The test track simulates the actual environment in which the automobiles are driven.

Monte-Carlo Technique or Monte – Carlo simulation

This technique involves the selection of random observations within the simulation model. It is constrained for application involving random numbers to solve deterministic and stochastic problems the underlying principle of this technique is

- 1) Replace the actual statistical universe by another universe described by some assumed probability distribution.
- 2) Sample from this theoretical population by mean of random numbers.

Steps in Simulation

Step 1: Clearly define the problem

- a) Identify the objectives of the problem.
- b) Identify the main factors which have the greatest effect on the objectives of the problem.

Step 2: Construct an appropriate model.

- a) Specify the variable and parameters of the model
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- a) Define the starting conditions for the simulation.
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Step 4: Using step 1 to 3, experiment with the model.

- a) Define a coding system that will correlate the factors defined in step with the random numbers to be generated for the simulation
- b) Select a random number generator and create the random numbers to used in the simulation.
- c) Associate the generated random numbers with the factors identified in step 1 and coded in step 4 (a).

Step 5: Summarizes and examine the results obtained in step 4.

Step 6: Evaluate the results of the simulation**Advantages to Simulation:**

- Can be used to study existing systems without disrupting the ongoing operations.
- Proposed systems can be “tested” before committing resources.
- Allows us to control time.
- Allows us to identify bottlenecks.
- Allows us to gain insight into which variables are most important to system performance.
- **Manufacturing:** Material handling systems, assembly lines, automated production facilities, inventory control systems, plant layout, etc..
- **Business:** Stock and commodity analysis, pricing policies, marketing strategies, cash flow analysis, forecasting, etc..
- **Government:** Military weapons and their use, military tactics, population forecasting, land use, health care delivery, fire protection, criminal justice, traffic control, etc.

Disadvantages to Simulation

- Model building is an art as well as a science. The quality of the analysis depends on the quality of the model and the skill of the modeler (Remember: *GIGO*)
- Simulation results are sometimes hard to interpret.
- Simulation analysis can be time consuming and expensive should not be used when an analytical method would provide for quicker results.

INVENTORY CONTROL

Inventory control is defined as the scientific method of providing the right type of material at the right time in the right quantities and at right price to sustain the given production schedules.

Inventory control is essentially
concerned with 2 aspects

- * minimising investments for the org. in materials
- * maximising the service levels to the customers and its own operating departments.

Objectives of Inventory Control

- * To support the production departments
- * materials of right quality
 - right quantity
 - right time
 - right price
 - right supplier.
- * To minimise investments in the materials by ensuring economies of storage and ordering costs.
- * To avoid accumulation of work in process
- * To ensure economy of costs by processing EOQ's.
- * To contribute directly to the overall profitability

Inventory Control
is a planned approach
of determining -

- what to order
- when to order
- how much to order
- how much to stock.

Factors Affecting Inventory Control

Inventory control function gets increasingly complex due to the following factors:

- * Sudden changes in the production plans
- * Increases in the material prices
- * Excessive storage costs
- * Stock-out costs [cost of breakdown in the production line as a result of non-availability of inventory]
- * Increasing lead time.

Lead time is the difference between the point of placing order and the time of procurement of stocks.

PROCESS OF INVENTORY CONTROL

Following stages are:

- * Formulate a clear-cut purchase procedure
- * classify in a comprehensive manner, the items of inventory & codify the same
- * Maintain store records to ensure continuous stock taking
- * Minimise the cost of each order by determining Economic Order Quantity [EOQ]

Other Costs

→ capacity costs

- * Over-time payments
- * Lay-offs + idle time

→ Set-up costs

- * Machine set-up
- * start-up scrap generated from getting a production run started

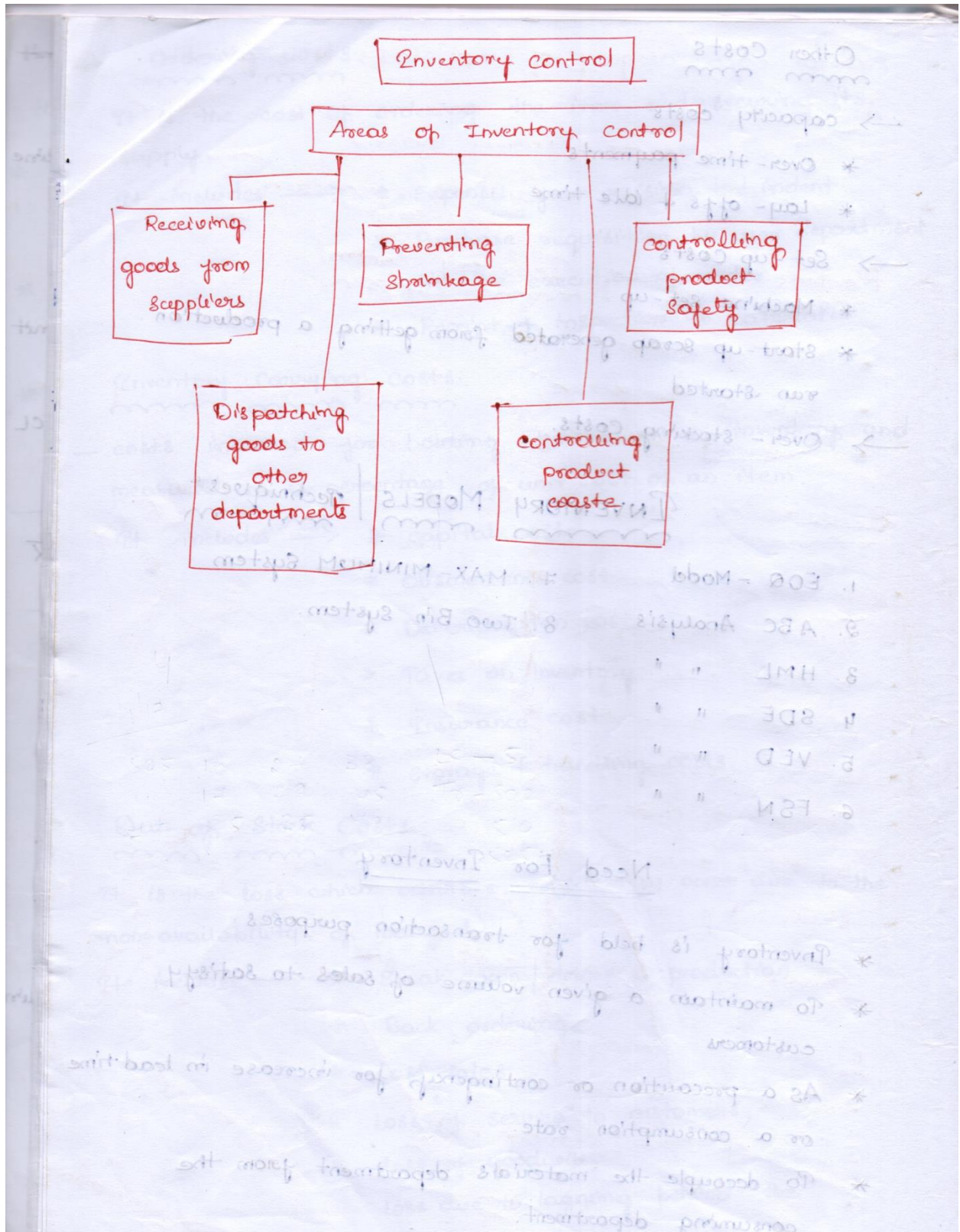
→ Over-stocking costs.

INVENTORY MODELS / Techniques

1. EOQ - Model
2. ABC Analysis
3. HML " "
4. SDE " "
5. VED " "
6. FSN " "
7. MAX-MINIMUM System
8. Two Bin System.

Need For Inventory

- * Inventory is held for transaction purposes
- * To maintain a given volume of sales to satisfy customers
- * As a precaution or contingency for increase in lead time or a consumption rate.
- * To decouple the materials department from the consuming department.



EOQ - ECONOMIC ORDER QUANTITY

[Barabab EOQ Model]

Introduction :

Economic Order Quantity is the order quantity that minimizes the total holding costs and ordering costs. It is one of the oldest classical production scheduling models.

The framework used to determine this order quantity is also known as Wilson EOQ Model, Wilson Formula or Andler Formula.

The model was developed by Ford W. Harris in 1913, but R.H. Wilson, a consultant who applied it extensively, and K. Andler are given credit for their in-depth analysis.

→ In other words, the EOQ is the amount of inventory to be ordered at one time for purposes of minimizing annual inventory cost.

→ The quantity to order at a given time must be determined by balancing two factors.

- 1) The cost of possessing or carrying materials
- 2) The cost of acquiring or ordering materials.

{ EOQ is also called Fixed Order Quantity. }

* EOQ is the technique of ordering materials whenever stock reaches the reorder point.

* In this technique, the order quantity is larger than a single period's requirement so that ordering costs + holding costs balance out.

Method of determining EOQ

Let us define the EOQ variables as below :

A = Annual Demand

S = Size of each Order

O = ordering cost per order

C = carrying cost per unit

Step 1 → Find out the total ordering cost per year

$$\begin{aligned} \text{Total ordering cost per year} &= \text{Number of orders placed per year} \times \text{Ordering costs per order} \\ &= \frac{A}{S} \times O \end{aligned}$$

Step 2 → Find out total carrying cost per year

$$\begin{aligned} \text{Total carrying cost per year} &= \text{Average Inventory level} \times \text{carrying cost per year} \\ &= \frac{S}{2} \times C \end{aligned}$$

Step 3 → Determine EOQ

EOQ is one where the total ordering cost is equal to total carrying cost or

$$\frac{A}{S} \times O = \frac{S}{2} \times C$$

$$2AO = S^2 C$$

$$S^2 = \frac{2AO}{C}$$

$$S = \sqrt{\frac{2AO}{C}}$$

ABC Analysis

In materials management, the ABC analysis (or Selective Inventory Control) is an inventory categorization technique.

ABC analysis divides an inventory into three categories —

- A items — with very tight control & accurate records
- B items — with less tightly controlled & good records
- C items — with the simplest controls possible and minimal records.

ABC analysis provides a mechanism for identifying items that will have a significant impact on overall inventory cost, while also providing a mechanism for identifying different categories of stock that will require different mgt & controls.

The ABC analysis suggests that inventories of an org are not of equal value.

Thus, the inventory is grouped into three categories (A, B and C) in order of their estimated importance.

- 'A' items — 20% of the items accounts for 70% of the annual consumption value of items.
- 'B' items — 30% of the items accounts for 25% of the annual consumption value of the items.
- 'C' items — 50% of the items accounts for 5% of the annual consumption value of the items.

PROCEDURE FOR ABC ANALYSIS

- * Make the list of all items of inventory.
- * Determine the annual volume of usage & money value of each item.
- * Multiply each item's annual volume by its rupee value.
- * Compute each item's percentage of the total inventory in terms of annual usage in rupees.
- * Select the top 10% of all items which have the highest rupee percentages & classify them as 'A' items.
- * Select the next 20% of all items with the next highest rupee percentages & designate them 'B' items.
- * The next 70% of all items with the lowest rupee percentage are 'C' items.

ADVANTAGES OF ABC ANALYSIS

- * Helps to exercise selective control.
- * Gives recording results quickly.
- * Helps to point out obsolete stocks easily.
- * Helps better planning of inventory control.
- * Provides sound basis for allocation of funds & human resources.
- * In case of 'A' items careful attention can be paid at every step such as estimate of requirements, purchase, safety stock, receipts, inspections, issues etc & close control is maintained.
- * In case of 'C' items, recording & follow up etc may be dispensed with or combined.

Disadvantages of ABC Analysis

- * Proper standardisation & codification of inventory items needed
- * Periodic review becomes difficult if only ABC analysis is recalled.
- * Considers only money value of items & neglects the importance of items for the production process or assembling or functioning.
- * When other important factors make it obligatory to concentrate on "c" items more, the purpose of ABC analysis is defeated.

A - items

1. Tight controls
2. Rigid estimates of Requirements
3. Strict and close watch
4. Safety stocks should be low
5. Management of items should be done at top mgt level.

B - items

1. Moderate control
2. Purchase based on rigid requirements
3. Reasonably strict watch & control
4. Safety stock moderate
5. Mgt. be done at middle level.

Dynamic Programming

Introduction: Dynamic programming is to perform optimization (Minimization or Maximization). It is to divide the original problem into many sub problems and solve the sub problems individual obtain the optimum solution of the last problem by integrating the optimum solution of original problem.

Characteristic of DPP

The features which characterize the dynamic programming problem are as follows:

1. The problem can be divided into stages with a policy decision required at each stage.
2. Every stage consists of a number of states associated with it. The states are the different possible condition in which the system may find itself at that stage of the problem
3. Decision at each stage converts the current stage into state associated with the next stage.
4. The state of the system at a stage is described by a set of variables called state variables.
5. When the current state is known, an optimal policy for the remaining stages is independent of the policy of the previous ones.
6. The solution procedure begins by finding the optimal policy for each state to the last stage.
7. A recursive relationship which identifies the optimal policy for each state with n stages remaining, given the optimal policy for each state with $(n-1)$ stages left
8. Using recursive equation approach, each time the solution procedure moves backward stage by stage by stage for obtaining the optimum policy of each state for the particular stage, till it attains the optimum policy beginning at the initial stage.

Dynamic Programming Algorithm

The computational procedure for solving a problem by dynamic programming approach can be summarized in the following steps:

Step 1: Identify the decision variables and specify objective function to be optimized under certain limitations, if any

Step 2: Decompose (or divide) the given problem into a number of smaller sub-problems (or stages). Identify the state variables at each stage and write down the transformation function as a function of the state variables and decision variables at the next stage.

Step 3 : write down a general recursive relationship for computing the optimal policy decide whether forward or Backward method is to follow to solve the problem

Step 4: Construct appropriate stages to show the required values of the return function.

Step 5: Determine the overall optimal policy or decisions and its value at each stage there may be more than one such optimal policy.

Applications of Dynamic Programming:

- In the production area, this technique has been used for production scheduling and employment smoothing problems
- It has been used to determine the inventory level and for formulating the inventory recording
- It can be applied for allocating the scarce resources to different alternative uses such as, allocating the salesmen to different sales districts etc.
- It is used to determine the optimal combination of advertising media(TV, Radio, News papers) and the frequency of advertising.
- It can apply in Replacement theory to determine at which age the equipment is to replace for optimal return from the facilities.
- Spare part level determination to guarantee high efficiency utilization of expensive equipment.
- Other areas: Scheduling methods, Monrovia decision models, infinite stage system, probabilistic decision problem etc.

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