Code: 9ABS104

1

B.Tech I Year (R09) Regular & Supplementary Examinations, June 2013

MATHEMATICS - I

(Common to all branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE questions

All questions carry equal marks

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1 (a) Solve : $\left(1+e^{x/y}\right)dx+\left(1-\frac{x}{y}\right)e^{\frac{x}{y}}dy=0$.

(b) Solve : x dx + y dy = $\frac{xdy-ydx}{x^2+y^2}$.

2 (a) Solve : $(D^3 - 1)y = (e^x + 1)^2$.

(b) Solve : $(D^2 - k^2)y = \cos h\, kx$.

3 (a) Find the points on the surface $z^2 = xy + 1$ that are nearest to the origin.

(b) Find the stationary points of $u(x, y) = \sin x \sin y \sin(x + y)$ where $0 < x < \pi$, $0 < y < \pi$ and find the maximum u.

4 (a) Trace the curve $y = x^3$.

(b) Trace the curve $y = (x - 1)(x - 2)(x - 3)$.

5 (a) Evaluate: $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)dx\,dy$.

(b) Evaluate the integral by changing the order of integration: $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$.

6 (a) Find the Laplace transform of $\left\{\left(\sqrt{t}-\frac{1}{\sqrt{t}}\right)^3\right\}$.

(b) Find: $L^{-1}\left\{\frac{1}{2}\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}$.

7 (a) Solve the D.E. $y''+2y'+5y=8\sin t+4\cos t,\ y(0)=1,\ y(\frac{\pi}{4})=\sqrt{2}$. Using Laplace transform.

(b) Using Laplace transform, evaluate $\int_0^\infty e^{-4t}\sin^3 t\ dt$.

8 (a) Use divergence theorem to evaluate $\int_s \overline{F}\cdot\overline{N}ds$, where $\overline{F}=x^3\boldsymbol{i}+y^3\boldsymbol{j}+\boldsymbol{z}^3\boldsymbol{k}$, and S is the surface of the sphere $x^2+y^2+z^2=a^2$.

(b) Evaluate: $\nabla\left[\nabla\cdot\left(\frac{\overline{R}}{r}\right)\right]$, where $\overline{R}$ = xi + yj + zk , r = $|\overline{r}|$.

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1 (a) Solve : $\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$.

(b) Solve : $2y \cos y^2 \frac{dy}{dx} - \frac{2}{x+1} \sin y^2 = (x+1)^3$.

2 (a) Solve : $(D^2 - 1)y = 2e^x + 3x$.

(b) Solve : $(D^2 + 1)y = \text{cosec } x$.

3 (a) Find the shortest and the longest distance from the point (1, 2, -1) to the sphere $x^2 + y^2 + z^2 = 24$.

(b) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $4x^2 + 4y^2 + 9z^2 = 36$.

4 (a) Trace the curve $r = a(1 + \cos\theta)$.

(b) Trace the curve $r = a + b\cos\theta$, $a > b$.

5 (a) Evaluate: $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\,dx\,dy$.

(b) Evaluate the integral by changing the order of integration: $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2)\,dx\,dy$.

6 (a) Find the Laplace transform of: (i) $\left\{\frac{\sin^2 t}{t}\right\}$. (ii) $\left\{\frac{1-\cos at}{t}\right\}$.

(b) Find: $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ using convolution theorem.

7 (a) Solve the D.E. $y'' + 4y' + 4y = 4e^{-2t}$, $y(0) = -1$, $y'(0) = 4$. Using Laplace transform.

(b) Solve the D.E $\frac{dy}{dt} + 2y + \int_0^t y\,dt = 2\cos t$, $y(0) = 1$. Using Laplace transform.

8 (a) If $\bar{A}$ is a constant vector and $\bar{R} = x\bar{i} + y\bar{j} + z\bar{k}$, prove that $\nabla \times \left(\frac{\bar{A}\times\bar{r}}{r^n}\right) = \frac{(2-n)\bar{A}}{r^n} + \frac{n(\bar{r}\cdot\bar{A})\bar{r}}{r^{n+2}}$.

(b) If $\bar{F} = (5xy - 6x^2)\boldsymbol{i} + (2y - 4x)\boldsymbol{j}$, evaluate $\int_c \bar{F}\cdot d\bar{R}$, where C is the curve in the xy-plane $y = x^3$ from (1, 1) to (2, 8).

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MATHEMATICS - I

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1 (a) Solve : $x\frac{dy}{dx} + y = x^3 y^6$.

(b) Solve : $\frac{dy}{dx} + \frac{y}{x} = y^2 x \sin x$.

2 (a) Solve : $(D^2 - 3D + 2)y = \cos hx$.

(b) Solve : $(D + 2)(D - 1)^2 4 = e^{-2x} + 2 \sin hx$.

3 (a) Prove that the maximum value of $x^m y^n z^p$ under the condition $x + y + z = a$ is $m^m n^n p^p a^{m+n+p} / (m + n + p)^{m+n+p}$.

(b) Find the maximum and minimum distance from the origin to the curve $5x^2 + 6xy + 5y^2 - 8 = 0$.

4 (a) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$.

(b) Trace the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

5 (a) Evaluate: $\int_0^2 \int_0^x e^{x+y}\, dy\, dx$.

(b) Change the order of integration in $\int_0^a \int_y^a \frac{x\, dx\, dy}{x^2+y^2}$ and hence evaluate the same.

6 (a) Find the Laplace transform of $\left\{e^{-4t} \int_0^t \frac{\sin 3t}{t}\, dt\right\}$.

(b) Find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$. Using convolution theorem.

7 (a) Solve the D.E. $y'' + 2y' + y = t$, $y(0) = -3$, $y(1) = -1$. Using Laplace transform.

(b) Solve the integral equation $y(t) = t^2 + \int_0^t y(u) \sin(t - u)\, du$, using Laplace transform.

8 (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1, 2).

(b) Apply Greens theorem to evaluate $\int_C [(2x^2 - y^2)\, dx + (x^2 + y^2)\, dy]$, where C is the boundary of the area enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$.

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1 (a) Solve : $\frac{dy}{dx} + yx = y^2 e^{x^2/2} \sin x.$

(b) Solve : $x\frac{dy}{dx} + y = y^2 x^3 \cos x.$

2 (a) Solve : $(D^2 - 5D + 6)\, y = xe^{4x}$.

(b) Solve : $(D^2 + a^2)\, y = \text{Sec } ax$.

3 (a) Find a point on the plane 3x + 2y + z - 12 = 0, which is nearest to the origin.

(b) Prove that it the perimeter of triangle is constant there its area is maximum when the triangle is equilateral.

4 (a) Prove that the volume of revolution of $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{\pi a^3}{6\sqrt{2}}\left[3\log\left(\sqrt{2}+1\right)-\sqrt{2}\right]$.

(b) Determine the volume of the solid generated by revolving the lemicon r = a + b cosθ (a > b) about the initial line.

5 (a) Evaluate: $\int_0^1 \int_y^1 \frac{dx\, dy}{\sqrt{(1-x^2)(1-y^2)}}$.

(b) Change the order of integration in $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \; dy\, dx$ and evaluate.

6 (a) Find the Laplace transform of $\left\{ 2^t + \frac{\cos 2t - \cos 3t}{t} + t \sin t \right\}$.

(b) Apply convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$.

7 (a) Solve the D.E. $y'' + 9y = cos2t$, $y(0) = 1,\ y(\frac{\pi}{2}) = -1$. Using Laplace transform.

(b) Solve the integral equation $y(t) = 1 + \int_0^t y(u)\sin(t-u)\, du$, using Laplace transform.

8 (a) A vector field is given by $\vec{A} = (x^2 + xy^2)\bar{i} + (y^2 + x^2 y)\bar{j}$. Show that the field is irrotational and find the scalar potential.

(b) Use the divergence theorem to show that $\int_S \nabla r^2 . d\bar{S}$ = 6V, where S is any closed surface enclosing a volume V and $\bar{r}$ = xi + yj + zk.

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