

Code: 9ABS105

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B.Tech I Year (R09) Regular &amp; Supplementary Examinations, June 2013

**MATHEMATICAL METHODS**

(Common to EEE, ECE, CSE, EIE, E.Con.E, ECM, IT &amp; CSS)

Time: 3 hours

Max. Marks: 70

Answer any FIVE questions  
 All questions carry equal marks

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- 1 (a) Reduce the  $\begin{bmatrix} 1 & -2 & 1 & 2 \\ 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \end{bmatrix}$  matrix into the normal form and hence find its rank.
- (b) Test the system of equations  $2x + y + 5z = 4$ ;  $3x - 2y + 2z = 2$ ;  $5x - 8y - 4z = 1$  consistency. If consistent solve them.
- 2 Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation.  $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$
- 3 (a) Using the Newton – Raphson method find the root of the equation  $f(x) = e^x - 3x$  that lies between 0 and 1.
- (b) State appropriate interpolation formula which is to be used to calculate the value of  $\exp(1.75)$  from the following data and hence evaluate it from the given data.

|                    |       |       |       |       |
|--------------------|-------|-------|-------|-------|
| x                  | 1.7   | 1.8   | 1.9   | 2.0   |
| y = e <sup>x</sup> | 5.474 | 6.050 | 6.686 | 7.389 |

- 4 Determine the constants a and b by the method of least squares such that  $y = ae^{bx}$ .

|     |       |        |        |        |        |
|-----|-------|--------|--------|--------|--------|
| x:  | 2     | 4      | 6      | 8      | 10     |
| y : | 4.077 | 11.084 | 30.128 | 81.897 | 222.62 |

- 5 Given  $\frac{dy}{dx} - \sqrt{xy} = 2$ ,  $y(1) = 1$  find the value of  $y(2)$  in steps of 0.2 using modified Euler's method.
- 6 (a) Obtain the Fourier series for the function  $f(x)$  is given by  $f(x) = \begin{cases} 0 & , -\pi < x < 0, \\ \sin x, & 0 < x < \pi. \end{cases}$  Deduce that  $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} \dots = \frac{1}{4}(\pi - 2)$ .
- (b) Find the Fourier cosine transform of  $f(x) = e^{-ax} \cos ax$ ,  $a > 0$ .
- 7 Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ , which satisfies the conditions  $u(0, y) = 0$ ,  $u(L, y) = 0$ ,  $u(x, 0) = 0$  and  $u(x, a) = f(x)$ .
- 8 (a) Find  $Z \left\{ \cos \left( \frac{n\pi}{2} \right) \right\}$  and  $Z \left\{ \sin \left( \frac{n\pi}{2} \right) \right\}$ .
- (b) State and prove convolution theorem for Z-transform.

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- 1 (a) Reduce the  $\begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & 5 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & -2 & 0 \end{bmatrix}$  matrix, to normal form and find its rank.  
 (b) Solve the system  $2x - y + 4z = 12$ ;  $3x + 2y + z = 10$ ;  $x + y + z = 6$ ; if it is consistent.
- 2 Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation.  $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$
- 3 (a) Evaluate  $\sqrt{28}$  to four decimal places by Newton's iterative method.  
 (b) Using Newton's forward interpolation formula, and the given table of values.
 

|      |      |      |      |      |      |
|------|------|------|------|------|------|
| x    | 1.1  | 1.3  | 1.5  | 1.7  | 1.9  |
| f(x) | 0.21 | 0.69 | 1.25 | 1.89 | 2.61 |

 Obtain the value of  $f(x)$  when  $x = 1.4$ .
- 4 Find the curve of best fit of the type  $y = ae^{bx}$  to the following data by the method of least squares.
 

|     |    |    |    |    |    |
|-----|----|----|----|----|----|
| x:  | 1  | 5  | 7  | 9  | 12 |
| y : | 10 | 15 | 12 | 15 | 21 |
- 5 Determine  $y(0.8)$  and  $y(1.0)$  by Milne's predictor-corrector method when  $\frac{dy}{dx} = x - y^2$ ,  $y(0) = 0$ .
- 6 (a) Obtain a half-range cosine series for  $f(x)$  is given by  $f(x) = \begin{cases} kx & , 0 \leq x \leq \frac{L}{2} \\ k(L-x), & \frac{L}{2} \leq x \leq L. \end{cases}$  Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .  
 (b) Prove that Fourier cosine and sine transforms are linear.
- 7 Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  which satisfies the conditions  $u(0, y) = 0$ ,  $u(L, y) = 0$ ,  $u(x, 0) = 0$  and  $u(x, a) = \sin\left(\frac{n\pi x}{L}\right)$ .
- 8 (a) Prove that  $Z(n^p) = -z \frac{d}{dz} Z(n^{p-1})$ ,  $p$  being a +ve integer. Hence evaluate  $Z(n)$  and  $Z(n^2)$ .  
 (b) Find:  $Z^{-1} \left\{ \frac{2z}{(z-1)(z^2+1)} \right\}$ .

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1 (a) Determine the rank of the matrix  $A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

- (b) Test the following system for consistency and if consistent solve it.  
 $u + 2v + 2w = 1$ ,  $2u + v + w = 2$ ,  $3u + 2v + 2w = 3$ ,  $v + w = 0$ .

- 2 Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation.
- $$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

- 3 (a) Find the root between 0 and 1 of the equation  $x^3 - 6x + 4 = 0$  correct to five decimal places.  
 (b) Find the values of  $\cos 1.747$  using the values given in the table below:

|        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| x:     | 1.70   | 1.74   | 1.78   | 1.82   | 1.86   |
| sin x: | 0.9916 | 0.9857 | 0.9781 | 0.9691 | 0.9584 |

- 4 Obtain a relation of the form  $y = ae^{bx}$  for the following data by the method of least squares.

|     |     |      |      |      |       |
|-----|-----|------|------|------|-------|
| x:  | 2   | 3    | 4    | 5    | 6     |
| y : | 8.3 | 15.4 | 33.1 | 65.2 | 127.4 |

- 5 Find  $y(0.8)$  by Milne's method for  $\frac{dy}{dx} = y - x^2$ ,  $y(0) = 1$ . Obtaining the starting values by Taylor's series method.

- 6 (a) Obtain the Fourier series for the function  $f(x)$  is given by  $f(x) = \begin{cases} x & , 0 \leq x \leq \pi, \\ 2\pi - x & , \pi \leq x \leq 2\pi. \end{cases}$  and  
 Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ .

- (b) Find the Fourier transform of  $f(x) = e^{-\frac{x^2}{2}}$ ,  $-\infty < x < \infty$ .

- 7 Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  within the rectangle  $0 \leq x \leq a$ ,  $0 \leq y \leq b$ , given that  $u(0, y) = u(a, y) = u(0, y) = 0$  and  $u(0, y) = x(a - x)$ .

- 8 (a) Find  $Z\{(\cos \theta + i \sin \theta)^n\}$ . Hence evaluate  $Z(\cos n\theta)$  and  $Z(\sin n\theta)$ .

- (b) Find  $Z^{-1}\left\{\frac{3z^2+z}{(5z-1)(5z+2)}\right\}$ .

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- 1 (a) Reduce the matrix  $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$  to canonical form (normal) and hence find its rank.  
 (b) Solve the system of homogeneous equations given by  $2x + y + 2z = 0$ ,  $x + y + 3z = 0$ ,  $4x + 3y + 8z = 0$ .
- 2 Diagonalize the following matrix by an orthogonal transformation and also find the matrix of transformation.  $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$
- 3 (a) Find the root for the following equation using bisection method correct to two decimal places:  $e^x - x + 2 = 0$  in 1, 1.4  
 (b) Find  $f(2.5)$  using the following table:

|      |   |   |    |    |
|------|---|---|----|----|
| x    | 1 | 2 | 3  | 4  |
| f(x) | 1 | 8 | 27 | 64 |

- 4 Fit the curve  $y = ae^{bx}$  for the following data.

|     |    |    |    |    |     |     |     |     |      |
|-----|----|----|----|----|-----|-----|-----|-----|------|
| x:  | 0  | 1  | 2  | 3  | 4   | 5   | 6   | 7   | 8    |
| y : | 20 | 30 | 52 | 77 | 135 | 211 | 326 | 550 | 1052 |

- 5 Using Runge-Kutta method of 4<sup>th</sup> order, find the solution of  $\frac{dy}{dx} = x^2 + 0.25y^2$ ,  $y(0) = -1$  on  $[0, 0.5]$  with  $h = 0.1$ .
- 6 (a) Express  $f(x) = x$  as a half-range sine series in the interval  $0 < x < 2$ .  
 (b) Find the Fourier cosine transform of  $f(x) = \begin{cases} x & , \text{for } 0 < x < 1 \\ 2 - x & , \text{for } 1 < x < 2 \\ 0 & , \text{for } x > 2 \end{cases}$
- 7 Solve  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  for  $0 \leq x \leq \pi$ ,  $0 \leq y \leq \pi$  which satisfies the conditions  $u(0, y) = 0$ ,  $u(\pi, y) = 0$ ,  $u(x, \pi) = 0$  and  $u(x, 0) = \sin^2 x$ .
- 8 (a) If  $Z(u_n) = \bar{u}(z)$  prove that  $Z(a^n u_n) = \bar{u}\left(\frac{z}{a}\right)$ .  
 (b) Using Z- transform, solve  $4u_n - u_{n+2} = 0$ , given that  $u_0 = 0$ ,  $u_1 = 2$ .

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