B.Tech II Year I Semester (R15) Regular & Supplementary Examinations November/December 2018

PROBABILITY THEORY & STOCHASTIC PROCESSES

(Electronics and Communication Engineering)

Time: 3 hours Max. Marks: 70

PART – A

(Compulsory Question)

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1 Answer the following: (10 X 02 = 20 Marks)

(a) Define the distribution function of a discrete random variable X.

(b) When two events are said to be independent?

(c) How to compute the probability of an event $P\{x_1 < X \leq x_2\}$ by using distribution function $F_x(X)$?

(d) If the variance of a random variable X is Var(X), then the variance of a random variable Y = aX is.

(e) Statistically independent zero-mean random processes X(t) and Y(y) have autocorrelation functions $R_{XX}(\tau)$ and $R_{yy}(\tau)$, then ACF of 'X(t) + Y(t)' is.

(f) What is the second order moment of the random processes X(t) if $R_{XX}(\tau) = \frac{16}{1+6\tau^2}$?

(g) Why the function $S_{XY}(w) = 3 + jw^2$ is a valid CPSD?

(h) Average power of Random processes X(t) = Acos (wt+Θ), where Θ is RV.

(i) Define narrow band process.

(j) Obtain the ratio between output PSD $S_{YY}(w)$ to input PSD $S_{XX}(w)$ from magnitude spectrum: $|H(w)| = \frac{4}{\sqrt{3+\omega^2}}$.

PART – B

(Answer all five units, 5 X 10 = 50 Marks)

UNIT – I

2 A random variable X has the distribution function: $F_X(X) = \sum_{n=1}^{12} \frac{n^2}{650} u(x-n)$ evaluate the probability: (i) $P\{-\infty < X \leq 6.5\}$. (ii) P(X> 4} (iii) P{6 < $X \leq 9\}$.

OR

3 Assume automobile arrivals at a gasoline station are Poisson and occurs at an average rate of 50per/Hour. The station has only one gasoline pump. If all cars are assumed to require one minute to obtain fuel, What is the probability that a weighting line will occur at the pump.

UNIT – II

4 Two random variables X and Y have means $\bar{X} = 1$ and $\bar{Y} = 2$ variances $\sigma_X^2 = 4$ and $\sigma_Y^2 = 1$ and a correlation coefficient $\rho_{XY} = 0.4$. New random variables W and V are defined by V = -X + 2Y, W = X + 3Y. Find: (i) The means. (ii) The variances. (iii) The correlations. (iv) The correlation coefficient ρ_{VW} of V and W.

OR

5 Let X & Y be statically independent random variables with $\bar{X} = \frac{3}{4}, \overline{X^2} = 4, \bar{Y} = 1, \overline{Y^2} = 5$. For a random variable W = X-2Y+1, then calculate: (i) R_{XY}. (ii) R_{XW}. (iii) C_{XY} and verify X & Y are uncorrelated or not.

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UNIT – III

6 If $X(t) = A\,Cos(\omega_0 t + \theta), where\ A,\ \omega_0$ are constants, and θ is a uniform random variable on $(-\pi, \pi)$. A new random process is defined by $Y(t) = X^2(t)$.
(i) Obtain the mean and auto correlation function of $X(t)$.
(ii) Obtain the mean and auto correlation function of $Y(t)$.
(iii) Find the cross correlation function of $X(t)$ & $Y(t)$.
(iv) Are $X(t)$ and $Y(t)$ are WSS.
(V) Are $X(t)$ & $Y(t)$ are jointly WSS.

OR

7 Two random process $X(t)$ & $Y(t)$ are defined as:
$X(t) = A\,Cos(\omega_0 t) + B\,Sin(\omega_0 t),\ Y(t) = B\,Cos(\omega_0 t) - A\,Sin(\omega_0 t),$ A, B are uncorrelated, zero mean random variables with same variance, ω_0 is constant: (i) Determine $R_{XY}(t, t+\tau)$. (ii) check $X(t),\ Y(t)$ are jointly WSS or not.

UNIT – IV

8 Suppose the cross power spectrum is defined by:

$$S_{XY}(\omega) = a + \frac{jb\omega}{W}, -W \le \omega \le W$$
$$0, \quad Otherwise$$

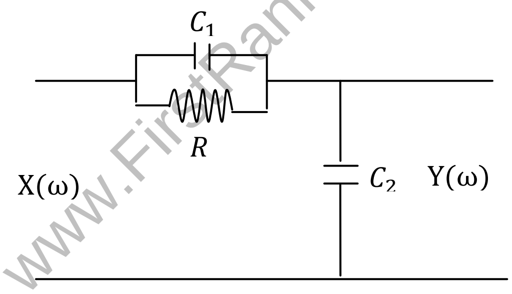
Where a, b are real constants, then obtain cross correlation functions $R_{XY}(\tau)$ and $R_{YX}(\tau)$.

OR

9 Determine the cross correlation function, whose cross PSD is
$S_{XY}(\omega) = \frac{8}{(\alpha + j\omega)^3}$ and also find $S_{YX}(\omega),\ R_{YX}(\omega)$.

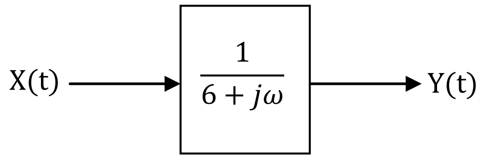
UNIT – V

10 Obtain the transfer function $H(\omega)$ of the network as shown in figure below if C_1 = 5F, C_2 = 10F and $R = 10\Omega$, then determine $S_{XY}(\omega)$ if $R_{XX}(\tau) = 5\,\delta(\tau)$.



OR

11 Consider a linear system as shown in figure below.



If ACF of input $R_{XX}(\tau) = 5\,\delta(\tau)$, then determine: (i) ACF of response. (ii) PSD of response. (iii) Mean square value of response.

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