

Code No: 133BC

R16**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD****B.Tech II Year I Semester Examinations, April/May - 2018****MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE****(Common to CSE, IT)****Time: 3 Hours****Max. Marks: 75****Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A**(25 Marks)**

- 1.a) Construct the truth table for the following formula:
 $\neg(P \vee (Q \wedge R)) \leftrightarrow ((P \vee Q) \wedge (P \vee R))$ [2]
- b) Explain duality law. [3]
- c) Give the formal definition for the composition of binary relations. [2]
- d) What are the properties of a group? [3]
- e) State addition principle and give an example of a problem solved by addition principle. [2]
- f) State pigeon-hole principle. [3]
- g) What is the general form of a first-order recurrence relation? [2]
- h) What is the generating function of $1, -1, 1, -1, \dots$ [3]
- i) If a simple graph G contains n vertices and m edges, how many number of edges are present in Graph G' (complement of G). [2]
- j) How many edges are present in a complete graph with n vertices? Explain. [3]

PART- B**(50 Marks)**

- 2.a) Show the following equivalence without constructing the truth table.
 $((P \wedge Q \wedge A) \rightarrow C) \wedge (A \rightarrow (P \vee Q \vee C)) \leftrightarrow (A \wedge (P \leftrightarrow Q)) \rightarrow C$
- b) Without constructing a truth table, show that $A \wedge E$ is not a valid consequence of
 $A \leftrightarrow B, B \leftrightarrow (C \wedge D), C \leftrightarrow (A \vee E), A \vee E$ [5+5]
- OR**
- 3.a) Obtain the principal disjunctive and conjunctive normal form of the following formula.
 $(P \rightarrow (Q \wedge R)) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$
- b) For the following formulas, let the universe be \mathbb{R} . Translate each of the following sentences into a formula (using quantifiers):
- i) There is a smallest number.
- ii) Every positive number has a square root. (Do not use the square root symbol; use only multiplication.) [5+5]

- 4.a) Consider the following Hasse diagram of a partially ordered set $\langle P, R \rangle$, where $P = \{x_1, x_2, x_3, x_4, x_5\}$. Find the least and greatest members in P if they exist. Also find the maximal and minimal elements of P . Find the upper and lower bounds of $\{x_2, x_3, x_4\}$, $\{x_2, x_4, x_5\}$ and $\{x_1, x_2, x_3\}$. Also indicate the LUB and GLB of these subsets if they exist.

- b) Let $n \in \mathbb{N}^+$ and G_1, G_2, \dots, G_n be groups, and consider

$$\prod_{i=1}^n G_i := G_1 \times G_2 \times \dots \times G_n = \{(a_1, a_2, \dots, a_n) : a_i \in G_i \forall i = 1, 2, \dots, n\} \text{ with the operation } \dagger$$

where if $x = (a_1, a_2, \dots, a_n)$ and $y = (b_1, b_2, \dots, b_n)$, then $x \dagger y = (a_1 b_1, a_2 b_2, \dots, a_n b_n)$, where each product $a_i b_i$ is performed according to the operation of the group G_i . Show

that $\prod_{i=1}^n G_i$ is a group.

[5+5]

OR

- 5.a) Find the transitive closure of the relation $R = \{(1,2), (2,3), (3,4), (4,1)\}$. Show R^i for all values of i that give new elements of the transitive closure.

- b) Find all the subgroups of (i) $(\mathbb{Z}_{12}, +_{12})$; and (ii) $(\mathbb{Z}_7^*, \times_7)$.

[5+5]

6. In the United States and Canada, a telephone number is a 10-digit number of the form $NXX - NXX - XXXX$ where $N \in \{2,3,\dots,9\}$ and $X \in \{0,1,2,\dots,9\}$. How many telephone numbers are possible? The first three digits of a telephone number are called an area code. How many different area codes must a city with 23,000,000 phones have? A previous scheme for forming a telephone numbers required a format of $NYX - NXX - XXXX$ where N and X are defined as above and Y is either a 0 or a 1. How many more phone numbers are possible under the new format than under the old format?

[10]

OR

- 7.a) How many four letter words can be formed using the letters $a, a, a, b, b, c, c, c, c, d, d$?

- b) Expand $(2x - y)^7$ using the Binomial Theorem.

[5+5]

- 8.a) Solve the recurrence relation $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 2$ where $a_0 = 2$ and $a_1 = 2$.

- b) Using generating function find a_n in terms of n if $a_0 = 1, a_1 = 2$ and $a_{n+2} = 5a_{n+1} - 4a_n$ for $n \geq 0$.

[5+5]

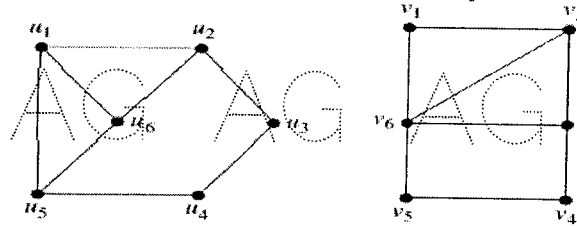
OR

- 9.a) Solve the recurrence relation $T(n) = 4T(n-1) + 2^n$, with $T(0) = 6$.

- b) Find the coefficient of x^{2005} in the generating function $\frac{1}{(1+5x)^2}$.

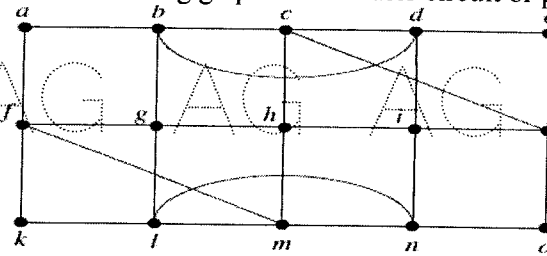
[5+5]

10.a) Determine whether the given pair of graphs is isomorphic?



b) Determine whether the following graph has an Euler circuit or path.

[5+5]



OR

11.a) How do you test the planarity of a graph? Explain.

b) What are the chromatic numbers of the graph G and H ?

[5+5]

