

**R13**

Code No: 113AH

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year I Semester Examinations, November - 2015**
**MATHEMATICS-III**

(Common to EEE, ECE, EIE, AGE, ETM)

**Time: 3 Hours**
**Max. Marks: 75**
**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART-A**
**(25 Marks)**

- 1.a) Determine the nature of the point  $x=0$  for the equation  $x^2(x^2+1)y''+(x^2-1)y'+2y=0$ . [2M]
- b) Find the series solution of the equation  $y''-y=0$ . [3M]
- c) Express  $2-3x+4x^2$  in terms of Legendre polynomial. [2M]
- d) Express  $J_3(x)$  in terms of  $J_0$  and  $J_1$ . [3M]
- e) Prove that  $f(z)=\bar{z}$  is not analytic at any point. [2M]
- f) Show that the function  $f(z)=\sin x \cosh y + i \cos x \sinh y$  is continuous as well as analytic every where. [3M]
- g) State the Cauchy's Residue theorem. [2M]
- h) Expand  $\log z$  by Taylor's series about  $z=1$ . [3M]
- i) Define conformal transformation. [2M]
- j) Find the points at which  $w=\cosh z$  is not conformal. [3M]

**PART-B**
**(50 Marks)**

- 2.a) Obtain the series solution of the equation  $y''+xy'+y=0$
- b) Find the series solution of  $4xy''+2y'+y=0$ . [4+6]

**OR**

- 3.a) Solve in series the equation  $y''+xy=0$
- b) Solve in series the equation  $(1-x^2)y''-2xy'+n(n+1)y=0$  about  $x=0$ . [4+6]
- 4.a) Prove that  $nP_n=(2n-1) \times P_{n-1}-(n-1)P_{n-2}$ ,  $n \geq 2$
- b) State and prove generating function of Bessel's function. [5+5]

**OR**

- 5.a) Prove that  $\int_{-1}^1 P_n(x)P_m(x)dx=0$ , if  $m \neq n$ ,  $2/(2n+1)$  if  $m=n$ .
- b) Prove that  $J_0^2+2(J_1^2+J_2^2+J_3^2+\dots)=1$ . [5+5]
- 6.a) Find the analytic function whose real part is  $u=e^{2x}(x \cos 2y - y \sin 2y)$ .
- b) Evaluate  $\int_C \operatorname{Re} z \, dz$  where  $C$  is the shortest path from  $1+i$  to  $3+2i$ . [5+5]

**OR**

- 7.a) State and prove Cauchy's integral theorem.
- b) Evaluate using Cauchy's integral formula  $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ , where  $C$  is the circle  $|z|=3$ . [5+5]

8.a) Express  $f(z) = \frac{z}{(z-1)(z-3)}$  in series of positive and negative powers of  $(z-1)$ .

b) Evaluate  $\int_0^{2\pi} \frac{1}{(5-3\cos\theta)} d\theta$  using residue theorem. [5+5]

OR

9.a) Give two Laurent's series expansions in powers of  $z$  for  $f(z) = \frac{1}{(1-z)z^2}$  and specify the region in which these expansions are valid.

b) Evaluate  $\int_C \frac{z^2 + 2z - 2}{z(z-4)(z-1)} dz$  where  $C$  is  $|z| = 1.5$ . [5+5]

10.a) Under the transformation  $w = 1/z$  find the image of the circle  $|z - 2i| = 2$ .

b) Find the bilinear transformation which maps the points  $(-1, 0, 1)$  into the points  $(0, i, 3i)$  [5+5]

OR

11.a) Find the image of the region in the  $z$ -plane between the line  $y = 0$  and  $y = \pi/2$  Under the transformation  $w = e^z$ .

b) Show the bilinear transformation preserves the cross ratio. [5+5]

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