

R16

Code No: 131AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year I Semester Examinations, December - 2017
MATHEMATICS-II

(Common to CE, ME, MCT, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART-A
(25 Marks)

- 1.a) Find the Laplace transform of $\cosh^3 2t$. [2]
- b) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$. [3]
- c) Evaluate the improper integral $\int_0^{\infty} \sqrt{x} e^{-x^2} dx$ using Gamma function. [2]
- d) Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ using Beta and Gamma functions. [3]
- e) Find the area bounded by the curves $x^2 = y^3, x = y$ using double integration. [2]
- f) Change the order of the integration $\int_{y=0}^1 \int_{x=0}^{y+4} \frac{2y+1}{x+1} dx dy$ and evaluate the integral. [3]
- g) Find $\nabla \phi$, when $\phi = 3x^2y - y^3z^2$ at the point $(1, -2, -1)$. [2]
- h) Find the directional derivative of the function $f(x, y, z) = 2xy + z^2$ at the point $(1, -1, 3)$ in the direction of the vector $i + 2j + 2k$. [3]
- i) If $R = t\bar{i} - t^2\bar{j} + (t-1)\bar{k}$ and $S = 2t^2\bar{i} + 6t\bar{k}$, evaluate $\int_0^2 R \cdot S dt$. [2]
- j) Evaluate the line integral $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region $y = \sqrt{x}, y = x$. [3]

PART-B
(50 Marks)

- 2.a) Find the Laplace transform of $\sin \sqrt{t}$. Hence find $L\left(\frac{\cos \sqrt{t}}{\sqrt{t}}\right)$
- b) Prove that $\int_{t=0}^{\infty} \int_{u=0}^t e^{-t} \left(\frac{\sin u}{u}\right) du dt = \frac{\pi}{4}$. [5+5]

OR

3.a) Find the inverse Laplace transform of $\ln\left(\frac{s+1}{s-1}\right)$.

b) Find the inverse Laplace transform of $\frac{1}{s^3(s^2+a^2)}$ using the convolution theorem.

4.a) Prove that $\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n} \operatorname{cosec}\left(\frac{\pi}{n}\right)$. [5+5]

b) Evaluate $\int_0^\pi x \sin^7 x \cos^4 x dx$ using Beta and Gamma functions. [5+5]

5. Prove that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \frac{1}{2} \sin^2 \theta}} = \frac{[\Gamma(1/4)]^2}{4\sqrt{\pi}}$. OR [10]

6.a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$, by changing to spherical polar coordinates.

b) Evaluate the integral $\int_{-1}^1 \int_0^1 \int_{x-z}^{x+z} (x+y+z) dz dy dx$. [5+5]

OR

7. Find by triple integration, the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. [10]

8. Prove the following vector identities:

a) $\nabla(\phi_1 \phi_2) = \phi_1 \nabla(\phi_2) + \phi_2 \nabla(\phi_1)$ b) $\nabla\left(\frac{\phi_1}{\phi_2}\right) = \frac{\phi_2 \nabla \phi_1 - \phi_1 \nabla \phi_2}{\phi_2^2}, \phi_2 \neq 0$. [5+5]

OR

9. If $R = xi + yj + zk$, show that: a) $\nabla r = \frac{R}{r}$ b) $\nabla\left(\frac{1}{r}\right) = -\frac{R}{r^3}$ c) $\nabla r^n = nr^{n-2}R$.
 d) $\nabla(a.R) = a$, where a is a constant vector and $r = |R|$. [10]

10. State the Stokes' theorem. Verify it for the vector field $F = (2x-y)i - yz^2j - y^2zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy -plane. [10]

OR

11. State the Green's theorem in a plane. Verify it for $\oint_C e^{-x}(\sin y dx + \cos y dy)$ where C is the rectangle with the vertices $(0,0)$, $(\pi,0)$, $\left(\pi, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$. [10]