

**R16**
**Code No: 132AB**
**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech I Year II Semester Examinations, August/September - 2017**
**MATHEMATICS-II**
**(Common to EEE, ECE, CSE, EIE, IT)**
**Time: 3 hours**
**Max. Marks: 75**
**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A**
**(25 Marks)**

- 1.a) Find  $L^{-1}\left(\frac{1}{s^2 - 2s + 5}\right)$  [2]
- b) Let  $L\{f(t)\} = \tilde{f}(s)$ , Prove that  $L\{f(at)\} = \frac{1}{a} \tilde{f}\left(\frac{s}{a}\right)$ . [3]
- c) Define Gamma function. [2]
- d) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^5 x dx$  using Beta and Gamma function. [3]
- e) Evaluate  $\int_0^1 \int_0^3 \int_0^4 (x + y + z) dx dy dz$  [2]
- f) Find the area of the circle using double integral. [3]
- g) Give short note on gradient of a scalar point function. [2]
- h) Find the scalar potential function of an irrotational vector  $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$  [3]
- i) Evaluate  $\oint_C x dx + y dy + z dz$  where C is a circle  $x^2 + y^2 = 1$  in  $xy$  - plan. [2]
- j) Apply Gauss divergence theorem to evaluate  $\iint x dy dz + y dz dx + z dx dy$  over the surface of the sphere of radius 'a' units. [3]

**PART-B**
**(50 Marks)**

- 2.a) Find Laplace transform of  $\frac{1 - e^t}{t}$ .
- b) Find the inverse Laplace transform of  $\frac{1}{(s^2 + 1)^2}$ . [5+5]

**OR**

- 3.a) Solve  $y'' + 4y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 6$  using Laplace transform.
- b) Solve the integral equation  $f(t) = at + \int_0^t f(u) \sin(t-u) du$ ,  $t > 0$ . [5+5]

4.a) Show that  $\int_0^{\infty} x^n e^{-a^2 x^2} dx = \frac{1}{2a^{n+1}} \Gamma\left(\frac{n+1}{2}\right), (n > -1).$

b) Evaluate  $\int_0^1 x^4 \left(\log \frac{1}{x}\right)^3 dx.$  [5+5]

OR

5.a) Prove that  $\beta(m, n) = \frac{\sqrt{\pi} \Gamma(n)}{2^{2n-1} \Gamma(n + \frac{1}{2})}$

b) Evaluate  $\int_0^1 \frac{x^3}{\sqrt{1-x}} dx.$  [5+5]

6. Evaluate  $\iint_R xy \, dx \, dy$  where R is the region bounded by x-axis and  $x = 2a$  and the curve  $x^2 = 4ay.$  [10]

OR

7.a) Using triple integral, find the volume of the sphere whose radius is 'a' units.

b) Evaluate  $\int_0^{\pi} \int_0^{a \sin \theta} \int_0^{\theta} r \, dr \, d\theta.$  [5+5]

8.a) Find  $\text{Curl } \vec{F}$  at the point (1,2,3), given that  $\vec{F} = \text{grad}(x^3 y + y^3 z + z^3 x - x^2 y^2 z^2).$

b) Find the Directional derivative of  $\phi = x^4 + y^4 + z^4$  at the point A(1,-2,1) in the direction of AB where B is (2, -6, -1). [5+5]

OR

9.a) If the vector field  $\vec{F} = (2xyz^2)\vec{i} + (x^2z^2 + z \cos yz)\vec{j} + (2x^2yz + y \cos yz)\vec{k}$  is conservative then find its scalar potential function.

b) Prove that  $\text{div}(r^n \vec{r}) = (n+3)r^n$  Where  $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$  [5+5]

10.a) Use Divergence theorem to evaluate  $\int_S \vec{F} \cdot \vec{n} \, ds$  where  $\vec{F} = 3xi + 3yj + 3zk$  and surface of sphere is  $x^2 + y^2 + z^2 = a^2.$

b) Calculate the work done by a force  $\vec{F} = 3xy \vec{i} - y^2 \vec{j}$  in moving a particle in xy- plane from (0, 1) to (1, 2) along the parabola  $y = x^2.$  [5+5]

OR

11. Verify Stokes theorem for  $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$  taken around a rectangle bounded by the lines  $x = a, x = -a, y = 0$  and  $y = b.$  [10]

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