

**R16**

Code No: 133BD

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech II Year I Semester Examinations, November/December - 2018**
**MATHEMATICS – IV**

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, ETM, MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 Hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART - A**
**(25 Marks)**

- 1.a) State the necessary and sufficient conditions for a function  $f(z)$  to be analytic. [2]
- b) Show that the function  $f(z) = xy + iy$  is everywhere continuous but is not analytic. [3]
- c) Show that  $f(z) = \frac{1}{1-e^z}$  has a simple pole at  $z = 2\pi i$ . [2]
- d) State Cauchy's integral formula and use it to evaluate  $\oint_C \frac{z^2+4}{z-3} dz$  where  $C$  is the circle  $|z| = 5$ . [3]
- e) Find the fixed points of the mapping  $w = z + 2i$ . [2]
- f) Find the residues at the poles of the function  $f(z) = \frac{2z+1}{(z-1)^2}$ ,  $C: |z| \leq 4$ . [3]
- g) If  $f(x) = x^3$  in  $[-\pi, \pi]$ , find the Fourier coefficient  $b_n$ . [2]
- h) Find  $f(x)$  if its finite sine transform is given by  $\bar{f}_s(s) = \frac{1+\cos s\pi}{s\pi}$  where  $0 < x < \pi$ ,  $s = 1, 2, 3, \dots$  [3]
- i) Classify the PDE:  $xu_{xx} - u_{xy} + yu_{yy} = 1$ . [2]
- j) Write the possible three solutions of the partial differential equation  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ . [3]

**PART-B**
**(50 Marks)**

- 2.a) Define analyticity of a function. Show that the function defined by  $f(z) = \sqrt{|xy|}$  is not analytic at the origin although the C-R equations are satisfied at that point.
- b) Find the analytic function  $f(z) = u(r, \theta) + iv(r, \theta)$ , when  $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$ . [5+5]

**OR**

- 3.a) Show that both the real and imaginary parts of an analytic function are harmonic.
- b) If  $f(z) = u + iv$  be an analytic function of  $z$  and if  $u - v = (x - y)(x^2 + 4xy + y^2)$  find  $f(z)$  in terms of  $z$ . [5+5]

4.a) State Cauchy integral theorem and use it to evaluate the integral  $\int_C \frac{e^{2z}}{(z-1)^2(z-3)} dz$  where  $C$  is the circle  $|z| = 4$ .

b) If  $\Phi(a) = \int_C \frac{3z^2+7z+1}{z-a} dz$ , where  $C$  is the circle  $x^2 + y^2 = 4$ , find  $\Phi(3)$ ,  $\Phi'(1-i)$  and  $\Phi''(1-i)$ . [5+5]

OR

5.a) Expand  $f(z) = \frac{1}{z^2-4z+3}$  in the region  $1 < |z| < 3$ . Also name the series so obtained.

b) Find the nature and location of the singularities of the function  $f(z) = \frac{e^{2z}}{(z-2)^4}$  by finding its Laurent's series expansion. [5+5]

6. State Residues theorem. Evaluate the integral by contour integration:  $\int_0^\pi \frac{d\theta}{13+5\cos\theta}$ . [10]

OR

7.a) Find the residue of  $f(z) = \frac{z^3}{z^2-1}$  at  $z = \infty$ .

b) Define bilinear transformation. Find the bilinear transformation which maps the points  $z = 1, i, -1$  onto the points  $w = i, 0, -i$  and hence find the image of  $|z| < 1$ . [5+5]

8.a) Find the Fourier series for the function  $f(x) = \frac{\pi x}{2}$  in  $0 \leq x \leq 2$ .

b) Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$ . Hence prove that  $\int_0^\infty \frac{\sin^2 ax}{x^2} dx = \frac{\pi a}{2}$ . [5+5]

OR

9.a) Develop  $f(x) = \begin{cases} 2, & -2 < x < 0 \\ x, & 0 < x < 2 \end{cases}$  in a series of sines and cosines and deduce the series for  $\pi^2$ .

b) Find the Fourier cosine transform of  $f(x) = e^{-x}$ ,  $x > 0$ . [5+5]

10. The ends A and B of a rod 20 cm long have the temperature at  $30^\circ\text{C}$  and  $80^\circ\text{C}$  until steady state conditions prevail. The temperature at the ends are suddenly changed to  $40^\circ\text{C}$  and  $60^\circ\text{C}$  respectively. Find the temperature distribution in the rod at time  $t$ . [10]

OR

11. Write down one dimensional wave equation. A string is stretched and fastened to two points  $l$  cm apart. Motion is started by displacing the string in a sinusoidal arch of height  $y_0$  and then released from rest at time  $t = 0$ . Find the displacement at point  $x$  and at any time  $t$ . [10]