

Code No: 132AB

**R16**
**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**
**B.Tech I Year II Semester Examinations, May - 2019**
**MATHEMATICS-II**
**(Common to EEE, ECE, CSE, EIE, IT, ETM)**
**Time: 3 hours**
**Max. Marks: 75**
**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A**
**(25 Marks)**

- 1.a) Find  $L^{-1}\left(\frac{1}{(s-2)^2}\right)$ . [2]
- b) Define Unit step function and find its Laplace transform. [3]
- c) Evaluate  $\Gamma\left(-\frac{3}{2}\right)$ . [2]
- d) Evaluate  $\int_0^1 x^5(1-x)^6 dx$  [3]
- e) Using triple integral, find the volume of a rectangular box whose length is 6 ft, breadth is 5 ft and height is 4 ft. [2]
- f) Evaluate  $\int_1^2 \int_0^x (x+y^2) dy dx$  [3]
- g) Define solenoidal vector. [2]
- h) Prove that  $\vec{r}$  is an irrotational where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  [3]
- i) State stokes theorem. [2]
- j) Evaluate  $\iiint_V \text{div} \vec{f} dx dy dz$  where  $V$  is the volume of the sphere whose radius is 'a' units and  $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ . [3]

**PART-B**
**(50 Marks)**

- 2.a) Find the Laplace transform of  $(\sin t + \cos t)^2$
  - b) Find the inverse Laplace transform of  $\frac{1}{(s^2+1)(s+1)}$ . [5+5]
- OR**
3. Solve  $y'' + 2y' + 5y = e^{-t}$ ,  $y(0)=1$ ,  $y'(0)=1$  using Laplace transform. [10]
  - 4.a) Evaluate  $\int_0^\infty e^{-x/3} x^3 dx$ .
  - b) Evaluate  $\int_0^1 \frac{x dx}{\sqrt{1-x^4}}$ . [5+5]

**OR**

- 5.a) Evaluate  $\int_0^{\infty} e^{-x^3} x^7 dx$ .
- b) Evaluate  $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}}$ . [5+5]
- 6.a) Evaluate  $\int_0^{2\sqrt{2x-x^2}} \int (x^2 + y^2) dx dy$  by changing to polar coordinates.
- b) Evaluate  $\iint_R y dx dy$  where R is the region bounded by the parabola  $y^2 = 4x$  and  $x^2 = 4y$ . [5+5]

**OR**

- 7.a) Evaluate  $\iiint xy^2 z dx dy dz$  taken through the positive octant of the sphere  $x^2 + y^2 + z^2 = a^2$ .
- b) Evaluate  $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$ . [5+5]
- 8.a) Find the directional derivative to the surface  $f(x,y,z) = xy^2z - 4$ , at the point  $(1, -1, 2)$  along  $i+j+k$ .
- b) A butterfly is located at  $(2, -1, 3)$  and desires to fly towards fragrance surface  $f(x,y,z) = x^2 + yz^2$ . Along which direction should it fly to get fragrance at the earliest? [5+5]

**OR**

- 9.a) Show that  $\nabla^2 r^n = n(n+1)r^{n-2}$  where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $|\vec{r}|^2 = r$ .
- b) Prove that  $\nabla \left( \frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$  where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $|\vec{r}|^2 = r$ . [5+5]
10. Verify Greens theorem for  $\oint_C (y - \sin x) dx + \cos x dy$  where C is the triangle enclosed by the lines  $y = 0, x = \frac{\pi}{2}$  and  $\pi y = 2x$ . [10]

**OR**

11. Verify stokes theorem for a vector field defined by  $\vec{F} = -y^3\vec{i} + x^3\vec{j}$  in the region  $x^2 + y^2 \leq 1, z = 0$ . [10]

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