## Code No: 123AH

## R15

## JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD <br> B.Tech II Year I Semester Examinations, May/June - 2019 <br> MATHEMATICS - III <br> (Common to EEE, ECE, EIE, ETM)

Time: 3 Hours
Max. Marks: 75
Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have $\mathrm{a}, \mathrm{b}, \mathrm{c}$ as sub questions.

## PART- A

(25 Marks)
1.a) Determine the nature of the point $x=0$ for the equation
$\left(x^{2}+1\right) y^{\prime \prime}+\left(x^{2}-1\right)+2 y=0$
b) Find the indicial equation of $x^{2} y^{\prime \prime}-2 x y^{\prime}-\left(x^{2}-2\right) y=0$.
c) Write the value of $J_{\frac{1}{2}}(x)$
d) Obtain the value of $P_{2}(x)$.
e) Write the Cauchy Riemann equations in polar form.
f) Show that the function $f(z)=\sin x \cosh y+i \cos x \sinh y$ is continuous and analytic everywhere.
g) Define essential singularity.
h) Expand $\log z$ by Taylor's series about $z=1$.
i) Find the image of $\mathrm{z}=2$-i under the transformation $w=z+2-3 i$.
j) Prove that $w=\frac{1}{z}$ is circle preserving.

## PART-B

(50 Marks)
2. Find the series solution of $4 x y^{\prime \prime}+2 y^{\prime}+y=0$.

## OR

3. Solve the equation $3 x \frac{d^{2} y}{d x^{2}}+(1-x) \frac{d y}{d x}-y=0$ in power series.
4.a) Prove that $\int_{-1}^{1} x P_{n}(x) P_{n-1}(x) d x=\frac{2 n}{4 n^{2}-1}$.
b) Show that $J_{0}^{2}+2\left(J_{1}^{2}+J_{2}^{2}+J_{3}^{2}+\cdots\right)=1$.

OR
5. If $m_{1}, m_{2}$ are roots of $J_{n}(x)=0$, then prove that $\int_{0}^{1} x J_{n}\left(m_{1} x\right) J_{n}\left(m_{2} x\right) d x=0$.
6. State and prove Cauchy's Integral formula.

## OR

7. Evaluate $\int_{c}\left(y-x-3 x^{2} i\right) d z$, where $c$ consists of the line segments from $z=0$ to $z=i$ and the other from $z=i$ to $z=1+i$.
8. Evaluate $\int_{-\infty}^{\infty} \frac{z^{2}-z+2}{z^{4}+10 z^{2}+9} d z$.

## OR

9. Find Laurent expansion of $\frac{1}{z^{2}-4 z+3}$ for $1<|z|<3$.
10. Determine the region of the $w$ - plane into which the first quadrant of $z$-plane is mapped by the transformation $w=z^{2}$.
[10]

## OR

11. Show that every bilinear transformation maps the circles in the $z$ - plane onto the circles in the $w$-plane.
