

R16

Code No: 133BD
JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech II Year I Semester Examinations, May/June - 2019
MATHEMATICS – IV
(Common to CE, EEE, ME, ECE, CSE, EIE, IT MCT, ETM MMT, AE, MIE, PTM, CEE, MSNT)
Time: 3 Hours
Max. Marks: 75
Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

PART- A
(25 Marks)

- 1.a) State the necessary and sufficient conditions for a function $f(z) = u + iv$ to be analytic. [2]
- b) Show that $f(z) = |z|^2$ is not analytic at any point. [3]
- c) State Cauchy's integral theorem. [2]
- d) Find the poles and the residues at the poles of the function $f(z) = \frac{e^z}{\cos \pi z}$. [3]
- e) Define bilinear transformation and cross ratio. [2]
- f) Find the image of the circle $|z| = 2$, under the transformation $w = z + 3 + 2i$. [3]
- g) State Fourier integral theorem. [2]
- h) Expand $f(x) = \pi x - x^2$ in a half range sine series in $(0, \pi)$. [3]
- i) Classify the partial differential equation $u_{xx} + 6u_{xy} + 2u_{yy} + 2u_x - 2u_y + u = x^2y$. [2]
- j) Write the three possible solutions of the heat equation.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$
 [3]

PART-B
(50 Marks)

- 2.a) If $f(z)$ is a regular function of z , prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2.$$
 - b) Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function $f(z)$ in terms of z . [5+5]
- OR**
- 3.a) Show that the function $f(z)$ defined by

$$f(z) = \frac{x^2 y^3 (x + iy)}{x^6 + y^{10}} \quad \text{for } z \neq 0$$
 , is not analytic at the origin, even though it satisfies the $f(0) = 0$ Cauchy-Riemann equations at the origin.
 - b) Determine the analytic function whose real part is $\log \sqrt{x^2 + y^2}$. [5+5]

4. Represent the function $\frac{1}{z^2-4z+3}$ in the domain
 (a) $1 < |z| < 3$ (b) $|z| < 1$. [10]

OR

- 5.a) Expand the function $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$, and name the series thus obtained.
 b) Evaluate $\oint_C \frac{e^z}{(z+3)(z+2)} dz$, where C is the circle $|z - 1| = \frac{1}{2}$. [5+5]

6. Evaluate the integral using contour integration $\int_0^{2\pi} \frac{d\theta}{2+\cos \theta}$. [10]

OR

7. Show that the transformation $w = i \frac{1-z}{1+z}$ transforms the circle $|z| = 1$ into the real axis of w plane and the interior of the circle $|z| < 1$ into the upper half of the w plane. [10]

8. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1. \end{cases}$ Hence evaluate $\int_0^\infty \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$. [10]

OR

- 9.a) Obtain the half range cosine series for $f(x) = \begin{cases} kx, & \text{for } 0 \leq x < L/2 \\ k(L-x), & \text{for } L/2 \leq x \leq L. \end{cases}$
 b) Find the Fourier sine transform of $f(x) = e^{-|x|}$. Hence show that $\int_0^\infty \frac{x \sin mx}{x^2+1} dx = \frac{\pi}{2} e^{-m}$. [5+5]

10. A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{L}$ from which it is released at time $t = 0$. Find the displacement of any point at a distance x from one end at time t . [10]

OR

11. Write down the one dimensional heat equation. Find the temperature $u(x, t)$ in a slab whose ends $x = 0$ and $x = L$ are kept at zero temperature and whose initial temperature $f(x)$ is given by $f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{1}{2}L \\ 0, & \text{when } \frac{1}{2}L < x < L. \end{cases}$ [10]

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