Code No: R21016 **R10** SET - 1

II B. Tech I Semester Regular Examinations, March – 2014
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $\int_0^1 x\, J_n(m_1 x)\, J_n(m_2 x)\, dx = 0,\ m_1 \neq m_2$.

   b) Using the generating functions of Legendre polynomials, prove that
   $(2n+1)x\,P_n = (n+1)\,p_{n+1}(x) + n\,P_{n-1}(x)$. (8M+7M)

2. a) State the necessary conditions for analyticity of a function f (z).
   Show that the function f (z) = $u + iv$, where

   $$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, \quad z \neq 0;$$
   $$= 0, \qquad z = 0$$

   satisfy the Cauchy-Riemann conditions at z = 0. Is the function analytic at z = 0? Justify your answer.

   b) If $u(x,y) = (x-1)^3 + xy^2 + 3y^2$, determine $v(x,y)$ so that $u + iv$ is a regular function of $x + iy$.
   (8M+7M)

3. a) Find all the roots of tan z = 2.

   b) If $\cos h\,(u + iv) = x + iy$, prove that $\frac{x^2}{\cosh^2 u} + \frac{y^2}{\sinh^2 u} = 1$ and $\frac{x^2}{\cos^2 v} - \frac{y^2}{\sin^2 v} = 1$.
   (8M+7M)

4. a) Evaluate $\int_0^{3+i} z^2 dz$ along i) the line $y = x/3$ ii) along the curve $x = 3y^2$, does the integration depend upon the path?

   b) Evaluate the integral $\int_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$, where C is the circle $|z| = 1$. (8M+7M)
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5. a) Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in the region $|z|<1$. Also name the series so obtained.

   b) Find the nature and location of the singularities of the function $f(z)=\frac{e^{2z}}{(z-1)^4}$ by finding its Laurent's series expansion. (8M+7M)

6. State Residue theorem. Evaluate the residues at the poles of the function $f(z)=\frac{1}{z^4+a^4}$.

   Hence evaluate the integral $\int_0^{\infty}\frac{dx}{x^4+a^4}$, $a>0$ using suitable contour. (15M)

7. a) State and prove Argument Principle.

   b) Use Rouche's theorem to show that the equation $f(z)=z^5+15z+1=0$ has one root in the disc |z|<3/2 and four roots in the annulus $\frac{3}{2}<|z|<2$. (8M+7M)

8. a) Find the mapping of x-axis under the transformation $w=\frac{i-z}{i+z}$ onto the w - plane.

   b) Define bilinear transformation. Find the bilinear transformation that maps the points $z_1=-i,\ z_2=0,\ z_3=i$ into the points $w_1=-1,\ w_2=i,\ w_3=2$, respectively. (8M+7M)

Code No: R21016 **R10** SET - 2

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1. a) Show that $\frac{d}{dx}(xJ_nJ_{n+1}) = x(J_n^2 - J_{n+1}^2)$

   b) Prove that $\int_{-1}^{+1}[P_n(x)]^2 dx = \frac{2}{2n+1}$. (8M+7M)

2. a) Define analyticity of a function. Show that the function defined by $f(z) = \sqrt{|xy|}$ is not analytic at the origin although the C-R equations are satisfied at that point.

   b) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of x and y, but are not harmonic conjugates. (8M+7M)

3. a) Find the modulus and principle value of the argument of $\frac{(1+i\sqrt{3})^{13}}{(\sqrt{3}-i)^{11}}$.

   b) Separate the real and imaginary parts of the function $\sinh(x + iy)$. (8M+7M)

4. State and prove Cauchy's Integral Formula. Use it to evaluate the integral

   $\int_C \frac{e^{2z}}{(z-1)^2(z-3)} dz$ where C is the circle $|z| = 4$. (15M)
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Code No: R21016 **R10** SET - 2

5. a) Represent the function $f(z)=\dfrac{3z+4}{z(z-3)(z+2)}$ in Laurent series in the annular region between $|z|=2$ and $|z|=3$.

   b) Find the singularities of the function $f(z)=\dfrac{\cot \pi z}{(z-a)^2}$ indicating the character of each singularity. (8M+7M)

6. State Residues theorem. Prove by calculus of residues $\int_0^\infty \dfrac{\cos mx\, dx}{(a^2+x^2)}=\dfrac{\pi}{2a}e^{-ma}$. (15M)

7. a) If $a>e$, then prove with the help of Rouche's theorem that the equation $e^z = az^n$ has n roots inside the circle $|z|=1$.

   b) If $f(z)$ is analytic within and on a closed contour C except at a finite number of poles and is not zero on C, then $\dfrac{1}{2\pi i}\int_C \dfrac{f'(z)}{f(z)} = N-P$, where N is the number of zeros and P is the number of poles inside . (8M+7M)

8. a) Find the image of the semi-infinite strip $x>0,\ 0<y<1$, under the transformation $w=iz+1$.

   b) Define bilinear transformation. Find the bilinear transformation that maps the points $z_1=2,\ z_2=i,\ z_3=-2$ into the points $w_1=1,\ w_2=i,\ w_3=-1$, respectively. (8M+7M)

Code No: R21016 **R10** SET - 3

II B. Tech I Semester Regular Examinations, March – 2014
MATHEMATICS - III
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Time: 3 hours Max. Marks: 75

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1. a) Prove Rodriguez' formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$. Hence derive Legendre polynomials of first three orders.

   b) Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1}(x)$ (8M+7M)

2. a) Discuss the analyticity of the function $f(z) = z\,\bar{z}$

   b) If $u + v = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = u + iv$ is an analytic function of z, then find $f(z)$ in terms of z. (8M+7M)

3. a) Separate the real and imaginary parts of $\tanh(x+iy)$.

   b) Find the general value of i^i. (8M+7M)

4. a) State and prove Cauchy theorem.

   b) Evaluate the integral $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$, where C is $|z-i| = 2$. (8M+7M)

5. a) Explain zeros and singularities of an analytic function. What type of singularities does the function $f(z) = (z-a)\sin\left(\frac{1}{z-a}\right)$ have?.

   b) Find the Laurant series expansion of $f(z) = \frac{7z-2}{z^s - z^2 - 2z}$ in the region given by $|z+1| < 1$. Write down the principal part in the series. (8M+7M)
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Code No: R21016 **R10** SET - 3

6. a) Find the residue at the singular points of the function $f(z)=\dfrac{1-e^{2z}}{z^4}$.

   b) Using suitable contour, evaluate the real integral $\int_0^\infty \dfrac{\cos 3x}{(x^2+1)(x^2+4)}dx$ (8M+7M)

7. a) If $f(z)$ is analytic within and on a closed contour C except at a finite number of poles and is not zero on C, then $\dfrac{1}{2\pi i}\int_C \dfrac{f'(z)}{f(z)} = N-P$, where N is the number of zeros and P is the number of poles inside C.

   b) State Rouche's theorem. Using it prove that all the roots of $z^7-5z^3+12=0$ lie between the circles $|z|=1$ *and* $|z|=2$. (8M+7M)

8. a) For the conformal transformation $w=z^2$, show that the circle $|z-1|=1$ transforms into the cardioids $R=2(1+\cos\phi)$, where $w=\mathrm{Re}^{i\phi}$ in the w plane.

   b) Find the bilinear transformation that maps the points $z_1=2,\ z_2=i,\ z_3=-2$ into the points $w_1=1,\ w_2=i,\ w_3=-1$, respectively. What are its critical points? (8M+7M)

Code No: R21016 **R10** SET - 4

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MATHEMATICS - III
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Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) Using generating functions of Legendre's polynomials, prove that
   $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$

   b) Prove that $\int_0^1 xJ_n(ax)\, J_n(\beta x)dx = 0.$ (8M+7M)

2. a) State the necessary conditions for analyticity of function *f (z)*.
   If $f(z) = \dfrac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, z \neq 0,\ f(0) = 0,$ then show that *f (z)* is not analytic at origin although Cauchy-Riemann equations are satisfied there.

   b) If $f(z) = u(r,\theta) + iv(r,\theta)$ be an analytic function and given that $v = \left(r - \dfrac{1}{r}\right)\sin\theta,$ then find *f (z)* in terms of *z*. (8M+7M)

3. a) Prove that the real part of the principal value of $i^{\log(1+i)}$ is equal to $e^{-\pi^2/8}\cos\left(\dfrac{\pi}{4}\log 2\right).$

   b) Find all the roots of $\sin z = 2.$ (8M+7M)

4. a) i) State Cauchy integral theorem.
   ii) Find the value of the integral $\int_C (x+y)\,dx + x^2 y dy$ along $y = x^2$ having (0,0) and(3,9) as end points.

   b) Evaluate $\int_C \dfrac{z}{z^2+1}\,dz$ over the circular path $C : |z+i| = 1.$ (8M+7M)
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Code No: R21016 **R10** SET - 4

5. a) Obtain the expression for $f(z)=\frac{1}{z^2-3z+2}$ which are valid in the region $1<|z|<2$. Also name the series thus obtained.

   b) Obtain the nature and location of singularities of the function $f(z)=\frac{z-\sin z}{z^2}$. (8M+7M)

6. a) Compute the residue of $f(z)=\frac{z^2-z+1}{z^4+10z^2+9}$ at $z=3i$.

   b) Use contour integration to evaluate the real integral $\int_0^\infty \frac{\sin mx}{1+x^2}dx$ (8M+7M)

7. a) State and prove the fundamental theorem of algebra.

   b) Prove that the polynomial $f(z)=z^5+z^3+2z+3$ has just one zero in the first quadrant of the complex plane. (8M+7M)

8. a) Find the image of $|z-2i|=2$ under the mapping $w=\frac{1}{z}$.

   b) What do you mean by bilinear transformation? Find the bilinear transformation that maps the points $z=1,i,-1$ into the points $w=1,\ 0,\ -i$, respectively. Hence find the image of $|z|<1$. (8M+7M)