Code No: R21053 **R10** SET - 1

II B. Tech I Semester Regular Examinations, March – 2014
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE AND ENGINEERING
(Com. to CSE, IT, ECC)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Prove that $(\exists x)P(x)\wedge Q(x) \Rightarrow (\exists x)P(x)\wedge(\exists x)Q(x)$. Does the converse hold?
   b) Obtain the principal conjunctive normal form of the formula
   $(\neg P\rightarrow R)\wedge(Q\rightleftarrows P)$.
   (8M+7M)

2. a) If a=1820 and b=231, find GCD (a, b). Express GCD as a linear combination of a and b.
   b) Find 11^7 mod 13 using modular arithmetic.
   c) Use mathematical induction to prove that 5 divides n^2-n whenever n is non-negative integer.
   (5M+5M+5M)

3. a) Let A be a given finite set and ρ(A) its power set. Let ⊆ be the inclusion relation on the elements of ρ(A). Draw Hasse diagrams of <ρ(A), ⊆> for A={a}; A={a,b}; A={a,b,c} and A={a,b.c.d}.
   b) Let F_x be the set of all one-to-one onto mappings from X onto X, where X={1,2,3}. Find all the elements of F_x and find the inverse of each element. (8M+7M)

4. a) Apply BFS algorithm to find a spanning tree for the following graph in Figure 4a.

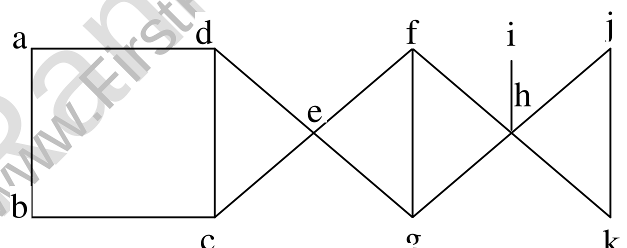

Figure 4a

   b) Determine whether the graphs in Figure 4b, are isomorphic. (8M+7M)

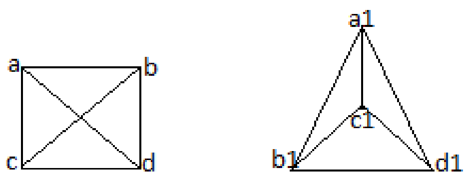

Fig: 4b
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5. a) Verify Euler's formula for Figure 5.

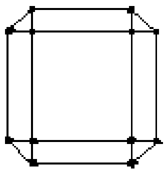

Fig: 5

b) Draw the bipartite graph $K_{3,3}$ and find its chromatic number. (5M+10M)

6. a) If <G,∗> is an abelian group, then for all a,b ∈ G show that $(a*b)^n = a^n * b^n$

b) We are given the ring <{a,b,c,d}, +, . > whose operations are given by the following table:

+	a	b	c	d
a	a	b	C	d
b	b	c	D	a
c	c	d	A	b
d	d	a	B	c

.	A	b	C	D
A	A	a	A	A
B	a	c	A	C
C	a	a	A	A
D	a	c	A	A

Is it a commutative ring? Does it have an identity? What is the zero of this ring? Find the additive inverse of each of its elements. (5M+10M)

7. a) In how many ways five students can be selected from 12 students:
   i) Without replacement.
   ii) If two students not be included.
   iii) A particular student to be included.

   b) Find the sum of all 4-digit numbers that can be obtained by using digits 2, 3, 5 and 7 (without repetition). (8M+7M)

8. a) Solve the recurrence relation $u_{n+3} + 5u_{n+2} - 18u_n = 0$. Given that

$u_0 = 4, u_1 = -\frac{1}{6}, u_2 = 4.$

b) Solve the recurrence relation $u_n = u_{n-1} + \frac{n(n+1)}{2}, n \geq 1.$ (8M+7M)

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1. a) Show that from
   i) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$
   ii) $(\exists y)\,(M(y) \wedge \urcorner W(y))$ the conclusion $(x)(F(x) \rightarrow \urcorner S(x))$ follows.
   b) Obtain the principal disjunctive and conjunctive normal forms of $(P \rightarrow (Q \wedge R)) \wedge (\urcorner P \rightarrow (\urcorner Q \wedge \urcorner R))$. Is this formula a tautology? (7M+8M)

2. a) Find the integers x such that
   i) $5x \equiv 4 \pmod 3$ ii) $7x \equiv 6 \pmod 5$ iii) $9x \equiv 8 \pmod 7$
   b) Determine GCD (1970, 1066) using Euclidean algorithm. (8M+7M)

3. a) If $f:A \rightarrow B$ and $g:B \rightarrow C$ are bijections, prove that $gof:A \rightarrow C$ is a bijection.
   b) Given the relation matrices M_R and M_S, find M_{RoS}, $M_{\tilde R}$, $M_{\tilde S}$, $M_{\widetilde{RoS}}$ and show that $M_{\widetilde{RoS}} = M_{\tilde S o \tilde R}$

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \quad M_S = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$
(7M+8M)

4. a) Find the depth first search spanning tree for the following graph in Figure 1.
   b) Prove that in a graph G, $\delta(G) \le \frac{2|E|}{|V|} \le \Delta(G)$. (10M+5M)

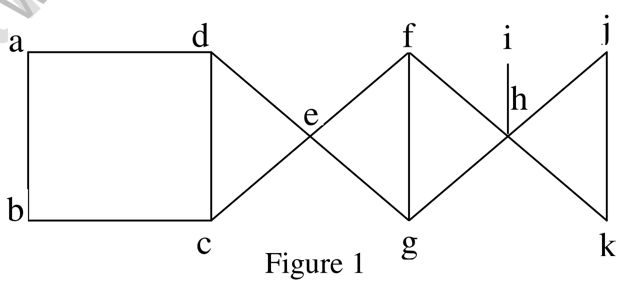

Figure 1
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5. a) Prove that a Star graph S_5 is isomorphic to bipartite graph $K_{1,4}$. Also Draw the graphs S_5 and $K_{1,4}$.
   b) Prove whether K_4 and K_5 are planar or non-planar. (7M+8M)

6. a) Find all the subgroups of S_4 generated by the permutation
   $$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 2 \end{pmatrix}$$
   b) Let A=(6,12,18,24,36,72), a≤b if and only if a divides b. Draw Hasse diagram for it and prove that it is a lattice, but not a distributive lattice. (7M+8M)

7. a) How many six character passwords in a computer are possible, if the first two characters are the letters and the others are digits?
   b) Define Multinomial theorem. Find number of integers <250 and divisible by 3 or 5 or 11. (7M+8M)

8. a) Find the generating function of the sequence $0, 1, 2^2, 3^2, 4^2, \ldots$
   b) Solve the recurrence relation $u_n + 5u_{n-1} + 6u_{n-2} = 3n^2-2n+1$. (7M+8M)

Code No: R21053 **R10** SET - 3

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1. a) Using normal forms, show that the formula $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.
   b) Show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ (5M+10M)

2. a) Let a, b $\in$ Z so that 2a+3b is a multiple of 17. Prove that 17 divides 9a+5b.
   b) Use Euler's theorem to find a number a between 0 and 9 such that a is congruent to 7^{1000} modulo 10.
   c) If n is a positive integer, using mathematical induction prove that
   $$1\cdot 2 + 2\cdot 3 + 3\cdot 4 + \ldots. + n\,(n+1) = \frac{n(n+1)\,(n+2)}{3}$$ (5M+5M+5M)

3. a) If A={1,2,3,4} and P={{1,2},{3},{4}} is a partition of A, find the equivalence relation determined by P.
   b) Let X={1,2,3} and f, g, h and s be functions from X to X given by f={<1,2>, <2,3>, <3,1>} g={<1,2>, <2,1>, <3,3>} h={<1,1>, <2,2>, <3,1>} and s={<1,1>, <2,2>, <3,3>}. Find fog, fohog, gos, fos.
   c) Let X={1,2,3,4} and R={<1,1>, <1,4>, <4,1>, <4,4>, <2,2>, <2,3>, <3,2>, <3,3>}. Write the matrix of R and sketch its graph. (5M+5M+5M)

4. a) Apply Krushal's algorithm to determine a minimal spanning tree for the weighted graph given below in Figure 4.

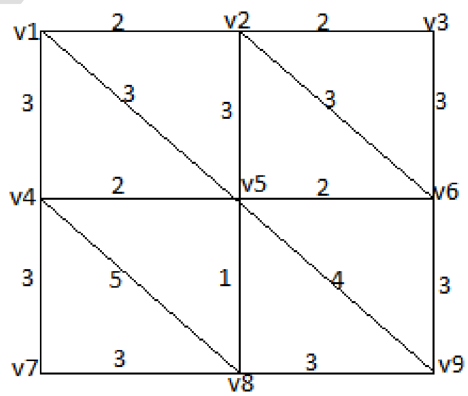

Fig: 4

   b) Consider a full binary tree of n vertices and k levels. Give the formulae for total number of vertices, total number of edges, total number of leaves and total number of internal vertices. Draw a full binary tree of level 3 and verify the above formulae. (7M+8M)
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5. a) Find the Chromatic polynomial of the graph of Figure 5.

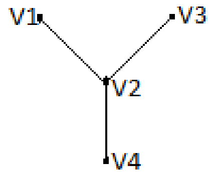

Fig. 5

b) Show that the maximum number of edges in a complete bipartite graph with n vertices is $n^2/4$.

c) Find the Hamiltonian circuit for Figure 6. (5M+5M+5M)

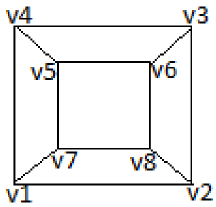

Fig: 6

6. a) Show that in a group <G, ∗>, if for any a,b ∈ G, $(a*b)^2 = a^2 * b^2$, then <G,∗> must be abelian.

b) Prove that (S≤) is a lattice, where S= (1, 2, 3, 6) and ≤ is for divisibility. Prove that this is a distributive lattice. (7M+8M)

7. a) In how many different orders can three men and three women be seated in a row of six seats if:

i) Any one may sit in any one of the seats.
ii) The first and last must be filled by men.
iii) Men and women are seated alternate.
iv) All members of same sex be seated in adjacent seats.

b) Find the number of integers < 500 and divisible by 9 or 11 or 13. (8M+7M)

8. a) Solve the recurrence relation $a_n = a_{n-1} + a_{n-2} =$ where $n \geq 2$, $a_0 = 1$, $a_1 = 1$ using generating function.

b) Determine the sequence generated by $\frac{1}{1-x} + 3x^7 - 11$. (10M+5M)

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1. a) Show that
$\neg(P\wedge Q) \rightarrow (\neg P\vee(\neg P\vee Q)) \Leftrightarrow (\neg P\vee Q)$
$(P\vee Q)\wedge(\neg P\wedge(\neg P\wedge Q)) \Leftrightarrow (\neg P\wedge Q)$
b) Prove that
$(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ (8M+7M)

2. a) State Division algorithm and apply it for a dividend of 170 and divisor of 11.
b) Using Fermat's theorem, find 3^{201} mod 11.
c) Show that $2^n > n^2$ whenever n is positive integer greater than 4. (5M+5M+5M)

3. a) Show that the function
$$f(x)=\begin{cases} x/2 & \text{when } x \text{ is even} \\ (x-1)/2 & \text{when } x \text{ is odd} \end{cases}$$ is primitive recursive.
b) Let X = {a,b,c,d,e} and let C = {{a,b},{c},{d,e}}. Show that the partition C defines an equivalence relation on X. (8M+7M)

4. a) Write the Adjacency matrix and Incidence matrix for the graph in Figure 2. List the Advantages of Adjacency matrix.

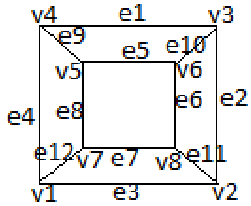

Fig: 2.

b) Draw a complete binary tree with 19 vertices. For a complete binary tree of n vertices, give the formulae for number of leaves and internal vertices. Verify these formulae with the above tree. (8M+7M)
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5. a) Draw eight regular graphs with six vertices. How many of them are connected.
   b) Define a planar graph. Does the graph in Figure 3 satisfy all the properties of planar graph? If so, draw the planar graph for it. (8M+7M)

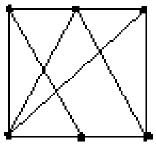

Fig: 3

6. a) Show that the groups <G, $*$> and <S,Δ> given by the following table are isomorphic.

$*$	p1	p2	p3	p4
p1	p1	p2	p3	p4
p2	p2	p1	p4	p3
p3	p3	p4	p1	p2
p4	p4	p3	p2	p1

Δ	q1	q2	q3	q4
q1	q3	q4	q1	q2
q2	q4	q3	q2	q1
q3	q1	q2	q3	q4
q4	q2	q1	q4	q3

   b) Prove that the addition modulo 5 is a group. (8M+7M)

7. a) Define Binomial theorem. Find the coefficient of x^5y^7 in the expansion of $(x+3y)^{12}$.
   b) Find the number of distinct triples (x_1,x_2,x_3) of non-negative integers satisfying $x_1+x_2+x_3<15$. (7M+8M)

8. a) Solve the recurrence relation $u_n=3u_{n-1}$, $n\geq 1$ using generating function.
   b) Solve the recurrence relation using the generating function.
   $a_n - 7a_{n-1} + 10a_{n-2} = 0$, $n \geq 2$, $a_0=10$, $a_1=41$. (5M+10M)