

Code No: R21016

R10
SET - 1
II B. Tech I Semester Supplementary Examinations, September - 2014
MATHEMATICS - III

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
 All Questions carry Equal Marks
 ~~~~~

1. a) Prove that  $\int_{-1}^1 p_m(x) \cdot p_n(x) dx = 0$  if  $m \neq n$ .  
 b) Prove that  $(1-x^2)p_n'(x) = n[p_{n-1}(x) - xp_n'(x)]$
2. a) Show that when  $n$  is a positive integer  
 i)  $J_{-n}(x) = (-1)^n J_n(x)$       ii)  $J_n(-x) = (-1)^n J_n(x)$  for +ve or -ve integers  
 b) Prove that  $J_3(x) = \frac{8-x^2}{x^2} J_1(x) - \frac{4}{x} J_0(x)$
3. a) Show that the function given by  $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$  is continuous at the origin.  
 b) Find the analytic function whose imaginary part is  $f(x, y) = x^3y - xy^3 + xy + x + y$  where  $z = x + iy$ .
4. a) Evaluate  $\int_C \frac{z+1}{z^2 + 2z + 4} dz$ , where  $C : |z+1+i| = 2$ , using Cauchy's integral formula  
 b) Evaluate  $\int_{-1+i}^{2+i} (x^2 + y^2 - ixy) dz$  along  $y = x^2$
5. a) State and prove Laurent's theorem  
 b) Obtain all the Laurent expansions of the function  $\frac{7z-2}{(z+1)z(z-2)}$  about  $z = -2$
6. a) Determine the poles and the corresponding residues of  $f(z) = \frac{2z+1}{z^2 - z - 2}$   
 b) Evaluate  $\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$  where  $C : |z| = 4$ .

Code No: R21016

**R10****SET - 1**

7. a) Evaluate by contour integration  $\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta}$ .

b) Evaluate  $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$ .

8. a) Show that the transformation  $\omega = \frac{1}{z}$  maps a circle to a circle (or) to a straight line if the former goes through the origin.

b) Find the image of  $1 < |z| < 2$  under the transformation  $w = 2iz + 1$

Code No: R21016

**R10**
**SET - 2**
**II B. Tech I Semester Supplementary Examinations, September - 2014**
**MATHEMATICS - III**

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions  
 All Questions carry Equal Marks

~~~~~

1. a) Prove that $\int_{-1}^1 x^m p_n(x) dx = 0$ if $m < n$ using Rodrigue's formula.
 b) Prove that $(2n+1)p_n(x) = p_{n+1}'(x) - p_{n-1}'(x)$

2. a) Prove that
 i) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin(x)}{x} - \cos(x) \right]$
 ii) $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{3-x^2}{x^2} \sin(x) - \frac{3}{x} \cos(x) \right]$
 b) Prove that $\int J_3(x) dx = C - J_2(x) - \frac{2}{x} J_1(x)$

3. a) If $W = \phi + i\psi$ represents the complex potential for an electric field and
 $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function ϕ .
 b) If $f(z) = u + iv$ is an analytic function of z and $u - v = e^x (\cos(x) - \sin(y))$ find $f(z)$ in terms of z .

4. a) Determine $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle.
 i). $|z| = 1$ ii). $|z+1+i| = 2$ iii). $|z+1+i| = 2$
 b) Evaluate $\int_C \frac{z^2 - z + 1}{z-1} dz$ where C is the circle
 i). $|z| = 2$ ii). $|z| = \frac{1}{2}$

5. a) Find the Taylor's expansion for the function $f(z)$ where $f(z) = \frac{1}{(1+z)^2}$ with center at $z = -i$.
 b) Expand $f(z) = \frac{1}{z^3 - z - 6}$ about i). $z = -1$ ii). $z = 1$

Code No: R21016

R10**SET - 2**

6. a) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z|=3/2$ using residue theorem.

b) Evaluate $\int_C \frac{e^z}{z^2+1} dz$ over the circular path $|z|=2$.

7. a) Show that by the method of contour integration,

$$\int_0^\infty \frac{\cos(mx)}{(a^2+x^2)^2} dx = \frac{\pi}{4a^3} (1-ma)e^{-ma}, \quad a > 0, \quad b > 0$$

b) Determine the poles and residues at each pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$

8. a) Find the image of a circle $|z-2i|=2$ under the transformation $w=1/z$.

b) Find the image of an infinite strip $0 < y < 1/2$ under the transformation $w=1/z$.

Code No: R21016

R10
SET - 3
II B. Tech I Semester Supplementary Examinations, September - 2014
MATHEMATICS - III

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions
 All Questions carry Equal Marks
 ~~~~~

1. a) Prove that  $\int_{-1}^1 [p_n(x)]^2 dx = \frac{2}{2n+1}$   
 b) State and prove Rodrigue's formula.
2. a) Prove that  $\frac{d}{dx}(xJ_n J_{n+1}) = x(J_n^2 - J_{n+1}^2)$   
 b) Express  $J_5(x)$  in terms of  $J_0(x)$  and  $J_1(x)$
3. a) Show that the function  $f(z) = \begin{cases} \frac{x^3 y^3 (y - ix)}{x^6 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$  is not differentiable at the origin.  
 b) If  $u + iv = \frac{2 \sin(2x)}{e^{2y} + e^{-2y} - 2 \cos(2x)}$  and  $f(z) = u + iv$  is an analytic function of  $z$ , find  $f(z)$  in terms of  $z$ .
4. a) Evaluate  $\int_C (x^2 + ixy) dz$  from  $A(1,1)$  to  $B(2,8)$  along  
 i) the straight line AB                      ii) The curve  $x=t, y=t^3$   
 b) Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along the paths i).  $y = x$  ii).  $y = x^2$
5. a) Find the Laurent's series of  $\frac{7z-2}{(z+1)z(z-2)}$  in the annulus  $1 < |z+1| < 3$   
 b) Find the Taylor's expansion of  $f(z) = \frac{2z^3+1}{z^2+z}$  about the point  $z = i$ .
6. a) Find the poles and residues at each pole of  $\tanh(z)$ .  
 b) Evaluate  $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$  where  $C$  is the circle  $|z|=3/2$  using residue theorem

Code No: R21016

**R10****SET - 3**

7. a) Prove by contour integration  $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2$
- b) Evaluate  $\int_C \frac{e^{2z} + z^2}{(z-1)^5} dz$  where  $C: |z| = 2$  using Cauchy's Residue theorem.
8. a) Find the image of the triangle with vertices at  $i, 1+i, 1-i$  in the  $z$ -plane under the transformation  $w=3z+4-2i$ .
- b) Find the image of the rectangle  $-\pi < x < \pi, 1/2 < y < 1$  under the transformation  $w=\sin(z)$ .

Code No: R21016

**R10**
**SET - 4**
**II B. Tech I Semester Supplementary Examinations, September - 2014**
**MATHEMATICS - III**

(Com. to CE, CHEM, BT, PE)

Time: 3 hours

Max. Marks: 75

Answer any FIVE Questions  
 All Questions carry Equal Marks

1. a) Using generating functions of Legendre's polynomial, prove that  $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$ .  
 b) Express  $x^3$  using Legendre's polynomials.  $P_1(x)$  and  $P_3(x)$ .
2. a) Show that i)  $\cos(x) = J_0 - 2J_2 + 2J_4$  ii)  $\sin(x) = 2J_1 - 2J_3 + 2J_5$   
 b) Prove that  $\frac{d}{dx}[J_n^2 + J_{n+1}^2] = \frac{2}{x}[2J_n^2 - (n+1)J_{n+1}^2]$  or  

$$J_n J_n' + J_{n+1} J_{n+1}' = \frac{1}{x}[nJ_n^2 - (n+1)J_{n+1}^2]$$
3. a) Find the analytic function whose imaginary part is  $f(x, y) = x^3 y - xy^3 + xy + x + y$  where  $z = x + iy$   
 b) Prove that  $\left[\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right] |\operatorname{Re} f(z)|^2 = 2 |f'(z)|^2$  where  $w = f(z)$  is analytic
4. a) Find the analytic function whose imaginary part is  $e^x(x \sin(x) + y \cos(y))$   
 b) Find the Principal value of  $\left[\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right]^{1+i\sqrt{3}}$ .
5. a) Evaluate  $\int_C (x+y)dx + x^2 y dy$  from (0,0) to (3,9) i). along  $y = x^2$  ii). along  $y = 3x$   
 b) If  $F(a) = \int_C \frac{3z^2 + 7z + 1}{z-a} dz$  where C is the circle  $|z|=2$ . Find the value of  $F(1)$ ,  $F(3)$ ,  $F^{11}(1-i)$  and  $F^{11}(1-i)$

Code No: R21016

**R10****SET - 4**

6. a) Obtain the Laurent's expansion of the function  $\frac{e^z}{(z-1)^2}$  in the neighborhood of its singular points and hence find its residue.
- b) Evaluate  $\int_C \frac{\coth(z)}{z-i} dz$  where  $C$  is  $|z|=2$
7. a) Show that  $\int_0^{2\pi} \frac{d\theta}{a+b\sin(\theta)} = \frac{2\pi}{\sqrt{a^2-b^2}}$ ,  $a > b > 0$  using residue theorem
- b) Evaluate by contour integration  $\int_0^\infty \frac{dx}{(1-x^2)}$
8. a) Find the image of the infinite strip bounded by  $x=0$  and  $x=\pi/4$  under the transformation  $w=\cos(z)$
- b) Find the image of infinite strips  $1/4 < y < 1/2$  under the transformation  $w=1/z$ . Show the regions graphically.