Code No: R21043 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, September - 2014
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) A random experiment consists of tossing a die and observing the number of dots showing up. Let
   A_1: number of dots showing up =3
   A_2: even number of dots showing up
   A_3: odd number of dots showing up
   i) Find $P(A_1)$ and $P(A_1 \cap A_3)$ ii) Find $P(A_2 \cup A_3), P(A_2 \cap A_3), P(A_1 | A_3)$
   iii) Are A_1 and A_3 disjoint? iv) Are A_2 and A_3 disjoint?
   b) State and prove theorem of total probability.

2. a) Show that the mean and variance of a binomial random variable X are μ_x = np and σ_x^2 = npq, where $q = 1 - p$
   b) Let X be a continuous random variable with density function f(x) = x/5+k for $0 \le x \le 3$ otherwise zero. Find k and $P(1 \le X \le 2)$.

3. a) A random variable X is defined by
$$X = \begin{cases} -2 \text{ with a probability of } 1/3 \\ 3 \text{ with a probability of } 1/2 \\ 1 \text{ with a probability of } 1/6 \end{cases}$$
   Find E(X), E(2X+5) and E(X^2)?
   b) Find the moment generating function of the random variable X whose moments are $m_r = (r+1)!\,2^r$.

4. a) The joint pdf of the random variables X and Y is
$$f(x,y) = \frac{1}{4} e^{-|x|-|y|} \text{ for } -\infty < x < \infty \text{ and } -\infty < y < \infty$$
   i) Are X and Y statistically independent variables?
   ii) Find $P(X \le 1 \cap Y \le 0)$
   b) If X and Y are independent random variables, show that
   E[XY] = E[X].E[Y]
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5. a) $X_1, X_2, X_3, \ldots X_n$ are n independent zero mean Gaussian random variables with equal variances, $\sigma_{xi}^2 = \sigma^2$. Show that $Z = \frac{1}{n}[X_1 + X_2 + \cdots + X_n]$

   b) Define joint characteristic function and also write its properties.

6. X(t) is a real WSS random process with an autocorrelation function $R_{XX}(\tau)$. Prove the following:

   i) If X(t) has periodic components, then $R_{XX}(\tau)$ will also have periodic components.

   ii) If $R_{XX}(0) < \infty$, and if $R_{XX}(\tau)$ is continuous at $\tau = 0$, then it is continuous for every τ.

7. a) The power spectral density of a stationary random process is given by

$$S_{XX}(\omega) = \begin{cases} A & -K < \omega < K \\ 0 & otherwise \end{cases}$$

   Determine the auto correlation function.

   b) Define cross power density spectrum and write its properties.

8. a) Derive the relation between PSDs of input and output random processes of LTI system.

   b) A white noise signal of zero mean and PSD $\eta/2$ is applied to an ideal LPF whose bandwidth is B. Find the auto correlation of the output noise signal?

Code No: R21043 **R10** SET - 2

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1. a) A box contains three 100 ohm resistors labeled R_1, R_2 and R_3 and two 1000 ohm resistors labeled R_4 and R_5. Two resistors are drawn from this box with replacement.
   i) List all the outcomes of this random experiment. [A typical outcome may be listed as (R_1,R_2) to represent that R_1 was drawn first followed by R_2]
   ii) Find the probability that both resistors are 100 ohm resistors
   iii) Find the probability of drawing one 100 ohm resistor and one 1000 ohm resistor.
   iv) Find the probability of drawing a 100 ohm resistor on the first draw and a 1000 ohm resistor on the second draw
   b) State and prove multiplication law of probability.

2. a) Show that the mean and variance of a Poisson random variable are $\mu_x = \lambda$ and $\sigma^2_x = \lambda$
   b) A random variable X has pdf $f(x)=k(1+x^2)$, for $0\le x\le 1$. Find the constant k and distribution function of random variable.

3. a) A random variable X is defined by density function
   $$f(x) = \begin{cases} 3x^2 & for \ \ 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$
   Find E(X), E(3X) and E(X^2)
   b) Find the characteristic function of a Gaussian random variable with mean μ and variance σ^2.

4. a) State central limit theorem and write its applications.
   b) Let X and Y be defined as X= cosθ and Y = sinθ. Where θ is a random variable, uniformly distributed over $(0,2\pi)$. Show that X and Yare not independent.

5. a) X_1, X_2, X_3,….X_n are n independent Gaussian random variables with zero means and unit variances. Let $Y = \sum_{i=1}^{n} x_i^2$ find the pdf of Y.
   b) Write properties of joint distribution function.

6. X(t) is a WSS process and let $Y(t) = X(t+a) - X(t-a)$. Show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$.

7. a) Determine the PSD of sum of two random processes with suitable example.
   b) Find the PSD of a stationary random process for which auto correlation is $R_{XX}(\tau) = e^{-\alpha|\tau|}$.

8. a) The noise figure of an amplifier is 0.2dB. Find the equivalent temperature?
   b) A WSS random process X(t) is applied to the input of an LTI system whose impulse response is $5t.e^{?2t}.u(t)$. The mean of X(t) is 3. Find the mean of the output of the system.
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Code No: R21043 **R10** SET - 3

II B. Tech I Semester Supplementary Examinations, September - 2014
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Time: 3 hours Max. Marks: 75

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1. a) The probability that a student passes a certain exam is 0.9, given that he studied. The probability that he passes the exam without studying is 0.2. Assume that the probability that the student studies for an exam is 0.75 (a somewhat lazy student). Given that the student passed the exam, what is the probability that he studied?
   b) Explain the various Axioms of probability.

2. a) Let $Z = X + Y - c$, where X and Y are independent random variables with variances σ_x^2 and σ_y^2 and c is constant. Find the variance of Z in terms of σ_x^2, σ_y^2 and c?
   b) Find the maximum value of Rayleigh's density function?

3. a) A random variable X has the probability function
   $$P(x) = \frac{1}{2^x} \quad x = 1, 2, 3 \ldots \ldots n$$
   Find its moment generating function.
   b) Find the characteristic function of a random variable X defined by the probability density function $f(x) = \begin{cases} 0 & for \quad x < 0 \\ 1 & for \quad 0 \le x \le 1 \\ 0 & for \quad x > 0 \end{cases}$

4. a) X is uniformly distributed in the interval $[-\pi, \pi]$. Find the pdf of Y=a sin(X).
   b) X_1 and X_2 are two independent random variables with uniform pdfs in the interval [0, 1]. Let $Y_1=X_1+X_2$ and $Y_2 = X_1 - X_2$
   i) Find the joint pdf $f_{Y_1Y_2}(y_1, y_2)$ and clearly identify the domain where this joint pdf is nonzero.
   ii) Find $E\{Y_1|Y_2 = 0.5\}$.

5. a) Let $X_1, X_2, X_3, \ldots X_n$ be n random variables which are identically distributed such that
   $$X_K = \begin{cases} 1 \; with\ a\ probability\ of\ 1/2 \\ 2 \; with\ a\ probability\ of\ 1/3 \\ 1 \; with\ a\ probability\ of\ 1/6 \end{cases}$$
   Find $E[X_1, X_2, X_3, \ldots X_n]$
   b) Write the properties of marginal distribution function
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6. X(t) and Y(t) are real random processes that are jointly WSS. Prove the following:

   i) $R_{XX}(\tau) = \sqrt{R_{XX}(0)R_{YY}(0)}$ ii) $R_{XX}(\tau) \le \frac{1}{2}[R_{XX}(0) + R_{YY}(0)]$

7. a) For a WSS random process, show that $S_{XX}(0)$ equal to the area under $R_{XX}(\tau)$

   b) Find the PSD of the random process X(t)=A cos (w_ct+θ) where θ is uniform random variable over (0, 2π).

8. a) A WSS process X(t) with $R_{XX}(\tau) = Ae^{-a|\tau|}$ where 'A' and 'a' are real positive constants is applied to the input of an LTI system with $h(t) = e^{-bt}u(t)$ where 'b' is a real positive constant. Find the PSD of the output of the system.

   b) Determine the overall noise figure of the cascaded systems.

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1. a) A random experiment can terminate in one of three events A, B or C with probability 1/2, 1/4 and 1/4 respectively. The experiment is repeated three times. Find the probability that events A, B and C each occur exactly one time.
   b) State and prove Baye's Theorem.

2. a) Write the properties of probability density function.
   b) Two discrete random variables X and Y have
   P(X=0, Y=0) = 2/9; P(X=0,Y=1) =1/9;
   P(X=1, Y=0) = 1/9; P(X=1,Y=1) =5/9;
   Examine whether X and Y are independent.

3. a) What is Chebychev's Inequality? Why this is used in random process?
   b) Find the expected value of the number on a die when thrown.
   c) Write the properties of characteristic function.

4. a) The joint pdf of random variables X and Y is
   $$f_{X,Y}(x, y) = \frac{1}{2} \qquad 0 \le x \le 1, \qquad 0 \le y \le 2.$$
   i) Find the marginal pdfs, $f_X(x)$ and $f_Y(y)$?
   ii) Find the conditional pdfs $f_{X/Y}(x|y)$ and $f_{Y|X}(y|x)$
   iii) Find E{X|Y = 1} and E{X|Y = 0.5}
   iv) Are X and Y statistically independent?
   v) Find ρ_{XY}
   b) Explain how central limit theorem is used in sum of a number of independent random ariables?

5. a) If X is a random variable for which E(X)=10 and Var(X)=25, for what positive value of 'a' and 'b' does $Y = aX - b$ have expectation 0 and variance 1.
   b) Determine the 3rd central moment about the mean in terms of the moments about origin of the random variable.
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6. a) Consider a random process X(t) defined by X(t) = cosΩt. Where Ω is a random variable distributed uniformly over $(0,w_0)$. Determine whether X(t) is stationary.
   b) Define cross correlation function and write its properties.

7. a) For the following autocorrelation function, find the power spectral density function
$$\frac{1}{4}e^{-|\tau|}[\cos\tau + \sin\tau]$$
   b) For a WSS random process, show that $R_{XX}(0)$ equal to the area under $S_{XX}(f)$

8. a) White noise with two sided spectral density η/2 is passed through a low pass RC network with time constant τ = RC and thereafter through an ideal amplifier with voltage gain of 10. Write the expression for the power spectral density of the noise at the output of the amplifier.
   b) Derive the relation between the noise figure and noise equivalent temperature.