Subject Code: R13102/R13 **Set No - 1**

I B. Tech I Semester Supplementary Examinations August - 2015

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(a) Find Laplace transform of Dirac Delta function?

(b) At the start of an experiment, there are 100 bacteria. If the bacteria follow an exponential growth pattern with rate k = 0.02, what will be the population after 5 hours? How long will it take for the population to double?

(c) Discuss about Jacobian?

(d) Explain about Laplace equation?

(e) Find $\frac{1}{D^3}(\cos x)$

(f) Define extreme value?

[4+4+4+3+4+3]

PART-B

2.(a) Find the maximum and minimum values of x + y + z subject to $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

(b) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^x \sin 2x$

[8+8]

3.(a) Solve $(1+y^2)+(x-e^{tan^{-1}y})\frac{dy}{dx}=0$

(b) Using Laplace transforms, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{2t}\sin 3t$.

[8+8]

4. Solve the following heat problem for the given initial conditions.

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

$u(x,0)=f(x)$ $\quad u(0,t)=0$ $\quad u(L,t)=0$

(a) $f(x)=6\,sin\left(\frac{\pi x}{L}\right)$

(b) $f(x)=12\,sin\left(\frac{9\pi x}{L}\right)-7\,sin\left(\frac{4\pi x}{L}\right)$

[8+8]

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5.(a) Find the Laplace transforms of the given functions

(i) $\left\{\frac{1-\cos at}{t^2}\right\}$ (ii) $3\sinh(2t)+3\sin(2t)$

(b) A bottle of soda pop at room temperature (92^0F) is placed in a refrigerator where the temperature is 64^0F. After half an hour the soda pop has cooled to 81^0F.
(i) What is the temperature of the soda pop after another half hour?
(ii) How long does it take for the soda pop to cool to 70^0F?

[8+8]

6.(a) Form the differential equation form $z = yf(x^2 + z^2)$.
(b) Solve $(p^2 - q^2)z = x - y$.

[8+8]

7. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity$\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time *t*.

[16]

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Subject Code: R13102/R13 **Set No - 2**

I B. Tech I Semester Supplementary Examinations August - 2015

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART-A

1.(a) State convolution theorem?
(b) Find the Laplace transform of Heaviside's function?
(c) Discuss about Bernoulli's equation?
(d) Find $\frac{1}{D}(x^2)$
(e) Explain about one dimensional heat equation?
(f) Explain chain rule of partial differentiation?

[3+4+3+3+5+4]

PART-B

2.(a) Solve the following IVP and find the interval of validity for the solution.
$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \qquad y(0) = -3$$
(b) Determine the orthogonal trajectories of the family of circles $x^2 + (y - c)^2 = c^2$ tangent to the x- axis at the origin.

[8+8]

3.(a) Suppose that the population of a colony of bacteria increases exponentially. At the start of an experiment, there are 6,000 bacteria, and one hour later, the population has increased to 6,400. How long will it take for the population to reach 10,000? Round your answer to the nearest hour.
(b) Find and classify all the critical points of $f(x,y) = 4 + x^3 + y^3 - 3xy$

[8+8]

4.(a) Find the Laplace transforms of the given functions.
(i) $f(t) = 6e^{-5t} + e^{3t} + 5t^3 - 9$ (ii) $g(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$
(b) Find the Laplace transform of $f(t) = |t - 1| + |t + 1|,\ t \geq 0$

[8+8]

5.(a) Solve $D^4y - y = \cos x \cosh x$?
(b) Using Laplace transforms, solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$.

[8+8]

Subject Code: R13102/R13 **Set No - 2**

6.(a) Solve $2x^4p^2 - yzq - 3z^2 = 0$.

(b) Solve $p\ tan\ x + q \tan y = \tan z$

[8+8]

7. A tightly stretched string with fixed end points x = 0 and x = p is initially in a position given by $y = y_0\, sin^3 \frac{\pi x}{p}$. If it is released from rest from this position, find the displacement y(x, t).

[16]

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Subject Code: R13102/R13 **Set No - 3**

I B. Tech I Semester Supplementary Examinations August - 2015

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**

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PART-A

1.(a) Define saddle point?

(b) Explain about law of natural decay?

(c) Find $\frac{1}{D^2+5D+6}e^x$

(d) Give the statement of Convolution theorem?

(e) Explain about wave equation?

(f) Define functional dependence?

[3+4+3+4+5+3]

PART-B

2.(a) Show that the functions $u = xy + yz + zx,\ v = x^2 + y^2 + z^2\ and\ w = x + y + z$ are functionally dependent.

(b) Form the partial differential equation by eliminating the arbitrary function ϕ from: $\phi\,(x^2 + y^2 + z^2, z^2 - 2xy) = 0$

[8+8]

3.(a) Solve $1 + p^2 = qz$

(b) Solve the differential equation $y^{11} + 4y = \tan 2x$

[8+8]

4.(a) Find the inverse transform of each of the following.

(i) $F(s) = \dfrac{1-3s}{s^2+8s+21}$ (ii) $G(s) = \dfrac{s+7}{s^2-3s-10}$

(b) A bottle of soda pop at room temperature (72^0F) is placed in a refrigerator where the temperature is 44^0F. After half an hour the soda pop has cooled to 61^0F.
(i) What is the temperature of the soda pop after another half hour?
(ii) How long does it take for the soda pop to cool to 50^0F?

[8+8]

5.(a) Using Laplace transforms, solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^t \sin 2t$

(b) Find the orthogonal trajectories of the confocal and coaxial parabolas

$$r = \frac{2a}{1 + cos\theta}$$

[8+8]

Subject Code: R13102/R13 **Set No - 3**

6. A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x,0) = \begin{cases} x & ; 0 \le x \le 50 \\ 100 - x; & 50 \le x \le 100 \end{cases}$$

Find the temperature $u(x, t)$ at any time.

[16]

7.(a) Obtain the Taylor's series expansion of $\sin x$ in powers of $x - \frac{\pi}{4}$

(b) Solve $\frac{dy}{dx} - y \tan x = \frac{\sin x \cos^2 x}{y^2}$

[8+8]

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Subject Code: R13102/R13 **Set No - 4**

I B. Tech I Semester Supplementary Examinations August - 2015

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours **Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**
Answering the question in **Part-A** is Compulsory,
Three Questions should be answered from **Part-B**
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PART-A

1.(a) Form the differential equation from $z = ax^3 + by^3$
(b) Explain about Newton's law of cooling?
(c) Find the inverse Laplace transform of $F(s) = \frac{6s-5}{s^2+7}$
(d) Obtain Maclaurin's series for e^x
(e) Solve $z = px + qy + pq$
(f) Find the particular integral of $\frac{1}{D^2+6D+9}(2e^{-3x})$

[3+3+4+4+4+4]

PART-B

2.(a) Solve $y^2p - xyq = x(z - 2y)$.
(b) Solve $(x + 1)\frac{dy}{dx} - xy = e^x(x + 1)^{n+1}$

[8+8]

3.(a) Use a convolution integral to find the inverse transform of the following transform.

$$H(s) = \frac{1}{(s^2 + a^2)^2}$$

(b) Write the following function (or switch) in terms of Heaviside functions and its Laplace transform.

$$f(t) = \begin{cases} -4 & \text{if } t < 6 \\ 25 & \text{if } 6 \le t < 8 \\ 16 & \text{if } 8 \le t < 30 \\ 10 & \text{if } t \ge 30 \end{cases}$$

[8+8]

4.(a) Solve the following differential equation.$(D^2 + 4)y = \sec 2x$.
(b) Investigate for the maxima and minima, if any of $x^3y^2(1 - x - y)$.

[8+8]

Subject Code: R13102/R13 **Set No - 4**

5. A homogenous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x,0) = \begin{cases} x & ; 0 \le x \le 5 \\ 100 - x & ; 50 \le x \le 100 \end{cases}$$

Find the temperature u(x, t) at any time.

[16]

6.(a) An object cools from 120^0 to 95^0 F in half an hour when surrounded by air whose temperature is 70^0F. Find its temperature at the end of another half an hour.

(b) Form the differential equation from $z = xf(2x + 3y) + g(2x + 3y)$.

[8+8]

7.(a) Solve $p^3 + q^3 = 8z$

(b) Expand the function $f(x, y) = e^x \log(1 + y)$ in terms of x and y up to the terms of 3rd degree using Taylor's thereom?

[8+8]

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