Code No: R21016 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Jan - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $J_n^{'}(x) = \frac{n}{x} J_n(x) - J_{n+1}(x)$.

   b) Prove that $x\, \mathrm{P}_n^{'}(\mathrm{x}) = \mathrm{nP}_n(\mathrm{x}) + \mathrm{P}_n^{'}(\mathrm{x})$ (7M+8M)

2. a) Show that $f(z) = \begin{cases} \frac{(x^3+y^3)+i(x^3+y^3)}{x^2+y^2}, & z \neq 0 \\ 0 \quad , & z = 0 \end{cases}$

   satisfier Cauchy-Riemann equations at the origin, but the derivative of *f(z)* does not exist at origin.

   b) Find the analytic function $f(z) = u + iv$ where $v = e^{-x}(x \sin y - y \cos y)$ (9M+6M)

3. a) Find the all the roots of the equation $e^z=-2$.

   b) Separate into real and imaginary parts of f (z)=cothz

   c) Find the principal value of i^i. (5M+5M+5M)

4. a) Integrate $f(z)=x^2+ixy$ from A(1,1) to B(2,4) along the curve x = t, y = t^2.

   b) Use Cauchy's integral formula to evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)\,(z-2)} dz$ where C is $|z| = 3$

   (7M+8M)

5. a) Find Taylor's expansion of f(z)=log(1+z) about the point z=0

   b) Find the Laurent series of $f(z) = \frac{1}{z^2 - 4z + 3}$, for $1 < |z| < 3$.

   c) Discuss the type of singularity of the function $f(z) = \frac{z - \sin z}{z^2}$. (5M+5M+5M)
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Code No: R21016 **R10** SET - 1

6. a) Determine the poles of the function $f(z)=\dfrac{2z+1}{z^2-z-2}$ and the residue at each pole.

   b) Evaluate $\oint_C \dfrac{e^z}{\cos z}\,dz$ where C is $|z|=4.5$.

   c) Use residue theorem to evaluate $\int_{-\infty}^{\infty}\dfrac{dx}{1+x^2}$ (5M+5M+5M)

7. a) State and prove Rouche's theorem.

   b) Use Rouche's theorem to determine the number of zeros of the polynomial $p(z)=z^7-5z^4+z^2-2$ which lie inside the circle $|z|=1$. (9M+6M)

8. a) Determine the image of the region $|z-3|=5$ under the transformation $w=\dfrac{1}{z}$.

   b) Find the fixed points of the transformation w= $(z-i)^2$.

   c) Find the bilinear transformation that maps the points $z_1=-1$, $z_2=i$, $z_3=1$ into the points $w_1=0$, $w_2=I$, $w_3=\infty$ respectively. (7M+8M)

Code No: R21016 **R10** SET - 2

II B. Tech I Semester Supplementary Examinations, Jan - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$.
   b) Prove that $(n+1)\,P_{n+1}(x) = (2n+1)x\ P_n(x) - nP_{n-1}(x)$. (7M+8M)

2. a) Show that for $f(z) = \begin{cases} \dfrac{(xy^2) + (x+iy)}{x^2+y^4}, & z \neq 0 \\ 0 & , z = 0 \end{cases}$
   the Cauchy –Riemann equations are satisfied at the origin but the derivative of f(z) at origin does not exist.
   b) Find the analytic function f(z)=u+iv where $v=e^{-x}(x\cos y+y\sin y)$. (9M+6M)

3. a) Find the all the roots of the equation $e^{4z}=i$.
   b) Separate into real and imaginary parts of f(z)= tanh z. (5M+5M+5M)

4. a) Integrate $f(z)=x^2+ixy$ from A(1,1) to B(2,8) along the curve x=t, $y=t^3$
   b) Use Cauchy's integral formula to evaluate $\oint_C \dfrac{e-2^z}{(z+1)^3}\,dz$ where C is $|z| = 2$. (7M+8M)

5. a) Find Taylor's expansion of f(z)=sin z about the point $z=\dfrac{\pi}{4}$.
   b) Find the Laurent series of $f(z) = \dfrac{1}{z(z-1)(z-2)}$, *for* $|z| > 2$.
   c) What type of singularity does the function $f(z) = \dfrac{1-e^{2z}}{z^4}$ possess? (5M+5M+5M)
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Code No: R21016 **R10** SET - 2

6. a) Determine the poles of the function $f(z)=\dfrac{z+1}{z^2(5-z)}$ and the residue at each pole.

   b) Evaluate $\oint_C \dfrac{e^z}{\cos \pi z}$ where C is $|z-i|=1.5$.

   c) Use residue theorem to evaluate $\int_0^{2\pi} \dfrac{d\theta}{7+6\cos\theta}$. (5M+5M+5M)

7. a) State and prove Fundamental theorem of Algebra.
   b) Use Rouche's theorem to determine the number of zeros of the polynomial $P(z)=2z^6+z^2-8z-2$ which lie inside the circle $|z|=1$. (8M+7M)

8. a) Find and sketch the image of the region $-1\le x\le 0,\ 0\le y\le \dfrac{\pi}{2}$ under $w = e^z$.

   b) Find the fixed points of the transformation $w=\dfrac{z+i}{z-i}$

   c) Determine the bilinear transformation that maps the points $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$, $w_1 =- 1$, $w_2 = -i$, $w_3=1$ respectively. (5M+4M+6M)

Code No: R21016 **R10** SET - 3

II B. Tech I Semester Supplementary Examinations, Jan - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $J_n(x) = \frac{x}{2n}(J_{n-1}(x) + J_{n+1}(x))$.

   b) Prove that $\left[1-2xt+t^2\right]^{-1/2} = \sum_{n=0}^{\infty} p_n(x)t^n$ (7M+8M)

2. a) Show that $f(z) = \sqrt{|xy|}$ is not analytic at $z = 0$, although the Cauchy-Riemann equations are satisfied at the origin.

   b) Determine the analytic function whose real part is $u = e^{x^2-y^2}\cos 2xy$. (9M+6M)

3. a) Find the all the roots of the equation sinz=cosh4 into real and imaginary parts
   b) Separate real and imaginary parts of f(z)=tan z.
   c) Find the real part of the principal value of $i^{\ln(1+i)}$. (5M+5M+5M)

4. a) Integrate $f(z)=x^2+ixy$ from A(1,1) to B(2,8) along the curve x=t, $y = t^3$

   b) Use Cauchy's integral formula to evaluate $\int_C \frac{e^{2z}}{(z+1)^4}dz$ where C is the circle $|z|=2$.

   (8M+7M)

5. a) Find Taylor's expansion of $f(z) = \frac{z}{(z+1)(z+2)}$ about the point z=2.

   b) Find the Laurent series of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$, for $|z| > 3$.

   c) What type of singularity does the function $f(z) = \frac{e^{2z}}{(z-1)^4}$ possess? (5M+5M+5M)
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Code No: R21016 **R10** SET - 3

6. a) Determine the poles of the function $f(z)=\dfrac{z^2}{(z-2)^2(z-1)}$ and the residue at each pole.

b) Evaluate $\oint_C \dfrac{\cosh z}{z^2-3iz}dz$ where C is $|z|=1$.

c) Use residue theorem to evaluate $\int_{-\infty}^{\infty}\dfrac{dx}{x^4+16}$ (9M+6M)

7. a) State and prove Liouville's theorem.

b) Use Rouche's theorem to determine the number of zeros of the polynomial $p(z)=2z^4-2z^3+2z^2-2z+9$ which lie inside the circle $|z|=1$. (9M+6M)

8. a) Find and sketch the image of the region $-0.5\le x\le 0.5, \dfrac{3\pi}{4}\le y\dfrac{5\pi}{4}$ *under* $w=e^z$.

b) Find the invariant points of the transformation $w=\dfrac{2iz-1}{z+2i}$.

c) Determine the bilinear transformation that maps the points $z_1=0$, $z_2=1, z_3=2$ into the points $w_1=1$, $w_2=\dfrac{1}{2}$, $w_3=\dfrac{1}{3}$ respectively. (5M+5M+5M)

Code No: R21016 **R10** SET - 4

II B. Tech I Semester Supplementary Examinations, Jan - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) Express $J_{\frac{7}{2}}(x)$ in terms of sine and cosine functions.

   b) Show that $(2n+1)\,P_n(x) = P'_{n+1}(x) - P'_{n-1}(x)$. (7M+8M)

2. a) Show that $f(z)=\begin{cases}\dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z\neq 0\\ 0 & ,z\neq 0\end{cases}$ is not analytic at z = 0, although the Cauchy-Riemann equations are satisfied at the origin.

   b) Determine the analytic function whose real part is $u=\dfrac{\sin 2x}{(\cosh 2y-\cos 2x)}$ (9M+6M)

3. a) Find the all the roots of the equation sinhz=i.

   b) Separate into real and imaginary parts of f(z)=cschz.

   c) Find all values and the principal value of = $\ln i^i$ (5M+5M+5M)

4. a) Evaluate $\int_0^{3+i} z^2\,dz$ along

   i) The line $y=\dfrac{x}{3}$ ii) the real axis to 3 and then vertically to 3+i

   b) Use Cauchy's integral formula to evaluate $\int_C \dfrac{\cos \pi z^2}{(z-1)(z-2)}$ where C is the circle $|z|=3$.

   (8M+7M)

5. a) Find Taylor's expansion of f(z)=cos z about the point $z=\dfrac{\pi}{4}$.

   b) Find the Laurent series of $f(z)=\dfrac{(z-2)(z+2)}{(z+1)(z+4)}$, $for 1<|z|<4$.

   c) What type of singularity has the function $f(z)=ze^{1/z^2}$ (5M+5M+5M)
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Code No: R21016 **R10** SET - 4

6. a) Determine the poles of the function $f(z) = \frac{e^z}{z^2 + \pi^2}$ and the residue at each pole.
   b) Evaluate $\oint_C \coth z dz$, where c is $|z| = 1$.
   c) Use residue theorem to evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$. (5M+5M+5M)

7. a) State and prove Argument principle.
   b) Use Rouche's theorem to determine the number of zeros of the polynomial $p(z) = z^7 - 4z^3 + z - 1$ which lie inside the circle $|z| = 1$. (8M+7M)

8. a) Find and sketch the image of the region $-1 \le x \le 0, 0 \le \frac{\pi}{2}$ *under* $w = e^z$.
   b) Find the invariant points of the transformation $w = \frac{-3iz - 5}{z + i}$.
   c) Determine the bilinear transformation that maps the points $z_1 = 0,\ z_2 = 2i,\ z_3 = -2i$ into the points $w_1 = -1, w_2 = 0, w_3 = \infty$ respectively.
   (5M+5M+5M)