Code No: R21016 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, June - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
All Questions carry Equal Marks
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1. a) Prove that $\int_0^1 x^{n-1}\left(\log\frac{1}{x}\right)^{m-1} dx = \frac{\Gamma(m)}{n^m}, m>0, n>0$

   b) Show that $\int J_3(x)dx = -J_2(x) - \frac{2}{x}J_1(x)$

2. a) Show that $f(z) = xy + iy$ is everywhere continuous but is not analytic

   b) Find a and b if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic. Hence find f(z) in terms of z.

3. a) If tan (log(x+iy)) = a+ib, where $a^2+b^2 \neq 1$ show that $\frac{2a}{1-a^2-b^2} = \tan(\log(x^2+y^2))$

   b) Find all the roots of sin z=2.

4. a) Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ (i) along the straight line from z=0 to z=1+i. (ii) along the real axis from z=0 to z=1 and then along a line parallel to imaginary axis from z=1 to z=1+i.

   b) State and Prove Cauchy's integral theorem?

5. a) Obtain the Taylor series expansion of f (z) = $\frac{e^z}{z(z+1)}$ about z=2.

   b) Find the Laurent series of $\frac{7z-2}{(z+1)(z-2)}$ in the annulus $1<|z+1|<3$.

6. a) Find the poles of f(z) and the residues of the poles which lie on imaginary axis if

   $$f(Z) = \frac{z^2+2z}{(z+1)^2(z^2+4)}$$

   b) Evaluate $\int_0^\infty \frac{x\sin mx}{x^4+16}$ dx using Residue theorem.
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7. a) Show that the polynomial $z^5 + z^3 + 2z + 3$ has just one zero in the first quadrant of the complex plane.

   b) Use Rouche's theorem to show that the equation $z^4 + 6z + 3 = 0$ has three out of four roots in the annulus $1<|z|<2$.

8. a) Find the image of the triangle with vertices i,1+i, 1-i in the z- plane under the transformation w=3z+4-2i

   b) Find the bilinear transformation which maps vertices ((1+i,-i, 2-i) of the triangle T of the z-plane into the points (0,1,i) of the w-plane.

Code No: R21016 **R10** SET - 2

II B. Tech I Semester Supplementary Examinations, June - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) show that $\Gamma(n)=\int_0^1\left(\log\frac{1}{x}\right)^{n-1}dx, n>0$

   b) Prove that $\int_{-1}^{1}xP_n(x)P_{n-1}(x)dx=\frac{2n}{4n^2-1}$

2. a) Show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\left|f'(z)\right|=0, where f(z)$ is an analytic function.

   b) Show that the function $u=e^{-2xy}\sin(x^2-y^2) is$ harmonic, find the conjugate.

3. a) Find the general and principal values of (i) log (1+i$\sqrt{3}$) (ii) log (-i) (iii) log(-1)

   b) Determine all values of $(1 - i)^{1+i}$

4. a) Evaluate 0 $\int_0^{1+i}(x^2-iy)dx$ along the paths (i) y=x (ii) y=x^2

   b) Evaluate $\int_c\frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$, where C is the circle $|z|=3$ using Cauchy's
      i. integral formula.

5. a) Obtain the Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$, in the region $|z|<2$.

   b) Expand f(z) = $\frac{1}{z^2-3z+2}$ in the region (i) 0<$|z-1|<1$. (ii) 1<$|z|<2$

6. a) Evaluate $\int_c\frac{\coth z}{z-1}dz$ where C is |z|=2.

   b) Evaluate $\int_c\frac{2e^z dz}{z(z-3)}$ where C is |z|=2 by Residue theorem.
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Code No: R21016 **R10** SET - 2

7. a) State and Prove Rouche's theorem?
   b) Show that one root of the equation $z^4+z+1=0$ lie in the first quadrant

8. a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$
   b) Find the linear fractional transformation that maps the points $z_1=-2$, $z_2=0$, $z_3=2$ onto the points $w_1=\infty$, $w_2=1/4$, $w_3=3/8$ respectively.

Code No: R21016 **R10** SET - 3

II B. Tech I Semester Supplementary Examinations, June - 2015
MATHEMATICS - III
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Time: 3 hours Max. Marks: 75

Answer any FIVE Questions
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1. a) Define Beta and Gamma functions?
   b) Write the properties of Beta function?
   c) show that

$$x^3 = \frac{2}{5}P_3(x) + \frac{3}{5}P_1(x)$$

2.
2. a) Prove that the function f(z) defined by f(z) = $\frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, (z \neq 0) =$ 0, (z=0) is continuous and the Cauchy-Riemann equations are satisfied at the origin but $f'(0)$ does not exist.
   b) If $v = x^2 - y^2$ imaginary of an analytic function, find analytic function and its real part.

3. a) Find all principal values of $\left(1+i\sqrt{3}\right)^{\left(1+i\sqrt{3}\right)}$
   b) Find all values of z which satisfy cos z = 2

4. a) Evaluate $\int_c \frac{z}{z^2+1}\,dz$ where c is $\left|z + \frac{1}{z}\right| = 2.$
   b) Evaluate $\int_c \frac{z-3}{z^2+2z+5}$ where C is the circle (i) $|z| = 1$ (ii) $|z+1-i = 2|$

5. a) Find the Laurent expansion of $\frac{1}{z^2-4z+3}$, *for* $1 < |z| < 3$.
   b) Find Taylor's expansion for the function f(z) = $\frac{1}{(1+z)^2}$ with centre at -i.

6. a) Find the residue of the function $f(z) = \frac{1-e^{2z}}{z^4}$ at the poles.

   b) Show that $\int_0^{\pi} \frac{d\theta}{a^2 + \sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}}$ $(a > 0)$ using Residue theorem.
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Code No: R21016 **R10** SET - 3

7. a) Show that the equation $z^4 + 4(1+i)z + 1 = 0$ has one root in each quadrant
   b) State and prove fundamental theorem of algebra?

8. a) Under the transformation $w = \frac{1}{z}$ find the image of the circle $|z - i1 = 2|$.
   b) Write about transformation W = e^z .

Code No: R21016 **R10** SET - 4

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1. a) Show that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{B(m,n)}{a^n(1+a)^m}$$

b) Prove that

$$J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0\left(x\right)$$

2. a) If u=$e^x\left[(x^2-y^2)\cos y - 2xy\sin y\right]$ is real part of an analytic function, find the analytic function.

b) Find a and b if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic. Hence find f(z) in terms of z.

3. a) Find all principal values of $\left(\frac{\sqrt{3}}{2} + \frac{i}{\sqrt{2}}\right)^{(1+i\sqrt{3})}$

b) Find all the roots of the equation tanhz+2=0

4. a) State and prove Cauchy's integral formula?

b) Evaluate $\int_c \frac{dz}{z^3(z+4)}$ where C is $|z| = 2$ using Cauchy's integral formula.

5. a) Expand log (1-z) when $|z < 1|$ using Taylor series.

b) Obtain Laurent's expansion for f(z) = $\frac{1}{(z+2)(1+z)^2}$ (i) $|z| < 2$ (ii) $|1+z| > 1$

6. a) Determine the poles of the function and the corresponding residues: $\frac{z+1}{z^2(z-2)}$

b) Evaluate $\int_0^\infty \frac{dx}{(x^2+9)(x^2+4)^2}$ using residue theorem.
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Code No: R21016 **R10** SET - 4

7. a) Use Rouches's theorem to show that the equation $z^5+15z+1=0$ has one root in the disc $|z| < \frac{1}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$.
   b) State and Prove Lowville's theorem?

8. a) Find and plot the image of the triangular region with vertices at (0, 0), (1, 0), (0, 1) under the transformation w= (1 - i) z + 3.
   b) Find the bilinear transformation that maps the points (0, 1, ∞) in z-plane onto the points (-1,-2, -i) in the w-plane.