Code No: R21016 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Dec - 2015
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. Prove that i) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$ (15M)

   ii) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}(\sin x)$

2. a) Find the regular function $w = u + iv$ where $u = e^{-x}[(x^2 - y^2)\cos y + 2xy \sin y]$. (8M)

   b) If $f(z) = \begin{cases} \frac{x^3 y(y - ix)}{x^6 + y^2}, z \neq 0 \\ 0, z = 0 \end{cases}$ prove that $\frac{f(z) - f(0)}{z} \to 0$ as $z \to 0$ along any radius vector but not as $z \to 0$ along the curve $y = ax^3$ (7M)

3. a) i) Expand $\cosh 5x$ in a series of powers of hyperbolic cosines of x. (8M)

   ii) Expand $\sinh^5 x$ in a series of powers of hyperbolic sines of multiples of x.

   b) If $\cos(x + iy) = \cos\theta + i\sin\theta$, show that $\cos 2x + \cosh 2y = 2$. (7M)

4. a) Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2, y = \pm 2$ where c is described in the positive sense. (8M)

   i) $\int_c \frac{Tan(z/2)}{(z - x_o)} dz$ $(|x_o| < 2)$ ii) $\int_c \frac{\cos z}{z(z^2 + 8)} dz$

   b) Evaluate $\int_0^{2+i} z^2 dz$ along (i) the real axis from z = 0 to 2 and then vertically to (2+i) ii) The imaginary axis to i and then horizontally to (2+i). (7M)
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5. a) Expand $f(z)=\frac{(z-1)(z+2)}{(z+1)(z+4)}$ in the region i) $1<|z|<4$ ii) $|z|<1$ (8M)

b) Explain different types of singularities with examples (7M)

6. a) Show by the method of contour integration (8M)

that $\int_0^\infty \frac{\cos mx}{(x^2+a^2)^2}dx=\frac{\pi}{4a^3}(1+ma)e^{-ma},(a>0,b>0)$.

b) Find the poles and residues at each pole of $\tanh z$. (7M)

7. i) If a>e, use Rouche's theorem to prove that $e^z=az^n$ has n roots inside the circle $|z|=1$. (15M)

ii) State and prove that Fundamental theorem of Algebra.

8. a) Find the bilinear transformation which maps the points $\infty, i, 0$ in the z-plane into $-1,-i,1$ in the w-plane (8M)

b) Show that the transformation $w=\frac{2z+3}{z-4}$ change the circle $x^2+y^2-4x=0$ into the straight line 4u+3=0. (7M)