# II B. Tech I Semester Supplementary Examinations, Dec - 2015 <br> PROBABILITY AND STATISTICS 

(Com. to CSE, IT)
Time: 3 hours
Max. Marks: 75

Answer any FIVE Questions<br>All Questions carry Equal Marks

1. a) If $A$ and $B$ are any events in $S$ then prove that
$P(A \cup B)=P(A)+P(B)-P(A \cap B)$.
b) Two cards are drawn at random from an ordinary deck of 52 playing cards. What is the probability of getting two aces if (i) the first card is replaced before the second card is drawn; (ii) the first card is not replaced before the second card is drawn?
2. a) Define discrete random variable and discrete probability distribution.
b) Let X have the density function
$f(x)= \begin{cases}k\left(1-x^{2}\right) & \text { for } 0 \leq x \leq 1 \\ 0 & \text { elsewhere }\end{cases}$
Find the distribution function. Find the Probabilities
$P(0.1 \leq X \leq 0.2)$ and $P(X>0.5)$.
3. a) Define Moment Generating Functions. Find Moment Generating Function for Binomial distribution.
b) Let X be normally distributed with mean $\mu=10$ and standard deviation $\sigma=2$. Find
i) $P(9<X<13)$
ii) $P(X<12)$
iii) $P(X>12)$ and $P(X<10)$.
4. a) Determine the probability that $\bar{X}$ will be between 66.8 and 68.3 if a random sample of size 25 is taken from an infinite population having the mean $\mu=68$ and $\sigma=3$.
b) Determine a $95 \%$ confidence interval for the mean of a normal distribution with variance $\sigma^{2}=0.25$, using a sample of $n=100$ values with mean $\bar{x}=212.3$.
5. a) A Company claims that its bulbs are superior to those of its main competitor. If a study showed that a sample of $n_{1}=40$ of its bulbs has a mean lifetime of 1647 hours of continuous use with a standard deviation of 27 hours, while a sample of $n_{2}=40$ bulbs made by its main competitor had a mean lifetime of 1638 hours of continuous use with a standard deviation of 31 hours, does this substantiate the claim at the 0.05 level of significance?
b) Explain the test procedure for large sample test concerning one proportion.
6. As part of the investigation of the collapse of the roof of a building, a testing laboratory is given all the available bolts that connected the steel structure at 3 different positions on the roof. The forces required to shear each of these bolts (coded values) are as follows:

| Position-1 | 90 | 82 | 79 | 98 | 83 | 91 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Position-2 | 105 | 89 | 93 | 104 | 89 | 95 | 86 |
| Position-3 | 83 | 89 | 80 | 94 |  |  |  |

Perform an analysis of variance to test at the 0.05 level of significance whether the differences among the sample means at the 3 positions are significant.
7. Consider the following data taken on subgroups of size 5 . The data contain 20 averages and ranges on the diameter (in millimeters) of an important component part of an engine. Display $\bar{X}$ and $R$ Charts. Does the process appear to be in control?

| Sample | $\bar{X}$ | $R$ | Sample | $\bar{X}$ | $R$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 2.3972 | 0.0052 | $\mathbf{1 1}$ | 2.3887 | 0.0082 |
| $\mathbf{2}$ | 2.4191 | 0.0117 | $\mathbf{1 2}$ | 2.4107 | 0.0032 |
| $\mathbf{3}$ | 2.4215 | 0.0062 | $\mathbf{1 3}$ | 2.4009 | 0.0077 |
| $\mathbf{4}$ | 2.3917 | 0.0089 | $\mathbf{1 4}$ | 2.3992 | 0.0107 |
| $\mathbf{5}$ | 2.4151 | 0.0095 | $\mathbf{1 5}$ | 2.3889 | 0.0025 |
| $\mathbf{6}$ | 2.4027 | 0.0101 | $\mathbf{1 6}$ | 2.4107 | 0.0138 |
| $\mathbf{7}$ | 2.3921 | 0.0091 | $\mathbf{1 7}$ | 2.4109 | 0.0037 |
| $\mathbf{8}$ | 2.4171 | 0.0059 | $\mathbf{1 8}$ | 2.3944 | 0.0052 |
| $\mathbf{9}$ | 2.3951 | 0.0068 | $\mathbf{1 9}$ | 2.3951 | 0.0038 |
| $\mathbf{1 0}$ | 2.4215 | 0.0048 | $\mathbf{2 0}$ | 2.4015 | 0.0017 |

8. A self service canteen employs one cashier at its counter. 8 customers arrive per
every 10 minutes on an average. The cashier can serve on average one per unit.
Assuming that the arrivals are Poisson and the service time distribution is exponential. Determine
(i) The average number of customers in the system
(ii) The average queue length
(iii) Average time a customer spends in the system
(iv) Average waiting time of each customer.
