Code No: R21043 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Jan - 2015
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Define probability and explain axioms of probability
   b) A rifleman can achieve a "Marksman" award if he passes a test. He is allowed to fire six shots at a target's bull's eye. If he hits the bull's eye with at least five of his six shots he wins a set. He becomes a marksman only if he can repeat the feat three times straight, that is; if he can win three straight sets. If his probability is 0.8 of hitting a bull's eye on any one shot, find the probability of becoming a Marksman.

2. a) Show that $P(x_1 < X \le x_2) = F_X(x_2) - F_X(x_1)$.
   b) Explain the following random variables
      i) Binomial ii) Rayleigh

3. a) Find mean and variance of uniform random variable.
   b) A random variable X can have values -4, -1, 2, 3, and 4, each with probability 0.2. Find (i) the density function (ii) the mean (iii) the variance of the random variable $Y= X^2$

4. a) Random variables X and Y are joint Gaussian and normalized if
   $$f_{X,Y}(x,y)=\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left[-\frac{x^2-2\rho xy+y^2}{2\left(1-\rho^2\right)}\right]$$
   where $-1\le\rho\le1$. Show that the marginal density functions are $f_X(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-x^2/2\right)$ and $f_Y(y)=\frac{1}{\sqrt{2\pi}}\exp\left(-y^2/2\right)$.
   b) Random variables X and Y have probability densities $f_X(x)=\begin{cases}\frac{3}{32}\left(4-x^2\right) & -2\le x\le 2\\ 0 & \text{elsewhere in } x\end{cases}$
   $$f_Y(y)=\frac{1}{2}\left[u(y+1)-u(y-1)\right]$$
   Find whether the X and Y are independent are not.
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5. a) Two random variables X and Y are related by the expression $Y = aX + b$, where a and b are any real numbers. Show that the covariance is $C_{XY} = a\sigma_X^2$ where σ_X^2 is the variance of X.
   b) Random variables X and Y have the joint density function

$$f_{X,Y}(x,y) = \begin{cases} (x^2 + y^2)/40 & -1 < x < 1 \ \ and -3 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

   Find σ_X^2 , σ_Y^2 and covariance C_{XY}

6. a) Define random process and explain different types of random processes.
   b) Explain the following terms
      i) Ergodicity ii) Wide-Sense Stationary iii) Gaussian Random Processes

7. A random process is given by $X(t) = A\cos(\Omega t + \theta)$ where A is a real constant, Ω is a random variable with density function $f_{\Omega}(\Omega)$ and θ is a random variable uniformly distributed over the interval $(0, 2\pi)$ independent of Ω. Show that the power spectrum of $X(t)$ is

$$S_{XX}(\omega) = \frac{\pi A^2}{2}[f_{\Omega}(\omega) + f_{\Omega}(-\omega)]$$ and also find P_{YY}.

8. Signal $X(t) = u(t)e^{-\alpha t}$ is applied to a network having an impulse response $h(t) = \delta(t - t_0)$, where α and t_o are real positive constants and $u(t)$ is a step function. Find:
   a) autocorrelation of $Y(t)$
   b) mean square value of $X(t)$ and $Y(t)$

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1. a) Define probability and explain fundamental axioms.
   b) A pair of fair dice are thrown in a gambling problem. Person A wins if the sum of numbers showing up is six or less and one of the dice shows four. Person B wins if the sum is five or more and one of the dice shows a four. Find the probability that both A and B win.

2. a) A random variable X is known to be Poisson with b = 4. Plot the density and distribution functions for this random variable. What is the probability of the event $\{0 \le X \le 5\}$?
   b) Find a value for constant A such that
   $$f_X(x) = \begin{cases} 0 & x < -1 \\ A(1-x^2)\cos(\pi x/2) & -1 \le x \le 1 \\ 0 & 1 < x \end{cases}$$
   is a valid probability density function.

3. a) A random variable X has a probability density
   $$f_X(x) = \begin{cases} (1/2)\cos(x) & -\pi/2 < x < \pi/2 \\ 0 & elsewhere \end{cases}$$
   Find the mean value of the function $g(X) = 4X^2$.
   b) A random variable X is uniformly distributed on the interval $(-\pi/2, \pi/2)$. X is transformed to the new random variable $Y = T(X) = a\tan(X)$, where $a > 0$. Find the probability density function of Y.

4. a) A joint sample space for two random variables X and Y has four elements (1, 1), (2,2), (3,3) and (4,4). Probabilities of these elements are 0.1, 0.35, 0.05 and 0.5 respectively. Determine through logic and sketch the distribution function $F_{X,Y}(x, y)$. Also find the probability of the event $\{X \le 2.5, Y \le 6\}$.
   b) Three statistically independent random variables X_1, X_2, and X_3 are defined by
   $\overline{X_1} = -1.2$ $\sigma^2_{X_1} = 1.3$
   $\overline{X_2} = 0.8$ $\sigma^2_{X_2} = 1.8$
   $\overline{X_3} = 1.0$ $\sigma^2_{X_3} = 1.2$
   Write the equation describing the Gaussian approximation for the density function of the sum $X = X_1 + X_2 + X_3$.
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5. a) Random variables X and Y have the joint density function

$$f_{X,Y}(x,y) = \begin{cases} (x+y)^2/40 & -1 < x < 1 \text{ and } -3 < y < 3 \\ 0 & \text{elsewhere.} \end{cases}$$

Find all the second-order moments of X and Y. What is the correlation coefficient?

b) For two random variables X and Y

$$f_{X,Y}(x,y) = 0.15\delta(x+1)\delta(y)+0.1\delta(x)\delta(y)+ 0.1\delta(x)\,\delta(y-2)+0.4\delta(x-1)\delta(y+2)+ 0.2\delta(x-1)\delta(y-1)+0.5\delta(x-1)\delta(y-3).$$

Find the correlation coefficients of X and Y.

6. a) A random process is defined by X(t) = A, where A is a continuous random variable uniformly distributed on (0, 1). Determine the form of the sample functions, classify the process.

b) Define a random process by X(t) = A cos (πt), where A is a Gaussian random variable with zero mean and variance σ_A^2. Find the density functions of X(0) and X(1). Is X(t) stationary?

7. a) A signal $x(t)$ = u(t) exp (- αt) is applied to a network having an impulse response $h(t)$ = Au (t) exp $(-\omega t)$. Here α and ω are real positive constants and $u(t)$ is the unit step function. Find the system's response.

b) Derive the relationship between power spectrum and autocorrelation

8. Derive an expression for noise figure of two cascaded amplifier.

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1. a) A man fires 12 shots independently at a target. What is the probability that he hits the target at least once if he has probability 9/10 of hitting the target on any given shot? What is the probability that the target is hit at least twice if it is known that it is hit at least once?
   b) What are the three axioms of the theory of probability? Explain by taking an event 'A' and its probability as $P(A)$.
   c) Given that two events A_1 and A_2 are statistically independent, show that A_1 is independent of $\overline{A_2}$.

2. a) A random voltage can have any value defined by the set S = {a ≤ s ≤ b}. A quantizer, divides S into 6 equal-sized contiguous subsets and generates random variable X having values {-4, -2, 0, 2, 4, 6}. Each value of X is earned to the midpoint of the subset of S from which it is mapped
      i) Sketch the sample space and the mapping to the line that defines the values of X
      ii) Find a and b?
   b) Write at least three properties of
      i) Probability distribution function
      ii) Conditional density function of a random variable

3. a) Let X be a Poisson random variable then find out its mean and variance.
   b) Show that the second moment of any random variable 'X' about arbitrary point 'a' is minimum when $a = \overline{X}$.

4. a) Show that the jointly Gaussian random variables X and Y are statistically independent.
   b) Find the density function of $W=X+Y$, where the densities of X and Y are assumed to be: $f_X(x) = 0.5[u(x)-u(x-2)]$; $f_Y(y) = 0.2[u(y)-u(y-5)]$
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5. a) Two random variables having joint characteristic function
$\phi_{XY}(\omega_1,\omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$. Find moments m_{10}, m_{01}, m_{11}.
b) Gaussian random variables X and Y have first and second order moments m_{10}=-1.1, m_{20}=1.16, m_{01}=1.5, m_{02} =2.89, R_{XY} = -1.724. Find C_{XY}, ρ.

6. a) Explain about stationary random process.
b) The random process is given by $X(t)=A\cos(w_0t) + B\sin(w_0t)$,
where w_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance. Show that $X(t)$ is wide sense stationary but not strictly stationary.

7. a) A Gaussian random process is known to be a WSS process with mean $\bar{X} = 4$ and
$R_{XX}(\tau) = 25e^{-3|\tau|}$ where $\tau = \frac{|t_k - t_i|}{2}$ and i, k=1, 2. Find joint Gaussian density function.
b) Find the autocorrelation of a Poisson random process.

8. A random noise $X(t)$, having a power spectrum $S_{XX}(\omega) = \frac{3}{49+\omega^2}$ is applied to a differentiator and the differentiator's output is applied to a network for which
$h_2(t) = u(t)t^2\exp(-7t)$.
The network's response is a noise denoted by $Y(t)$.
i) What is the average power in $X(t)$
ii) Find the power spectrum of $Y(t)$
iii) Find the average power in $Y(t)$

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1. a) Define probability and explain fundamental axioms.
   b) State and prove total probability theorem.

2. a) A random current is described by the sample space $S=\{-4\le i\le 12\}$. A random variable X is defined by
$$X(i)=\begin{cases}-2 & i\le -2\\ i & -2<i\le 1\\ 1 & 1<i\le 4\\ 6 & 4<i\end{cases}$$
   Show, by a sketch, the value x into which the values of i are mapped by x. What type of random variable is X?
   b) Explain Gaussian random variable with neat sketches.

3. a) State and prove Chebchev's inequality.
   b) Find the expected value of the function $g(X)=X^3$ where X is a random variable defined by the density
$$f_X(x)=\left(\frac{1}{2}\right)u(x)\exp(-x/2).$$

4. a) Define marginal density function? Find the marginal density functions of X and Y from.
$$f_{XY}=\frac{1}{12}u(x)u(y)e^{-x/3}e^{-y/4}$$
   b) Find the density function of $W=X+Y$, where the densities of X and Y are assumed to be: $f_X(x)=4u(x)e^{-4x}$; $f_Y(y)=5u(y)e^{-5y}$.
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5. a) Two statistically independent random variables X and Y have mean values $\overline{X} = E(X) = 2$ and $\overline{Y} = E(Y) = 4$. Thus have second moments $\overline{X^2} = E(X^2) = 8$ and $\overline{Y^2} = E(Y^2) = 25$. Find the mean values, the variance of the random variable $W = 3X - Y$.

   b) Two random variables Y_1 and Y_2 are defined by

$$Y_1 = X \cos\theta + Y \sin\theta$$
$$Y_2 = X \sin\theta + Y \cos\theta.$$

   Determine the covariance of Y_1 and Y_2.

6. A random process $X(t)$ has periodic sample functions as show in Figure 6; where B, T and $4t_0 \le T$ are constants but $\in$ is a random variable uniformly distributed on the interval (0, T). Find first order density function and distribution function of $X(t)$.

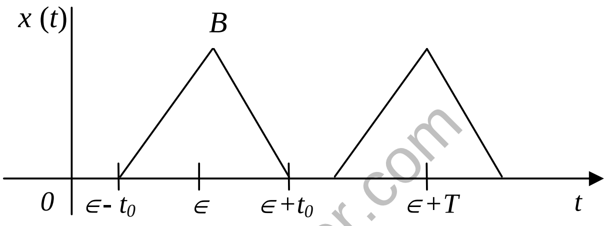

Figure 6

7. a) Derive the relationship between power spectrum density and autocorrelation function

   b) Explain properties of cross-power spectrum density.

8. a) Explain the generalized Nyquist theorem to model a resistive noise source.

   b) Derive a relationship between input and output power spectral densities of a linear time invariant system with the transfer function H (ω).