Code No: R21043 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, June - 2015
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1 a) Explain the term independent events and also write the properties of independent events. 5

b) A binary communication channel carries data as one of the two types of signals denoted by 0 and 1. Owing to noise a transmitted 0 is sometimes received as 1 and a transmitted 1 is sometimes received as a 0. For a given channel, assume a probability of 0.91 that a transmitted 0 is correctly received as a 0 and a probability of 0.94 that a transmitted 1 is received as a 1. Further assume a probability of 0.4 of transmitting a 0. If a signal is sent, Determine 10
(i) Probability that a 1 was transmitted, given that a 1 was received
(ii) Probability that a 0 was transmitted, given that a 0 was received

2 a) Write the expressions for cdf and pdf of uniform and exponential random variables and also sketch them. 5

b) Check whether the following function is a valid distribution function $G_X(x) = 4[u(x-2a) - u(x-3a)]$. Mention the properties used for justification. 4

c) Given k is a constant and X is a random variable with pdf 6
$$f_X(x) = \begin{cases} cx & o < x < 1 \\ 0 & else\ where \end{cases}$$
Find the value of c and P[1/2 < ¾]

3 a) Obtain the mean of Poisson random variable. 8

b) Define characteristic function of a random variable and write explain how moments can be generated using it. 7

4 a) Define joint characteristic function and write its properties. 5

b) Two random variables X and Y have a joint probability density function 10
$$f_{X,Y}(x,y) = \frac{5}{16}x^2 y, 0 < y < x < 2$$
Check whether X and Y are statistically independent or not with supporting expressions / values derived.
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5 a) Define covariance of random variables X and Y and explain correlation coefficient. 5

b) The density function of two random variables X and Y is $f_{XY}(x,y) = u(x)u(y)4e^{-2(x+y)}$. Find the mean value of the function $e^{-(X+Y)}$. 10

6 a) State and prove any three properties of cross correlation function. 8

b) If a random process, $X(t) = A\cos\omega t + B\sin\omega t$ is given, where A and B are uncorrelated zero mean random variables having the variance σ^2. Find autocorrelation function of X(t). 7

7 a) Determine whether the following functions can be a valid power density function or not? Support your claim with proper workout. 3

$\exp[-(\omega-1)^2]$

b) State and prove Wiener-Khintchine relations 12

8 a) An LTI system is excited with a random process X(t). Obtain the expression for the autocorrelation function of the response. 7

b) Write short notes on thermal noise. 8

Code No: R21043 **R10** SET - 2

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1 a) Write the axioms of probability. 5

b) Two boxes B1 and B2 contain 100 and 200 light bulbs respectively. B_1 and B_2 have 10 and 5 defective bulbs respectively. 10
i) Suppose a box is selected at random and one bulb is picked out. What is the probability that it is defective?
ii) Suppose we test the bulb and it is found to be defective. What is the probability, which it came from B_1?

2 a) A random variable X has the pdf 8

$$f_X(x) = \begin{cases} Cx(1-x) & ; \quad 0 \le x \le 1 \\ 0 & ; \quad else\ where \end{cases}$$

i) Find C ii) Find $P\left[\frac{1}{2} \le X \le \frac{3}{4}\right]$ iii) Find $F_X(x)$

b) Define a random variable. Classify random variables and explain. 7

3 a) Obtain the mean of Gaussian random variable. 8

b) A random variable X is uniformly distributed on the interval (-4, 12). Another random variable is defined as $Y = e^{-X/4}$. Find E(Y). 7

4 a) Write the properties of joint distribution function. 5

b) Obtain the expression for pdf of sum of two statistically independent random variables. 10

5 a) Prove that mean value of a weighted sum of random variables equals the weighted sum of mean values. 8

b) Prove that any correlated Gaussian random variables are statistically independent 7
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6 a) Check whether the random process $X(t) = A\cos(\omega_o t + \Theta)$ is WSS process or not, for 'A' and ω_o being constant and Θ uniformly distributed between $(0, \pi)$. 8

b) Explain the terms SSS process and Correlation ergodic process. 7

7 a) Determine whether the following functions can be a valid power density function or not? Support your claim with proper workout. 3

$$\frac{\omega^4}{j\omega^6 + \omega^2 + 1}$$

b) Derive the expression for cross power density spectrum. 12

8 a) Write short notes on thermal noise. 7

b) Derive the expression for noise figure of a cascaded two port network. 8

Code No: R21043 **R10** SET - 3

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Time: 3 hours Max. Marks: 75

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1 a) With reference to mathematical modeling of experiments explain how is an experiment defined mathematically. 5

b) An verification plan calls for verification of five chips and for either accepting each chip, rejecting each chip, or submitting it for reverification, with probabilities of p_1= 0.70, p_2= 0.20, p_3= 0.10 respectively. What is the probability that all five chips must be reverified? What is the probability that none of the chips must be reverified? 6

c) Explain the terms continuous and discrete sample spaces 4

2 a) Write the expressions for cdf and pdf of Rayleigh and Poisson random variables and also sketch them. 5

b) Given the function $g_X(x) = 4\cos\left(\frac{\pi x}{2b}\right) rect\left(\frac{x}{2b}\right)$, find the value of b so that $g_X(x)$ is a valid probability density. 4

c) Write the properties of CDF and explain. 6

3 a) Obtain the mean of Uniformly distributed random variable. 8

b) Find the characteristic function of the random variable with pdf 1/b($e^{-(X-a)/b}$). Also find the first moment from it. 7

4 a) State and prove central limit theorem for equal distributions case. 10

b) Write the properties of joint density function. 5

5 a) The joint density function of two random variables X and Y is 10

$$f_{XY}(x,y) = \begin{cases} \frac{(x+y)^2}{40} & ; \quad -1 < x < 1 \quad and \quad -3 < y < 3 \\ 0 & ; \quad elsewhere \end{cases}$$

Find the variances of X and Y.

b) Define joint characteristic function and write its properties. 5
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6 a) Given that the autocorrelation function for a stationary ergodic process with no periodic components is

$$R(T) = 25 + \frac{4}{(1+6T^2)}$$

Find the mean and variance of the process {X(t)}.

b) Explain the terms WSS process and Mean ergodic process.

7 a) Determine whether the following functions can be a valid power density function or not? Support your claim with proper workout. 3

$$\frac{\omega^2}{\omega^4 + 1} - \delta(\omega)$$

b) Derive the expression for power density spectrum of a random process. 12

8 a) Write short notes on thermal noise. 8

b) Write short notes on average Noise figure. 7

Code No: R21043 **R10** SET - 4

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1 a) Explain the terms sample space, outcome, event and mutually exclusive events with an example. 7
b) State and prove Baye's Theorem. 8

2 a) Define conditional density and write its properties. Also discuss different methods of defining the conditioning event obtain the respective density functions. 10
b) Write the expressions for cdf and pdf of Gaussian and binomial random variables and also sketch them. 5

3 a) Obtain the mean of Binomial random variable. 8
b) Define moment function of a random variable and write explain how moments can be generated using it. 7

4 a) The joint pdf of random variables X and Y is given b 7

$$f_{X,Y}(x,y)=\begin{cases}\frac{xy}{9} & 0<x<2 \ \ and \ \ 0<y<3 \\ 0 & , else\end{cases}$$

Check whether X and Y are correlated or not.
b) Joint density function of random variables X and Y are given by 8

$$f_{X,Y}(x,y)=\begin{cases}\left(\frac{x^2+y^2}{8\pi}\right) & , x^2+y^2<b \\ 0 & else\ where\end{cases}$$

Find constant b so that this is a valid joint density function. Also find $P\{0.5b<x^2+y^2\leq 0.8b\}$

5 a) Two random variables X and Y have joint characteristic function 7
$\Phi_{X,Y}(\omega_1,\omega_2)=\exp(-2\omega_1^2-8\omega_2^2)$
Show that X and Y are zero mean random variables and uncorrelated.
b) For the transformations $Y_1 = aX_1 + bX_2$, $Y_2 = cX_1 + dX_2$ where a, b, c, d are real constants and X_1 and X_2 are random variables. Derive the expression for joint density of Y_1 and Y_2 in terms of joint density of X_1 and X_2. 8
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6 a) Prove that random process $X(t)=A\ Cos(\omega_c t+\theta)$ is a wide sense stationary process if it is assumed that A, ω_c are constants and Θ is uniformly distributed over interval $0\leq\theta\leq2\pi$. 8

b) Classify random processes and explain. 7

7 a) Determine whether the following functions can be a valid power density function or not? Support your claim with proper workout. 3

$$\frac{\omega^2}{\omega^6+3\omega^2+3}$$

b) Derive the relationship between cross correlation function and cross power spectrum. 12

8 a) Write short notes on Narrowband random process. 8

b) Write short notes on Effective Noise Temperature. 7