Code No: R21043 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Dec - 2015
PROBABILITY THEORY AND STOCHASTIC PROCESSES
(Electronics and Communications Engineering)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1\. a) State and prove total probability theorem. (8M)

b) A bin contain the 100 resistors as detailed below (7M)

R/Tolerance	10 Ω	47 kΩ	1 kΩ	Total
5%	5	20	10	35
10%	10	15	20	45
20%	10	5	5	20
Total	25	40	35	100

If a resister is picked randomly with replacement, find i) p (20% resister) ii) p(10 Ω and 1k Ω) iii) p (4 kΩ and 10%) iv) p (5% and 1 kΩ)

2\. a) A sample space is defined by $S = \{1, 2 \le s \le 3, 4, 5\}$. A random variable is defined by X= 2 for $0 \le s \le 2.5$, $X = 3$ for $205 < s < 3.5$, and $X = 5$ for $3.5 \le s \le 6$ (7M)
i) Is X discrete, continuous, or mixed?
ii) give a set that defines the values X can have

b) A random variable X is Gaussian with a_X=0 and σ =1 (8M)
i) What is the probability that $|X| > 2$?
ii) What is the probability that X>2?

3\. a) Find mean and variance of exponential random variable. (7M)

b) A random variable X is uniformly distributed on the interval $(-\pi, \pi)$ X is transformed to the new random variable Y = T(x) = a Tan(X), where a > 0. Find the probability density function of Y. (8M)

4\. a) Random variables X and Y have the joint density (7M)

$$f_{X,Y}(x,y) = \frac{1}{12}u(x)u(y)e^{-(x/4)-(y/3)}$$

Find (i) $P\{2 < X \le ,4, -1 < Y \le 5\}$ (ii) $P\{0 < X < \infty, -\infty < Y \le -2\}$

b) Random variables X and Y have respective density functions (8M)

$$f_x(x) = \frac{1}{a}[u(x) - u(x-a)] \text{ and}$$

$$f_y(y) = bu(y)e^{-by}$$

where a>0 and b>0. Find and sketch the density function of W= X + Y if X and Y are statistically independent.
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5. a) Two Gaussian random variables X and Y have variance $\sigma_X^2 = 9$ and $\sigma_Y^2 = 4$ respectively, and correlation coefficient ρ. It is known that a coordinate rotation by an angle $-\pi/8$ results in new random variables Y1 and Y2 that are uncorrelated. Determine ρ. (8M)
   b) Two random variables having joint characteristic function $\varnothing_{XY}(\omega_1, \omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$. Find moment's m_{10}, mo_1, m_{11}. (7M)

6. a) Given the random process by (8M)
   $X(t) = A\cos(w_0 t) + B\sin(w_0 t)$
   where w_0 is a constant, and A and B are uncorrelated zero-mean random variables having different density functions but the same variance. Show that X(t) is wide sense stationary but not strictly stationary.
   b) Let X (*t*) be a stationary continuous random process that is differentiable. Denote its time derivative by $\dot{X}(t)$. Show that $E\left[\dot{X}(t)\right] = 0$. (7M)

7. a) If X(t) is a stationary process, find the power spectrum of $Y(t) = A_0 + B_0 X(t)$ in terms of the power spectrum of X(t) if A_0 and B_0 are real constants. (7M)
   b) A random process is given by $X(t) = A\cos(\Omega t + \theta)$ where A is a real constant, Ω is a random variable with density function $f_\Omega(\Omega)$ and θ is a random variable uniformly distributed over the interval (0, 2π) independent of Ω. Show that the power spectrum of $X(t)$ is $S_{XX}(\omega) = \frac{\pi A^2}{2}[f_\Omega(\omega) + f_\Omega(-\omega)]$ and also find P_{YY}. (8M)

8. A system's power transfer function is (15M)

$$|H(\omega)|^2 = \frac{16}{256 + \omega^4}$$

   a) What is its noise bandwidth?
   b) If white noise with power density nW/Hz is applied to the input, find the noise power in the system's output.