Code No: R21053 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Jan - 2015
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE AND ENGINEERING
(Com. to CSE, IT, ECC)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Prove the implication: $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$.
   b) Prove that $(\exists x)\ (P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

2. a) What is the least common multiple of $2^3 3^5 7^2$ and $2^4 3^3$?
   b) Define relatively prime and pairwise relatively prime? Determine whether 17 and 22 are relatively prime and whether 10, 17 and 21 are pairwise relatively prime?
   c) Explain Division theorem?

3. a) Let A = {a,b,c,d,e} and Let P = {{a,b}, {c}, {d,e}}. Show that the partition P defines an equivalence relation on A.
   b) Let A,B,C be arbitrary sets. Show that
   i) $(A - B) - C = (A - C) - (B - C)$. ii) $(A - B) - C = A - (B \cup C)$

4. a) Which of the simple graphs shown in fig 1 have a Hamiltonian Circuit or, if not, a Hamiltonian path

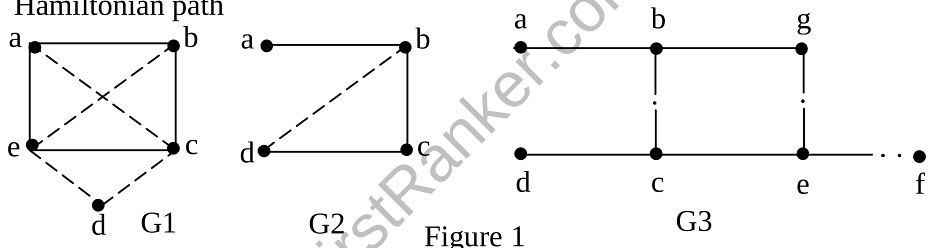

Figure 1

   b) If a connected r-regular graph is Eulerian, What can you say about r?

5. a) Give the adjacency matrix of the digraph G = ({a,b,c,d},R), where R ={(a,b),(b,c),(d,c),(d,a)}.
   b) Determine if bipartite graph $K_{2,2}$ is planar or not? Determine the number of edges in a complete bipartite graph $K_{m,n}$.

6. a) Let L be a Lattice. Then prove that the relation $a \le b$ defined by either $a \wedge b = a$ (or) $a \vee b = b$ is a partial ordering relation on L.
   b) Define a group, giving atleast two examples. If G is a group, then show that the identity element of G is unique and every $a \in G$ has a unique inverse in G.

7. a) If x>2, y>0, z>0 then find the number of solutions of x+y+z+w=21
   b) Using the binomial theorem to prove that $3^n = \sum_{r=0}^{n} c(n,r)2^r$.

8. a) Solve the recurrence relation $y_{n+3} - 6y_{n+2} + 11y_{n+1} - 6y_n = 0$ with $y_0=2$, $y_1=0$ and $y_2=-2$.
   b) Using generating function solve $y_{n+2} - 2y_{n+1} + y_n = 2^n$, $n \ge 0$ with $y_0=2$ and $y_1=1$.
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Code No: R21053 **R10** SET - 2

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1. a) Show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$

   b) Using normal forms, show that the formula $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology.

2. a) Use mathematical induction to prove that $1+2+2^2+...+2^n = 2^{n+1}-1$ for all nonnegative integers n.

   b) State Fermat's Theorem and Euler's Theorem and explain with examples?

3. a) Let $f:R \to R$ & $g: R \to R$. Where R is a set of real numbers find fog and gof, where $f(x) = x^2-2$, $g(x) = x+4$. State whether these functions are injective, surjective or bijective.

   b) Draw the Hasse diagram of $< x, \leq >$ where $x = \{2,3,6,12,24,36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y.

4. a) Find an Eulerian cycle in the graph (Figure 2).

   b) Determine whether the directed graphs shown in Figure 3 are isomorphic?

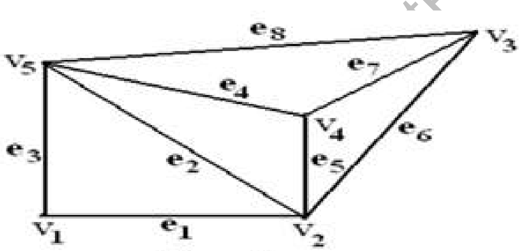

Figure 2

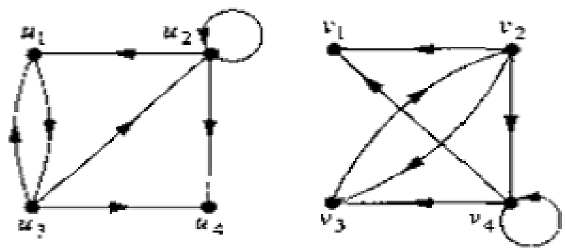

Figure 3.
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5. a) Use depth-first search and breadth-first search to find the spanning tree for the graph shown in Figure 4.

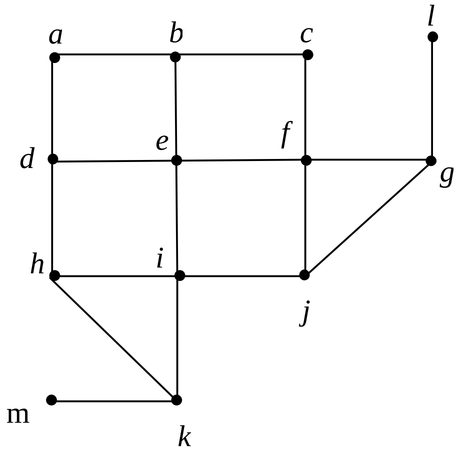

Figure 4

b) Explain Binary search trees? Construct a binary search tree for the words mathematics, physics, geography, zoology, meteorology, geology, psychology and chemistry (using alphabetical order).

6. If (G, *) and (H, Δ) are two groups and f: G→H is homorphism, then prove that the kernel of `f' is a normal subgroup.

7. a) State Pigeonhole principle. Give any one application?
b) How many numbers greater than 1000 but not greater than 4000 can be formed with the digits 0, 1, 2, 3, 4?
c) Determine the coefficient of $x^5y^{10}z^5w^5$ in $(x-7y+3z-w)^{25}$

8. a) Solve the recurrence relation using generating function $a_n - 6a_{n-1} = 0$ for $n \geq 1$ and $a_0 = 1$
b) Write a generating function of a_r, where a_r is the number of integers between 0 and 999 whose sum of digits is r

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1. a) Show that from
   i) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$
   ii) $(\exists y)\ (M(y) \wedge \neg W(y))$
   the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows.

   b) Show that the proposition: $\sim P \rightarrow (P \rightarrow Q)$ is a universally valid formula.

2. a) Using Euclidean algorithm determine GCD (1970,1066)
   b) How many integers between 10^5 and 10^6 have no digits other than 2, 5 or 8?

3. a) What is a partial order relation? Let S = { x,y,z} and consider the power set P(S) with relation R given by set inclusion. Is R a partial order?
   b) Let A = {0,1,2,3,4}. Show that the relation R = {(0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4,)} is an equivalence relation. Find the distinct equivalence classes of R.

4. a) Determine whether the directed graph shown in Figure 2 has an Euler circuit. Construct an Euler circuit if it exists.
   b) Find all the cut edges in the graph (Figure 3).

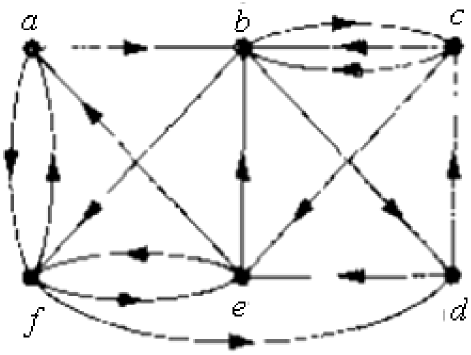

Figure 2

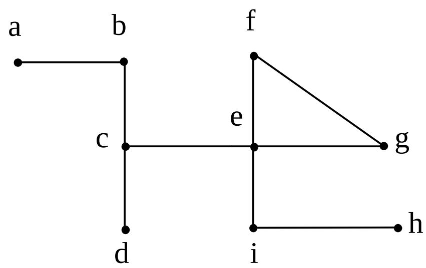

Figure 3

   c) Define a walk, distance between two vertices eccentricity, radius of a graph.
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5. a) Give the Kruskal's algorithm and apply it to the following graph in Figure 4.

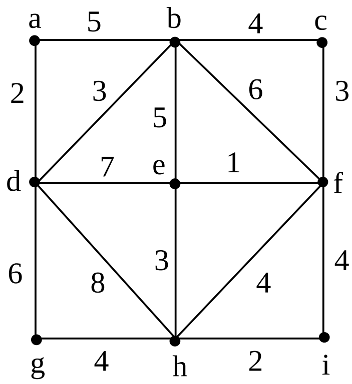

Figure 4

b) Find the minimum number of edges that must be removed from the complete graph K6, so that the resulting graph is planar.

6. a) Discuss various properties of binary operations * and + on a non empty set 'S' with a suitable example.
b) Consider the group G = {1,2,4,7,8,11,13,14} under multiplication modulo 15. Construct the multiplication table of G and verify whether G is cycle or not.

7. a) How many different 8-digit numbers can be formed by arranging the digits 1, 1, 1, 1, 1, 2, 3, 3, 3?
b) Find the coefficient of $x^3y^2z^3$ in the expansion of $(x+y+z)^8$.

8. Find the solution for the Fibonacci sequence F1, F2, ... satisfying the recurrence relation $F_k = F_{k-1}+F_{k-2}$, for all integers $K \geq 2$ with initial conditions $F_0 = F_1 = 1$.

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1. Solve the recurrence relation $a_n - 9a_{n-1} + 26a_{n-2} - 24a_{n-3} = 0$ for $n \geq 3$, $a_0 = 0$.

2. a) Use Pascal's identity to prove that $C(n,1)+2C(n,2)+\ldots+nC(n,n) = n2^{n-1}$
   b) Find the number of integers between 1 and 250 both inclusive that are
   i) Divisible by 2, 3, 5 ii) Not divisible by 2,3,5

3. a) IF <S1, *> and <S2, *> are monoids having e1 and e2 as the respective identity elements. Prove that the direct product S1 x S2 is a monoid with (e1, e2) as the identity element.
   b) Prove that H = {0,2,4} forms a sub group of $<Z_6, +_6>$.

4. a) Write Prim's algorithm? Find the minimum spanning tree for the following graph using prim's algorithm (Figure 1).

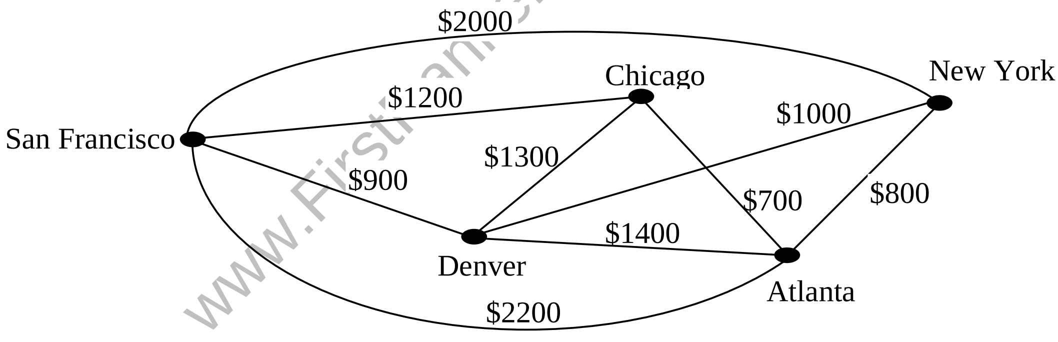

Figure 1

   b) Find the chromatic numbers of the graphs G and H shown in Figure 2?

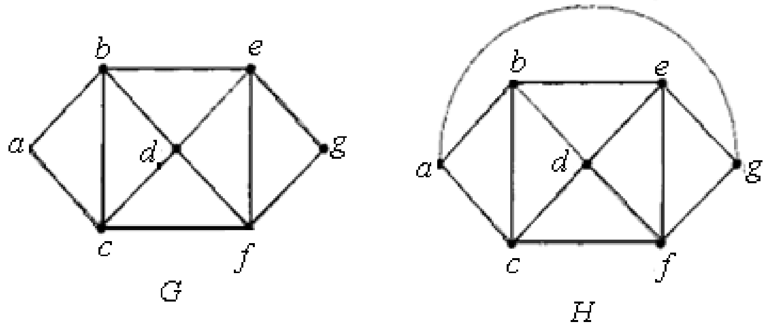

Figure 2
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5. a) Determine whether the directed graphs in Figure 3 are isomorphic?

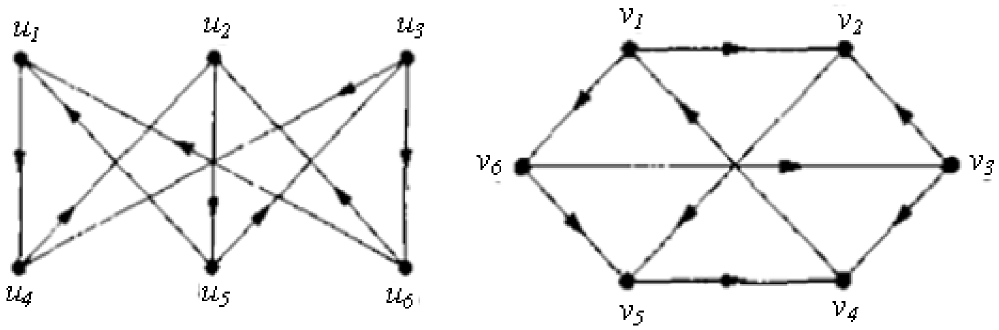

Figure 3

b) Which graphs shown in Figure 4 has an Euler path?

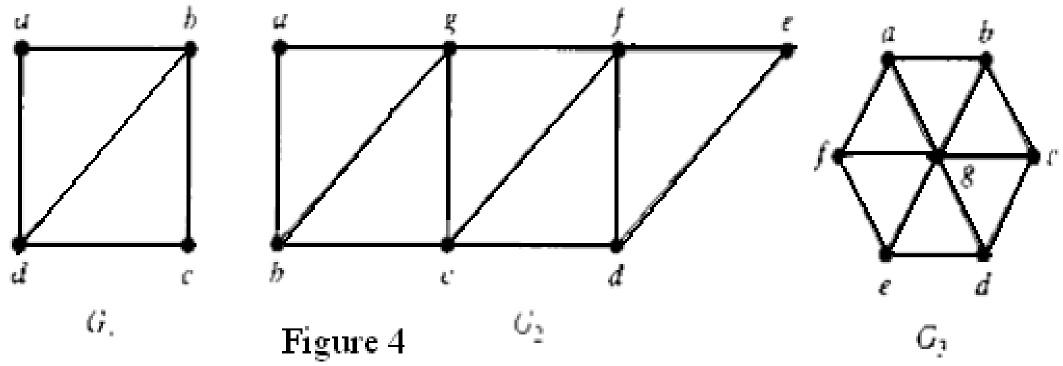

Figure 4

6. a) A function f(ZxZ) → Z is defined by f(x,y) = 4x + 5y. Prove that f is not one-to-one, but onto
b) If A,B,C are three sets such that A ⊆ B. Show that (A x C) ⊆ (B x C)
c) If A = {1,2,3}, B = {4,5}. Find: i) AxB ii) BxA

7. a) Find the number of positive integers less than are equal to 2076 and divisible by 3 or 4.
b) Use mathematical induction to prove that the sum of the first n odd positive integers is n^2.

8. a) Prove that $(\exists x)P(x) \wedge Q(x) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$. Does the converse hold?
b) Obtain the principal conjunctive normal form of the formula $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$.