Code No: RT21024 **R13** SET - 1

II B. Tech I Semester Supplementary Examinations, June - 2015
COMPLEX VARIABLES AND STATISTICAL METHODS
(Electrical and Electronics Engineering)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B)**
2. Answer **ALL** the question in **Part-A**
3. Answer any **THREE** Questions from **Part-B**

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PART-A

1. a) Find orthogonal trajectories of the family of curves $x^4-6x^2y^2+y^4=c_1$

b) The evaluate $\int_{(0,0)}^{(1,1)}\left(3x^2+5y+i\left(x^2-y^2\right)\right)dz$ along $y^2=x$

c) Determine and classify the singular points of $ze^{1/z}$

d) Find invariant points of the transformation $\omega=\dfrac{1+z}{1-z}$

e) A population consists of five numbers 2,3,6,8 and 11. Consider all possible samples of size two which can be drawn with replacement from the population. Find (i) The mean of the population (ii) standard deviation of the population (iii) Mean of sampling distribution of means (iv) The standard deviation of the sampling distribution of means.

f) A coin was tossed 400 times and returned heads 216 times. Test the hypothesis that the coin is unbiased. Use a 0.05 level of significance. (4M+4M+3M+3M+4M+4M)

PART-B

2. a) Prove that if $f(z)=u(x,y)+iv(x,y)$ is differentiable at z then at this point the first order partial derivatives of u and v exists and satisfy Cauchy-Reimann equations

b) Find conjugate harmonic of $u=e^{x^2-y^2}\cos 2xy$. Hence find f (z) in terms of z.
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3. a) Represent the function $f(z)=\dfrac{4z+3}{z(z-3)(z+2)}$ in Laurent's series (i) within $|z|=1$

   (ii) in the annular region between $|z|=2$ and $|z|=3$ (iii) exterior to $|z|=3$

   b) Evaluate $\int_C \dfrac{e^{2z}}{(z-1)(z-2)}\,dz$ where C is the circle $|z|=3$.

4. a) Evaluate $\int_C \left[\dfrac{ze^z}{z^2+9}\right]dz$ where C: $|z|=5$ by residue theorem

   b) Evaluate $\int_{-\infty}^{\infty} \dfrac{x^2}{(x^2+1)(x^2+4)}\,dx$ using Residue theorem

5. a) Find the bilinear transformation which maps the points $(1,i,-1)$ into the points $(0,1,\infty)$

   b) Prove that the transformation $\omega=\sin z$ maps the families of lines $x=a$ and $x=b$ into two families of confocal central conics.

6. a) Ten bearings made by a certain process have a mean diameter of 0.5060cm with a standard deviation of 0.004cm. Assuming that the data may be taken as a random sample from a normal distribution, construct a 95% confidence interval for the actual average diameter of the bearings.

   b) A random sample of size 100 is taken from an infinite population having the mean $\mu=76$ and the variance $\sigma^2=256$. What is the probability that $\bar{x}$ will be between 75 and 78.

7. a) A machine is designed to produce insulating washers for electrical devices of average thickness of 0.025cm. A random sample of 10 washers was found to have a thickness of 0.024cm with standard deviation of 0.002cm. Test the significance of the deviation. Value of t for 9 degrees of freedom at 5% level is 2.262.

   b) Four coins were tossed 160 times and the following results were obtained.

No. of Heads	0	1	2	3	4
Observed frequencies	17	52	54	31	6

   Under the assumption that coins are balanced, find the expected frequencies of 0,1,2,3 or 4 heads, and test the goodness of fit $(\alpha=0.05)$

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2. Answer **ALL** the question in **Part-A**
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PART-A

1. a) Find orthogonal trajectories of the family of curves $e^{-x}(x\sin y - y\cos y) = c_1$

b) Evaluate $\int_0^{3+i} z^2 dz$ along the line $y = \frac{x}{3}$

c) Determine and classify the singular points of $\frac{e^{1/z}}{z-1}$

d) Find fixed points of the transformation $\omega = \frac{3z-4}{z-1}$

e) Samples of size two are taken from the population 1, 2, 3, 4, 5, 6 with replacement. Find (i) The mean of the population (ii) standard deviation of the population (iii) Mean of sampling distribution of means (iv) The standard deviation of the sampling distribution of means.

f) A die is tossed 256 times and it turns up with an even digit 150 times. Is the die biased at 0.01 level of significance? (4M+3M+4M+3M+4M+4M)

PART-B

2. a) If $f(z) = u(r,\theta) + iv(r,\theta)$ is differentiable at $z = re^{i\theta} \neq 0$ then

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

b) Find k such that $f(x, y) = x^3 + 3k\,xy^2$ is harmonic and find its conjugate

3. a) Find the Taylor's or Laurent's series which represents the function $\frac{z^2-1}{(z+3)(z+2)}$ (i) when $|z| < 2$ (ii) (i) when $2 < |z| < 3$ (iii) when $|z| > 3$

b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ around C: $|z-1| = 3$.
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4. a) Evaluate $\int_C \left[\frac{2e^z}{z(z-3)}\right] dz$ where C: $|z| = 2$ by residue theorem

   b) Show that $\int_0^{\pi} \frac{d\theta}{3+2\cos\theta} = \frac{\pi}{\sqrt{5}}$ by contour integration.

5. a) Find the image of the rectangle $R: -\pi < x < \pi, \frac{1}{2} < y < 1$ under the transformation $\omega = \sin z$

   b) Find the bilinear transformation which maps the points $(\infty, i, 0)$ in the z-plane into $(-1, -i, 1)$ in w-plane

6. a) A random sample of size 10 was taken from population. Standard deviation of sample is 0.3. Find maximum error with 99% confidence.

   b) If the mean of breaking strength of copper wire is 575lbs, with a standard deviation of 8.3lbs. How large a sample must be used in order that there will be one chance in 100 that the mean breaking strength of the sample is less than 572lbs.

7. a) The mean life time of a sample of 25 fluorescent light bulbs produced by a company is computed to be 157 hours with a standard deviation of 120 hours. The company claims that the average life of the bulbs produced by a company is 1600 hours using a level of significance of 0.005. Is the claim acceptable?

   b) A pair of dice are thrown 360 times and the frequency of each sum is indicated below

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	8	24	35	37	44	65	51	42	26	14	14

   Would you say that the dice are fair on the basis of χ^2 test at 0.05 level of significance?

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PART-A

1. a) Find orthogonal trajectories of the family of curves $x^3 - 3xy^2 = c_1$

b) Evaluate $\int_0^{1+i} z^2\, dz$ along the line $y = x^2$

c) Determine and classify the singular points of $\dfrac{z^2}{1+z}$

d) Find fixed points of the transformation $\omega = \dfrac{3iz+1}{z+i}$

e) Find the mean and standard deviation of sampling distribution of variances for the population 2,3,4,5 by drawing samples of size two with replacement

f) A die is tossed 960 times and it falls with 5 upwards 184 times. Is the die unbiased at 0.01 level of significance? (4M+3M+3M+4M+4M+4M)

PART-B

2. a) Prove that $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} = 4\dfrac{\partial^2}{\partial z\, \partial \bar{z}}$

b) Show that $u(x, y) = x^3 - 3\, xy^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function f(z) in terms of z.

3. a) Expand in the series the function $f(z) = \dfrac{1}{z^2 - 3z + 2}$ in the regions

(i) when $|z| < 1$ (ii) when $1 < |z| < 2$ (iii) when $|z| > 2$

b) Evaluate $\int_C \left[\dfrac{e^z}{z^3} + \dfrac{z^4}{(z+i)^2}\right] dz$ where C: $|z| = 2$ using Cauchy integral formula.

4. a) Evaluate $\int_C \left[\dfrac{(2z+1)^2}{4z^3 + z}\right] dz$ where C is the circle $|z| = 1$ using residue theorem

b) Prove that $\int_{-\infty}^{\infty} \dfrac{x^2 - x + 2}{(x^4 + 10x^2 + 9)}\, dx = \dfrac{5\pi}{12}$
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5. a) Plot the image $1 < |z| < 2$ under the transformation $\omega = 2iz + 1$

   b) Find the bilinear transformation which maps the points $(1, i, -1)$ into the points $(2, i, -2)$

6. a) A sample of 11 rats from a central population had an average blood viscosity of 3.92 with standard deviation of 0.61. Estimate the 95% confidence limits for the mean blood viscosity of the population.

   b) A normal population has a mean of 0.1 and standard deviation of 2.1. Find probability that mean of a sample size 900 will be negative.

7. a) A random sample from a company's very extensive files shows that the orders of the certain kind of machinery were filled respectively in 10,12,19,14,15,18,11 and 13 days. Use the level of significance $\alpha = 0.01$ to test the claim that on the average such orders are filled in 10.5 days. Choose an alternative hypothesis so that rejection on null hypothesis $\mu = 10.5$ days implies that it takes longer than indicated.

   b) The following figures show the distribution of digits in numbers chosen at random from a telephone directory.

Digits	0	1	2	3	4	5	6	7	8	9
Frequency	1026	1107	997	966	1075	933	1107	972	964	853

   Test whether the digits may be taken to occur equally frequently in the directory.

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PART-A

1. a) Show that $\underset{x \to 0}{Lt} \dfrac{x^2 y}{x^4 + 2y^2}$ does not exist even though this function approaches the same limit along every straight line through the origin.

b) State Cauchy integral theorem and evaluate $\int_C \dfrac{e^{2z}}{z-2} dz$ where C is $|z| = 1$.

c) Determine and classify the singular points of $\sin\left(\dfrac{1}{1-z}\right)$

d) Find invariant points of the transformation $\omega = \dfrac{z-3}{z+1}$

e) Samples of size two are taken from the population 3,6,9, 15, 27 with replacement. Find (i) The mean of the population (ii) standard deviation of the population (iii) Mean of sampling distribution of means (iv) The standard deviation of the sampling distribution of means.

f) A coin was tossed 960 times and returned heads 183 times. Test the hypothesis that the coin is unbiased. Use a 0.05 level of significance. (3M+3M+4M+4M+4M+4M)

PART-B

2. a) Show that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) \log|f'(z)| = 0$ where $f(z)$ is an analytic function.

b) Find the regular function whose imaginary part is $\log(x^2 + y^2) + x - 2y$

3. a) Find the Taylor's or Laurent's series which represents the function $f(z) = \dfrac{1}{(z^2+1)(z+2)}$ in the regions (i) when $|z| < 1$ (ii) when $1 < |z| < 2$ (iii) when 2 $|z| > 2$

b) Evaluate $\int_C \left[\dfrac{z^3 - \sin 3z}{\left(z - \dfrac{\pi}{2}\right)^3}\right]$ where C: $|z| = 2$ using Cauchy integral formula.

4. a) Evaluate $\int_C \left[\dfrac{z}{(z-1)(z-2)^2}\right] dz$ where C: $|z-2| = \dfrac{1}{2}$ by residue theorem

b) Show that $\int_0^{\pi} \dfrac{d\theta}{(a + b\cos\theta)^2} = \dfrac{\pi a}{(a^2 - b^2)^{s/2}}, a > b > 0$
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5. a) Show that the function $\omega = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane into a circle in the w - plane.

   b) Find the bilinear transformation which maps the points $(-i, 0, i)$ into the points $(-1, i, 1)$ respectively.

6. a) A random sample of size 100 taken from population with Standard deviation $(-i, 0, i)$ $\sigma = 5.1$. Given that the sample mean is $\bar{x} = 21.6$ construct a 95% confidence interval for the population mean.

   b) The mean height of the students in a college is 155 cms and standard deviation is 15. What is the probability that mean height of 36 students is less than 157cms.

7. a) The manufacturer of a certain make of electrical bulbs claim that his bulbs have a mean life time of a25 months with a standard deviation of 5 months. A random sample of 6 such bulbs gave the following values. Life months: 24, 26, 30, 20, 20, 18. Can you regard the producers, claim to be valid at 1% level of significance?

   b) A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class and 20 got a first class. Do these figures commensurate with general examination result which is in the ratio of 4:3:2:1 for the various categories respectively.