III B.Tech I Semester Supplementary Examinations, June - 2015 COMPLEX VARIABLEDS AND STATISTICAL METHODS
(Electrical and Electronics Engineering)
Time: $\mathbf{3}$ hours
Max. Marks: 75

## Answer any FIVE Questions

## All Questions carry equal marks

*****
1
a) Show that the function $f(z)=\left\{\begin{array}{cc}z^{5} & z \neq 0 \\ |z|^{4} & z=0\end{array}\right.$ satisfies C-R equations at $\mathrm{z}=0$, but $f(z)$ is not differentiable at $\mathrm{z}=0$.
b) Find the regular function whose imaginary part is $\frac{x-y}{x^{2}+y^{2}}$.
a) Using Cauchy integral formula, evaluate $\oint_{c} \frac{e^{z} d z}{(z+2)(z+1)^{2}}$ where $c:|z|=3$.
b) Obtain Laurent series for $f(z)=\frac{1}{(z+2)(1+z)^{2}}$ in the regions
i). $|1+z|>1$
ii). $\quad 1<|z|<2$.

3
a) Evaluate $\oint_{c} \frac{(2 z+1)^{2} d z}{4 z^{3}+z}$ where $c:|z|=1$ asing Residue theorem.
b) Evaluate $\int_{0}^{2 \pi} \frac{1+4 \cos \theta}{17+8 \cos \theta} d \theta$.

4 a) Find the image of the triangular region in the Z-plane bounded by the lines $x=0, y=0$ and $\mathrm{x}+\mathrm{y}=1$ under the transformation $\mathrm{w}=2 \mathrm{z}$.
b) Find the bilinear transformation which maps the points ( $2, i,-2$ ) into the points $(1, i,-1)$.

5 a) The mean and variance of a binomial variate X with parameters $n$ and $p$ are 16 and 8 respectively. Find $P(X \geq 1)$ and $P(2<X \leq 4)$.
b) If the masses of 300 students are normally distributed with mean 68 kg and standard deviation 3 kg , how many students have masses
(i) greater than 72 kg
(ii) between 65 and 71 kg .

6 a) It is desired to estimate the mean number of hours of continuous usage until a certain computer requires a repair. If it can be assumed that $\sigma=48$ hours, then
(i). how large a sample be needed to assert with $90 \%$ confidence that the sample mean is off by at most 10 hours.
(ii). Find $99 \%$ confidence interval for the actual average if sample mean is assumed as 500 hours.
b) Find the mean and standard deviation of sampling distribution of sample means by considering samples of size 2 (without replacement) from the numbers $16,14,12,824,20$.

7 a) An ambulance service claims that it takes on average less than 10 minutes to reach its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and variance 16. Test the significance at 0.05 level.
b) Experience had shown that $20 \%$ of manufactured product is of top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.

8 Three different machines are used for production. On the basis of the outputs, set up one-way ANOVA table and hence, test whether the machines are equally effective.

| OUTPUTS |  |  |
| :--- | :--- | :--- |
| Machine I | Machine II | Machine III |
| 10 | 9 | 20 |
| 5 | 7 | 16 |
| 11 | 5 | 10 |
| 10 | 6 | 14 |

## III B.Tech I Semester Supplementary Examinations, June - 2015 COMPLEX VARIABLEDS AND STATISTICAL METHODS

(Electrical and Electronics Engineering)
Time: $\mathbf{3}$ hours

## Answer any FIVE Questions

## All Questions carry equal marks

1
a) Show that the function $f(z)=\left\{\begin{array}{ll}\frac{x y^{2}(x+i y)}{x^{2}+y^{4}}, & z \neq 0 \\ 0 & z=0\end{array}\right.$ is not analytic at $\mathrm{z}=0$ although C R equations are satisfied at $\mathrm{z}=0$.
b) Find $f(z)=u+i v$ given that $u+v=\frac{\sin 2 x}{\cosh 2 y-\cos 2 x}$.

2
a) Using Cauchy integral formula, evaluate $\oint_{c} \frac{(z+1) d z}{z^{2}+2 z+4}$ where $c:|z+1+i|=2$.
b) Obtain series for $f(z)=\frac{1}{\left(z^{2}+2\right)\left(1+z^{2}\right)}$ in positive and negative powers of z in the region $1<|z|<\sqrt{2}$.

3 a) Evaluate $\oint_{c} \frac{d z}{\left(z^{2}+1\right)\left(z^{2}-4\right)}$ where $c:|z|=1.5$ using Residue theorem.
b)

Evaluate $\int_{0}^{\infty} \frac{d x}{x^{6}+1}$.
4 a) Find the image of the domain which is to the left of the line $x=-3$ in the $Z$-plane under the transformation $w=z^{2}$.
b) Find the bilinear transformation which maps the points ( $0,-i,-1$ ) into the points ( $i, 1,0$ ).
a) A bag P contains 2 white and 3 red balls and bag $Q$ contains 4 white and 5 red balls.
One ball is drawn at random from one of the bags and is found to be red. Find the probability that it is drawn from bag Q .
b) Suppose $2 \%$ of people are left handed on average. Find the probability of finding
(i). 3 or more left handed (ii). at most one left handed.

Code No: R31021

## R10

6 a) The efficiency expert of a computer company tested 40 engineers to estimate the average time it takes to assemble a certain computer component, getting a mean of 12.73 minutes and standard deviation of 2.06 minutes. If $\bar{x}=12.73$ is used as a point estimate of the actual average, find
(i). maximum error with $99 \%$ confidence
(ii). $95 \%$ confidence interval for the true average.
b) Find the mean and standard deviation of sampling distribution of sample means by considering samples of size 2 (without replacement) from the numbers $2,5,7,9,14,15$.

7 a) It is claimed that a random sample of 49 tyres has a mean life 15200 km . This sample was drawn from a population whose mean is 15150 km and standard deviation of 1200 km . Test the significance at 0.05 level.
b) The average marks scored by 32 boys is 72 with a standard deviation of 8 , while the average of 36 girls, the mean and standard deviation are 70 and 6 respectively. Does this indicate that the boys perform better than girls at 0.05 level of significance.

8 The following data gives the yields on 12 plots of land in three samples, each of 4 plots, under three varieties of fertilizers $\mathrm{A}, \mathrm{B}$ and C .

| A | B | C |
| :--- | :--- | :--- |
| 25 | 20 | 24 |
| 22 | 17 | 26 |
| 24 | 16 | 30 |
| 21 | 19 | 20 |

Is there any significant difference in the average yields of land under the three varieties of fertilizers?

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## Answer any FIVE Questions <br> All Questions carry equal marks <br> *****

1
a) Prove that the function $f(z)=\left\{\begin{array}{cc}\frac{x^{3}(1+i)-y^{3}(1-i)}{x^{2}+y^{2}}, & z \neq 0 \\ 0 \quad z= & 0\end{array}\right.$ is continuous and satisfies

Cauchy Riemann equations at the origin, yet $f^{\prime}(0)$ does not exist.
b) Find the analytic function $f(z)=u(r, \theta)+i v(r, \theta)$ where
$v(r, \theta)=\left(\mathrm{r}-\frac{1}{\mathrm{r}}\right) \sin \theta, \quad r \neq 0$.
2 a) Using Cauchy integral formula, evaluate $\oint_{c} \frac{e^{z} d z}{\left(z^{2}+\pi^{2}\right)^{2}}$ where $c:|z|=4$.
b) Expand $f(z)=\frac{1}{z^{2}-3 z+2}$ as Laurent's series inthe regions
(i). $0<|z-1|<1$
(ii). $1<|z|<2$.

3 a) Evaluate $\oint_{C} \frac{d z}{\sinh z}$ where $C:|z|=4$.
b)

Evaluate $\int_{0}^{\infty} \frac{\cos m x}{\left(1+x^{2}\right)^{2}} d x$ using Residue theorem.
4 a) Find the image of the region between the lines $\mathrm{y}=0$ and $\mathrm{y}=\pi / 2$ in the z -plane under the transformation $w=e^{z}$.
b) Find the bilinear transformation which maps the points $(0,-\mathrm{i},-1)$ into the points $(\mathrm{i}, 1,0)$.

5 a) Fit a Poisson's distribution to the following frequency distribution

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| f | 13 | 25 | 52 | 58 | 32 | 16 | 4 |

b) The marks obtained in Mathematics by 1000 students are normally distributed with mean $78 \%$ and standard deviation $11 \%$. Find
(i) how many students got marks above $90 \%$
(ii) how many students got marks between $60 \%$ and $70 \%$.

6 a) Find the mean and standard deviation of sampling distribution of sample means by considering samples of size 2 (without replacement) from the numbers $2,5,7,9,14,15$.
b) The pulse rate of 50 yoga practitioners decreased on average by 20.2 beats/ minute with a standard deviation of 3.5 .
(i) If sample mean of 20.2 is used as point estimate of true average, what can we assert with $95 \%$ confidence about the maximum error?
(ii) Construct $99 \%$ confidence interval for the true average.

7 a) In a 64 randomly selected hours of production, the mean and standard deviation of the number of accepted items produced by automatic machine are 1.038 and 0.146 respectively. At 0.05 level of significance, does this information enable us to reject the claim that population mean is greater than 1.0 ?
b) The mean yield of wheat from a district A was 210 pounds with standard deviation 10 per acre from a sample of 100 plots. From district B, the corresponding mean and standard deviation were 220 and 12 pounds per acre from a sample of 150 . Assuming that standard deviation of entire state was 11 pounds, test whether there is any significant difference between the mean yield in the two districts.

8 The following data gives the yields on 12 plots of land in three samples, each of 4 plots, under three varieties of fertilizers $\mathrm{A}, \mathrm{B}$ and C .

| $A$ | $B$ | $C$ |
| :--- | :--- | :--- |
| 22 | 21 | 25 |
| 21 | 15 | 18 |
| 22 | 18 | 26 |
| 22 | 20 | 21 |

Is there any significant difference in the average yields of land under the three varieties of fertilizers?

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Time: $\mathbf{3}$ hours
Max. Marks: 75
Answer any FIVE Questions
All Questions carry equal marks
*****
1
a) Show that $f(z)=\left\{\begin{array}{c}\frac{x y^{2}(x+i y)}{x^{2}+y^{4}}, z \neq 0 \\ 0, \quad z=0\end{array}\right.$ is not analytic at $\mathrm{z}=0$ although Cauchy Riemann equations are satisfied at the origin.
b) Find the regular function whose imaginary part is $\log \left(x^{2}+y^{2}\right)+x-2 y$.
a) Using Cauchy integral formula, evaluate $\oint_{c} \frac{\cosh \pi z d z}{\left(z^{2}+1\right) z}$ where $c:|z|=2$.
b) Expand $f(z)=\frac{z+3}{z\left(z^{2}-z-2\right)}$ as Laurent's series in the regions
(i). $\quad 1<|z|<2$
(ii). $|z|>2$.

3 a)
Evaluate $\oint_{C} \frac{z-3}{z^{2}+2 z+5} d z$ where $C:|z+1+i|=2$.
b) Evaluate $\int_{0}^{\infty} \frac{x \sin m x}{x^{4}+16}$ using Residue theorem.

4 a) Find the image of the infinite strip $1 / 8<y<1 / 4$ under the transformation $\mathrm{w}=1 / \mathrm{z}$.
b) Find the bilinear transformation which maps the points ( $1+\mathrm{i}, \mathrm{i}, 2-\mathrm{i}$ ) into the points $(0,1, i)$.

5 a) Fit a binomial distribution to the following frequency distribution.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| f | 13 | 25 | 52 | 58 | 32 | 16 | 4 |

b) In a normal distribution $31 \%$ of the items are under 45 and $8 \%$ of items are under 64 .
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6 a) Find the mean and standard deviation of sampling distribution of sample means by considering samples of size 2 (without replacement) from the numbers $12,1418,22$, 14, 25.
b) The mean and standard deviation of a population are 11,795 and 14,054 respectively. What can we assert with $95 \%$ confidence about the maximum error if sample mean is 11,795 and size is 50 . Also construct $95 \%$ confidence interval for the mean.

7 a) A manufacturer claimed that at least $98 \%$ of the steel pipes which he supplied to a factory are up to the specification. An examination of a sample of 500 pipes revealed that 30 were defective. Test his claim at a significant level $5 \%$.
b) The mean life of a sample of 10 electric bulbs of Brand A was found to be 1456 hours with a standard deviation of 423 hours. A second sample of 17 from Brand B has a mean of 1280 hours and standard deviation of 398 hours. Is there a significant difference between the means of the two Brands.

8 The following data gives the yields on 12 plots of land in three samples, each of 4 plots, under three varieties of fertilizers $\mathrm{A}, \mathrm{B}$ and C .

| A | B | C |
| :--- | :--- | :--- |
| 20 | 21 | 4 |
| 22 | 18 | 18 |
| 19 | 18 | 17 |
| 22 | 21 | 28 |

Is there any significant difference in the average yields of land under the three varieties of fertilizers?

