

Code No: R10102

R10
SET - 1
I B. Tech I Semester Supplementary Examinations, May - 2017
MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours

Max. Marks: 75

 Answer any **FIVE** Questions
 All Questions carry **Equal** Marks
 ~~~~~

1. a) Solve  $\frac{dy}{dx} = \frac{1}{(1+y^2)}(e^{\tan^{-1}x} - y)$ . (7M)
- b) A body is heated to  $110^\circ\text{C}$  and placed in air at  $10^\circ\text{C}$ . After 1 hour its temperature is  $60^\circ\text{C}$ . How much additional time is required for it to cool to  $30^\circ\text{C}$ ? (8M)
2. a) Solve  $(D^2 + 2)y = x^2e^{3x} + e^x \cos 2x$ , where  $D = \frac{d}{dx}$ . (7M)
- b) Solve  $(D^2 + 3D + 2)y = e^{-x} + x^2 + \cos x$ , where  $D = \frac{d}{dx}$ . (8M)
3. a) If  $u = x + y + z$ ,  $uv = y + z$ ,  $uvw = z$ , find  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ . (7M)
- b) Examine the function  $f(x, y) = \sin x + \sin y + \sin(x + y)$  for extreme. (8M)
4. a) Trace the curve  $x^3 + y^3 = 3axy$ . (7M)
- b) Trace the curve  $r^2 = a^2 \cos 2\theta$ . (8M)
5. a) Find the length of the arc of parabola  $y^2 = 4ax$  cut-off by latus rectum. (7M)
- b) Find the surface area of the solid generated by the revolution of the asteroid  $x = a \cos^3 t$ ,  $y = a \sin^3 t$  about the  $y$ -axis. (8M)
6. a) Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing to polar coordinates. (5M)
- b) Evaluate  $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz$ . (5M)
- c) Change the order of integration and hence evaluate  $I = \int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2) dx dy$ . (5M)
7. a) Evaluate divergence of  $(2x^2z \mathbf{i} - xy^2z \mathbf{j} + 3yz^2 \mathbf{k})$  at the point  $(1, 1, 1)$ . (5M)
- b) Show that  $\nabla^2 r^n = n(n+1)r^{n-2}$  where  $r^2 = x^2 + y^2 + z^2$ . (5M)
- c) Evaluate  $\text{Curl of } \vec{V} = yz \mathbf{i} + 3zx \mathbf{j} + z \mathbf{k}$  at the point  $(2, 3, 4)$ . (5M)
8. Verify Stoke's theorem for a vector field defined  $\vec{F} = -y^3 \mathbf{i} + x^3 \mathbf{j}$ , in the region  $x^2 + y^2 \leq 1, z = 0$ . (15M)