

Code No: **R13202**
R13
SET-1
I B. Tech II Semester Supplementary Examinations, Nov/Dec - 2017
MATHEMATICS-III

(Com. to All Branches)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answering the question in **Part-A** is Compulsory
 3. Answer any **THREE** Questions from **Part-B**

PART -A

1. a) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ in to Echelon form and hence find the rank. (4M)
- b) If λ is an Eigen value of a non singular matrix A. Show that $1/\lambda$ is an Eigen value of A^{-1} (3M)
- c) Trace the curve $r\theta = a$ ($a > 0$). (3M)
- d) Show that $\int_0^{\pi/2} \sin^m \theta \cos^n \theta d\theta = \frac{1}{2} B\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$ (4M)
- e) Find the angle between the surfaces $x^2 + y^2 + z^2 = 12$ and $x^2 + y^2 - z^2 = 6$ at $(2, -2, 2)$ (4M)
- f) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ If $\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ along the curve C in xy plane $y = x^3$ from $(1, 1)$ to $(2, 8)$. (4M)

PART -B

2. a) Determine whether the following equations will have a non-trivial solution if so solve them $4x+2y+z+3w=0$, $6x+2y+4z+7w=0$, $2x+y+w=0$ (8M)
- b) Test for Consistency the set of equations and solve them if they are consistent. $x+2y+2z=2$; $3x-2y-z=5$; $2x-5y+3z=-4$; $x+4y+6z=0$ (8M)
3. a) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4xz$ to the canonical form hence find the rank, index and signature. (8M)
- b) Determine the characteristic roots and the corresponding characteristic vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (8M)

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4. a) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^a \cos \theta \, r \sqrt{a^2 - r^2} \, dr \, d\theta$ (8M)
- b) Find the Length of the curve $3x^2=y^3$ between $y=0$ & $y=1$. (8M)
5. a) Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} \, dx$ in terms of Beta-Gamma function. (8M)
- b) Show that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (8M)
6. a) Find the directional derivation of $\phi(x, y, z) = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of $2\bar{i} + \bar{j} - 2\bar{k}$ (8M)
- b) If \bar{f}, \bar{g} are two vector point functions then show that $\nabla \times (\bar{f} \times \bar{g}) = \bar{f}(\nabla \cdot \bar{g}) - \bar{g}(\nabla \cdot \bar{f}) + (\bar{g} \cdot \nabla)\bar{f} - (\bar{f} \cdot \nabla)\bar{g}$ (8M)
7. a) Verify Green's theorem in the plane for $\int_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is a square with vertices $(0, 0), (2, 0), (2, 2), (0, 2)$. (8M)
- b) Evaluate $\iint_s f \cdot \bar{n} \, ds$ where $f = y^2\bar{i} + y\bar{j} - xz\bar{k}$ where s is the upper half of the sphere with radius 'a' units. (8M)