

Code No: R161203

R16
SET - 1
I B. Tech II Semester Regular Examinations, April/May – 2017
MATHEMATICS-III

(Com. to CE, EEE, ME, ECE, CSE, CHEM, EIE, IT, ECC, AE, AME, MM, PE, PCE, MET, AGE)

Time: 3 hours

Max. Marks: 70

 Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)

 2. Answering the question in **Part-A** is Compulsory

 3. Answer any **FOUR** Questions from **Part-B**

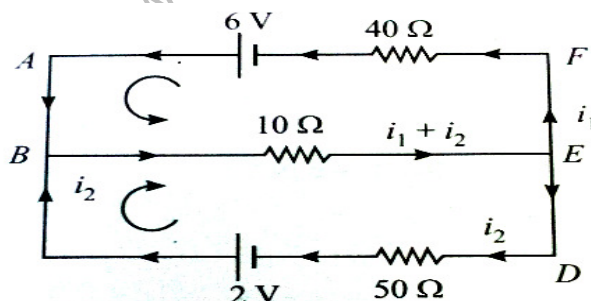
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**PART -A**

1. a) Find the rank of a matrix  $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7 \end{bmatrix}$  (2M)
- b) Prove that if  $\lambda$  is an eigen value of a matrix A then  $\lambda^{-1}$  is an eigen value of the matrix  $A^{-1}$  if it exists. (2M)
- c) Evaluate  $\int_0^1 \int_0^1 \int_{\sqrt{x^2+y^2}}^y xyz \, dz dy dx$ . (2M)
- d) Find the value of  $\Gamma\left(\frac{5}{2}\right)$ . (2M)
- e) Find the angle between the surface  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at (2, -1, 2). (2M)
- f) If  $\vec{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$  then evaluate  $\int \vec{F} \cdot d\vec{R}$  along the curve  $y = x^3$  from the point (1, 1) to (2, 8). (2M)
- g) Write the quadratic form corresponding to the symmetric matrix  $\begin{bmatrix} 1 & 0 & 4 \\ 0 & -2 & -1 \\ 4 & -1 & 3 \end{bmatrix}$ . (2M)

**PART -B**

2. a) Solve the system of equations  $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$  by Gauss Jacobi method. (7M)
- b) Find the currents in the following circuit (7M)



Code No: R161203

**R16**
**SET - 1**

3. a) Verify Cayley-Hamilton theorem and find the inverse of the matrix (7M)
- $$A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$
- b) Reduce the quadratic form  $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$  to canonical form by orthogonal transformation and hence find rank, index, signature and nature of the quadratic form. (7M)
4. a) Trace the curve  $r^2 = a^2 \cos 2\theta$ . (7M)
- b) Evaluate  $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} x y^2 dy dx$  by changing the order of integration. (7M)
5. a) Express  $\int_0^1 x^m (1-x^n)^p dx$  in terms of  $\Gamma$  functions and hence evaluate  $\int_0^1 x^5 (1-x^3)^{10} dx$ . (6M)
- b) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^5 \theta \cos^7 \theta d\theta$  by using  $\beta, \Gamma$  functions. (4M)
- c) Express  $\int_0^4 \sqrt{x}(4-x)^{3/2} dx$  in terms of  $\beta$  function. (4M)
6. a) Show that the vector field  $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$  is conservative and find the scalar potential function corresponding to it. (7M)
- b) Show that  $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot (\nabla \times \vec{F}) - \vec{F} \cdot (\nabla \times \vec{G})$  (7M)
7. State Stoke's theorem and verify the theorem for  $\vec{F} = (x+y)\vec{i} + (y+z)\vec{j} - x\vec{k}$  and S is the surface of the plane  $2x + y + z = 2$ , which is in the first octant. (14M)

Code No: R161203

**R16**
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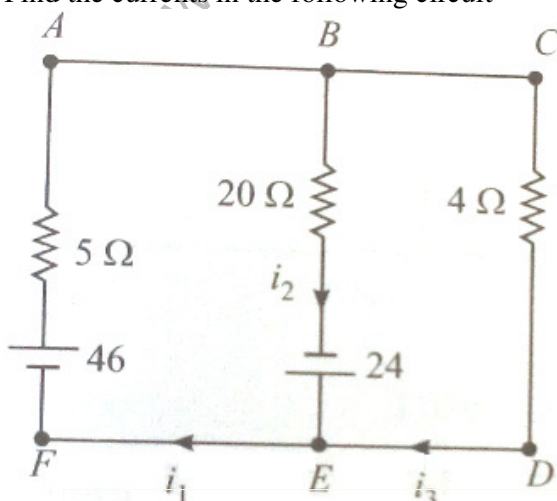
 2. Answering the question in **Part-A** is Compulsory

 3. Answer any **FOUR** Questions from **Part-B**
**PART -A**

1. a) Determine the rank of a matrix  $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ . (2M)
- b) Use Cayley-Hamilton theorem to find  $A^8$  if  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ . (2M)
- c) Evaluate  $\int_0^1 \int_0^1 \int_0^y xyz \, dx dy dz$ . (2M)
- d) Find the value of  $\Gamma\left(-\frac{5}{2}\right)$ . (2M)
- e) Find unit normal vector to the surface  $x^2y + 2xz^2 = 8$  at the point  $(1, 0, 2)$ . (2M)
- f) If  $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz\vec{k}$  then evaluate  $\int \vec{F} \cdot d\vec{R}$  from  $(0, 0, 0)$  to  $(1, 1, 1)$  along the path  $x = t, y = t^2, z = t^3$ . (2M)
- g) Write the quadratic form corresponding to the symmetric matrix  $\begin{bmatrix} 0 & 5/2 & 3 \\ 5/2 & 7 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ . (2M)

**PART -B**

2. a) Show that the system of equations is consistent  $2x - y - z = 2, x + 2y + z = 2, 4x - 7y - 5z = 2$  and solve. (7M)
- b) Find the currents in the following circuit (7M)



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Code No: R161203

**R16**
**SET - 2**

3. a) Reduce the quadratic form  $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$  to (7M)  
canonical form and hence state nature, rank, index and signature of the quadratic form.
- b) Determine the natural frequencies and normal modes of a vibrating system for (7M)  
which mass  $m = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$  and stiffness  $k = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ .
4. a) Trace the curve  $y^2(2a - x) = x^3$ . (7M)
- b) Evaluate  $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$  by changing in to polar coordinates and hence (7M)  
deduce  $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ .
5. a) Show that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ . (6M)
- b) Evaluate  $\int_0^\pi \sin^4 \theta \cos^2 \theta d\theta$  by using  $\beta, \Gamma$  functions. (4M)
- c) Express  $\int_0^1 \frac{1}{(1-x^3)^{1/3}} dx$  in terms of  $\beta$  function. (4M)
6. a) Show that the vector field  $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$  is conservative and (7M)  
find the scalar potential function.
- b) Show that  $\nabla(\nabla \cdot \vec{F}) = \nabla \times (\nabla \times \vec{F}) + \nabla^2 \vec{F}$ . (7M)
7. State Greens theorem in plane and verify the theorem for  $\oint_C [(y - \sin x)dx + (14M)$   
 $\cos x dy]$ , where C is the plane triangle formed by the lines  $y = 0, x = \frac{\pi}{2}$ ,  
 $y = \frac{2}{\pi}x$ .

Code No: R161203

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**SET - 3**
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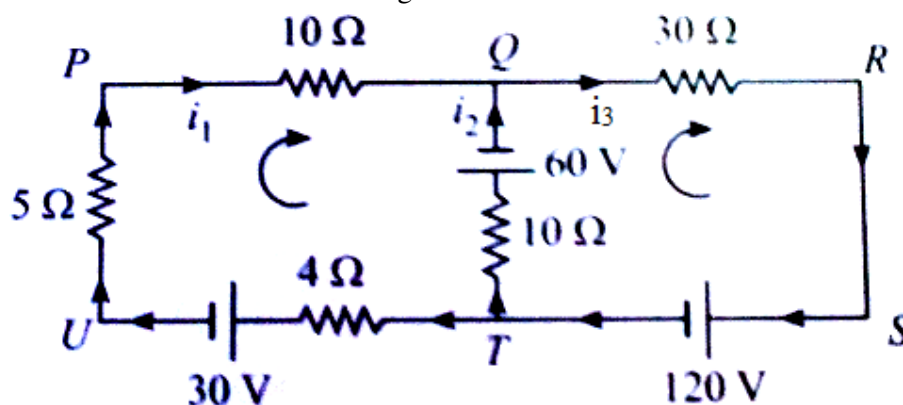
 2. Answering the question in **Part-A** is Compulsory

 3. Answer any **FOUR** Questions from **Part-B**
**PART -A**

1. a) Determine the rank of a matrix  $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ . (2M)
- b) Use Cayley-Hamilton theorem and find  $A^{-1}$  if  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ . (2M)
- c) Evaluate  $\int_0^{\frac{\pi}{2}} \int_0^{a \sin \theta} \int_0^{\frac{(a^2-r^2)}{a}} r \, dz \, dr \, d\theta$ . (2M)
- d) Show that  $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$ . (2M)
- e) Find directional derivative of  $\phi = xy^2 + yz^2$  at the point (2,-1,1) in the direction of the vector  $\bar{i} + 2\bar{j} + 2\bar{k}$ . (2M)
- f) If  $\vec{F} = (x^2 - y)\bar{i} + (2xz - y)\bar{j} + z^2\bar{k}$  then evaluate  $\int_C \vec{F} \cdot d\vec{R}$  where C is the straight line joining the points (0, 0, 0) to (1, 2, 4). (2M)
- g) Write the quadratic form corresponding to the symmetric matrix  $\begin{bmatrix} 3 & 5 & 0 \\ 5 & 5 & 4 \\ 0 & 4 & 7 \end{bmatrix}$ . (2M)

**PART -B**

2. a) Solve the system of equations  $10x + y + z = 12$ ,  $2x + 10y + z = 13$ ,  $2x + 2y + 10z = 14$  by Gauss Seidel method. (7M)
- b) Find the currents in the following circuit (7M)



Code No: R161203

**R16****SET - 3**

3. a) Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$  to canonical form by orthogonal transformation and hence find the rank, index signature and nature of the quadratic form. (7M)
- b) Find the natural frequencies and normal modes of a vibrating system (7M)  
 $mx'' + kx = 0$  for mass  $m = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  and stiffness  $k = \begin{bmatrix} 9 & -3 \\ -3 & 3 \end{bmatrix}$ .
4. a) Trace the curve  $a^2y^2 = x^2(a^2 - x^2)$ . (7M)
- b) Evaluate  $\int_0^1 \int_{\sqrt{y}}^{2-y} xy \, dx \, dy$  by changing the order of integration. (7M)
5. a) Show that  $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ . (6M)
- b) Evaluate  $\int_0^{\pi} \sin^5 \theta \, d\theta$  by using  $\beta, \Gamma$  functions. (4M)
- c) Express  $\int_0^1 \frac{x \, dx}{\sqrt{1+x^4}}$  in terms of  $\beta$  function. (4M)
6. a) Find the constants a, b such that the surfaces  $5x^2 - 2yz - 9x = 0$  and  $ax^2y + bz^3 = 4$  cut orthogonally at (1,-1,2). (7M)
- b) Show that  $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla \cdot \vec{F}) - \nabla^2 \vec{F}$ . (7M)
7. State Gauss divergence theorem in plane and verify the theorem for  $\vec{F} = 4xz\vec{i} - y^2\vec{j} + zy\vec{k}$  over the cube  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ . (14M)

Code No: R161203

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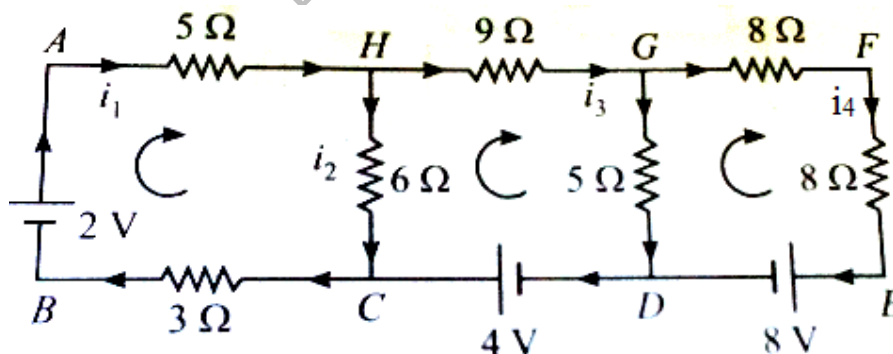
 2. Answering the question in **Part-A** is Compulsory

 3. Answer any **FOUR** Questions from **Part-B**
**PART -A**

1. a) Find inverse of the matrix  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$  by elementary operations. (2M)
- b) Prove that if  $\lambda$  is an eigen value of a matrix A then  $\frac{|A|}{\lambda}$  is an eigen value of  $\text{adj}A$ . (2M)
- c) Evaluate  $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dz \, dx \, dy$ . (2M)
- d) Determine the value of  $\beta(2, 3)$ . (2M)
- e) Show that  $\nabla f g = f \nabla g + g \nabla f$ . (2M)
- f) If  $\vec{F} = x^2 y^2 \vec{i} + y \vec{j}$  then evaluate  $\int_C \vec{F} \cdot d\vec{R}$  where C is the curve  $y^2 = 4x$  in the XY plane from (0, 0) to (4, 4). (2M)
- g) Write the quadratic form corresponding to the symmetric matrix (2M)
 
$$\begin{bmatrix} 2 & -3 & 5 \\ -3 & 2 & -2 \\ 5 & -2 & 2 \end{bmatrix}$$

**PART -B**

2. a) Solve the system of equations (7M)
 
$$x + 10y + z = 6, 10x + y + z = 6, x + y + 10z = 6$$
 by Gauss Seidel method.
- b) Find the currents in the following circuit (7M)



Code No: R161203

**R16**
**SET - 4**

3. a) Verify Cayley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$  and hence find  $A^4$ . (7M)
- b) Reduce the quadratic form  $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$  to canonical form and hence state nature, rank, index and signature of the quadratic form. (7M)
4. a) Trace the curve  $r = a \sin 3\theta$ . (7M)
- b) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} y \sqrt{x^2+y^2} dy dx$  by transforming to polar coordinates. (7M)
5. a) Establish a relation between  $\beta$  and  $\Gamma$  functions. (6M)
- b) Evaluate  $\int_0^{\frac{\pi}{2}} \cos^7 \theta d\theta$  by using  $\beta, \Gamma$  functions. (4M)
- c) Express  $\int_0^1 \frac{x dx}{\sqrt{1-x^5}}$  in terms of  $\beta$  function. (4M)
6. a) Find the angle between the surfaces  $ax^2 + y^2 + z^2 - xy = 1$  and conservative  $bx^2y + y^2z + z = 1$  at  $(1, 1, 0)$ . (7M)
- b) Show that  $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$  is both solenoidal and irrotational. (7M)
7. a) State Greens theorem in plane and apply the theorem to evaluate  $\oint_C x^2y dx + y^3dy$ , where C is the closed path formed by  $y = x$ ,  $y = x^3$  from  $(0, 0)$  to  $(1, 1)$ . (7M)
- b) Evaluate  $\int_S \vec{F} \cdot \overrightarrow{ds}$  using Gauss divergence theorem, where  $\vec{F} = 2xy\vec{i} + yz^2\vec{j} + z\vec{k}$  and S is the surface of the region bounded by  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $x + 2z = 6$ . (7M)