Code No: R21016 **R10** SET - 1

II B. Tech I Semester Supplementary Examinations, Oct/Nov- 2017
MATHEMATICS - III
(Com. to CE, CHEM, BT, PE)

Time: 3 hours Max. Marks: 75

Answer any **FIVE** Questions
All Questions carry **Equal** Marks
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1. a) Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. (8M)

   b) If m_1, m_2 are roots $J_n(x)=0$, then prove that $\int_0^1 xJ_n(m_1x)J_n(m_2x)dx = 0$. (7M)

2. a) If $u-v=(x-y)(x^2+4xy+y^2)$ and $f(z)=u+iv$ is an analytic function of $z=x+iy$ Find f(z) in terms of z. (8M)

   b) If $f(z)=w=u+iv$,then prove that $\frac{dw}{dz}=(\cos\theta - i\sin\theta)\frac{\partial w}{\partial r}$. (7M)

3. a) If $\tan\frac{x}{2}=\tanh\frac{y}{2}$ prove that $i)\tan x = \sinh y, ii)\cos x\cosh y = 1$ (8M)

   b) Prove that $\log\left(\frac{a+ib}{a-ib}\right)=2i\tan^{-1}\frac{b}{a}$. Evaluate $\cos\left[i\log\left(\frac{a+ib}{a-ib}\right)\right]$ (7M)

4. a) Evaluate $\int_c \left[(x-2y)dx+(y^2-x^2)dy\right]$ where C is the boundary of the circle $x^2+y^2=4$ in the first octant. (8M)

   b) Evaluate $\int_c \left[\frac{e^z}{z^3}+\frac{z^4}{(z+i)^2}\right]dz$ where c: $|z|=2$ using Cauchy's integral formula. (7M)
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5. a) Find the poles and residues of $\frac{1}{z^2-1}$. (8M)

   b) Expand the Laurent series of $\frac{z^2-1}{(z+2)(z+3}$ for $|z|>3$. (7M)

6. a) Evaluate $\int_c \frac{dz}{\sinh z}$, where C is the circle $|z|=4$, using residue theorem. (8M)

   b) Use the method of contour integration to evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)^3}dx$ (7M)

7. a) State and prove Liouville's theorem. (8M)

   b) Determine the poles of the function $f(z)=\frac{z^2}{(z-1)^2(z+1)}$ and the residue at each pole. Hence evaluate $\oint_c f(z)\,dz$, where C in the circle $|z|=2.5$. (7M)

8. a) Find the image of the lines x=-3 in the z-plane under the transformation $w=z^2$. (8M)

   b) Determine the bilinear transformation that maps the point's 1-2i, 2+i, 2+3i, respectively into 2+2i, 1+3i, 4. (7M)