Code No: R161203 **R16** SET - 1

I B. Tech II Semester Regular/Supplementary Examinations, April/May - 2018
MATHEMATICS-III
(Com. to CE,CSE,IT,AE,AME,EIE,EEE,ME,ECE,Metal E,Min E,E Com E,Agri E,Chem E,PCE,PE)

Time: 3 hours Max. Marks: 70

Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
2. Answer **ALL** the question in **Part-A**
3. Answer any **FOUR** Questions from **Part-B**

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PART –A

1. a) Write the working procedure to reduce the given matrix into Echelon form. (2M)

b) Find the Eigen value of the matrix A = $\begin{bmatrix} 4 & 2 \\ 1 & 5 \end{bmatrix}$. (2M)

c) Find the point of the curve $r = a\,(1 + \cos\theta)$ where tangent coincide with the radius vector. (2M)

d) Evaluate $\int_1^2\int_3^4 (xy + e^y)\,dxdy$ (2M)

e) Show that $\Gamma(n+1) = n\Gamma(n)$ for ` $n > 0$ (2M)

f) Find grad ϕ where $\phi = x^3 + y^3 + 3xyz$ at $(1,1,-2)$ (2M)

g) Find the work done in moving particle in the force field $\bar{F} = 3x^2\,\bar{i} + \bar{j} + z\bar{k}$ along the straight line (0, 0, 0) to (2, 1, 3). (2M)

PART -B

2. a) Reduce the matrix $A = \begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$ in to normal form hence find the rank. (7M)

b) If consistent, solve the system of equations. (7M)

$$x + y + z + t = 4$$
$$x - z + 2t = 2$$
$$y + z - 3t = -1$$
$$x + 2y - z + t = 3.$$

3. a) Determine the diagonal matrix orthogonally similar to the matrix. (7M)

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

b) Find the Nature , index and signature of the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$ (7M)
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4\. a) By change of order of integration evaluate $\int_0^a \int_x^a (x^2 + y^2)\, dy\, dx$ (7M)

b) Evaluate $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{(a^2-r^2)/a} r\, dr\, d\theta\, dz$ (7M)

5\. a) Evaluate $\int_0^{\infty} 3^{-4x^2} dx$ (7M)

b) Show that $\int_0^{\infty} \sin x^2 dx = \int_0^{\infty} \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$ (7M)

6\. a) Show that $\overline{f} = r^n (\overline{a} \times \overline{r})$ is solenoidal where $\bar{a} = a_1\bar{\imath} + a_2\bar{\jmath} + a_3\bar{k}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ (7M)

b) Prove that $\nabla\left(r\nabla\left(\frac{1}{r^3}\right)\right) = \frac{3}{r^4}$ (7M)

7\. a) Verify stoke's theorem for $\overline{F} = y\bar{i} + z\bar{j} + x\bar{k}$ for the upper part of the sphere $x^2 + y^2 + z^2 = 1$. (7M)

b) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2dy$. Where c is the closed curve of the region bounded by y=x & y=x^2 (7M)

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PART –A

1. a) Write the working procedure to reduce the given matrix into Normal form. (2M)
b) Write quadratic form $x^2+3y^2+3z^2-2yz$ (2M)
c) Write the tangents at the origin of the curve $a^2y^2=x^2(a^2-x^2)$. (2M)
d) Evaluate $\int_0^1\int_0^1\int_0^1 dx\,dy\,dz$ (2M)
e) Prove that $\beta(m,n)=\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta$ (2M)
f) Find the maximum value of the directional derivative of $\phi=2x^2-y-z^4$ at $(2,-1,1)$ (2M)
g) Write Stoke's theorem. (2M)

PART -B

2. a) For what value of k the matrix $A=\begin{bmatrix}4 & 4 & -3 & 1\\ 1 & 1 & -1 & 0\\ k & 2 & 2 & 2\\ 9 & 9 & k & 3\end{bmatrix}$ has rank 3. (7M)
b) Solve the following system of equations $\begin{matrix}8x-3y+2z=20\\ 4x+11y-z=33\\ 6x+3y+12z=35\end{matrix}$ by using. Gauss – Seidel method. (7M)

3. a) Determine the characteristic roots and the corresponding characteristic vectors of the matrix. (7M)
$A=\begin{bmatrix}3 & 10 & 5\\ -2 & -3 & -4\\ 3 & 5 & 7\end{bmatrix}$
b) Find the Nature , index and signature of the quadratic form $4x^2+3y^2+z^2-8xy+4xz-6yz$ (7M)
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4. a) Trace the curve $r^2 = a^2 \cos 2\theta$ (7M)

   b) Evaluate $\int\int (x^2 + y^2)\, dx\, dy$ over the area bounded by the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (7M)

5. a) Evaluate $\int_0^\infty a^{-bx^2} dx \;\; b > 0, a > 1$ (7M)

   b) Show that $\int_0^1 x^m (\log x)^n\, dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ (7M)

6. a) Find the constants 'a' and 'b' such that the surfaces $5x^2$-2yz-9x=0 and ax^2y+bz^3=4 cuts orthogonally at (1,-1,2) (7M)

   b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential. (7M)

7. a) If $\bar{f} = (3x^2 - 2z)\bar{i} - 4xy\,\bar{j} - 5x\bar{k}$ Evaluate $\int_V Curl\, \bar{F}\, dv$, where v is volume bounded by the planes x = 0; $y = 0$; $z = 0$ and $3x + 2y + 3z = 6$. (7M)

   b) Evaluate $\oint_c \cos y\, dx + x(1 - \sin y)\, dy$ over a closed curve c given by $x^2 + y^2 = 1$; $z = 0$ using Green's theorem. (7M)

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PART –A

1. a) Write the working procedure to find the inverse of the given matrix by Jordan method. (2M)

b) Find the Eigen value of Adj A if the 'λ' is the Eigen value of A. (2M)

c) Write the symmetry of the curve $y^2(2a-x)=x^3$ (2M)

d) Evaluate $\int_0^3\int_{-x}^{x} xy\,dx\,dy$ (2M)

e) Find the value of $\beta\left(\frac{1}{2},\frac{1}{2}\right)$ (2M)

f) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. (2M)

g) Write the physical interpretation of Gauss divergence theorem. (2M)

PART -B

2. a) Reduce the matrix to Echelon form and find its rank $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (7M)

b) Solve the equations $\begin{array}{l} 10x+y+z=12, \\ 2x+10y+z=13, \\ x+y+5z=7. \end{array}$ by Gauss – Jordan method. (7M)

3. a) Find the Natural frequencies and normal modes of vibrating system for which mass $M=\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and stiffness $K=\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ (7M)

b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Hence find A^{-1} (7M)
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4. a) Find the volume of region bounded by the surface $z = x^2 + y^2$ and $z = 2x$. (7M)

b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ by changing in to polar co-ordinates. (7M)

5. a) Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1} dx = (b-a)^{m+n-1} \beta(m,n) m > 0, n > 0$ (7M)

b) Evaluate $\int_0^1 (x \log x)^4 dx$ (7M)

6. a) Find the directional derivative of $\phi = xyz$ at (1,-1, 1) along the direction which makes equal angles with the positive direction of x , y , z axes (7M)

b) Prove that div $curl\,\overline{f} = 0$ (7M)

7. a) Verify Green's theorem for $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region enclosed by the lines. $x = 0$ $y = 0$ $x + y = 1$. (7M)

b) Find the flux of vector function $\overline{F} = (x-2z)\overline{i}\ (x+3y)\overline{j} + (5x+y)\overline{k}$ through the upper side of the triangle ABC with vertices (1,0,0), (0,1,0), (0,0,1). (7M)

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PART –A

1. a) Find the Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (2M)

b) Prove the AB and BA has same Eigen values. (2M)

c) Write the Asymptote of the curve **y=** $\frac{x^2+1}{x^2-1}$ (2M)

d) Evaluate $\int_0^3 \int_1^2 xy\,(x+y)\,dx\,dy$ (2M)

e) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ (2M)

f) Show that $\nabla\left(r^2\right) = 2\bar{r}$ (2M)

g) Write Green's theorem. (2M)

PART -B

2. a) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$ into PAQ form and hence find the rank of the matrix. (7M)

b) Solve the equations $\begin{aligned} x + y + z &= 8, \\ 2x + 3y + 2z &= 19 \\ 4x + 2y + 3z &= 23 \end{aligned}$ by Gauss – Elimination method. (7M)

3. a) Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ if possible. (7M)

b) Find the Nature, index and signature of the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy + 2xz - 2yz$ by orthogonal reduction. (7M)
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Code No: R161203 **R16** SET - 4

4. a) Trace the curve $x = a\cos t + \frac{a}{2}\log\tan^2 t/2, \quad y = a\sin t$ (7M)

   b) Find the area between the circles $r = a\cos\theta$ and $r = 2a\cos\theta$. (7M)

5. a) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{\beta(m,n)}{(a+b)^m a^n}$ (7M)

   b) Evaluate $\int_0^\infty e^{-x^6} x^4 dx$ (7M)

6. a) Find the directional derivative of the function $e^{2x}\cos yz$ at the origin in the direction to the tangent to the curve $x = a\sin t, y = a\cos t, z = at$ at $t = \frac{\pi}{4}$ (7M)

   b) Show that $curl\ curl\ \overline{f} = \nabla\times\left(\nabla\times\overline{f}\right) = \nabla\left(\nabla.\overline{f}\right) - (\nabla.\nabla)\overline{f}$ if $\overline{f}(x,y,z)$ is vector point function. (7M)

7. a) Verify Gauss Divergence theorem for $\overline{F} = (x^2 - yz)\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$ taken over the rectangular parallelepiped $0 \le x \le a\,;\, 0 \le y \le b\,;\, 0 \le z \le c$. (7M)

   b) Evaluate $\iint_s (\nabla\times\overline{F}).\overline{n}\,ds$ where $\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\,\overline{j} + (2xy + z^2)\overline{k}$ and s in the surface of the paraboloid $z = 4 - x^2 - y^2$ above the xy plane. (7M)