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## SYSTEM OF PARTICLES AND ROTATIONAL MOTION

(1). According to the theorem of perpendicular axes moment of inertia of a body about perpendicular axis is  $I_z = I_x + I_y$







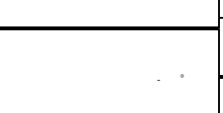

where  $I_x$  and  $I_y$  are the moment of inertia of the rigid body about x, y and z axes respectively and x and y axes lie in the plane of the body and z-axis lies perpendicular to the plane of the body and passes through the point of intersection of x and y.

(2). According to the theorem of parallel axes  $I = I_c + Md^2$

where  $I_c$  is the moment of inertia of the body about an axis passing through its centre of mass and d is the perpendicular distance between the two axes.

Table 1: Moment of inertia of some symmetrical bodies

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	Body	Axis	Figure	Point
1211	Rod (Length $L$ )	Perpendicular to rod, at the midpoint of rod.		MC 12
211	Circular ring (radius $R$ )	Passing through centre and perpendicular to the plane		MR2
011	Circular disc (radius $R$ )	Diameter		MRa 2
4	Circular Disc radius $R$	Perpendicular to the disc at centre.		MR2 2
51	Circular Disc radius $R$	Diameter		MR2 4
51	Hollow cylinder (radius $R$ )	Axis of cylinder		MR2
711	Solid cylinder (radius $R$ )	Axis of cylinder		IVIFE2
all	Solid sphere (radius $R$ )	Diameter		2 5

Q9	Hollow sphere (radius R)	Diameter		2
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(3). Relation between moment of inertia (I) and angular momentum L is given by  $L = I\omega$  (7)

Relation between moment of inertia and kinetic energy of rotation is given by

$$K = \frac{1}{2} I \omega^2$$

(5). Relation between of inertia (I) and torque  $\tau = I \alpha$

If no external torque acts on the system, the total angular momentum of the system remains unchanged  $I_1 \omega_1 = I_2 \omega_2$

(7). Position vector of centre of mass of a discrete particle system

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_n \vec{r}_n}{M}$$

Where  $m$  is the mass of the particle and  $\vec{r}$  is the position of the particle corresponding.

$x_{cm}$  and  $y_{cm}$  co-ordinates are

$$x_{cm} = \frac{\sum_{i=1}^n m_i x_i}{M}, \quad y_{cm} = \frac{\sum_{i=1}^n m_i y_i}{M}$$

Velocity of centre of mass,  $\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt}$

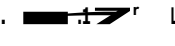
(9). Acceleration of CM,  $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt}$

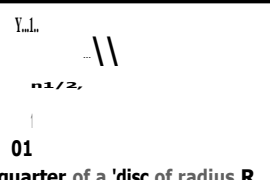
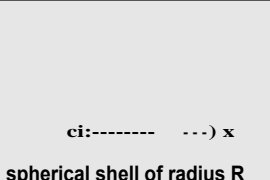

DC). Momentum of system,  $\vec{p} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_n \vec{v}_n = \left( \sum_{i=1}^n m_i \right) \vec{v}_{cm}$

{11}. Centre of mass of continuous mass distribution

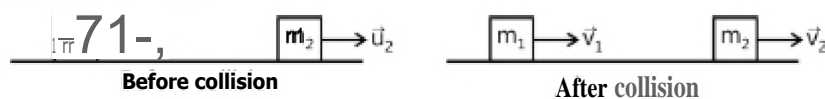
$$x_{cm} = \frac{\int x dm}{\int dm}, \quad y_{cm} = \frac{\int y dm}{\int dm}, \quad z_{cm} = \frac{\int z dm}{\int dm}$$

02). Given below are the positions of centre of mass of some objects.

S.No.	Object	Location of centre of mass
i.	<p>YA</p> <p>0</p> <p>*Uniform rod of length L</p>	L
ii.	<p>yt</p>  <p>Uniform semicircular ring of radius R</p>	<p>2R/π</p>
iii.	<p>Y.L.</p> <p>ay.</p> <p>01 ..... ) x</p> <p>Uniform Semicircular disc of radius R</p>	<p><math>x_{cm} = 0, y_{cm} = \frac{4R}{3\pi}, z_{cm} = 0</math></p>
iv.	<p>Y</p> <p>Or</p> <p>Uniform quarter of a ring of radius R</p>	<p><math>x_{cm} = \frac{2R}{\pi}, y_{cm} = \frac{2R}{\pi}, z_{cm} = 0</math></p>

$V_{cm} = \frac{4R}{311} \cdot \frac{4R}{311} = 0$	 <p>Uniform quarter of a disc of radius R</p>
$V_i =$	 <p>Uniform spherical shell of radius R</p>
$V_{cm} = \frac{3R}{8r} \cdot \frac{3R}{8r} = 0$	 <p>Uniform Spherical of radius R.</p>

(13). Head -ors col



Velocity of bodies  $m_1$  and  $m_2$  after collision are

$$\vec{v}_1 = \left( \frac{m_1 - e m_2}{m_1 + m_2} \right) \vec{u}_1 + \left( \frac{m_2 (1 + e)}{m_1 + m_2} \right) \vec{u}_2 ; \quad \vec{v}_2 = \left( \frac{m_2 - e m_1}{m_1 + m_2} \right) \vec{u}_2 + \left( \frac{m_1 (1 + e)}{m_1 + m_2} \right) \vec{u}_1$$

Here  $e$  is coefficient of reglution.

$$\text{Loss in kinetic energy, } \Delta E_k = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - \left( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \right) = \frac{1}{2} m_1 m_2 (1 - e^2) \frac{(u_1 - u_2)^2}{m_1 + m_2}$$

(14). For elastic collision  $\Delta E_k = 0$  and  $e = 1$ , then velocities after collision are

$$\vec{v}_1 = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) \vec{u}_1 + \left( \frac{2m_2}{m_1 + m_2} \right) \vec{u}_2 ; \quad \vec{v}_2 = \left( \frac{m_2 - m_1}{m_1 + m_2} \right) \vec{u}_2 + \left( \frac{2m_1}{m_1 + m_2} \right) \vec{u}_1$$

(15). For perfectly inelastic collision,  $e = 0$ , then velocities after collision are

$$\vec{v}_1 = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \text{ and loss in kinetic energy is } \Delta E_k = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (u_1 - u_2)^2$$