Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Tech. (2011 to 2017) (Sem.–1)

ENGINEERING MATHEMATICS – I

Subject Code : BTAM-101

Paper ID : [A1101]

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B & C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B & C.**
5. **Symbols used have their usual meanings. Statistical tables, if demanded, may be provided.**

SECTION-A

Q1 a) Find the curvature at any point of the curve $y^2 = x^3 + 8$ at (1,3)

b) Find the radius of curvature at any point (r, θ) of polar curve $r = a(1 + \cos\theta)$.

c) Write down the formula for finding the volume of solid by revolving the area bounded by the curve y=f(x) and the line x=a, x=b and y=p about the line y=p.

d) Find the area of ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$.

e) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2}{y^2-x^2}$ does not exist.

f) Find $\frac{\partial w}{\partial r}$ *if* $w = x + 2y + z^2, x = \frac{r}{s}, \ y = 2r + s^2, \ z = 2r$.

g) Write down the equation of hyperboloid of two sheet and draw its rough sketch.

h) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 5$, where t is the time variable. Find the velocity at time t=l.

i) Determine whether $\vec{A} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational or not?

j) State Stoke's theorem.

SECTION-B

Q2 a) Sketch the curve by considering all the salient features $y = x + \frac{1}{x}$.

b) Trace the polar curve : $r = a(1 + \sin\theta)$, $a > 0$.

Q3 a) Find the perimeter of the circle $x^2 + y^2 = 9$.

b) Find the surface of the solid generated by the revolution of Lemniscate $r^2 = a^2 \cos 2\theta$ about initial line.

Q4 a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$.

b) If u = f(y - z, z - x, x - y)), then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

Q5 a) Expand $e^x \log(1 + y)$ in. powers of x and y upto third degree.

b) Find all the local maxima and minima of the function :

$$f(x,y) = x^3 + y^3 - 63\,(x + y) + 12\,xy$$

SECTION-C

Q6 a) Evaluate $\iint (x^2 + y^2)\; dxdy$ over the circle $x^2 + y^2 = a^2$ by changing into polar coordinates.

b) Evaluate the volume of the sphere $x^2 + y^2 + z^2 = 1$ by using triple integration.

Q7 a) If $\vec{A}$ is vector function and Φ is a scalar function then prove that :

$$\nabla \times (\phi \vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla \phi) \times \vec{A}$$

b) if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, *prove taht* $\nabla \cdot (r^n \vec{r}) = (n+3) r^n$, where $r = |\vec{r}|$.

Q8. Evaluate $\iint_s \vec{A}.\hat{n}\, ds$ where $\vec{A} = 12x^2 y\hat{i} - 3yz\hat{j} + 2z\hat{k}$ and S is the portion of the plane $x + y + z = 1$ included in the first quadrant.

Q9. Verify Green's theorem for $\oint_c (xy + y^2)\, dx + x^2 dy$, where c is the boundary of the closed region bounded by $y = x^2$ and $y = x$.