

[illegible]

Max. Marks : 60

SECTION-B

Q2. a) For what values of a , m , and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b & 1 \leq x \leq 2 \end{cases}$$

satisfy the hypothesis of Mean value theorem on the interval $[0, 2]$.

b) Use L 'Hospital's rule to find the limit $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

Q3. a) Evaluate the integral $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$, if it exists.

b) The line segment $x = 1 - y$, $0 \leq y \leq 1$, is revolved about the y -axis to generate the cone. Find its lateral surface area.

Q4 a) If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$.

b) Find the plane tangent to the surface: $z = x \cos y - y e^x$ at $(0, 0, 0)$

Q5 a) Find the area of the region bounded between the curve $y = e^x$ and the lines $x = 0$ and $x = \ln 2$.

b) Evaluate $\iint (x^2 + y^2) dx dy$ over the circle $x^2 + y^2 = a^2$ by changing into polar coordinates.

SECTION-C

Q6 a) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$

b) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1.3.5 \dots (2n-1)}{2.4.6 \dots (2n)(3^n + 1)}$

Q7 For the series $\sum_{n=1}^{\infty} \frac{nx^n}{4^n(n^2+1)}$ Find:

- a) For what values of x does the series converge absolutely?
- b) For what values of x does the series converge conditionally?
- c) Find the interval of convergence.

Q8. Determine whether the given system of equations is consistent or not? if consistent solve it

$$x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2, x - y + z = -1$$

Q9. Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$