Roll No. | | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 09

B.Tech.(Automation & Robotics) (2011 & Onwards) (Sem.–3)

MATHEMATICS – III

Subject Code : BTAR-301

Paper ID : [A0130]

Time : 3 Hrs. Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION–A

1. Write briefly :

(a) Find the Laplace transforms of $(t-1)^2 u(t-1)$

(b) Evaluate $\int_0^\infty \phi(t)\delta'(t-a)dt$.

(c) Show that $P_n(1) = 1$.

(d) Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

(e) Show $\text{erf}(x) + \text{erf}_c(f) = 1$.

(f) Write necessary and sufficient conditions for $f(z)$ to be analytic.

(g) Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic.

(h) Define conformal transformation.

(i) Define Cauchy's integral formula.

(j) Discuss singularity of $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$.

SECTION–B

2. Find the inverse Laplace transform of $f(p) = \frac{2p^2 - 1}{(p^2 + 1)(p^2 + 4)}$.

3. State and prove Rodrigue's formula.

4. If $U + V = \frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ and $f(z) = U + iV$ is an analytic function of $Z = X + iY$, Find $f(z)$ in the terms of Z.

5. Consider the map $W = \frac{1}{Z}$ and determine the region R in W-plane of the infinite strip R bounded by $\frac{1}{4} < y < \frac{1}{2}$.

6. Evaluate by using Cauchy-integral formula $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$ where C is $|z - i| = 2$.

SECTION–C

7. The Co-ordinates (x, y) of a particle moving along a plane curve at any time t are given by

$$\frac{dy}{dt} + 2x = \sin 2t, \frac{dx}{dt} - 2y = \cos 2t; (t > 0)$$

It is given that at $t = 0$, $x = 1$ and $y = 0$. Show using transforms that the particle moves along the curve $4x^2 + 4xy + 5y^2 = 4$.

8. State and Prove/orthogonality of Bessel's function.

9. Using Contour integration, evaluate $\int_0^\infty \frac{dx}{1 + x^2}$.